Effect of combustion on the frequency response of jets in crossflow

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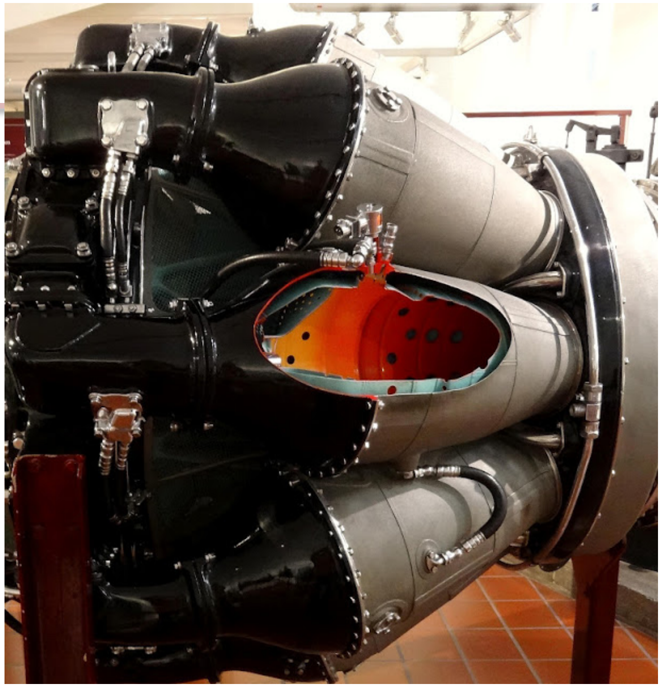
Motivation

- Jets in crossflow (JICF) are common feature
  - Combustion chambers
  - Dilution jets
  - Film cooling, …
- Two important aspects
  - **Flow related**
    - Interaction of the jet with the cross stream: Absolute vs. convective instability (dependent on the velocity ratio of the incoming jet)
  - **Combustion related**
    - Impact of reactions on overall instability of the jet



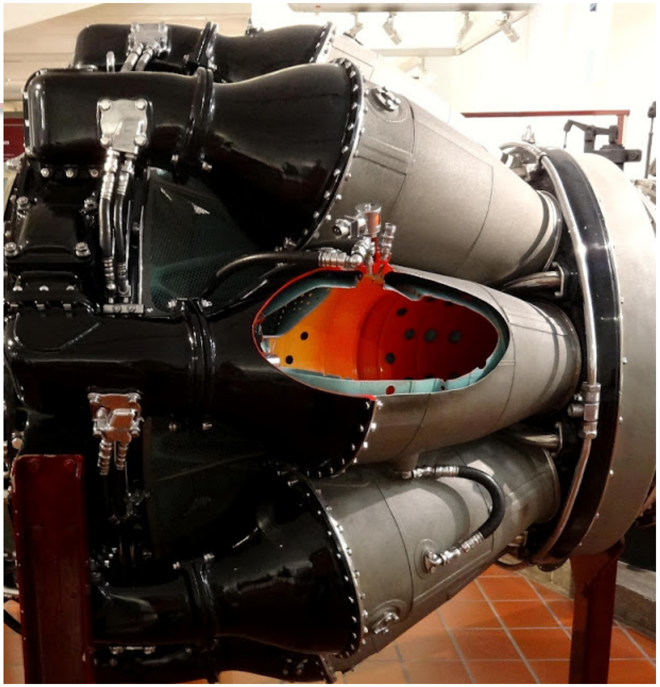
Rolls-Royce Nene 103

Jeff Swensen, Getty

Motivation

- JICF are common feature
  - Combustion chambers
  - Dilution jets
  - film cooling, …
- Two important aspects
  - **Flow related**
    - Bagheri *et al.* (2009), Megerian *et al.* (2007 & 2008), Mahesh (2013), Karagozian (2014)
  - **Combustion related**
    - Kolla *et al.* (2012), Lyra *et al.* (2015)

Difference in the **nature of instability** and the **impact of combustion** on the overall dynamics and interaction is less understood



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Objective

Quantifying the effect of reactions

1. Considering both reactive and non-reactive setup
2. Dominant frequencies in the flow
3. Dominant modal shapes extracted from the data
4. Parametric sensitivity analysis
5. Optimal gain for various frequencies

Analyzing the forward solution

Methods

- ✓ Spectral and data decomposition methods (DMD)
- ✓ Adjoint-based method for sensitivity analysis

Numerical framework

- **Compressible NS** equations, ideal gas, **Ma = 0.2**
- **Fourth-order** finite differences in three spatial directions
- **Staggered** variables, **Curvilinear** coordinates
- Time advancement scheme
  - NS equations : low storage **RK3**
  - Chemistry : **fifth-order** backward differentiation (**DVODE**)
- **Sponges** are used at all the boundaries except at the wall

NS equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_i u_j + p\delta_{ij}) = \frac{\partial \sigma_{ij}}{\partial x_j}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j}[(E+p)u_j] = \boxed{\dot\omega_T} - \frac{\partial q_j}{\partial x_j} + \frac{\partial}{\partial x_k}(u_j \sigma_{jk})$$

$$\frac{\partial \rho Y_k}{\partial t} + \frac{\partial(\rho u_j Y_k)}{\partial x_j} = -\boxed{\dot\omega_f} + \frac{\partial}{\partial x_j}\left[\frac{\nu}{Sc}\frac{\partial \rho Y_k}{\partial x_j}\right]$$

$$\dot\omega_T = \Sigma_k \dot\omega_k \Delta h_k^0$$

Chemistry

$$\boxed{\dot\omega_f} = Da\, \rho Y_f \exp\left(-\frac{Ze}{T}\right),$$

$$\boxed{\dot\omega_T} = Ce\, \dot\omega_f,$$

Da : Damkohler number
Ce : Heat release parameter
Ze : Zeldovich number

State variable **q** & combustion parameters **g**:

$$\mathbf{q} = [\rho \quad \rho u_1 \quad \rho u_2 \quad \rho u_3 \quad E \quad \rho Y_f]^\top,$$

$$\mathbf{g} = [Da \quad Ze \quad Ce]^\top,$$

Formulating the optimization problem

- State equation

q, state variable
f, forcing fucntion

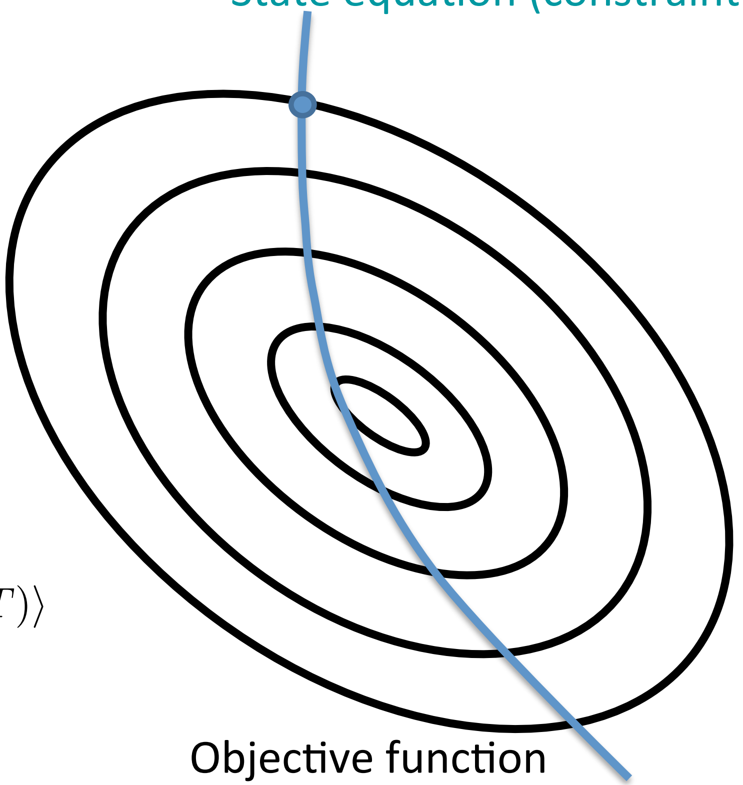
$$\frac{d\mathbf{q}}{dt} = A\mathbf{q} + B\mathbf{f}sin(2\pi\omega t) = R(\mathbf{q},\mathbf{f},\omega), \quad \mathbf{q}(0) = 0$$

- Objective function

$$J(\mathbf{q}) = \boxed{\phi(\mathbf{q}(T))} + \boxed{\int_0^T \psi(\mathbf{q}(t))dt,}$$

Final time **Integrated** time

- Here, $\psi(\mathbf{q}(t)) = 0$ and $\phi(\mathbf{q}(T)) = \langle \mathbf{q}(T) \cdot \mathbf{q}(T) \rangle$ a simple **energy norm.**
- Forming the Lagrange function
  - Transforms the constrained optimization to an unconstrained system



Formulating the optimization problem

- State equation

q, state variable
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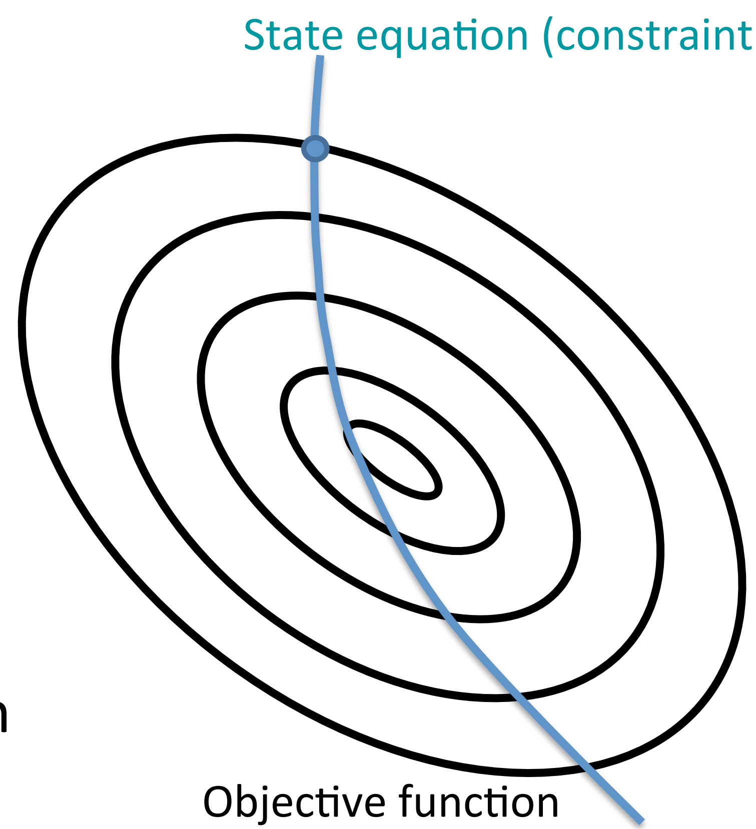
$$\frac{d\mathbf{q}}{dt} = A\mathbf{q} + B\mathbf{f}sin(2\pi\omega t) = R(\mathbf{q},\mathbf{f},\omega), \quad \mathbf{q}(0) = 0$$

- Objective function

$$J(\mathbf{q}) = \boxed{\phi(\mathbf{q}(T))} + \boxed{\int_0^T \psi(\mathbf{q}(t))dt,}$$

