Making Kerr quasinormal mode frequency computation robust

Sashwat Tanay, Leo Stein (Univ. of MS)

APS April meeting 2023

stanay@olemiss.edu

Background

Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).

Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).
- Final Kerr BH oscillates at the quasinormal mode (QNM) frequencies.

Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).
- Final Kerr BH oscillates at the quasinormal mode (QNM) frequencies.
- Determining QNM frequencies is essential for GW data analysis.

Background

- **Physical system:** A spinning (Kerr) BH in the ringdown stage (result of a BBH merger).
- Final Kerr BH oscillates at the quasinormal mode (QNM) frequencies.
- Determining QNM frequencies is essential for GW data analysis.
- **Objective:** work towards improving the spectral variants of Leaver's method of *arXiv: 1410.7698 (Cook & Zalutskiy)* and *arXiv: 1908.10377 (Leo Stein)*.

Quasinormal mode frequencies

Quasinormal mode frequencies

Quasinormal mode frequencies

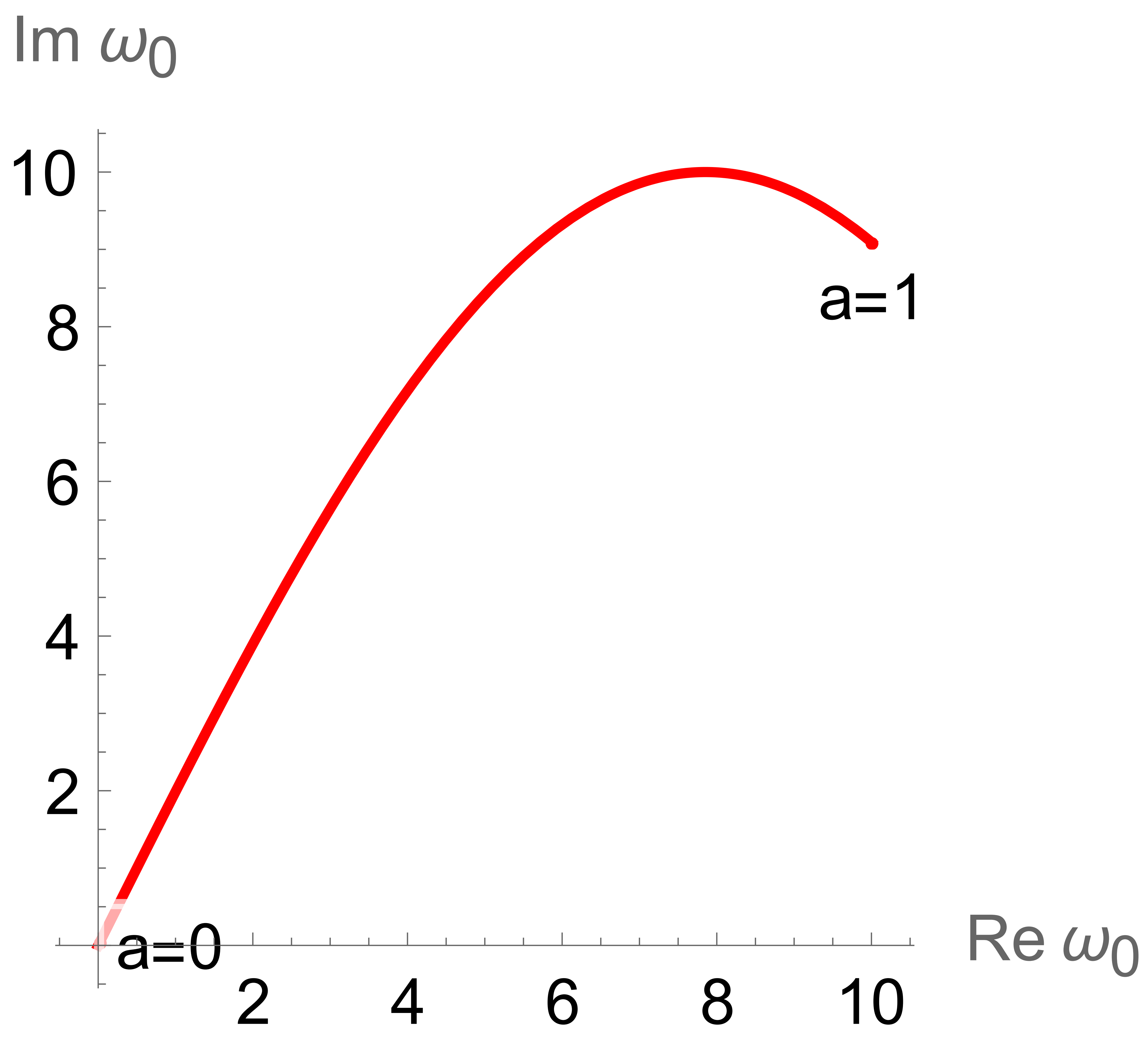
- **Notation:** ω = QNM frequency.
  $a =$ BH spin.

Quasinormal mode frequencies

- **Notation:** ω = QNM frequency.
  $a =$ BH spin.
- QNM frequencies are complex.

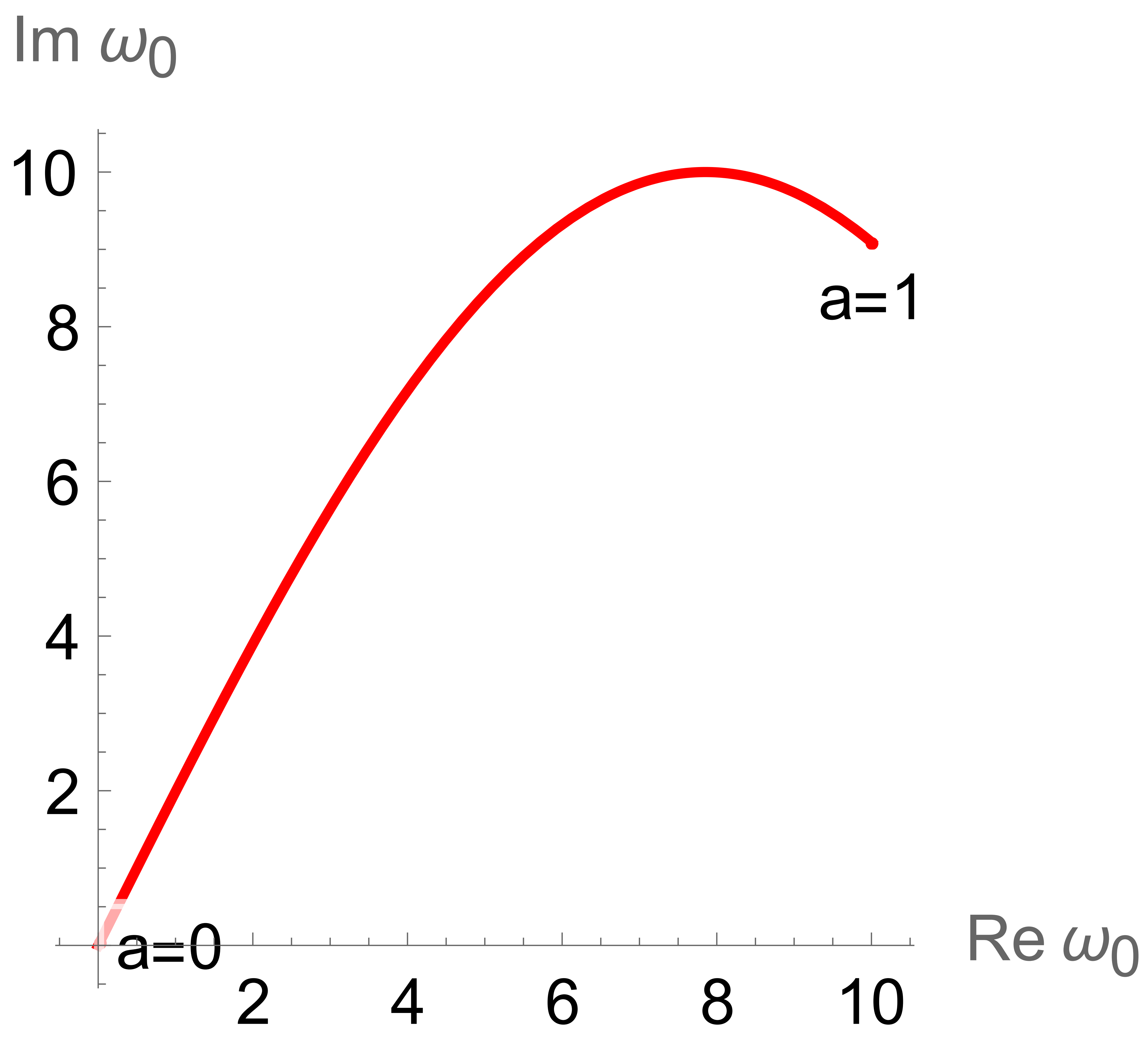
Quasinormal mode frequencies

- **Notation:** ω = QNM frequency. a = BH spin.
- QNM frequencies are complex.
- $\omega_0 = \omega_0(a)$ with $0 < a < 1$.

