Notes on Classical Mechanics

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February 2021

Outline

Preface

/* **TODO**: add some interesting citation here */

Recommended Bibliography

Beside this notes, the reader is encouraged to seek other sources, among which there are some classics (and other more obscure textbooks):

- H. Goldstein, C. Poole and J. Safko, *Classical Mechanics*
  It is the main reference of this notes and of almost every first course on Classical Mechanics, a must-read for everyone interesting in studying the subject.

  /* **TODO**: add some comments about how Goldstein aproaches the problem */

- Arnold, A. Weinstein, K. Vogtmann - *Mathematical Methods Of Classical Mechanics*

What is this book about

Parte I

Newtonian Mechanics

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1| Kinematics and a introduction to Mechanics

We'll first start studying the simplest system possible, a infinitesimal point moving through empty space. But before that, we must first define some concepts.

I Key concepts to Kinematics

a Vectors and Scalars

2| Newtonian Mechanics in 1-D

Parte II

Lagrangian and Hamiltonian Formulations

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3| Elementary Principles

In this chapter, we'll do a quick review on some concepts from the previous part while also adding some key notions for understanding other formulations of mechanics.

I Mechanics of a Particle

Let $\vec{r}$ be the position of an given particle to the origin. We know that the velocity vector $\vec{v}$ is given by:

$$\vec{v} = \frac{d\vec{r}}{dx}.$$

The linear momentum of the particle is defined as the product of the velocity with it's mass:

$$\vec{p} = m\vec{v}.$$

This equations alone can't describe more interesting systems, because they don't account for forces external to the particle. Therefore, we need some relation between the linear momentum and the forces acting on the mass. This is given by **Newton's Second Law**:

$$\sum_{n=1}^{N} \vec{F}_n = \frac{d\vec{p}}{dx}.$$

This equation of motion is a diferential equation (which can be tranformed to a diferential equation of second order with respect to $\vec{r}$, assuming that $\vec{F}$ does not depend on higher order derivatives)

A reference frame in which Newton's Law is valid is called a inertial frame. And one which it isn't is called a non-inertial frame.
Note that for diferentiating inertial"to non-inertial ones we need **Newton's Third Law**. *Only by detecting forces without an action-reaction pair* one may detect that they are on a non-inertial frame.

II Conservation Laws

a Mechanics of a System of Particles

III Degrees of Freedom

We are motivated to define a dynamical variable as any set of variables that *fully* describe our system in its totality (including the change of it undet the action of the forces). We obviously also want to it be a minimal set. Therefore, what we are looking are some variables that are fully indepentent between them (and are themselves functions of time). This functions can be found by solving Newton's Law together with some initial conditions corresponding to each variable.

Usually the dynamic variables are chosen to be positions and/or angles. For a free particle on 3D space for example, the dynamic variables are the cartesian coordinates $\vec{r}(t) = (x(t), y(t), z(t))$.
Unspecified dynamical variables q_k are referred to as *generalized coordinates*. Note that the number of degrees of freedom is an intrisic proprierty of the system, however what are the actual coordinates is up to us. This have to do withe the vector structure of the phase space, more on that later.

IV Constraints

Constraints reduce the number of degrees of freedom. Originally, a N dimensional space with M particles have $N * M$ degrees of freedom. This is a lot of variables to take track of, so the concept of constrains help us to minimize this number.
Take for example a 2D pendullum with radius R. Even thou it is on 2D space,it only takes a single angle θ to fully describe the system.
Therefore if there are j *indepentent* constrains, the system has only $V = N * M - j$ degrees of freedom.
In general, we have two different types of degrees of freedom. The *explicit time dependence* constrains are the ones we call *holonomic*, while time indepentent constrains are called *scleronomic*.

4| Lagrangian Formalism

At this point, one may ask: "Why do we need other formulations for Classical Mechanics? Isn't Newton's formulation complete? (in a sense that it can describe all physical systems)".
The answer for the first question is highlighted by example (???), It shows that albeit Newton's formulation is indeed complete,when deriving the equations of motion starting from the diagram of forces it cannot account correctly for one thing, **Human mistake**.

The *Action* S is defined as:

$$S = \int_{t_0}^{t_1} \mathcal{L}(q, \dot{q}, t) \, dt \tag{4.1}$$

The Principle of Least Action states as follows.

Principle of Least Action. *The evolution of a physical system in the time interval* $[t_0, t_1]$ *corresponds to a stationary point of the action.*

Even thou the action usually takes a minimum on the true trajectory of the system, this isn't always true, as this example shows.

Before we go any further, I would like to make a brief discussion about the intuition behind the Least Action principle.

/* **TODO**: add discurssion about Least action principle */

5| Exoteric formulations of Classical Mechanics