MATHS - ASSIGNMENT ①

PART - B

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ECE B

1) Given curve is

$y = 4\sin x$

diff wrt 'x'

$\frac{dy}{dx} = 4(-\cos x)$

$y_1 = -4\cos x$

at $x = \pi/2$; $y_1 = -4\cos \pi/2$

$y_1 = 0$

diff wrt 'x'

$\frac{d^2y}{dx^2} = -4\sin x$

at $x = \pi/2$

$y_2 = \frac{d^2y}{dx^2} = -4$

$y_1 = 0 \quad y_2 = -4$

Radius of curvature . $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

$= \left(\frac{(1+0)}{-4}\right)^{3/2}$

$\rho = -1/4$

radius of curvature can't be negative so, radius of curvature of curve $y = 4\sin x$ is $1/4$ ©

② Given curve is

$r = e^\theta$

differentiate wrt θ

$\frac{dr}{d\theta} = e^\theta$

$r_1 = \frac{dr}{d\theta} = e^\theta$

diff wrt θ

$\frac{d^2r}{d\theta^2} = e^\theta$

$r_2 = e^\theta$

Radius of curvature $\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - (r r_2)}$

$= \frac{(e^{2\theta} + e^{2\theta})^{3/2}}{e^{2\theta} + 2e^{2\theta} - e^{2\theta}}$

$= \frac{(2e^{2\theta})^{3/2}}{2e^{2\theta}}$

3) Given;

$y = mx + a/m$

$y = \frac{m^2 x + a}{m}$

$my = m^2 x + a$

$m^2 x - my + a = 0$

This is of the form $Am^2 + Bm + c = 0$

The envelope is $B^2 - 4AC = 0$

Here, $B = -y$, $A = x$, $C = a$

$\boxed{y^2 - 4ax = 0}$ ⓒ The envelope

④ Given; To find the value of $\Gamma[-5/2]$

WKT: $\Gamma(n+1) = n\Gamma(n) \rightarrow ①$

$n = -5/2$ in ①

$\Gamma(-5/2 + 1) = -5/2\, \Gamma(-5/2)$

$\Gamma(-3/2) = -5/2\, \Gamma(-5/2)$

$-2/5\, \Gamma(-3/2) = \Gamma(-5/2)$ - ②

$n = -3/2$ in ①;

$\Gamma(-3/2 + 1) = -3/2\, \Gamma[-3/2]$

$-2/3\, \Gamma(-1/2) = \Gamma(-3/2)$ - ③

Sub eqn ③ in ②;

$4/5\, \Gamma(-1/2) = \Gamma(-5/2)$ - ④

Put $n = -1/2$ in ①;

$\Gamma(-1/2+1) = -1/2\,\Gamma(-1/2)$

$\cancel{-2}\,(-2\Gamma(1/2) = \Gamma(-1/2)$

$-2\sqrt{\pi} = \Gamma(-1/2) \rightarrow (5)$

sub (5) in (4)

$4/15 \times -2\sqrt{\pi} = \Gamma(-5/2)$

(D) $\frac{-8}{15}\sqrt{\pi} = \Gamma(-5/2)$

(5) Given $B(5/2, 1/2)$

WKT. $B(M,N) = \frac{\Gamma(M)\Gamma(N)}{\Gamma(M+N)}$

$B(5/2, 1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(5/2+1/2)}$

$= \frac{3/2 \cdot 1/2\,\sqrt{\pi}\cdot\sqrt{\pi}}{\Gamma(3)}$

$= \frac{3\pi/4}{2+1}$

$B(5/2, 1/2) = \frac{3\pi}{8}$ (A)

PART - C

(1) Given

$x\cos x + y\sin x = a\sec\alpha$ x is the paramater $\div \cos\alpha$

$x + y\tan\alpha = \frac{a\sec\alpha}{1/\sec\alpha}$

$x + y\tan\alpha = a\sec^2\alpha$

$x + y\tan\alpha = a(1+\tan^2\alpha)$

$a\tan^2\alpha - y\tan\alpha + a - x = 0$

This is in the form of $Am^2 + Bm + c = 0$

$A = a$; $B = -y$; $C = a - x$

The enclopes is $B^2 - 4Ac = 0$

$y^2 - 4a(a-x) = 0$

② Given:

$$\int_0^1 x^6(1-x)^9$$

Hence we know that

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$$

By comparing we get

~~m=6~~ $m-1=6$ $\quad n-1=9$

$m=7$ $\quad n=10$

$$B(m,n) = B(7,10)$$

$$B(7,10) = \frac{\Gamma(7)\Gamma(10)}{\Gamma(7+10)}$$

$$= \frac{6! \times 9!}{16!}$$

$$B(7,10) = 1/80080$$

③ Given:

$$\int_0^{\pi/2} \sin^6\theta \cos^6\theta d\theta$$

Hence we need to compare with:

$$\int_0^{\pi/2} \sin^m\theta \cos^n\theta d\theta = 1/2\, B\left[\frac{m+1}{2}, \frac{n+1}{2}\right]$$

now, $m=n=6$

$$\cancel{\int_0^{\pi/2} \sin^6\theta} \cos^6\theta d\theta$$

Hence

$$\int_0^{\pi/2} \sin^6\theta \cos^6\theta d\theta = 1/2\, B[7/2, 7/2]$$

$$1/2\, B[7/2, 7/2] = 1/2 \frac{\Gamma(7/2)\Gamma(7/2)}{\Gamma(7/2+7/2)}$$

$$= 1/2 \frac{\left[5/2 \cdot 3/2 \cdot 1/2\, \Gamma(1/2)\right]^2}{\Gamma(7)}$$

$= \frac{1}{2} \cdot \frac{223\pi}{64} + \frac{1}{6!}$

$\frac{1}{2} B\left[\frac{7}{2}, \frac{7}{2}\right] = \frac{5\pi}{32 \times 64} = \frac{5\pi}{2048}$

By using Beta gamma functions ;

$\int_0^{\pi/2} \sin^6\theta \cos^6\theta \, d\theta$ is $5\pi/2048$