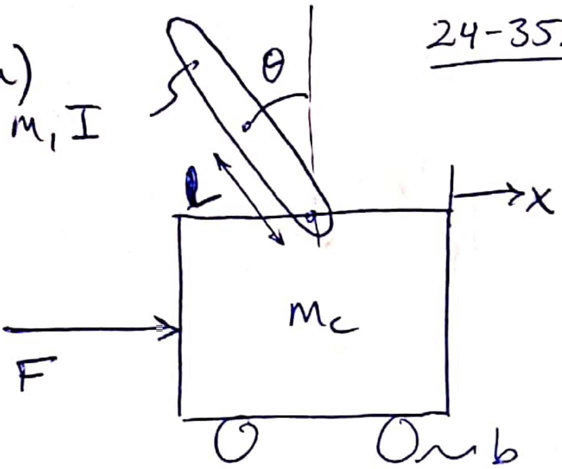
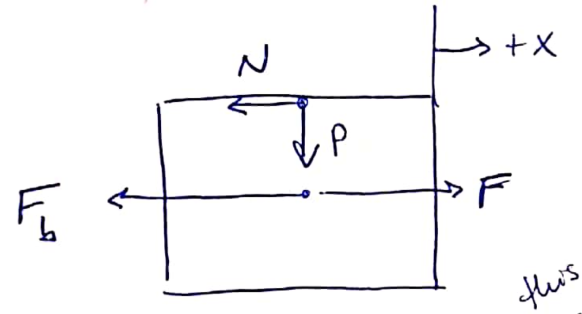
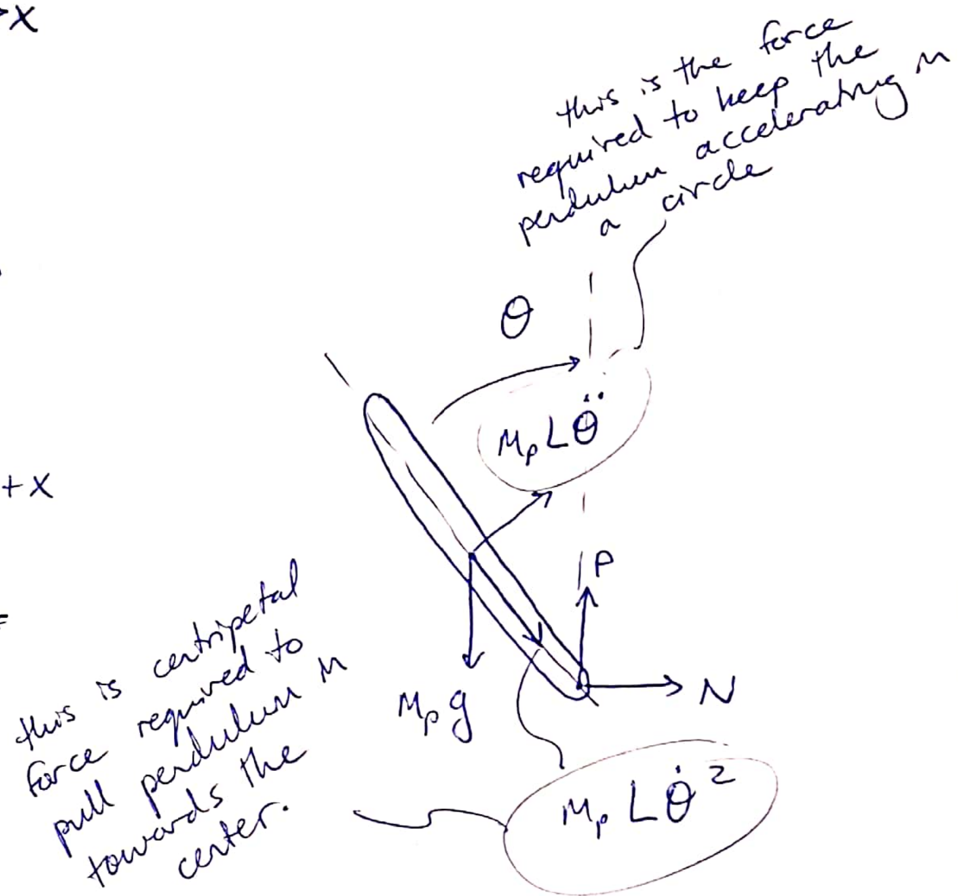
2. a)







* SOME QUICK INTUITION ON CENTRIPETAL & ACCELERATION FORCES:

① centripetal:

