Low-Rank Dynamic Mode Decomposition using Riemannian Manifold Optimization.

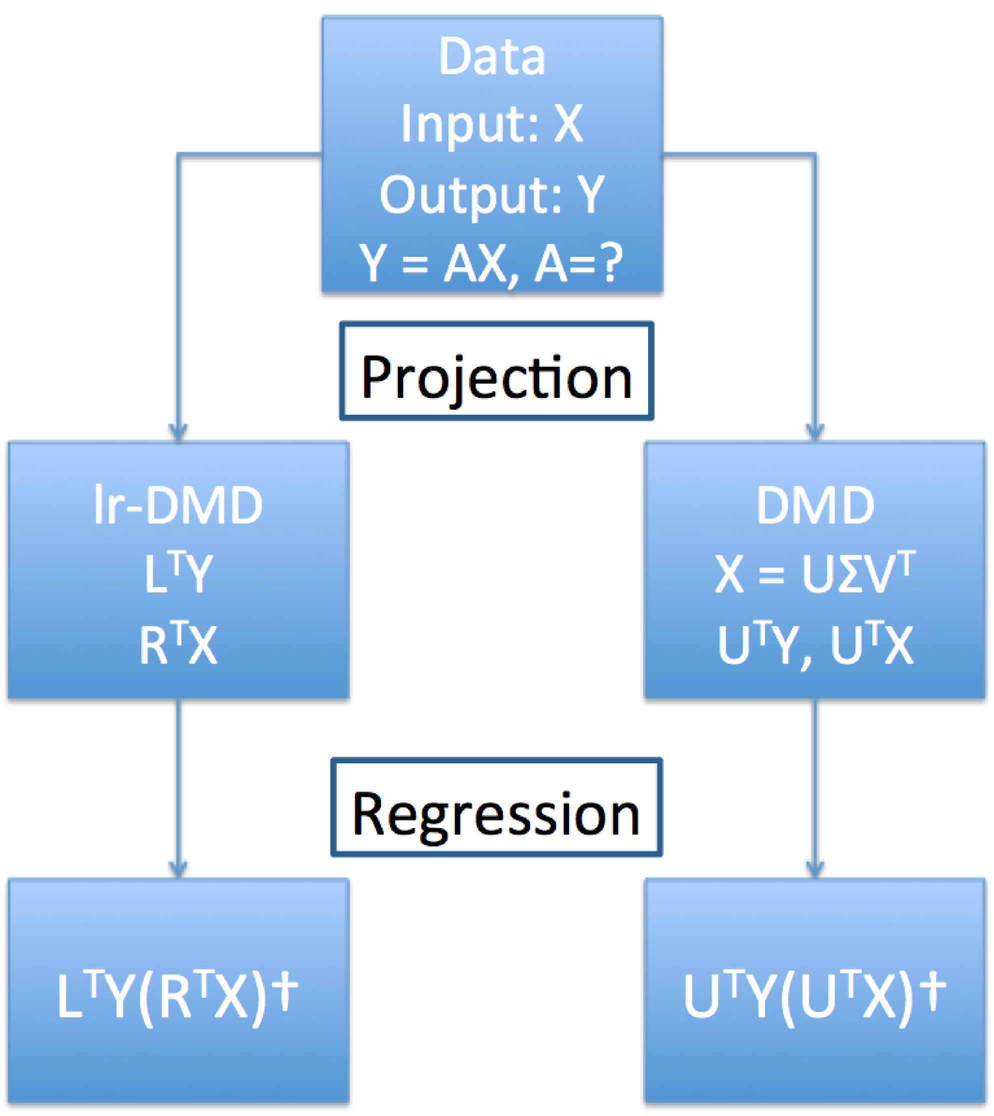
Palash Sashittal and Daniel J. Bodony
Department of Aerospace Engineering, University of Illinois at Urbana-Champaign
(This work was sponsored by the Office of Naval Research (ONR) as part of the Multidisciplinary University Research Initiatives (MURI) Program, under grant number N00014-16-1-2617.)

1. Introduction

We present a method for non-intrusive data-driven reduced order modeling of high-dimensional dynamical systems using a new low-rank extension of Dynamic Mode Decomposition (DMD). A matrix optimization problem with a rank-constraint on the solution is formulated and results in a non-convex optimization problem. We propose two methods to solve the optimization problem. The first is an iterative subspace projection method that is computationally efficient but can only give the optimal solution under certain conditions. In the second method we perform Riemannian optimization on Grassmanian manifolds. Using a model equation for fluid flows, we evaluate the performance of the proposed methods on the complex linearized Ginzburg-Landau equation in the supercritical globally unstable regime.

