Making Kerr quasinormal mode frequency computation more robust

Sashwat Tanay, Leo Stein (Univ. of MS)

APS April meeting 2023, Minneapolis

stanay@olemiss.edu

Background

Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).

Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).
- Final Kerr BH oscillates at the quasinormal mode (QNM) frequencies.

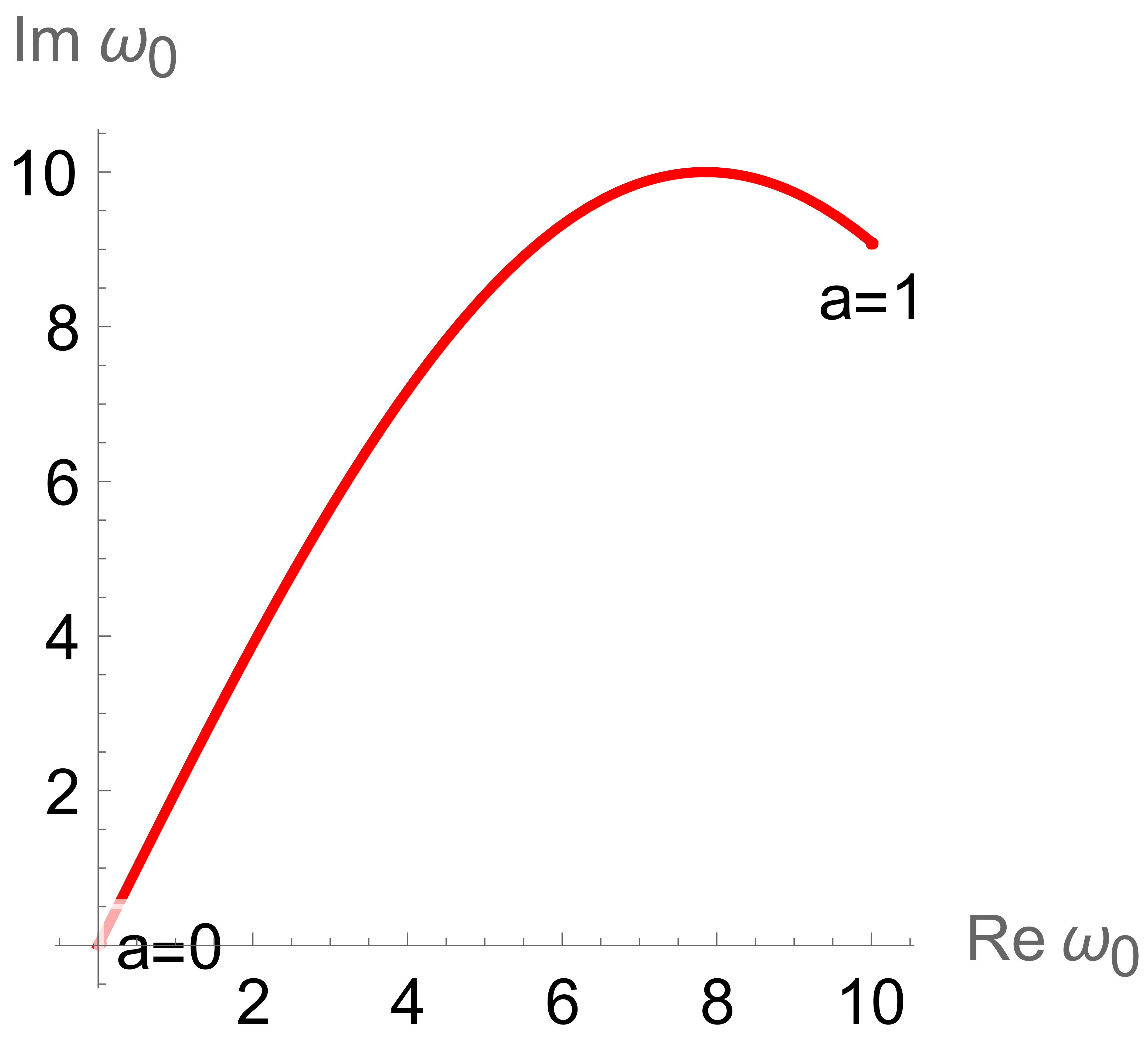
Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).
- Final Kerr BH oscillates at the quasinormal mode (QNM) frequencies.
- Determining QNM frequencies is essential for GW data analysis.

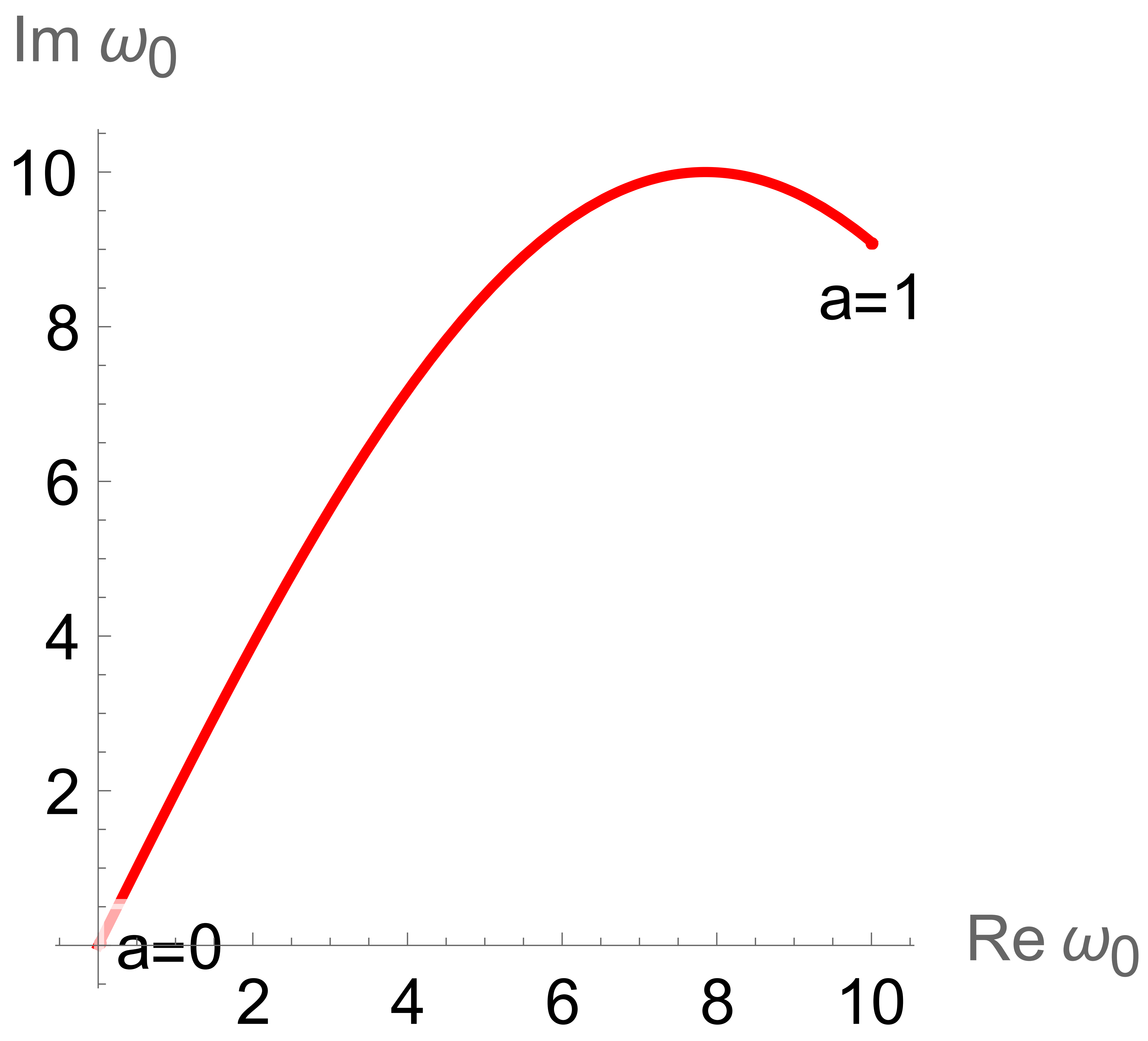
Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).
- Final Kerr BH oscillates at the quasinormal mode (QNM) frequencies.
- Determining QNM frequencies is essential for GW data analysis.
- **Objective:** work towards improving the spectral variants of Leaver's method of *arXiv: 1410.7698 (Cook & Zalutskiy)* and *arXiv: 1908.10377 (Leo Stein)*.

QNM frequency ω_0 as a function of BH spin a.

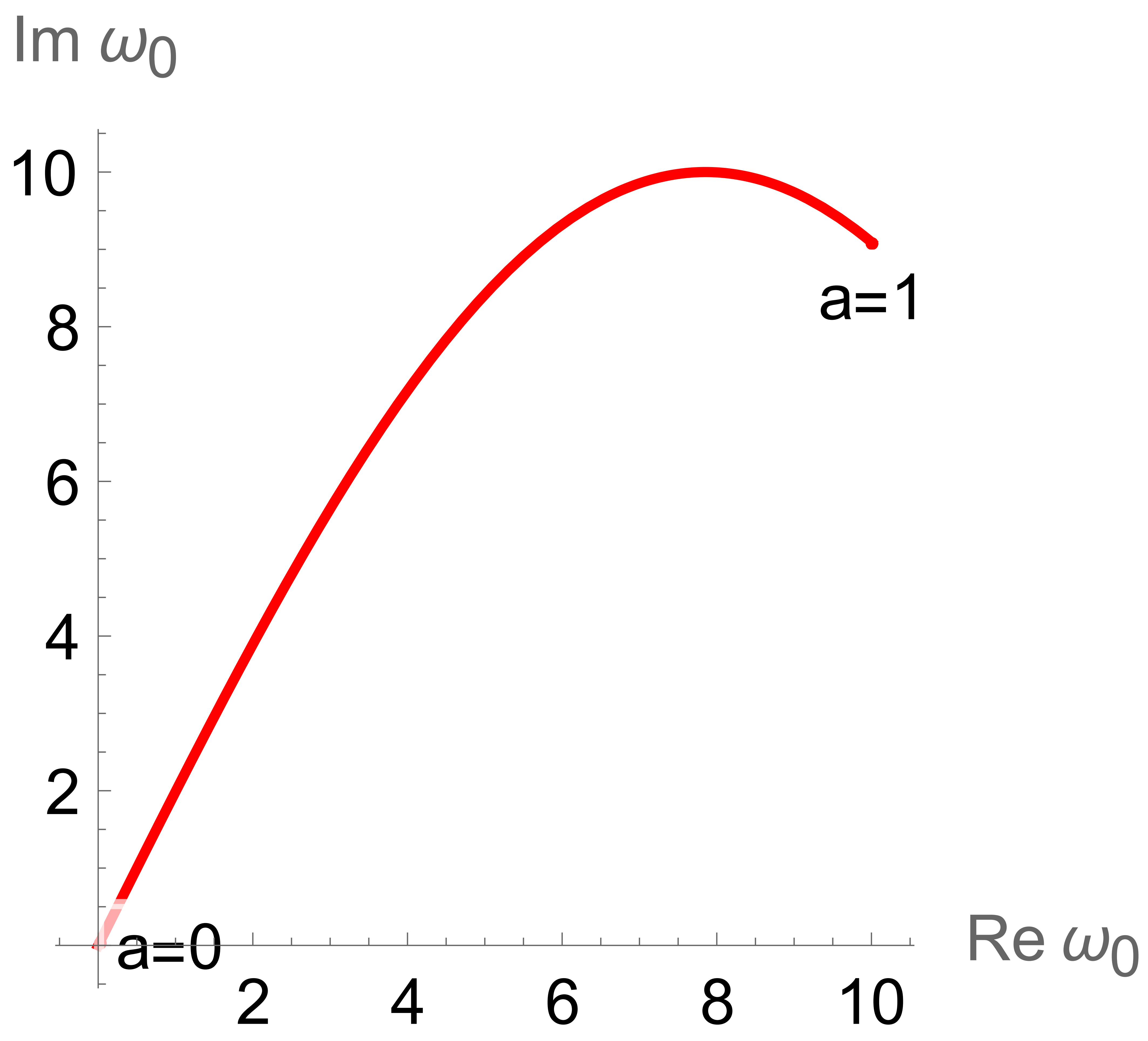


QNM frequency ω_0 as a function of BH spin a.



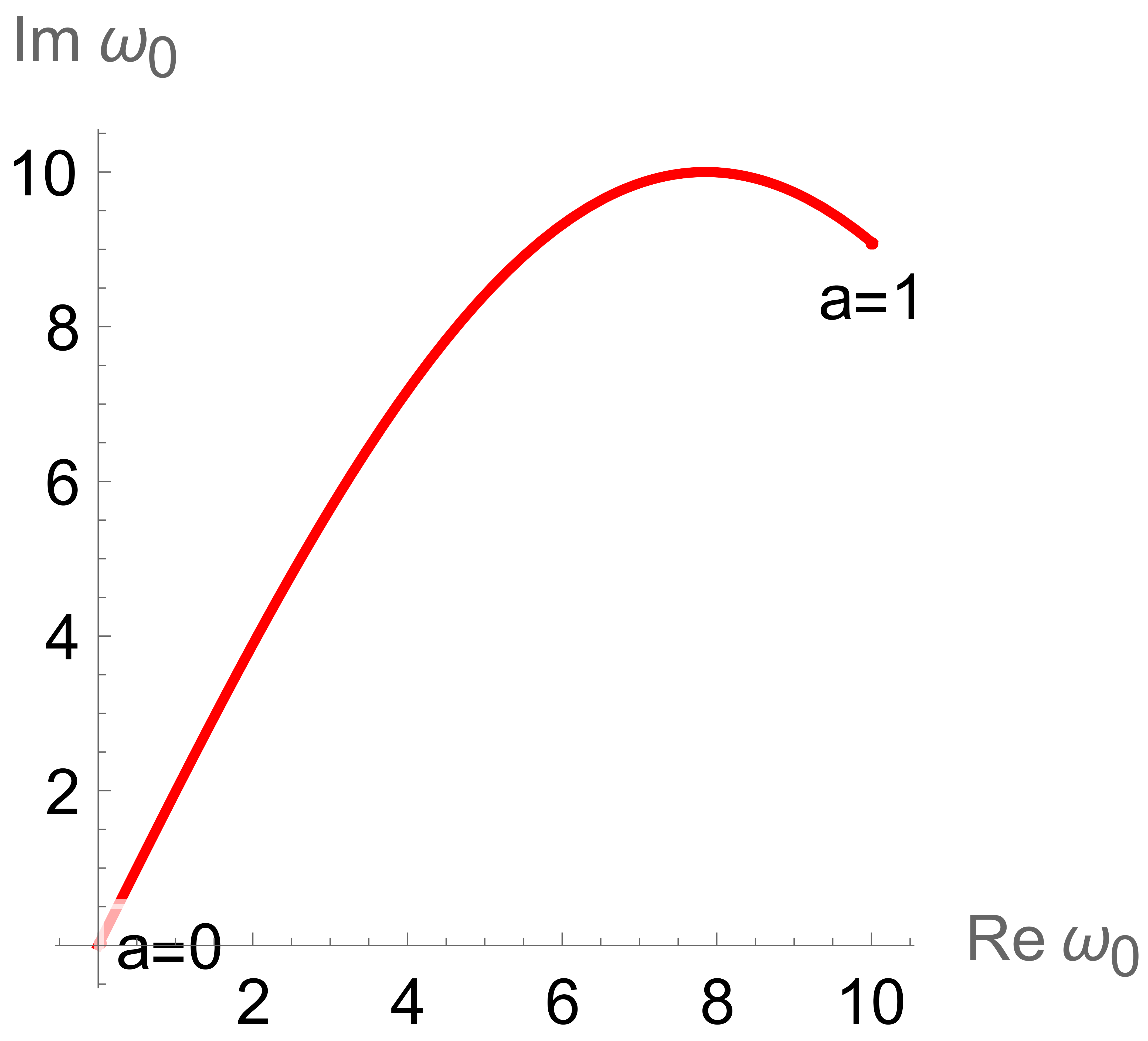
QNM frequency ω_0 as a function of BH spin a.

