$A_{3\times3}$ $\quad \lambda_1 = 1, \quad \lambda_2 = 2, \quad \lambda_3 = 3$

$\quad\quad\quad x_1 \quad\quad x_2 \quad\quad x_3$ are LI

LI : If : $\sum_{i=1}^{n=3} \alpha_i x_i = 0 \Rightarrow \alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0$ as the only solution.

(λ, x) EV Evector pair iff $Ax = \lambda x$ $\quad x \neq 0$

Let A be a $n\times n$ matrix & $\lambda_1, \lambda_2, \ldots \lambda_n$ be its distinct eigenvalues. $\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq \ldots \neq \lambda_n$

Let $x_1, x_2, \ldots x_n$ be the corr. eigenvectors

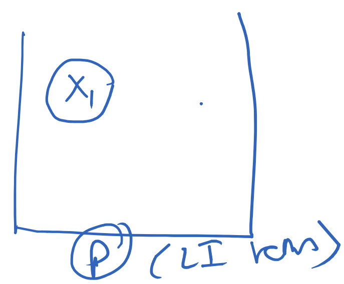
$\therefore Ax_1 = \lambda_1 x_1; \; Ax_2 = \lambda_2 x_2; \; \ldots \; Ax_n = \lambda_n x_n$

$P = \{ \overset{\neq 0}{x_1}, \overset{\neq 0}{x_{3,4}}, \ldots x_n \}$ is LI Then result is proved.

LI

LD

$P = \{ x_1, x_2, \ldots x_r \}$ $\quad r < n \quad$ is LI



Cooking : x_1 = turmeric

x_2 = Salt

x_3 = Sambar powder

x_4 = garam masala powder

x_5 = Hing

x_6 = Sugar $\quad x_7$ = Cinnamon $\ldots$

$P = \{$ turmeric, Salt, Hing, .. Cinnamon $\}$ is LI

---

x to x_i

$x_1, x_2 \dots x_n$ as eigenvectors $\lambda_i \neq$ to λ_2

$P = \{x_1, x_2 \dots x_r\}$ $r < n$ to be LI.

$$\boxed{x_{r+1} = \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_r x_r} \quad (1)$$

$$A x_{r+1} = A\alpha_1 x_1 + A\alpha_2 x_2 + \dots + A\alpha_r x_r$$

$$\lambda_{r+1} x_{r+1} = \boxed{\alpha_1 \lambda_1 x_1} + \alpha_2 \lambda_2 x_2 + \dots + \alpha_r \lambda_r x_r \quad (2)$$

Multiply (1) with λ_{r+1} throughout

$$\lambda_{r+1} x_{r+1} = \boxed{\alpha_1 \lambda_{r+1} x_1} + \alpha_2 \lambda_{r+1} x_2 + \dots + \alpha_r \lambda_{r+1} x_r \quad (3)$$

(2) − (3)

$$\boxed{0} = \alpha_1 x_1 (\lambda_1 - \lambda_{r+1}) + \dots + \alpha_r x_r (\lambda_r - \lambda_{r+1})$$

$$\alpha_1 \underbrace{\boxed{(\lambda_1 - \lambda_{r+1})}}_{\neq 0} = 0 \Rightarrow \alpha_1 = 0$$

$$\alpha_2 \underbrace{\boxed{(\lambda_2 - \lambda_{r+1})}}_{\neq 0} = 0 \Rightarrow \alpha_2 = 0$$

$\vdots$

$$\alpha_r \underbrace{\boxed{(\lambda_r - \lambda_{r+1})}}_{\neq 0} = 0 \Rightarrow \alpha_r = 0$$

$\Rightarrow x_{r+1} = 0$, a contradiction because x_{r+1} being an eigenvector $\neq 0$

x_1 = turmeric, x_2 = salt,

x_3 = hing, x_4 = thoor dhal

$\dots$ x_5 = $\boxed{\text{Sambar}}$

$P = \{x_1, x_2, x_3, x_4$.

$P = \{ x_1, x_2, x_3, x_4,$

$\lambda_1 \neq \lambda_2, \dots \neq \lambda_n$

$P : \{ x_1, x_2, \dots, x_n \}$ LI

$A_{n \times n}$ eigenvalues $\boxed{\lambda_1, \lambda_2, \dots \lambda_n}$ 1, 2, 3, 6, 8, 3

corr. ev $x_1, x_2, \dots x_n$

$\therefore A x_i = \lambda_i x_i \quad \forall i$

$P_{n \times n}$ non-singular. P^{-1} exists QAQ^{-1}.

$\hat{A}_{n \times n} = P^{-1} A P \; ; \; P^{-1} = Q, \; P = Q^{-1}$

Theorem: Eigenvalues of $\hat{A}$ are $\lambda_1, \lambda_2 \dots \lambda_n$

If λ is an eigenvalue of A, then λ is an eigenvalue of $\hat{A}$.

