Assignment No 9

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1 Introduction

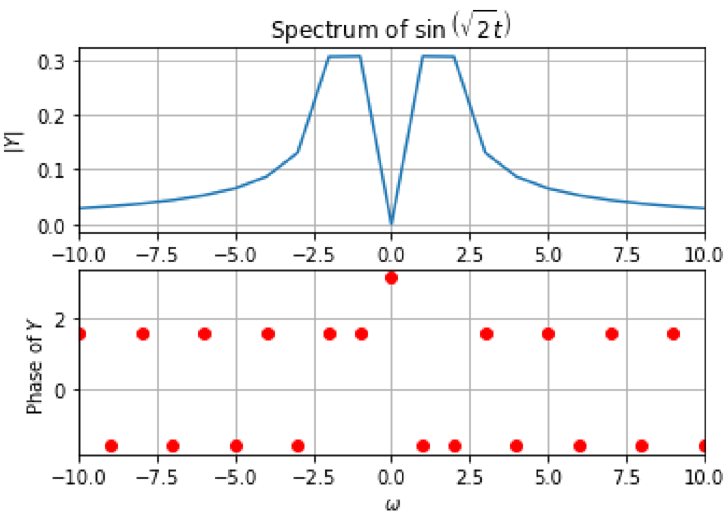
- We extend our analysis of signals using FT to aperiodic signals.
- We also see the Gibbs Phenomenon at work, we also resolve the problem using hamming window approach.
- We use this windowed transform to analyse signals known to contain a sinusoid of unknown frequencies and extract its phase and frequency.
- We also perform a sliding DFT on a chirped signal and plot the results

2 Problems and Solutions

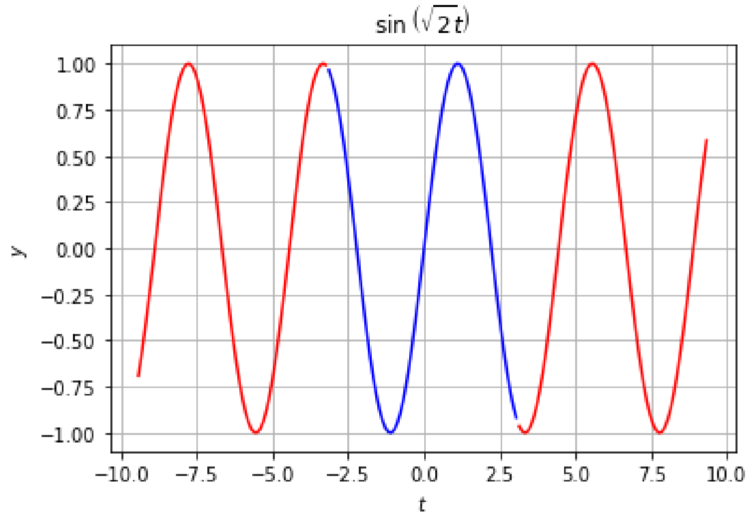
2.1 Working out examples

2.1.1 Spectrum of $\sin(\sqrt{2}t)$

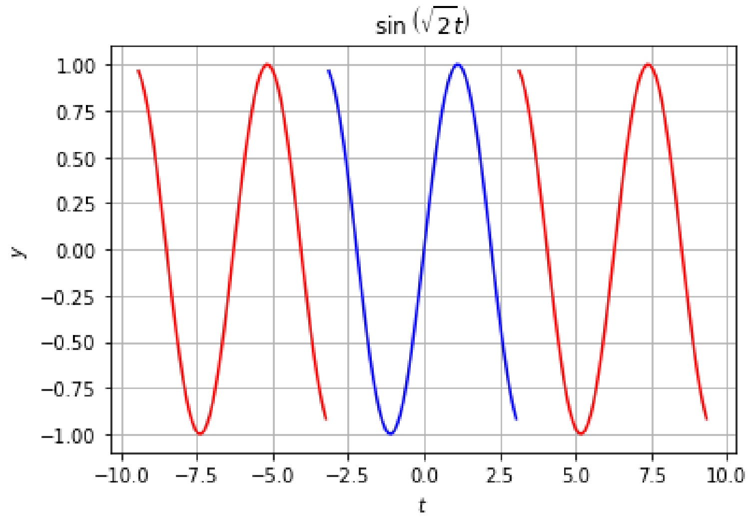
We start by plotting the spectrum of $\sin\left(\sqrt{(}2)t\right)$:



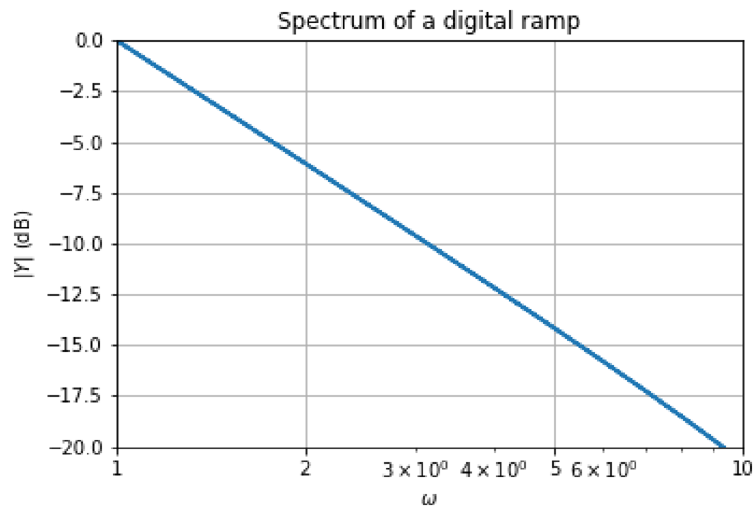
We want DFT for the function:



Since the DFT is plotted for a finite time interval we plot the DFT on the same graph as well.



As we can see that there are discontinuities, which leads to non harmonic components in the FFT, which decay as $\frac{1}{\omega}$, we also plot the spectrum of a periodic ramp below to confirm this:

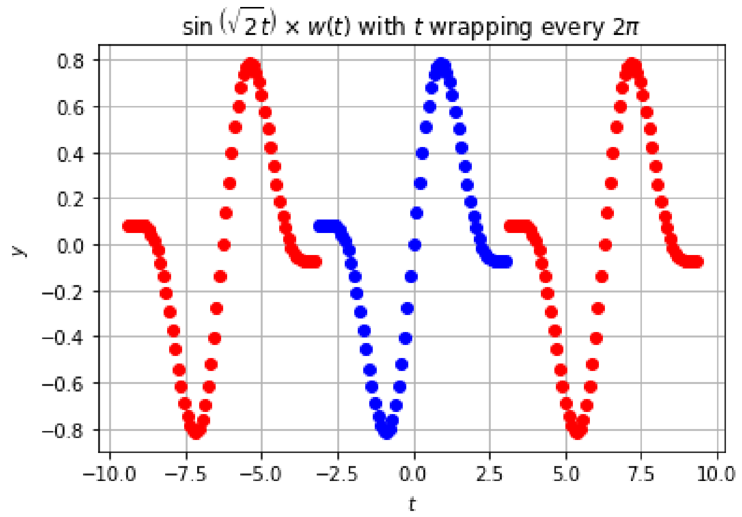


2.1.2 Hamming Window

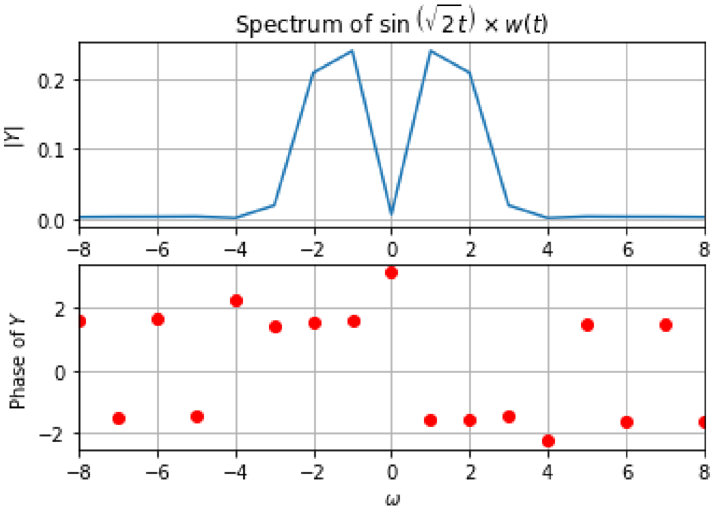
We use the Hamming window to remove the discontinuities by attenuating the high frequency components that are major causes of discontinuities. The Hamming window function is given by:

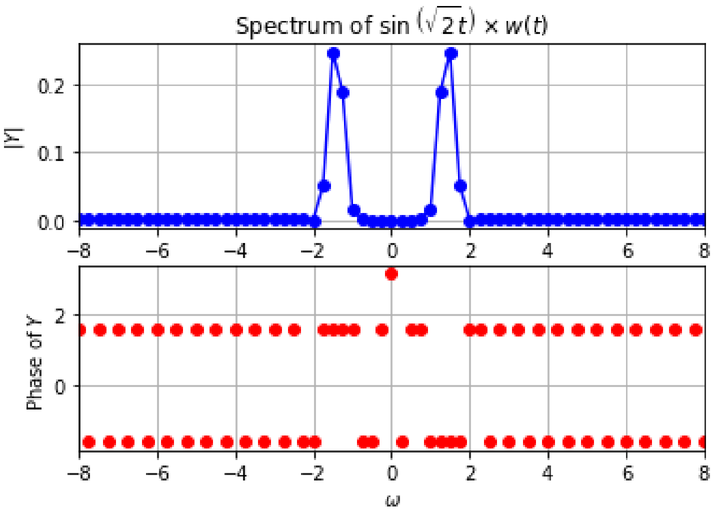
$$x[n] = 0.54 + 0.46cos(\frac{2\pi n}{N-1}$$

We multiply our signal with this window and extend it periodically.



We see that the discontinuities starts to vanish.
We plot the spectrum with time periods 2π and 8π respectively:





We see the spectrum with the time period of 8π has slightly more sharper peaks.

2.1.3 Helper Functions

We used the following helper functions to help us simply the various problems we are tasked with:

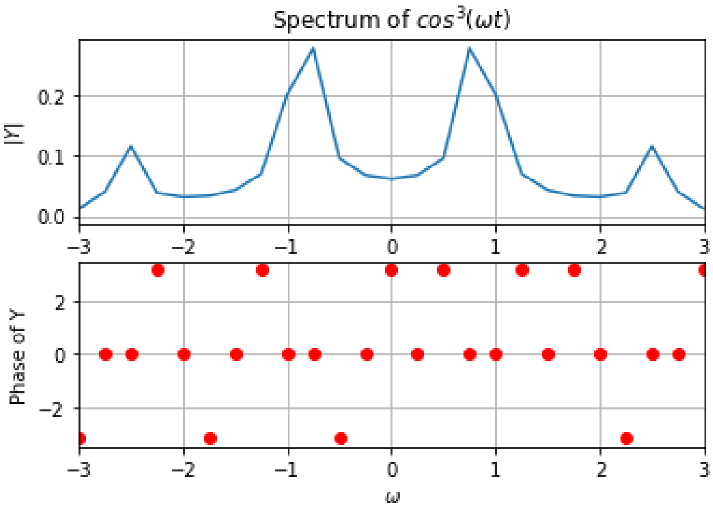
```
def solve(f, lim, n, xlim, ylabel, ylabel_, xlabel,
            title, window = True, show = True, last = False, t2 = None):
    t, dt = np.linspace(-lim, lim, n, False, retstep = True)
    if last:
```

```
        t = t2;dt = t2[1] - t2[0]
    fmax = 1/dt
    y = f(t)
    if window:
        a = np.arange(n)
        wnd = fft.fftshift(0.54+0.46*np.cos(2*np.pi*a/n))
        y = y*wnd
    y[0] = 0
    y = fft.fftshift(y)
    Y = fft.fftshift(fft.fft(y))/n
    w = np.linspace(-np.pi*fmax, np.pi*fmax, n, False)

    if show:
        plt.figure()
        plt.subplot(2,1,1)
        plt.plot(w, abs(Y))
        plt.xlim([-xlim, xlim])
        plt.ylabel(ylabel)
        plt.title(title)
        plt.grid()
        plt.subplot(2,1,2)
        phase = np.angle(Y)
        phase[np.where(abs(Y)<3e-3)] = 0
        plt.plot(w, phase, "ro")
        plt.xlim([-xlim, xlim])
        plt.ylabel(ylabel_)
        plt.xlabel(xlabel)
        plt.grid()
        plt.show()

    return Y,w

def estimator(Y, w, window = 1, s = 1e-4):
    ii = np.where(w>0)
    ii_ = np.where(np.logical_and(abs(Y)>s, w>0))[0]
    np.sort(ii_)
    omega = sum(abs(Y[ii])**2*w[ii])/sum(abs(Y[ii])**2)
    delta = np.sum(np.angle(Y[ii_[1:window+1]]))/(window+1)
    print(f"Estimate Delta and Omega are {delta}, {omega} respectively.")
```