- QNM frequencies are complex.



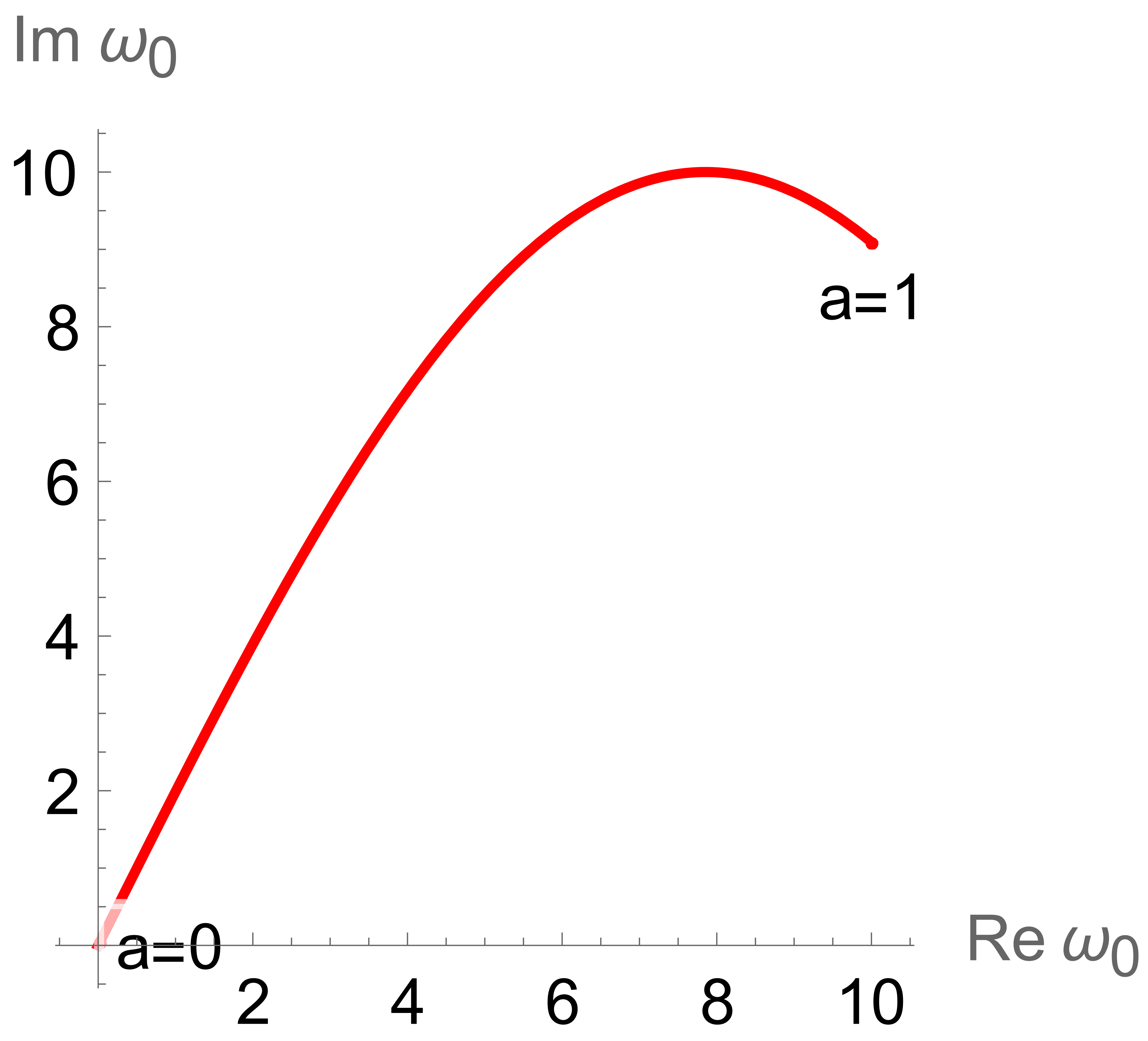
QNM frequency ω_0 as a function of BH spin a.

- QNM frequencies are complex.
- $\omega_0 = \omega_0(a)$ with $0 < a < 1$.

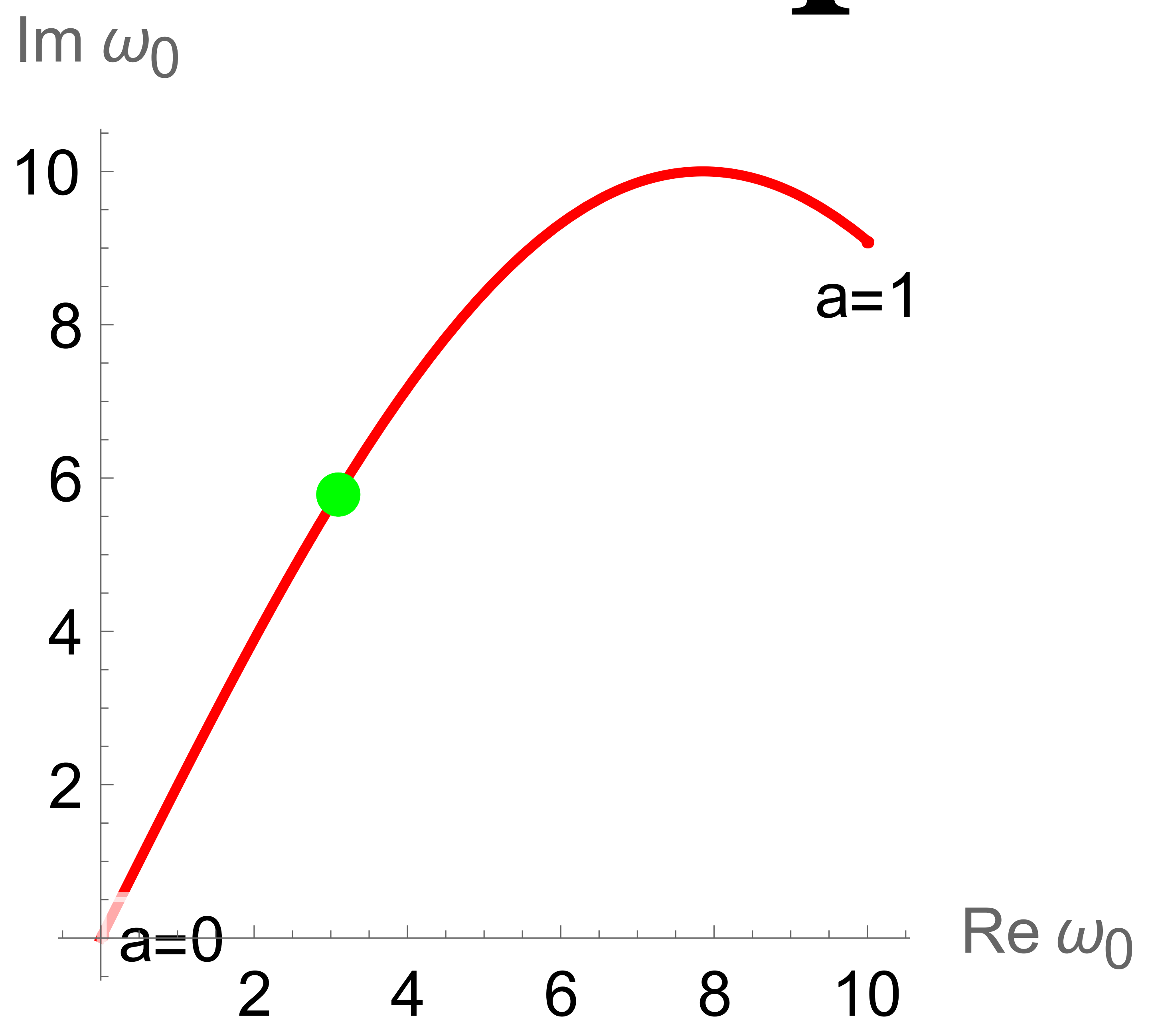


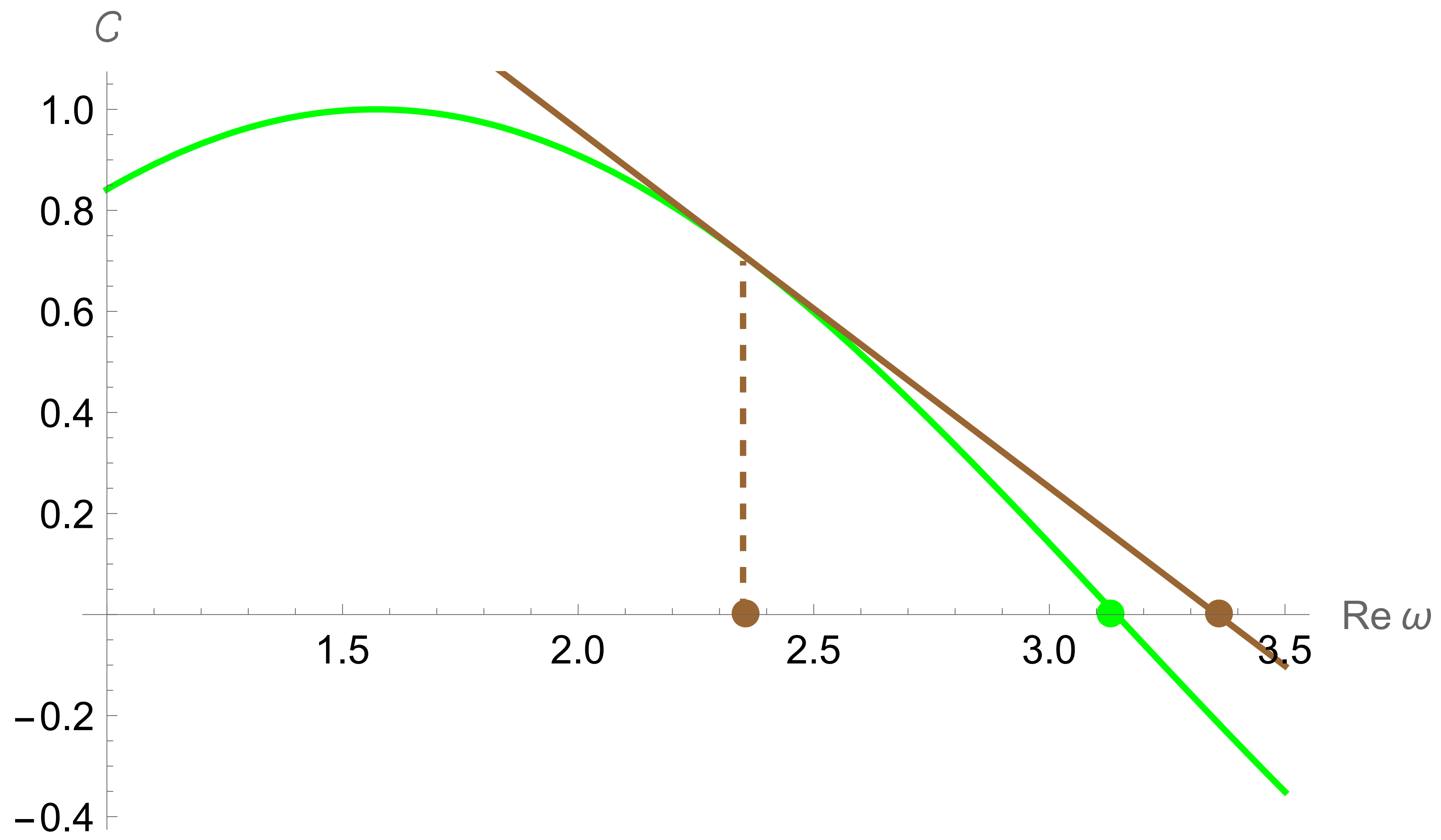
QNM frequency ω_0 as a function of BH spin a.

- QNM frequencies are complex.
- $\omega_0 = \omega_0(a)$ with $0 < a < 1$.
- Won't show actual QNM curves; will use fake curves for simplicity.



