

Generalized Additive Models: Allowing for some wiggle room in your models

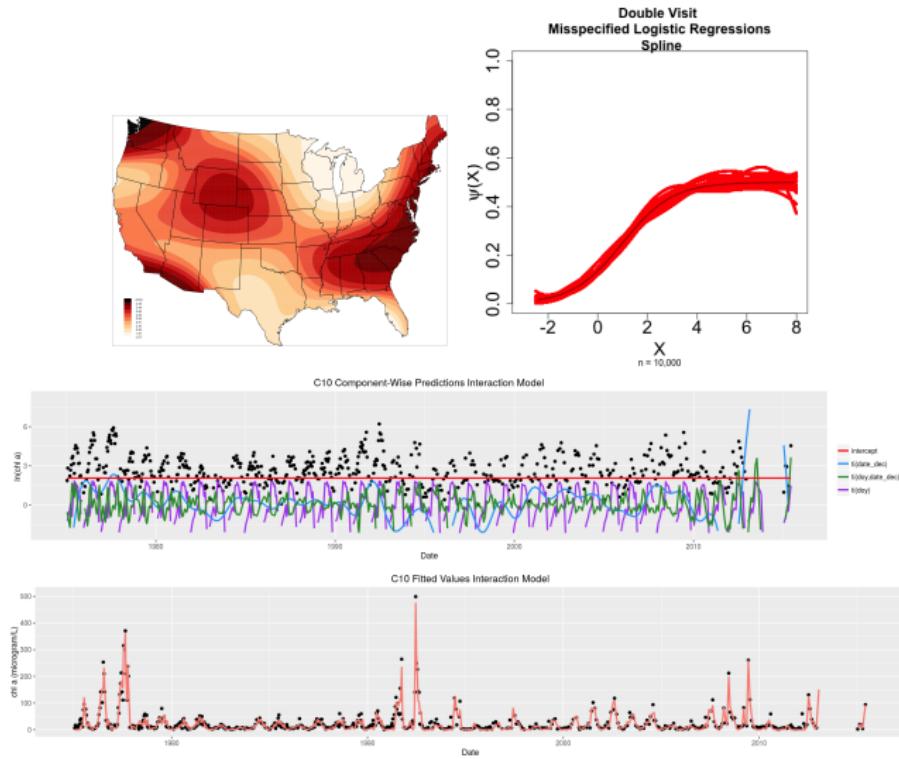
Sara Stoudt

March 17, 2021

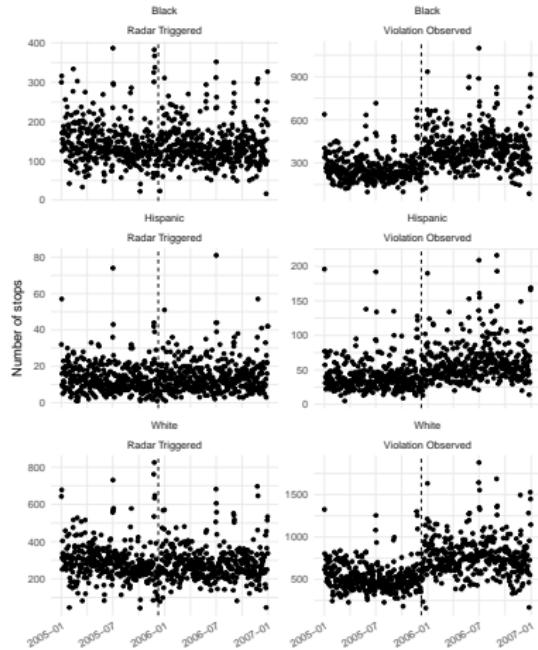
About Me

- currently teaching in the Statistical and Data Sciences Program at Smith College
- PhD in Statistics at Berkeley
 - ecology: evaluating fitness for purpose of a variety of data collection protocols for species distribution and abundance models
 - ecology: evaluating model fit in terms of community metrics for joint species distribution models
 - statistics communication: co-authored a book *Communicating with Data: The Art of Writing for Data Science* with Deborah Nolan

