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HYBRIDIZATION OF COMPACT AND WENO SCHEMES FOR SIMULATION OF TURBULENT FLOWS WITH SHOCKS

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Sashwat Mishra

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To Family and Friends, for all their love and support.

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SYMBOLS

	Roman symbols
e_t	Total energy per unit volume
$\mathbf{F}, \mathbf{G}, \mathbf{H}$	Inviscid flux vectors
$\mathbf{F_v}, \mathbf{G_v}, \mathbf{H_v}$	Viscous flux vectors
$\mathfrak{F}, \mathfrak{G}, \mathfrak{H}$	Inviscid flux vectors in characteristic form
M_0	Reference Mach number
P	Pressure
Pr_0	Reference Prandtl number
q_i	Heat flux along i-direction
$\mathbf{Q}$	Vector of primitive flow variables
Re_0	Reference Reynolds number
S_{ij}	Strain rate tensor
u, v, w	Fluid velocity components
w_k	Non-linear weights for stencil k in a WENO scheme
	Greek symbols
α_k	Non-linear weight coefficients for stencil k in a WENO scheme
β_k	Smoothness indicator for stencil k in a WENO scheme
γ	Ratio of specific heats
ϵ	Small constant($\sim 10^{-40}$)
μ	Dynamic viscosity
ρ	Density
τ	Overall smoothness indicator for super stencil S_{sup} in a WENO scheme
τ_{ij}	Stress tensor component acting on i-face along j-direction

ABBREVIATIONS

DNS	Direct Numerical Simulation
ENO	Essentially Non-Oscillatory
LES	Large Eddy Simulation
RANS	Reynolds Averaged Navier-Stokes
TVD	Total Variation Diminishing
WCNS	Weighted Compact Nonlinear Schemes
WENO	Weighted Essentially Non-Oscillatory

ABSTRACT

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Computation of flow through shock waves was extremely difficult to handle, as shocks present sharp changes in flow variables like density, pressure and velocity. One could have handled such changes by using a very fine mesh that could capture these gradients, but this would have led to higher computational costs.

The use of shock capturing schemes has been a useful innovation towards handling flow through shocks while still keeping the mesh relatively coarse. These schemes, namely ENO, WENO etc., smear the shock over a few points in the computational domain, in order to better handle the near infinite gradients. This smearing is a result of numerical (artificial) dissipation added by these schemes. On the other hand, while simulating high speed turbulent flows, this artificial dissipation adds to the natural turbulent dissipation and dissipates the turbulence excessively. Consequently, the accuracy of the simulation results at areas away from shocks, suffers.

To accurately capture both natural turbulence and shocks, a hybridization between a non-dissipative and a dissipative scheme is devised. A sixth order basal compact scheme is hybridized with several variations of the WENO schemes. The switch between the compact and WENO scheme is controlled by a shock detector. The resulting hybrids are tested on well-known benchmark problems and validated by comparison with results from using a WENO scheme on a fine grid.

1. INTRODUCTION

1.1 Motivation

Jet noise, caused by high speed turbulent flows, is responsible for some of the loudest sounds produced by man. Its adverse effects range from mild annoyance to severe long-term hearing disabilities, especially in veterans working on aircraft carriers who are regularly exposed to such noise. The primary sources of such noise are shocks in the flow field behind a jet. These shocks interact with the turbulence passing through them and can generate noise.

Direct numerical simulation (DNS) would produce the most accurate representations of the flow field, but would require computational power that is unlikely to be attained in the near future. Reynolds averaged Navier-Stokes (RANS) computations are cheaper but do not account for unsteady fluctuations in the turbulent flow as they are based on averaged or mean values of flow variables. In contrast, Large eddy simulation (LES) modeling has proven to be an effective computational tool for analyzing and predicting jet noise.

The compact discretization schemes, widely used in jet simulations, manage to preserve the natural vortices but find the regions near a shock computationally challenging to simulate. Artificial dissipation can be added through characteristic based filters to smoothen the shock, but the lack in capability to accurately reproduce the interaction between turbulence and shocks, diminishes the accuracy of jet noise predicted by such methods. Alternatively, the shock capturing schemes dissipate the shock over a few grid points and successfully capture it, but cause excessive dissipation elsewhere in the flow field, weakening or killing naturally occurring vortices.

This serves as motivation to work on developing better numerical methods to capture shocks, while still preserving naturally occurring turbulent vortices.

1.2 Literature Review

A lot of effort has been made towards developing shock capturing schemes that work well for shock-turbulence interactions - schemes that would get rid of artificial Gibbs phenomena near shock while still maintaining high resolution of turbulent structures away from shocks. Since turbulence and shocks represent different physical phenomena they could be treated by different numerical methods. This is how the hybrid schemes were conceived. Over the years, several hybrid schemes have been developed, most of which differ in the method used for one or more of the three major components of a hybrid scheme:

- the finite differencing scheme used away from shocks,
- the shock capturing scheme used near shocks, and
- the switching mechanism.

One of the earliest works in this area is the development of a hybrid scheme by Pirozzoli [1] which used a compact fifth-order upwind scheme as the finite differencing scheme to be used away from shocks, the WENO scheme developed by Jiang and Shu [2] as the scheme near the shocks and a switching mechanism based on whether the absolute difference between primitive variables at two adjacent nodes crosses a threshold. While this hybrid scheme captures the shock well and preserves turbulence structures in smooth region, the simple switching mechanism may not accurately predict the correct scheme to be used in certain regions of the flow field.

The work by Costa and Don [3] is also very similar with the difference in methodology being as follows — a sixth order central finite difference scheme as the finite differencing scheme to be used away from shocks, the WENO scheme developed by Jiang and Shu [2] as the scheme near the shocks and a switching mechanism based on high order multi-resolution analysis illustrated by Harten [4]. The multi-resolution analysis used might in this approach can be computationally expensive.

Li and Qiu [5] illustrate the difference in performance of hybrid schemes based on switching mechanisms based on different discontinuity indicators. A central differencing scheme at the half-points is used as the finite differencing scheme to be used away from shocks and the WENO scheme developed by Jiang and Shu [2] is used near the shocks.

A common trend is observed in the above studies. The accuracy of the hybrid methods always drops to fifth order near shocks due to the implementation of a fifth order WENO scheme. Fifth order WENO schemes use three upwind stencils of three points each to obtain a the fifth order interpolated flux. Sixth order accuracy can be achieve by using another stencil downwind of the shock. However, this downwind stencil suffers from loss of information upwind of the shock and provides inaccurate interpolation.

More recently, sixth order WENO schemes have been developed [6, 7] where the smoothness indicator for the downwind stencil is modified to contain some information from the points upwind of the shock in addition to the three downwind points. This helps to control the contribution of the interpolated flux from the three downwind points. The development of such schemes enable us to use these as the shock capturing method in a hybrid scheme.

Jung and Nguyen [8] observe that these sixth order WENO schemes suffer from a phase lag near extrema. They develop a different hybrid WENO scheme which switches between a fifth order and sixth order WENO scheme based on smoothness indicators of the super stencil containing all points used to get the interpolated flux at a location.

This study focusses on a hybrid scheme where a sixth order compact finite difference scheme by Lele [9] is used as the finite differencing scheme away from shocks. A shock detector as suggested by Ren et al. [10] or Ducros et al. [11] is used as the switching mechanism. The shock capturing scheme used for the hybrid scheme is either a fifth order WENO scheme or a sixth order WENO scheme. Additionally,