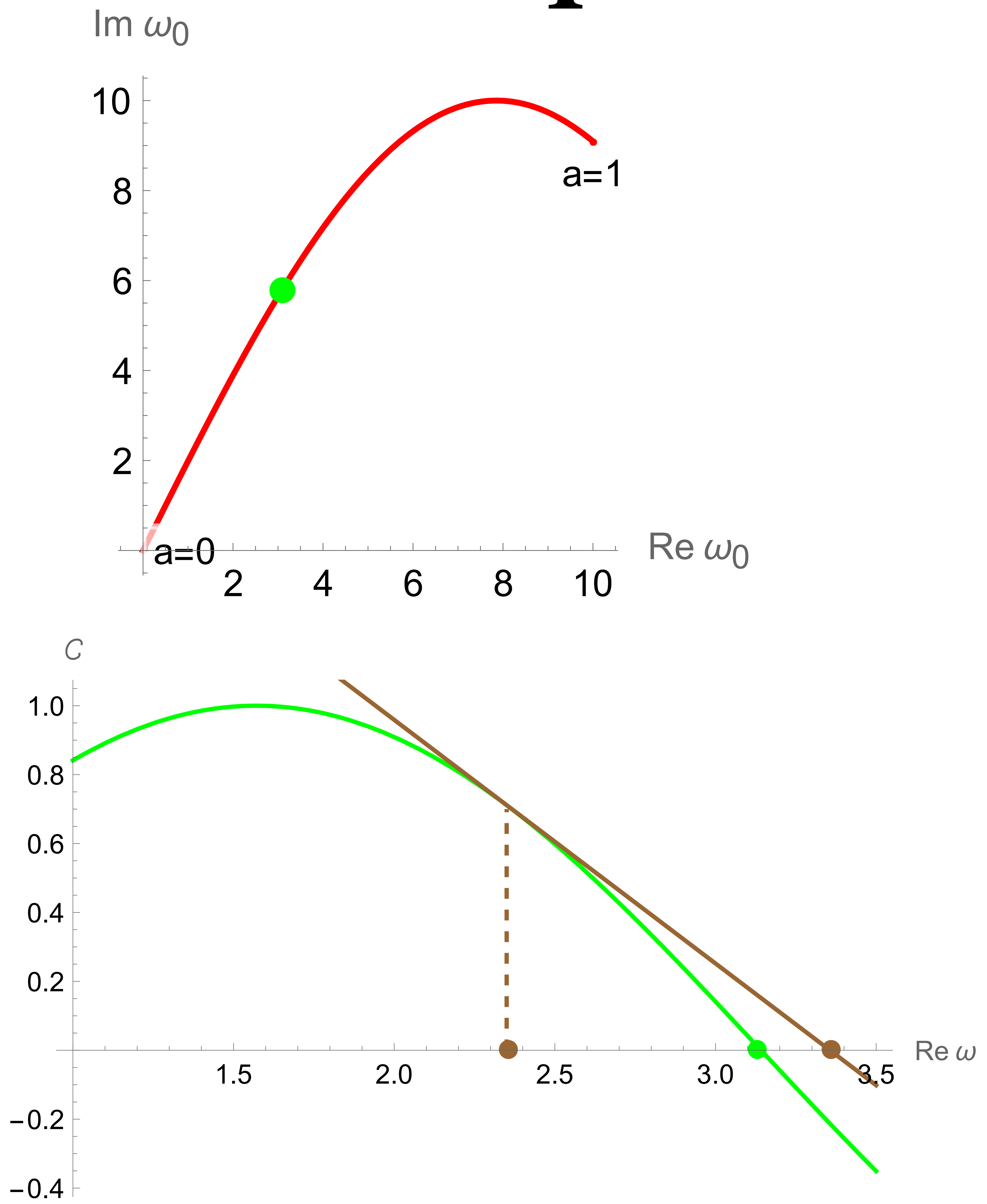
QNM frequency ω_0 as roots of an equation



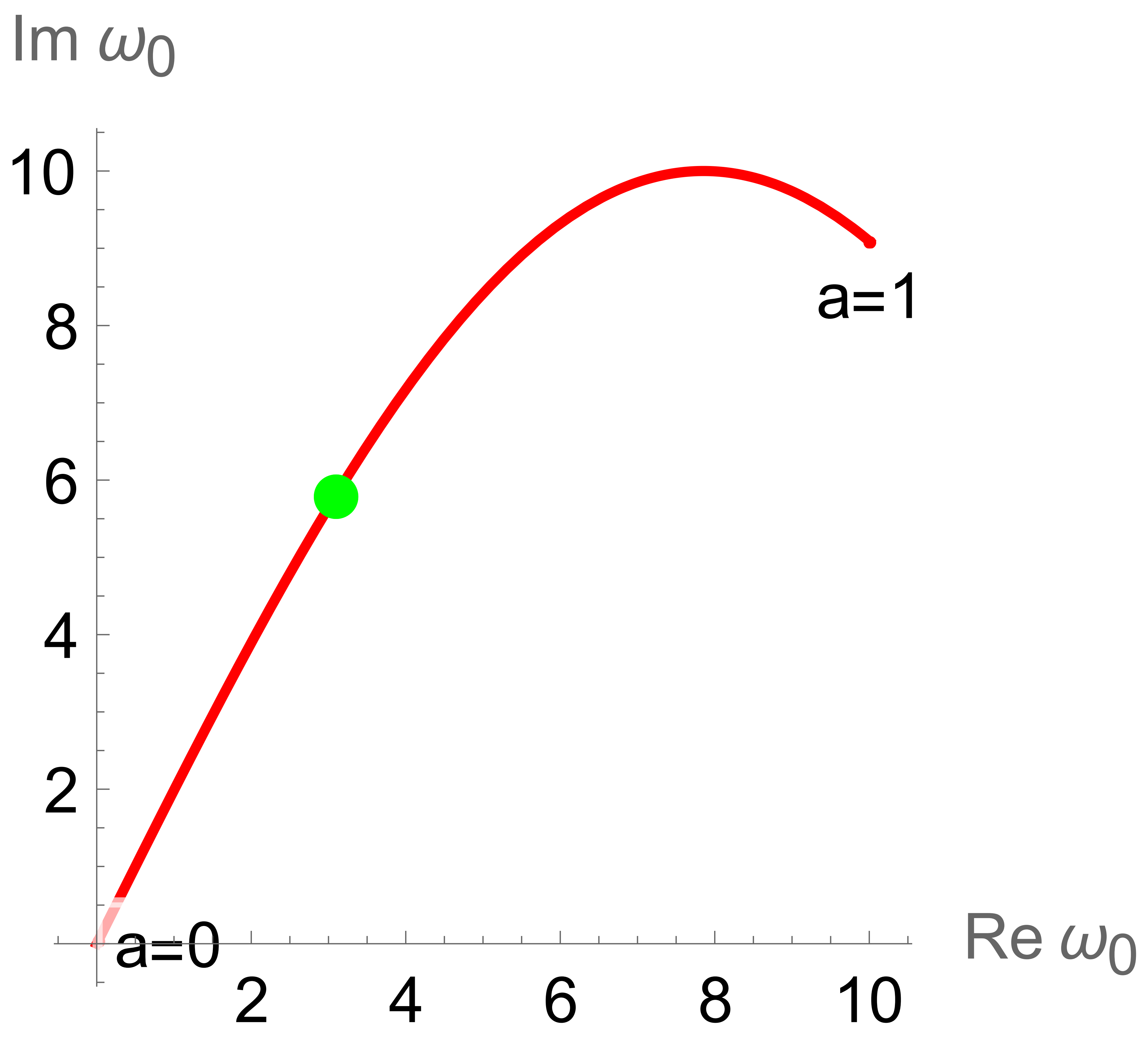


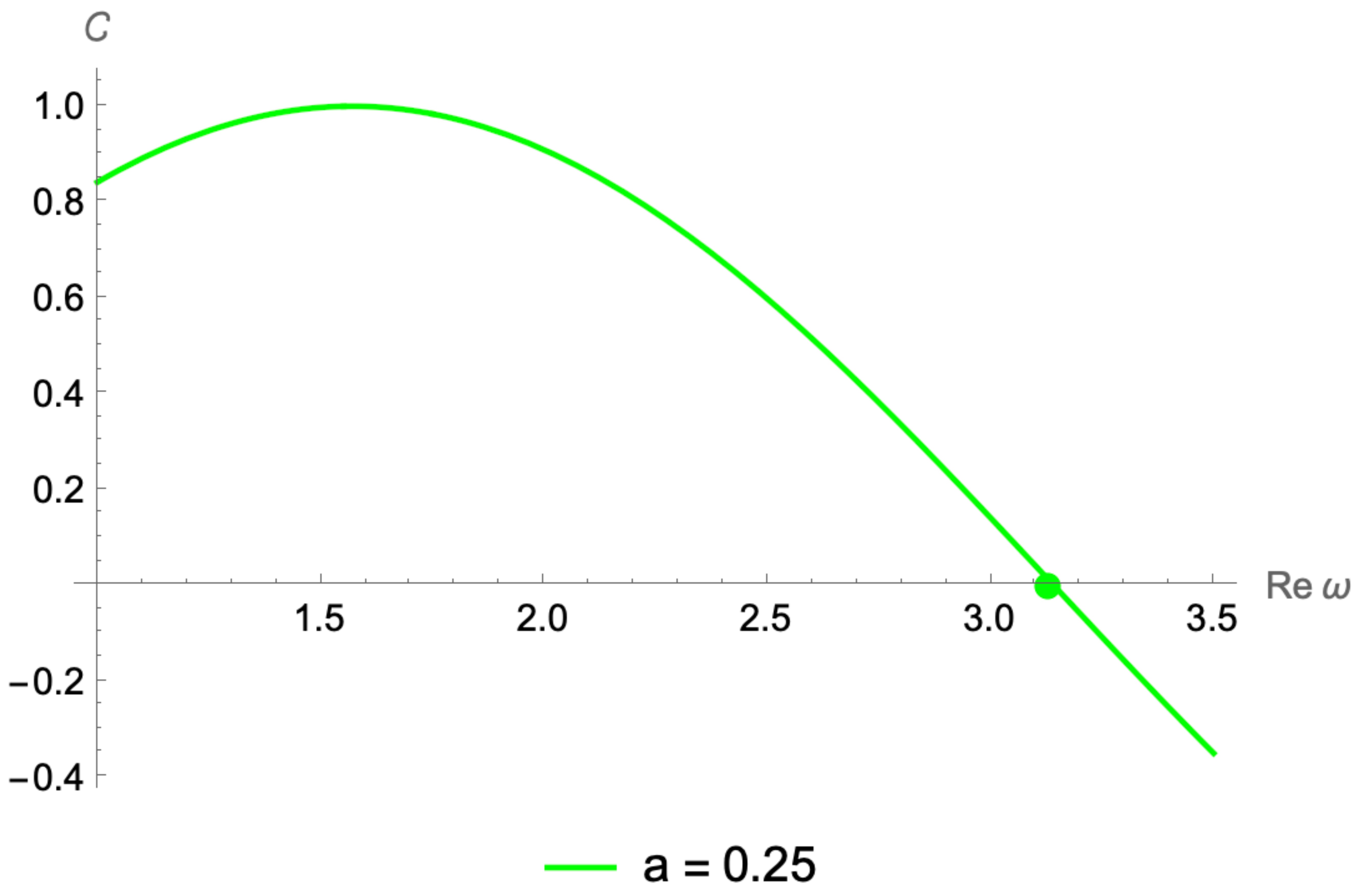
QNM frequency ω_0 as roots of an equation

- ω_0's are solutions of $\mathscr{C}(\omega; a) = 0$ (via Newton-Raphson root finding)