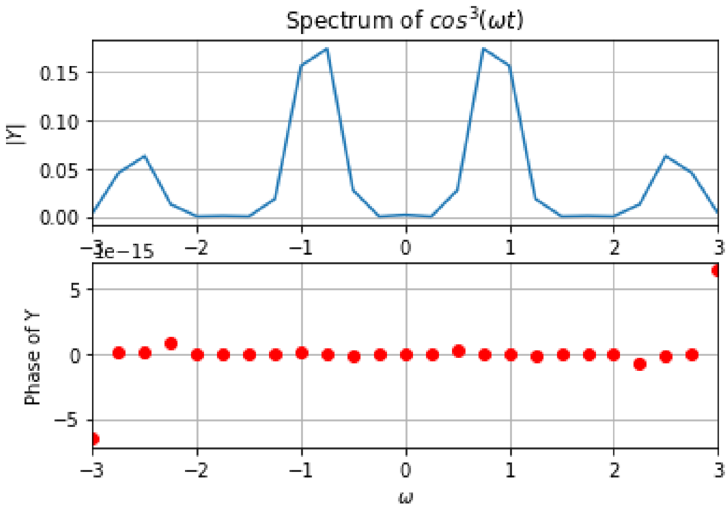
2.2 Question 2

We are tasked with plotting the FFT of $\cos^3(0.86t)$:

The FFT without the Hamming window is:



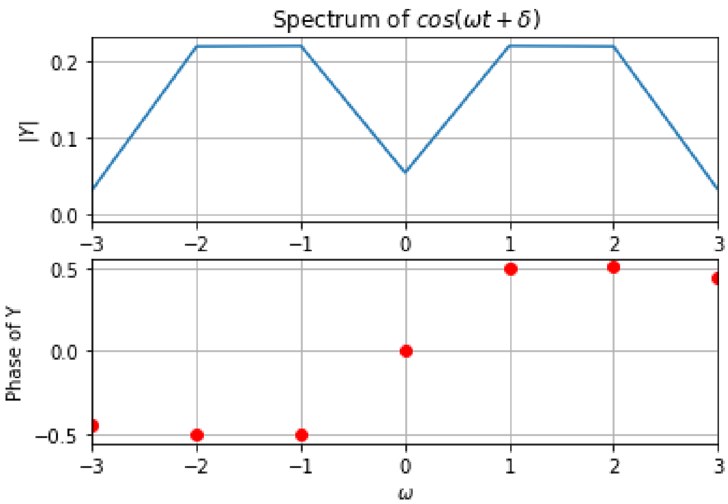
The FFt with the hamming Window:



We see that the energy of the signal is distributed in many high frequencies that aren't part of the actual signal, these energy gets attenuated after performing windowing.

2.3 Question 3

We want to estimate ω and δ for the signal $\cos(\omega t + \delta)$, we estimate omega using weighted average of $|Y|$ and for δ we consider a window on each half of ω and extract the mean slope.



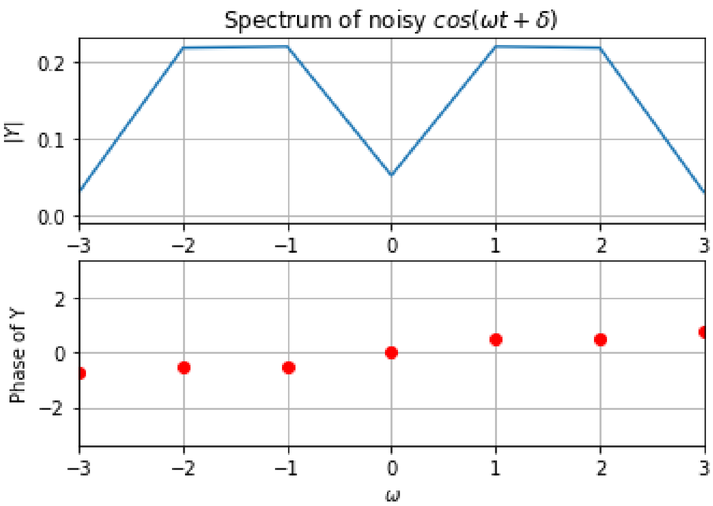
2.4 Question 4

In this question we take a similar approach to question 3 as it is same a question 3 but with noise added.

