Assignment No 6

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1 Introduction

In this assignment, we learnt:

- How to analyze LTI systems.
- How to use scipy.signals for the same.
- How RLC systems are used as low pass filters.
- How to analyze and plot Laplace Transforms.

2 Problems and Solutions

2.1 Question 1

We were given the following differential equation to solve with the initial conditions:

$$\ddot{x} + 2.25x = f(t)$$

x(0) = 0, $\dot{x(0)} = 0$

We converted it from time domain to Laplace domain and we got

$$s^2X(s) + 2.25X(s) = F(s)$$

We used impulse response of X(s) to get its inverse Laplace transform.

We got Fs and Xs and from a helper function `gib_Fs_Xs_TF()`.

```
Fs =gib_Fs_Xs_TF()
Xs = gib_Fs_Xs_TF(decay = 0.5, Fs = Fs,Xs = True)
t = np.linspace(1,50, 300) #numpy defaults to 50 points
t, xt = sp.impulse(Xs, T = t)
```

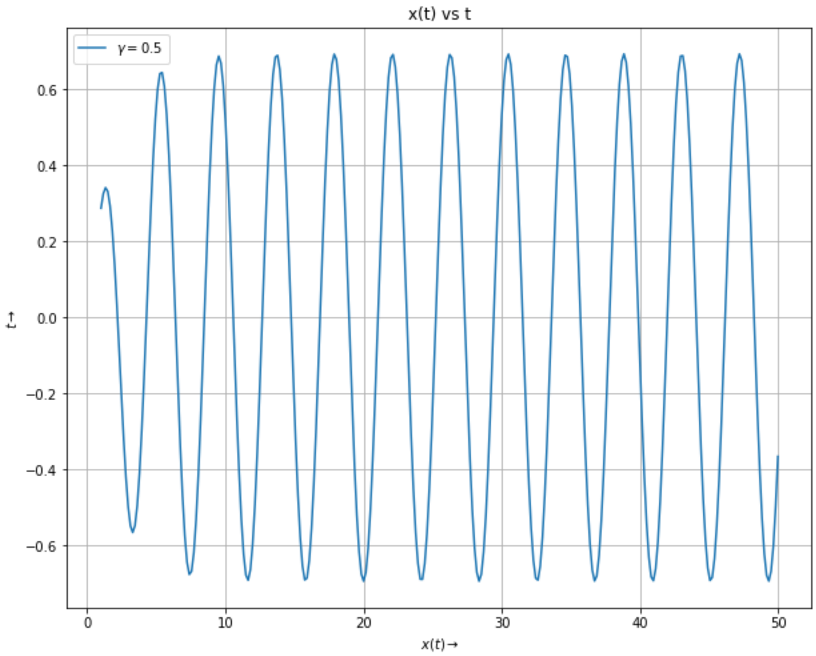

Figure 1: System response for decay = 0.5

2.2 Question 2

We now analyze the same system with a smaller decay constant

```
Fs = gib_Fs_Xs_TF(decay = 0.05)
Xs = gib_Fs_Xs_TF(decay = 0.05, Fs = Fs, Xs = True)
t = np.linspace(1,50, 300) #numpy defaults to 50 points
t, xt = sp.impulse(Xs, T = t)
```