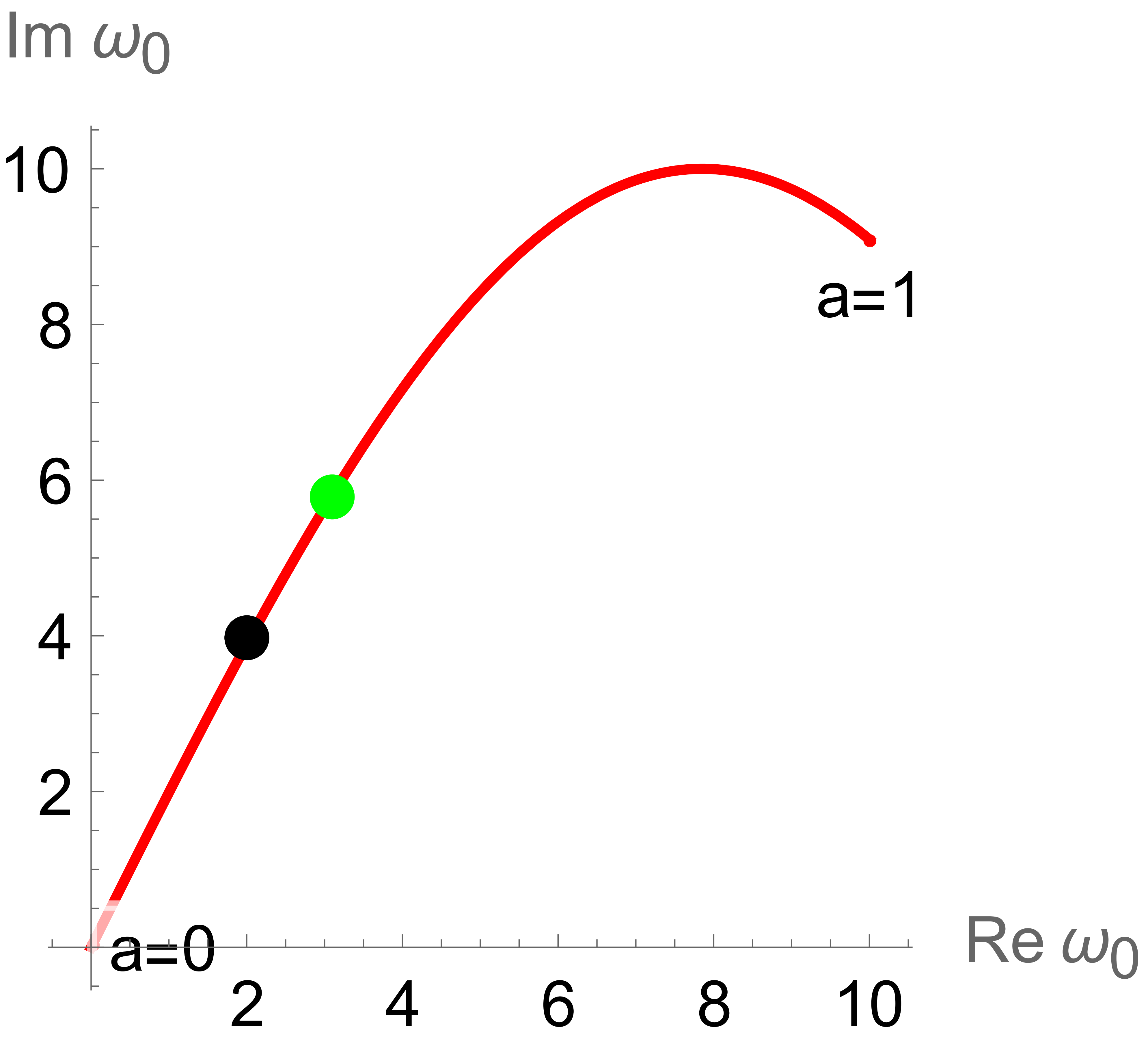


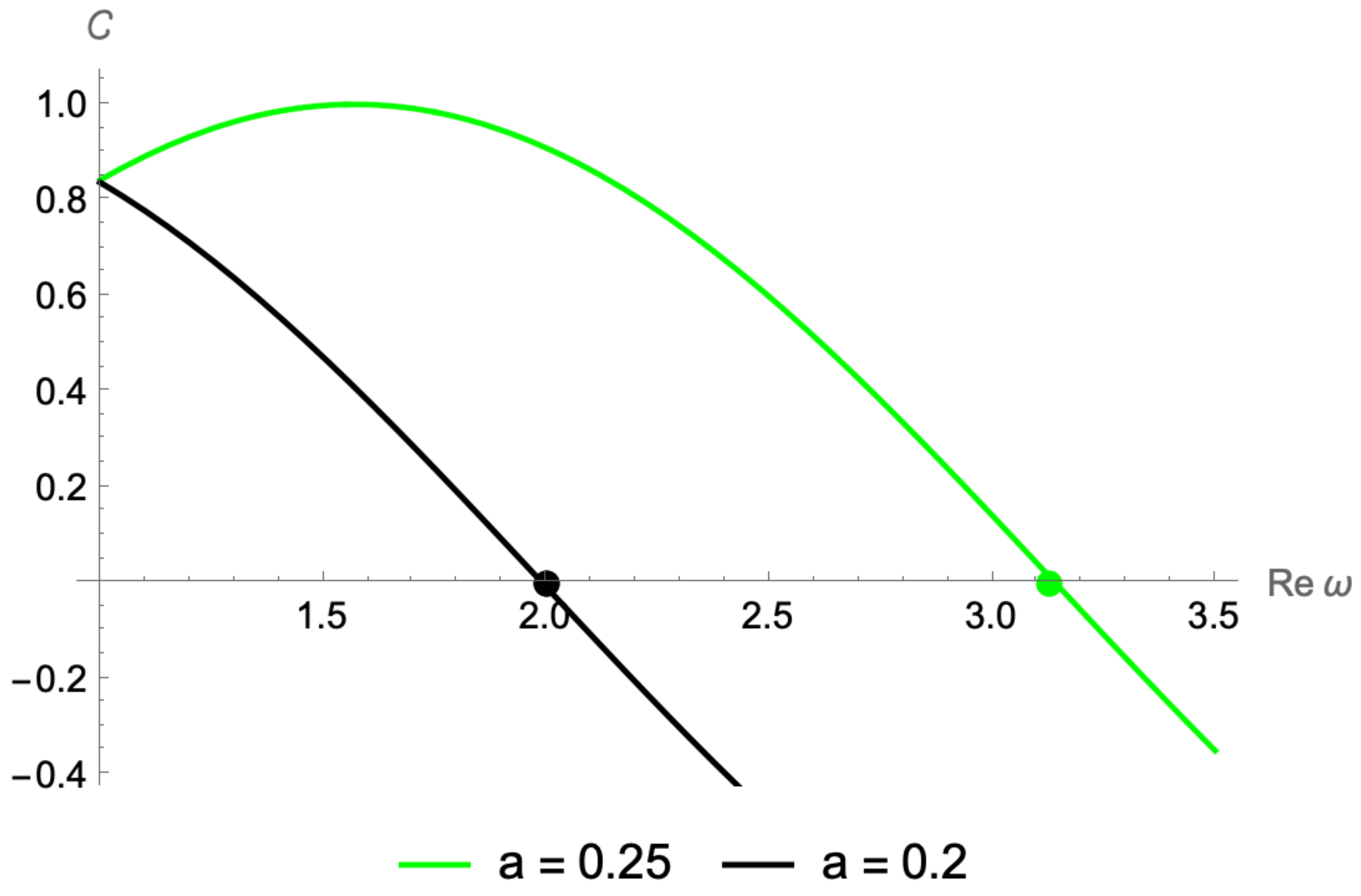
Overall picture



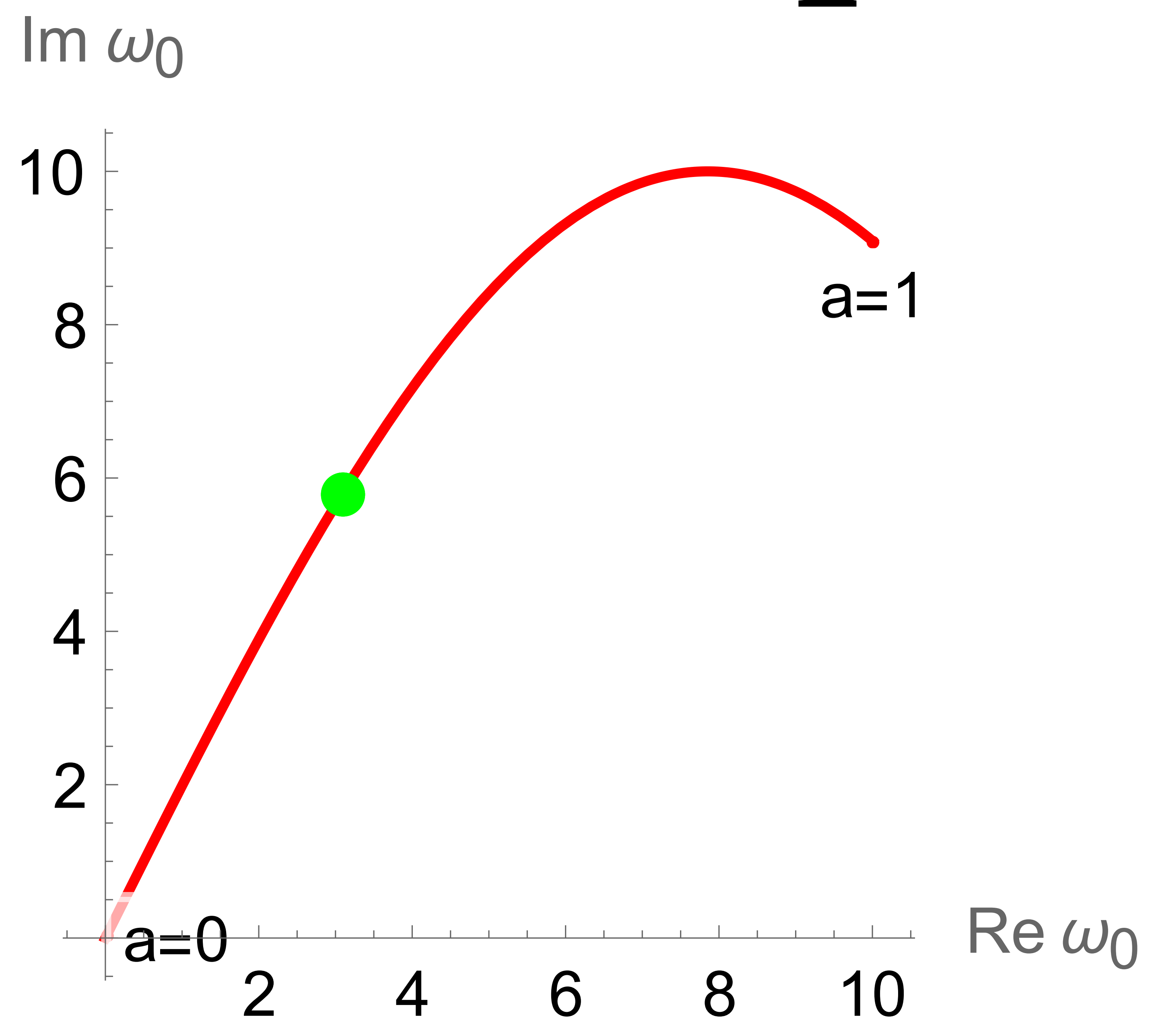


Finding QNM frequencies is a parameterized root-finding problem



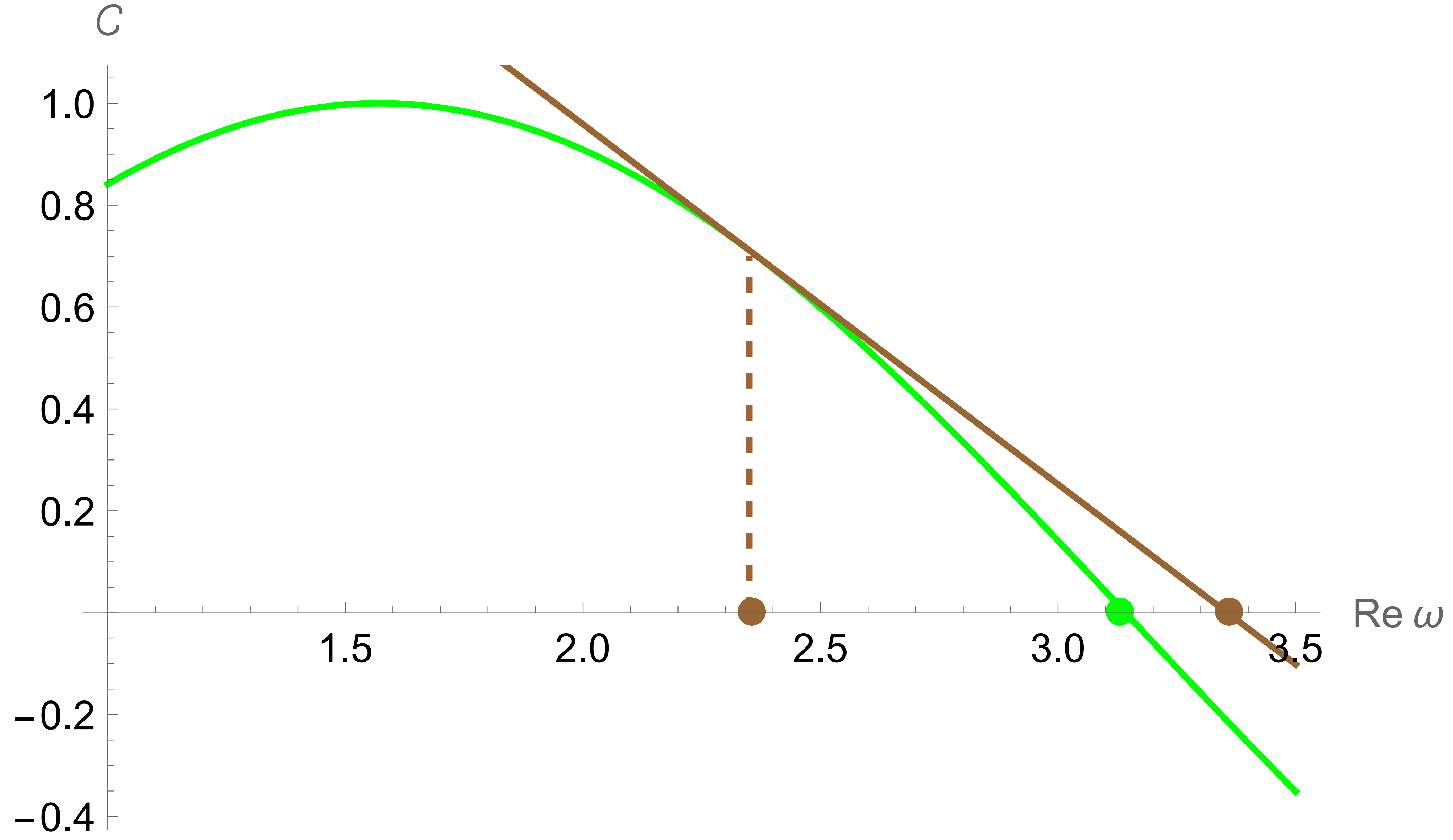


QNM frequency ω_0 as roots of an equation

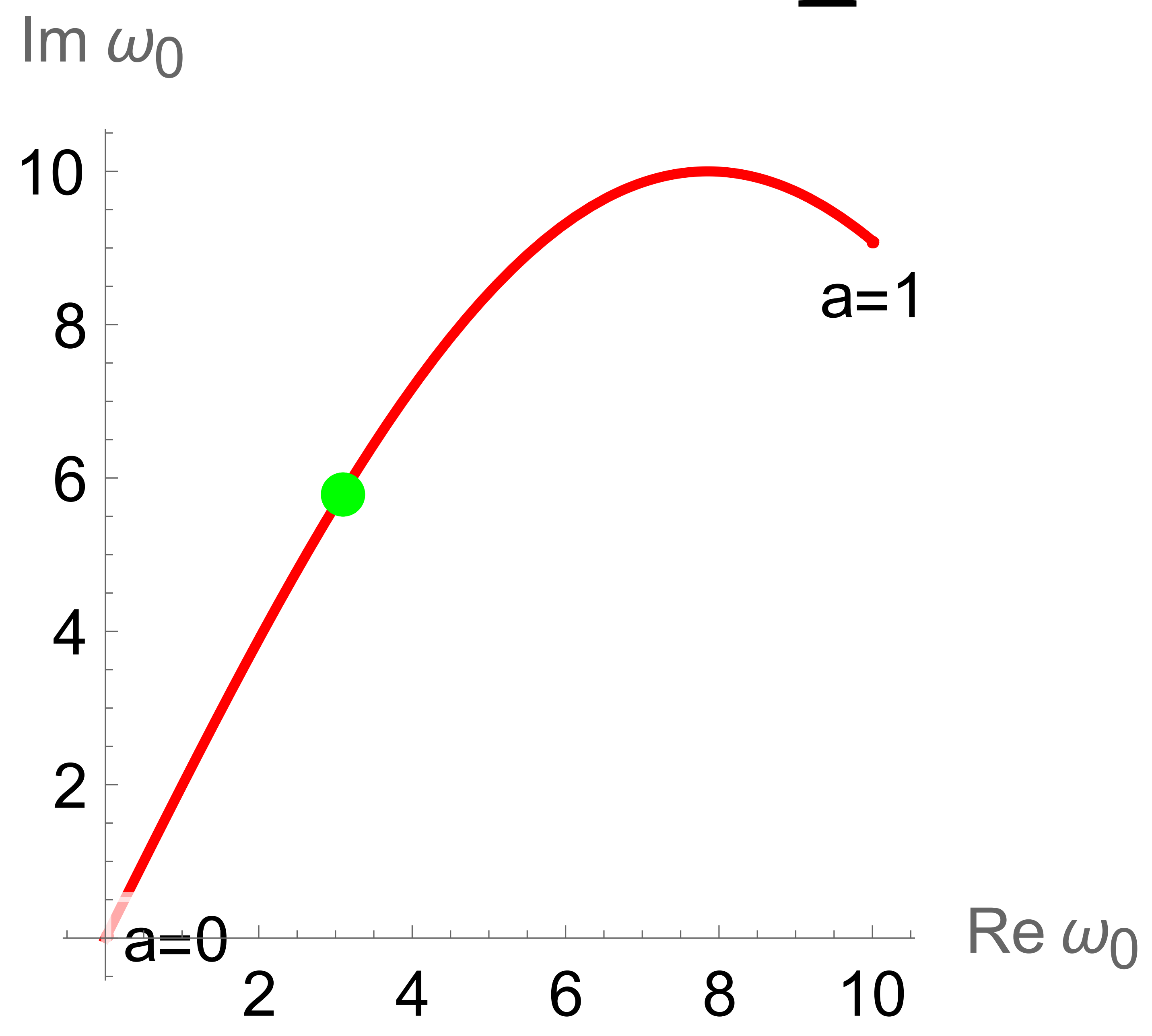


- ω_0's are solutions of $\mathscr{C}(\omega; a) = 0$. (via Newton-Raphson root finding)

