Birla Institute of Technology and Science, Pilani

Work Integrated Learning Programmes Division

Session Coverage Plan for DSE* ZC416

Mathematical Foundation for Data Science

Semester: I Sem 2021-22

Preface

Welcome to the course on Mathematical Foundation for Data Science.

About the course

This course introduces you to the basics of linear algebra, optimization (linear and non-linear) and to a formal setting in discrete structures, thereby equipping you to understand and apply concepts in Data Science from a mathematical view point. Essential calculus would also be covered to introduce the reader to non-linear optimization. The focus would be both on the theoretical aspects and practical applications, accompanied by exemplifications using MATLAB / Octave / Excel / TORA, wherever applicable. The emphasis would be more on the algorithmic aspects and some classical results would be proved in the "Theorem - Proof" style to a) introduce a formal setting and b) strengthen the logic. The reader is encouraged to code the algorithms discussed in the class to get a deeper insight into the complexity.

About this document

This document is a detailed version of the handout outlining the contents which would be covered in each of the 16 sessions. Expectations on pre-reading is outlined and it would be beneficial to come prepared so that the session time could be utilized to cover the content it is intended for. The reader is encouraged to do the homework problems given in the document and utilize the discussion forum to post doubts, if any.

Happy learning and all the very best.

The MFDS Team

Contents

Relevance of topics

S. No.	Module	Relevant in Course(s)
1	System of linear equations	ML
2	Vector spaces	ML / DL / NLP
3	Eigenvalues	ML / DL
4	SVD	ML / DL
5	Linear programming	Probabilistic graphical models
6	Calculus	ML / DL
7	Counting & Structural induction	ISM, Structural induction is relevant in decision tree induction, Probabilistic graphical models, graph mining.

0 Recapitulation of elementary concepts

0.1 Pre-reading to be done by students

Students are expected to come prepared with

a) matrices, their representations and operations

b) compatibility condition for addition and multiplication

c) storage and complexity in computation of the sum and product

0.2 What would be covered in the contact session

a) writing $S = A + B$ and $P = AB$ as $s_{ij} = a_{ij} + b_{ij}$ and $p_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$, where n is defined from the compatibility condition.

b) properties of determinants including $\det(AB) = \det A \ \det B$ and simpler ways of evaluating determinants

c) row reduction techniques for matrices

d) concept of rank of a matrix and deduction from determinants and row reduction

e) use of row reduction techniques to deduce the consistency and inconsistency of linear systems using exercises from the text book

0.3 Exemplification to be done in contact session

Use of MATLAB / Octave / Excel to perform some of the matrix operations like finding

a) matrix vector product and matrix matrix product

b) determinant of a matrix

c) rank of a matrix

d) inverse of a matrix

e) consistency of a linear system

0.4 Homework problems

Q1 Let $A_{m\times n}$ be a given matrix with $m > n$. If the time taken to compute the determinant of a square matrix of size j is j^3, find upper bound on the

a) total time taken to find the rank of A using determinants

b) number of additions and multiplications required to determine the rank using the elimination procedure.

Q2 Let $A_{n\times n}$ be a given square matrix. Compute the number of multiplications and additions required to evaluate A^{28} using

a) the naive method, $A^{28} = \underbrace{A \cdot A \cdots A}_{28 \text{ times}}$

b) A^2, $A^4 = A^2 \cdot A^2$, etc.

Q3 Modelling of electrical / traffic networks would lead to a linear system $Ax = b$. Refer to the text book / other resources and construct a network which has the following properties

a) the number of equations are 6

b) A has rank 5

c) the system is consistent.

Q4 Write a code to implement d digit significant arithmetic and solve the following system and compare the results for $d = 2, 3, 4, 5$ by eliminating one of the variables?

$$\begin{aligned} 0.1036x_1 + 0.2122x_2 &= 0.7381 \\ 0.2081x_1 + 0.4247x_2 &= 0.9327 \end{aligned}$$

Do you think that the solution would change if we interchange the two equations? Compare by Tabulating the results.