GAMs in my work



Setting the Scene



- “Using change in a seat belt law to study racially-biased policing in South Carolina” by Corinne A Riddell, Jay S Kaufman, Jacqueline M Torres, and Sam Harper
- <https://github.com/corinne-riddell/SCarolinaTrafficStops>

Linear Model - 1m

$$Y = X\beta + \epsilon$$

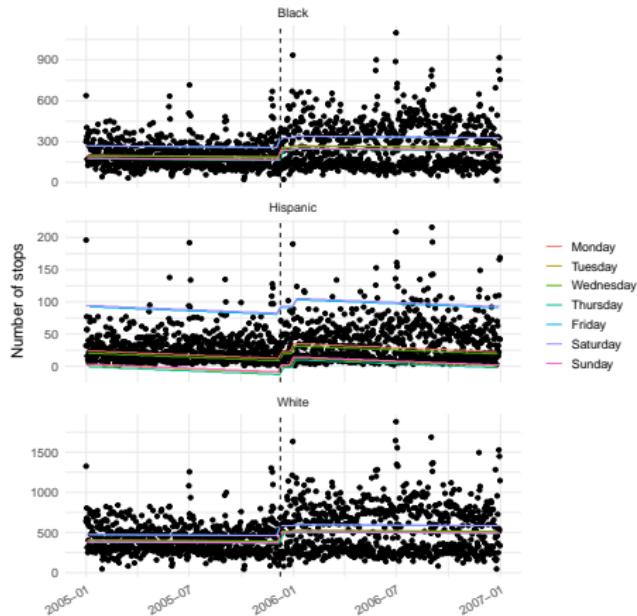
daily number of stops $\sim \beta_{driverRace} +$

$\beta_{isPostPolicy} + \beta_{driverRace, isPostPolicy} +$

$\beta_{dayOfWeek} + \beta * month$

Choices:

- which covariates X to use



Generalized Linear Model - `glm`

$$E[Y] \sim g^{-1}(X\beta)$$

$$g(E[Y]) \sim X\beta$$

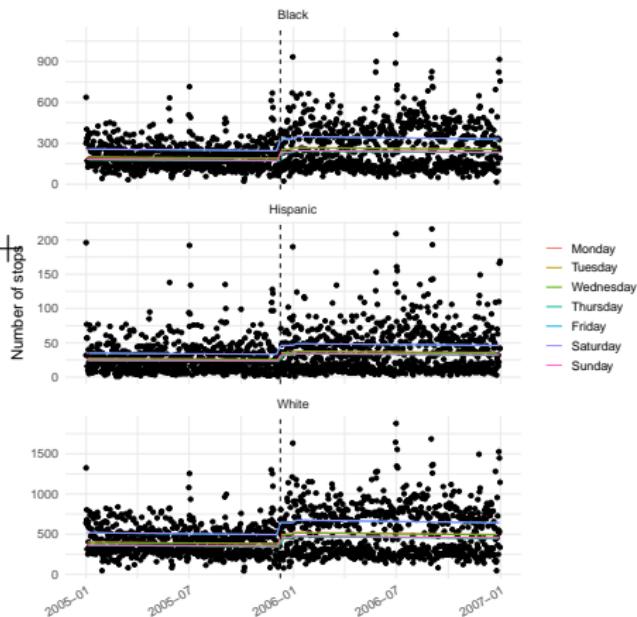
$$g(\text{daily number of stops}) \sim \beta_{\text{driverRace}} +$$

$$\beta_{\text{isPostPolicy}} + \beta_{\text{driverRace}, \text{isPostPolicy}} +$$

$$\beta_{\text{dayOfWeek}} + \beta * \text{month}$$

Choices:

- which covariates X to use
- response distribution
(quasipoisson) and link function g



