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Calculating status with negative relations

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Abstract

A new measure of status or network centrality that takes into account both positive and negative relationships is suggested. This measure is based on the eigenvector measure of centrality, a standard measure in network research. Its use is illustrated with data from Sampson's well-known study of monks in a monastery.

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1. Introduction

There are numerous standard measures of centrality in networks (Wasserman and Faust, 1994, Chapter 5), each appropriate under different circumstances. Eigenvector centrality is a valid measure when a valued quantity can be transferred through social osmosis between interacting individuals (Hubbell, 1965, Bonacich, 1972, 1987, 1991; Friedkin, 1991; Bonacich and Lloyd, 2001). In a communication network being connected to well-connected individuals adds more to one's knowledge pool. Having popular friends adds more to ones own popularity.

This conception is expressed in the formula for eigenvector centrality. Let $\boldsymbol{A}$ be a symmetric adjacency matrix, where $a_{ij} = a_{ji} = 1$ if i and j are connected in a network and $a_{ij} = a_{ji} = 0$

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Table 1
Comparison between balance theory and status achievement

	Balance theory	Status
1	Friends of friends are friends	A positive connection with a high status individual increases ones status
2	Friends of enemies are enemies	A positive connection to a disvalued individual decreases ones status
3	Enemies of friends are enemies	A negative relation to a high status individual reduces ones status
4	Enemies of enemies are friends	A negative relation to a disvalued individual increases one status

otherwise.[1] The eigenvector measure of centrality x is the solution to the following matrix equation:

$$\boldsymbol{A}x = \lambda x$$

The logic of the eigenvector measure of centrality could easily be extended to the determination of status (or popularity) in networks of positive and negative relationships.[2] It may be true that a negative or disassociate connection to a popular individual *reduces* ones own status. If being liked by popular individuals confers status, being disliked by these same individuals should be particularly harmful. Suppose that some individuals have negative status in the sense that contact with them reduces others' status. One might also expect that a rejection by a disvalued individual might increase ones status.

There is a similarity to this logic of status and the logic of affective relationships in balance theory (Table 1).[3]

As this table makes clear, there is a close association between balancing processes and the acquisition of status. When a sociometric structure is balanced there is an even closer relation between the two approaches: the eigenvector of centrality scores corresponds exactly to the clique structure of the balanced graph.[4]

Proposition 1. *The eigenvector of a balanced symmetric matrix of affective relations reveals the clique structure.*

Let $\boldsymbol{A}$ be a valued binary symmetric matrix representing both positive and negative relations among a set of n actors; $a_{ij} = a_{ji} = 1$ if actors i and j are in a positive relationship, $a_{ij} = a_{ji} = -1$ if they are in a negative relationship, and $a_{ij} = a_{ji} = 0$ if there is no relationship between them. Suppose that this structure is *balanced*; the set of actors can be divided into two sets (one of which may be empty, and the division is not necessarily unique) such that all positive relations are between members of the same set and all negative relations are

[1] Bonacich and Lloyd (2001) show that there is a generalization of the measure for nonsymmetrical networks.

[2] We are not equating centrality and status. The claim is only that both diffuse in networks.

[3] Note that status transmission has an implicit transitive-like effect; a high status individual adds to the status not only of his friends but indirectly to the status of the friends of his friends.

[4] A sociometric structure is balanced if there are no violations of the above principles: no friends of friends are enemies, and so on balanced structures can be divided into two cliques (not necessarily uniquely and one of the cliques may be empty) such that all positive relations connect members of the same clique and all negative relations connect members of different cliques.

between members of different sets. Rearrange the rows and column so that the n_1 members of the first set are followed by the n_2 members of the second set. The matrix $\boldsymbol{A}$ then has the following form:[5]

$$\boldsymbol{A} = \begin{pmatrix} 1/0 & -1/0 \\ -1/0 & 1/0 \end{pmatrix}$$

Let $\boldsymbol{D}$ be a diagonal matrix whose first n_1 diagonal elements are 1 with the remaining n_2 elements -1. Pre- and post-multiply $\boldsymbol{A}$ by $\boldsymbol{D}$ to create matrix $\boldsymbol{B}$.