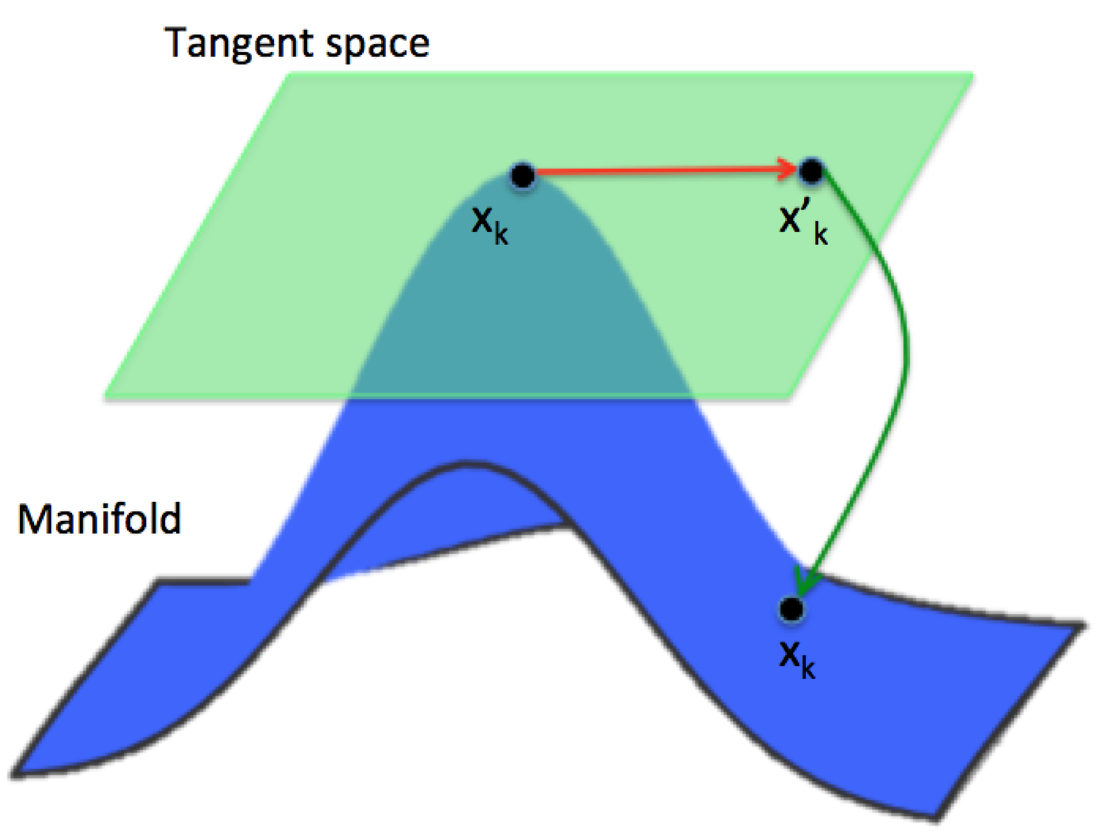
2. DMD → lr-DMD

The two major steps for DMD[1] algorithm are (a) projection/convolution and (b) regression (usually least-square estimate). In DMD the data matrices are projected onto their leading left singular vectors. In lr-DMD[2] we project the data matrices onto subspaces that provide the optimal least-sqaures estimate.