1 Analytical techniques for systems of linear equations

1.1 Pre-reading to be done by students

Students are expected to come prepared with

a) elementary row operations

b) result on consistency of linear systems

c) closed form expressions for $\sum_{i=1}^{n} i, \ \sum_{i=1}^{n} i^2$

1.2 What would be covered in the contact session

a) recapitulation of solutions of linear systems

b) condition number and its significance

c) example of a 2×2 linear system for pivoting using Gauss elimination

d) operation count for Gauss elimination

e) LU decomposition methods - an overview

1.3 Exemplification to be done in contact session

Use of MATLAB / Excel to perform some of the matrix operations like finding

a) norm and condition numbers

b) LU decomposition

1.4 Homework problems

Q1 Consider a matrix $A_{n\times n}$ and do the following.

i) Write a code (in any language of your choice) to implement the Gauss elimination method with and without partial pivoting.

ii) Check the code with the examples given in the text book.

iii) Create random matrices using the random number generator and plot the time taken for the forward elimination and back substitution against n. Consider cases for which $n = 5, 10, 15, 20, \ldots, 100$.

iv) Do you observe the orders for iii) with the orders mentioned in the text book?

Q2 Implement the method of Dolittle and Crout in solving $A_{n\times n}x = b$. Check the code by inserting data from the examples / exercises given in the book.

2 Numerical solution of linear systems

2.1 Pre-reading to be done by students

Students are expected to come prepared with

a) matrix algebra

b) the concept of diagonally dominant matrices

c) concepts covered in the previous session

2.2 What would be covered in the contact session

a) LU decomposition continued from previous session

b) writing A in $Ax = b$ as $L + D + U$

c) iterative solution with Gauss Jacobi method

d) iterative scheme with Gauss Seidel method

e) introduce to vector and matrix norms (row sum, column sum and Frobenius) with example

f) convergence criteria (only the sufficient part)

2.3 Exemplification to be done in contact session

Use of MATLAB / Excel to show example of

a) Row sum, column sum and Frobenius norms

b) Gauss Jacobi and Gauss Seidel schemes for an example

2.4 Homework problems

Q1 Consider a matrix $A_{n\times n}$ and do the following.

i) Write a code (in a language of your choice) to derive the matrices L and U such that $A = I + L + U$, as given in the text book.

ii) Check the code with the examples given in the text book.

iii) Add a subroutine to the code in i) to calculate the l_1, l_∞ and Frobenius norms.

iv Compute $\|(I+L)^{-1}U\|$ using the three norms mentioned in iii) to check the criteria for convergence.

iv) Calculate the spectral value (the largest eigenvalue in magnitude) and check if that value is < 1.

MATLAB / OCTAVE would be convenient as most of the routines are available.

Q2 Solve problems 19 and 20 in Problem Set 20.4 (page no. 868) in the text book T1. Write a code to compute the condition number of a matrix, using part iii) of problem 1 and use that to evaluate the required quantities for the two problems.

Q3 Implement the method of Dolittle and Crout in solving $A_{n\times n}x = b$. Check the code by inserting values from the examples given in the book.

3 Vector spaces and Linear transformations

3.1 Pre-reading to be done by students

Students are expected to come prepared with

a) concepts and properties of scalars and vectors

b) algebraic properties like associativity, commutativity and distributivity

c) scalar vector multiplication, vector vector addition

d) concept of rank introduced in session 1

3.2 What would be covered in the contact session

a) concept of a field (as an Abelian group with respect to addition and multiplication)

b) definition and examples of vector spaces and sub-spaces

c) inner product spaces

d) concept of linear dependence and independence of vectors using the rank

e) linear span of a finite set

f) concept of basis and dimension with examples including $\mathbb{R}^n$

g) dependence of basis and dimension on the choice of the field - examples

h) linear transformation - definition and examples

i) proving range and kernel as subspaces

j) the rank-nullity theorem (statement) with examples from matrices and linear transformations

k) NS(A), RS(A) and CS(A) - concepts and examples

3.3 Exemplification to be done in contact session

Use of MATLAB / Excel to perform some of the matrix operations like finding

a) linear dependence and independence of vectors

b) determining rank for the rank-nullity theorem

3.4 Homework problems

Q1 Let $\mathbf{B} = (\mathbf{b_1}, \mathbf{b_2}, \ldots, \mathbf{b_{r-1}}, \mathbf{b_r}, \mathbf{b_{r+1}}, \ldots, \mathbf{b_n})$ be a non-singular matrix. If column $\mathbf{b_r}$ is replace by $\mathbf{a}$ and that the resulting matrix is called $\mathbf{B_a}$ along with $\mathbf{a} = \sum_{i=1}^{n} y_i \mathbf{b_i}$, then state the necessary and sufficient condition for $\mathbf{B_a}$ to be non-singular.