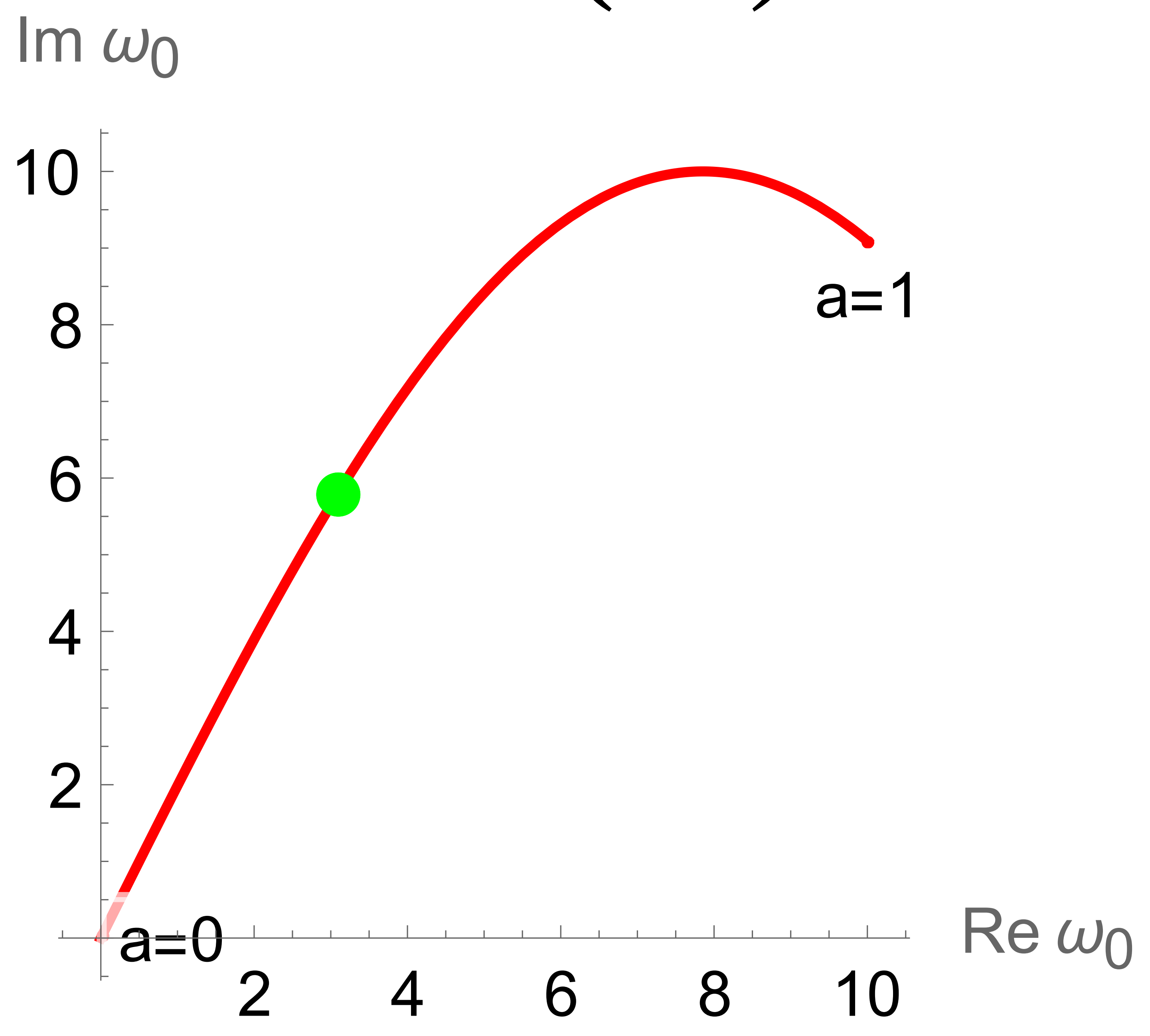


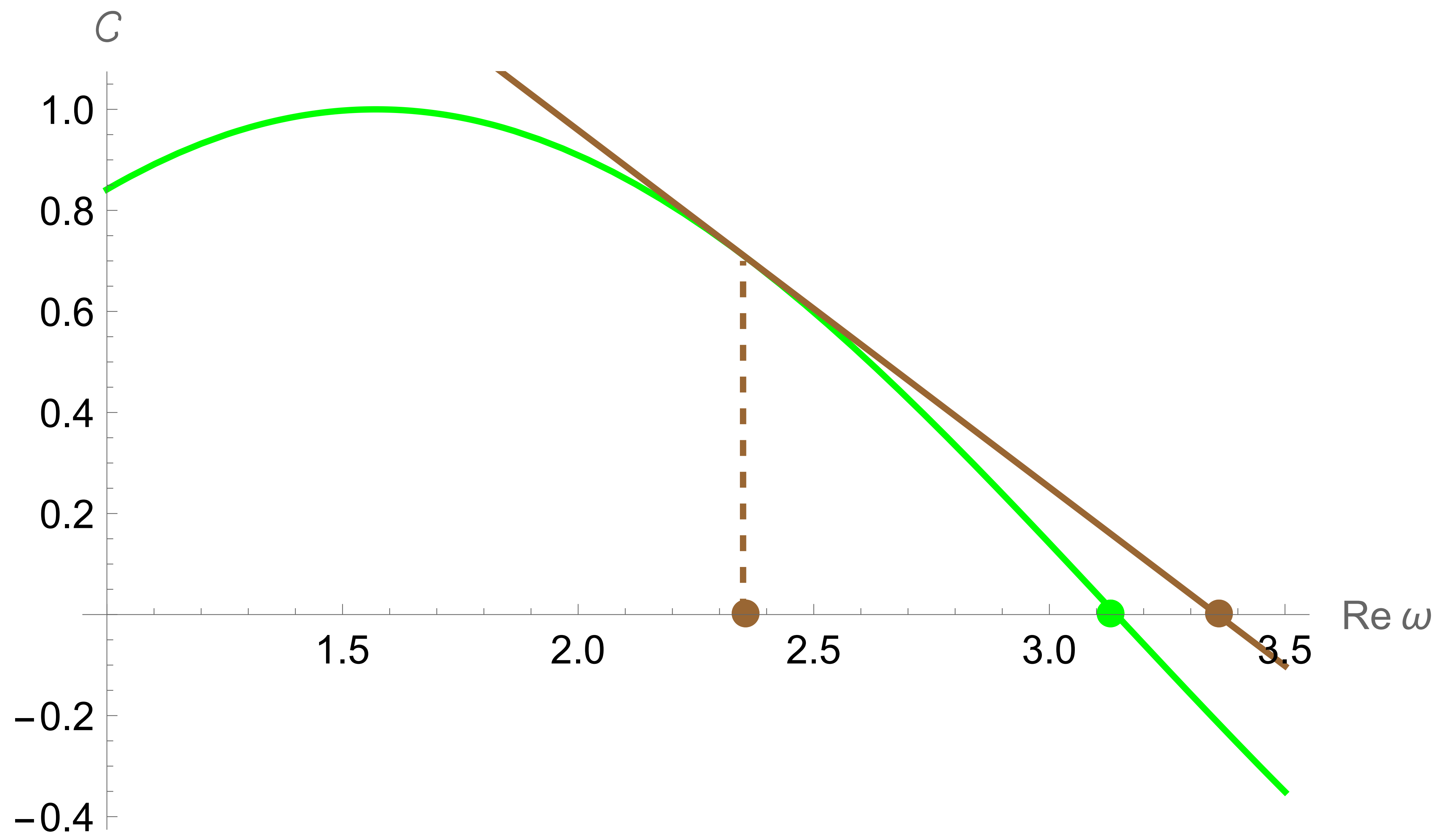
Quasinormal mode frequencies

- **Notation:** ω = QNM frequency.
  a = BH spin.
- QNM frequencies are complex.
- $\omega_0 = \omega_0(a)$ with $0 < a < 1$.
- **Note:** won't show actual QNM curves; will use fake curves for simplicity.



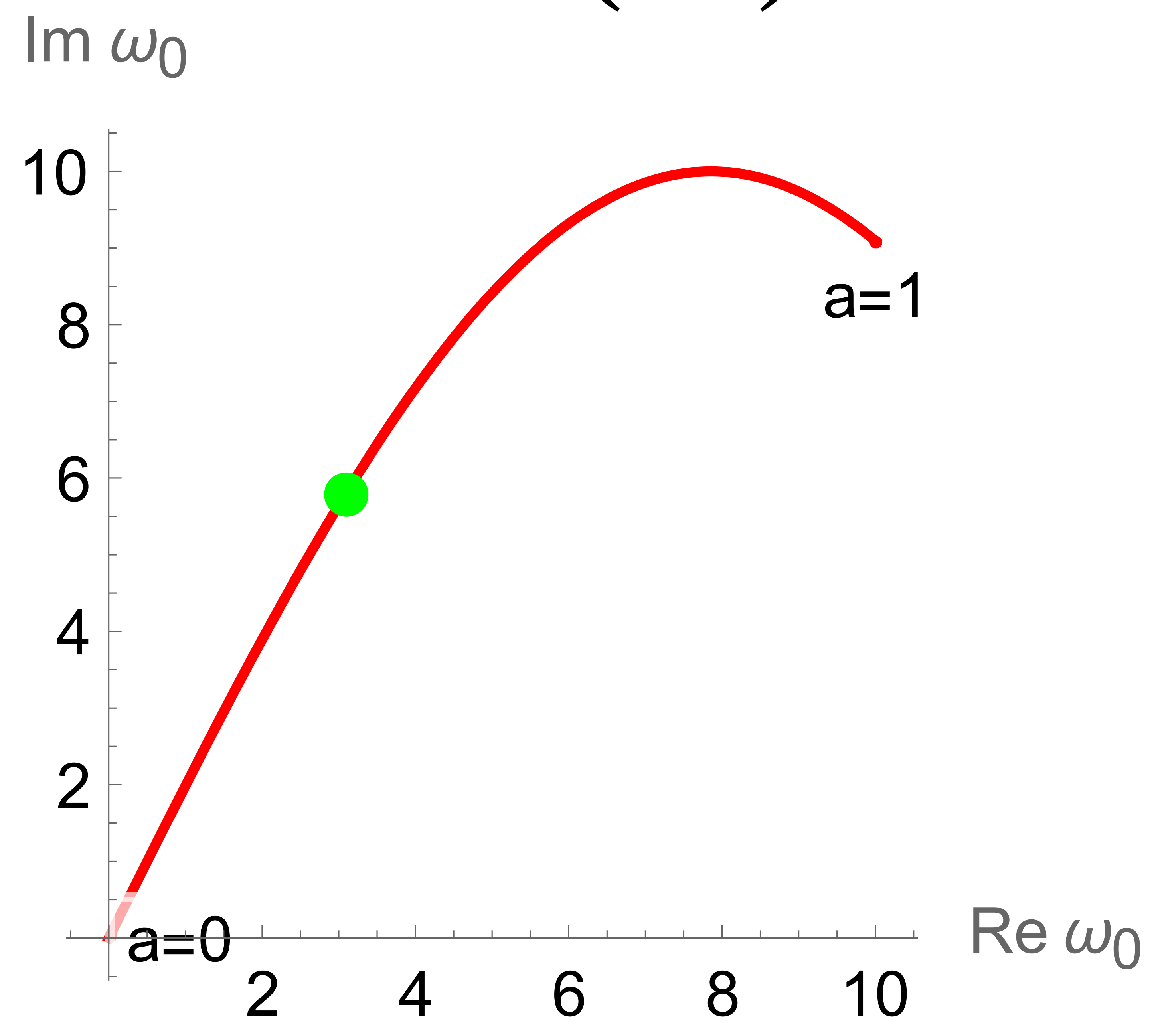
QNM frequency ω_0: root of $\mathscr{C}(\omega) = 0$

