Towards Data-Driven Control of Multiphase Flows

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Abstract

There are a number of engineering applications where the governing system of interest is of very high dimension or not known entirely. However it is quite possible to access some physical data for such systems. Complex multiphase flows belong to this class of systems. To design control strategies for such systems it is useful to develop a data-driven control method. As a model equation for our eventual spray control application, we use state-of-the-art data-driven system identification and model reduction techniques to design optimal control for the linear and non-linear Ginzburg Landau equation. The focus of this work is to develop reduced-order direct and adjoint operators that approximate the state transition operator of the high dimensional system. We then use these operators for optimal control and evaluate the performance of the controllers. Perspectives on applying the data-driven methods to multiphase flow control will also be given.

Introduction

The analysis and control of two-phase flows is a motivating and useful example of an application of model reduction-based control. The complexity of the governing equations at the boundary between the interacting fluids makes data-driven control techniques especially attractive. Figure 1 shows a practical scenario where liquid water is ejected from a central orifice surrounded by high-speed air in which data-driven optimal control is desirable.

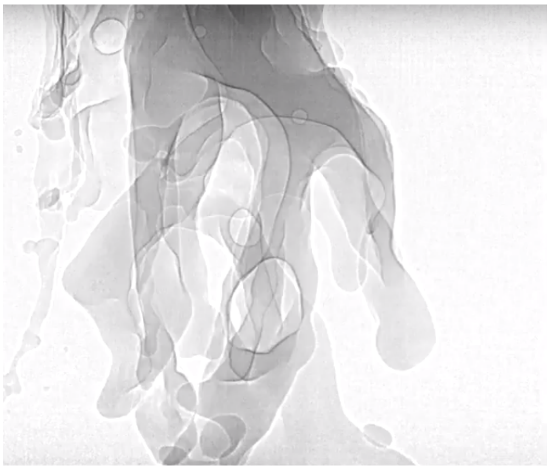

Figure 1: X-ray image of a liquid jet interacting with the air. This image was taken during an experiment at Argonne National Lab by the Ted Heindel group at Iowa State University [1].

When attempting to construct an optimal control for a two-phase flow problem, it is tempting to want to use the adjoint method. Using the adjoint method, the gradient of the cost functional with respect to inputs to the system is computed directly via the adjoint equations. The main benefit of the adjoint method is that it eliminates the need for an exhaustive search through the space of possible control parameters. However, the complex nature of the dynamics of a two-phase flow problem severely complicates the manipulations of the governing equations required to create the adjoint equations, and imposes challenges on their numerical implementation.

Model reduction techniques such as dynamic mode decomposition (DMD)[2] can be used to circumvent these problems. DMD takes as input either experimental or simulation-based flow data and gives as an output an approximate, reduced order state transition matrix $\tilde{A}$ and reduced order input matrix $\tilde{B}$. The resulting approximate model for the dynamical system is $\tilde{\mathbf{q}}_{i+1} = \tilde{A}\tilde{\mathbf{q}}_i + \tilde{B}\mathbf{u}$ where $\mathbf{q}_i$ is the fluid state at time t_i. From here, the discrete adjoint operator $\tilde{A}^H$ can be defined where the superscript H denotes the conjugate transpose. This operator is used

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to construct the corresponding reduced order adjoint equations, from which the adjoint method can be used to find an optimal control.

Mathematical Formulation

In this section we describe the mathematical formulation for DMD and how it can be used to find the optimal control using the adjoint method.

Dynamic Mode Decomposition

Dynamic mode decomposition is an algorithm used in capturing the dynamics of a system using only data [2]. This technique can also be used to project the system onto a low-order basis in order to improve the efficiency of integrating this system forward in time.

