Econometrics Test Exercise 7

ANSWERS to Case Project – House Prices

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Background

This project is of an applied nature and uses data that are available in the data file Capstone-HousePrices. The source of these data is Anglin and Gencay, "Semiparametric Estimation of a Hedonic Price Function"(Journal of Applied Econometrics 11, 1996, pages 633-648). We consider the modeling and prediction of house prices. Data are available for 546 observations of the following variables:

- **sell**: Sale price of the house
- **lot**: Lot size of the property in square feet
- **bdms**: Number of bedrooms
- **fb**: Number of full bathrooms
- **sty**: Number of stories excluding basement
- **drv**: Dummy that is 1 if the house has a driveway and 0 otherwise
- **rec**: Dummy that is 1 if the house has a recreational room and 0 otherwise
- **ffin**: Dummy that is 1 if the house has a full finished basement and 0 otherwise
- **ghw**: Dummy that is 1 if the house uses gas for hot water heating and 0 otherwise
- **ca**: Dummy that is 1 if there is central air conditioning and 0 otherwise
- **gar**: Number of covered garage places
- **reg**: Dummy that is 1 if the house is located in a preferred neighborhood of the city and 0 otherwise
- **obs**: Observation number, needed in part (h)

Questions

- **(a)** Consider a linear model where the sale price of a house is the dependent variable and the explanatory variables are the other variables given above. Perform a test for linearity. What do you conclude based on the test result?
- **(b)** Now consider a linear model where the log of the sale price of the house is the dependent variable and the explanatory variables are as before. Perform again the test for linearity. What do you conclude now?
- **(c)** Continue with the linear model from question (b). Estimate a model that includes both the lot size variable and its logarithm, as well as all other explanatory variables without transformation. What is your conclusion, should we include lot size itself or its logarithm?
- **(d)** Consider now a model where the log of the sale price of the house is the dependent variable and the explanatory variables are the log transformation of lot size, with all other explanatory variables as before. We now consider interaction effects of the log lot size with the other variables. Construct these interaction variables. How many are individually significant?
- **(e)** Perform an F-test for the joint significance of the interaction effects from question (d).
- **(f)** Now perform model specification on the interaction variables using the general-to-specific approach. (Only eliminate the interaction effects.)
- **(g)** One may argue that some of the explanatory variables are endogenous and that there may be omitted variables. For example, the ‘condition’ of the house in terms of how it is maintained is not a variable (and difficult to measure) but will affect the house price. It will also affect, or be reflected in, some of the other variables, such as whether the house has an air conditioning (which is mostly in newer houses). If the condition of the house is missing, will the effect of air conditioning on the (log of the) sale price be over- or underestimated? (For this question no computer calculations are required.)
- **(h)** Finally we analyze the predictive ability of the model. Consider again the model where the log of the sale price of the house is the dependent variable and the explanatory variables are the log transformation of lot size, with all other explanatory variables in their original form (and no interaction effects). Estimate the parameters of the model using the first 400 observations. Make predictions on the log of the price and calculate the MAE for the other 146 observations. How good is the predictive power of the model (relative to the variability in the log of the price)?

Notes

See website for how to submit your answers and how feedback is organized.

- This exercise uses the datafile CaseProject-HousePrices and requires a computer.
- The dataset CaseProject-HousePrices is available on the website.
- Perform all tests at a 5% significance level.

Goals and skills being used

- Experience the processes of variable transformation and model selection.
- Apply tests to evaluate models, including effects of endogeneity.
- Study the predictive ability of a model.

Exercise initialization

Loading & preparing data…

```
datafilename <- "../Housing-Prices.txt"
dat <- read.csv(datafilename, sep = '\t')

n <- nrow(dat)
print(paste(n, "data entries loaded.."))
```

```
## [1] "546 data entries loaded.."
```

```
dat$sell_LOG <- log(dat$sell)
dat$lot_LOG  <- log(dat$lot)

str(dat)
```

```
## 'data.frame':    546 obs. of  15 variables:
##  $ obs     : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ sell    : int  42000 38500 49500 60500 61000 66000 66000 69000 83800 88500 ...
##  $ lot     : int  5850 4000 3060 6650 6360 4160 3880 4160 4800 5500 ...
##  $ bdms    : int  3 2 3 3 2 3 3 3 3 3 ...
##  $ fb      : int  1 1 1 1 1 1 2 1 1 2 ...
##  $ sty     : int  2 1 1 2 1 1 2 3 1 4 ...
##  $ drv     : int  1 1 1 1 1 1 1 1 1 1 ...
##  $ rec     : int  0 0 0 1 0 1 0 0 1 1 ...
##  $ ffin    : int  1 0 0 0 0 1 1 0 1 0 ...
##  $ ghw     : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ ca      : int  0 0 0 0 0 1 0 0 0 1 ...
##  $ gar     : int  1 0 0 0 0 0 2 0 0 1 ...
##  $ reg     : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ sell_LOG: num  10.6 10.6 10.8 11 11 ...
##  $ lot_LOG : num  8.67 8.29 8.03 8.8 8.76 ...
```

```
summary(dat)
```

```
##       obs             sell             lot             bdms
##  Min.   :  1.0   Min.   : 25000   Min.   : 1650   Min.   :1.000
##  1st Qu.:137.2   1st Qu.: 49125   1st Qu.: 3600   1st Qu.:2.000
##  Median :273.5   Median : 62000   Median : 4600   Median :3.000
##  Mean   :273.5   Mean   : 68122   Mean   : 5150   Mean   :2.965
##  3rd Qu.:409.8   3rd Qu.: 82000   3rd Qu.: 6360   3rd Qu.:3.000
##  Max.   :546.0   Max.   :190000   Max.   :16200   Max.   :6.000
##        fb              sty              drv              rec
##  Min.   :1.000   Min.   :1.000   Min.   :0.000   Min.   :0.0000
##  1st Qu.:1.000   1st Qu.:1.000   1st Qu.:1.000   1st Qu.:0.0000
##  Median :1.000   Median :2.000   Median :1.000   Median :0.0000
##  Mean   :1.286   Mean   :1.808   Mean   :0.859   Mean   :0.1777
##  3rd Qu.:2.000   3rd Qu.:2.000   3rd Qu.:1.000   3rd Qu.:0.0000
##  Max.   :4.000   Max.   :4.000   Max.   :1.000   Max.   :1.0000
##       ffin             ghw                ca              gar
##  Min.   :0.0000   Min.   :0.00000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.00000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :0.00000   Median :0.0000   Median :0.0000
##  Mean   :0.3498   Mean   :0.04579   Mean   :0.3168   Mean   :0.6923
##  3rd Qu.:1.0000   3rd Qu.:0.00000   3rd Qu.:1.0000   3rd Qu.:1.0000
##  Max.   :1.0000   Max.   :1.00000   Max.   :1.0000   Max.   :3.0000
##       reg            sell_LOG        lot_LOG
##  Min.   :0.0000   Min.   :10.13   Min.   :7.409
##  1st Qu.:0.0000   1st Qu.:10.80   1st Qu.:8.189
##  Median :0.0000   Median :11.03   Median :8.434
##  Mean   :0.2344   Mean   :11.06   Mean   :8.467
##  3rd Qu.:0.0000   3rd Qu.:11.31   3rd Qu.:8.758
##  Max.   :1.0000   Max.   :12.15   Max.   :9.693
```

(a) First linear model

(a) Consider a linear model where the sale price of a house is the dependent variable and the explanatory variables are the other variables given above. Perform a test for linearity. What do you conclude based on the test result?

First model estimation

```
# Estimating first linear model.
modelA <- lm(sell ~ lot + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg,
             data = dat)
print(modelA.summary <- summary(modelA))
```

```
##
## Call:
## lm(formula = sell ~ lot + bdms + fb + sty + drv + rec + ffin +
##     ghw + ca + gar + reg, data = dat)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -41389  -9307   -591   7353  74875
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4038.3504  3409.4713  -1.184 0.236762
## lot             3.5463     0.3503  10.124  < 2e-16 ***
## bdms         1832.0035  1047.0002   1.750 0.080733 .
## fb          14335.5585  1489.9209   9.622  < 2e-16 ***
## sty          6556.9457   925.2899   7.086 4.37e-12 ***
## drv          6687.7789  2045.2458   3.270 0.001145 **
## rec          4511.2838  1899.9577   2.374 0.017929 *
## ffin         5452.3855  1588.0239   3.433 0.000642 ***
## ghw         12831.4063  3217.5971   3.988 7.60e-05 ***
## ca          12632.8904  1555.0211   8.124 3.15e-15 ***
## gar          4244.8290   840.5442   5.050 6.07e-07 ***
## reg          9369.5132  1669.0907   5.614 3.19e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15420 on 534 degrees of freedom
## Multiple R-squared:  0.6731, Adjusted R-squared:  0.6664
## F-statistic: 99.97 on 11 and 534 DF,  p-value: < 2.2e-16
```

Conclusion:

The first model considered is:

$$\begin{aligned} sell = & -4038.35 \\ & + 3.546 \cdot lot \\ & + 1832.003 \cdot bdms \\ & + 14335.558 \cdot fb \\ & + 6556.946 \cdot sty \\ & + 6687.779 \cdot drv \\ & + 4511.284 \cdot rec \\ & + 5452.386 \cdot ffin \\ & + 12831.406 \cdot ghw \\ & + 12632.89 \cdot ca \\ & + 4244.829 \cdot gar \\ & + 9369.513 \cdot reg \\ & + \epsilon \end{aligned}$$

Model characteristics: **R^2** = 0.6731 , **F-statistic**: 99.97 on 11 and 534 DF

Model linearity testing

Ramsey's RESET (linearity) testing

```
# Model Ramsey's RESET testing.
modelA.RESET <- resettest(modelA, power = 2, type = "fitted",
                          data = dat)
print(modelA.RESET)
```

```
##
##  RESET test
##
## data:  modelA
## RESET = 26.986, df1 = 1, df2 = 533, p-value = 2.922e-07
```

Conclusion:

> With a statistic of **~26.986** and a p-value of **~0**, the Ramsey's RESET test suggests that the linear model is **NOT correctly** specified (H_0 of correct/linear specification **rejected**).