2.5 Question 5

In this we analyze a chirp signal which is a Frequency modulated signal:

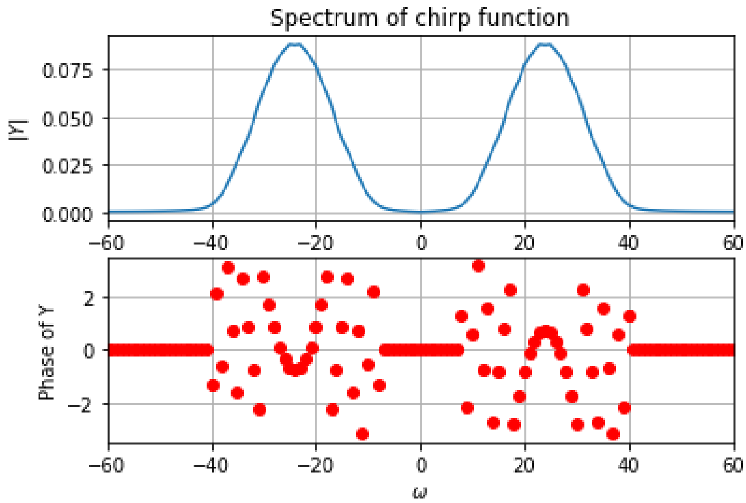
$$f(t) = \cos\left(16t(1.5 + \frac{t}{2\pi})\right)$$

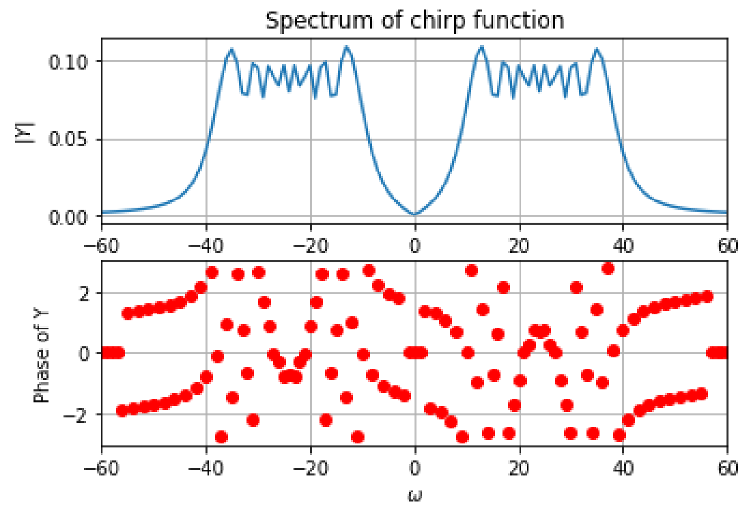


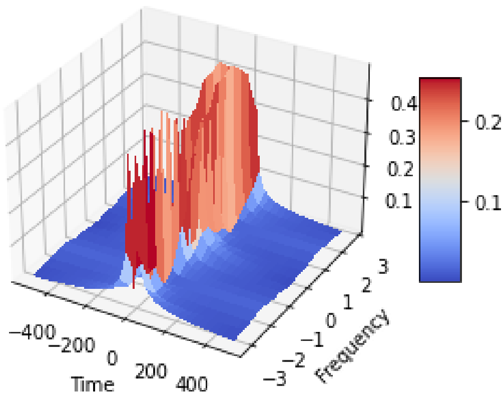
We see that a large frequency appears as Gibbs frequency.

3 Question 6

For the same chirped signal, we break the 1024 vector into pieces that are 64 samples wide. Then plot the array as a surface plot to show how the frequency of the signal varies with time.







3.1 Conclusion

In this assignment we say the use of hamming window to find the DFT's of aperodic signals. We do this to mitigate the effect of Gibbs phenomenon. The last question addresses the time varying spectra for a chirped signal, where we plot Fourier spectra for different time slices of a signal.