a) $m\ddot{x} = \nabla B x - C_h \dot{x}$ $\qquad x(0)=1$

$\dot{x}(0)=0$

$m\ddot{x} + C_h\dot{x} - \nabla B x = 0$

$m(s^2X(s) - sx(0) - \dot{x}(0)) + C_h(sX(s) - x(0)) - \nabla B X(s) = 0$

$ms^2X(s) - msx(0) - \cancel{m\dot{x}(0)}^{0} + C_h s X(s) - C_h x(0) - \nabla B X(s) = 0$

$X(s)(ms^2 + C_h s - \nabla B) = ms - C_h$

$$\boxed{X(s) = \frac{ms - C_h}{ms^2 + C_h s - \nabla B}}$$

b) $m=1$; $C_h=1$; $\nabla B = 20$:

$$X(s) = \frac{s-1}{s^2+s-20} = \frac{s-1}{(s+5)(s-4)} = \frac{a}{s+5} + \frac{b}{s-4}$$

$a(s-4) + b(s+5) = s-1$

$s=+4$: $0 + 9b = 3 \Rightarrow b = 1/3$

$s=-5$: $-9a = -6 \Rightarrow a = 2/3$

$$X(s) = \frac{2/3}{s+5} + \frac{1/3}{s-4} \Rightarrow \mathcal{L}^{-1}\{X(s)\} = \frac{2}{3}e^{-5t} + \frac{1}{3}e^{4t}$$

$$\boxed{x(t) = \frac{2}{3}e^{-5t} + \frac{1}{3}e^{4t}}$$

c) $X(s) = \dfrac{s-1}{s^2+s-20}$; $x(t) = \frac{2}{3}e^{-5t} + \frac{1}{3}e^{4t}$

Initial Value Theorem: $\lim_{s\to\infty} s \cdot X(s) = \lim_{t\to 0} x(t)$

$$\lim_{s\to\infty} s \cdot \frac{s-1}{s^2+s-20} = \lim_{s\to\infty} \frac{s^2-s}{s^2+s-20} = \boxed{1}$$

✓agree

$$\lim_{t\to 0}\left(\frac{2}{3}e^{-5t} + \frac{1}{3}e^{4t}\right) = \frac{2}{3}e^{(0)} + \frac{1}{3}e^{(0)} = \frac{2}{3} + \frac{1}{3} = \boxed{1}$$

IVT shows that we did our math right in both time & frequency domains.

d) Final Value Theorem only applies if the system is STABLE. To check this, we look at the poles of the transfer function X(s):

$$s^2+s-20 = (s+5)(s-4) \Rightarrow \text{roots: } -5, +4$$

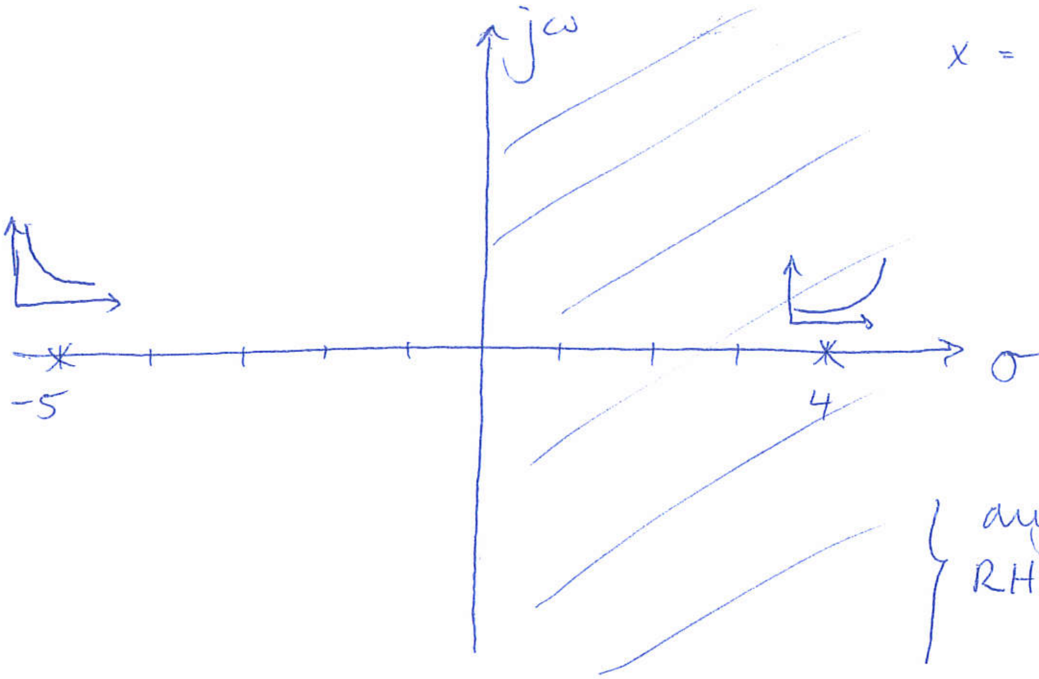
A positive real root means that our system is unstable. Why? Let's look in the time domain!

As $t\to\infty$, $\underbrace{\frac{2}{3}e^{-5t}}_{\text{this term goes to } 0} + \underbrace{\frac{1}{3}e^{4t}}_{\text{this term goes to } \infty}$

this term goes to 0
↳ STABLE

this term goes to ∞
↳ UNSTABLE.

We can also look @ frequency domain:



x = poles

anything in the RHP is UNSTABLE

anything on the Imaginary axis is "Marginally stable" and according to Bounded Input Bounded Output (BIBO) definition, that means poles on the Im axis are <u>also</u> UNSTABLE

e) Let $m=1$, $c_h = 5$, $\nabla B = -6$:

$$X(s) = \frac{s-5}{s^2+5s+6} = \frac{s-5}{(s+2)(s+3)}$$

⇒ now both roots are negative, $s = -2, -3$, so our system is <u>stable</u> and we can apply FVT:

$$\lim_{s\to 0} s \cdot X(s) = \lim_{t\to\infty} x(t)$$

$$\boxed{\lim_{s\to 0} s \cdot \frac{s-5}{s^2+5s+6} = 0}$$

~~$\lim_{t\to\infty}$~~

time domain →

$$X(s) = \frac{s-5}{(s+2)(s+3)} = \frac{a}{s+2} + \frac{b}{s+3}$$

$$a(s+3) + b(s+2) = s-5$$

$s=-3$: $0 + b(-1) = -8 \Rightarrow b = 8$

$s=-2$: $a + 0 = -7 \Rightarrow a = -7$

$$X(s) = \frac{-7}{s+2} + \frac{8}{s+3} \Rightarrow \mathcal{L}^{-1}\{X(s)\} = -7e^{-2t} + 8e^{-3t}$$

$$\boxed{\lim_{t\to\infty} \left(-7e^{-2t} + 8e^{-3t}\right) = 0}$$

$\Rightarrow$ time & frequency domains agree steady state value = 0.

Discussion to give intuition:

→ In Case 1, the magnetic force is pulling the swimmer while the drag resists its motion. The magnetic force is increasing at a faster rate so the swimmer continues to move in the +x-direction @ an ~~exponentially~~ increasing rate.

→ In case 2, the magnetic force & the drag force are acting in the same direction. ~~Actually, with the given IC's, the swimmer never moves~~ When the swimmer reaches $x=0$, there is no F_B, and no $\dot{x}$, and it stops moving.