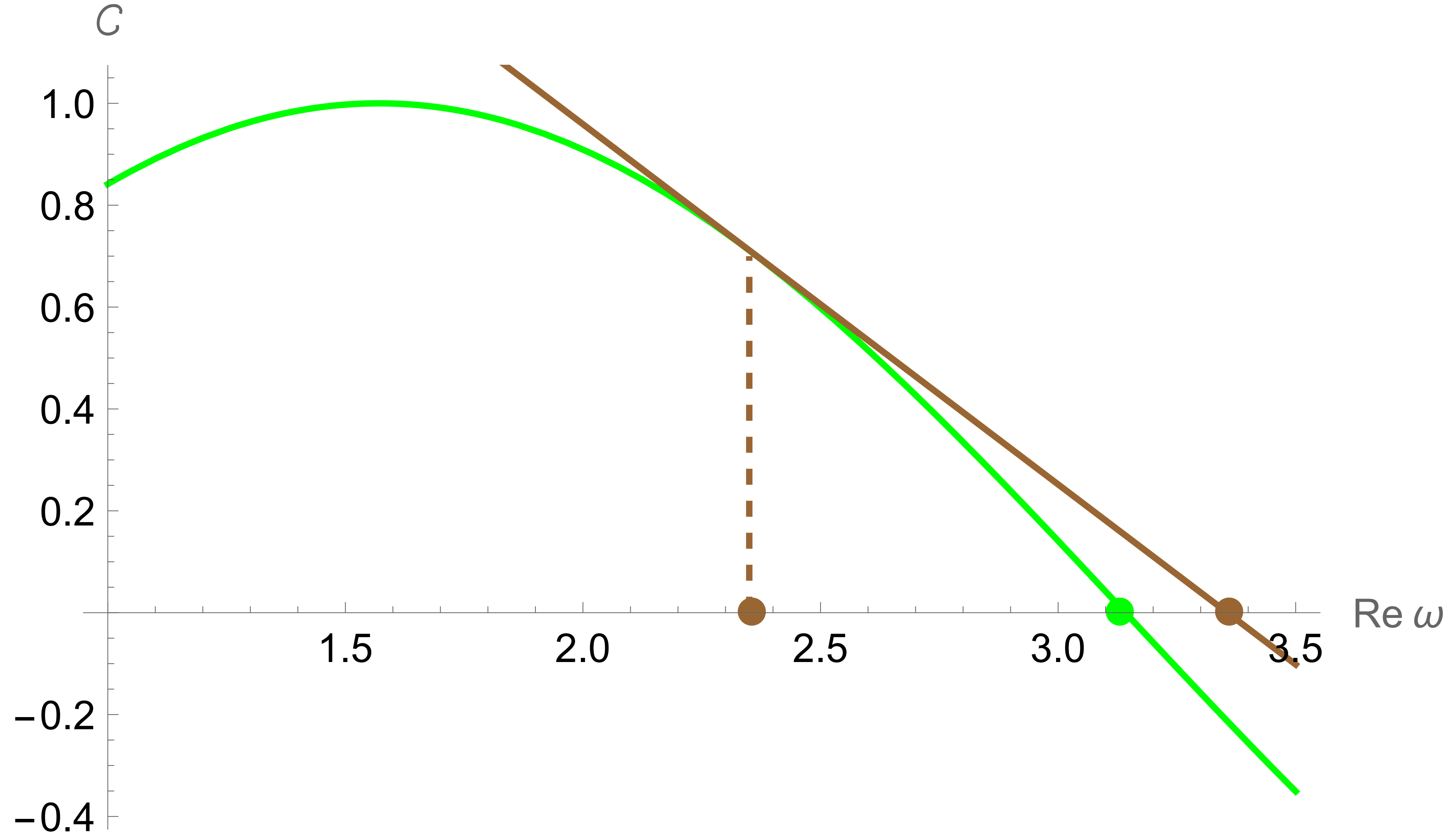
- Finding ω_0's ~ parameterized (by a) numerical root-finding problem.



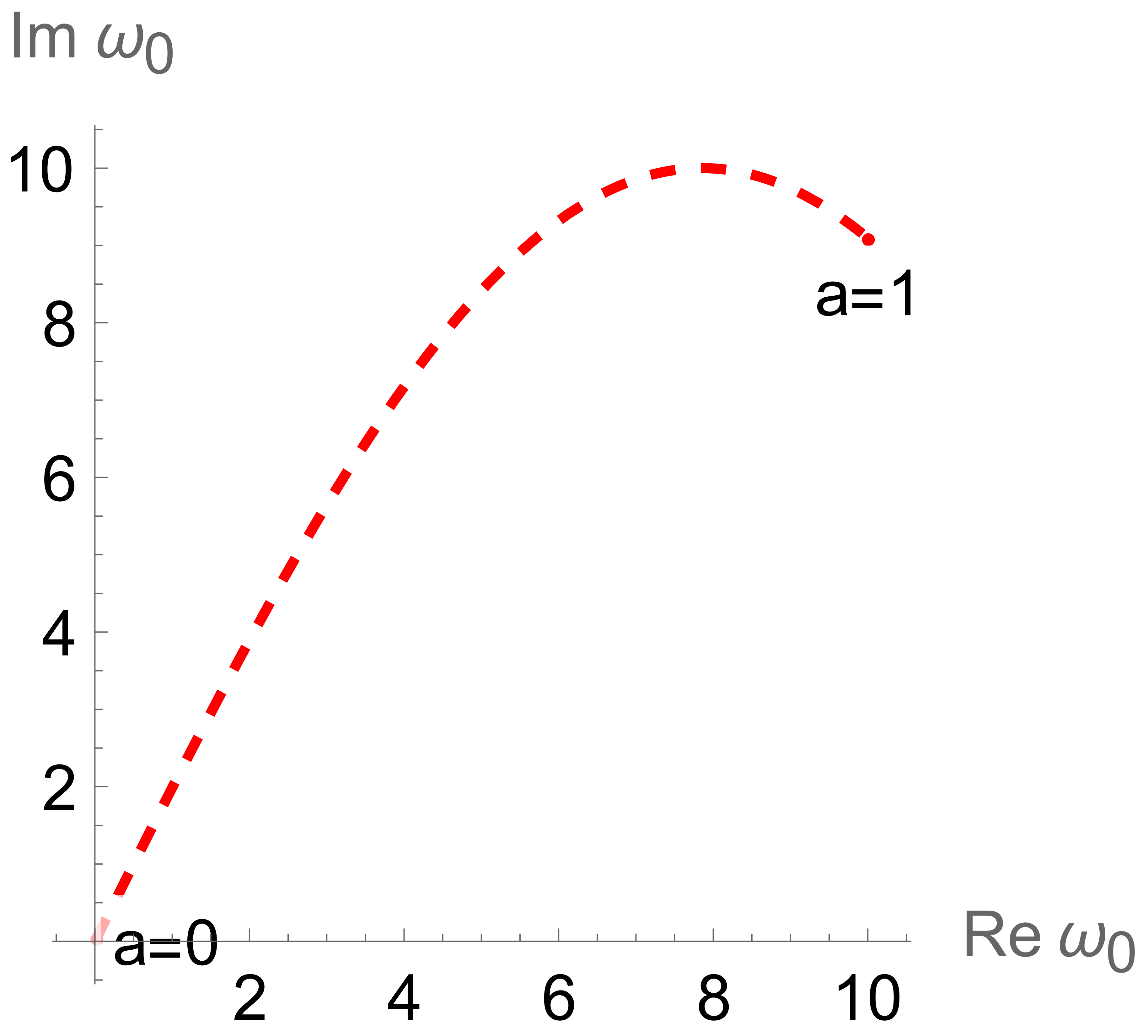
QNM frequency ω_0 as roots of an equation



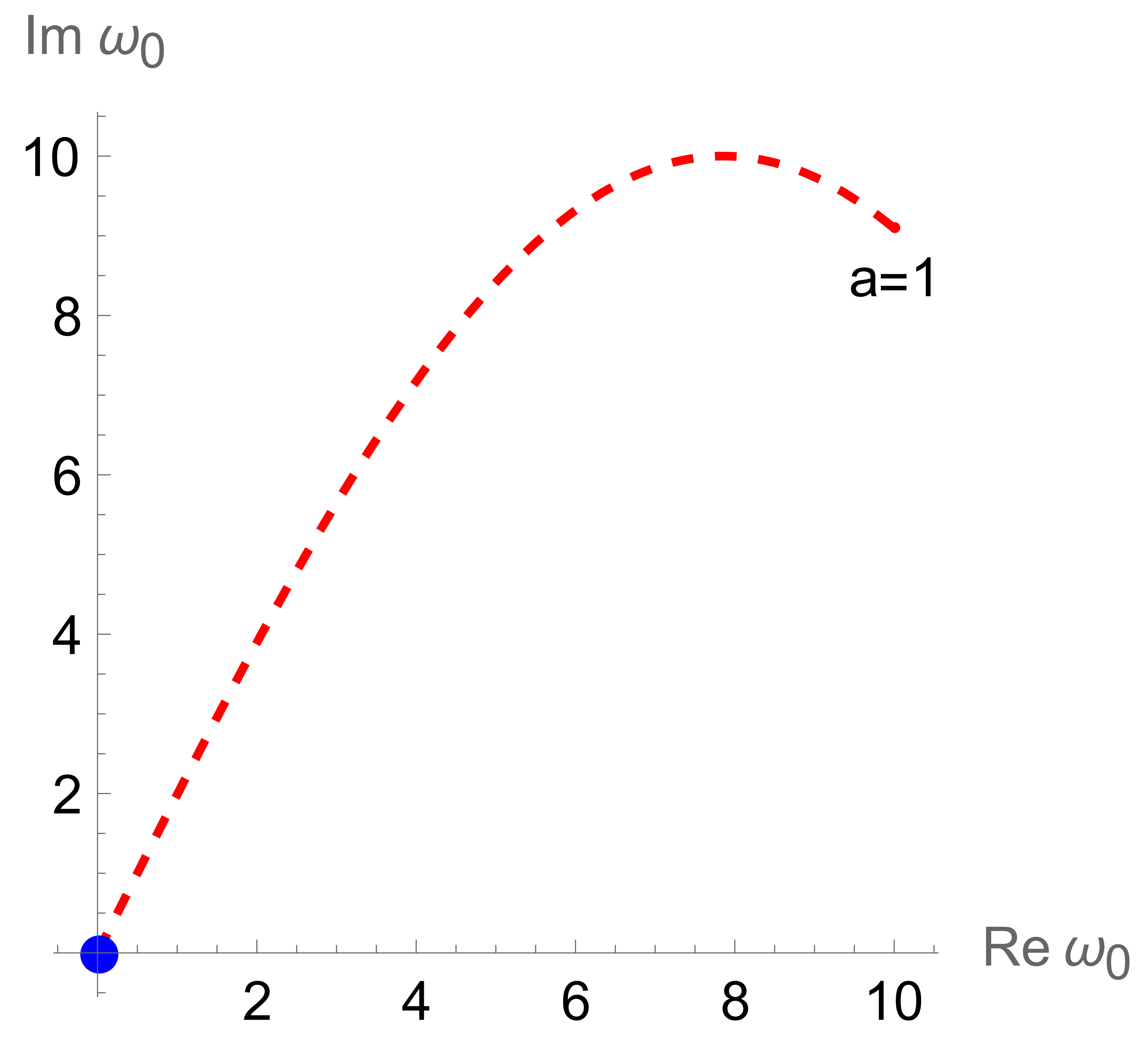
- ω_0's are solutions of $\mathscr{C}(\omega; a) = 0$. (via Newton-Raphson root finding)
- Finding ω_0's $\sim$ parameterized (by a) numerical root-finding problem.
- **VVI:** Distinguish b/w $\mathscr{C}(\omega; a)$ *(bottom)* and ω_0 *(up)*.



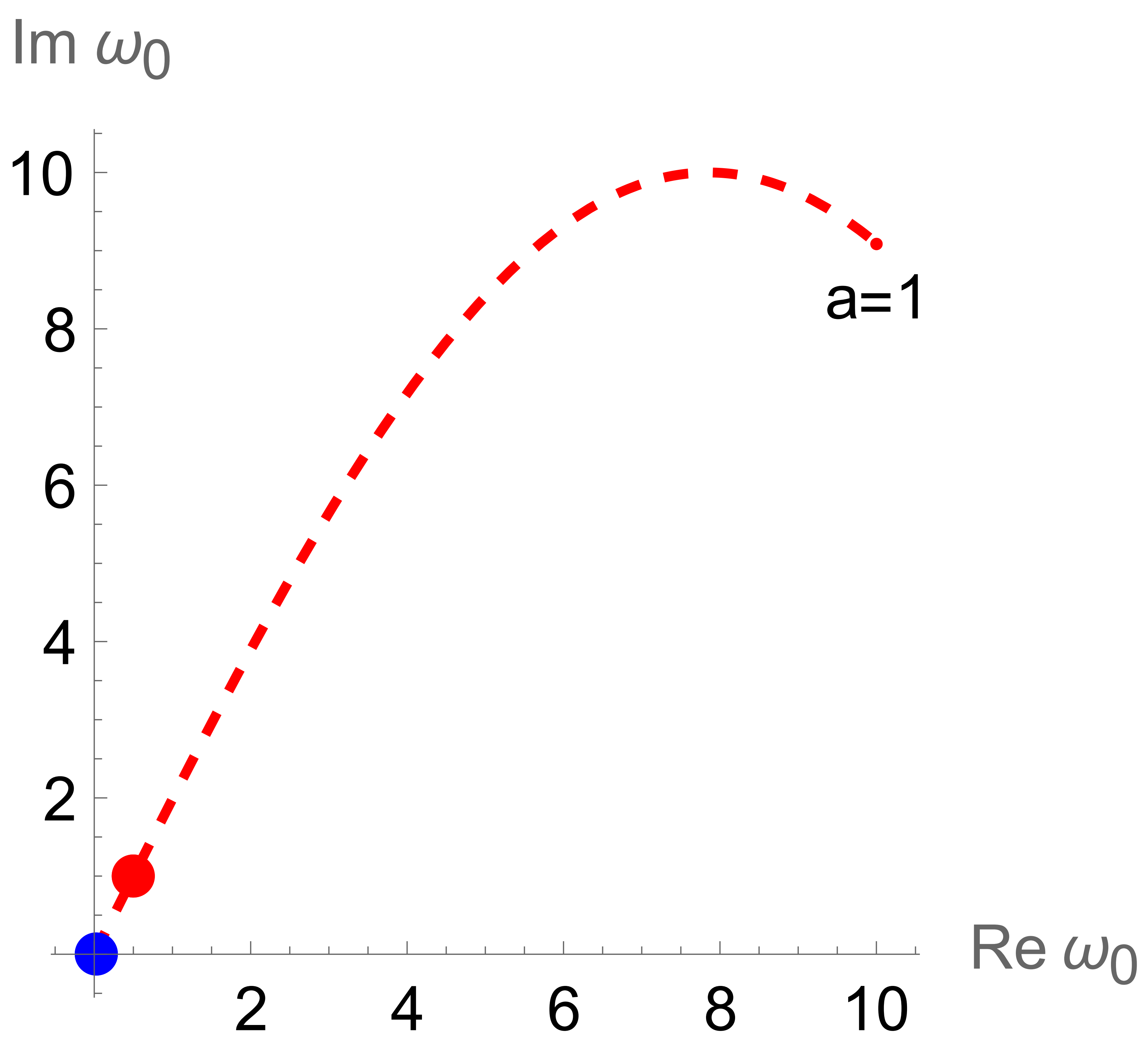
Using $\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$



