13 November 2021 11:00

elearn.bits-pilani.ac.in

→ Username : ____ @wilp.bits-pilani.ac.in

→ password : ____

Courses

$Ax = b$ $\quad A_{n\times n}$ $\quad x_{n\times 1}$ $\quad b_{n\times 1}$

$\cancel{A^{-1}Ax = A^{-1}b}$

3x3, 4x4, 5x5 → $\det(A) \neq 0$; $\boxed{x = A^{-1}b}$

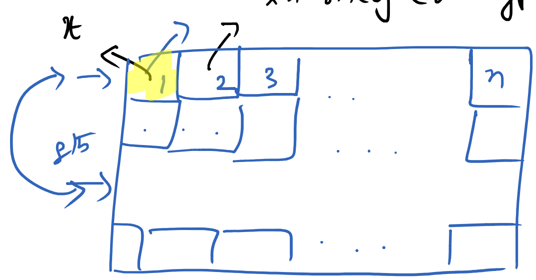
100x100 } → Algorithms

GE algorithm = $O\left(\frac{2}{3}n^3\right)$

partial pivoting → $\boxed{a_{11} = 0}$

x + size (datatype)

$\boxed{\text{double } **A;}$



$Ax = b$, $n = 10000$, FE ~ 11 min, BS ~ .01 sec

$Ax = b_1$, $Ax = b_2$ . . . $Ax = b_k$

GE FE 11 min, BS .01 s; 11 min, .01 s; . . . 11 min, 0.01 sec

BS .01 s .01 s

Total time : $(11 \cdot k)$ mins $+ \frac{(0.01) \cdot k}{60}$ mins

$= \left(11 + \frac{0.01}{60}\right) k$

$= 11k$

eg: $k = 25$; time : 275 min

$A = LU$ & solve L & U

Doolittle's method

$$L = \begin{bmatrix} 1 & & \\ l_{21} & 1 & \\ l_{31} & l_{32} & 1 \\ l_{n1} & l_{n2} \cdots & l_{n\,n-1} \; 1 \end{bmatrix} ; \quad U = \begin{bmatrix} \cancel{u_{11}} & \cancel{u_{12}} & \cdots & \cancel{u_{1n}} \\ 0 & u_{22} & \cdots & u_{2n} \\ & & & \\ 0 & \cdots & & u_{nn} \end{bmatrix}$$

$a_{11} \quad a_{12} \quad a_{1n}$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = [LU]$$

I row of U : ✓

I col of L ✓

II row of U

II col of L

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \cancel{l_{21}}\,4 & 1 & 0 \\ \cancel{l_{31}}\,3 & l_{32} & 1 \end{bmatrix} \begin{bmatrix} \cancel{u_{11}}\,1 & \cancel{u_{12}}\,2 & \cancel{u_{13}}\,3 \\ 0 & \cancel{u_{22}}\,-3 & \cancel{u_{23}}\,-6 \\ 0 & 0 & u_{33} \end{bmatrix}$$

9 = 3 + 6 = 9

$4 \cdot 2 + u_{22} = 5$

$8 + u_{22} = 5 \Rightarrow u_{22} = -3$

$l_{21} \cdot u_{13} + 1 \cdot u_{23} = 6$

$L \cdot z = c$

$O(n^2)$ steps

$4 \cdot 3 + u_{23} = 6$

$u_{23} = -6$

$L : O(n^3/3)$ steps

$U : O(n^3/3)$ steps $\therefore$ $L+U : O(\frac{2}{3}n^3)$

$$\boxed{\begin{array}{l} GE \sim O(\frac{2}{3}n^3) \text{ steps FE} \\ \quad O(n^2) \text{ steps BS} \end{array}}$$

$$\boxed{\begin{array}{l} LU : \text{decomposition } O(\frac{2}{3}n^3) \\ \quad FS, BS \sim O(n^2) \text{ steps} \end{array}}$$

$Ax = b \; ; \; A = LU$ (L → LT, U → UT)