$$\boldsymbol{B} = \boldsymbol{DAD}$$

$\boldsymbol{B}$ has only non-negative elements[6]. Thus, $\boldsymbol{B}$ has a real positive eigenvalue λ greater than or equal in absolute value to any other eigenvalue and λ has an eigenvector x all of whose elements are positive (Minc, 1988, p. 11). Since

$$\boldsymbol{B}x = \lambda x \quad \text{By substitution,} \quad \boldsymbol{DAD}x = \lambda x \quad \text{or,} \quad \boldsymbol{AD}x = \lambda \boldsymbol{D}^{-1}x = \lambda \boldsymbol{D}x$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ have exactly the same eigenvalues and $\boldsymbol{D}$x is the eigenvector of $\boldsymbol{A}$ corresponding to λ. In $\boldsymbol{D}$x the first n_1 values for the first clique are positive and the next n_2 values corresponding to the second clique are negative.[7]

Thus, in a balanced graph the eigenvector's positive and negative scores promise to reveal not only the clique structure (the positive score clique and the negative score clique) but also status scores within each clique as well. It is an assumption of our model that a negative tie to a member of the other clique increases one status. One way to think of this in balance terms is that in a balanced structure a negative tie to a member of the other clique precludes belonging to the other clique and is compatible only with membership in the given clique. The status score then becomes an index of how perfectly one is placed in a clique.

2. An example: Sampson data

Sampson (1968) reports sociometric data[8] on relations between monks in a monastery. We have selected for analysis the SAMPLK3 and SAMPDLK data sets contained in UCINET (Borgatti et al., 1999). Monks were asked for their top three selections for who they liked and disliked among the other monks at the monastery. These relations are given in Figs. 1 and 2.[9]

Looking just at the liking relations, there are clearly two cliques of 11 (clique B) and 7 (clique A) monks. In the diagram we have drawn a line separating the two sets. Ninety-three

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[5] '1/0' and '$-1/0$' refers to blocks with ones and zeros and -1 and zeros, respectively.

[6] We will also assume that B is irreducible (Minc, 1988, p. 7). This excludes pathologies such as unconnected components.

[7] If x is an eigenvector so is $-x$; which clique has negative values is arbitrary.

[8] We would like to thank Elisa Bienenstock for suggesting the applicability of this data set.

[9] The top choices are combined.

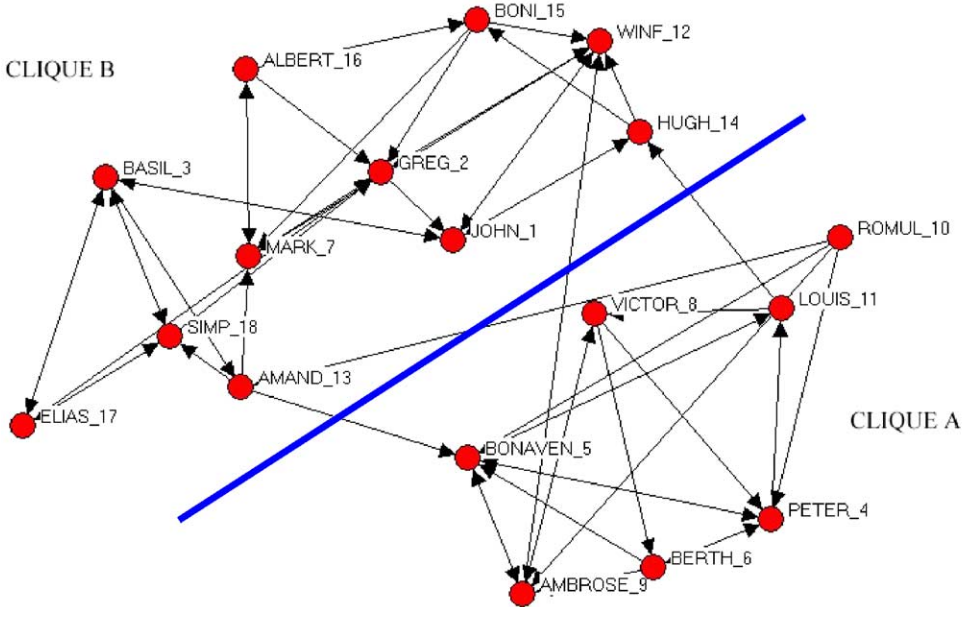

Fig. 1. Liking relations at time 3 in Sampson's monastery.