Generalized Additive Models: Intuition

$$g(E[Y]) = X\beta + f_1(x_{1i}) + f_2(x_{2i})$$

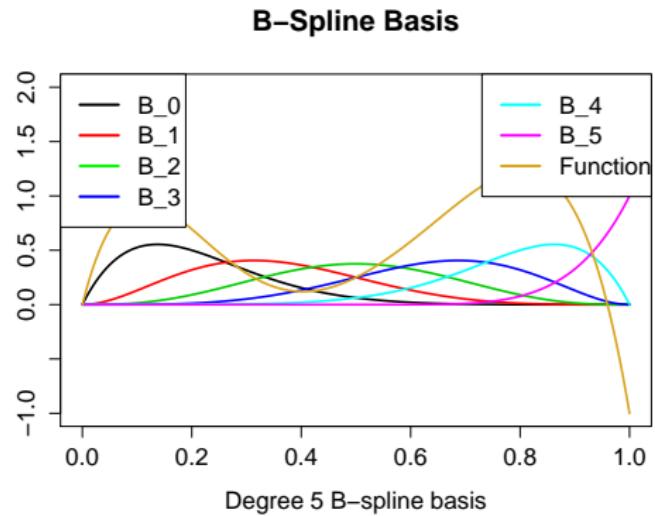
$$\begin{aligned} g(\text{daily number of stops}) \sim & \beta_{\text{driverRace}} + \\ & \beta_{\text{isPostPolicy}} + \beta_{\text{driverRace}, \text{isPostPolicy}} + \\ & f_1(\text{dayOfWeek}) + f_2(\text{month}) \end{aligned}$$

Choices:

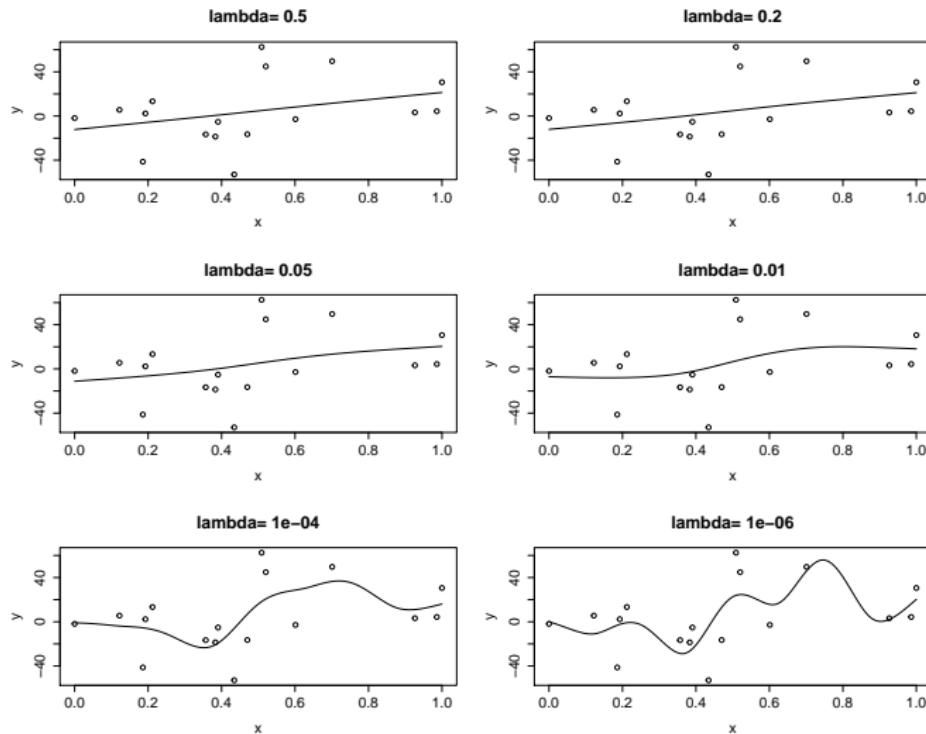
- which covariates X to use
- response distribution and link function g
- type of basis that defines f_i
- dimension of basis
- smoothing parameter

*Simon N. Wood. *Generalized Additive Models: An Introduction with R*. Chapman and Hall/CRC, 2006.