Using $\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

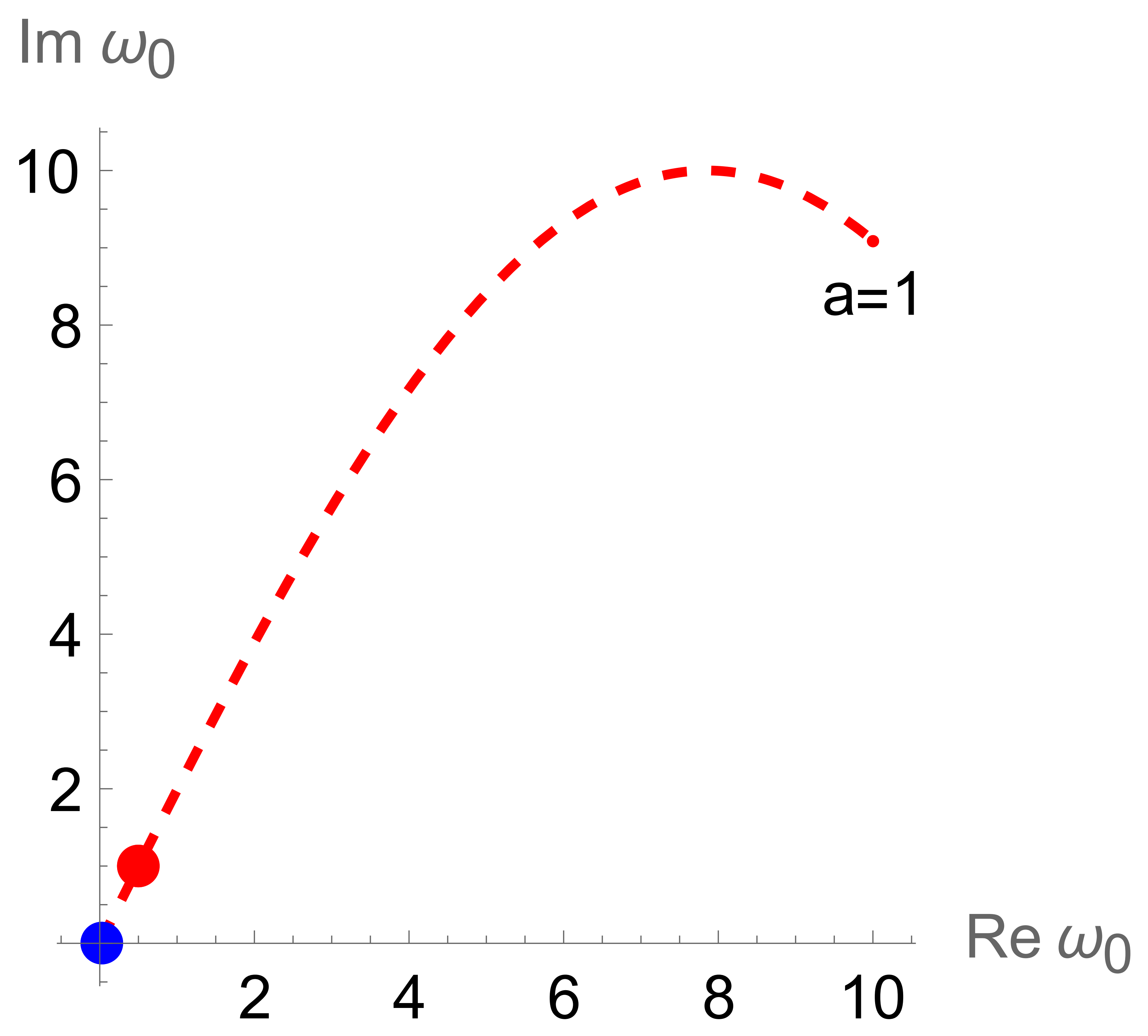


Using $\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$



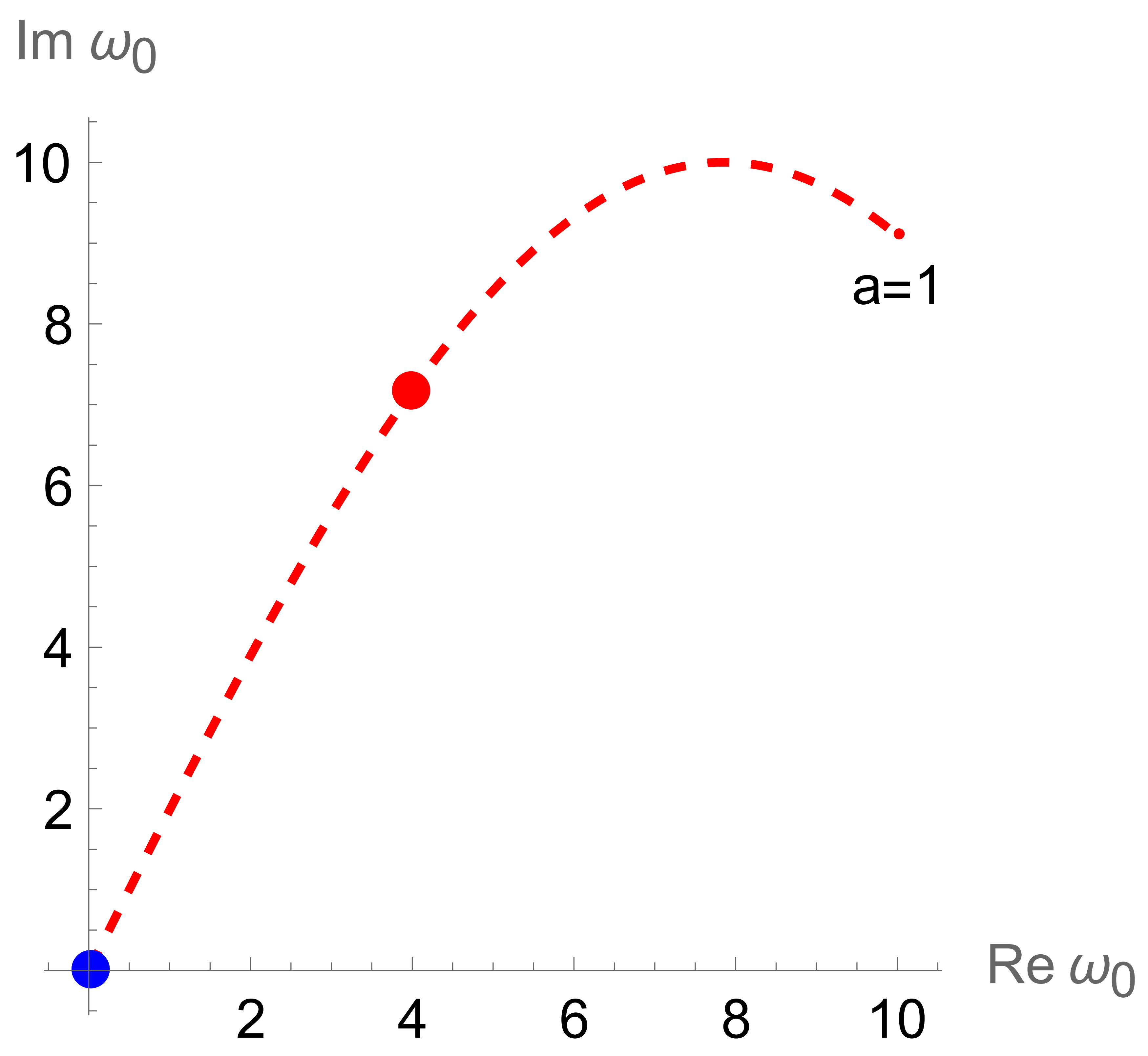
Using $\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.



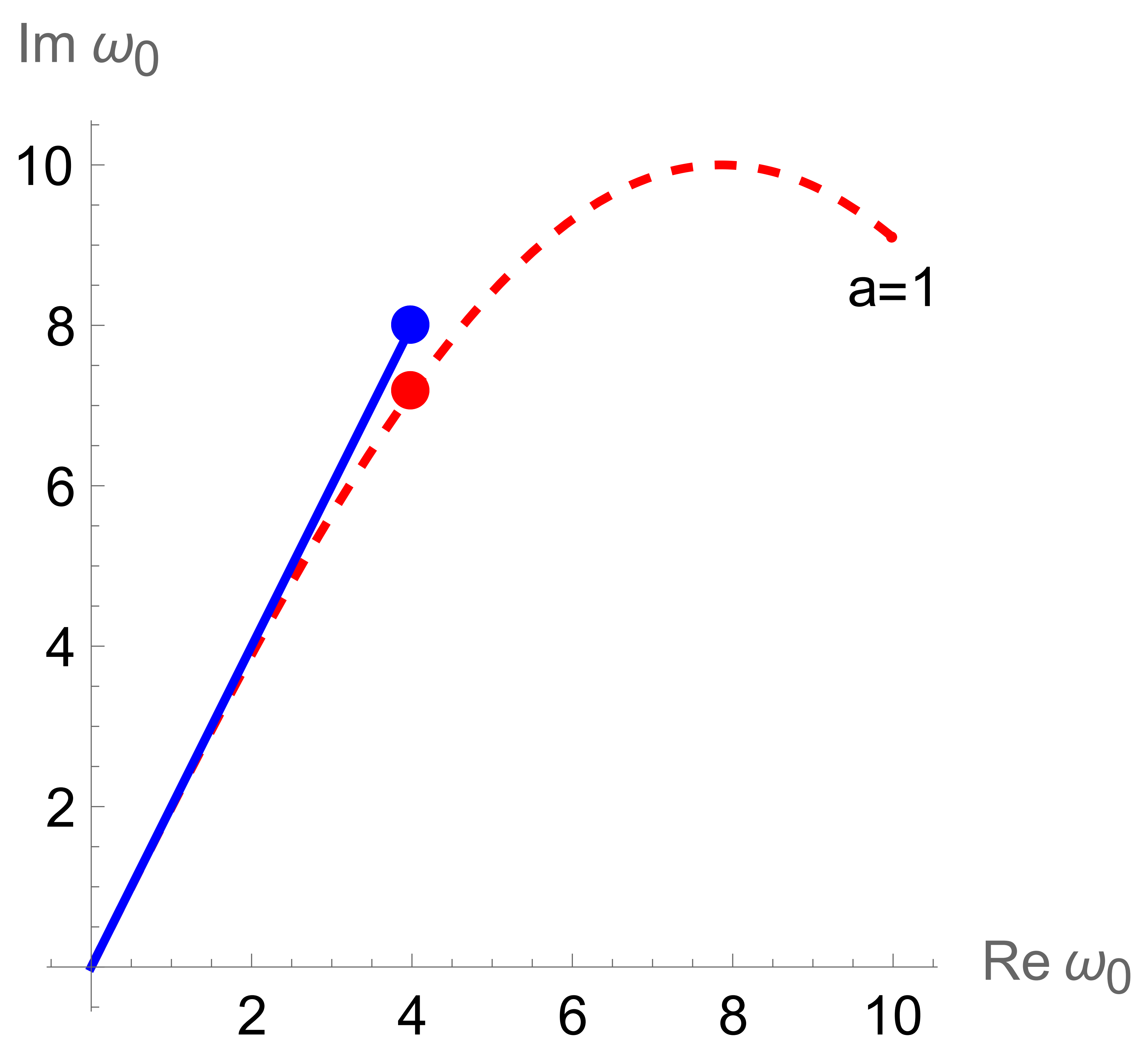
Using $\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\implies$ take small steps ($da \sim 0.02$) in BH spin a.



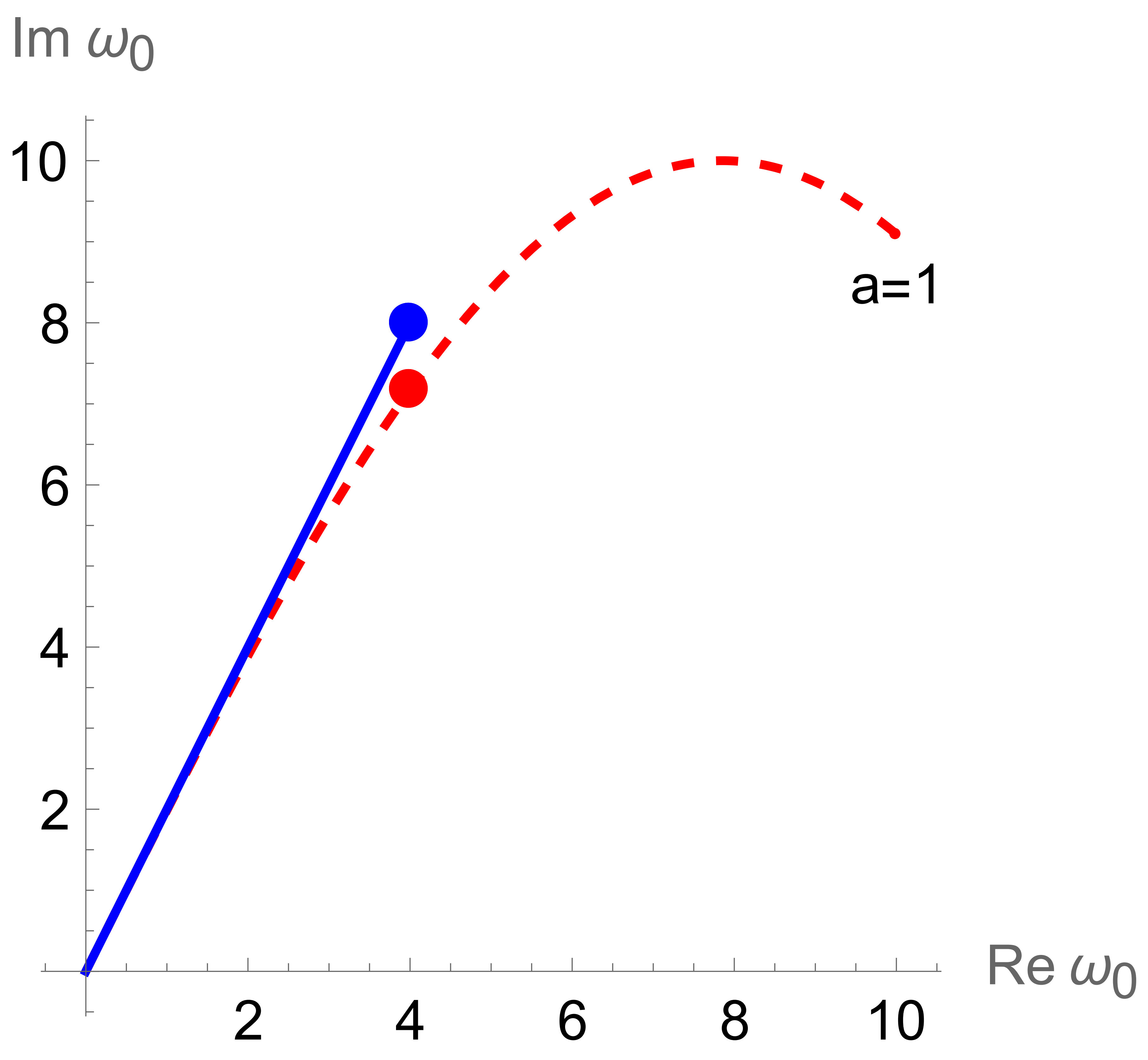
Using $\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\implies$ take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.