Q2 Let V be a finite dimensional vector space over $\mathbb{R}$. If S is a set of elements in V such that $\text{Span}(S) = V$, what is the relationship between S and the basis of V?

Q3 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$$

then,

1. show that T is a linear transformation
2. what are the conditions on a, b, c such that (a, b, c) is in the null space of T. Specifically, find the nullity of T.

Q4 Construct a linear transformation $T : V \to W$, where V and W are vector spaces over F such that the dimension of the kernel space of T is 666. Is such a transformation unique? Give reasons for your answer.

4 Eigenvalues and Eigenvectors of a Matrix - Part I

4.1 Pre-reading to be done by students

Students are expected to come prepared with

a) definition and properties of matrices including orthogonal, idempotent, symmetric, skew-symmetric; transpose and trace of matrix

4.2 What would be covered in the contact session

a) definition and examples of eigenvalues

b) computation of eigenvalues using the definition and properties of matrices - through examples

c) use of Greshgorin's theorem to determine the bounds for eigenvalues

d) eigenvalues of powers of matrices and inverse of a matrix

e) definition and determination of eigenvectors

4.3 Exemplification to be done in contact session

Use of MATLAB / equivalent software to perform some of the matrix operations like finding

a) eigen values and eigenvectors for a variety of matrices

4.4 Homework problems

Q1a) Let P be a real square matrix satisfying $P = P^T$ and $P^2 = P$.

i) Can the matrix P have complex eigenvalues? If so, construct an example, else, justify your answer.

ii) What are the eigenvalues of P?

b) Given the following matrix $A = \begin{pmatrix} 1 & 2 & r \\ c & 1 & 7 \\ c & 1 & 7 \end{pmatrix}$ where c and r are arbitrary real numbers and $5.5 < r \leq 6.5$, and the fact that $\lambda_1 = 3$ is one of the eigenvalues, is it possible to determine the other two eigenvalues? If so, compute them and give reasons for your answer.

Q4 Construct examples of matrices for which the **defect** is positive, negative and zero wherever possible.

Q4 How do you check if a matrix is positive semi-definite? Construct symmetric, positive semi-definite matrices and check their eigenvalues and eigenvectors. What do you observe?

5 Eigenvalues and Eigenvectors of a Matrix - Part 2

5.1 Pre-reading to be done by students

Students are expected to come prepared with

a) content covered in the previous session

b) linear dependence and independence concepts

5.2 What would be covered in the contact session

a) proof of the statement - eigenvectors corresponding to distinct eigenvalues are linearly independent

b) similarity of matrices and diagonalization

c) power method for eigenvalue and eigenvector pair determination numerically

5.3 Exemplification to be done in contact session

Use of MATLAB / equivalent software to perform some of the matrix operations like finding

a) eigen values and eigenvectors for a variety of matrices

b) similarity transformations for diagonalization

5.4 Homework problems

Q2 The Fibonacci sequence is defined by $V_n = V_{n-1} + V_{n-2}$ for $n \geq 2$ with starting values $V_0 = 1$ and $V_1 = 1$. Observe that the calculation of V_k requires the calculation of $V_2, V_3, \ldots, V_{k-1}$. To avoid this, could this problem be written as an eigenvalue problem and solved for V_n directly? If so, find the explicit formula for V_n.

Q3 Prove that if A is a square matrix of size $n \times n$, then $A^k \to 0$ as $k \to \infty$ if and only if $|\lambda_i| < 1 \quad \forall i$.