Jarque-Bera (residuals normality) testing

```
# Model Jarque-Bera testing.
modelA.JB <- jarqueberaTest(modelA.summary$residuals)
print(modelA.JB)
```

```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##   STATISTIC:
##     X-squared: 247.6198
##   P VALUE:
##     Asymptotic p Value: < 2.2e-16
##
## Description:
##  Sun Jun 05 00:13:19 2016 by user: Yiannis
```

Conclusion:

> With a statistic of **~247.62** and a p-value of **~0**, the Jarque-Bera test suggests that the linear model residuals are **NOT normally distributed**, therefore the linear model is **NOT correctly** specified.

Testing summary

First model linearity testing results:

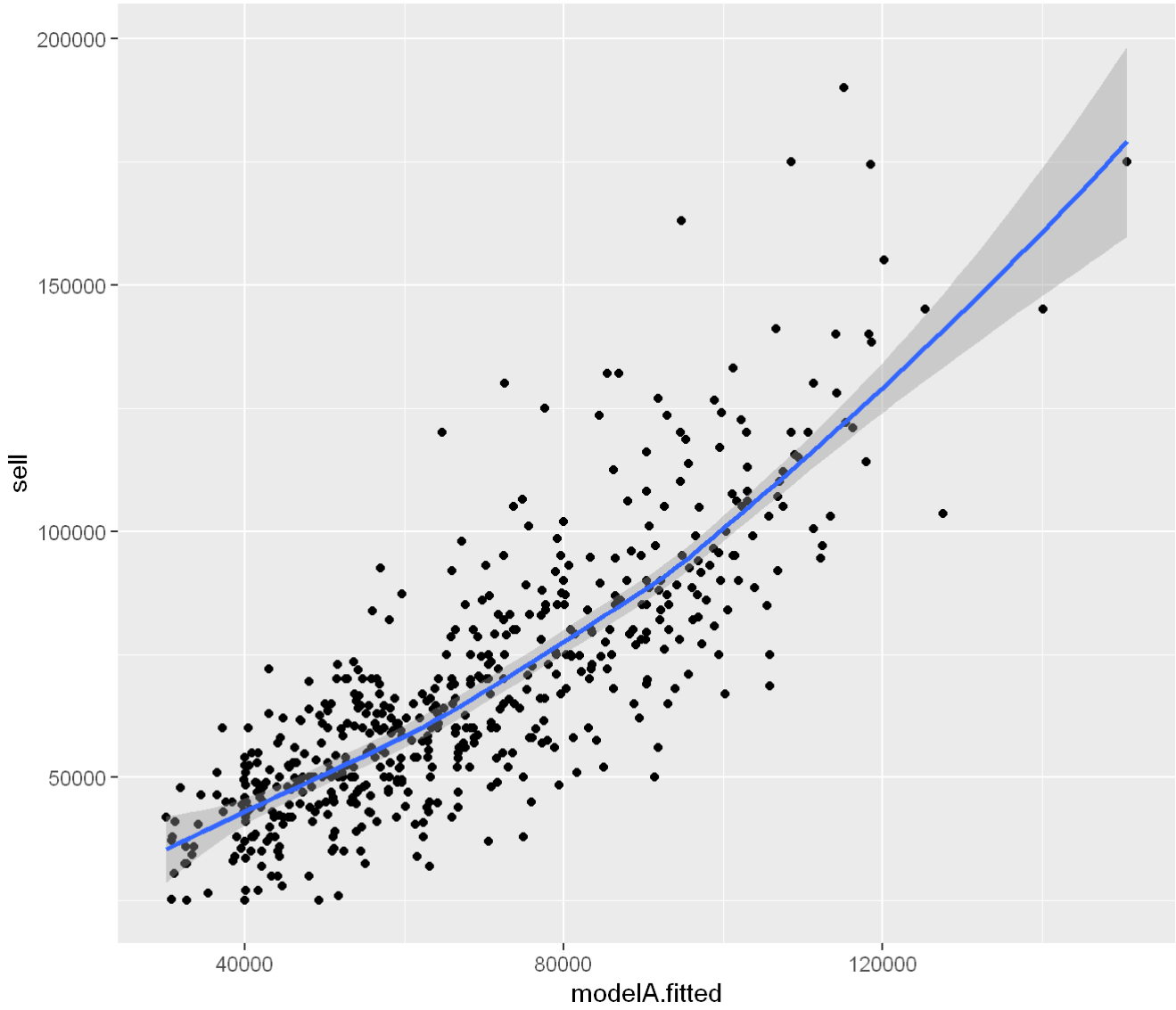
Test name	Test statistic	p-value	Test result
Ramsey's RESET	F= **26.986**	0	H_0: **Rejected**
Jarque-Bera	JB= **247.62**	0	H_0: **Rejected**

Conclusion:

> Both Ramsey's RESET and Jarque-Bera tests suggest that the considered linear model is **NOT correctly** specified.
>
> This is also intuitively demonstrated by the model real to fitted-values diagram shown at the next page (does NOT look like a linear relationship).

Real to fitted-values diagram

```
modelA.fitted <- fitted.values(modelA)

ggplot(dat, aes(x=modelA.fitted, y=sell)) +
    geom_point(shape=16) +
    geom_smooth()
```



(b) Logarithmic dependent variable

(b) Now consider a linear model where the log of the sale price of the house is the dependent variable and the explanatory variables are as before. Perform again the test for linearity. What do you conclude now?

Second model estimation

```
# Estimating second model.
modelB <- lm(sell_LOG ~ lot + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg,
             data = dat)
print(modelB.summary <- summary(modelB))
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot + bdms + fb + sty + drv + rec + ffin +
##     ghw + ca + gar + reg, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.67865 -0.12211  0.01666  0.12868  0.67737
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.003e+01  4.724e-02 212.210  < 2e-16 ***
## lot         5.057e-05  4.854e-06  10.418  < 2e-16 ***
## bdms        3.402e-02  1.451e-02   2.345  0.01939 *
## fb          1.678e-01  2.065e-02   8.126 3.10e-15 ***
## sty         9.227e-02  1.282e-02   7.197 2.10e-12 ***
## drv         1.307e-01  2.834e-02   4.610 5.04e-06 ***
## rec         7.352e-02  2.633e-02   2.792  0.00542 **
## ffin        9.940e-02  2.200e-02   4.517 7.72e-06 ***
## ghw         1.784e-01  4.458e-02   4.000 7.22e-05 ***
## ca          1.780e-01  2.155e-02   8.262 1.14e-15 ***
## gar         5.076e-02  1.165e-02   4.358 1.58e-05 ***
## reg         1.271e-01  2.313e-02   5.496 6.02e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2137 on 534 degrees of freedom
## Multiple R-squared:  0.6766, Adjusted R-squared:  0.6699
## F-statistic: 101.6 on 11 and 534 DF,  p-value: < 2.2e-16
```

Conclusion:

The second model considered is:

$$
\begin{aligned}
LOG(sell) = & + 10.026 \\
& \cancel{+ 0 \cdot lot} \\
& + 0.034 \cdot bdms \\
& + 0.168 \cdot fb \\
& + 0.092 \cdot sty \\
& + 0.131 \cdot drv \\
& + 0.074 \cdot rec \\
& + 0.099 \cdot ffin \\
& + 0.178 \cdot ghw \\
& + 0.178 \cdot ca \\
& + 0.051 \cdot gar \\
& + 0.127 \cdot reg \\
& + \epsilon
\end{aligned}
$$

Model characteristics: **R^2** = 0.6766 , **F-statistic**: 101.56 on 11 and 534 DF

> Notice the ~ zero (0) coefficient of variable lot.

Model linearity testing

Ramsey's RESET (linearity) testing

```
# Model Ramsey's RESET testing.
modelB.RESET <- resettest(modelB, power = 2, type = "fitted",
                          data = dat)
print(modelB.RESET)
```

```
##
##  RESET test
##
## data:  modelB
## RESET = 0.27031, df1 = 1, df2 = 533, p-value = 0.6033
```

Conclusion:

> With a statistic of **~0.27** and a p-value of **~0.6033**, the Ramsey's RESET test suggests that the second linear model **might be correctly** specified (H_0 of correct/linear specification **NOT rejected**, at the 5% level of significance).