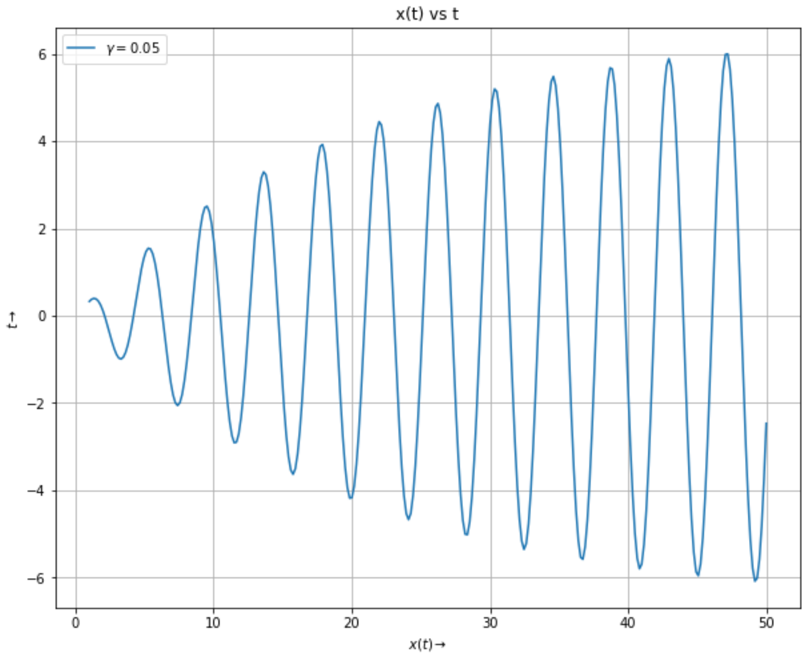

Figure 2: System response for decay = 0.05

2.3 Question 3

Next we analyze what happens when we vary the frequency, we vary the frequency from 1.4 to 1.6 with steps of 0.05 and plot the resulting values.

```
for freq in np.arange(1.4, 1.65, 0.05):
    t = np.linspace(1,150, 400)
    TF = gib_Fs_Xs_TF(freq = freq, decay = 0.05, TF = True)
    t,y,_ = sp.lsim(TF, U = ft(freq, 0.05, t), T = t)
```

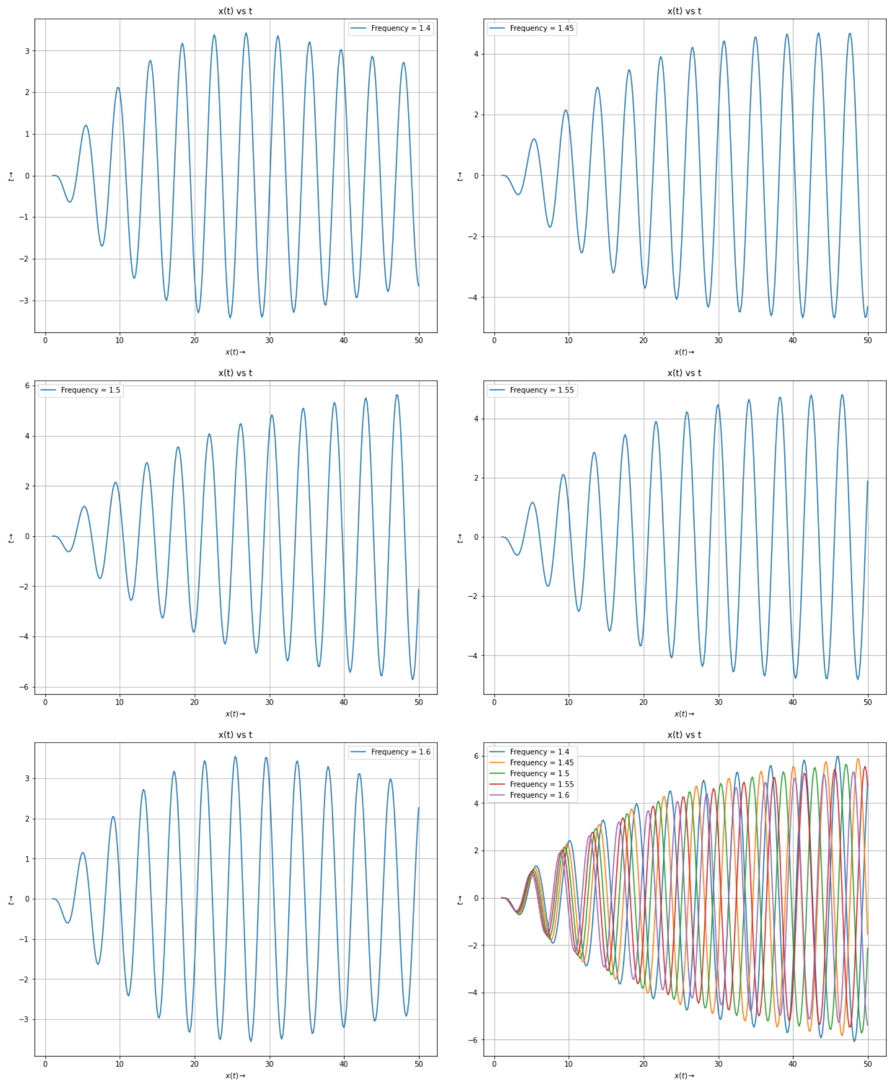

Figure 3: System response for different frequencies

We see that when the input frequency is at natural frequency the output amplitude is maximum and in other cases the output amplitude decreases, this is due to resonance effect.