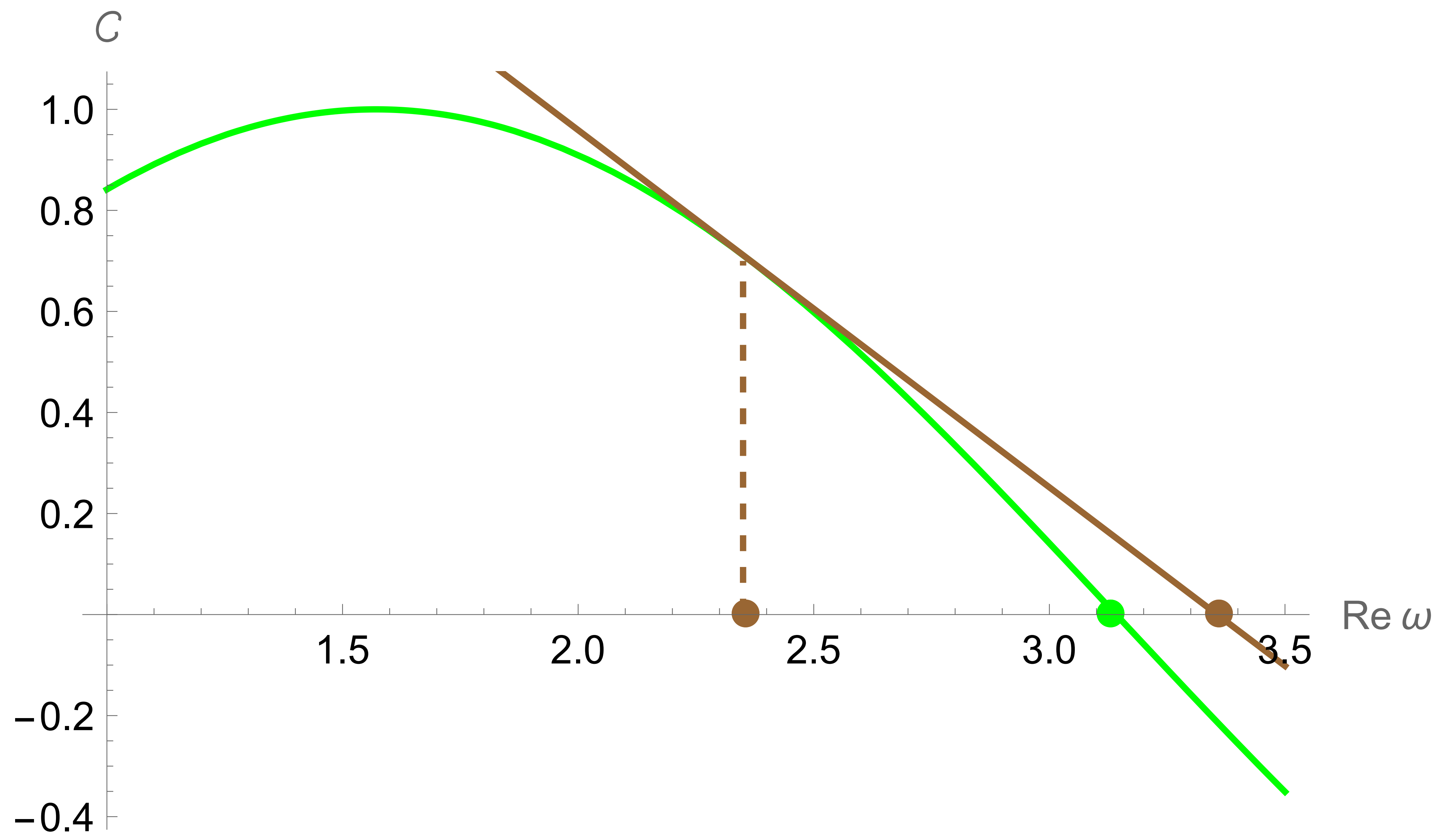




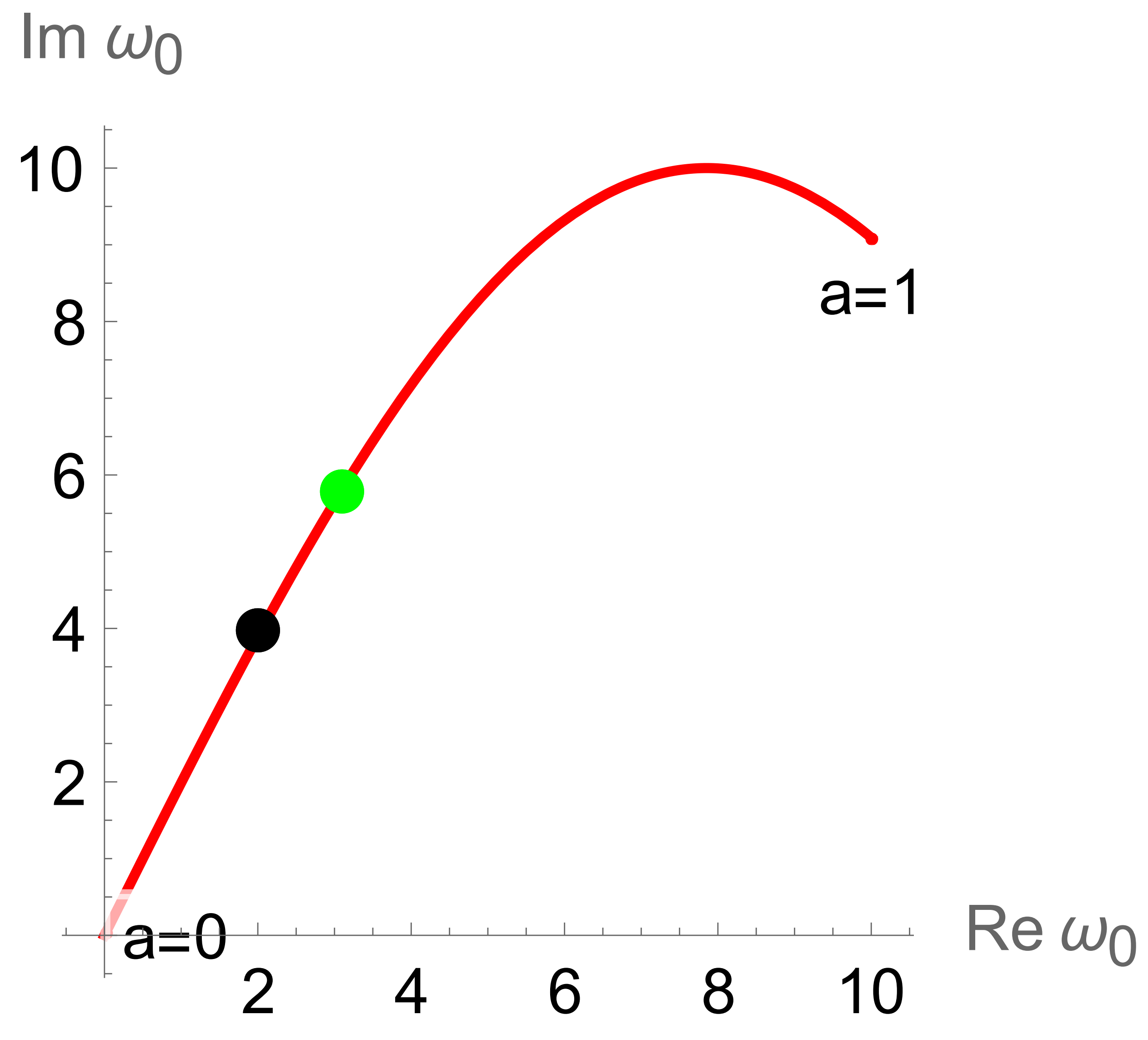
QNM frequency ω_0: root of $\mathscr{C}(\omega) = 0$

- ω_0's are solutions of $\mathscr{C}(\omega) = 0$ (via Newton-Raphson root finding)



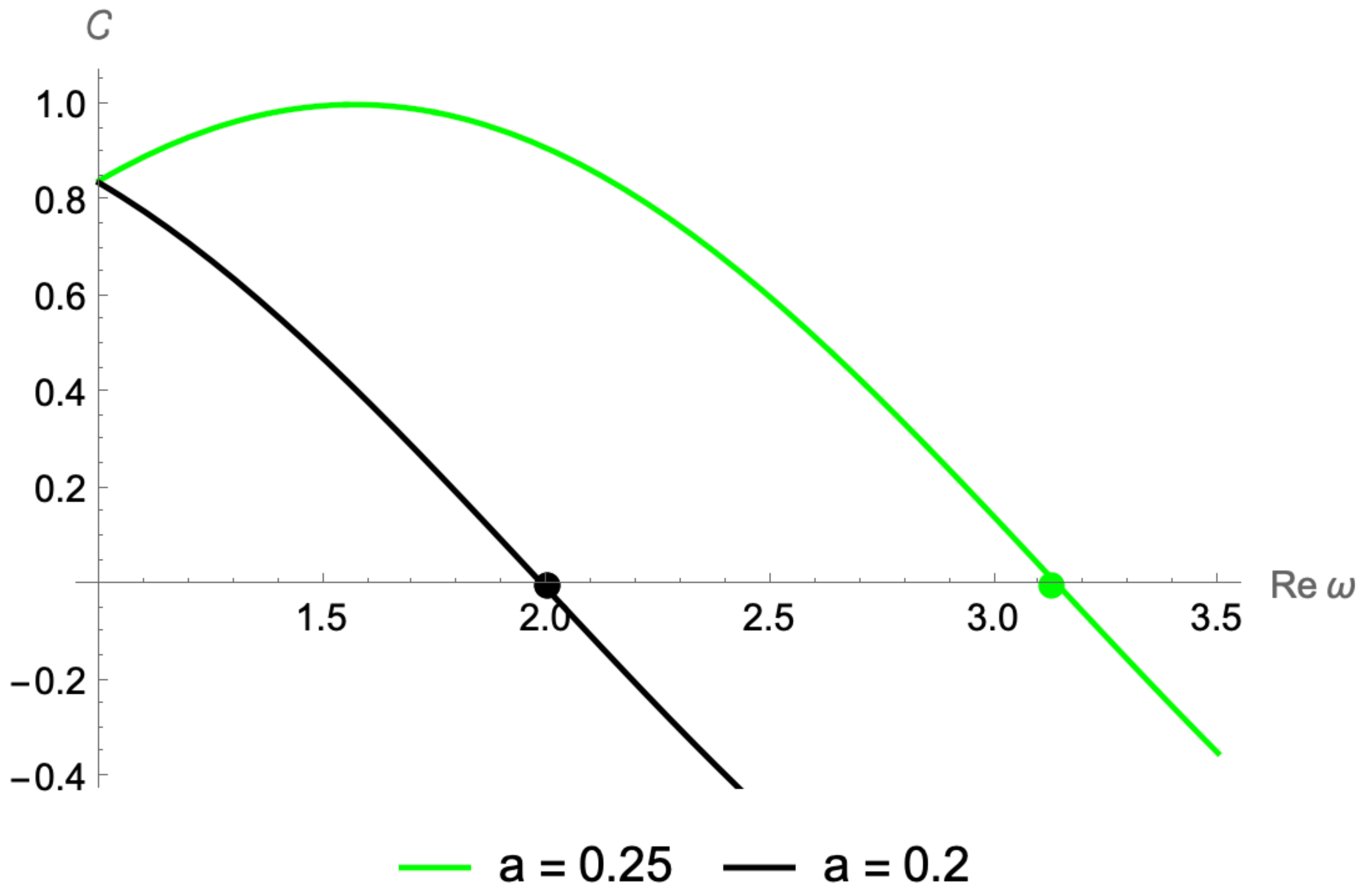


QNM frequency ω_0: root of $\mathscr{C}(\omega, a) = 0$

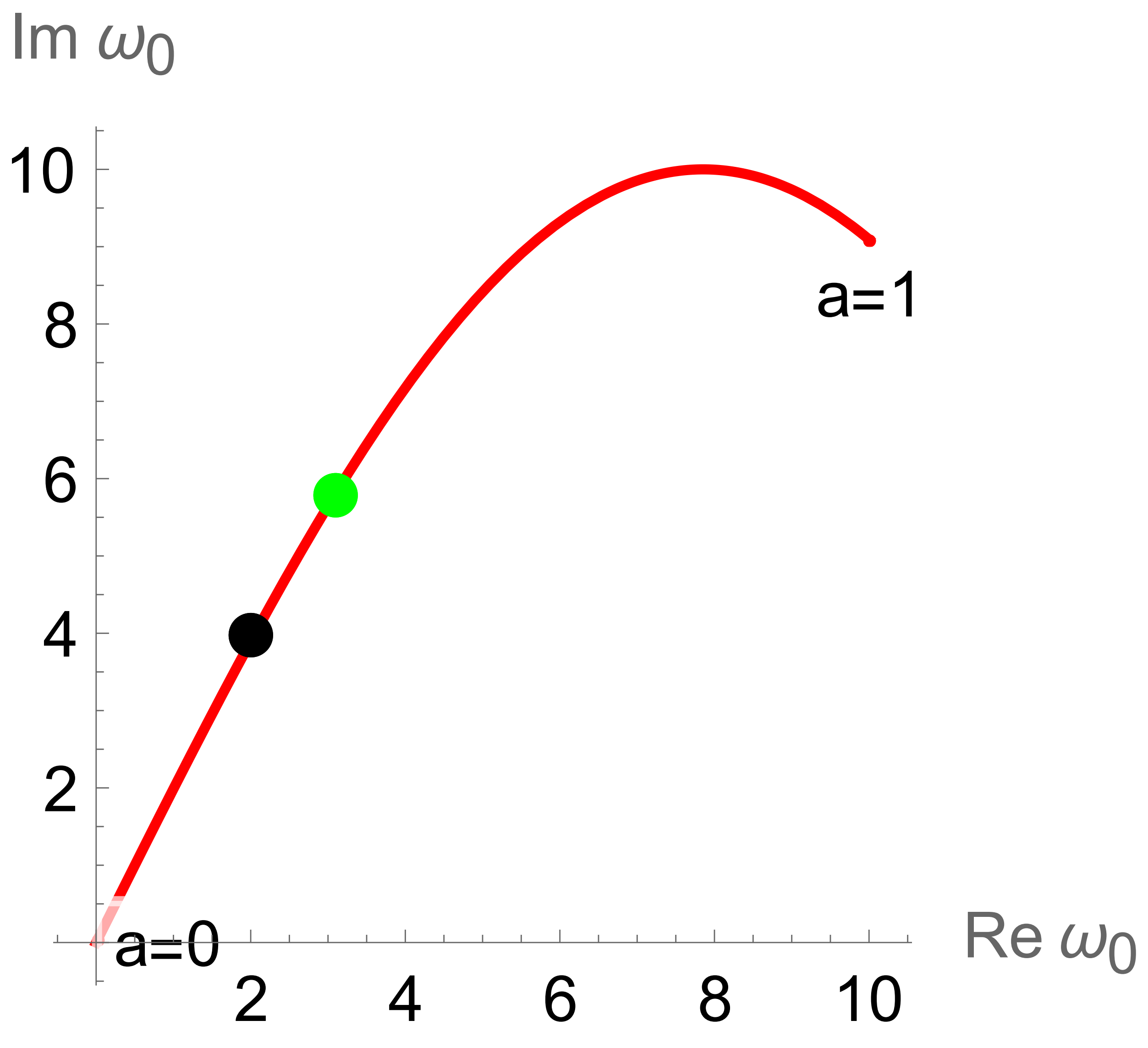


- ω_0's are solutions of $\mathscr{C}(\omega; a) = 0$ (via Newton-Raphson root finding).