Jarque-Bera (residuals normality) testing

```
# Model Jarque-Bera testing.
modelB.JB <- jarqueberaTest(modelB.summary$residuals)
print(modelB.JB)
```

```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##   STATISTIC:
##     X-squared: 8.4432
##   P VALUE:
##     Asymptotic p Value: 0.01467
##
## Description:
##  Sun Jun 05 00:13:20 2016 by user: Yiannis
```

Conclusion:

With a statistic of **~8.443** and a p-value of **~0.0147**, the Jarque-Bera test suggests that the linear model residuals are still **NOT normally distributed**, therefore the linear model is still **NOT correctly** specified, althought that the second model's JB statistic is significantly decreased (and therefore the model significantly improved).

Testing summary

Second model linearity testing results:

Test name	Test statistic	p-value	Test result
Ramsey's RESET	F= **0.27**	0.6033	H_0: **Not rejected**
Jarque-Bera	JB= **8.443**	0.0147	H_0: **Rejected**

Conclusion:

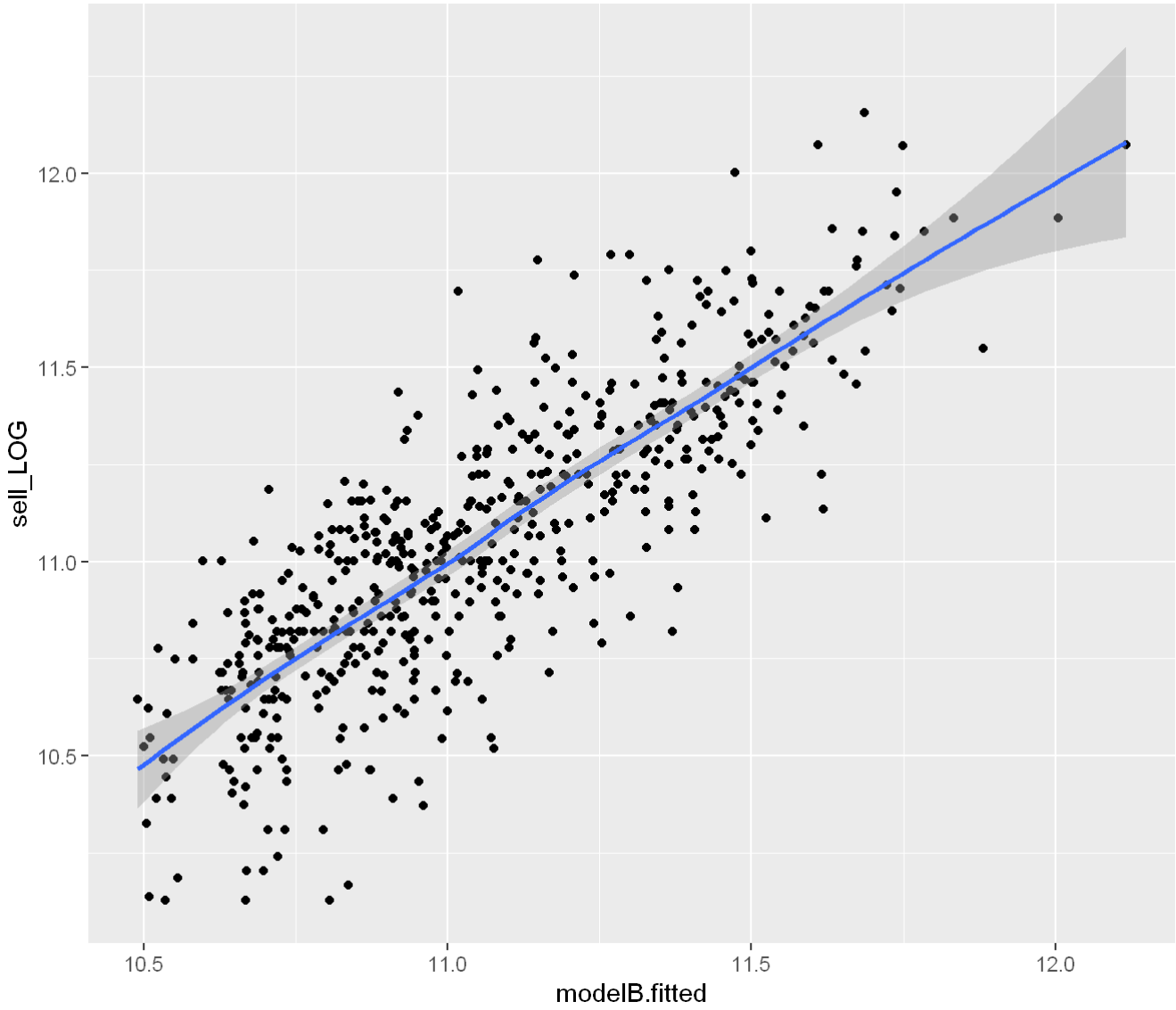
Both Ramsey's RESET and Jarque-Bera tests suggest that the second model is **significantly improved** than the model considered first.

The Ramsey's RESET test suggests that the second linear model **might be correctly** specified, while the Jarque-Bera test suggests that it is still **NOT correctly** specified (although significantly improved).

This is also intuitively demonstrated by the second model real to fitted-values diagram shown at the next page (looks much more like a linear relationship than before).

Real to fitted-values diagram

```
modelB.fitted <- fitted.values(modelB)

ggplot(dat, aes(x=modelB.fitted, y=sell_LOG)) +
    geom_point(shape=16) +
    geom_smooth()
```



(c) Logarithmic independent variable

(c) Continue with the linear model from question (b). Estimate a model that includes both the lot size variable and its logarithm, as well as all other explanatory variables without transformation. What is your conclusion, should we include lot size itself or its logarithm?

Third model estimation

```
# Estimating third model.
modelC <- lm(sell_LOG ~ lot + lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar +
 reg,
             data = dat)
print(modelC.summary <- summary(modelC))
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot + lot_LOG + bdms + fb + sty + drv +
##     rec + ffin + ghw + ca + gar + reg, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68573 -0.12380  0.00785  0.12521  0.68112
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.150e+00  6.830e-01  10.469  < 2e-16 ***
## lot         -1.490e-05  1.624e-05  -0.918 0.359086
## lot_LOG      3.827e-01  9.070e-02   4.219 2.88e-05 ***
## bdms         3.489e-02  1.429e-02   2.442 0.014915 *
## fb           1.659e-01  2.033e-02   8.161 2.40e-15 ***
## sty          9.121e-02  1.263e-02   7.224 1.76e-12 ***
## drv          1.068e-01  2.847e-02   3.752 0.000195 ***
## rec          5.467e-02  2.630e-02   2.078 0.038156 *
## ffin         1.052e-01  2.171e-02   4.848 1.64e-06 ***
## ghw          1.791e-01  4.390e-02   4.079 5.20e-05 ***
## ca           1.643e-01  2.146e-02   7.657 9.01e-14 ***
## gar          4.826e-02  1.148e-02   4.203 3.09e-05 ***
## reg          1.344e-01  2.284e-02   5.884 7.10e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2104 on 533 degrees of freedom
## Multiple R-squared:  0.687,  Adjusted R-squared:   0.68
## F-statistic: 97.51 on 12 and 533 DF,  p-value: < 2.2e-16
```

Conclusion:

The third model considered is:

$$\begin{aligned} LOG(sell) = & + 7.15 \\ & \cancel{+ 0 \cdot lot} \\ & + 0.383 \cdot LOG(lot) \\ & + 0.035 \cdot bdms \\ & + 0.166 \cdot fb \\ & + 0.091 \cdot sty \\ & + 0.107 \cdot drv \\ & + 0.055 \cdot rec \\ & + 0.105 \cdot ffin \\ & + 0.179 \cdot ghw \\ & + 0.164 \cdot ca \\ & + 0.048 \cdot gar \\ & + 0.134 \cdot reg \\ & + \epsilon \end{aligned}$$

Model characteristics: **R^2** = 0.687 , **F-statistic**: 97.51 on 12 and 533 DF

Notice the ~ zero (0) coefficient of variable lot.

Model linearity testing

Ramsey's RESET (linearity) testing

```
# Model Ramsey's RESET testing.
modelC.RESET <- resettest(modelC, power = 2, type = "fitted",
                          data = dat)
print(modelC.RESET)
```

```
##
##  RESET test
##
## data:  modelC
## RESET = 0.06769, df1 = 1, df2 = 532, p-value = 0.7948
```

Conclusion:

With a statistic of **~0.068** and a p-value of **~0.7948**, the Ramsey's RESET test suggests that the third linear model **might be correctly** specified (H_0 of correct/linear specification **NOT rejected**, at the 5% level of significance).

It also suggests that this is the best model constructed so far, as it has the lowest statistic and the highest p-value scored by all Ramsey's RESET tests ran so far.

Jarque-Bera (residuals normality) testing

```
# Model Jarque-Bera testing.
modelC.JB <- jarqueberaTest(modelC.summary$residuals)
print(modelC.JB)
```

```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##   STATISTIC:
##     X-squared: 9.3643
##   P VALUE:
##     Asymptotic p Value: 0.009259
##
## Description:
##  Sun Jun 05 00:13:21 2016 by user: Yiannis
```

Conclusion:

With a statistic of **~9.364** and a p-value of **~0.0093**, the Jarque-Bera test suggests that the linear model residuals are still **NOT normally distributed**; therefore the linear model is still **NOT correctly** specified.

No further model improvement is indicated by the Jarque-Bera residuals normality test; in fact the second model's residuals were slightly more normal than the third's.