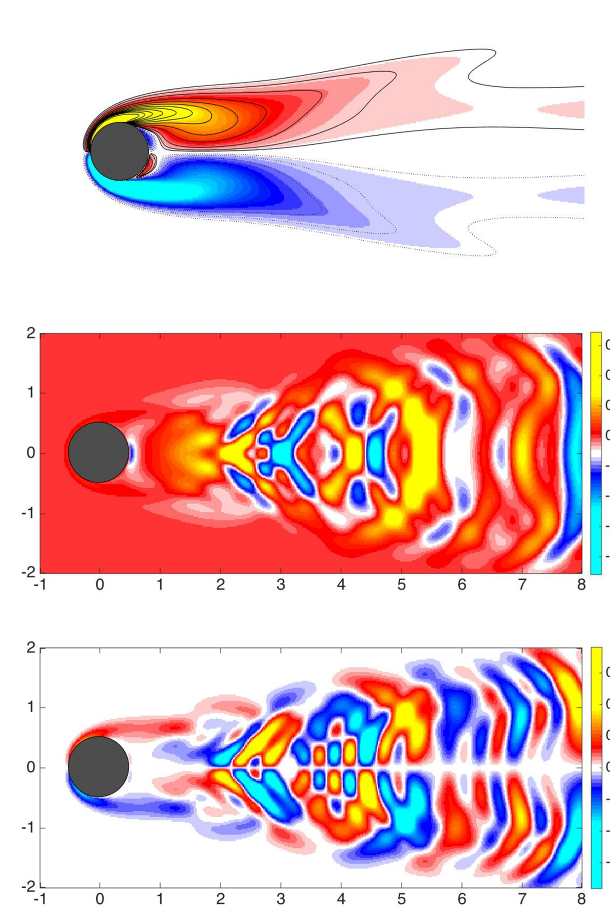
Riemannian Optimization

This schematic shows the two step process of computing the retracted Riemannian gradient on the manifold. First step is **translation** along the Riemannian gradient and second step computes the **retraction** onto the manifold.



3. Algorithm

See the sidebar on the left for a schematic on Riemannian Optimization. Details of both the algorithms are given in [2].



Mean flow and leading singular vectors of the state transition matrix for flow past cylinder. Data from 'http://dmdbook.com/'

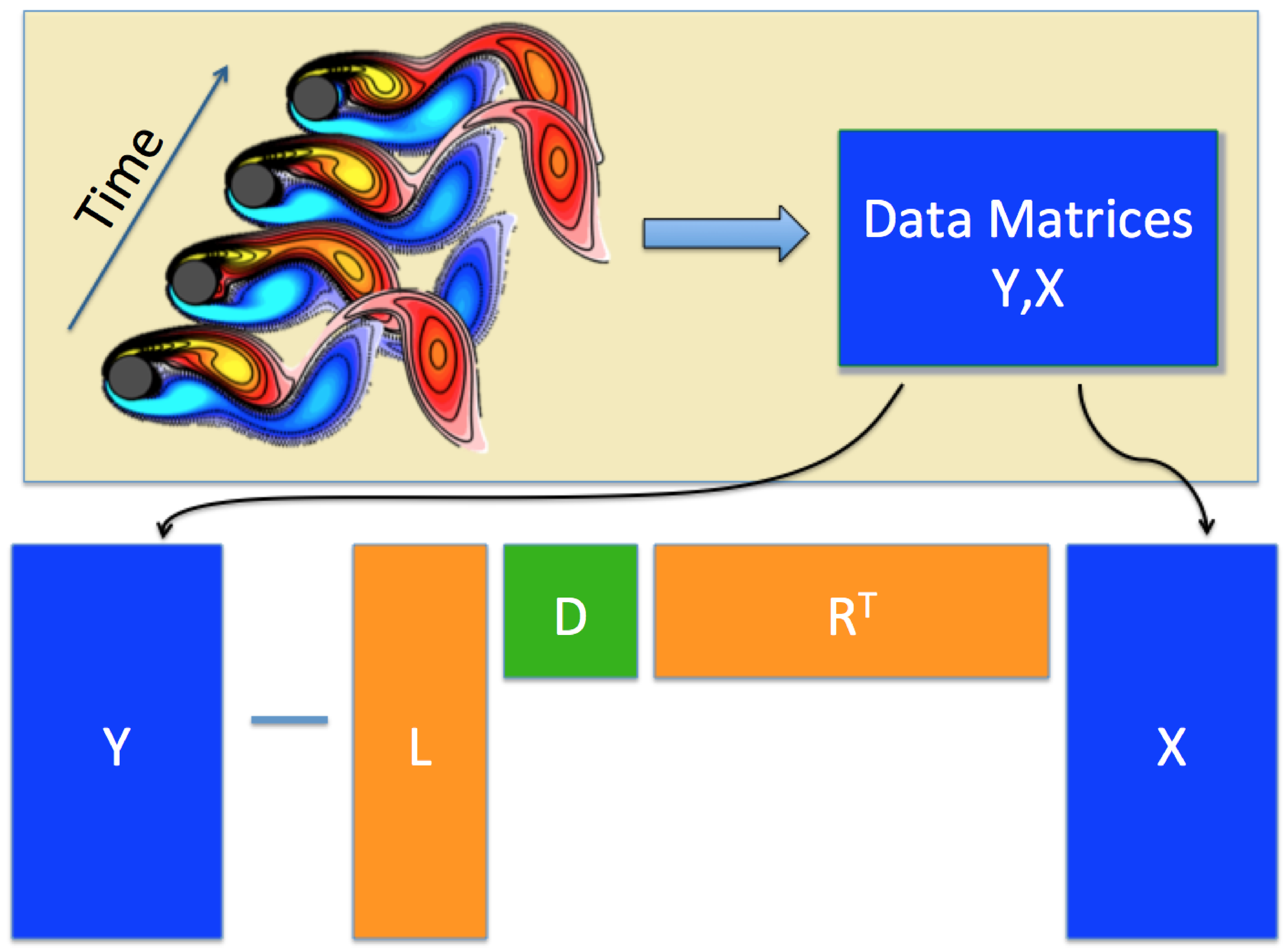
We consider the following optimization problem