6 Singular value decomposition

6.1 Pre-reading to be done by students

Students are expected to come prepared with

a) orthogonal matrices and their properties

b) eigenvalues, eigenvectors and similarity transformations

6.2 What would be covered in the contact session

a) Gram-Schmidt orthogonalization process

b) derivation of singular value decomposition of a matrix

c) determination of U, $\sum$ and V

d) a simple example for which the above are evaluated

e) dimensionality reduction

6.3 Exemplification to be done in contact session

Use of MATLAB / equivalent to exhibit

a) svd for a few non-square matrices

b) dimensionality reduction using svd

6.4 Homework problems

Q1 Consider a matrix A and do the following.

i) Enter the matrix A in MATLAB / Octave using

$$A = [a_{11}\ a_{12}\ \ldots a_{1n}; a_{21}\ a_{22}\ \ldots a_{2n}; \cdots ; a_{m1}\ a_{m2}\ \ldots a_{mn}];$$

for a fixed m and n which are small with $m < n$ (say $m = 2$ and $n = 3$).

ii) Evaluate $A^T A$ and AA^T and find their eigenvalues and eigenvectors. You could do it in MATLAB / Octave using the command $[E, V] = \text{eigs}(B)$ for a given matrix B. Are the eigenvectors orthogonal? Are they orthonormal?

iii) Use the command $[U\ S\ V] = \text{svd}(A)$ and compare the values of U and V with the eigenvector matrix obtained in step ii).

iv) Do you observe the decreasing order in which the singular values appear in S.

v) Repeat the above for the case $m = n = 3$ (say)

vi) Does the eigendecomposition of A in v) coincide with the SVD of A?

vii) Do you see some relationship between eigenvalues and singular values, in case of a square matrix?

Q2 The svd is derived from reduced svd. Refer to any web source and find out what reduced svd means and how svd is obtained from it.

7 Applications of Linear Algebra in Optimization - Part 1

7.1 Pre-reading to be done by students

Students are expected to come prepared with

a) basic modelling using linear systems - review from session 1

b) matrix vector products

c) rank of a matrix

7.2 What would be covered in the contact session

a) modelling of 3 problems using linear systems

Taken from the book of Operations Research by Hamdy A Taha, 8th edition, Pearson. These numbers in front of the problems correspond to the examples used in the book.

1. Example 2.1-1 (The Reddy Mikks Company)

Reddy Mikks produces both interior and exterior paints from two raw materials, M1 and M2. The following table provides the basic data of the problem:

	Total of raw material per ton of Exterior paint	Interior paint	Maximum daily availability (tons)
Raw material, M1	6	4	24
Raw material, M2	1	2	6
Profit per ton ($1000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that for a exterior paint by more than 1 ton. Also, the maximum daily demand for interior paint is 2 tons.

Reddy Mikks wants to determine the optimum(best) product mix of interior and exterior paints that maximizes the total daily profit.

2. Example 2.3-1 (Urban Renewal Model)

The city of Erstville is faced with a severe budget shortage. Seeking a long term solution, the city council votes to improve the tax base by condemning an inner city housing area and replacing it with a modern development.

The project involves two-phases (1) demolishing substandard houses to provide land for the new development, and (2) building the new development. The following is a summary of the situation.

i) Many as 300 substandard houses can be demolished. Each house occupies a 0.25-acre lot. The cost of demolishing a condemned house is \$2000.

ii) Lot sizes for new single,double, triple and quadruple family homes (units) are .18, .28, .4 and .5 acres respectively. Streets, open space, and utility easements accounts for 15% of available acreage.

iii) In the new development the triple and quadruple units account for at least 25% of the total. Single units must be at least 20% of all units and double units at least 10%.

iv) the tax levied per unit for single, double, triple and quadruple units is \$1,000, \$1,900, \$2,700 and \$3,400 respectively.

v) The cost of construction per unit for single, double, triple and quadruple family homes is \$50,000, \$70,000, \$130,000 and \$160,000 respectively. Financing through a local bank amount to a maximum of \$15 million.

How many units of each type should be constructed to maximize the tax collection?