Testing summary

Third model linearity testing results:

Test name	Test statistic	p-value	Test result
Ramsey's RESET	F= **0.068**	0.7948	H_0: **Not rejected**
Jarque-Bera	JB= **9.364**	0.0093	H_0: **Rejected**

Conclusion:

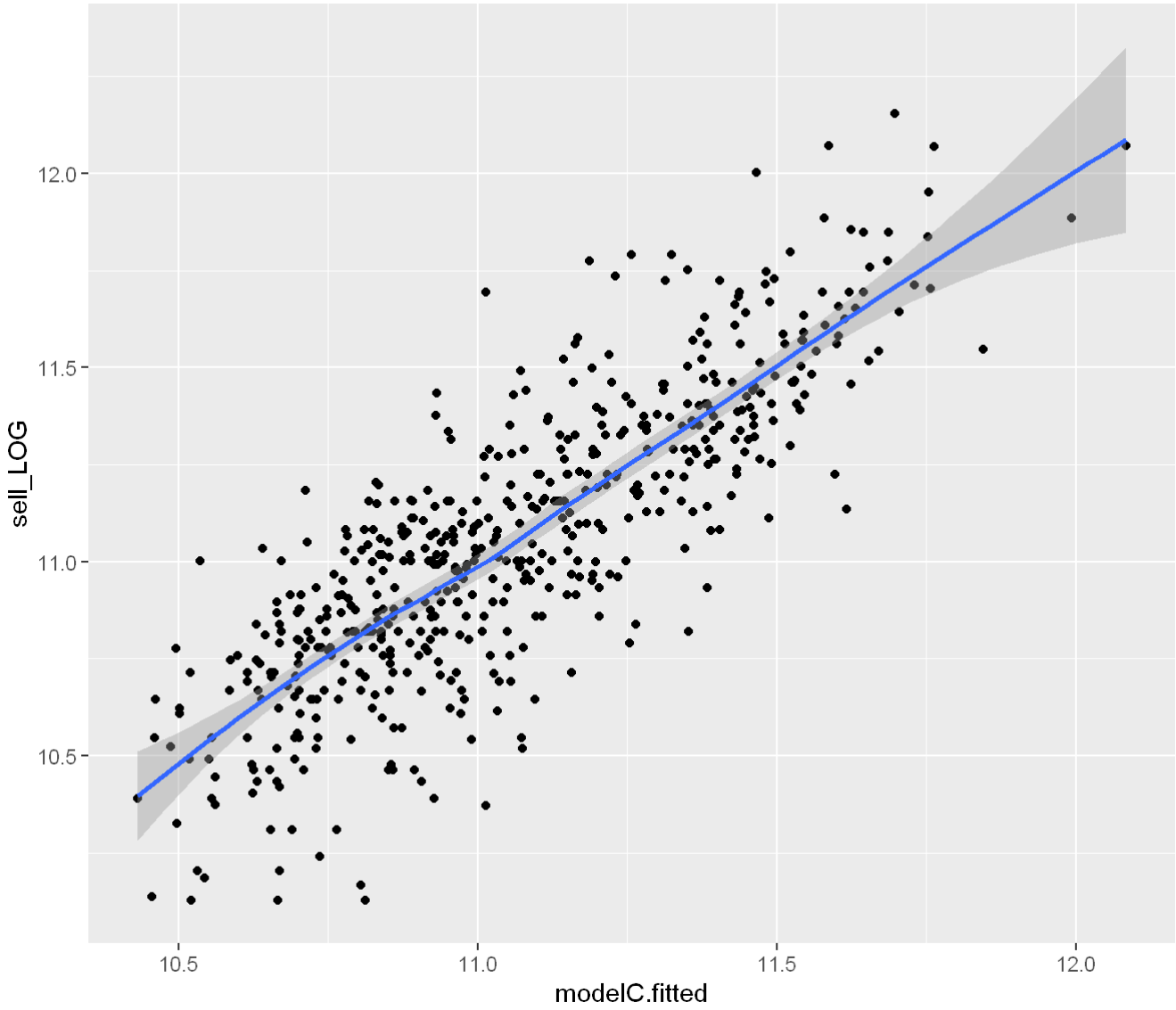
Both Ramsey's RESET and Jarque-Bera tests suggest that the third model is **significantly improved** than the model considered first, while the Ramsey's RESET test suggests that it is **even more improved** than the model considered second.

The Ramsey's RESET test suggests that the third linear model **might be correctly** specified, while the Jarque-Bera test suggests that it is still **NOT correctly** specified.

This is also intuitively demonstrated by the third model real to fitted-values diagram shown at the next page (looks about the same or more like a linear relationship than before).

Real to fitted-values diagram

```
modelC.fitted <- fitted.values(modelC)

ggplot(dat, aes(x=modelC.fitted, y=sell_LOG)) +
    geom_point(shape=16) +
    geom_smooth()
```



lot variable decision

Models linearity test results' comparison chart:

Mode	Test name	Test statistic	p-value	Test result
First	Ramsey's RESET	F= **26.986**	0	H_0: **Rejected**
Second	Ramsey's RESET	F= **0.27**	0.6033	H_0: **Not rejected**
Third	Ramsey's RESET	F= **0.068**	0.7948	H_0: **Not rejected**
——-	———————	——————————-	———————	———————
First	Jarque-Bera	JB= **247.62**	0	H_0: **Rejected**
Second	Jarque-Bera	JB= **8.443**	0.0147	H_0: **Rejected**
Third	Jarque-Bera	JB= **9.364**	0.0093	H_0: **Rejected**

Third model lot related variables' coefficients' comparison chart:

Variable	Coefficient	Std. Error	t-value	p-value

Variable	Coefficient	Std. Error	t-value	p-value
lot	**0**	0	-0.918	**0.359**
$LOG(lot)$	**0.383**	0.091	4.219	**0**

Conclusion:

It is concluded that it would be better to include the lot **size logarithm** in the model, rather than the lot *size* variable itself, due to the following reasons:

- The three models testing performed so far, see *Table 4* (above): *"Models linearity test results' comparison chart"*. The Ramsey's RESET tests showed that the lot size logarithm variable significantly improves the model linearity, while the Jarque-Bera tests showed that it produces a satisfactory (so far) level of residuals normality.
- The (much better) lot size logarithm variable coefficient p-value (**0**), compared to the lot size variable itself coefficient p-value (**0.359**), when used together. See *Table 5* (above): *"Third model lot related variables' coefficients' comparison chart"*.
- The fact that lot variable ended with a **~zero (0) coefficient** anyway at the (improved) second and third models.

(d) Interaction effects and variables

(d) Consider now a model where the log of the sale price of the house is the dependent variable and the explanatory variables are the log transformation of lot size, with all other explanatory variables as before. We now consider interaction effects of the log lot size with the other variables. Construct these interaction variables. How many are individually significant?

Fourth model estimation

```
# Estimating fourth model.
modelD <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                 lot_LOG * bdms + lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * re
c +
                 lot_LOG * ffin + lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar + lot_LOG * re
g,
             data = dat)
print(modelD.summary <- summary(modelD))
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * bdms + lot_LOG *
##     fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec + lot_LOG *
##     ffin + lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar + lot_LOG *
##     reg, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68306 -0.11612  0.00591  0.12486  0.65998
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.966499   1.070667   8.375 5.09e-16 ***
## lot_LOG       0.152685   0.128294   1.190   0.2345
## bdms          0.019075   0.326700   0.058   0.9535
## fb           -0.368234   0.429048  -0.858   0.3911
## sty           0.488885   0.309700   1.579   0.1150
## drv          -1.463371   0.717225  -2.040   0.0418 *
## rec           1.673992   0.655919   2.552   0.0110 *
## ffin         -0.031844   0.445543  -0.071   0.9430
## ghw          -0.505889   0.902733  -0.560   0.5754
## ca           -0.340276   0.496041  -0.686   0.4930
## gar           0.401941   0.258646   1.554   0.1208
## reg           0.118484   0.479856   0.247   0.8051
## lot_LOG:bdms  0.002070   0.038654   0.054   0.9573
## lot_LOG:fb    0.062037   0.050145   1.237   0.2166
## lot_LOG:sty  -0.046361   0.035942  -1.290   0.1977
## lot_LOG:drv   0.191542   0.087361   2.193   0.0288 *
## lot_LOG:rec  -0.188462   0.076373  -2.468   0.0139 *
## lot_LOG:ffin  0.015913   0.052851   0.301   0.7635
## lot_LOG:ghw   0.081135   0.106929   0.759   0.4483
## lot_LOG:ca    0.059549   0.058024   1.026   0.3052
## lot_LOG:gar  -0.041359   0.030142  -1.372   0.1706
## lot_LOG:reg   0.001515   0.055990   0.027   0.9784
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2095 on 524 degrees of freedom
## Multiple R-squared:  0.6951, Adjusted R-squared:  0.6829
## F-statistic: 56.89 on 21 and 524 DF,  p-value: < 2.2e-16
```

Conclusion:

The fourth model considered is:

$$\begin{aligned} LOG(sell) = & + 8.966 \\ & + 0.153 \cdot LOG(lot) \\ & + 0.019 \cdot bdms \\ & - 0.368 \cdot fb \\ & + 0.489 \cdot sty \\ & - 1.463 \cdot drv \\ & + 1.674 \cdot rec \\ & - 0.032 \cdot ffin \\ & - 0.506 \cdot ghw \\ & - 0.34 \cdot ca \\ & + 0.402 \cdot gar \\ & + 0.118 \cdot reg \\ & + 0.002 \cdot LOG(lot) \cdot bdms \\ & + 0.062 \cdot LOG(lot) \cdot fb \\ & - 0.046 \cdot LOG(lot) \cdot sty \\ & + 0.192 \cdot LOG(lot) \cdot drv \\ & - 0.188 \cdot LOG(lot) \cdot rec \\ & + 0.016 \cdot LOG(lot) \cdot ffin \\ & + 0.081 \cdot LOG(lot) \cdot ghw \\ & + 0.06 \cdot LOG(lot) \cdot ca \\ & - 0.041 \cdot LOG(lot) \cdot gar \\ & + 0.002 \cdot LOG(lot) \cdot reg \\ & + \epsilon \end{aligned}$$

Model characteristics: **R^2** = 0.6951 , **F-statistic**: 56.89 on 21 and 524 DF

Notice the absence of variable lot itself (as instructed).