$$\min_{L,D,R} \|Y - LDR^TX\|_F^2$$

where both $L, R \in \mathbb{R}^{m\times r}$ and $L^TL = R^TR = I_r$ ($r \times r$ identity matrix) and D is the r-ranked matrix approximating the dynamics of the underlying system. It can be simplified to

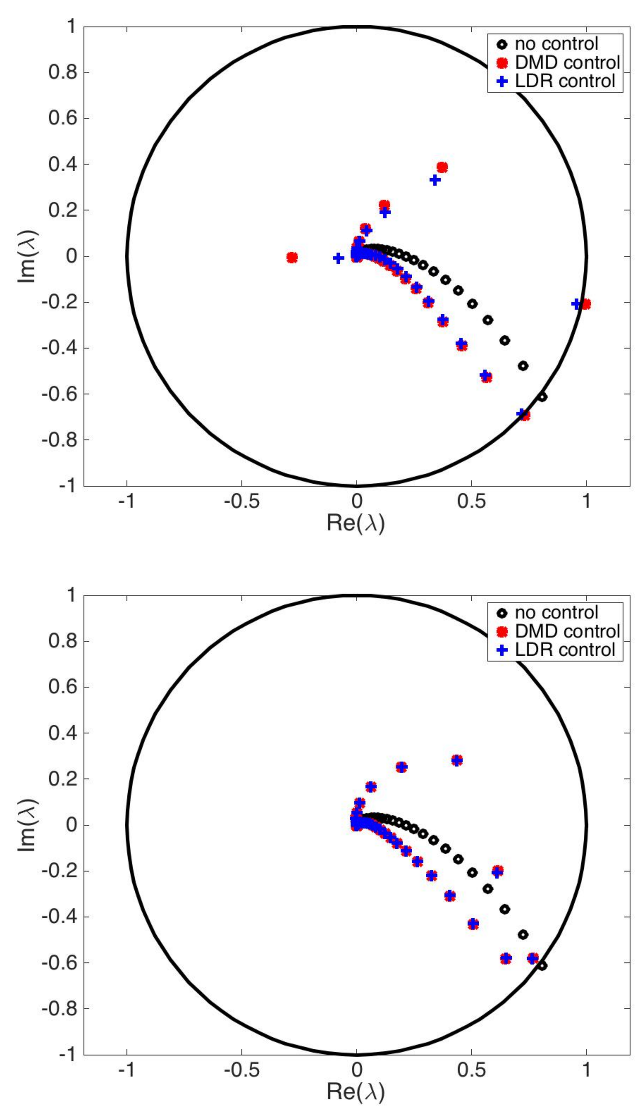
$$\min_{L,R} (-\|L^TYQ_R\|_F^2).$$

where $Q_R = X^TR(R^TXX^TR)^{-1}R^TX$ and $D(L,R) = (L^TYX^TR)(R^TXX^TR)^{-1}$.