- Forming the Lagrange function
- Setting the variation w.r.t state variable to zero $\longrightarrow$ adjoint equation

$$-\frac{d\eta}{dt} = \left(\frac{\partial R}{\partial \mathbf{q}}\right)^{\top}\eta, \quad \eta(T) = 2\mathbf{q}(T)$$



Formulating the optimization problem

q, state variable
g, model/control parameters

- State equation

$$\frac{d\mathbf{q}}{dt} = f(\mathbf{q}, \mathbf{g}), \quad \mathbf{q}(0) = \mathbf{q}_0$$

- Objective function

$$J(\mathbf{q}, \mathbf{g}) = \phi(\mathbf{q}(T), \mathbf{g}(T)) + \int_0^T \psi(\mathbf{q}(t), \mathbf{g}(t))\, dt$$

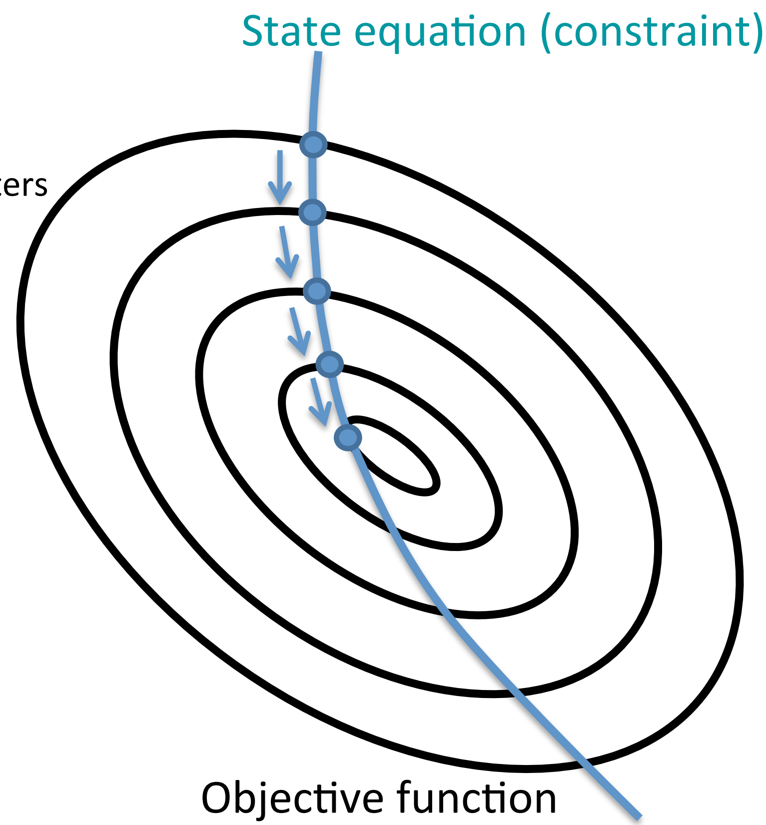
- Adjoint equation

$$-\frac{d\eta}{dt} = \left(\frac{\partial R}{\partial \mathbf{q}}\right)^\top \eta, \quad \eta(T) = 2\mathbf{q}(T)$$

- Derivative

$$\mathbf{f} = \frac{1}{2\xi} \int_0^T B^\top \eta \sin(2\pi\omega t)\, dt$$

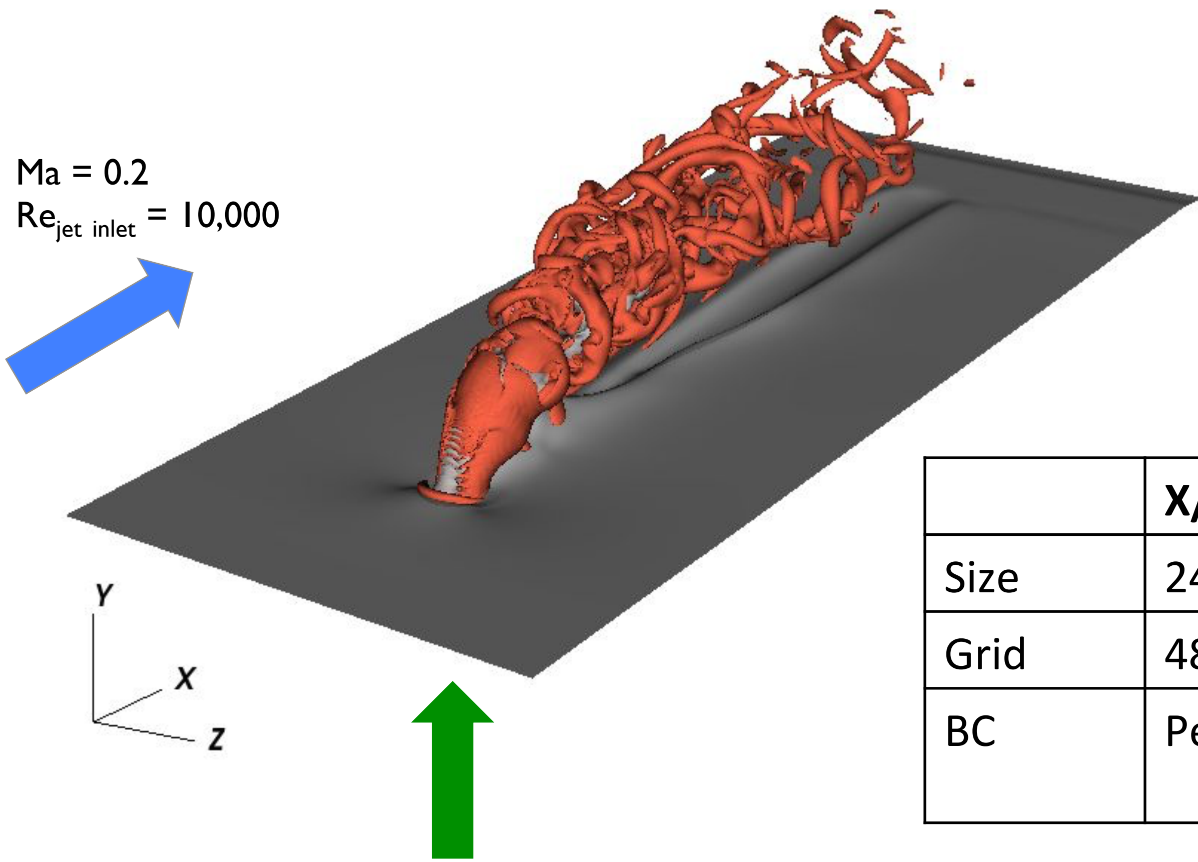
In effect normalizes the forcing function