2.4 Question 4

We now consider a coupled differential system

$$\ddot{x} + (x - y) = 0$$

and

$$\ddot{y} + 2(y - x) = 0$$

with the initial conditions: $\dot{x} = 0, \dot{y} = 0, x(0) = 1, y(0) = 0$. Taking Laplace Transform and solving for X(s) and Y(s), we got:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s}$$
$$Y(s) = \frac{2}{s^3 + 3s}$$

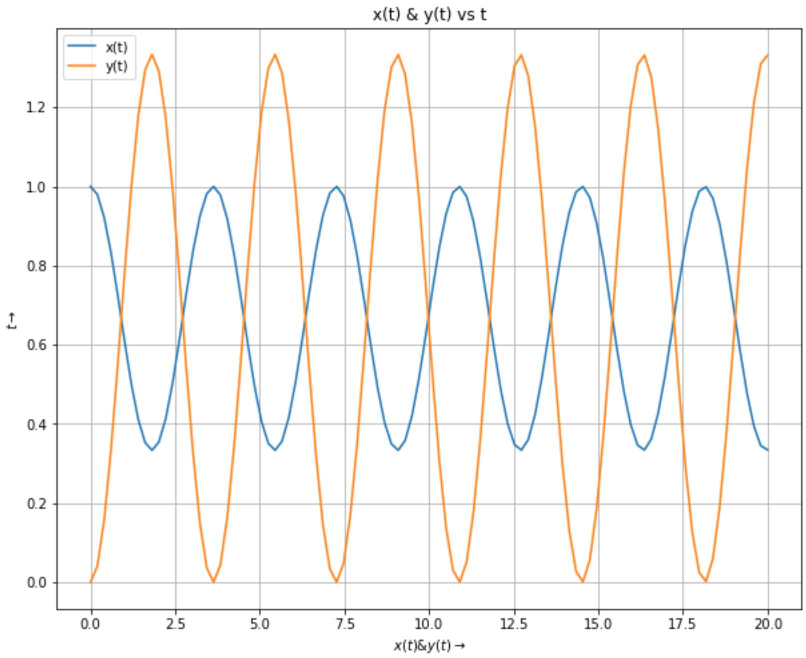

Figure 4: Coupled Oscillations

2.5 Question 5

Next we plot the bode plot for the low pass filter defined in the question:

We used a helper function called `TF_5_plotting` for plotting the bode plot.

```
H = sp.lti([1], [L*C, R*C, 1])
    if bode:
        w,S, phi = H.bode()
```

```
plt.figure(figsize = (70,70))
plt.subplot(10,8,1)
plt.title("Magnitude plot")
plt.ylabel(r"$|H(j\omega)|\rightarrow$")
plt.xlabel(r"$\omega\rightarrow$")
plt.semilogx(w, S, label = "Magnitude")
plt.grid(); plt.legend()
plt.subplot(10,8,2)
plt.title("Phase plot")
plt.ylabel(r"\phi\rightarrow")
plt.xlabel(r"\omega\rightarrow")
plt.semilogx(w,phi, label ="Phase")
plt.grid(); plt.legend()
plt.show()
```