$\therefore L(Ux) = b$, $\boxed{Ux = y}$

$Ly = b \Rightarrow y$ can be got in $O(n^2)$

$Ux = y \Rightarrow x$ can be obtained in $O(n^2)$

$Ax = b \quad LU \rightarrow \underbrace{O(\frac{2}{3}n^3)}_{L \cdot U} + \underbrace{O(n^2)}_{y} + \underbrace{O(n^2)}_{x}$

$Ax = b_1$ $Ax = b_2$ $\cdots$ $Ax = b_{25}$

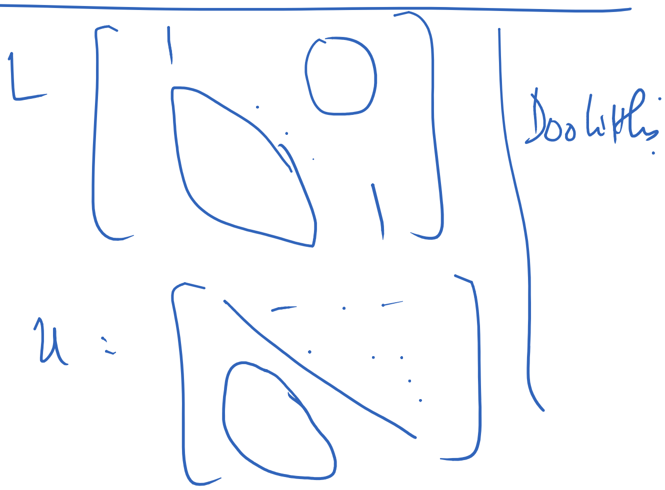
GE   11m + 0.01s ,   11m + 0.01s $\sim$ 275 mi + ~~0.01s~~ 0.25

GE $\quad 11_m + 0.01s \quad , \quad \| \quad 11m + 0.01s \sim \ldots + 2s\,\cancel{0.01s}$

LU $\quad \boxed{11min + 0.01s + 0.01s} \quad 0.01 + 0.01 \quad 0.01 + 0.01 \sim$

$$11m + (\cancel{(0.01)}\, 25s)$$

$A = L \cdot U$ $\quad$ Doolittle



$$A = LU$$

$$L : \begin{array}{c} 1 \\ 2 \\ \\ n \end{array} \begin{bmatrix} l_{11} & 0 & \cdots & \\ l_{21} & l_{22} & \cdots & \\ l_{31} & l_{32} & l_{33} & \cdots \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix} \qquad 1+2+\cdots n = \frac{n(n+1)}{2}$$

$+$

$$U : \begin{bmatrix} 1 & u_{12} & \cdots & u_{1n} \\ 0 & \ddots & \neq 0 & \\ 0 & \cdots & 0 & 1 \end{bmatrix} \begin{array}{c} n-1 \\ n-2 \\ \vdots \\ 0 \end{array} \qquad \frac{n(n-1)}{2}$$

$= n^2$

Crout decomp

$O(\frac{2}{3} n^3)$

FS $O(n^2)$

BS $O(n^2)$

$$A = \begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix} ; \quad A = A^T$$

$$A = U^T U = \begin{bmatrix} U^T \end{bmatrix} \begin{bmatrix} U \end{bmatrix}$$

$$A = U' U = \begin{bmatrix} U' \end{bmatrix} \begin{bmatrix} U \end{bmatrix}$$

$$U^T = \begin{bmatrix} 2 & 0 & 0 \\ 6 & 1 & 0 \\ -8 & 5 & 3 \end{bmatrix}$$

$$U : \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\overset{U^T}{\begin{bmatrix} u_{11} & 0 & 0 \\ u_{12} & u_{22} & 0 \\ u_{13} & u_{23} & u_{33} \end{bmatrix}} \overset{U}{\begin{bmatrix} u_{11} & \boxed{u_{12}} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}} = \begin{array}{ccc} -2 & -6 & 8 \\ \end{array} \begin{bmatrix} 2 & 6 & -8 \\ & & \\ & & \end{bmatrix}$$

