Assignment 1

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SM21MTECH12006

1 Problem IV (4II)

Show that the following triad of points form an equilateral triangle $\begin{pmatrix} a \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 2a \end{pmatrix}$, $\begin{pmatrix} 2a \\ a \end{pmatrix}$, axes being inclined at an angle of $60°$

1.1 Solution

Let the points be

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix}; \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2a \end{pmatrix}; \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2a \\ a \end{pmatrix} \tag{1.1.1}$$

In order to convert to rectangular coordinate system, the y-axis should be rotated by $30°$ in anti-clockwise. Transformed coordinates of $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ & $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ be $\begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$, $\begin{pmatrix} x_5 \\ y_5 \end{pmatrix}$ & $\begin{pmatrix} x_6 \\ y_6 \end{pmatrix}$ respectively.

$x_4 = \mathrm{O}X_1 + X_1X_4 = x_1 + y_1\cos 60°$
$y_4 = \mathrm{O}Y_1\cos 30° = \mathrm{y}_1\cos 30°$

$$\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & \cos 60° \\ 0 & \cos 30° \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{1.1.2}$$

The generalised equation for transformed coordinates $\begin{pmatrix} x_t \\ y_t \end{pmatrix}$ when the angle between axes 'θ is,

$$\boxed{\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 & \cos(\theta) \\ 0 & \sin(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}} \tag{1.1.3}$$

Let the transformed point be $\mathbf{X_t}$, $\mathbf{T}$ be the transformation matrix and the point in angular axes be $\mathbf{X}$, (1.1.3) can be written as

$$\mathbf{X_t} = \mathbf{T}\ \mathbf{X} \tag{1.1.4}$$

Substituting (1.1.1) in (1.1.3)

$$\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix}; \begin{pmatrix} x_5 \\ y_5 \end{pmatrix} = \begin{pmatrix} a \\ \sqrt{3}a \end{pmatrix}; \begin{pmatrix} x_6 \\ y_6 \end{pmatrix} = \begin{pmatrix} \frac{5a}{2} \\ \frac{\sqrt{3}a}{2} \end{pmatrix} \tag{1.1.5}$$

The distance between points is a norm of the distance vector,

$$d_{12} = \|\mathbf{X_{t1}} - \mathbf{X_{t2}}\| \tag{1.1.6}$$

Substituting (1.1.4) in (1.1.6),

$$d_{12} = \|\mathbf{TX_1} - \mathbf{TX_2}\| \tag{1.1.7}$$

$$d_{12} = \|\mathbf{T}(\mathbf{X_1} - \mathbf{X_2})\| \tag{1.1.8}$$

$$d_{12} = (\mathbf{X_1} - \mathbf{X_2})^\top\ \mathbf{T}^\top\ \mathbf{T}\ (\mathbf{X_1} - \mathbf{X_2}) \tag{1.1.9}$$

$$d_{12} = \sqrt{3}a \tag{1.1.10}$$

$$similarly, d_{23} = \sqrt{3}a \tag{1.1.11}$$

$$d_{31} = \sqrt{3}a \tag{1.1.12}$$

Therefore,

$$d_{12} = d_{23} = d_{31} \tag{1.1.13}$$

From (1.1.13) The three points form equilateral triangle

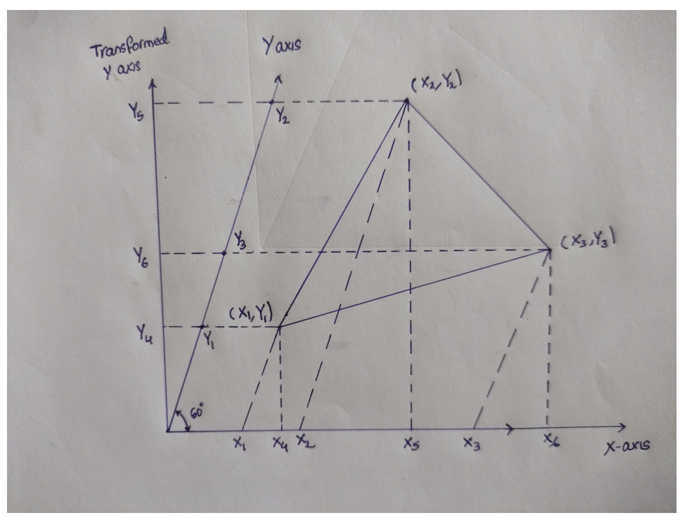

Fig. 1.1: Points defined on angular & rectangular axes

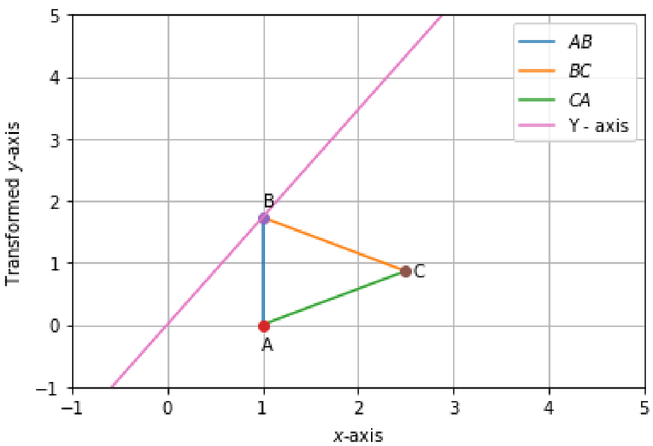

Fig. 1.2: Points plotted in Python