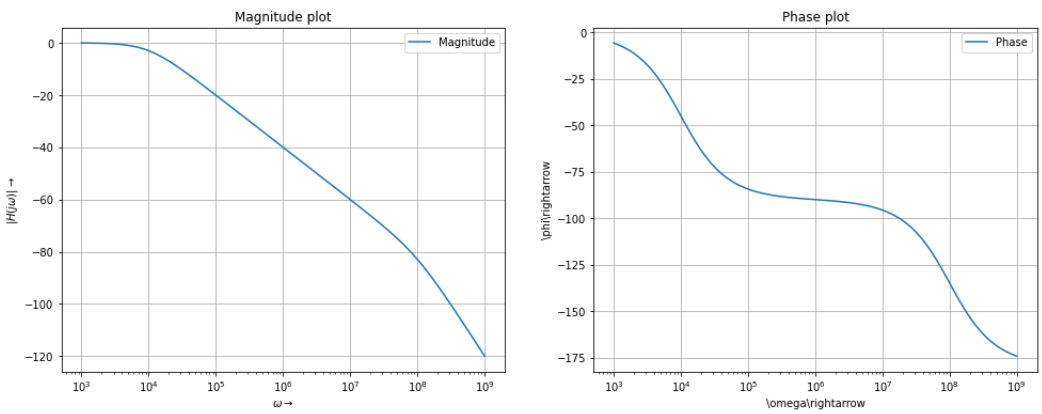

Figure 5: Bode Plots for RLC low pass filter

2.6 Question 6

We now give input to the low pass filter and plot the response for $0 < t < 30\mu s$ and $0 < t < 30ms$.
We also see that the poles are at 10^4 and 10^8.

```
t1 = np.linspace(0,30e-6,150)
t1,y1,_ = sp.lsim(H, inSignal(t1), T = t1)
t2 = np.linspace(0,30e-3, 10000000)
t2,y2,_ = sp.lsim(H, inSignal(t2), T = t2)
plt.figure(figsize = (70,70))
plt.subplot(10,8,1)
plt.title(r"$V_o$ till $30\mu s$")
plt.ylabel(r"$V_o(t)\rightarrow$")
plt.xlabel(r"$t\rightarrow$")
```

```
plt.plot(t1, y1, label = "y(t)")
plt.grid();plt.legend()
plt.subplot(10,8,2)
plt.title(r"$V_o$ till $30ms$")
plt.ylabel(r"$V_o(t)\rightarrow$")
plt.xlabel(r"$t\rightarrow$")
plt.plot(t2,y2, label = "y(t)")
plt.grid();plt.legend()
plt.show()
```

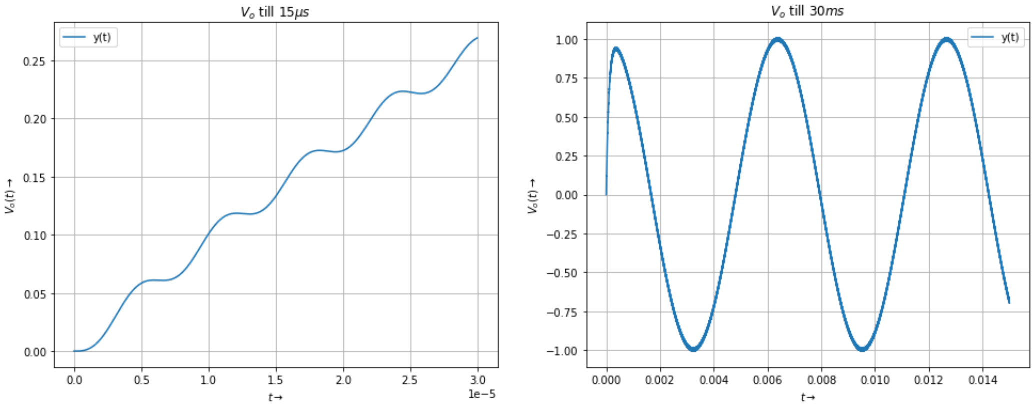

Figure 6: Caption

In the short term response we can see that the capacitor is charging up to input amplitude and there are small ripples due to the high frequency component. This component dies out in long term response plot hence is not visible.
We can see that the low frequency component passes almost unchanged as it is under 10^4 and rest all frequency that are greater than 10^4 are not passed.

3 Conclusion

- LTI systems are observed in all fields of engineering and are very important.
- We learnt about scipy's signal processing library.
- We analyzed how to solve differential equations using Laplace Transforms.
- We plotted the graphs for the better analysis.