GAM: Parameter Intuition



GAM: Parameter Intuition

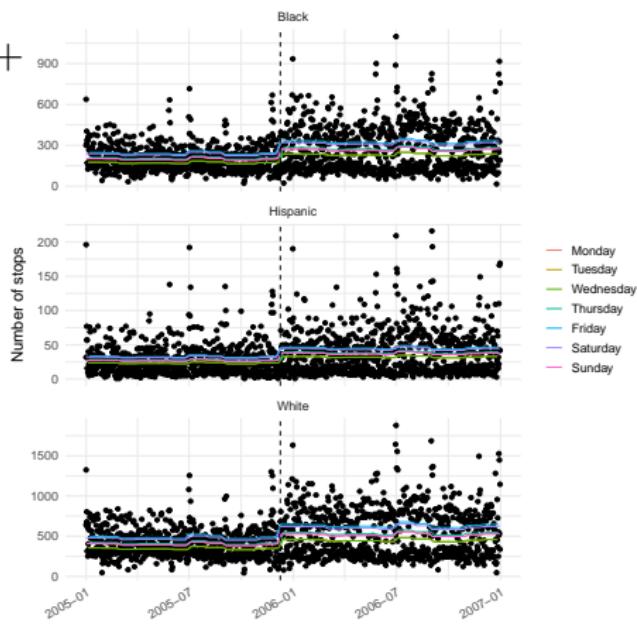


Generalized Additive Model - `mgcv:::gam`

$$g(\text{daily number of stops}) \sim \beta_{\text{driverRace}} + \beta_{\text{isPostPolicy}} + \beta_{\text{driverRace}, \text{isPostPolicy}} + f_1(\text{dayOfWeek}) + f_2(\text{month})$$

Choices:

- which covariates X to use
- response distribution (quasipoisson) and link function g
- type of basis that defines f_i
- dimension of basis
- smoothing parameter (default: GCV)



Under the Hood - Cyclic Basis

```
s(month, bs = "cc")
```

- constrained to have the same beginning and end
- helpful for time components
- when you aren't in the mood to do a formal time series analysis

Under the Hood - Choosing k

```
s(day_of_week_num, bs =  
"cc", k = 4)
```

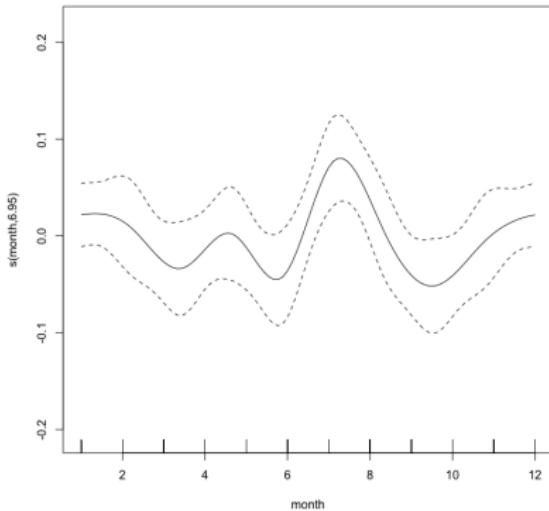
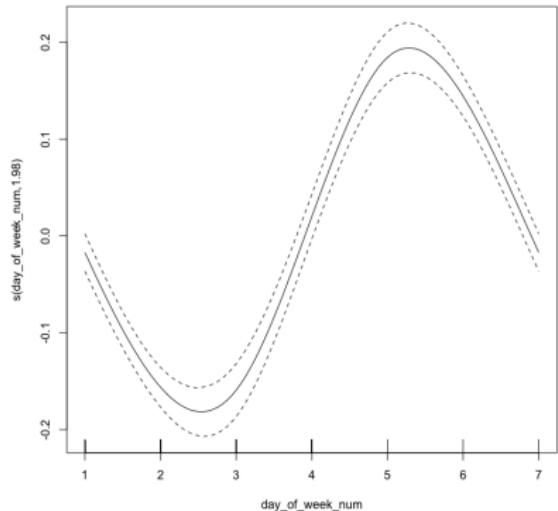
- k controls the maximum amount of flexibility
- bigger k means more computational complexity
- constrained by how much data you have (R will yell at you if you try to go too big)

```
gam.check(model)
```