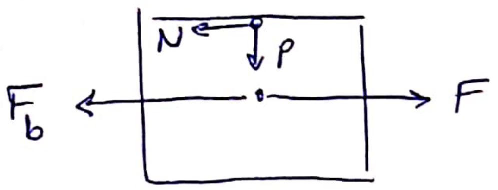
$F = m \cdot a$ where $a = \frac{v}{t}$, $v = l\dot{\theta}$

$a = \frac{l\dot{\theta}}{t}$, $\frac{l}{t} = v$

$a = v\dot{\theta} = (l\dot{\theta}) \cdot \dot{\theta} = \underline{l\dot{\theta}^2}$

② ACCELERATION:

$M_p \ddot{X} \approx M_p L\ddot{\theta}$ for small angles



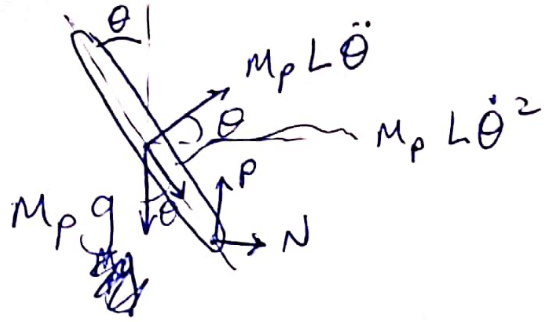
Sum of horizontal forces:

$$\Sigma F = M_c \ddot{x}$$

$$F - F_b - N = M_c \ddot{x}$$

$$F - b\dot{x} - N = M_c \ddot{x} \quad (1)$$

NOTE: no need to sum vertical forces, no motion in that direction for cart.



Sum of horizontal forces:

$$\Sigma F = M_p \ddot{x}$$

$$N + M_p L\ddot{\theta}\cos\theta + M_p L\dot{\theta}^2\sin\theta = M_p \ddot{x}$$

Solve for N & sub into (1)

$$N = M_p\ddot{x} - M_p L\ddot{\theta}\cos\theta - M_p L\dot{\theta}^2\sin\theta$$

$$\Rightarrow F - b\dot{x} - \left(M_p\ddot{x} - M_p L\ddot{\theta}\cos\theta - M_p L\dot{\theta}^2\sin\theta\right) = M_c\ddot{x}$$

$$\boxed{F = (M_c + M_p)\ddot{x} + b\dot{x} - M_p L\ddot{\theta}\cos\theta - M_p L\dot{\theta}^2\sin\theta}$$

↳ this is our first equation.

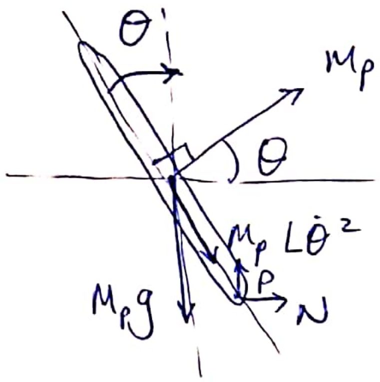
For our 2nd equation, lets sum the forces that are <u>perpendicular to the pendulum</u>:

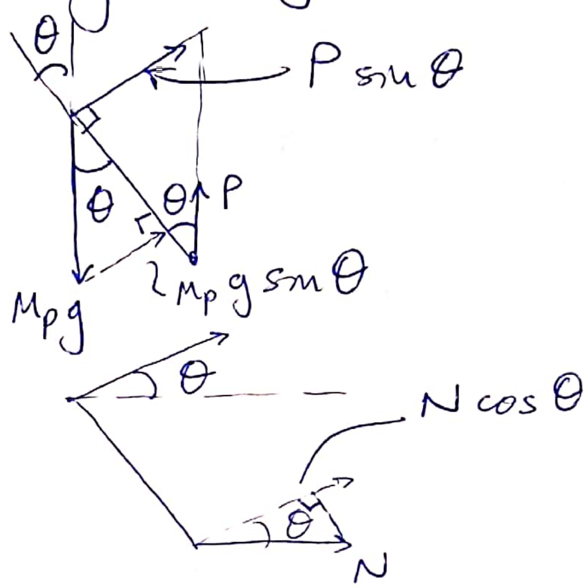
$$\Sigma F = M_p\ddot{x}\cos\theta$$

$$M_p L\ddot{\theta} + N\cos\theta + P\sin\theta + M_p g\sin\theta = M_p\ddot{x}\cos\theta \quad (2)$$

24-352 R2 Q2

NOTE: To better visualize the trigonometry:





⇒ now, we want to <u>eliminate</u> P & N (our reactive forces) so we can write an expression for the pendulum's net rotational acceleration which is due <u>solely</u> to the external reactive forces P & N:

$$\sum \tau = I\ddot{\theta} \qquad \tau_N = r \times F_N = L \cdot N\cos\theta$$

~~Mass~~ $$\tau_P = r \times F_P = L \cdot P\sin\theta$$

$$LN\cos\theta + LP\sin\theta = I\ddot{\theta}$$

$$(N\cos\theta + P\sin\theta) = \frac{I\ddot{\theta}}{L} \quad \rightarrow \text{sub into } (2):$$

$$M_p L\ddot{\theta} + \frac{I\ddot{\theta}}{L} + M_p g\sin\theta = M_p \ddot{x}\cos\theta$$

⇒ multiply through by L:

$$\boxed{M_p L^2\ddot{\theta} + I\ddot{\theta} + M_p L g\sin\theta = M_p L\ddot{x}\cos\theta}$$

this is our second equation

→

⇒ now apply some assumptions (small angle):

$\sin\theta \approx \theta$; $\cos\theta \approx 1$; $\dot{\theta}^2 \approx 0$

$$F = (m_c + m_p)\ddot{x} + b\dot{x} - m_p L\ddot{\theta}(1) - m_p L(0)(\theta)$$

$$\boxed{F = (m_c + m_p)\ddot{x} + b\dot{x} - m_p L\ddot{\theta}} \quad A$$

$$m_p L^2 \ddot{\theta} + I\ddot{\theta} + m_p g L(\theta) = m_p L\ddot{x}(1)$$

$$\boxed{(m_p L^2 + I)\ddot{\theta} + m_p g L\theta = m_p L\ddot{x}} \quad B$$

∴ A & B are my equations of motion.

2.b) First let's substitute in some numbers because if we don't this is going to get ugly. I am making approximations & choosing potentially unrealistic numbers because I think my arm is going to be really tired of writing by this point:

$m_p = 1\,kg$

$M_c = 1\,kg$

$L = 1\,m$

$I = 1\,kg \cdot m^2$

$g = 10\,m/s^2$

~~$b = 1\,N/m/s$~~

$b = 0$

★ Also, let's assume zero initial conditions.

Ⓐ : $F = (2)\ddot{x} + (0)\dot{x} - (1)\ddot{\theta}$

$F = 2\ddot{x} - \ddot{\theta}$

Ⓑ : $(1\cdot 1^2 + 1)\ddot{\theta} + (1)\cdot(10)(1)\theta = (1\cdot 1)\ddot{x}$

$2\ddot{\theta} + 10\theta = \ddot{x}$

→ apply Laplace transform:

$F(s) = 2(s^2X(s) - s\cancel{x(0)}^{0} - \cancel{\dot{x}(0)}^{0}) - (s^2\Theta(s) - s\cancel{\theta(0)}^{0} - \cancel{\dot{\theta}(0)}^{0})$

$\boxed{F(s) = 2s^2X(s) - s^2\Theta(s)}$ ①

$2(s^2\Theta(s) - s\cancel{\theta(0)}^{0} - \cancel{\dot{\theta}(0)}^{0}) + 10\Theta(s) = s^2X(s) - s\cancel{x(0)}^{0} - \cancel{\dot{x}(0)}^{0}$

$\Theta(s)(2s^2 + 10) = s^2X(s)$

$\boxed{X(s) = \Theta(s)\left(\frac{2s^2 + 10}{s^2}\right)}$ ②

sub ② into ①:

$F(s) = 2\cancel{s^2}\left[\Theta(s)\left(\frac{2s^2 + 10}{\cancel{s^2}}\right)\right] - s^2\Theta(s)$

$F(s) = \Theta(s)(4s^2 + 20 - s^2)$

$\boxed{\frac{\Theta(s)}{F(s)} = \frac{1}{3s^2 + 20}}$ Transfer function for the system

~~$\Theta(s)$~~

2. c) We will use MATLAB's ilaplace() function ~~A~~ to get the ~~transfer function~~ response to a step input in the time domain:

→ apply $F(s) = \frac{1}{s}$:

$$\Theta(s) = F(s) \cdot \left(\frac{1}{3s^2 + 20}\right)$$

$$\Theta(s) = \frac{1}{3s^3 + 20s}$$

$$\Theta(t) = \frac{1}{20} - \frac{1}{20}\cos\left(2\sqrt{\frac{5}{3}}\,t\right)$$

→ show plots of both analytical & step function in MATLAB.