3. Example 2.3-2 (Currency Arbitrage Model)

Suppose that a company has a total of 5 million dollars that can be exchanged for Euros, British Pounds, Yen and Kuwaiti Dinars(KD). Currency dealers set the following limits on the amount of any single transaction: 5 million dollars, 3 million euros, 3.5 million pounds, 100 million yen and 2.8 million KDs. The table below provides typical spot exchange rates. The bottom diagonal rates are the reciprocal of the top diagonal rates. For example, rate(euros - dollars) = 1/rate(dollars - euros) = 1/.769 = 1.30.

	Dollars	Euros	Pounds	Yen	KD
Dollars	1	0.769	0.625	105	0.342
Euros	1/.769	1	0.813	137	0.445
Pounds	1/.625	1/.813	1	169	0.543
Yen	1/105	1/137	1/169	1	0.0032
KD	1/.342	1/.445	1/.543	1/.0032	1

Is it possible to increase the dollar holdings (above the initial \$5 million) by circulating the currencies through the market?

b) graphical insight into the problem for 2 dimensions for maximization and minimization

c) standard form of the linear programming problem for simplex method - use of

   - slack variables
   - surplus variables
   - a transformation technique for variables unrestricted in sign

7.3 Exemplification to be done in contact session

Use of TORA to exhibit

a) graphical solution to 2d maximization / minimization

b) algebraic solution of simplex method

7.4 Homework problems

Q1 For the Reddy Mikks problem,

i) Use TORA or a simple graph paper to plot the feasible region.

ii) If the maximum daily availability of M1 is changed from 24 to 25, find the change in profit. This is called the shadow price for M1. In a similar way, find the shadow price of M2.

iii) If the profits are changed from \$5000 and \$4000 per liter of exterior paint and interior paint to \$c1 and \$c2 respectively, find the range for the ration $c1/c2$, keeping the same optimal value of x_1 and x_2.

iv Try to add a constraint that would change / would not change the present optimal solution.

Q2 For the Urban planning problem,

i) Write the complete set of constraints and convert the problem in the standard form.

ii) Would the optimal value change if the maximum number of houses we could demolish is changed to 400?

iii) Observe the decimal nature of the solution (given in the book) and understand the rounding.

8 Applications of Linear Algebra in Optimization-Part 2

8.1 Pre-reading to be done by students

Students are expected to come prepared with

a) what was done in session 7

b) homework problems given at the end of session 7

8.2 What would be covered in the contact session

a) motivation for the simplex method

b) simplex method in table form

c) simplex method to solve an LPP - including the interpretation of steps - for an example

d) computational effort of simplex method compared to naive method

e) special cases of simplex method - unique solution, unboundedness, infeasible cases

f) explain Gauss Jordan method implicitly deployed in simplex algorithm

g) sensitivity analysis for changes in the

   - right hand side of constraints
   - objective function coefficients

   in graphical solution for the 2d case.

h) shadow price - definition and its interpretation in point g) above

8.3 Exemplification to be done in contact session

Use of TORA to exhibit

a) cases of unique solution, unboundedness and infeasible cases

b) graphical sensitivity for changes in rhs and objective function coefficients

8.4 Homework problems

Q1 JOBCO produces two products on two machines.A unit of product 1 requires 2hours on machine 1 and 1 hour on machine 2. For product 2,a unit requires 1 hour on machine 1 and 3hours on machine 2. The revenues per unit of products 1 and 2 are \$30 and \$20 respectively. The total daily processing time available for each machine is 8 hours.

i) Formulate the above as a LPP.

ii) Solve the LPP using the simplex method and observe as to what happens in each of the tables, with the graphical method.

iii) Get the shadow price, minimum capacity and maximum capacity of machines 1 and 2, using TORA and by plotting the lines physically.

Q2 Show that the following objective function can be presented in equation form.

$$\text{Minimize } Z = \max\{|x_1 - x_2 + 3x_3|, |-x_1 + 3x_2 - x_3|\}; x_1, x_2, x_3 \geq 0$$

Hints:

i) $|a| \leq b$ is equivalent to $a \leq b$ and $a \geq -b$

ii) Set $y = \max\{|x_1 - x_2 + 3x_3|, |-x_1 + 3x_2 - x_3|\}$ and observe that in such a case $y \geq |x_1 - x_2 + 3x_3|$ and $y \geq |-x_1 + 3x_2 - x_3|$ and use i).