Configuration

- Crossflow (air) → Laminar boundary layer over a flat plate
- Jet (fuel) → Laminar circular jet in wall normal direction

Ma = 0.2
$Re_{jet\ inlet}$ = 10,000



V_{jet}/V_{∞}=3

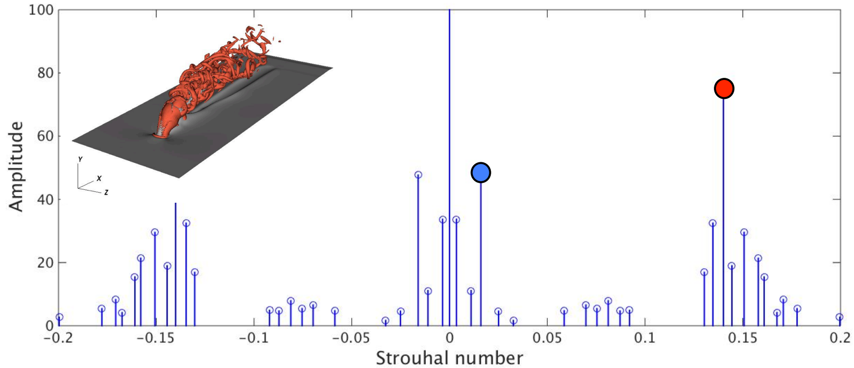
	X/D	Y/D	Z/D
Size	24	8	10
Grid	480	256	200
BC	Periodic	Wall/ outflow	Periodic

In the reactive case, the fuel is injected at the nozzle

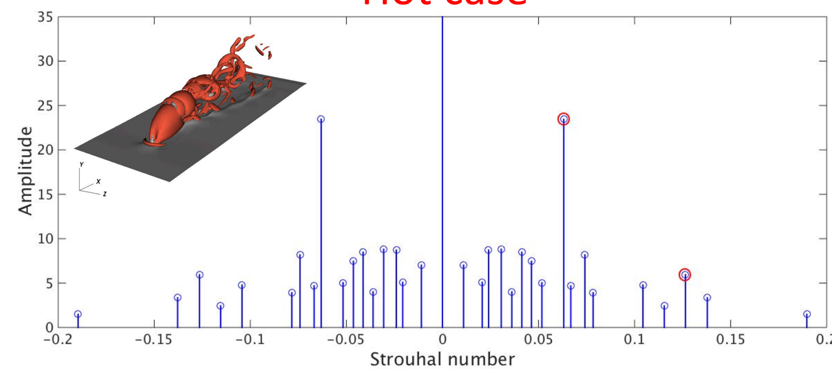
Dominant frequencies

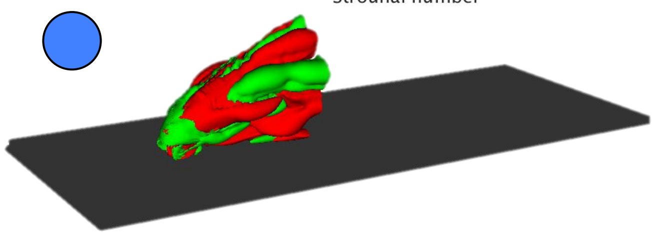
- Frequencies are extracted using DMD
- Compare well with the energy spectra of probe data
- Sparsity promoting algorithm (Jovanovic *et al* 2014) used to extract the amplitudes