- Finding ω_0's $\sim$ parameterized *(by a)* numerical root-finding problem.



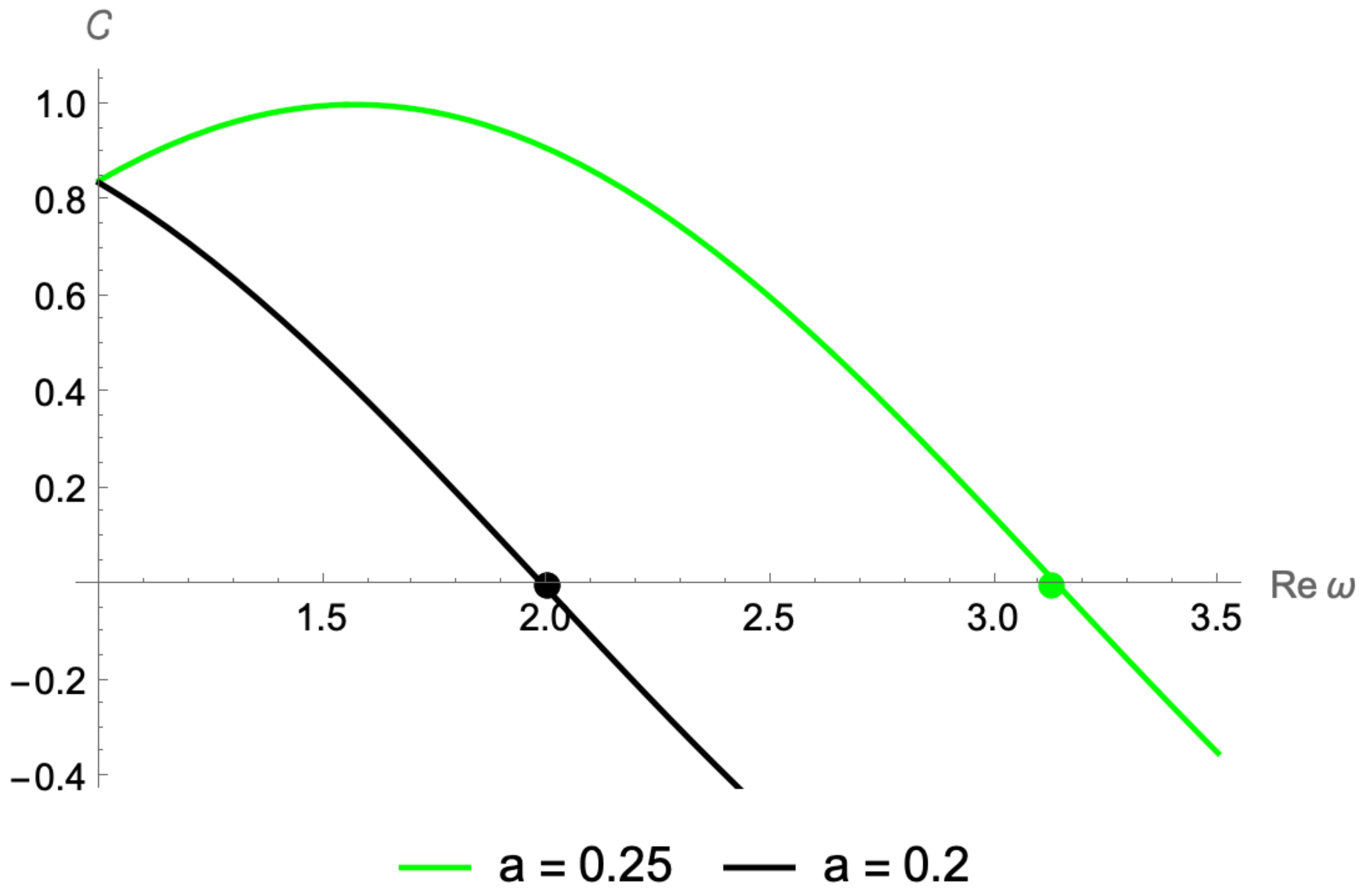
QNM frequency ω_0: root of $\mathscr{C}(\omega, a) = 0$



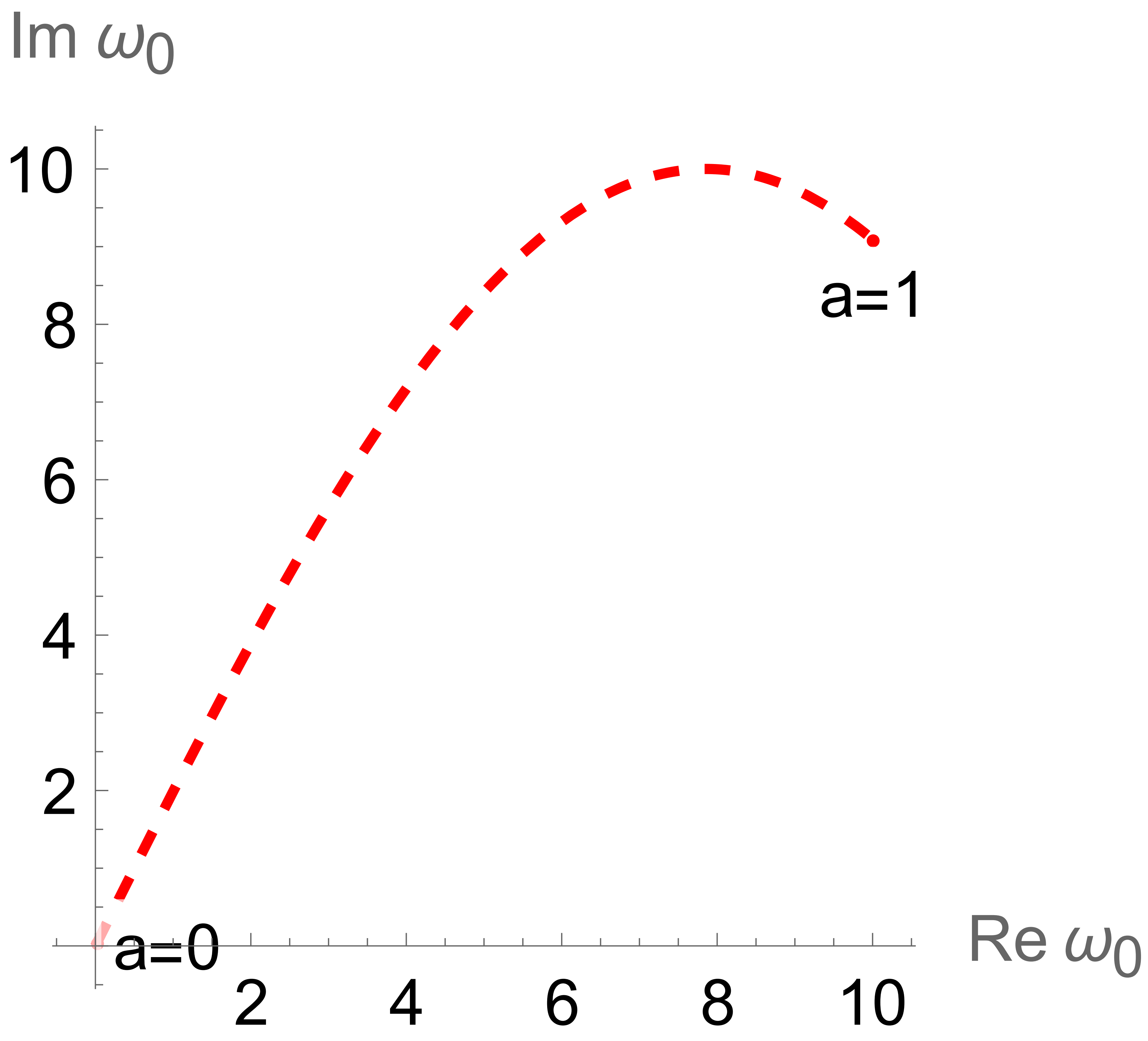
- ω_0's are solutions of $\mathscr{C}(\omega; a) = 0$. (via Newton-Raphson root finding)

- Finding ω_0's ~ parameterized *(by a)* numerical root-finding problem.

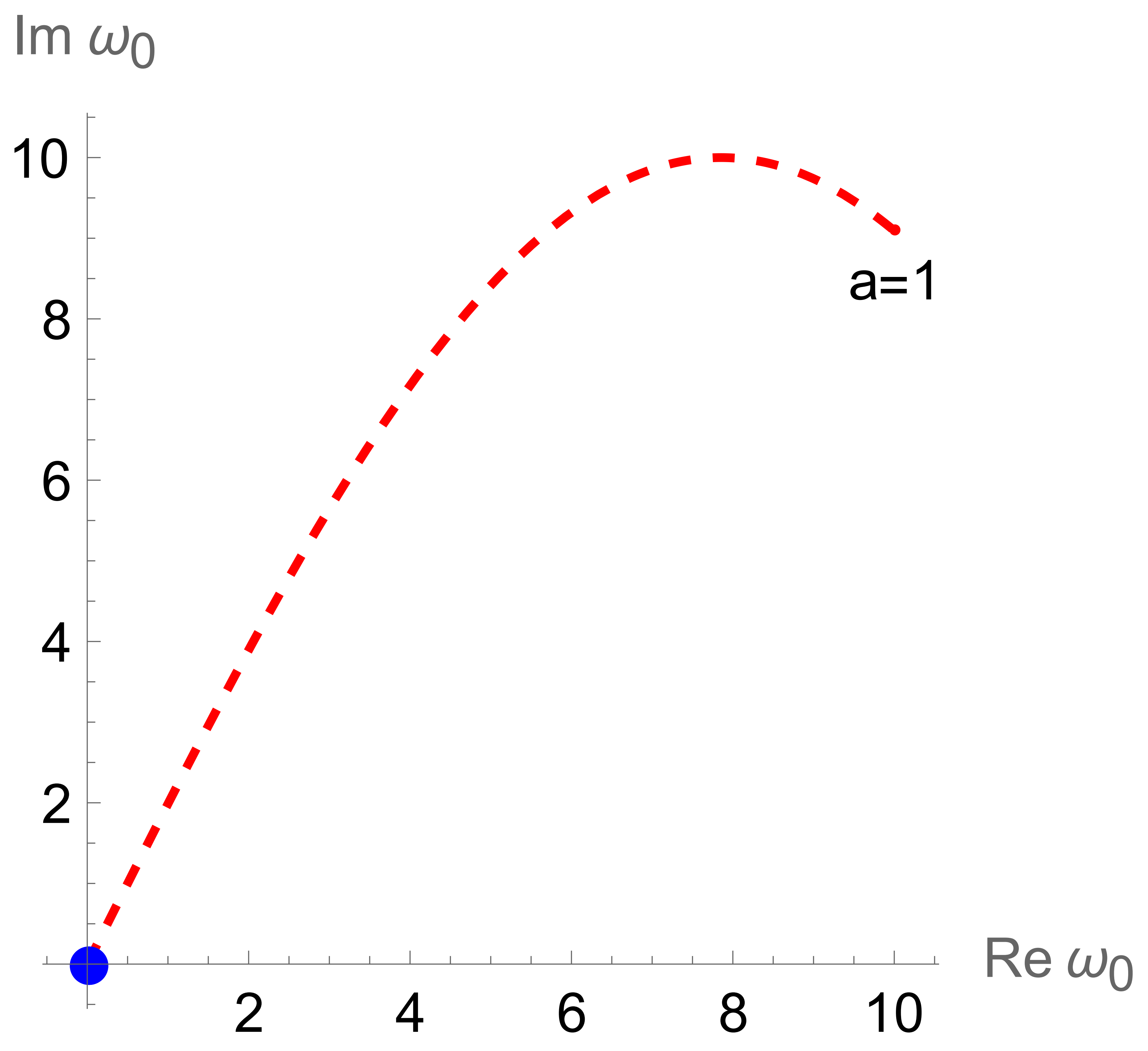
- **Important:** Distinguish b/w $\mathscr{C}(\omega, a)$ *(bottom)* and ω_0 *(up)*.