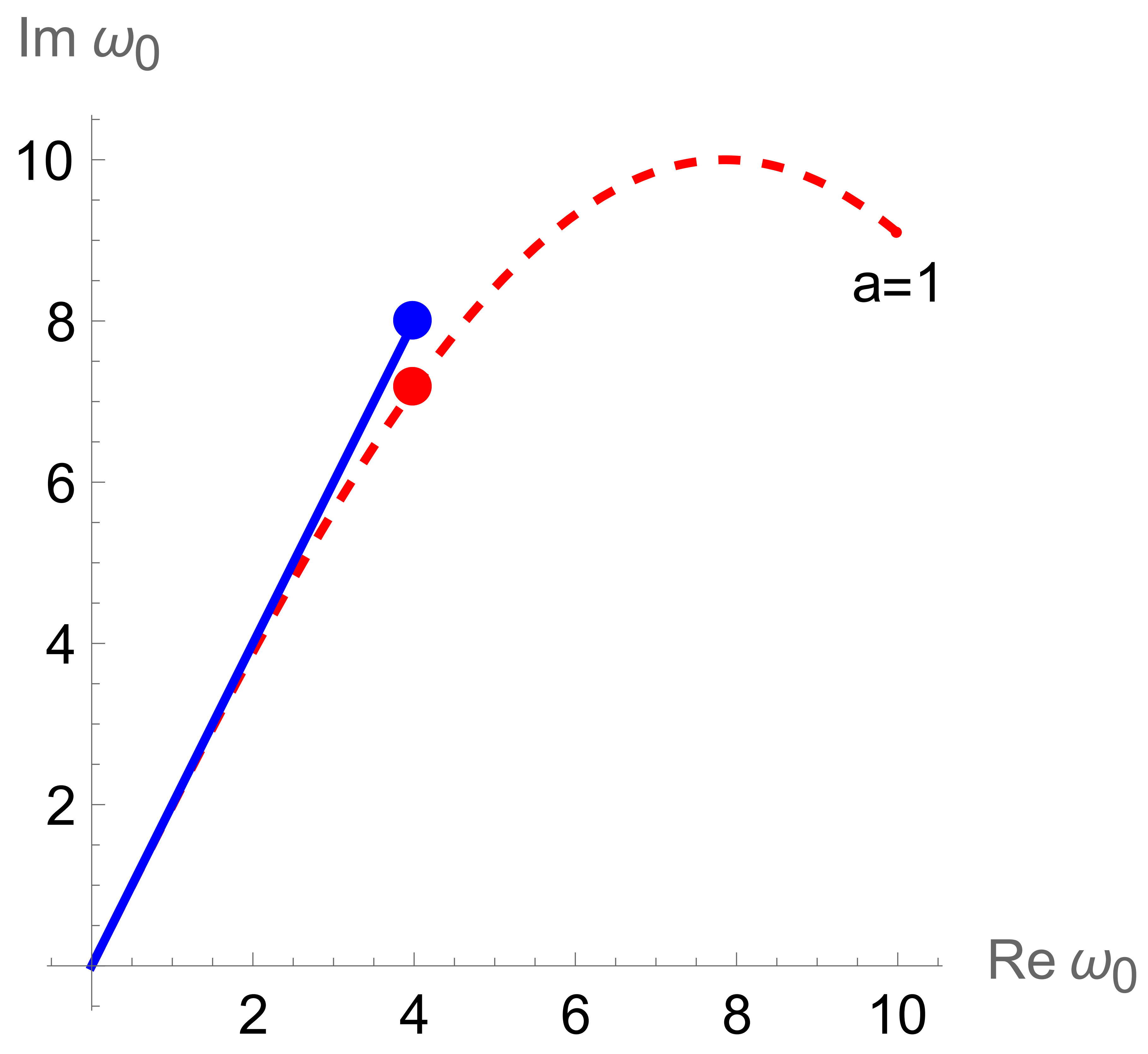
Using $\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\implies$ take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.



Results

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\Longrightarrow$ take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.
- We provide $d\omega_0/da$ analytically.

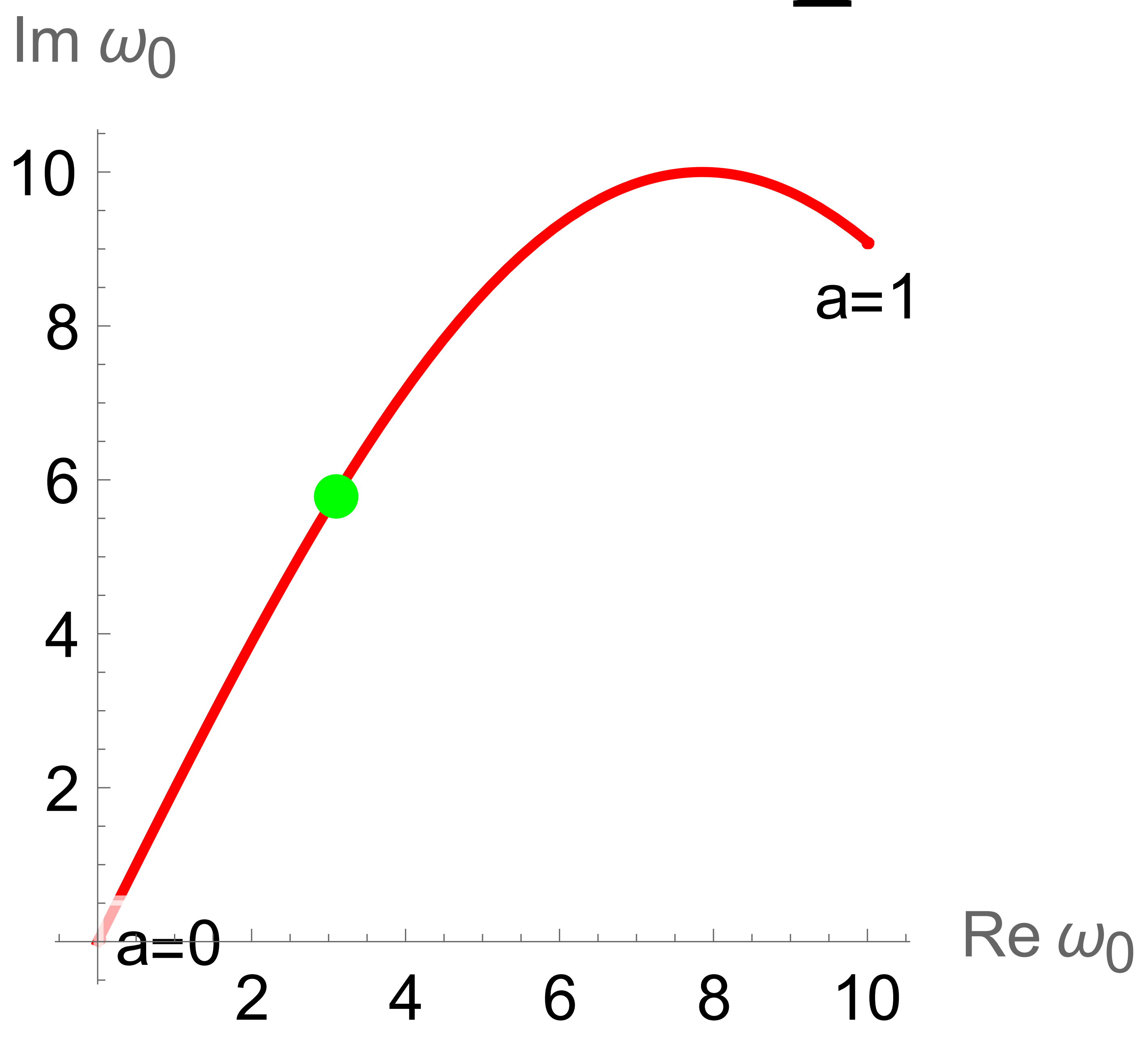


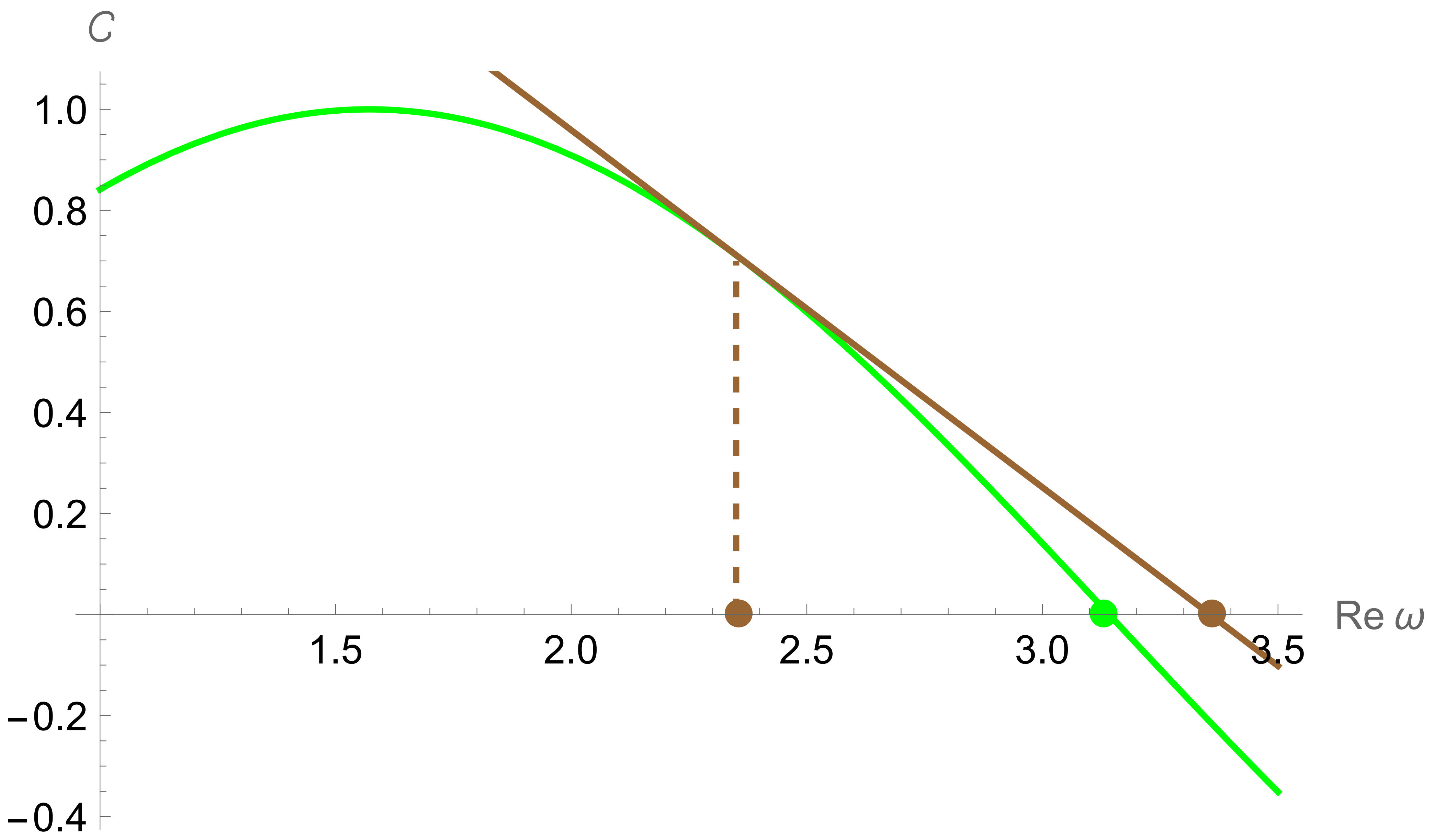
QNM frequency ω_0 as roots of an equation

- ω_0's are solutions of $\mathscr{C}(\omega; a) = 0$. (via Newton-Raphson root finding)

- Finding ω_0's ~ parameterized (by a) numerical root-finding problem; $0 < a < 1$.

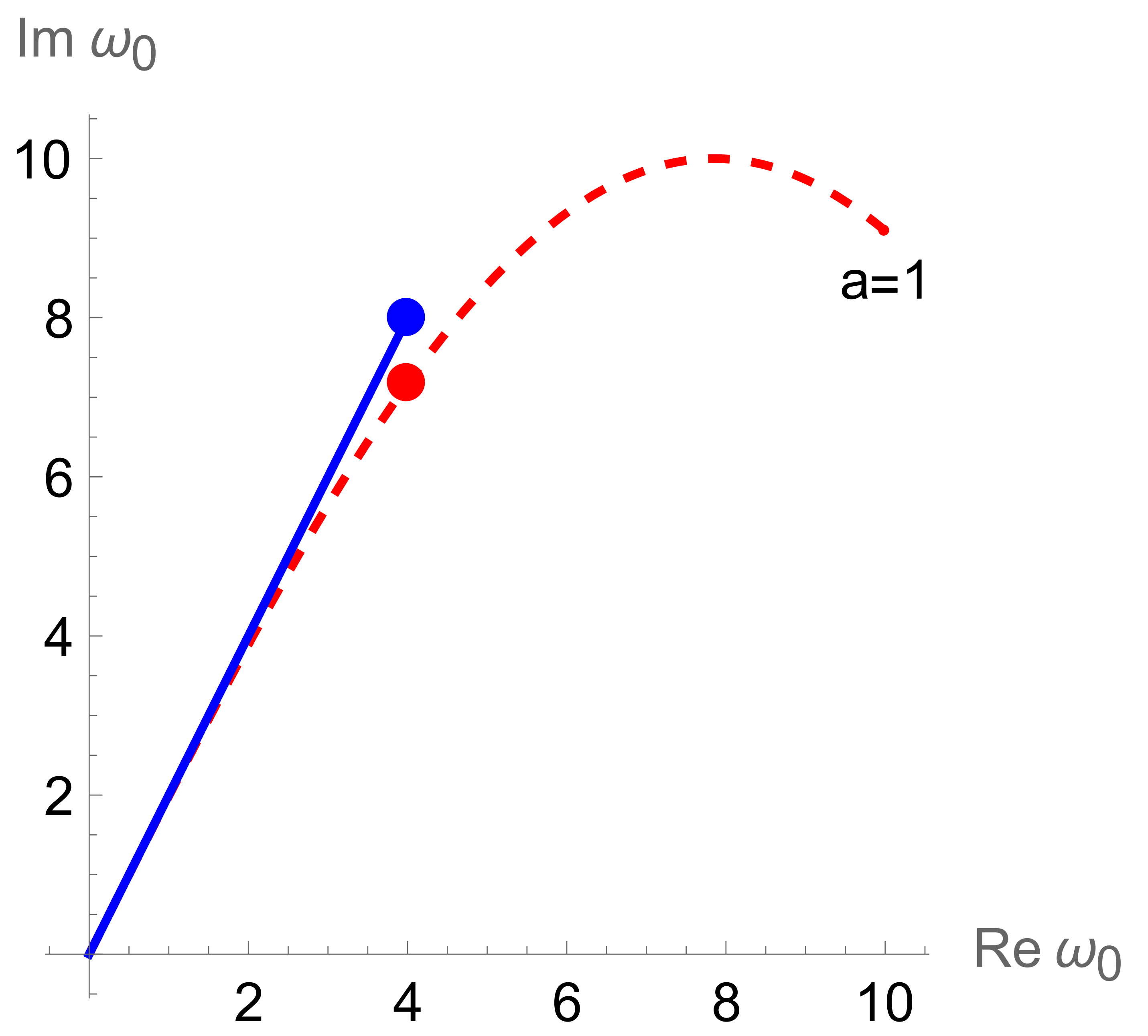
- **VVI:** Distinguish b/w $\mathscr{C}(\omega; a)$ *(bottom)* and ω_0 *(up)*.

