20 November 2021 10:59

Closure ✓
associativity ✓
identity
inverse.

Ex 1: $G = \underline{\mathbb{Z}}$, $\boxed{{}^{\backprime}*' = \underline{+}}$

$g_1 \in \mathbb{Z}$; $g_1 + g = g_1$, $g = 0$

$g_1 \in \mathbb{Z}$, $g_1 + g_2 = 0$

$g_2 = -g_1$

$\langle \mathbb{Z}, + \rangle$ is a group.

Ex: '$*$'; $a * b = \sqrt{ab}$

$6 * 8 = \sqrt{48}$

$\langle \mathbb{R}, {}^{\backprime}*' \rangle$, $2 * -4 = \sqrt{-8} \notin \mathbb{R}$

Ex: $\langle \mathbb{Z}^+, + \rangle$; closure ✓

asso ✓

identity No identity

inverse $\notin \mathbb{Z}^+$.

$\langle \mathbb{Z}^+ \cup \{0\}, + \rangle$ ✓

$5, -5$

Ex: $\boxed{M_{2\times 2}} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \ a, b, c, d \in \underline{\mathbb{R}} \right\}$

'$*$' = matrix addition.

Closure ✓

associativity ✓

identity $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ ✓

inverse $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} + \begin{pmatrix} -\alpha & -\beta \\ -\gamma & -\delta \end{pmatrix}$

$\boxed{{}^{\backprime}*' \text{ matrix multiplication}}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

INV $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(Ex): $V = \mathbb{R}^2 = \{ (a, b) \mid a, b \in \mathbb{R} \}$

$(a, b) + (c, d) = (a+c, b+d)$

$\langle M_{2\times 2}, + \rangle$, closed, associative

$\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ id. $\begin{bmatrix} -\alpha & -\beta \\ -\gamma & -\delta \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$= \begin{bmatrix} e+a & f+b \\ g+c & h+d \end{bmatrix} ?$$

$$= \begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

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$$\langle S_{2\times 2}, * \rangle = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{R} \right\}$$

closure

$(AB)C = A(BC)$ associativity

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ identity

$\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ inverse

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$V = \mathbb{R}^2 = \{(a, b) \mid a, b \in \mathbb{R}\}$

$F = \mathbb{R}$

I. 1 II. 1

$F = \mathbb{R}$

I 1
I 2
I 3
I 4

$\boxed{V}$ as a vector space over $\boxed{F}$

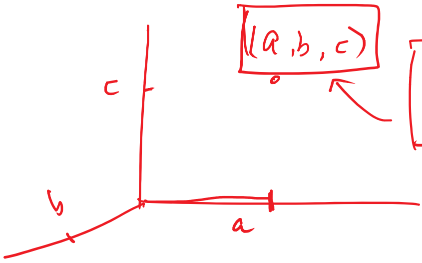
$\equiv$

$\mathbb{R}^2$ as a vector space over $\mathbb{R}$.

II.1
II 2
II 3
II 4

$\langle F, + \rangle$ is abelian

$\langle F, \cdot \rangle$ is abelian




$\boxed{(a,b,c)}$ ← $\boxed{a\hat{i} + b\hat{j} + c\hat{k}}$

$(a_1, a_2, \ldots a_n) \in \mathbb{R}^n$

$a_1 \hat{i}_1 + a_2 \hat{i}_2 \ldots \quad \hat{i}_n$

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NDA

$G \rightarrow$ set of students

(a) XII, 80 M 80 P 80 C

(b) Biceps ..., chest ...
height: ...

c) Eyesight ...

d) Cancer, TB, ...

e) Clear the test + interview

Cleared NDA

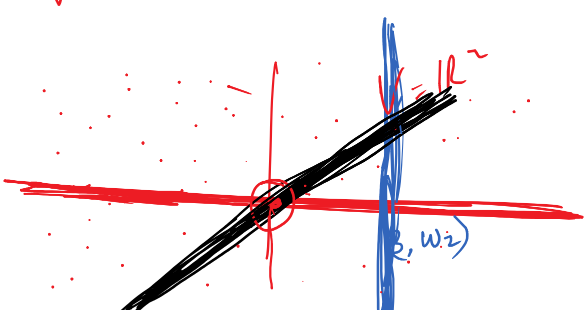
V

vectors
I.1 ...
I 4

Field

scalars
II.1
II 4

V as a vector space over F.