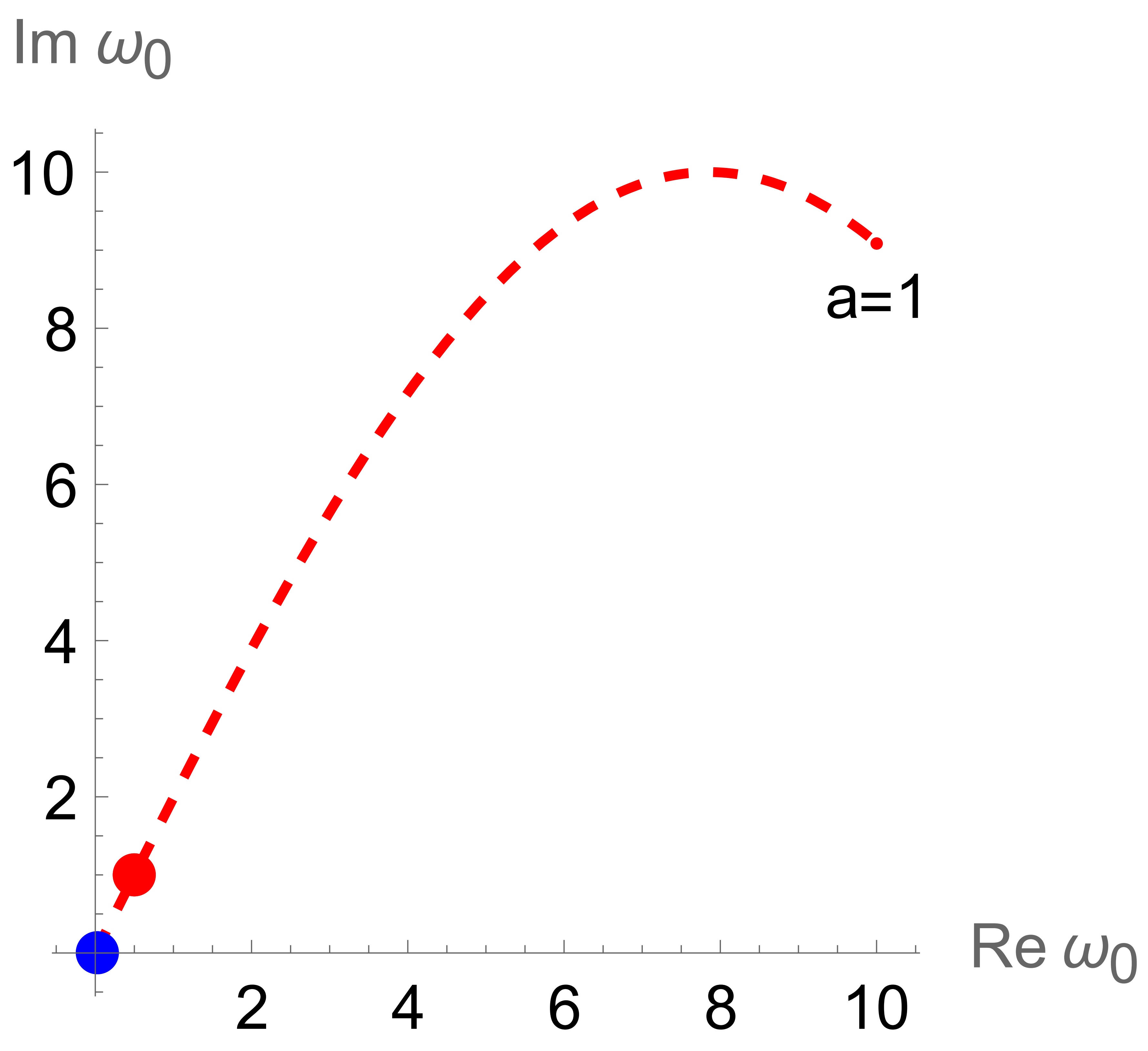
$\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$



$\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

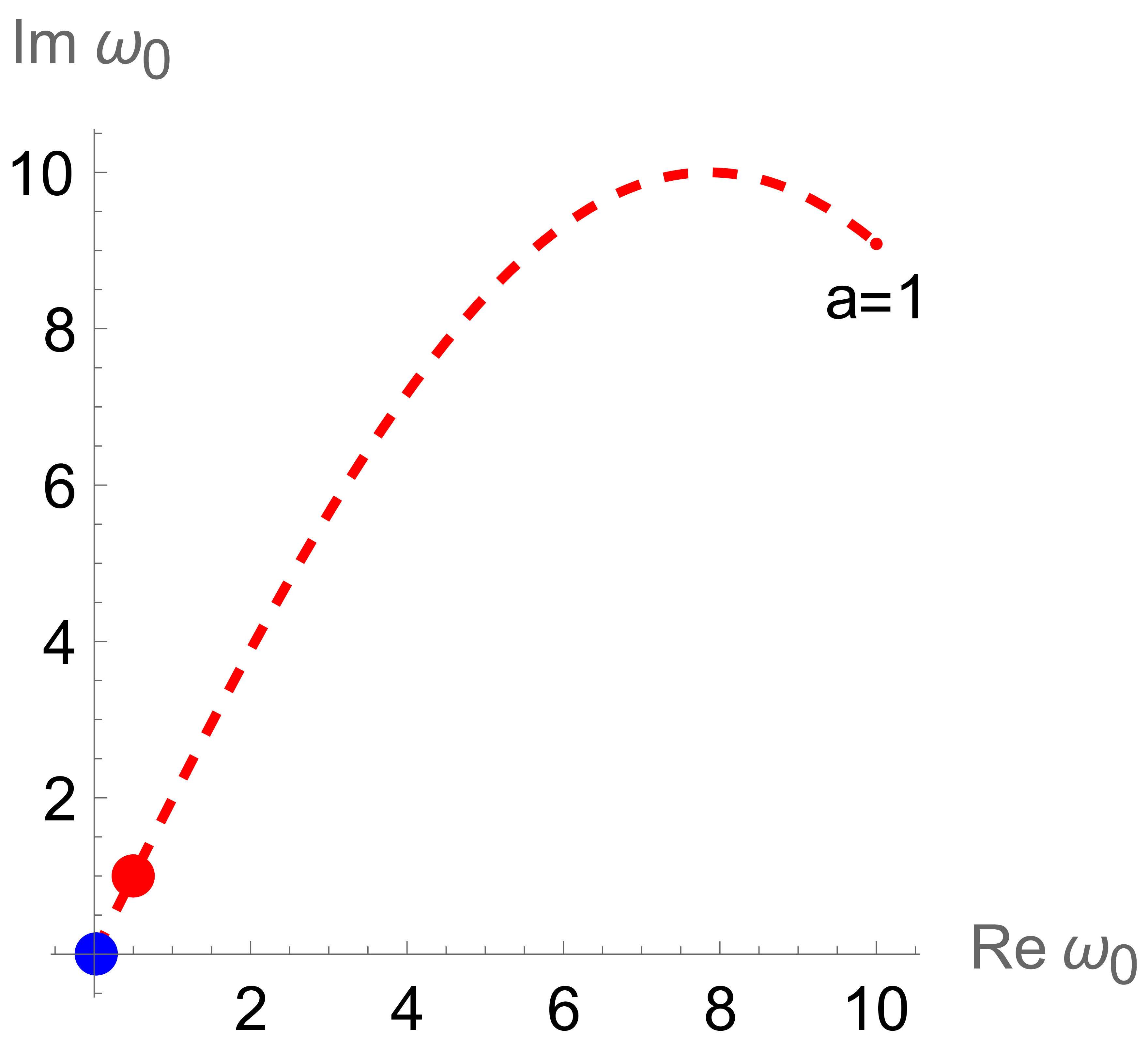


$\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$



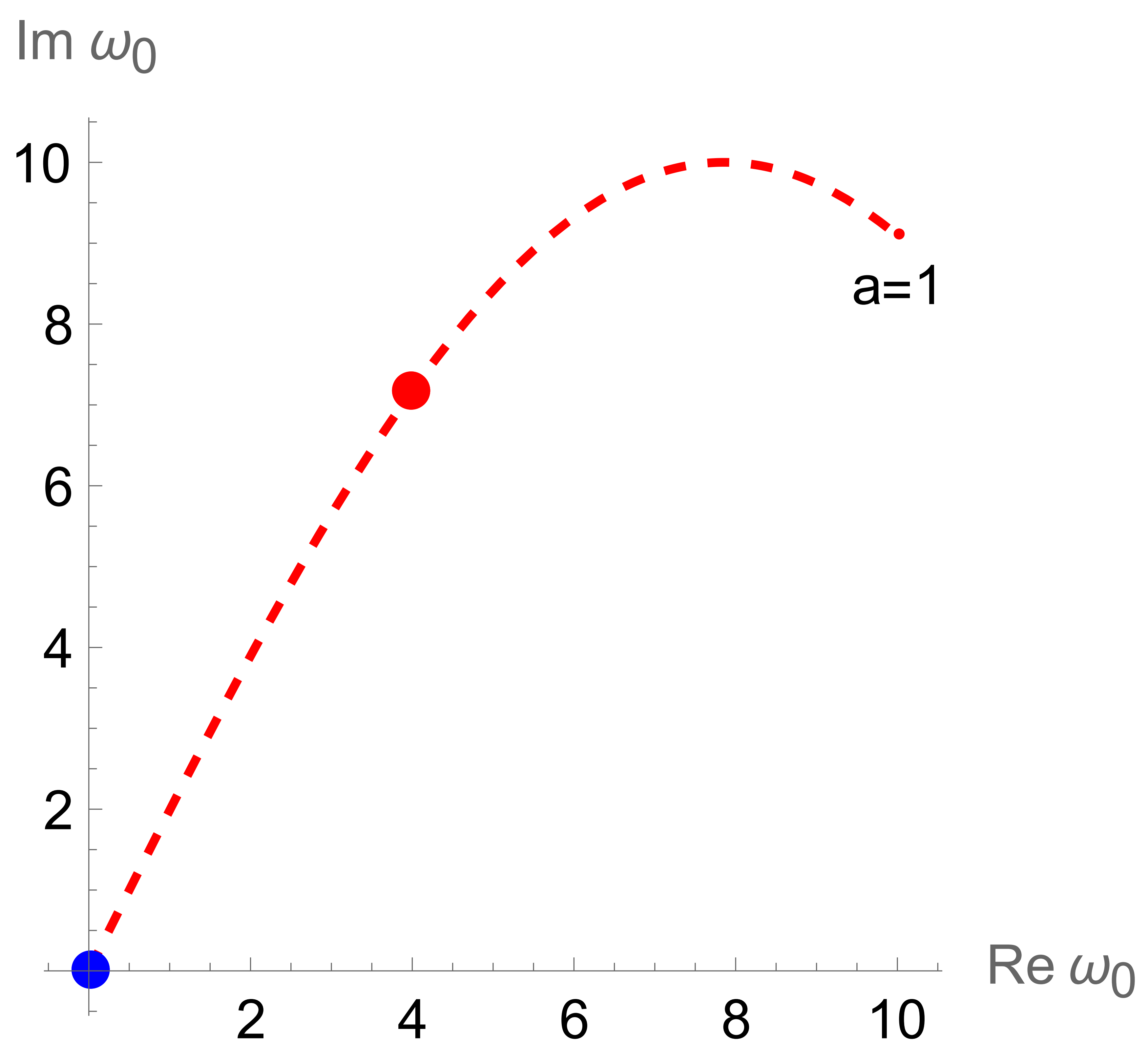
$\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.