- there isn't one magic k , robust to choice of k as long as in a reasonable range
- but did I pick a k big enough?
- rough guide: small p-value means you could probably benefit from increasing k

Under the Hood - Seeing the Smooths

```
plot(model, rug = T)
```



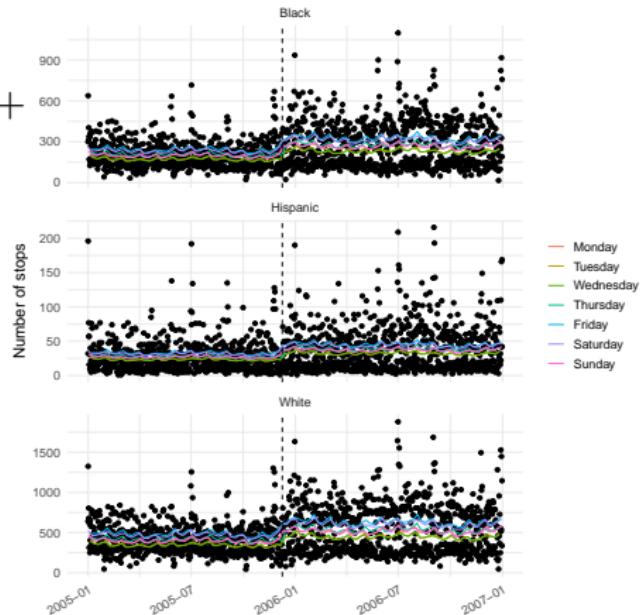
Generalized Additive Model - `mgcv:::gam`

$$g(\text{daily number of stops}) \sim \beta_{\text{driverRace}} +$$

$$\beta_{\text{isPostPolicy}} + \beta_{\text{driverRace}, \text{isPostPolicy}} +$$

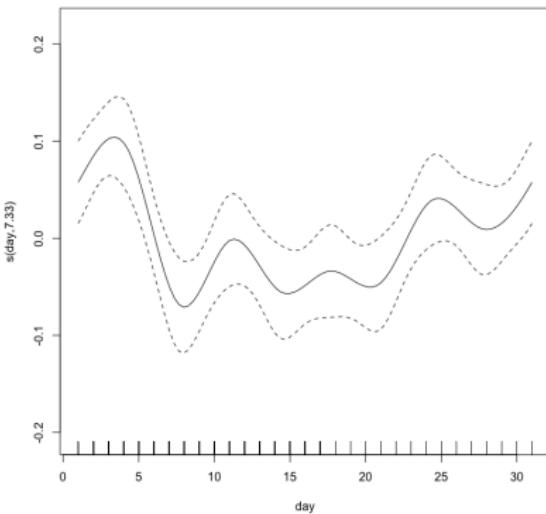
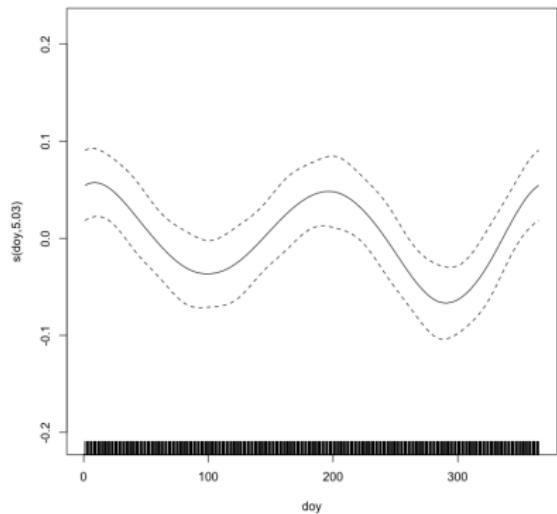
$$f_1(\text{dayOfWeek}) + f_2(\text{dayOfMonth}) +$$