In order to compute the DMD, first we must collect N_s snapshots of the uncontrolled system $\mathbf{q}_{i+1} = \bar{A}\mathbf{q}_i$ with nonzero initial condition $\mathbf{q}_0$. The data is organized into the snapshot matrices $X_1 = [\mathbf{q}_1\mathbf{q}_2...\mathbf{q}_{N_s-1}]$ and $X_2 = [\mathbf{q}_2\mathbf{q}_3...\mathbf{q}_{N_s}]$. The matrices X_1 and X_2 are the un-shifted and shifted snapshot matrices, respectively. DMD assumes that there exists a linear operator $\bar{A}_{svd}$ such that $X_2 = \bar{A}_{svd}X_1$. In order to find this operator, we compute the economy-sized singular value decomposition of the un-shifted snapshot matrix $X_1 \approx \tilde{U}\tilde{\Sigma}\tilde{V}^*$. The columns of the unitary matrices $\tilde{U}$ and $\tilde{V}$ are the left-singular vectors and the right-singular vectors of X_1, respectively. We use the resulting pseudo-inverse to solve for $\bar{A}_{svd} = X_2\tilde{V}\tilde{\Sigma}^{-1}\tilde{U}^*$.

In order to perfom model reduction, we project the (svd-based) discrete time system matrix $\bar{A}_{svd}$ and input matrix $\bar{B}$ onto the basis of POD modes spanned by $\tilde{U}$ as $\tilde{A} = \tilde{U}^*\bar{A}_{svd}\tilde{U}$ and $\tilde{B} = \tilde{U}^*\bar{B}$, respectively. The resulting DMD-reduced dynamical system becomes $\tilde{\mathbf{q}}_{i+1} = \tilde{A}\tilde{\mathbf{q}}_i + \tilde{B}\mathbf{u}$. Computing the DMD reduced system data $\tilde{\mathbf{q}}$ is much more computationally cost effective than integrating the full dimension system data $\mathbf{q}$ forward in time.

Adjoint Method

Given the original system $\mathbf{q}_{i+1} = \bar{A}\mathbf{q}_i + \alpha_i\bar{B}\mathbf{u}_i$, we wish to find the gradient in the space of all possible control parameters $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_{N_s}]$ which which will lead us to the set of optimal control parameters $\boldsymbol{\alpha}_{opt}$ which minimizes the cost functional

$$J = \sum_{i=1}^{N_t-1} j(\mathbf{q}_i, \alpha_i, \mathbf{u}_i) = \sum_{i=1}^{N_t-1} [\mathbf{q}_i^H\mathbf{q_i} + (\alpha_i\mathbf{u}_i)^H\alpha_i\mathbf{u}_i]$$

Following the analysis in [3], we first define the Hamiltonian $H(\mathbf{q}_i, \alpha_i, \mathbf{u}_i, \mathbf{z}_{i+1}) = j(\mathbf{q}_i, \alpha_i, \mathbf{u}_i) + \mathbf{z}_{i+1}^H f(\mathbf{q}_i, \alpha_i, \mathbf{u}_i)$. For the case of our particular system, we have the Hamiltonian $H(\mathbf{q}_i, \alpha_i, \mathbf{u}_i, \mathbf{z}_{i+1}) = [\mathbf{q}_i^H\mathbf{q_i} + (\alpha_i\mathbf{u}_i)^H\alpha_i\mathbf{u}_i] + \mathbf{z}_{i+1}^H(\bar{A}\mathbf{q}_i + \alpha_i\bar{B}\mathbf{u}_i)$. The set of Lagrange multipliers $\mathbf{z}$ will be known as the adjoint data.

The adjoint data is computed via the adjoint difference equations

$$\mathbf{z}_i^H = \frac{\partial H(\mathbf{q}_i, \alpha_i, \mathbf{u}_i, \mathbf{z}_{i+1})}{\partial \mathbf{q}_i} = \mathbf{z}_{i+1}^H\bar{A} + 2\mathbf{q}_i^H$$

which are marched backward in time starting from the adjoint terminal conditions $\mathbf{z}_i^H = \mathbf{0}$.