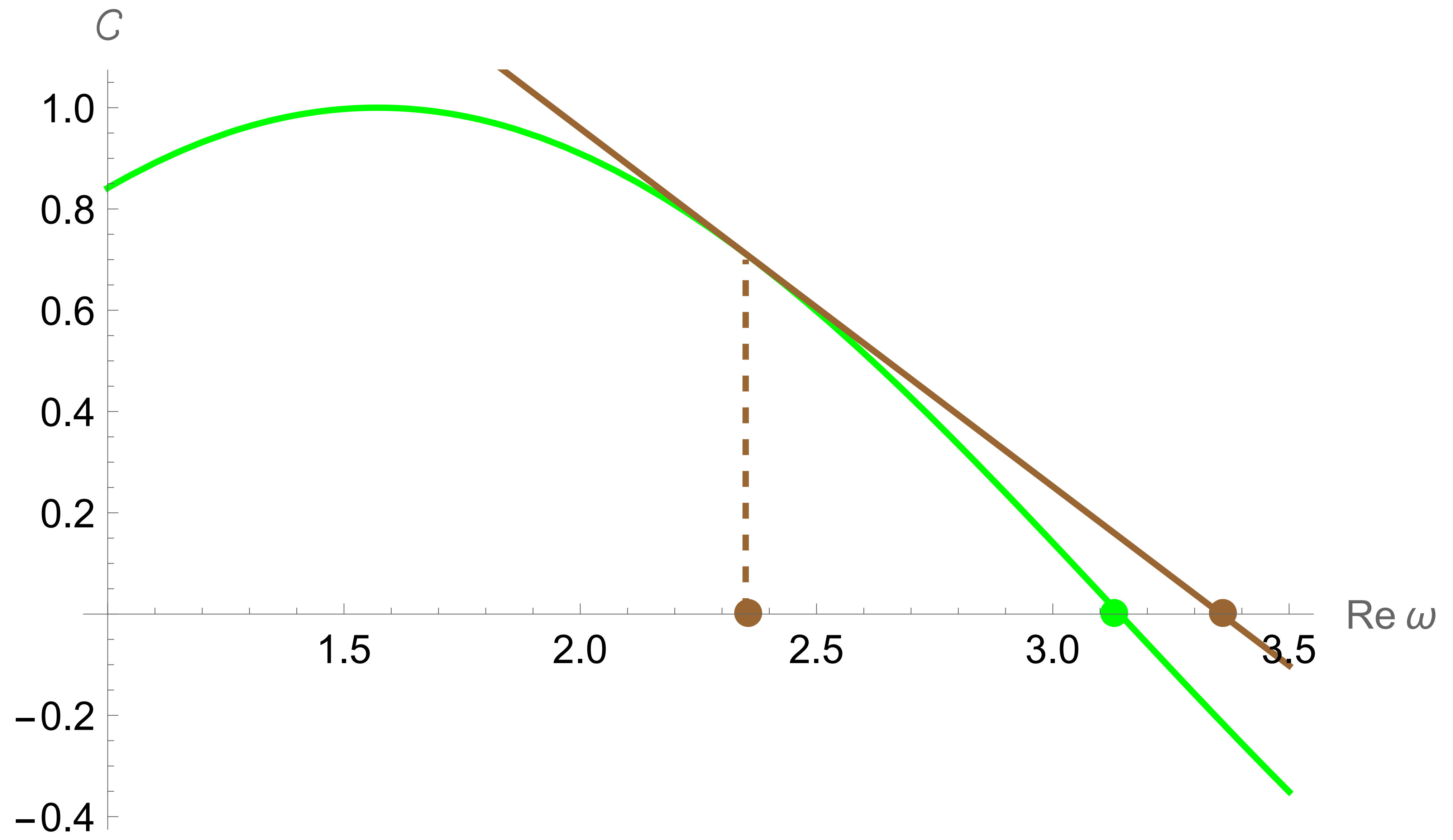




Results

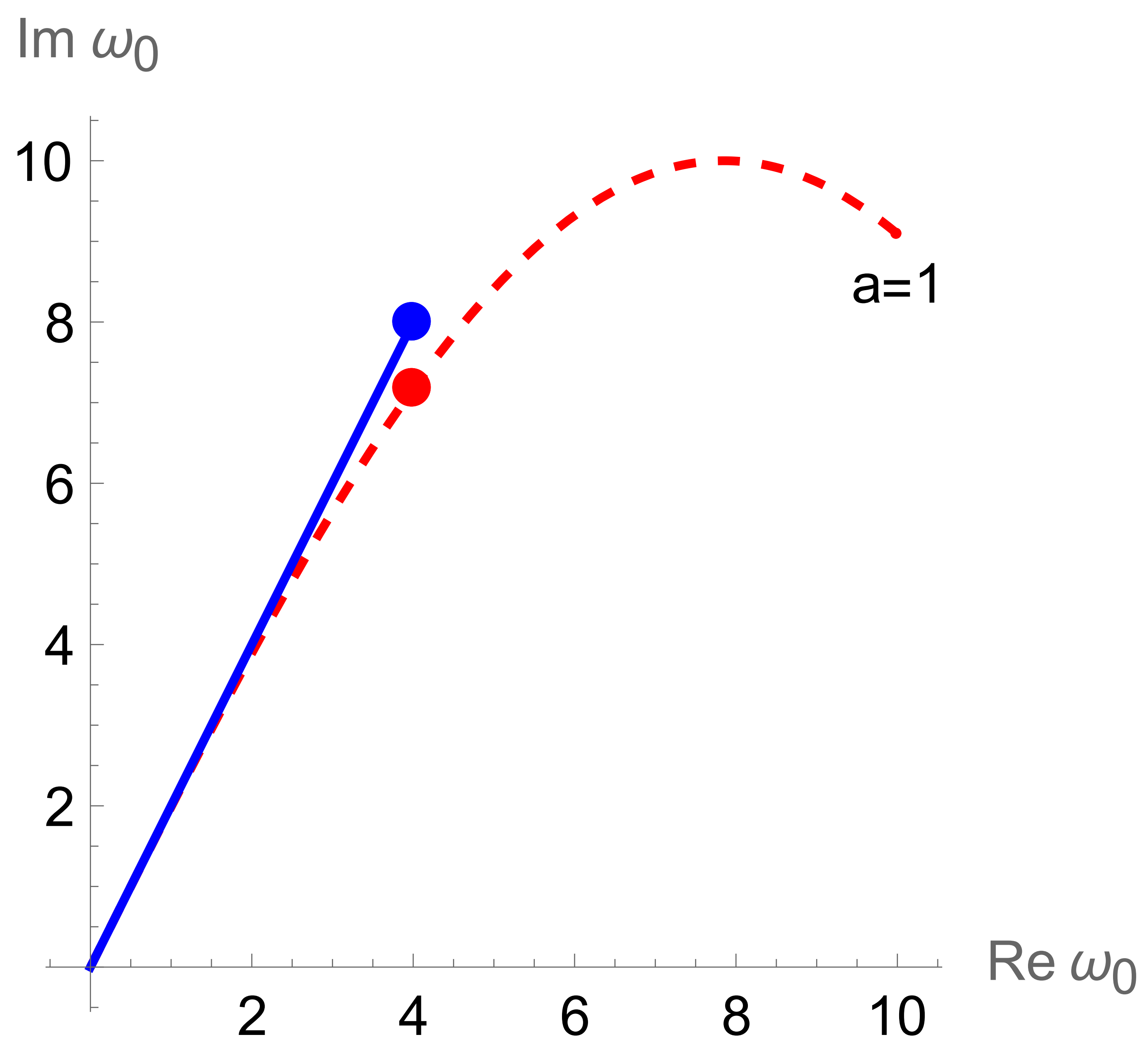
- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a+da)$.
- Root can't be found if the guess is too far $\Longrightarrow$ take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.
- We provide $d\omega_0/da$ analytically.
- We provide $d\mathscr{C}/d\omega$ for Newton-Raphson analytically.

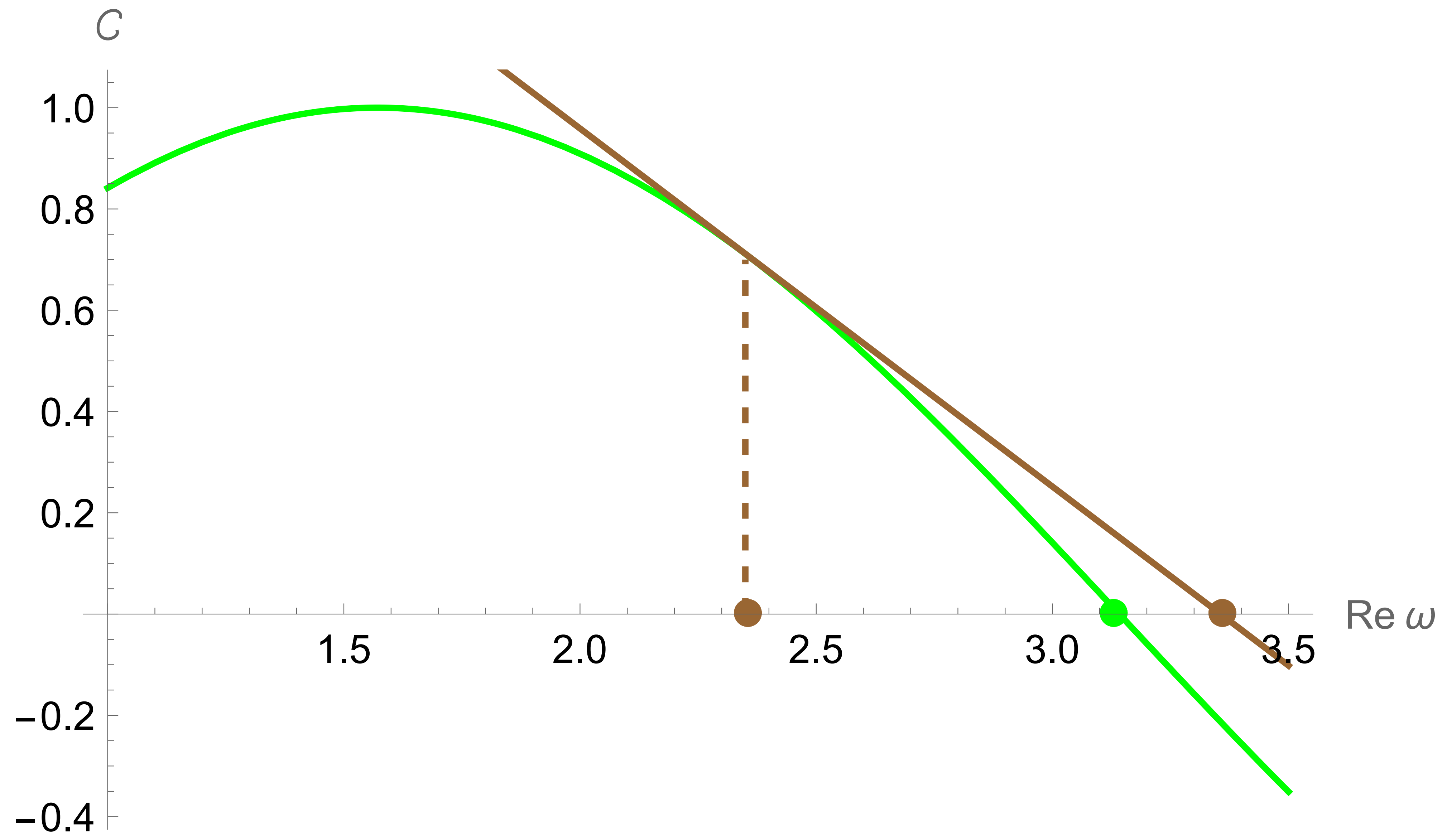




Results

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\Longrightarrow$ take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.
- We provide $d\omega_0/da$ analytically.
- We provide $d\mathscr{C}/d\omega$ for Newton-Raphson analytically.
- **Why analytical derivatives?** "Applied uncritically, the above procedure is almost guaranteed to produce inaccurate results": *Numerical Recipes in C; Sec. 5.7*





Details

Details

- Integration could be hard; isn’t differentiation trivial?

Details

- Integration could be hard; isn't differentiation trivial?
- Computing $d\mathscr{C}/d\omega$ *naively* is expensive and inefficient.

Details

- Integration could be hard; isn't differentiation trivial?
- Computing $d\mathscr{C}/d\omega$ *naively* is expensive and inefficient.
- $$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}$$

Details

- Integration could be hard; isn't differentiation trivial?
- Computing $d\mathscr{C}/d\omega$ *naively* is expensive and inefficient.
- $$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}$$
- **Problem:** We don't know a priori how many fractions to keep.

Details

- Integration could be hard; isn't differentiation trivial?
- Computing $d\mathscr{C}/d\omega$ *naively* is expensive and inefficient.
- $$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}$$
- **Problem:** We don't know a priori how many fractions to keep.
- Incorporated in qnm (Python package by Leo Stein *arXiv: 1908.10377*).

Summary

- Provided derivative information ($d\omega_0/da$ & $d\mathscr{C}/d\omega$) to make QNM frequency computation more efficient.
- $d\omega_0/da$: lets us take larger step sizes $da \sim 0.02 \to 0.25$.
- **Future work:** Calculate and incorporate $d^2\omega_0/da^2$; can let us take $da \sim 0.65$.
- **Future work:** Apply this method beyond Kerr QNMs (within GR) and beyond GR.
- **Preprint:** *arXiv: 1908.10377.*