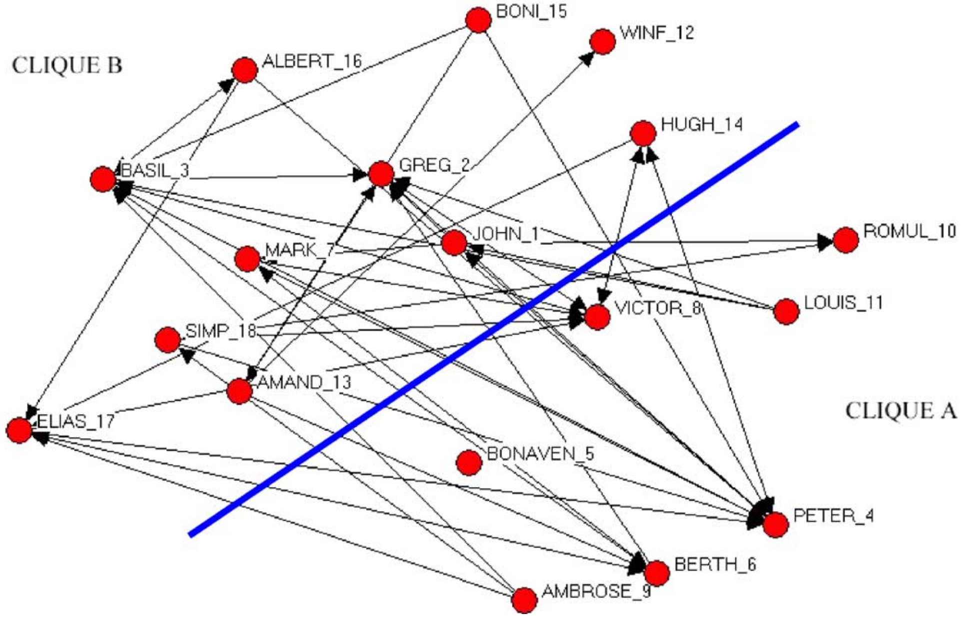

Fig. 2. Disliking relations in Sampson's monastery.

Table 2
Status scores for all 18 monks

	Name	Status
Clique A	Romul	.174
	Bonaventure	.188
	Ambrose	.248
	Berthold	.319
	Peter	.420
	Louis	.219
	Victor	.365
Clique B	Winfrid	−.081
	John	−.142
	Gregory	−.292
	Hugh	−.088
	Boniface	−.123
	Mark	−.217
	Albert	−.072
	Amand	−.030
	Basil	−.254
	Elias	−.282
	Simplicius	−.287

percent of the liking ties are between members of the same clique whereas 74% of the disliking ties are between members of different cliques. This structure is highly but not perfectly balanced.

Table 2 shows the eigenvector status scores for the 18 monks arranged by their clique membership; the seven members of clique A come first.[10]

There is a perfect association between the two cliques evident in the diagrams of the liking and disliking relations and this vector. All the members of clique B have negative scores, while all members of clique A have positive scores. Moreover, the four monks who were expelled—Basil, Elias, Simplicius, and Gregory—are precisely the four monks with the lowest status score.[11]

To explore further the effects of positive and negative scores on status we will focus on clique B. Table 3 shows eigenvector status scores for membership in clique B under three conditions: eigenvector status for membership in clique B using only the positive relations within clique B; eigenvector status scores for membership in clique B using only the positive and negative relations within clique B; eigenvector status scores for membership in clique B using all the relations among the eighteen members of cliques A and B (from Table 2).[12]

Fig. 3 shows the relations between status scores without and with the negative relations in clique B (the first two columns of Table 3). An equal scores line ($x = y$) has been added. Both of these sets of status scores are hypothetical and incomplete. Both assume that clique

[10] Both positive and negative relations were first made symmetric; a symmetric tie existed between two individuals if either chose the other. To perform a similar analysis on asymmetric data, see Bonacich and Lloyd (2001).
[11] This interesting fact was pointed out by an editor.
[12] The centrality scores in the third column are all reversed from Table 2; if x is an eigenvector, so is $-x$.

Table 3
Eigenvector status scores for members of clique B calculated from positive relations within clique B, positive and negative relations within clique B, and all positive and negative relations

	Positive and within B	Positive and negative within B	Positive and negative within A and B
Winfrid	.380	.345	.081
John	.280	−.023	.142
Gregory	.464	.405	.292
Hugh	.230	.196	.086
Boniface	.378	.476	.123
Mark	.368	.276	.217
Albert	.268	.351	.072
Amand	.175	−.298	.030
Basil	.194	−.374	.254
Elias	.196	−.127	.282
Simplicius	.227	−.084	.287

A, which will be shown to have an effect on status scores, did not exist. Moreover, the first analysis, based only on the positive relations within clique B is doubly defective because it ignores disliking relations. The scores are presented here merely to illustrate the formal properties of the measure.