Cold case

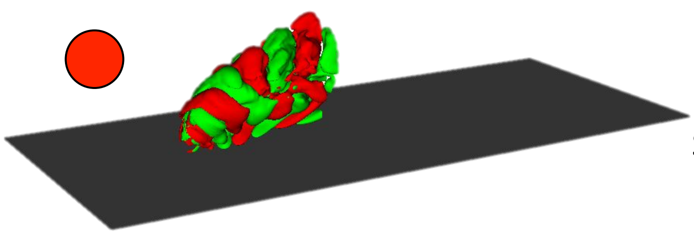


Hot case





St = 0.016 → Frequency of the mode related to the motion near the wall

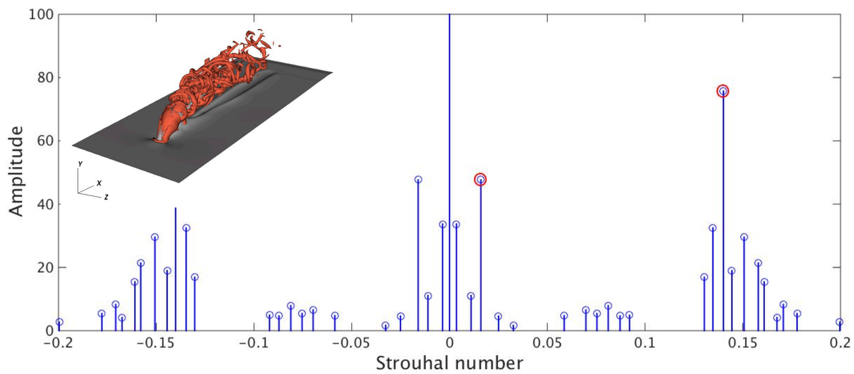


St = 0.14 → Frequency of the mode on the shear layer

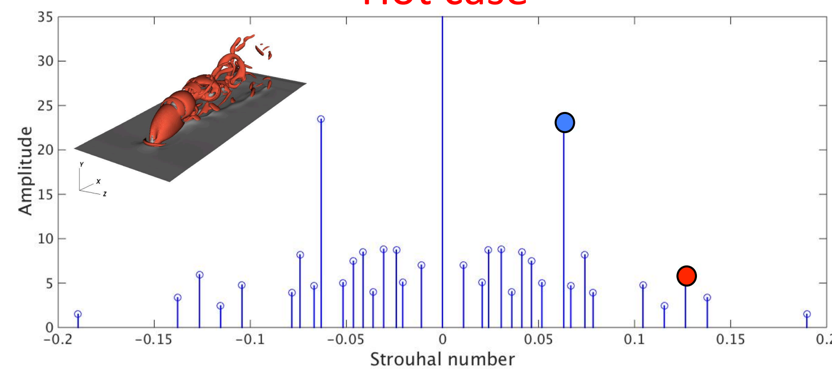
Dominant frequencies

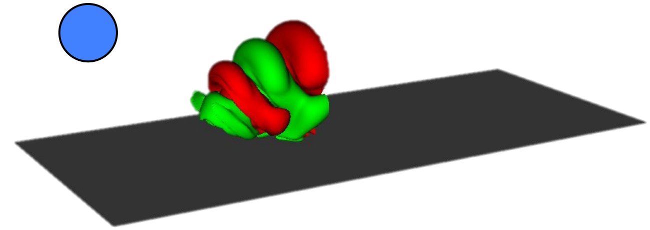
- Frequencies are extracted using DMD
- Compare well with the energy spectra of probe data
- Sparsity promoting algorithm (Jovanovic *et al* 2014) used to extract the amplitudes

Cold case

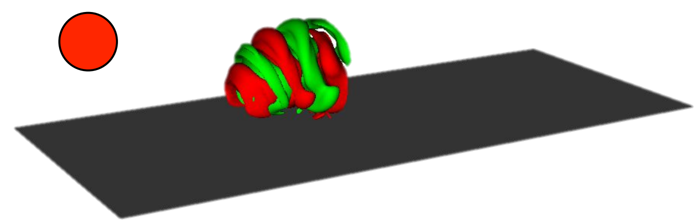


Hot case





Frequency of the mode related to the motion on the shear layer ← St = 0.06

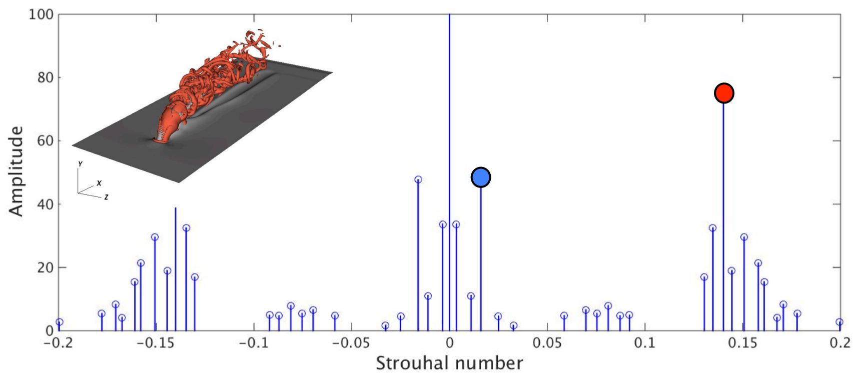


The subharmonic of the previous mode ← St = 0.12

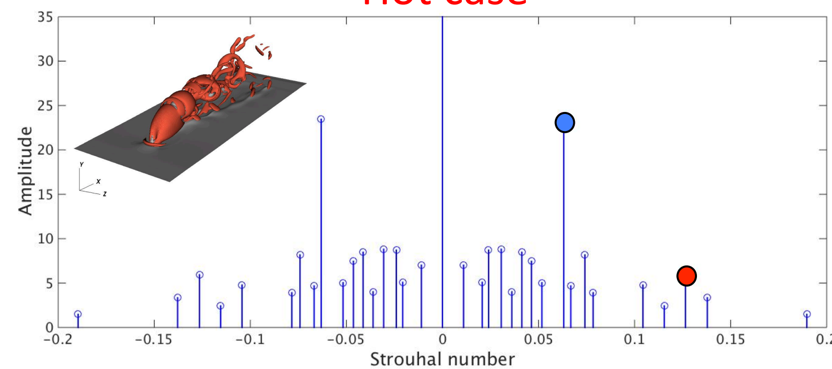
Dominant frequencies

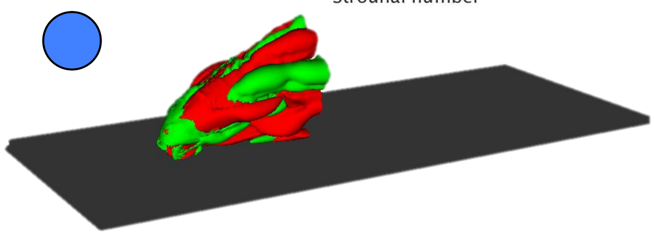
- Frequencies are extracted using DMD
- Compare well with the energy spectra of probe data
- Sparsity promoting algorithm (Jovanovic *et al* 2014) used to extract the amplitudes

