List 1

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Exercise 1

We want to find an $a^* = \arg\min_a \mathbb{E}(a-y)^2$, where y is a random variable. We know that because y is randomly distributed, by definition its expected value is $\mathbb{E}[y]$. This means that if $a^* = \mathbb{E}[y]$, then on average a^* minimizes $l(a,y)$.

Theorem. *if $l(a,y) = (a-y)^2$, then the Bayes argument that minimizes the loss function is $a^* = \mathbb{E}y$.*

Proof.

$$a^* = \arg\min_a \mathbb{E}(a-y)^2$$
$$a^* = \arg\min_a [\mathbb{E}[y^2] + a^2 - 2a\mathbb{E}[y]]$$

Deriving this expression with respect to a.

$$0 = 2a^* - 2\mathbb{E}[y]$$
$$a^* = \mathbb{E}[y]$$

□

Exercise 2

(a) Based on exercise 1, we know that $f^*(x) = \mathbb{E}[y|x]$.

(b)