$W \neq \phi$ as $\underline{(0,0) \in W}$

$\alpha, \beta \in F$

$W \subseteq V$

$(w_1, 0), (w_2, 0)$.

$W = \{(3, w_2) \mid w_2 \in \mathbb{R}\}$

$(0,0) \notin W$

$\alpha(w_1, 0) + \beta(w_2, 0)$

$= \boxed{(\alpha w_1 + \beta w_2, 0)} \in W$

$V$

$= \left( \alpha w_1 + \beta w_2, \ldots \right)$

$\Rightarrow W$ is a subspace of $V$

I.1 II.1

II.4 II.4

$(0,0) \notin W$

$$W = \{(x_1, kx_1) \mid x_1 \in \mathbb{R}\}$$

$$(0,0) \in W$$

$$(a, ka), \quad (b, kb)$$

$$\alpha(a, ka) + \beta(b, kb)$$

$$= (\alpha a + \beta b, \alpha k a + \beta k b)$$

$$= (\underbrace{\alpha a + \beta b}, \underline{k}(\underbrace{\alpha a + \beta b}))$$

$W$ is a subspace of $V$.

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$$a_1, a_2, \ldots, a_m \in V$$

$$\text{Span}(a_1, a_2, \ldots, a_m) = \{\alpha_1 a_1 + \alpha_2 a_2 + \ldots + \alpha_m a_m \mid \alpha_1, \ldots, \alpha_m \in F\}$$

Span { $a_1$ = tea powder, $a_2$ = milk, $a_3$ = water, $a_4$ = sugar, $a_5$ = masala, $a_6$ = dudh milk }

= { $\alpha_1$ . tea powder + $\alpha_2$ . milk + $\alpha_3$ . water + $\alpha_4$ . sugar + $\alpha_5$ . masala }

$\cancel{\alpha_1 = 0}$, $\cancel{\alpha_3 = 0}$, $\cancel{\alpha_4 = 0}$, $\cancel{\alpha_5 = 0}$

$$\boxed{\text{1400 ingredients} \qquad \text{PBM}}$$

$$\{a_1, a_2, \ldots, a_m \mid a_i \in V\} \leftarrow 0$$

$$\alpha_1 a_1 + \alpha_2 a_2 + \ldots + \alpha_m a_m =$$

$\alpha_1 = 0,\ \alpha_2 = 0 \ldots,\ \alpha_m = 0$ as the only solution

$V = \mathbb{R}^2 \quad \{\underline{(1,0)}\ \&\ \underline{(0,1)}\}$ — LI

$$\alpha_1(1,0) + \alpha_2(0,1) = (0,0)$$

$$(\alpha_1, 0) + (0, \alpha_2) = (0,0) \Rightarrow \alpha_1 = 0,\ \alpha_2 = 0$$

$$(\alpha_1, \alpha_2) = (0,0)$$

$$\{(1,2)\ \ (2,4)\} \qquad (2,4) = 2(1,2)$$

$$\alpha_1(1,2) + \alpha_2(2,4) = (0,0)$$

$$(\underbrace{\alpha_1 + 2\alpha_2}, \underbrace{2\alpha_1 + 4\alpha_2}) = (0,0)$$

$$\boxed{\begin{aligned} &\alpha_1 + 2\alpha_2 = 0 \\ &2\alpha_1 + 4\alpha_2 = 0 \Rightarrow \alpha_1 + 2\alpha_2 = 0 \end{aligned}}$$