Using this adjoint system, we can now compute the gradient of J with respect to α directly with the formula

$$\nabla_{\alpha_i} J = \frac{\partial H(\mathbf{q}_i, \alpha_i, \mathbf{u}_i, \mathbf{z}_{i+1})}{\partial \alpha_i} = \mathbf{z}_{i+1}^H\bar{B}\mathbf{u}_i + 2\alpha_i^H\mathbf{u}_i^H\mathbf{u}_i$$

The following are the reduced order adjoint equations corresponding to the Dynamic Mode Decomposition of the original system:

$$\tilde{\mathbf{z}}_i^H = \tilde{\mathbf{z}}_{i+1}^H\tilde{A} + 2\tilde{\mathbf{q}}_i^H,$$

with terminal conditions $\tilde{\mathbf{z}}_i^H = \mathbf{0}$. The DMD-based adjoint gradient is then

$$\nabla_{\alpha_i}\tilde{J} = \tilde{\mathbf{z}}_{i+1}^H\tilde{B}\mathbf{u}_i + 2\alpha_i^H\mathbf{u}_i^H\mathbf{u}_i.$$

This gradient can now be used in a gradient-descent search method until convergence on the optimal set of control parameters $\boldsymbol{\alpha}_{opt}$. These control parameters can be used to approximately minimize the full-state cost functional $J = \sum_{i=1}^{N_t-1} [\mathbf{q}_i^H\mathbf{q_i} + (\alpha_i\mathbf{u}_i)^H\alpha_i\mathbf{u}_i]$.

Numerical Experiment

The linearized Ginzburg-Landau system is a second order partial differential equation commonly used to study flow instability. This PDE can be tuned to produce a wide variety of different flow stability types by modifying a set of parameters. The following system is constructed following the subcritical implementation in [4]. The subcritial tuning of the system is globally stable but experiences interesting transient growth. This implementation uses a Hermite polynomial collocation technique.

A set of input parameters $\boldsymbol{\alpha}$ is added to attempt to dampen an initial disturbance and, in turn, minimize the cost functional $J = \sum_{i=1}^{N_t-1} [\mathbf{q}_i^H \mathbf{q_i} + (\alpha_i \mathbf{u}_i)^H \alpha_i \mathbf{u}_i]$. We wish to use the DMD-based adjoint method to find the optimal set of control parameters $\boldsymbol{\alpha}_{opt}$ which minimize J.

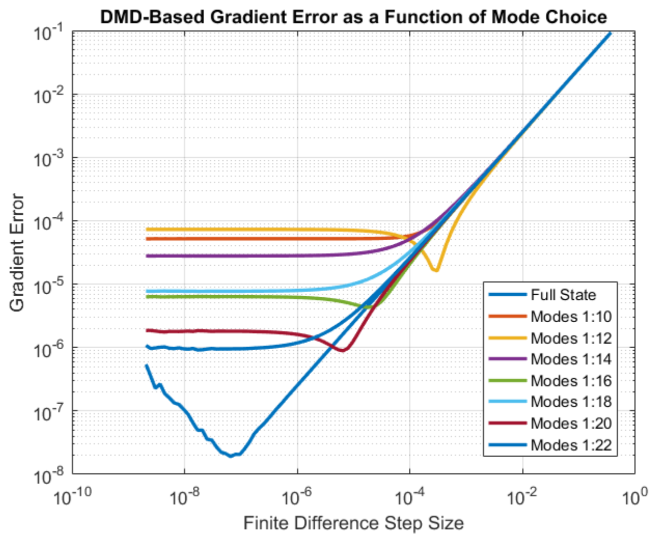


Figure 2: The error in the gradient of the DMD-informed adjoint method measured with respect to the finite difference gradient. The colored lines indicate the number of modes associated with each DMD-based adjoint operator.