$$f_3(\text{dayOfYear})$$



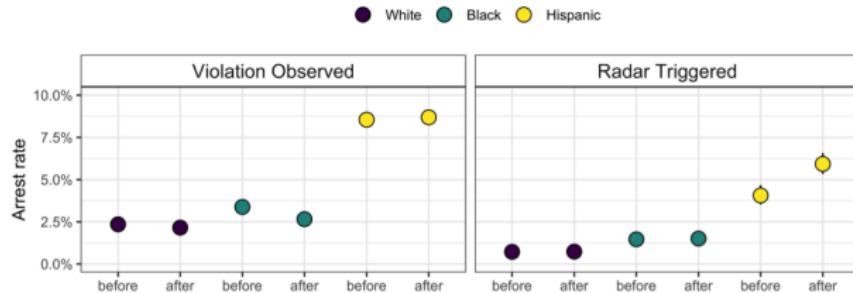
More Smoothes

```
plot(model, rug = T)
```



Are these wiggles “real”?

Setting the Scene



- “Using change in a seat belt law to study racially-biased policing in South Carolina” by Corinne A Riddell, Jay S Kaufman, Jacqueline M Torres, and Sam Harper
- https://github.com/corinne-riddell/SCarolinaTrafficStops

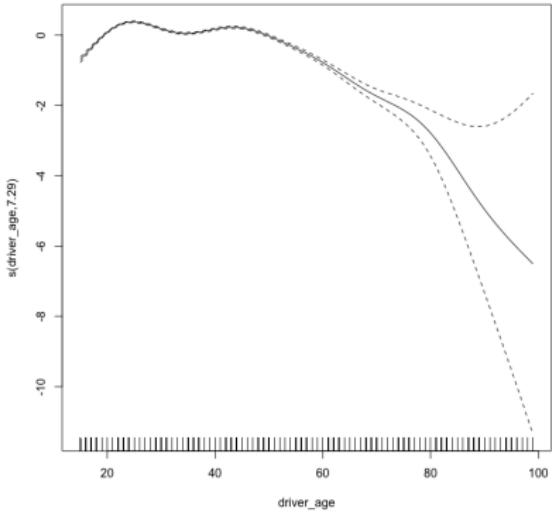
Big GAMs - mgcv::bam

$$g(\text{arrest}) \sim \beta_{\text{driverRace}} +$$

$$\beta_{\text{gender}} + f_1(\text{age}, bs = "cr") +$$

$$\beta_{\text{isPostPolicy}} + \beta_{\text{driverRace, isPostPolicy}}$$

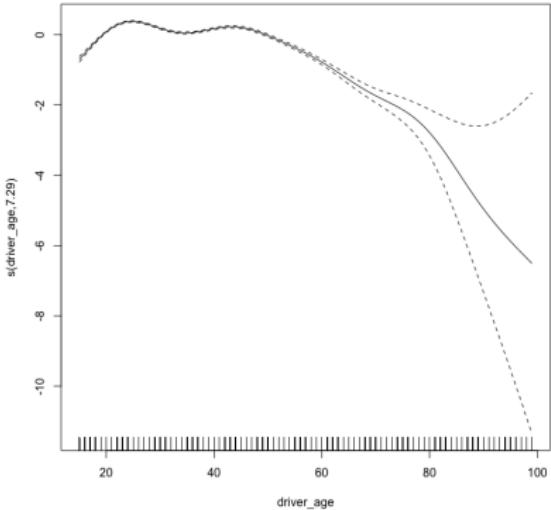
- response distribution
(binomial)
- link function g (logit)