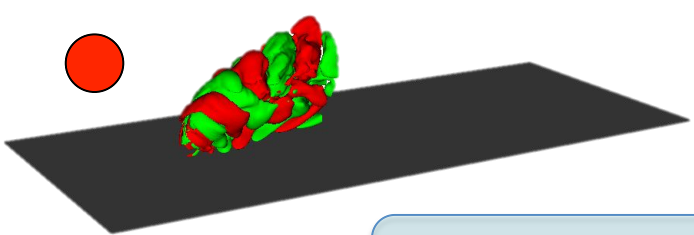
Cold case

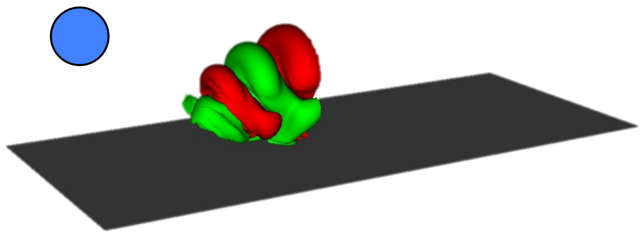


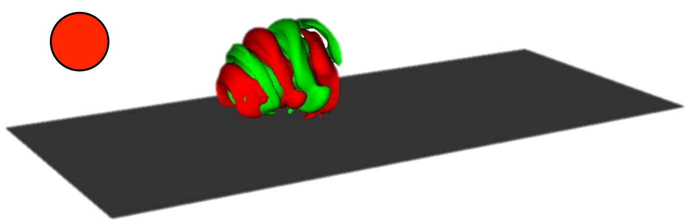
Hot case







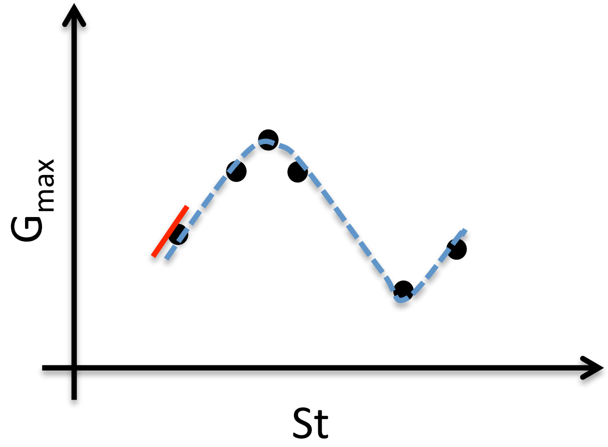




Both the **shape** and the **frequency** of the dominant modes are altered through combustion

Frequency response

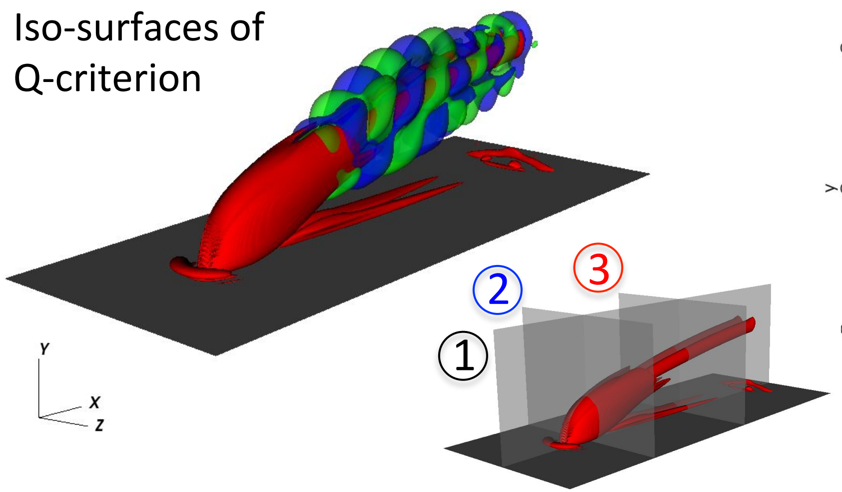
- From the extracted frequency an interval is selected
  - F = [0.05, … , 0.15]
  - Frocing at the selected frequencies
- Extract the optimal gain for these frequencies and the slopes: Adjoint-based algorithm
- Compare the interpolated results between the reactive and non-reactive case