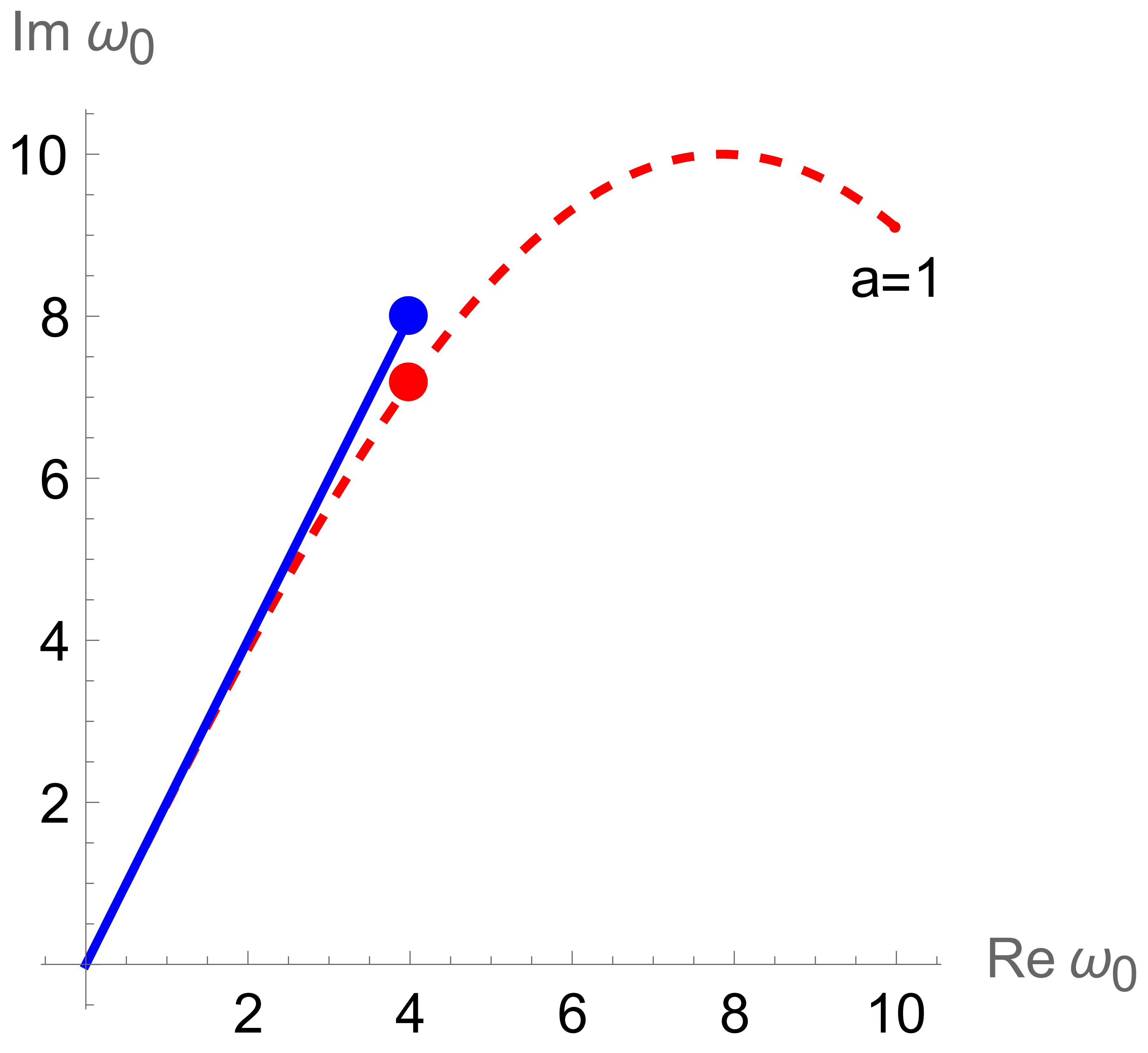
$\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\implies$ must take small steps ($da \sim 0.02$) in BH spin a.



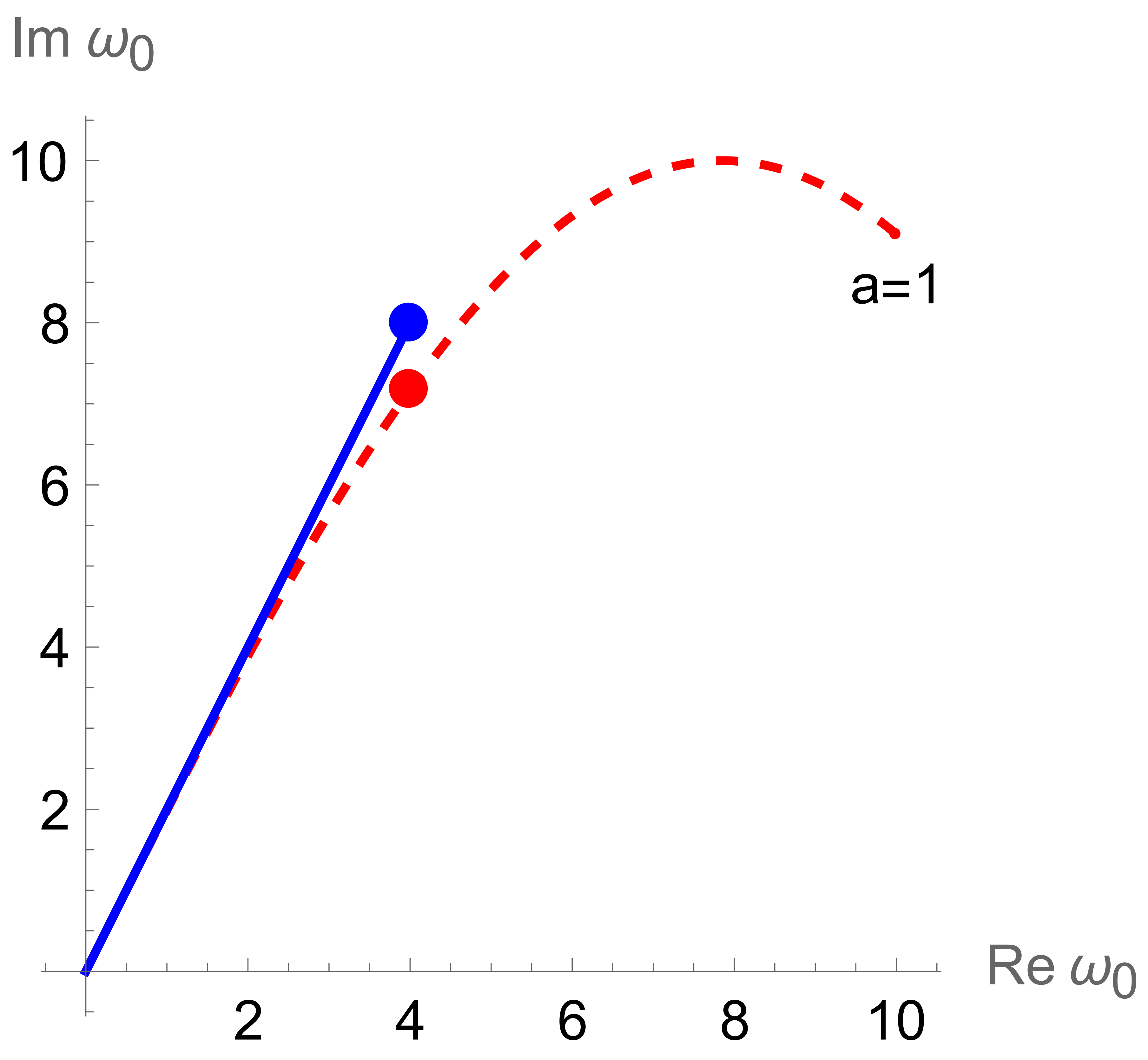
$\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\Longrightarrow$ must take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.