Notice also the ten (10) interaction variables introduction, between the **log lot size** and each one of all other variables.

Model linearity testing

Ramsey’s RESET (linearity) testing

```
# Model Ramsey's RESET testing.
modelD.RESET <- resettest(modelD, power = 2, type = "fitted",
                          data = dat)
print(modelD.RESET)
```

```
##
##  RESET test
##
## data:  modelD
## RESET = 0.011571, df1 = 1, df2 = 523, p-value = 0.9144
```

Conclusion:

With a statistic of **~0.012** and a p-value of **~0.9144**, the Ramsey's RESET test suggests that the fourth model **might be correctly** specified (H_0 of correct/linear specification **NOT rejected**, at the 5% level of significance).

It also suggests that this is the best model constructed so far, as it has an even lower statistic and the highest p-value scored by all Ramsey's RESET tests ran so far.

Jarque-Bera (residuals normality) testing

```
# Model Jarque-Bera testing.
modelD.JB <- jarqueberaTest(modelD.summary$residuals)
print(modelD.JB)
```

```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##   STATISTIC:
##     X-squared: 8.2029
##   P VALUE:
##     Asymptotic p Value: 0.01655
##
## Description:
##  Sun Jun 05 00:13:22 2016 by user: Yiannis
```

Conclusion:

With a statistic of **~8.203** and a p-value of **~0.0165**, the Jarque-Bera test suggests that the model residuals are still **NOT normally distributed**; therefore the model is still **NOT correctly** specified.

This Jarque-Bera test result, however, is the best scored so far.
It seems that the interaction variables introduction slightly improves the (previous best) second model residuals normality.

Testing summary

Fourth model linearity testing results (comparison chart):

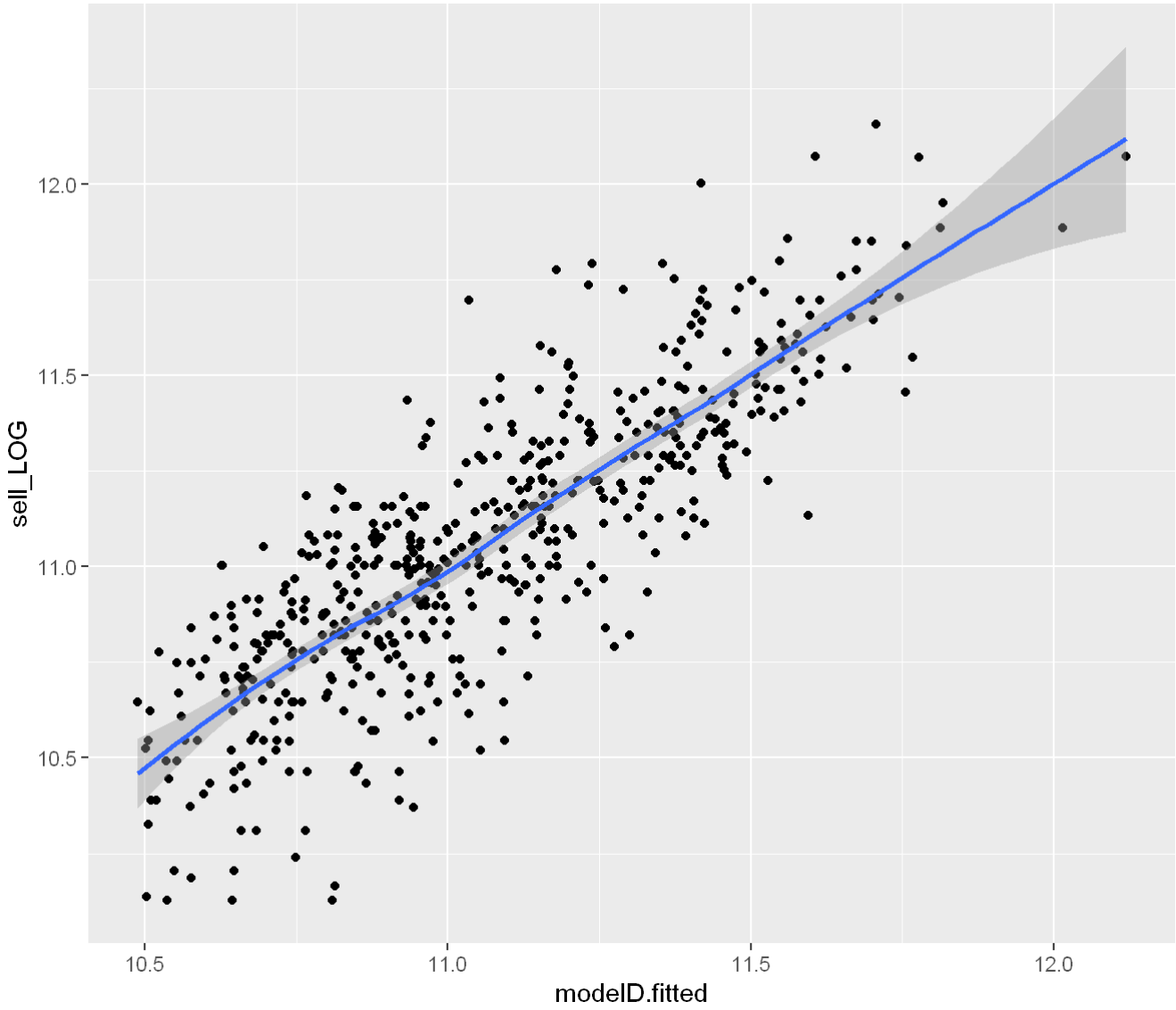
Mode	Test name	Test statistic	p-value	Test result
First	Ramsey's RESET	F= **26.986**	0	H_0: **Rejected**
Second	Ramsey's RESET	F= **0.27**	0.6033	H_0: **Not rejected**
Third	Ramsey's RESET	F= **0.068**	0.7948	H_0: **Not rejected**
Fourth	Ramsey's RESET	F= **0.012**	0.9144	H_0: **Not rejected**
——-	———————	——————————-	———————	———————
First	Jarque-Bera	JB= **247.62**	0	H_0: **Rejected**
Second	Jarque-Bera	JB= **8.443**	0.0147	H_0: **Rejected**
Third	Jarque-Bera	JB= **9.364**	0.0093	H_0: **Rejected**
Fourth	Jarque-Bera	JB= **8.203**	0.0165	H_0: **Rejected**

Conclusion:

> Both Ramsey's RESET and Jarque-Bera tests suggest that the fourth model is **significantly improved** than the models previously considered.
>
> The Ramsey's RESET test suggests that the fourth model **might be correctly** specified, while the Jarque-Bera test suggests that it is still **NOT correctly** specified.
>
> This is also intuitively demonstrated by the fourth model real to fitted-values diagram shown at the next page (looks about the same or more like a linear relationship).

Real to fitted-values diagram

```
modelD.fitted <- fitted.values(modelD)

ggplot(dat, aes(x=modelD.fitted, y=sell_LOG)) +
    geom_point(shape=16) +
    geom_smooth()
```



Interaction variables significance

Fourth model LOG(lot) interaction variables summary:

Interaction variable	Coefficient	Std. Error	t-value	p-value
$LOG(lot) \cdot bdms$	**0.002**	0.039	0.054	0.957
$LOG(lot) \cdot fb$	**0.062**	0.05	1.237	0.217
$LOG(lot) \cdot sty$	**-0.046**	0.036	-1.29	0.198
$LOG(lot) \cdot drv$ (*)	**0.192**	0.087	2.193	***0.029*** (*)
$LOG(lot) \cdot rec$ (*)	**-0.188**	0.076	-2.468	***0.014*** (*)
$LOG(lot) \cdot ffin$	**0.016**	0.053	0.301	0.763
$LOG(lot) \cdot ghw$	**0.081**	0.107	0.759	0.448
$LOG(lot) \cdot ca$	**0.06**	0.058	1.026	0.305
$LOG(lot) \cdot gar$	**-0.041**	0.03	-1.372	0.171
$LOG(lot) \cdot reg$	**0.002**	0.056	0.027	0.978

Conclusion:

> Using the 5% significance level, only two (2) of the ten (10) interaction variables used are individually significant:
>
> - $LOG(lot) \cdot drv$
> - $LOG(lot) \cdot rec$

(e) Interaction effects joint significance

> **(e) Perform an F-test for the joint significance of the interaction effects from question (d).**

In order to perform an F-test for the joint significance of the fourth model interaction effects, it is first needed to estimate the corresponding restricted model.

