Assignment - 1

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CHAPTER II EX-II Q.4-II

Show that the following triad of points form an equilateral triangle $\begin{pmatrix} a \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 2a \end{pmatrix}$, $\begin{pmatrix} 2a \\ a \end{pmatrix}$, **axes being inclined at an angle of 60°**

Answer: A triangle is said to be equilateral if length of all sides are equal.

The given points are:

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 2a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2a \\ a \end{pmatrix}$$

The distance between points A and B,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2a \end{pmatrix} = \begin{pmatrix} a \\ -2a \end{pmatrix}$$

$$\|\mathbf{A} - \mathbf{B}\|^2 = (\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B})$$
$$= \begin{pmatrix} a & -2a \end{pmatrix} \begin{pmatrix} a \\ -2a \end{pmatrix}$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(a)^2 + (-2a)^2}$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{5}a \qquad (1)$$

Similarly,

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 0 \\ 2a \end{pmatrix} - \begin{pmatrix} 2a \\ a \end{pmatrix} = \begin{pmatrix} -2a \\ a \end{pmatrix}$$

$$\|\mathbf{B} - \mathbf{C}\|^2 = (\mathbf{B} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C})$$
$$= \begin{pmatrix} a & -2a \end{pmatrix} \begin{pmatrix} a \\ -2a \end{pmatrix}$$

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{(-2a)^2 + (a)^2}$$

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{5}a \qquad (2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 2a \\ a \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix}$$

$$\|\mathbf{C} - \mathbf{A}\|^2 = (\mathbf{C} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A})$$
$$= \begin{pmatrix} a & a \end{pmatrix} \begin{pmatrix} a \\ a \end{pmatrix}$$

$$\|\mathbf{C} - \mathbf{A}\| = \sqrt{(a)^2 + (a)^2}$$

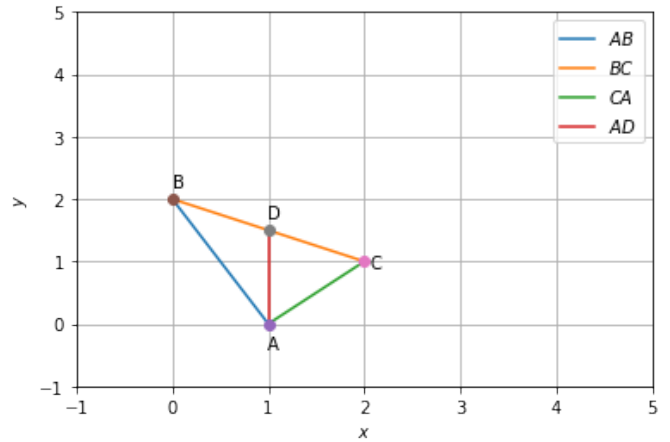$$\|\mathbf{C} - \mathbf{A}\| = \sqrt{2}a \qquad (3)$$

Fig. 1: Triangle formed for a=1

From (1), (2), (3)

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| \neq \|\mathbf{C} - \mathbf{A}\|$$

So the given triad of points does not form an equilateral triangle.