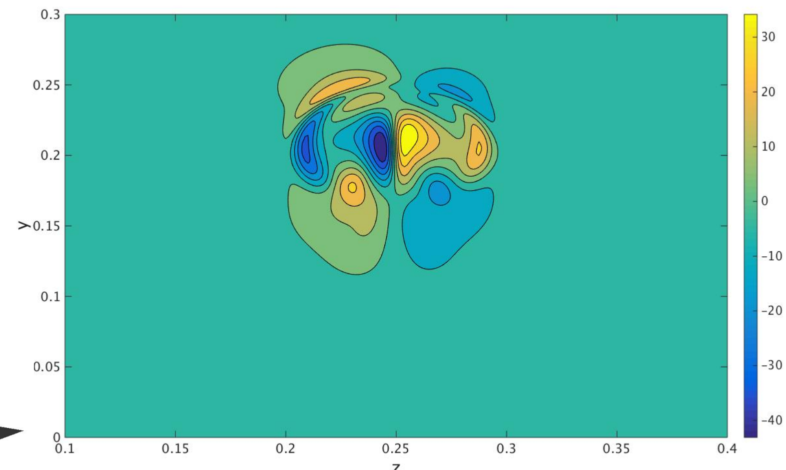


Response function- Cold case

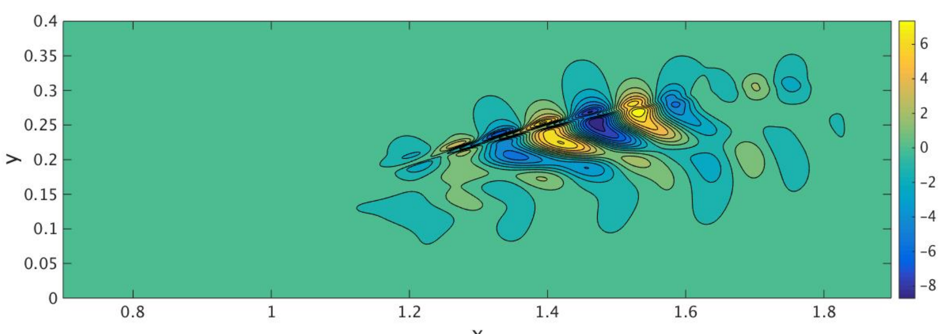
- Frequency St = 0.05, lower limit of the interval
- Shape of the mode evolves downstream

Iso-surfaces of Q-criterion





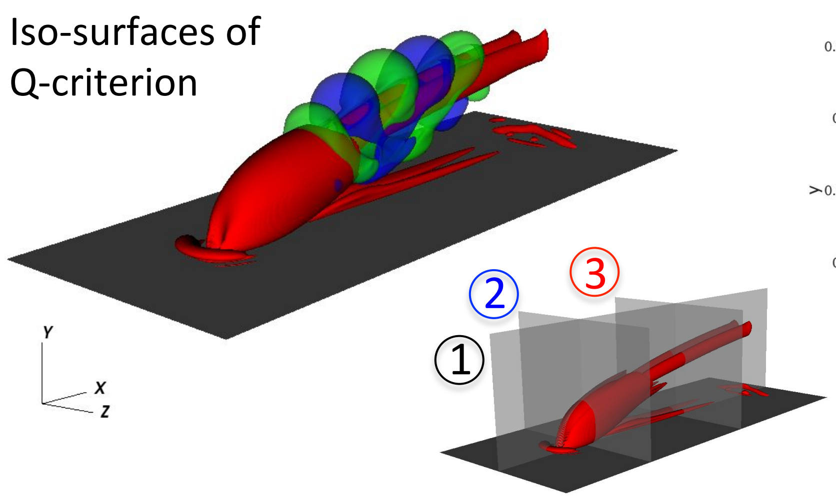
Slice 3 : contours of streamwise velocity

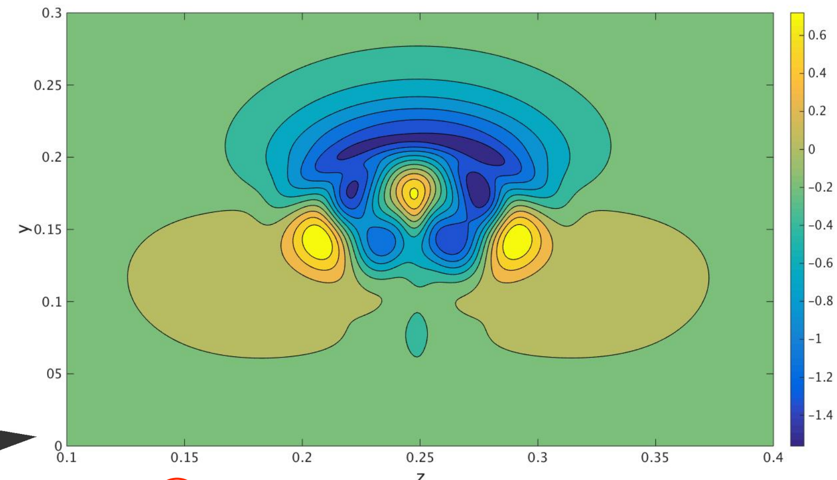


Slice 1 : contours of streamwise velocity

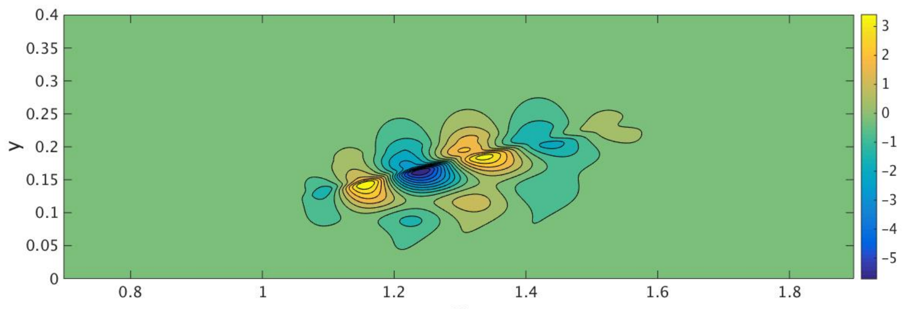
Response function- Hot case

- Frequency St = 0.05, lower limit of the interval
- Frequency close to the peak frequency in the spectra





Slice 3 : contours of streamwise velocity



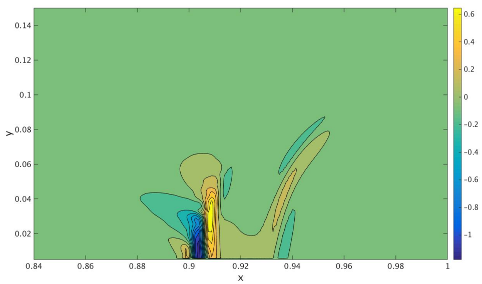
Slice 1 : contours of streamwise velocity