The corresponding restricted model (fifth model) will use only the two (2) significant interaction variables, as identified at the previous section **(d)**:

- $LOG(lot) \cdot drv$
- $LOG(lot) \cdot rec$

It will also use the rest of the fourth model variables (as is).

> *Note: Normally, a more structured model specification procedure (i.e. general to specific or specific to general) should had been employed for accurately identifying the restricted model. For this part of the exercise, however, this will be avoided as such a procedure is part of the following section (f).*

Fifth model estimation

```
# Estimating fifth model.
modelE <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * drv + lot_LOG * rec,
             data = dat)
print(modelE.summary <- summary(modelE))
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * drv + lot_LOG * rec,
##     data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.67934 -0.12225  0.00849  0.12259  0.65051
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.74189    0.62863  13.906  < 2e-16 ***
## lot_LOG      0.17906    0.07707   2.323  0.02053 *
## bdms         0.03881    0.01430   2.714  0.00686 **
## fb           0.16145    0.02025   7.971 9.62e-15 ***
## sty          0.09083    0.01254   7.242 1.56e-12 ***
## drv         -1.18996    0.66462  -1.790  0.07395 .
## rec          1.50253    0.62553   2.402  0.01665 *
## ffin         0.10276    0.02157   4.763 2.46e-06 ***
## ghw          0.18448    0.04368   4.223 2.83e-05 ***
## ca           0.16526    0.02121   7.792 3.48e-14 ***
## gar          0.04690    0.01142   4.107 4.65e-05 ***
## reg          0.13260    0.02255   5.880 7.24e-09 ***
## lot_LOG:drv  0.15943    0.08124   1.962  0.05024 .
## lot_LOG:rec -0.16826    0.07270  -2.314  0.02103 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2091 on 532 degrees of freedom
## Multiple R-squared:  0.6916, Adjusted R-squared:  0.6841
## F-statistic: 91.79 on 13 and 532 DF,  p-value: < 2.2e-16
```

Conclusion:

The fifth model considered is:

$$\begin{aligned} LOG(sell) = & + 8.742 \\ & + 0.179 \cdot LOG(lot) \\ & + 0.039 \cdot bdms \\ & + 0.161 \cdot fb \\ & + 0.091 \cdot sty \\ & - 1.19 \cdot drv \\ & + 1.503 \cdot rec \\ & + 0.103 \cdot ffin \\ & + 0.184 \cdot ghw \\ & + 0.165 \cdot ca \\ & + 0.047 \cdot gar \\ & + 0.133 \cdot reg \\ & + 0.159 \cdot LOG(lot) \cdot drv \\ & - 0.168 \cdot LOG(lot) \cdot rec \\ & + \epsilon \end{aligned}$$

Model characteristics: **R^2** = 0.6916 , **F-statistic**: 91.79 on 13 and 532 DF

Notice the absence of the not-significant interaction variables as well as the absence of the lot variable itself.

Model linearity testing

Ramsey's RESET (linearity) testing

```
# Model Ramsey's RESET testing.
modelE.RESET <- resettest(modelE, power = 2, type = "fitted",
                          data = dat)
print(modelE.RESET)
```

```
##
##  RESET test
##
## data:  modelE
## RESET = 0.045677, df1 = 1, df2 = 531, p-value = 0.8308
```

Conclusion:

With a statistic of **~0.046** and a p-value of **~0.8308**, the Ramsey's RESET test suggests that the fifth model **might be correctly** specified (H_0 of correct/linear specification **NOT rejected**, at the 5% level of significance).

It also suggests that this is not the best model constructed so far, as the fourth model had scored an even lower statistic and a higher p-value than this.

Jarque-Bera (residuals normality) testing

```
# Model Jarque-Bera testing.
modelE.JB <- jarqueberaTest(modelE.summary$residuals)
print(modelE.JB)
```

```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##   STATISTIC:
##     X-squared: 9.2372
##   P VALUE:
##     Asymptotic p Value: 0.009866
##
## Description:
##  Sun Jun 05 00:13:22 2016 by user: Yiannis
```

Conclusion:

With a statistic of **~9.237** and a p-value of **~0.0099**, the Jarque-Bera test suggests that the model residuals are still **NOT normally distributed**; therefore the model is still **NOT correctly** specified.

This Jarque-Bera test result is not the best scored so far either, as the fourth model related test had indicated an even better residuals normality.

Testing summary

Fifth model linearity testing results (comparison chart):

Mode	Test name	Test statistic	p-value	Test result
First	Ramsey's RESET	F= **26.986**	0	H_0: **Rejected**
Second	Ramsey's RESET	F= **0.27**	0.6033	H_0: **Not rejected**
Third	Ramsey's RESET	F= **0.068**	0.7948	H_0: **Not rejected**
Fourth	Ramsey's RESET	F= **0.012**	0.9144	H_0: **Not rejected**
Fifth	Ramsey's RESET	F= **0.046**	0.8308	H_0: **Not rejected**

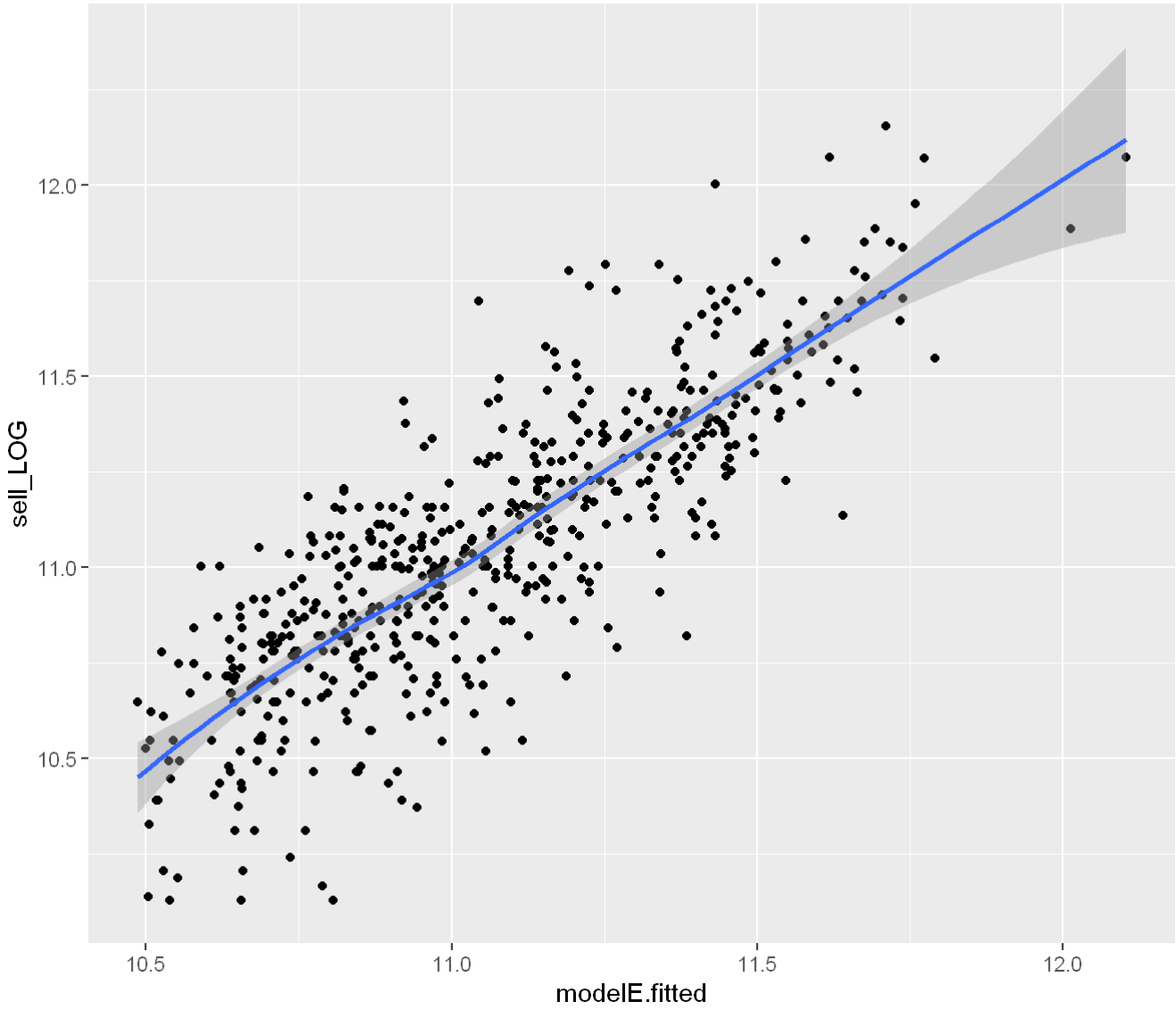
Mode	Test name	Test statistic	p-value	Test result
First	Jarque-Bera	JB= **247.62**	0	H_0: **Rejected**
Second	Jarque-Bera	JB= **8.443**	0.0147	H_0: **Rejected**
Third	Jarque-Bera	JB= **9.364**	0.0093	H_0: **Rejected**
Fourth	Jarque-Bera	JB= **8.203**	0.0165	H_0: **Rejected**
Fifth	Jarque-Bera	JB= **9.237**	0.0099	H_0: **Rejected**

Conclusion:

> Both Ramsey's RESET and Jarque-Bera tests suggest that the fourth model was better than this fifth one (restricted model); which, however, seems to be the second best (according to Ramsey's RESET testing) or the third best (according to Jarque-Bera testing) model constructed so far.
>
> The Ramsey's RESET test suggests that the fourth model **might be correctly** specified, while the Jarque-Bera test suggests that it is still **NOT correctly** specified.
>
> This is also intuitively demonstrated by the fifth model real to fitted-values diagram shown at the next page (looks about the same or more like a linear relationship).