Q3 Solve the following LPP

$$\text{Maximize } z = 15x_1 + 6x_2 + 9x_3 + 2x_4$$

subject to the constraints

$$2x_1 + x_2 + 5x_3 + 0.6x_4 \leq 10$$

$$3x_1 + x_2 + 3x_3 + 0.25x_4 \leq 12$$

$$7x_1 + x_4 \leq 35$$

$$x_j \geq 0, \quad j = 1, 2, 3, 4$$

using TORA and by hand and verify the solution.

9 One Dimensional Calculus

9.1 Pre-reading to be done by students

Students are expected to come prepared with

a) concepts of functions at high-school level.

b) basic understanding of mathematical functions and their plots such as $\sin(x), \cos(x), \log(x), \exp(x)$ etc.

c) differentiation of common functions and their rules

9.2 What would be covered in the contact session

a) define function and explain domain, codomain and range with examples

b) draw the graph of known functions

c) Limits & continuity through graphical and algebraic approaches

d) to get a basic understanding of derivative using the first principles of derivatives

e) differentiation rules for common functions and combined functions - for the univariate case

f) concepts of critical points of a function, maxima, minima, global and local maxima/minima with examples.

g) integral properties (cdf and pdf, even and off integrands, integration by parts)

h) linear approximation and Taylor series expansion of a function

9.3 Exemplification to be done in contact session

a) graphing a few functions using ceiling and floor function

9.4 Homework problems

Refer Discrete Maths by Kenneth Rosen 7th edition
Exercise problems

a) 14,15,16 and 63 - Page nos. 154-156

Q2 For each pair of functions, find $a.) f + g,\ b.) f - g,\ c.) f * g,\ d.) \frac{f}{g}$. Determine the domain of each of these new functions.

1. $f(x) = 3x + 4, \quad g(x) = x - 2$
2. $f(x) = x - 8, \quad g(x) = x^2$

Q3 Find the Taylor series of the given function centered at the indicated point.

1. $f(x) = \sin x$ at $x = \frac{pi}{2}$
2. $f(x) = x^3$ at $x = 1$

Q4 Evaluate the limit.

1. $\lim_{x\to 4} \dfrac{x^2 - 16}{x - 4}$
2. $\lim_{x\to 2} \dfrac{x - 2}{x^2 - 2x}$

10 Multivariable calculus

10.1 Pre-reading to be done by students

Students are expected to come prepared with

a) concepts at high-school level calculus.

10.2 What would be covered in the contact session

a) vector calculus and some identities

b) multivariate function of n variables is represented as $f(x1, x2, ..., xn)$.

c) partial derivatives - the rate of change of multivariate functions with respect to the independent variables.

d) gradient decent algorithm, classification of algorithms and challenges faced in executing gradient decent approach.

e) constrained optimization covering only equality constraints and the method of Lagrange multipliers.

10.3 Exemplification to be done in contact session

Using MATLAB / PYTHON to draw some of the basic functions and show properties like local and global maxima / minima

10.4 Homework problems

Q1 Investigate the nature of critical points for the following functions

1. $f(x, y) = x^3 - 3x^2 + y^2$
2. $f(x, y) = x^2 + xy + y^2 + \dfrac{1}{x} + \dfrac{1}{y}, x \neq 0, y \neq 0$

Q2 Using Lagrange multipliers, show that

1. the maximum value of $x^2y^3z^4$ subject to $2x + 3y + 4z = a$is $(a/9)^9$.
2. the minimum value of $yz + zx + xy$ subject to $xyz = a^2(x + y + z)$ is $9a^2$

Q3 Find the minimum of $f(x, y) = \alpha x^2 + \beta y^2$ for various values of α and β, by

a) computing the gradient of f and τ

b) coding the iterations in Python with initial values $x_0 = 3$ and $y_0 = 4$ and using the stopping criteria as $|f(j+1) - f(j)| < \epsilon = 10^{-6}$

Estimate the order of convergence by plotting the error against the number of iterations for a few cases.

