

Lake water mass balance calculations for Lake Turkana, Kenya

Based on equations developed by Gibson et al. (2016)

First, there are a number of ‘input’ values that you must define for the model:

1. Temperature, T ($^{\circ}\text{C}$ @ lake surface, *though technically this should be the air-water interface temperature, which is rarely known and can vary diurnally*)
2. Humidity, h (%rh, normalized to the water surface temperature, *though a general air mass value should work just as well*)
3. $\delta^8\text{O}_i$ and δD_i of the input (‰ SMOW , *in this our case this would be dominated by the Omo River by volume, but I think we would be justified to use an average value from Turkwel and Omo River measurements*).
4. The degree to which there is isotopic equilibrium between precipitation and atmospheric water vapor, k (%), *there is lots of room for play in this value, but the end result isn't all that sensitive so ~75% is a good place to start*).
5. The $\delta^8\text{O}_A$ and δD_A of atmospheric water vapor, which can be calculated by

$$(1) \quad \delta_A = \frac{(\delta_P - k\varepsilon^+)}{(1 + (10^{-3}k\varepsilon^+))} \quad \text{Gea16 eqn. 18}$$

where

$$(2) \quad \varepsilon^+ = (\alpha^+ - 1) \times 1000$$

and

$$(3) \quad 1000\ln^{18}\alpha^+ = -7.685 + \frac{6.7123 \times 10^3}{T} - \frac{1.6664 \times 10^6}{T^2} + \frac{0.35041 \times 10^9}{T^3} \quad \text{Horita \& Wesolowski (1994)}$$

$$(4) \quad 1000\ln^2\alpha^+ = \frac{1158.8 \times T^3}{10^9} - \frac{1620.1 \times T^2}{10^6} + \frac{794.84 \times T}{10^3} - 161.04 + \frac{2.992 \times 10^9}{T^3} \quad \text{Horita \& Wesolowski (1994)}$$

where temperature is in kelvin for both equations.

Next, you need to define, or you may elect to iterate over, the ratio of evaporation to inflow ($x = E/I$). These values are just #'s from 0 to 1, with 1 being a full evaporated lake. In practice, this ('1') will be the maximum possible $\delta^8\text{O}_L$ and δD_L values for the lake water. One interesting outcome of this little modeling exercise will be seeing where the average measured values line up relative to this water balance term (x).

The balance term may be modified by what *Gea16* call the temporal enrichment slope (m):

$$(5) \quad m = \left(h - 10^{-3} \left(\varepsilon_K + \frac{\varepsilon^+}{\alpha^+} \right) \right) / (1 - h + (10^{-3} \varepsilon_K)) \quad \text{Gea16 eqn. 6}$$

where

$$(6) \quad \varepsilon_K = \theta C_K (1 - h)$$

θ in this equation is a so-called “transport resistance parameter” which is assumed to be 1 when humidity and the water vapor isotopes are measured or estimated close to the air-water interface (Section 2.1., *Gea16*). C_K is a kinetic fractionation constant for lakes that is 14.2‰ for $\delta^{18}\text{O}$ and 12.5‰ for δD , representing turbulent, open-water conditions (Horita et al., 2008). Kinetic isotope fractionation constants can be very different for laminar flow and other diffusion controlled settings, for example.

The limiting isotope ratios, which are approached as $x \rightarrow 1$ for $\delta^{18}\text{O}$ and δD , are calculated as

$$(7) \quad \delta^* = \left(h\delta_A + \varepsilon_K + \left(\frac{\varepsilon^+}{\alpha^+} \right) \right) / \left(h - \left(10^{-3} \left(\varepsilon_K + \left(\frac{\varepsilon^+}{\alpha^+} \right) \right) \right) \right) \quad \text{Gea16 eqn. 7}$$

Finally, the lake water isotope ratios ($\delta^{18}\text{O}_L$ and δD_L) can be calculated:

$$(8) \quad \delta_L = \left(\delta_i (mx\delta^i) \right) / (1 + (mx)) \quad \text{Gea16 eqn. 10}$$

and the isotope ratios of the evaporate or vapor ($\delta^{18}\text{O}_V$ and δD_V) are:

$$(9) \quad \delta_V = \left(\left(\frac{\delta_L - \varepsilon^+}{\alpha^+} \right) - h\delta_A - \varepsilon_K \right) / (1 - h + (10^{-3}\varepsilon_K)) \quad \text{Gea16 eqn. 3}$$

The slope of the resulting local evaporation line (on which sits both the lake water and the evaporate δ values) is thus

$$(10) \quad S_{LEL} = \left(\frac{\delta\text{D}^* - \delta\text{D}_i}{\delta^{18}\text{O}^* - \delta^{18}\text{O}_i} \right) \quad \text{Gea16 eqn. 19}$$