$$\alpha_1 = -2\alpha_2, \quad \alpha_2 = 1,\ \alpha_1 = -2$$

$$\therefore\ (-2)(1,2) + (1)(2,4) = (0,0)$$

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$V: \mathbb{R}^2 \qquad \underline{S = \{(0,0), (1,0)\}}$

$$\alpha(0,0) + \beta(1,0) = (0,0)$$

$\alpha = 5,\ \beta = 0$, linearly dep

**Basis** $\quad S = \{(1,0), (2,4), \cancel{(5,5)}\}$

$$\alpha(1,0) = (0,0) \Rightarrow \alpha = 0$$

$$\alpha(1,0) + \beta(2,4) = (0,0)$$

$$((\alpha + 2\beta), 4\beta) = (0,0)$$

$$\beta = 0, \quad \alpha = 0$$

S is LI

$$\alpha(1,0) + \beta(2,4) + \gamma(5,5) = (0,0) \qquad \text{LD}$$

$$\alpha + 2\beta + 5\gamma = 0$$

$$4\beta + 5\gamma = 0$$

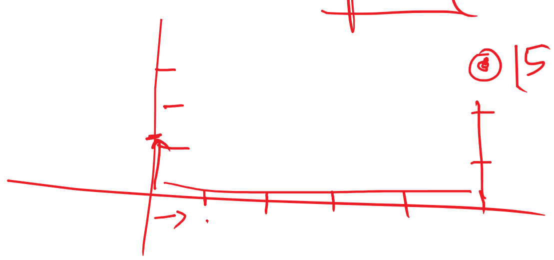
$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & 4 & 5 \end{pmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\gamma$ = arbitrary $\quad \gamma = t$

In $\mathbb{R}^2$ $\quad \{(1,0), (0,1)\}$

$$\text{Span}(\alpha(1,0) + \beta(0,1)) = \boxed{(\alpha, \beta)}$$

(5,3)



$$\{(1,2), (8,-12)\}$$

$$(3,4) = \alpha(1,2) + \beta(8,-12)$$

$$3 = \alpha + 8\beta$$

$$4 = 2\alpha - 12\beta$$

$$6 = 2\alpha + 16\beta$$

$$-2 = 0 - 28\beta \Rightarrow \beta = 1/14$$

$\boxed{4 \text{ cents}}$ $\quad$ 5 cents

$\geq 12$ cents $\quad 12 = 4 \times 3$

$13 \rightarrow 2 \cdot 4 + 5$

$14 - 1 \cdot 4 + 2 \cdot 5$

$15 = 0 \cdot 4 + 3 \cdot 5$

$16$

$17$

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$$\boxed{5+17i} = \underline{\alpha}\,\boxed{(2+3i)}$$

$$\alpha = \frac{(5+17i)}{(2+3i)} \times \frac{(2-3i)}{(2\cdot 3i)}$$

$$= \frac{\_\,i + \_\,i}{13}$$

$\underline{\mathbb{C}}$ over $\underline{\mathbb{R}}$ $\quad \text{Span}\{1, i\}$

$\alpha \cdot 1 + \beta \cdot i \qquad \alpha, \beta \in \mathbb{R}$

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$R(T) = \{\underline{T(v)} \mid v \in V\}$

$\text{Ker}(T) = \{v \in V \mid T(v) = 0\}$ is a subsp of $V$

$\underline{T(0)} = T(0+0) = \underline{T(0) + T(0)} \Rightarrow T(0) = 0$

$\therefore R(T)$ is non-empty

& $0 \in R(T)$

$T(v_1) \quad T(v_2) \in R(T)$

$\alpha T(v_1) + \beta T(v_2) = T(\alpha v_1 + \beta v_2)$

$\in \underline{R(T)}$

$R(T)$ is a subspace of $W$