$$u_{11}^2 = 4 \Rightarrow u_{11} = 2, \quad u_{11} = -2$$

Fixed pt theorem

$$X_{i+1} = \alpha x_i + \beta$$

$$7x_i = x_{i+1} + 18; \qquad x = 3$$

$$x_{i+1} = 7x_i - 18$$

$$\left.\begin{aligned} X_1 &= 7X_0 - 18 \\ X_2 &= 7X_1 - 18 \\ &\vdots \\ X_{1000} &= 7X_{999} - 18 \end{aligned}\right\} \quad \boxed{X_0}$$

$$x_2 = 7x_1 - 18 = 7(7x_0 - 18) - 18$$
$$= 7^2 x_0 - 7 \cdot 18 - 18$$

$$x_3 = 7x_2 - 18 = 7^3 x_0 - 7^2 \cdot 18 - 7 \cdot 18 - 18$$

$$x_n = 7^n x_0 - 7^{n-1} \cdot 18 - 7^{n-2} \cdot 18 \dots - 18$$

$$\boxed{7x_i = x_{i+1} + 18} \qquad = \underline{\underline{7^n x_0}} - 18\underbrace{(1 + 7 + 7^2 \dots + 7^{n-1})}$$

$$\boxed{7x_{i+1} = x_i + 18} \Rightarrow x_{i+1} = \frac{x_i + 18}{7}$$

$$x_1 = \frac{x_0}{7} + \frac{18}{7}$$

$$1 + r + \dots + r^n = \frac{1}{1-r}$$

$$\underline{\underline{|r| < 1}}$$

$$x_2 = \frac{x_0}{7^2} + \frac{18}{7^2} + \frac{18}{7}$$

$$\vdots$$

$$x_n = \frac{1 x_0}{7^n} + \frac{18}{7}\left(\frac{1}{7^{n-1}} + \frac{1}{7^{n-2}} + \dots + 1\right)$$

$n \sim 1000$ $\quad \to 0$

$$+ \frac{18}{7}\left(\frac{1}{1 - 1/7}\right)$$

$$= \frac{\cancel{18}^{3}}{\cancel{7}}\left(\frac{\cancel{7}}{\cancel{6}}\right) = 3$$

$$x_1 + 2x_2 = 7$$
$$\frac{3}{4}x_1 + \frac{4}{4}x_2 = \frac{11}{4}$$

$$Ax = b$$

$$A = \begin{bmatrix} 1 & 2 \\ 3/4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 3/4 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 3/4 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= I + L + U$$

$$A = I + L + U \;;$$

$$Ax = b$$

$$\Rightarrow (I + L + U)x = b$$

$$\boxed{(I+L)\,x_{i+1}} = -\boxed{Ux_i + b}$$

$$x_{i+1} = \boxed{-(I+L)^{-1}U}\, x_i + (I+L)^{-1} b$$

$7x_{i+1} = x_i + 18$

$7x_{i+1} = -x_i + 18$

$x_{i+1} = \frac{x_i}{7} + \frac{18}{7}$

$$\boxed{\text{G. Seidel algorithm}}$$

$$\|(I+L)^{-1}U\|_{1,\, \text{inf},\, \text{fro}} < 1$$

$$\begin{aligned} 7x_1 + x_2 + 3x_3 &= 23 \\ 2x_1 + 6x_2 + x_3 &= 18 \\ x_1 + x_2 + 4x_3 &= 11 \end{aligned}$$

$$x_1 = \frac{23 - x_2 - 3x_3}{7}$$

$$x_2 = \frac{18 - 2x_1 - x_3}{6}$$

$$x_3 = \frac{11 - x_1 - x_2}{4}$$

$$\begin{bmatrix} 1 & 1/7 & 3/7 \\ 2/6 & 1 & 1/6 \\ 1/4 & 1/4 & 1 \end{bmatrix} \begin{bmatrix} 23/7 \\ 3 \\ 11/4 \end{bmatrix}$$

$$Ax = b \;;\; (I + L + U)x = b \,.$$

$$Ix = -Lx - Ux + b$$

$$Ix_{i+1} = -(L+U)x_i + b$$

1/2

$$\|L+U\|_{1,\infty\ldots} < 1$$