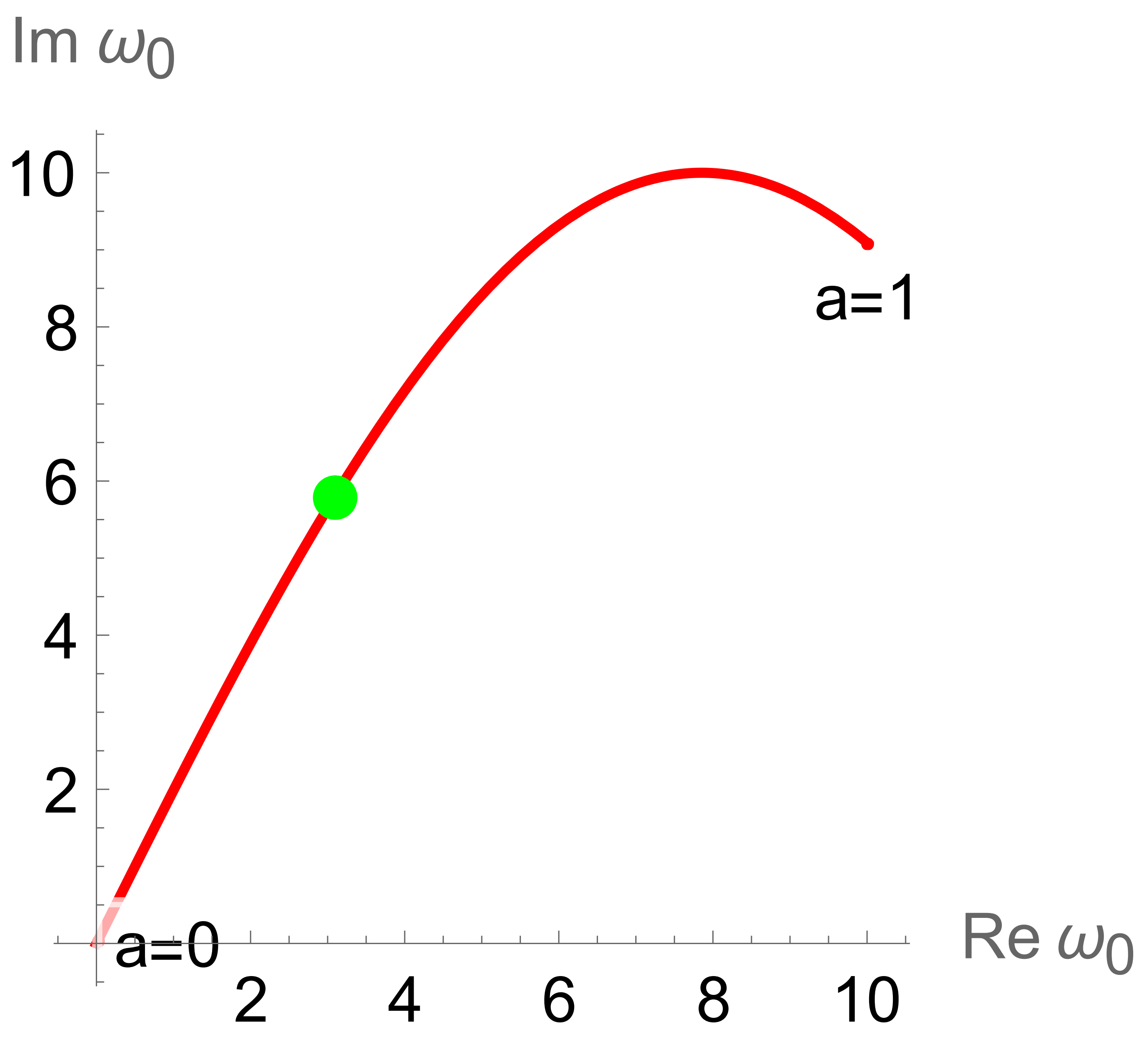


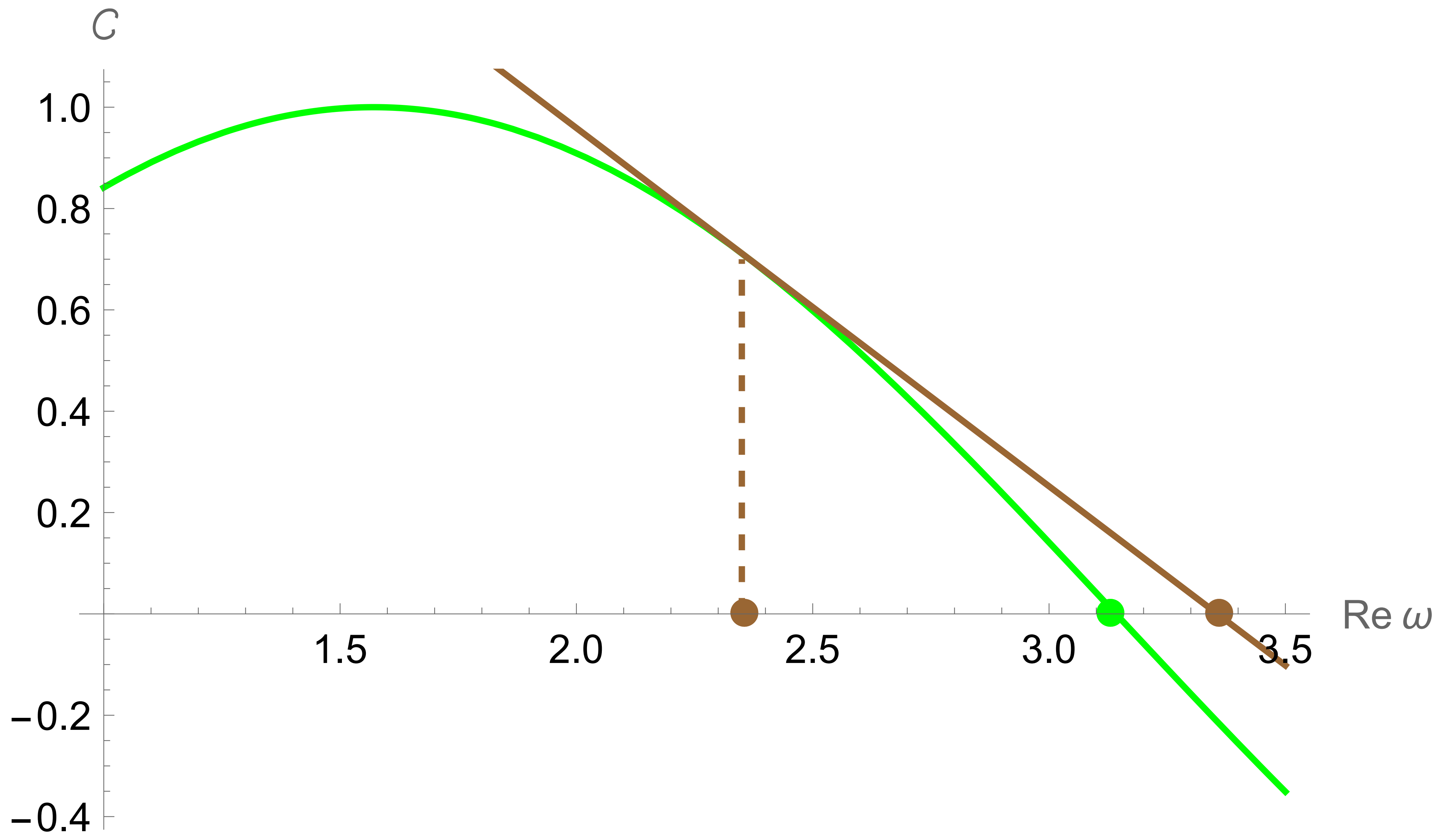
Results

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\implies$ must take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.
- **Result:** we provide $d\omega_0/da$ analytically.



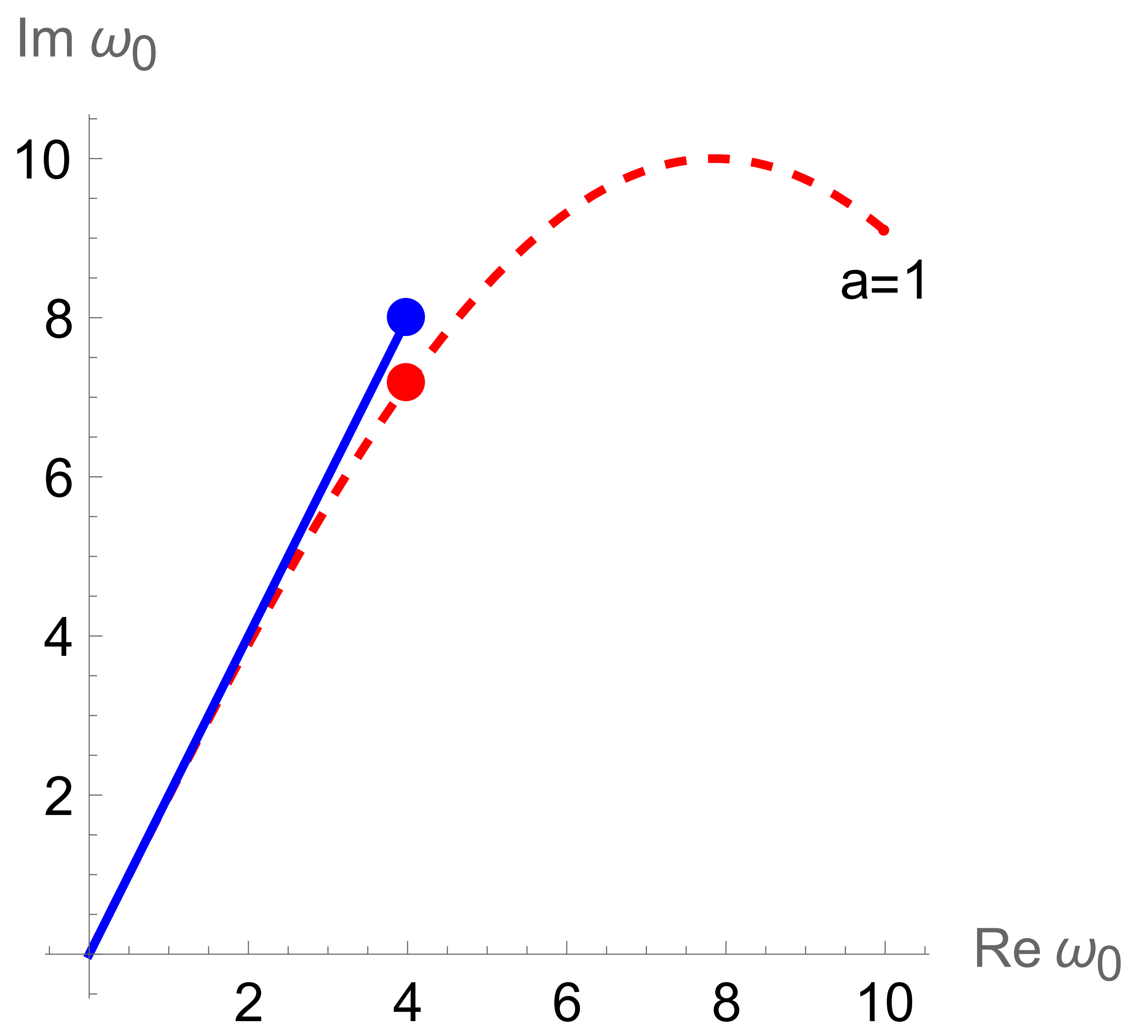
Recall...

