Recitation 1, Spring 2020

24-352 Dynamic Systems and Controls

January 21, 2020

Robot Landing

A robot with mass m is dropped and hits the ground at time $t = 0$ with a velocity of $v(0) = -v_0$. The robot's leg motors are programmed to exert a downward force of $F(t)$ on the ground, resulting in a upward force of $F(t)$ on the robot. The force $F(t)$ can be written as

$$F(t) = F_g + F_s(t), \tag{1}$$

where F_g is a gravity compensation force used to hold the robot's weight at rest ($F_g = mg$), and $F_s(t)$ is a stopping force used to slow the robot's fall until the robot comes to rest. Note that $F_s(t)$ is equivalent to the total sum of forces on the robot.

When the robot's legs hits the ground, but its body is still traveling downwards (legs are compressing), $F_s(t)$ is programmed to be a constant force, f_s. When the robot's body stops traveling downwards and comes to rest, $F_s(t)$ is programmed to be 0.

$$F_s(t) = \begin{cases} f_s & \text{if } t \geq 0 \text{ and } v(t) < 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

(a) Plot $F_s(t)$.

(b) Find the Laplace transform of $F_s(t)$.

(c) Find $\int_0^\infty F_s(t)dt$. What is this value equivalent to?

(d) What happens to $F_s(t)$ as f_s is increased? What happens to $\int_0^\infty F_s(t)dt$?

(e) If the robot's motors are replaced with fixed joints, then the robot will hit the ground and come to rest immediately. In this case, what does the plot of $F_s(t)$ look like? What happens to $\int_0^\infty F_s(t)dt$?

Magnetic Microswimmer

A magnetic microswimmer can be pulled through a fluid in the x-direction using a force due to a magnetic field gradient, $F_{\nabla B}$. $F_{\nabla B}$ is dependent on the position of the swimmer, x. The swimmer will also experience some hydrodynamic drag, F_h, which is proportional to its velocity, $\dot{x}$, and it will ultimately accelerate towards the source of the magnetic field. We can write the equation of motion of the swimmer in the x-direction as:

$$
\begin{aligned}
m\ddot{x} &= F_{\nabla B} - F_h && (3)\\
&= \nabla B x - c_h \dot{x} && (4)
\end{aligned}
$$

For this problem, let us assume that the following initial conditions are true:

$$x(0) = 1, \dot{x}(0) = 0 \tag{5}$$

(a) Write the equation of motion in the frequency domain as an expression for $X(s)$.

(b) Let the coefficients have the following values: $m = 1, c_h = 1, \nabla B = 20$. Now write the equation of motion in the time domain as an expression for $x(t)$.

(c) Use the Initial Value Theorem to check your work.

(d) Can you use the Final Value Theorem to calculate the steady state value for this second-order system? Why or why not?

(e) If we change the drag coefficient and the magnetic field gradient coefficient to $c_h = 5$ and $\nabla B = -6$, what happens to the stability of the system? Can you use Final Value Theorem in this case?