Real to fitted-values diagram

```
modelE.fitted <- fitted.values(modelE)

ggplot(dat, aes(x=modelE.fitted, y=sell_LOG)) +
    geom_point(shape=16) +
    geom_smooth()
```



Interaction effects joint significance

An interaction effects joint significance F-test can be performed either:

- using the **sum of square residuals (SSR)** of the restricted and the unrestricted models, or
- using the **R^2** of both the restricted and the unrestricted models.

Using SSR

```
SSR_u <- sum(modelD.summary$residuals ^ 2)
SSR_r <- sum(modelE.summary$residuals ^ 2)

SSR_nom <- (SSR_r - SSR_u) / g
SSR_den <- SSR_u / (n - k)
SSR_ <- SSR_nom / SSR_den
```

$$F = \frac{(SSR_R - SSR_U) \Big/ g}{SSR_U \Big/ (n-k)} = \frac{(23.2547 - 22.9927) \Big/ 10}{22.9927 \Big/ (546-22)} = \frac{0.0262}{0.0439} = 0.5971$$

```
SSR_p <- pf(SSR_, g, n - k)
```

$$p.value\ for\ F \sim F(10,\ 524) = 0.1833$$

Using R^2

```
R2_u <- modelD.R2
R2_r <- modelE.R2

R2_nom <- (R2_u - R2_r) / g
R2_den <- (1 - R2_u) / (n - g)
R2_ <- R2_nom / R2_den
```

$$F = \frac{(R_U^2 - R_R^2) \Big/ g}{(1 - R_U^2) \Big/ (n - g)} = \frac{(0.6951 - 0.6916) \Big/ 10}{(1 - 0.6951) \Big/ (546 - 10)} = \frac{3 \times 10^{-4}}{6 \times 10^{-4}} = 0.6108$$

```
R2_p <- pf(R2_, g, n - k)
```

$$p.value\ for\ F \sim F(10,\ 524) = 0.1948$$

Evaluation

Interaction effects joint significance F-test results:

Method	t-statistic	p-value	Test result
SSR	F= **0.5971**	p= **0.1833**	H_0: **Not rejected**
R^2	F= **0.6108**	p= **0.1948**	H_0: **Not rejected**

Conclusion:

> Both SSR and R^2 methods produced an F-test statistic of $0.6\ -\ 0.61$, with a p-value of $0.18\ -\ 0.19$.
>
> The above results conclude that interactions are **jointly significant** at the 5% significance level.

(f) Model specification

> **(f) Now perform model specification on the interaction variables using the general-to-specific approach. (Only eliminate the interaction effects.)**

General-to-specific model

The results of ten (10) regression rounds performed (see Appendix A for full details), are shown below:

Variable	round1	round2	round3	round4	round5	round6	round7	round8	round9	round10
Constant	**8.966**	**8.968**	**8.935**	**8.877**	**8.809**	**8.782**	**8.774**	**8.298**	**8.742**	**7.591**
	se=1.07	se=1.07	se=0.86	se=0.84	se=0.84	se=0.84	se=0.84	se=0.76	se=0.63	se=0.23
	t=8.38	t=8.39	t=10.38	t=10.55	t=10.53	t=10.5	t=10.49	t=10.98	t=13.91	t=33.51
	p=0	p=0	p=0	p=0	p=0	p=0	p=0	p=0	p=0	p=0
$LOG(lot)$	**0.153**	**0.153**	**0.156**	**0.164**	**0.172**	**0.175**	**0.176**	**0.231**	**0.179**	**0.32**
	se=0.13	se=0.13	se=0.1	se=0.1	se=0.1	se=0.1	se=0.1	se=0.09	se=0.08	se=0.03
	t=1.19	t=1.19	t=1.51	t=1.62	t=1.71	t=1.74	t=1.75	t=2.53	t=2.32	t=11.56
	p=0.24	p=0.23	p=0.13	p=0.11	p=0.09	p=0.08	p=0.08	p=0.01	p=0.02	p=0
$bdms$	**0.019**	**0.019**	**0.037**	**0.037**	**0.037**	**0.035**	**0.035**	**0.036**	**0.039**	**0.038**
	se=0.33	se=0.33	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.01	se=0.01
	t=0.06	t=0.06	t=2.51	t=2.51	t=2.51	t=2.43	t=2.43	t=2.5	t=2.71	t=2.68
	p=0.95	p=0.95	p=0.01	p=0.01	p=0.01	p=0.02	p=0.02	p=0.01	p=0.01	p=0.01
fb	**-0.368**	**-0.368**	**-0.377**	**-0.382**	**-0.376**	**-0.38**	**-0.34**	**0.165**	**0.161**	**0.163**
	se=0.43	se=0.43	se=0.39	se=0.38	se=0.38	se=0.38	se=0.38	se=0.02	se=0.02	se=0.02
	t=-0.86	t=-0.86	t=-0.98	t=-0.99	t=-0.98	t=-0.99	t=-0.89	t=8.03	t=7.97	t=8.04
	p=0.39	p=0.39	p=0.33	p=0.32	p=0.33	p=0.32	p=0.38	p=0	p=0	p=0
sty	**0.489**	**0.487**	**0.482**	**0.489**	**0.491**	**0.472**	**0.468**	**0.384**	**0.091**	**0.091**
	se=0.31	se=0.3	se=0.29	se=0.28	se=0.28	se=0.28	se=0.28	se=0.28	se=0.01	se=0.01
	t=1.58	t=1.61	t=1.68	t=1.71	t=1.72	t=1.66	t=1.64	t=1.38	t=7.24	t=7.22
	p=0.12	p=0.11	p=0.09	p=0.09	p=0.09	p=0.1	p=0.1	p=0.17	p=0	p=0
drv	**-1.463**	**-1.468**	**-1.463**	**-1.45**	**-1.433**	**-1.429**	**-1.237**	**-1.255**	**-1.19**	**0.113**
	se=0.72	se=0.7	se=0.69	se=0.69	se=0.69	se=0.69	se=0.67	se=0.67	se=0.66	se=0.03
	t=-2.04	t=-2.11	t=-2.12	t=-2.11	t=-2.09	t=-2.08	t=-1.85	t=-1.88	t=-1.79	t=4.02
	p=0.04	p=0.04	p=0.03	p=0.04	p=0.04	p=0.04	p=0.06	p=0.06	p=0.07	p=0
rec	**1.674**	**1.675**	**1.676**	**1.621**	**1.631**	**1.557**	**1.514**	**1.473**	**1.503**	**1.443**
	se=0.66	se=0.66	se=0.65	se=0.63	se=0.63	se=0.63	se=0.63	se=0.63	se=0.63	se=0.63
	t=2.55	t=2.56	t=2.56	t=2.57	t=2.58	t=2.48	t=2.42	t=2.35	t=2.4	t=2.3
	p=0.01	p=0.01	p=0.01	p=0.01	p=0.01	p=0.01	p=0.02	p=0.02	p=0.02	p=0.02
$ffin$	**-0.032**	**-0.035**	**-0.037**	**0.102**	**0.104**	**0.103**	**0.103**	**0.1**	**0.103**	**0.104**
	se=0.45	se=0.43	se=0.43	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02