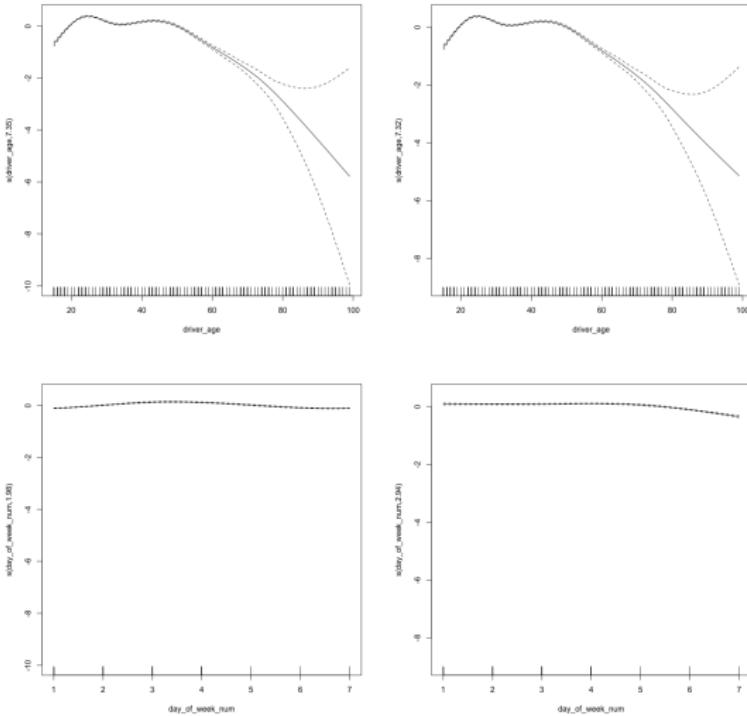
Shrinkage Splines - bs = "cs"

$$\begin{aligned}g(\text{arrest}) \sim & \beta_{\text{driverRace}} + \\& \beta_{\text{gender}} + f_1(\text{age}) + \\& \beta_{\text{isPostPolicy}} + \beta_{\text{driverRace}, \text{isPostPolicy}} + \\& f_1(\text{dayOfWeek}) + f_2(\text{dayOfMonth}) + \\& f_3(\text{dayOfYear})\end{aligned}$$

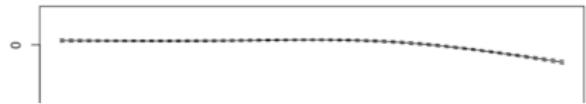
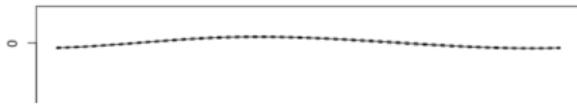
- response distribution
(binomial)
- link function g (logit)



Real wiggles?



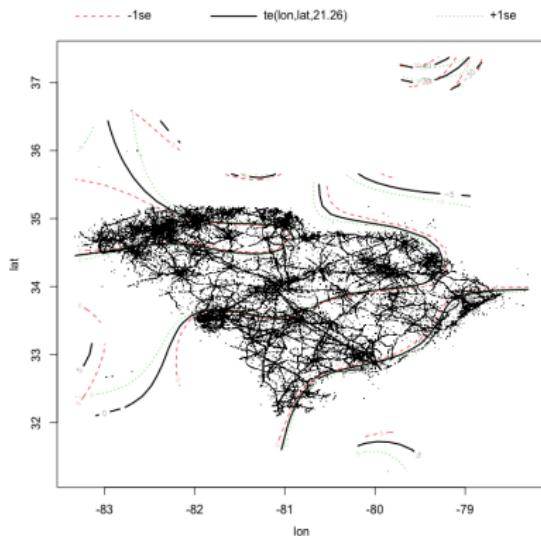
Subtle shrinkage happening...