$A \quad P^{-1} = \begin{bmatrix} 3 & -1 \\ 2 & -6 \end{bmatrix}$

$\boxed{\lambda_1 = 2}, \; x_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$P^{-1} A P$

Proof: $(\lambda, x) \rightarrow$ pair of A

$\Rightarrow A x = \lambda x \quad (x \neq 0)$

$A I x = \lambda x \qquad I$ is the identity matrix

$A \underbrace{P P^{-1}} x = \lambda x$

$P^{-1} A P P^{-1} x = P^{-1} \lambda x$

$\hat{A} (P^{-1} x) = \lambda (P^{-1} x)$

λ is an eigenvalue of $\hat{A}$? Yes

λ is an eigenvalue of $\hat{A}$? Yes

$P^{-1}x$ is the corr. eigenvector

A ~~for nxn~~ any general matrix

$\boxed{\lambda_1, \lambda_2, \ldots \lambda_n \text{ evalues}}$

$x_1, x_2, \ldots x_n$ are LI eigenvectors

$$P = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ x_1 & x_2 & \cdots & x_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

$\det(P) \neq 0$

P is invertible

$$P^{-1}AP = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

$A_{n \times n}$ $\lambda_1, \lambda_2, \ldots \lambda_n$ evalues

$x_1, x_2, \ldots x_n$ evecs

$Ax_i = \lambda_i x_i \quad 1 \leq i \leq n$ ✓

$$\boxed{\begin{aligned} Ax_1 &= \lambda_1 x_1 \\ Ax_2 &= \lambda_2 x_2 \\ &\vdots \\ Ax_n &= \lambda_n x_n \end{aligned}}$$

$$A \begin{bmatrix} x_1 & & \end{bmatrix} = \begin{bmatrix} x_1 & & \end{bmatrix} \begin{bmatrix} \lambda_1 & \cdots \\ 0 & \\ \vdots & \end{bmatrix}$$

$$A \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ x_1 & x_2 & \cdots & x_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ x_1 & x_2 & \cdots & x_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix} \begin{bmatrix} \lambda_1 & \cdots & & 0 \\ \vdots & \lambda_2 & & \\ \vdots & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

$$A \quad P = P \cdot \Lambda$$

$$\Rightarrow \quad P^{-1} A P = \Lambda \rightarrow \text{diagonal matrix.}$$

Suppose $A_{n \times n} \rightarrow$ find $\underbrace{A^{28}, A^{51}, A^{68}, A^{79} \dots}$

$$A \cdot A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \vdots & & \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

(ii) $\quad n^3 M \quad n^2(n-1) A$

$A^4 \rightarrow$ total M :

$$A^4 = \underbrace{\underbrace{\underbrace{A \cdot A}_{\substack{n^3 M \\ n^2(n-1) A}} \cdot A}_{\substack{n^3 M \\ n^2(n-1) A}} \cdot A}_{\substack{n^3 M \\ n^2(n-1) A}}$$

$3 n^3 M, \quad 3 (n^2(n-1)) A$

$A^k \quad (k-1) \cdot n^3 M, \quad (k-1) \cdot n^2 (n-1) A$?

$$A^4 = A^2 \cdot A^2 = \begin{matrix} n^3 M, & n^2(n-1) A \\ n^3 M, & n^2(n-1) A \end{matrix}$$

$2n^3$ M , $2n^2(n-1)$ A

$A \to$ Ev. $\to$ Evecs $\to P \to P^{-1} \to$ Ske

$$P^{-1} A P = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

$$P^{-1} A \cancel{P} \cdot \cancel{P^{-1}} A P = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

$$P^{-1} A^2 P = \begin{pmatrix} \lambda_1^2 & & 0 \\ & \ddots & \\ 0 & & \lambda_n^2 \end{pmatrix}$$

$\vdots$

$$P^{-1} A^k P = \begin{pmatrix} \lambda_1^k & & 0 \\ & \ddots & \\ 0 & & \lambda_n^k \end{pmatrix} , \quad k \in \mathbb{N}.$$

A^{28}

$$P^{-1} A^{28} \cdot P = \begin{pmatrix} \lambda_1^{28} & & \\ & \ddots & \\ & & \lambda_n^{28} \end{pmatrix}$$

$$A^{28} = P \begin{pmatrix} \lambda_1^{28} & & \\ & \ddots & \\ & & \lambda_n^{28} \end{pmatrix} P^{-1}$$

$$P^{-1} A \cancel{P}\, \cancel{P^{-1}} A \cancel{P}\, \cancel{P^{-1}} A \cancel{P}\, \cancel{P^{-1}} A P$$

$$= P^{-1} A^4 P = \begin{pmatrix} \lambda_1^4 & & 0 \\ & \ddots & \\ 0 & & \lambda_n^4 \end{pmatrix}$$

$A_{n \times n}$ $(\lambda_1, x_1) \ldots (\lambda_n\ x_n)$.

$A x_1 = \lambda_1 \underline{x_1}$, $A x_2 = \lambda_2 \underline{x_2} \quad \ldots \quad A x_n = \lambda_n \underline{x_n}$

$x_1, x_2, \ldots, x_n$ are LI

$\mathbb{R}^n$

x_0 as an initial approximation.

$$x_0 = c_1 x_1 + \cdots + c_n x_n$$

λ_1 d.e.v

$$A x_0 = A c_1 x_1 + \cdots + A c_n x_n$$

$$= c_1 . \lambda_1 x_1 + \cdots + c_n \lambda_n x_n$$

$$A^2 x_0 = A . c_1 \lambda_1 x_1 + \cdots + A c_n \lambda_n x_n$$

$$= c_1 . \lambda_1^2 x_1 + \cdots + c_n \lambda_n^2 x_n$$

$$A^k x_0 = c_1 \lambda_1^k x_1 + c_2 \lambda_2^k x_2 + \cdots + c_n \lambda_n^k x_n$$

$$= c_1 \lambda_1^k \left[x_1 + \frac{c_2}{c_1} \left(\frac{\lambda_2}{\lambda_1}\right)^k x_2 + \cdots + \frac{c_n}{c_1} \left(\frac{\lambda_n}{\lambda_1}\right)^k . x_n \right]$$

$\to 0$

$\to 0$

$k = 25$

$\frac{\lambda_2}{\lambda_1} = (0.1)$

$(0.1)^{25}$

(0.88)