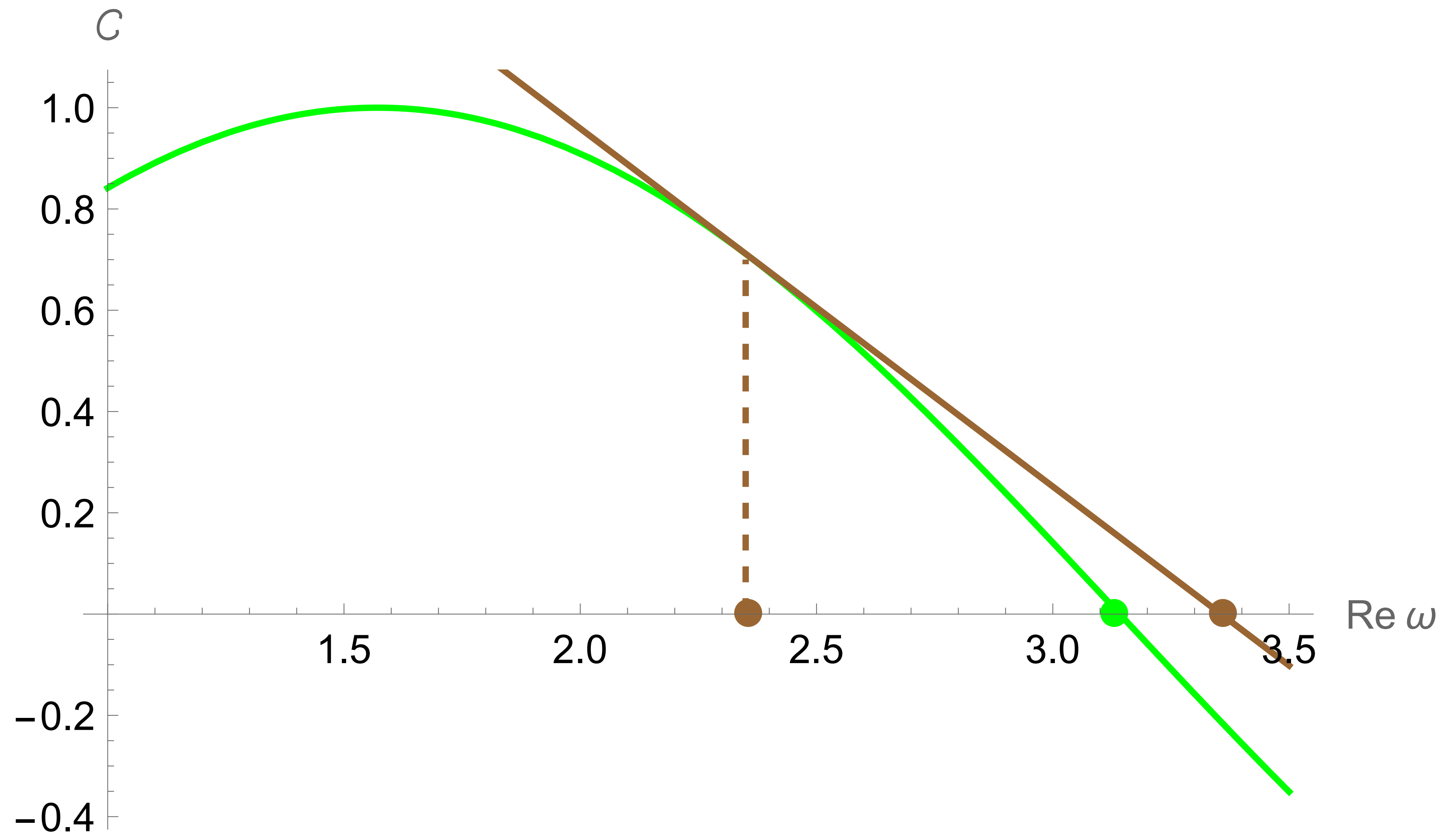

Im ω_0
10
8
6
4
2
a=1
a=0
2
4
6
8
10
Re ω_0


C
1.0
0.8
0.6
0.4
0.2
−0.2
−0.4
1.5
2.0
2.5
3.0
3.5
Re ω

Results

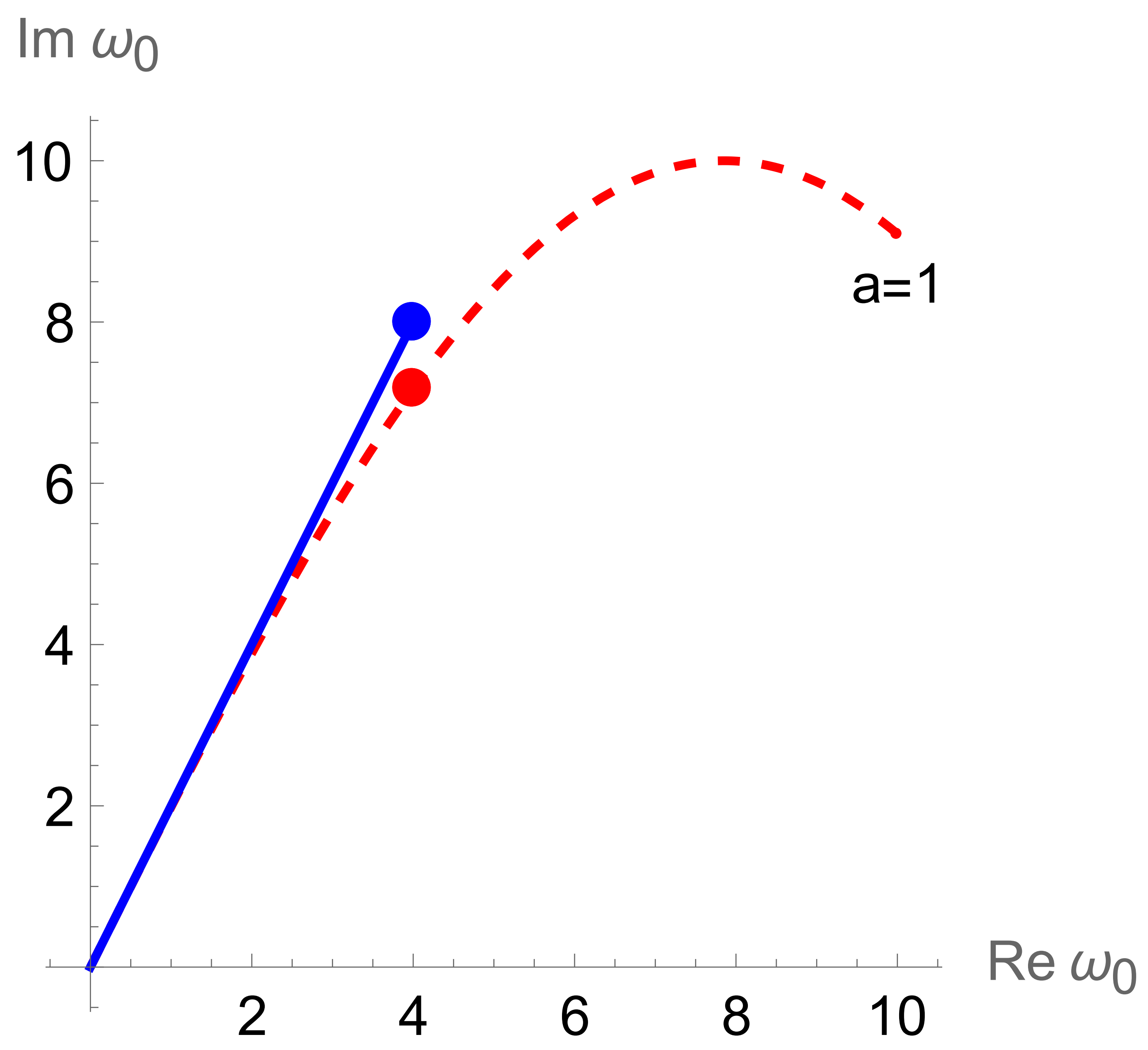
- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\implies$ take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.
- **Result:** we provide $d\omega_0/da$ analytically.
- **Result:** we provide $d\mathscr{C}/d\omega$ for Newton-Raphson analytically.

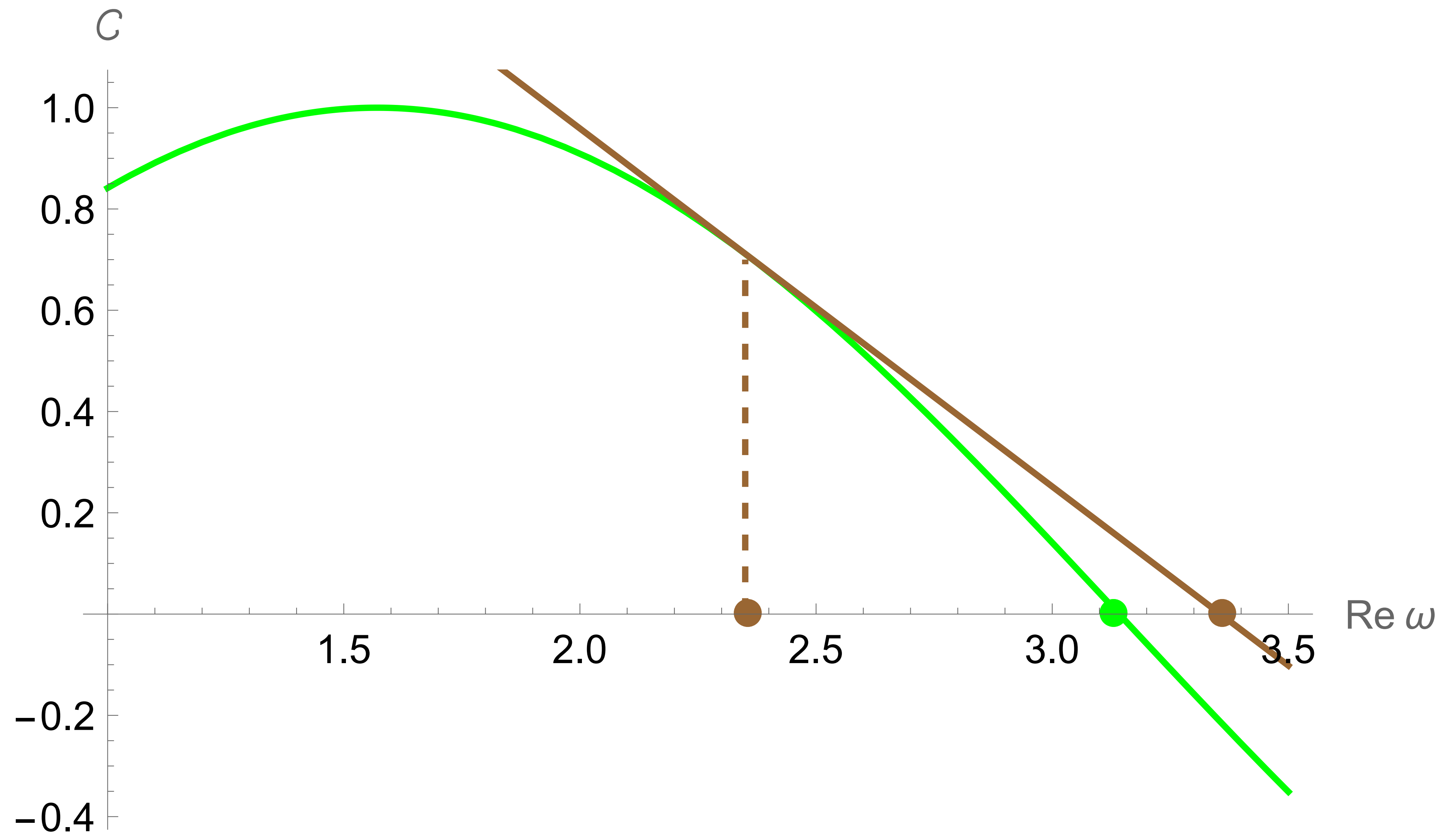




Results

- Previously found $\omega_0(a)$ is used as a guess to find $\omega_0(a + da)$.
- Root can't be found if the guess is too far $\Longrightarrow$ take small steps ($da \sim 0.02$) in BH spin a.
- We can take large steps ($da \sim 0.25$) in a if we have $d\omega_0/da$.
- **Result:** we provide $d\omega_0/da$ analytically.
- **Result:** we provide $d\mathscr{C}/d\omega$ for Newton-Raphson analytically.
- Analytical derivatives preferred over numerical ones *(see Secs. 5.7, 9.4, 9.6, and 9.7 of Numerical Recipes in C).*





Technical details

Technical details

- Integration could be hard; isn't differentiation trivial?

Technical details

- Integration could be hard; isn't differentiation trivial?

$$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}.$$

(we don't know a priori how many fractions to keep)

-

Technical details

- Integration could be hard; isn't differentiation trivial?
- $$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}.$$ (we don't know a priori how many fractions to keep)
- **Problem 1:** Naive $d\mathscr{C}/d\omega$ computation is *non-iterative* & inefficient.

Technical details

- Integration could be hard; isn't differentiation trivial?
- $$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}.$$ (we don't know a priori how many fractions to keep)
- **Problem 1:** Naive $d\mathscr{C}/d\omega$ computation is *non-iterative* & inefficient.
- I lied; we have 2 equations in 2 unknowns (ω_0, A); $A =$ some eigenvalue.

Technical details

- Integration could be hard; isn't differentiation trivial?
- $$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}.$$ (we don't know a priori how many fractions to keep)
- **Problem 1:** Naive $d\mathscr{C}/d\omega$ computation is *non-iterative* & inefficient.
- I lied; we have 2 equations in 2 unknowns (ω_0, A); $A =$ some eigenvalue.
- We deliver $(d\omega_0/da, d\mathscr{C}/d\omega, dA/da)$.

Technical details

- Integration could be hard; isn't differentiation trivial?
- $$\mathscr{C} = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \cdots}}}}}.$$ (we don't know a priori how many fractions to keep)
- **Problem 1:** Naive $d\mathscr{C}/d\omega$ computation is *non-iterative* & inefficient.
- I lied; we have 2 equations in 2 unknowns (ω_0, A); $A =$ some eigenvalue.
- We deliver $(d\omega_0/da, d\mathscr{C}/d\omega, dA/da)$.
- **Problem 2:** Redo Griffiths' quantum mech. pert. theory with *non-hermitian* matrix.

Summary

- **Result:** we provided derivatives ($d\omega_0/da$, $d\mathscr{C}/d\omega$ & dA/da) to make QNM frequency computation more efficient.

- **Result:** $d\omega_0/da$ lets us take larger step sizes $da \sim 0.02 \to 0.25$.

- **Future:** Calculate and incorporate $d^2\omega_0/da^2$; can let us take $da \sim 0.65$.

- **Future:** apply this method to beyond Kerr QNMs (within GR) and beyond GR.

- **Refs:** *arXiv: 2210.03657, github.com/sashwattanay/qnm*