Multidimensional Smoothing

`te(lon, lat)`

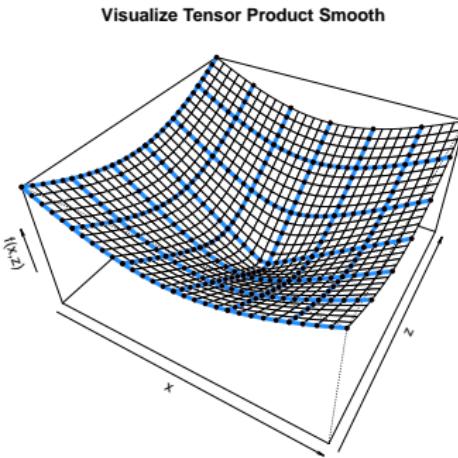
- two-dimensional smooth
- helpful for spatial components
- when you aren't in the mood to do a formal spatial analysis



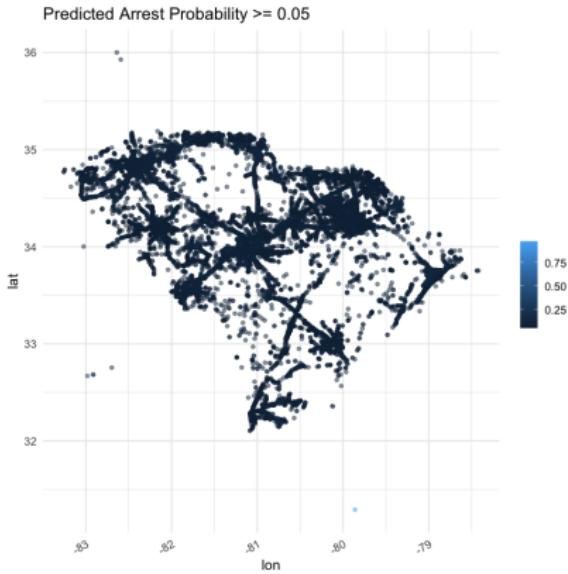
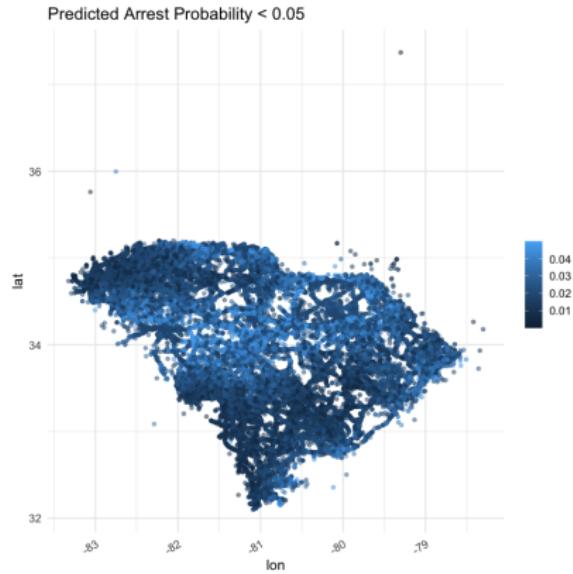
GAM: Parameter Intuition

$$f(x) = \sum_{i=1}^q a_i(x) \alpha_k$$

$$f(x, z) = \sum_i \beta_i(z) a_i(x) = \sum_i \sum_j \beta_{ij} b_j(z) a_i(x)$$



Spatial Predictions



More Resources

-
-

Time to GAM-ify your own work?

Questions?

sstoudt@smith.edu

[@sastoudt](https://twitter.com/sastoudt)

I'll leave you with some other Wiggles...



Figure: <https://www.youtube.com/watch?v=a13WnqsRc5g>