11 Induction principles

11.1 Pre-reading to be done by students

Students are expected to come prepared with

a) basic understanding of sets from Section 2.1 of Rosen

11.2 What would be covered in the contact session

a) introduction to the principle of mathematical induction

b) classical proofs by mathematical induction

c) proving inequalities by mathematical induction

d) creative use of mathematical induction

e) guidelines for proofs by mathematical induction

11.3 Exemplification to be done in contact session

a) NA

11.4 Homework problems

Refer Discrete Maths by Kenneth Rosen 7th edition
Exercise problems

a) 3, 5 - Page No. 329

b) 18, 31, 32, 38 - Page No. 330-331

b) 45, 69 - Page No.332

12 Strong Induction, Recursive Definitions and Structural Induction

12.1 Pre-reading to be done by students

Students are expected to come prepared with

a) content covered in the session 11

12.2 What would be covered in the contact session

a) principle of strong induction

b) examples of proofs using strong induction

c) using strong induction in computational geometry

d) proofs using well-ordering property

e) explain recursively defined functions and Lame's theorem

f) explain recursively defined sets and structures

g) principle of structural induction

h) examples of proofs using structural induction

12.3 Exemplification to be done in contact session

a) NA

12.4 Homework problems

Refer Discrete Maths by Kenneth Rosen 7th edition
Exercise problems

a) 1.a, 2.a, 3.a, 4.a, 5.a - Page No. 357

b) 12, 18, 56 - Page No. 358

c) 1, 2, 7,8 - Page No. 370

d) 23, 32, 43 - Page No. 371

13 Counting : Part 1

13.1 Pre-reading to be done by students

Students are expected to come prepared with

a) Ideas of induction and recursion

13.2 What would be covered in the contact session

a) basics of Counting

b) inclusion exclusion principle

c) pigeonhole principle

d) permutations and combinations - introduction and examples

13.3 Exemplification to be done in contact session

Nothing in particular

13.4 Homework problems

Refer Discrete Maths by Kenneth Rosen 7th edition Chapter 6 Counting Exercise problems

a) Section 6.1 Problem No : 23, 25, 29

b) Section 6.2 Problem No : 1, 3, 5

c) Section 6.3 Problem No : 11, 15, 17

14 Counting : Part 2

14.1 Pre-reading to be done by students

Students are expected to come prepared with

a) basic counting

b) permutations and combinations

14.2 What would be covered in the contact session

a) binomial Coefficients

b) binomial theorem

c) generalized permutations and combinations

d) generating Permutations and combinations

14.3 Exemplification to be done in contact session

Nothing in particular

14.4 Homework problems

Refer Discrete Maths by Kenneth Rosen 7th edition Chapter 6 Counting Exercise problems

a) Section 6.4 Problem No : 11, 27, 29

b) Section 6.5 Problem No : 9, 11, 13

c) Section 6.6 Problem No : 1, 3, 5

15 Advanced Counting : Part 1

15.1 Pre-reading to be done by students

Students are expected to come prepared with

a) Topics covered in lecture 13 and 14

15.2 What would be covered in the contact session

a) recurrence Relations

b) modeling with recurrence relations

c) examples of common recurrence relation based models

d) solving linear recurrence relations

e) linear homogeneous recurrence relations with constant coefficients

f) characteristic equation and multiplicity

15.3 Exemplification to be done in contact session

Nothing in particular

15.4 Homework problems

Refer Discrete Maths by Kenneth Rosen 7th edition Chapter 8 Advanced Counting
Exercise problems

a) Section 8.1 Problem No : 7, 9, 11

b) Section 8.2 Problem No : 1, 2, 3, 4

16 Advanced Counting : Part 2

16.1 Pre-reading to be done by students

Students are expected to come prepared with

a) Topics covered in Lecture 15

16.2 What would be covered in the contact session

a) linear non-homogeneous recurrence relations with constant coefficients

b) generating functions

c) extended binomial theorem

d) counting problems and generating functions

e) solving recurrence relations using generating functions

16.3 Exemplification to be done in contact session

Nothing in particular

16.4 Homework problems

Refer Discrete Maths by Kenneth Rosen 7th edition Chapter 8 Advanced Counting
Exercise problems

a) Section 8.2 Problem No : 23, 25, 27

b) Section 8.4 Problem No : 3, 5, 7, 17