Distance below the line indicates how far the status of an individual dropped when negative relations within clique B were taken into account. The two individuals who dropped the most when negative scores were added (Elias and Basil) had negative relations with the most popular members of their clique, Gregory and Boniface.

Fig. 4 shows the effects on the status scores of members of clique B of adding their relation so clique A. A line indicating equal status on the two measures has been added. The presence of relations between cliques has a profound effect on the status scores to the extent that there is a slight negative correlation between the two sets of statuses. The two individuals with the greatest increase in status score, Elias and Basil, benefited in status by having many negative relations with members of the other clique A. Two members of clique B, Albert

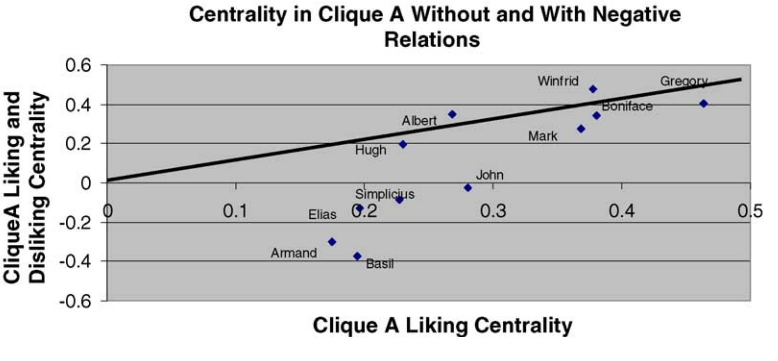

Fig. 3. Status scores in clique A without and with negative relations.

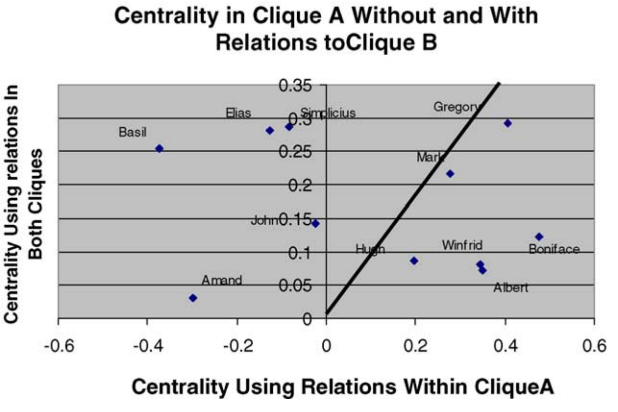

Fig. 4. Within clique status vs. total status, for clique A.

and Boniface, declined in status because they were involved in few negative relations with members of clique A. These changes are consistent with the dynamics outlined in Table 1 and show the expected effects on status of taking into account negative relationships.

3. Status scores in the absence of cliques

The same general reasoning can apply to sociometric structures that are not balanced. Take the following pair of networks, in which solid and broken lines represent positive and negative relations, respectively (Fig. 5).

In the first diagram the most central or highest status individual, 3, has a positive connection to all the others. In the second structure the relation between 3 and 5 is negative. The following table gives the status of the vertices in the two structures, normalized so that the maximum score is 1.00 (Table 4).

Because of its negative connection to the highly central individual 3, the score for individual 5 becomes negative in network B. The positive connection to the disvalued 5 now reduces 4's status score as well.

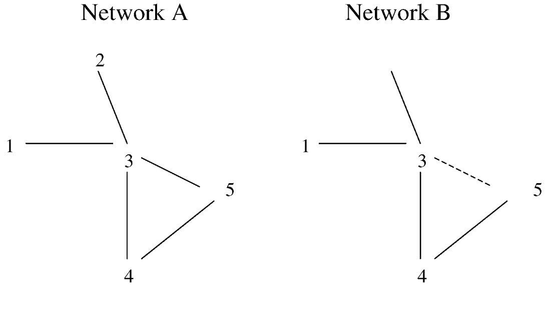

Fig. 5. Two networks illustrating effect of a negative relation.

Table 4
Status scores for the networks of Fig. 5

Position	Network A status	Network B status
1	.43	.55
2	.43	.55
3	1.00	1.00
4	.74	.35
5	.74	−.35

4. Conclusions

The implications and feasibility of using eigenvectors to examine the effects of negative relations on status is examined in this paper. The proposed measures assume an anti-symmetry of the effects of positive and negative relationships on status. One adds to ones status by having a positive connection to a high status individual or a negative connection to a rejected individual. One reduces ones status through a negative connection to a high status individual or a positive connection to a disvalued individual. If one is willing to make these assumptions the eigenvector of the valued adjacency matrix provides a reasonable measure of status.

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