16-811 Assignment 1: Re-Submission 1

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1 Problem 1

Please see code contained in folder code/q1.m. The implementation was tested on the following matrices. The results are listed after the test matrix:

Test matrix:

$$A_1 = \begin{pmatrix} 10 & 9 & 2 \\ 5 & 3 & 1 \\ 2 & 2 & 2 \end{pmatrix} \tag{1}$$

Code output for matrix A_1 (taken from Problem 2):

$$L_{A_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.2 & -0.133 & 1 \end{pmatrix} \tag{2}$$

$$D_{A_1} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & -1.5 & 0 \\ 0 & 0 & 1.6 \end{pmatrix} \tag{3}$$

$$U_{A_1} = \begin{pmatrix} 1 & 0.9 & 0.2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

$$P_{A_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

Test matrix:

$$A_3 = \begin{pmatrix} 10 & 6 & 4 \\ 5 & 3 & 2 \\ 1 & 1 & 0 \end{pmatrix} \tag{6}$$

Code output for matrix A_3 (taken from Problem 2):

$$L_{A_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix} \tag{7}$$

$$D_{A_3} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

$$U_{A_3} = \begin{pmatrix} 1 & 0.6 & 0.4 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \tag{9}$$

$$P_{A_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{10}$$

2 Problem 3

Please see code contained in folder code/q3.m.

2.1 Part (b)

This matrix is singular (non-invertible) and has a non-trivial null-space. I can say that the column space is spanned by the vectors:

$$v_1 = \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} \tag{11}$$

and:

$$v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{12}$$

Knowing this, and looking at the vector b, I can see that the vector b is in the column space of the matrix, i.e.:

$$b = 2 \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{13}$$

In this case, since b is in the column space of A, there are an infinite number of affine solutions that can be written as:

$$\bar{x} + span(nullspaceofA) = \bar{x} + span(\begin{pmatrix} 0.5774 \\ -0.5774 \\ -0.5774 \end{pmatrix}) \tag{14}$$

Where, from MATLAB, I have calculated that $\bar{x}$ is:

$$\bar{x} = \begin{pmatrix} 0.33 \\ -1.333 \\ 1.667 \end{pmatrix} \tag{15}$$

3 Problem 4

Please note: I had help for this problem from the TA's (thank you!) and another student, Sophie (last name unknown).

3.1 Part (c)

The rank of A (i.e. the column space of A) and the null space of A should sum to n. Since we already know that A negates the component of v in the direction of u and keeps all the other components, we know that the size of the null space of A is equal to 1. And therefore we also know the column space (or rank) of A is of size $n-1$.

The null space of A is spanned by the vector u.

3.2 Part (d)

We can solve for A^2 as follows:

$$A^2 = (I - uu^T)(I - uu^T) = I - 2uu^T + (uu^T)^2 \tag{16}$$

$$A^2 = I - 2uu^T + u(u^Tu)u^T \tag{17}$$

We know that $u^Tu = 1$ so we can solve this to be:

$$A^2 = I - 2uu^T + u(1)u^T = I - uu^T \tag{18}$$

Therefore:

$$A^2 = A \tag{19}$$