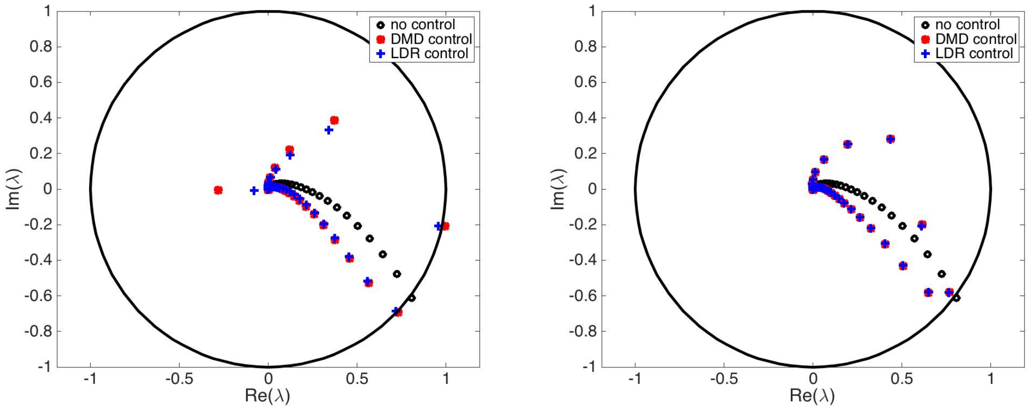
As can be seen from figure 2, the DMD-based adjoint method is able to capture the desired gradient to within a reasonable margin of error. This gradient was computed using only the uncontrolled data. The underlying dynamics of the system were not needed. The gradients provided by this method are effective enough to inform a rapid and accurate gradient descent search for the set of optimal control parameters $\boldsymbol{\alpha}_{opt}$ which minimize the cost J.

Alternative formulation with improved model reduction

So far we have only considered state transition matrix projected onto the leading singular vectors of the data matrices. For a more extensive search we need to consider all possible subspaces of dimension r in $\mathbb{R}^m$ space or in the range of the data matrix. Such a search is performed in Optimal Mode Decomposition[5] but with the restriction of the state transition matrix having the same left and right images. The authors of [5] do this by formulating a matrix optimization problem which is solved using gradient based optimization on Grassmanian manifolds. In [6], similar problem is solved for a more general optimization problem

$$\min_{rank(\bar{A}) \leq r} \left\| X_2 - \bar{A} X_1 \right\|_F^2$$

where $X_2, X_1 \in \mathbb{R}^{m \times n}$ are data matrices and X_2 is shifted in time with respect to X_1. m is the dimension of the state space and n is the number of snapshots in time. Since the adjoint modes reside in the right image of the state transition matrix, unrestricted optimization over all possible subspaces could result in more accurate adjoint modes and therefore better control performance. We will refer to the controller using the solution in [6] as 'LDR control'.



(a) (b)

Figure 3: The eigenvalues of controlled and uncontrolled dynamics using reduced order controllers for (a) rank 4 and (b) rank 9 approximation of the state transition matrix. The black border denotes a unit circle around the origin.

We evaluate the performance of LDR and DMD on the complex Ginzburg-Landau system. Figure 3 shows the performance of LQR controllers designed using the reduced order approximation given by DMD and LDR in controlling the true dynamical system. Eigenvalues inside the unit circle means that the dynamical system is stable and if the eigenvalues lie outside the circle, the system is unstable. Our goal is to stabilize the system with a low rank approximation of the true dynamical system. In figure 3a we can see that with rank 4 approximation the LDR controller is able to stabilize the true dynamical system whereas the DMD controller is not. It takes a rank 9 approximation shown in figure 3b for the DMD controller to stabilize the system. The controllers given by both DMD and LDR for rank 9 approximation are very close to the optimal controller of the true dynamical system.

Summary and Outlook

In, this paper, we demonstrated an effective and efficient method for performing optimal control of a fluid system without information of the governing dynamics. Using Dynamic Mode Decomposition, we used fluid data to construct an accurate approximation of the adjoint system corresponding to the system in question. The adjoint system can be used to find the gradient in the space of all possible controls that points to the best possible control and minimize a cost functional J. Dynamic Mode Decomposition is a versatile tool which can be adapted and modified in variety of different ways. In particular, we use a specialized version of DMD called LDR to improve our approximation of the Ginzburg-Landau system.

As we move forward, we would like to apply our method to a two-phase flow optimization problem. The intent is to show that we can use the DMD-based adjoint method to improve the atomization of a liquid in a gas crossflow. This will have an impact on the efficiency of the fuel burn in a combustion chamber.

Acknowledgements

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