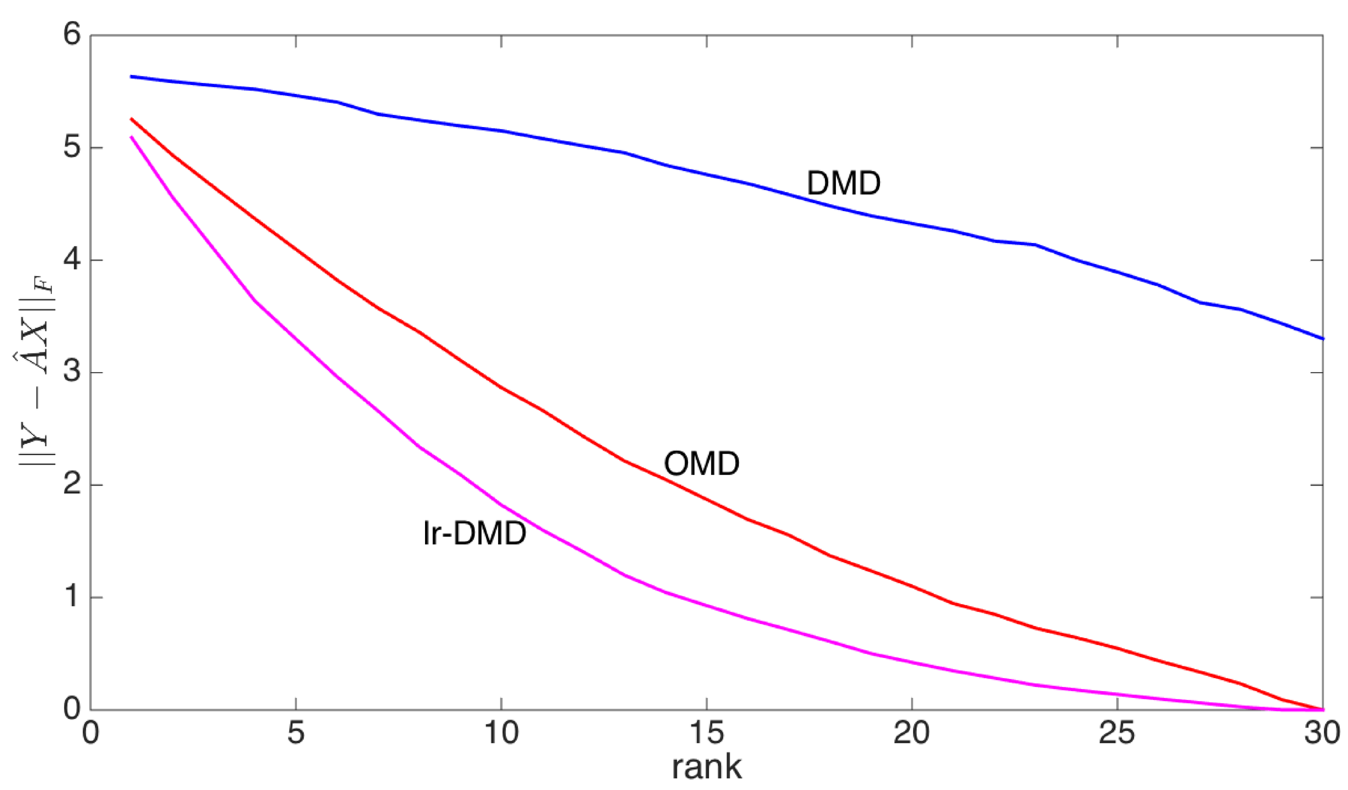


4. lr-DMD vs DMD

We compare the performance of lr-DMD vs DMD (and OMD[3]) for (a) control of linearized complex Ginzbug-Landau system (b) toy controlled dynamics problem fabricated with random matrices.



(a) Complex Ginzburg-Landau system rank 4 (above), 9 (below)



(b) Reconstruction error for increasing rank r of the reduced order model applied to a random linear system with control

5. Conclusions

1. lr-DMD has reduced error in reconstruction in Frobenius norm than DMD
2. Performance of lr-DMD on a fabricated controlled dynamical system is better than DMD
3. lr-DMD stabilizes linearized complex Ginzburg-Landau system with a lower rank approximation than DMD

6. References

[1] P. J. Schmid, "Dynamic mode decomposition of numerical and experimental data," *Journal of fluid mechanics*, vol. 656, pp. 5–28, 2010.

[2] P. Sashittal and D. Bodony, "Low-rank dynamic mode decomposition using Riemannian manifold optimization." submitted to 57th IEEE (CDC), 2018.

[3] P. J. Goulart, A. Wynn, and D. Pearson, "Optimal mode decomposition for high dimensional systems," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*, pp. 4965–4970, IEEE, 2012.