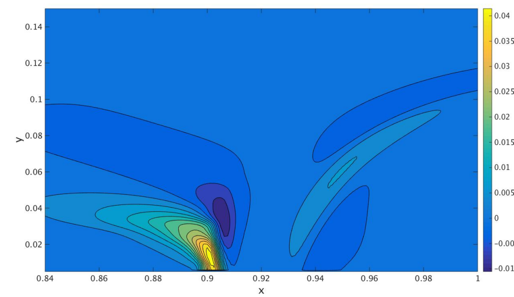
Comparing forcing and response function, St = 0.05

Cold case

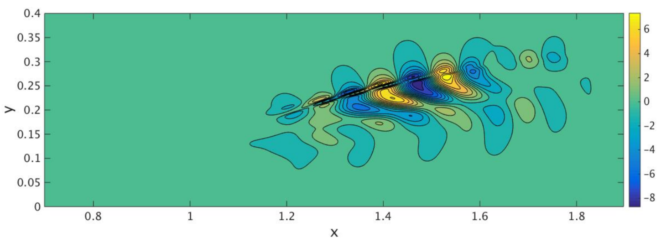
Hot case

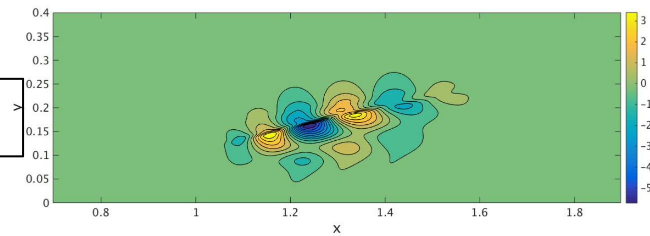
Forcing





response





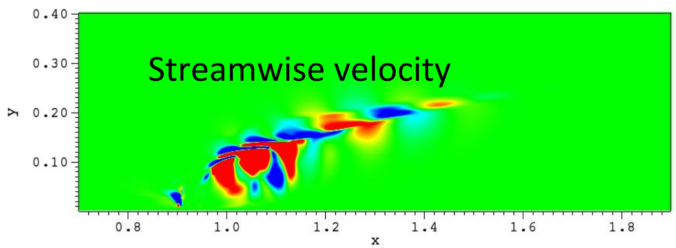
- Response in the cold case is of higher wavelength in all spatial direction, travels further on the CVP
- Forcing function is also of higher frequency in the cold case

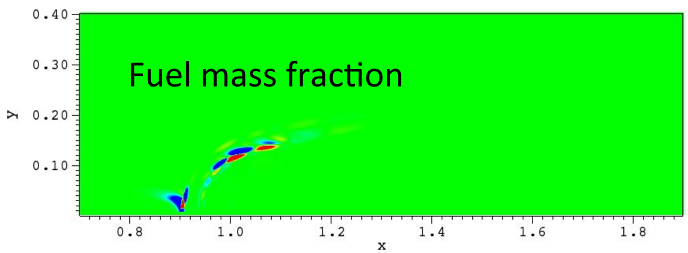
	G_{max}	Slope
Cold case	34	89 degrees
Hot Case	1.2	60 degrees

Outlook and conclusions

- Combustion has an obvious effect on the forcing and response functions
- The response and forcing functions are both of lower wavelength in the reactive case

- Analyze the optimal gain of various frequencies and compare between the hot and cold cases
- Assess the contribution of each component to the overall sensitivity





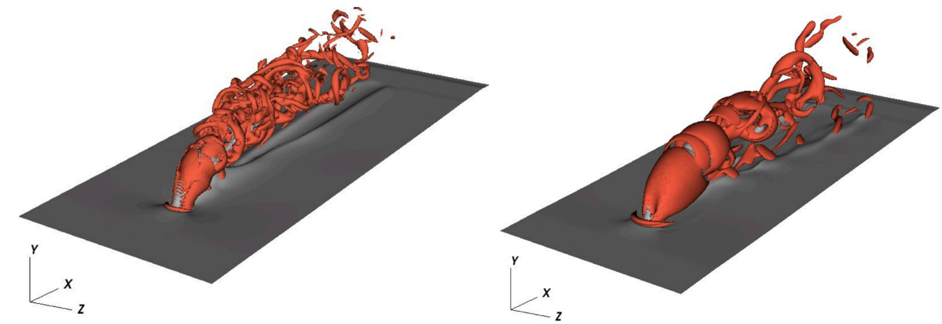
Base flow

- **Selective frequency damping (SFD)** (Espen Åkervik *et al.* 2006) on the non-lin forward equations

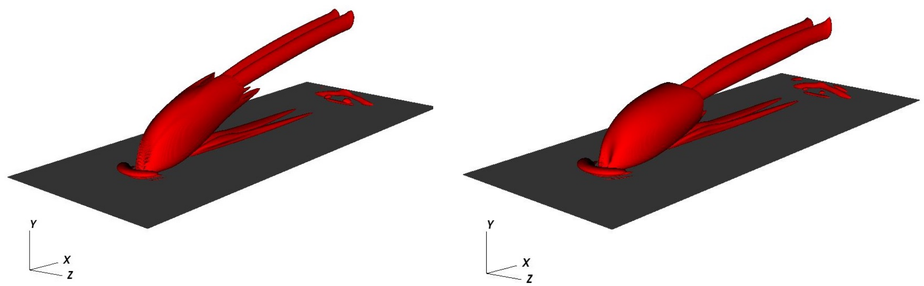
$$\frac{d\mathbf{q}}{dt} = f(\mathbf{q}, \mathbf{g}) - \chi(\mathbf{q} - \bar{\mathbf{q}}),$$
$$\frac{d\bar{\mathbf{q}}}{dt} = \frac{(\mathbf{q} - \bar{\mathbf{q}})}{\Delta}.$$

- This ensures that the base flow is a **stationary solution** of the nonlinear governing equations

Instantaneous



Base-flow



Iso-surfaces of Q criterion indicate vortical structures in the flow