Variable	round1	round2	round3	round4	round5	round6	round7	round8	round9	round10
	t=-0.07	t=-0.08	t=-0.09	t=4.69	t=4.77	t=4.76	t=4.73	t=4.63	t=4.76	t=4.84
	p=0.94	p=0.94	p=0.93	p=0	p=0	p=0	p=0	p=0	p=0	p=0
ghw	**-0.506**	**-0.504**	**-0.502**	**-0.516**	**0.18**	**0.177**	**0.18**	**0.181**	**0.184**	**0.184**
	se=0.9	se=0.9	se=0.9	se=0.9	se=0.04	se=0.04	se=0.04	se=0.04	se=0.04	se=0.04
	t=-0.56	t=-0.56	t=-0.56	t=-0.58	t=4.1	t=4.04	t=4.11	t=4.13	t=4.22	t=4.21
	p=0.57	p=0.57	p=0.58	p=0.56	p=0	p=0	p=0	p=0	p=0	p=0
ca	**-0.34**	**-0.34**	**-0.339**	**-0.354**	**-0.34**	**0.167**	**0.167**	**0.166**	**0.165**	**0.166**
	se=0.5	se=0.5	se=0.49	se=0.49	se=0.49	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02
	t=-0.69	t=-0.69	t=-0.68	t=-0.72	t=-0.69	t=7.88	t=7.87	t=7.83	t=7.79	t=7.8
	p=0.49	p=0.49	p=0.49	p=0.47	p=0.49	p=0	p=0	p=0	p=0	p=0
gar	**0.402**	**0.402**	**0.401**	**0.401**	**0.397**	**0.339**	**0.048**	**0.048**	**0.047**	**0.048**
	se=0.26	se=0.26	se=0.26	se=0.26	se=0.26	se=0.25	se=0.01	se=0.01	se=0.01	se=0.01
	t=1.55	t=1.56	t=1.56	t=1.57	t=1.55	t=1.36	t=4.2	t=4.16	t=4.11	t=4.21
	p=0.12	p=0.12	p=0.12	p=0.12	p=0.12	p=0.18	p=0	p=0	p=0	p=0
reg	**0.118**	**0.131**	**0.131**	**0.132**	**0.131**	**0.133**	**0.13**	**0.131**	**0.133**	**0.134**
	se=0.48	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02	se=0.02
	t=0.25	t=5.7	t=5.71	t=5.79	t=5.75	t=5.85	t=5.75	t=5.81	t=5.88	t=5.92
	p=0.8	p=0	p=0	p=0	p=0	p=0	p=0	p=0	p=0	p=0
$LOG(lot)$	**0.002**	**0.002**								
$\cdot bdms$	se=0.04	se=0.04								
	t=0.05	t=0.05								
	p=0.96	p=0.96								
$LOG(lot)$	**0.062**	**0.062**	**0.063**	**0.064**	**0.063**	**0.064**	**0.059**			
$\cdot fb$	se=0.05	se=0.05	se=0.04	se=0.04	se=0.04	se=0.04	se=0.04			
	t=1.24	t=1.24	t=1.4	t=1.42	t=1.4	t=1.42	t=1.32			
	p=0.22	p=0.22	p=0.16	p=0.16	p=0.16	p=0.16	p=0.19			
$LOG(lot)$	**-0.046**	**-0.046**	**-0.046**	**-0.046**	**-0.047**	**-0.044**	**-0.044**	**-0.034**		
$\cdot sty$	se=0.04	se=0.04	se=0.03	se=0.03	se=0.03	se=0.03	se=0.03	se=0.03		
	t=-1.29	t=-1.31	t=-1.37	t=-1.4	t=-1.41	t=-1.34	t=-1.33	t=-1.06		
	p=0.2	p=0.19	p=0.17	p=0.16	p=0.16	p=0.18	p=0.18	p=0.29		
$LOG(lot)$	**0.192**	**0.192**	**0.191**	**0.19**	**0.188**	**0.187**	**0.164**	**0.167**	**0.159**	
$\cdot drv$	se=0.09	se=0.08	se=0.08	se=0.08	se=0.08	se=0.08	se=0.08	se=0.08	se=0.08	

Variable	round1	round2	round3	round4	round5	round6	round7	round8	round9	round10
	t=2.19	t=2.26	t=2.28	t=2.26	t=2.24	t=2.23	t=2.02	t=2.05	t=1.96	
	p=0.03	p=0.02	p=0.02	p=0.02	p=0.02	p=0.03	p=0.04	p=0.04	p=0.05	
$LOG(lot)$	**-0.188**	**-0.189**	**-0.189**	**-0.182**	**-0.183**	**-0.175**	**-0.169**	**-0.165**	**-0.168**	**-0.161**
$\cdot rec$	se=0.08	se=0.08	se=0.08	se=0.07	se=0.07	se=0.07	se=0.07	se=0.07	se=0.07	se=0.07
	t=-2.47	t=-2.47	t=-2.48	t=-2.48	t=-2.5	t=-2.4	t=-2.33	t=-2.26	t=-2.31	t=-2.21
	p=0.01	p=0.01	p=0.01	p=0.01	p=0.01	p=0.02	p=0.02	p=0.02	p=0.02	p=0.03
$LOG(lot)$	**0.016**	**0.016**	**0.017**							
$\cdot ffin$	se=0.05	se=0.05	se=0.05							
	t=0.3	t=0.32	t=0.33							
	p=0.76	p=0.75	p=0.74							
$LOG(lot)$	**0.081**	**0.081**	**0.081**	**0.082**						
$\cdot ghw$	se=0.11	se=0.11	se=0.11	se=0.11						
	t=0.76	t=0.76	t=0.76	t=0.78						
	p=0.45	p=0.45	p=0.45	p=0.44						
$LOG(lot)$	**0.06**	**0.059**	**0.059**	**0.061**	**0.059**					
$\cdot ca$	se=0.06	se=0.06	se=0.06	se=0.06	se=0.06					
	t=1.03	t=1.03	t=1.03	t=1.07	t=1.03					
	p=0.3	p=0.3	p=0.3	p=0.29	p=0.3					
$LOG(lot)$	**-0.041**	**-0.041**	**-0.041**	**-0.041**	**-0.041**	**-0.034**				
$\cdot gar$	se=0.03	se=0.03	se=0.03	se=0.03	se=0.03	se=0.03				
	t=-1.37	t=-1.38	t=-1.38	t=-1.38	t=-1.37	t=-1.16				
	p=0.17	p=0.17	p=0.17	p=0.17	p=0.17	p=0.24				
$LOG(lot)$	**0.002**									
$\cdot reg$	se=0.06									
	t=0.03									
	p=0.98									
…										
R^2:	0.6951	0.6951	0.6951	0.695	0.6947	0.6941	0.6933	0.6923	0.6916	0.6894

> *[..] you start with the most general model, including as many variables as are at hand. Then, check whether one or more variables can be removed from the model. This can be based on individual t-tests, or a joint F-test in case of multiple variables. In case you remove one variable at a time, the variable with the lowest absolute t-value is removed from the model. The model is estimated again without that variable, and the procedure is repeated. The procedure continues until all remaining variables are significant.*

Variables elimination after regression, one at a time, produced the following results:

- After regression round #1: $LOG(lot) \cdot reg$ interaction variable was chosen to be removed.
- After regression round #2: $LOG(lot) \cdot bdms$ interaction variable was chosen to be removed.
- After regression round #3: $LOG(lot) \cdot ffin$ interaction variable was chosen to be removed.
- After regression round #4: $LOG(lot) \cdot ghw$ interaction variable was chosen to be removed.
- After regression round #5: $LOG(lot) \cdot ca$ interaction variable was chosen to be removed.
- After regression round #6: $LOG(lot) \cdot gar$ interaction variable was chosen to be removed.
- After regression round #7: $LOG(lot) \cdot fb$ interaction variable was chosen to be removed.
- After regression round #8: $LOG(lot) \cdot sty$ interaction variable was chosen to be removed.
- After regression round #9: $LOG(lot) \cdot drv$ interaction variable was chosen to be removed.
- After regression round #10: all remaining variables were found to be significant; variables removal stops here. Conclusively, **the only interaction variable found to be significant is** $LOG(lot) \cdot rec$.

Sixth model estimation

The sixth model considered (after a general-to-specific model specification) is:

$$\begin{aligned} LOG(sell) = & + 7.591 \\ & + 0.32 \cdot LOG(lot) \\ & + 0.038 \cdot bdms \\ & + 0.163 \cdot fb \\ & + 0.091 \cdot sty \\ & + 0.113 \cdot drv \\ & + 1.443 \cdot rec \\ & + 0.104 \cdot ffin \\ & + 0.184 \cdot ghw \\ & + 0.166 \cdot ca \\ & + 0.048 \cdot gar \\ & + 0.134 \cdot reg \\ & - 0.161 \cdot LOG(lot) \cdot rec \\ & + \epsilon \end{aligned}$$

Model characteristics: **R^2** = 0.6894 , **F-statistic**: 98.59 on 12 and 533 DF

Notice the absence of all interaction variables except of $LOG(lot) \cdot rec$, according to the above general-to-specific model specification process.

Model linearity testing

Ramsey's RESET (linearity) testing

```
# Model Ramsey's RESET testing.
modelF.RESET <- resettest(modelF, power = 2, type = "fitted",
                          data = dat)
print(modelF.RESET)
```

```
##
##  RESET test
##
## data:  modelF
## RESET = 0.43102, df1 = 1, df2 = 532, p-value = 0.5118
```

Conclusion:

> With a statistic of **~0.431** and a p-value of **~0.5118**, the Ramsey's RESET test suggests that the sixth model **might be correctly** specified (H_0 of correct/linear specification **NOT rejected**, at the 5% level of significance).
>
> It also suggests that this is far from a linear model, as all the previous models (except from the first) had scored a lower statistic and a higher p-value than this.

Jarque-Bera (residuals normality) testing

```
# Model Jarque-Bera testing.
modelF.JB <- jarqueberaTest(modelF.summary$residuals)
print(modelF.JB)
```

```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##   STATISTIC:
##     X-squared: 10.3483
##   P VALUE:
##     Asymptotic p Value: 0.005661
##
## Description:
##  Sun Jun 05 00:13:24 2016 by user: Yiannis
```

Conclusion:

> With a statistic of **~10.348** and a p-value of **~0.0057**, the Jarque-Bera test suggests that the model residuals are still **NOT normally distributed**; therefore the model is still **NOT correctly** specified.
>
> This Jarque-Bera test result is not the best scored so far. All the previous models (except from the first) related test had indicated an even better residuals normality.

Testing summary

Sixth model linearity testing results (comparison chart):

Mode	Test name	Test statistic	p-value	Test result
First	Ramsey's RESET	F= **26.986**	0	H_0: **Rejected**
Second	Ramsey's RESET	F= **0.27**	0.6033	H_0: **Not rejected**
Third	Ramsey's RESET	F= **0.068**	0.7948	H_0: **Not rejected**
Fourth	Ramsey's RESET	F= **0.012**	0.9144	H_0: **Not rejected**
Fifth	Ramsey's RESET	F= **0.046**	0.8308	H_0: **Not rejected**

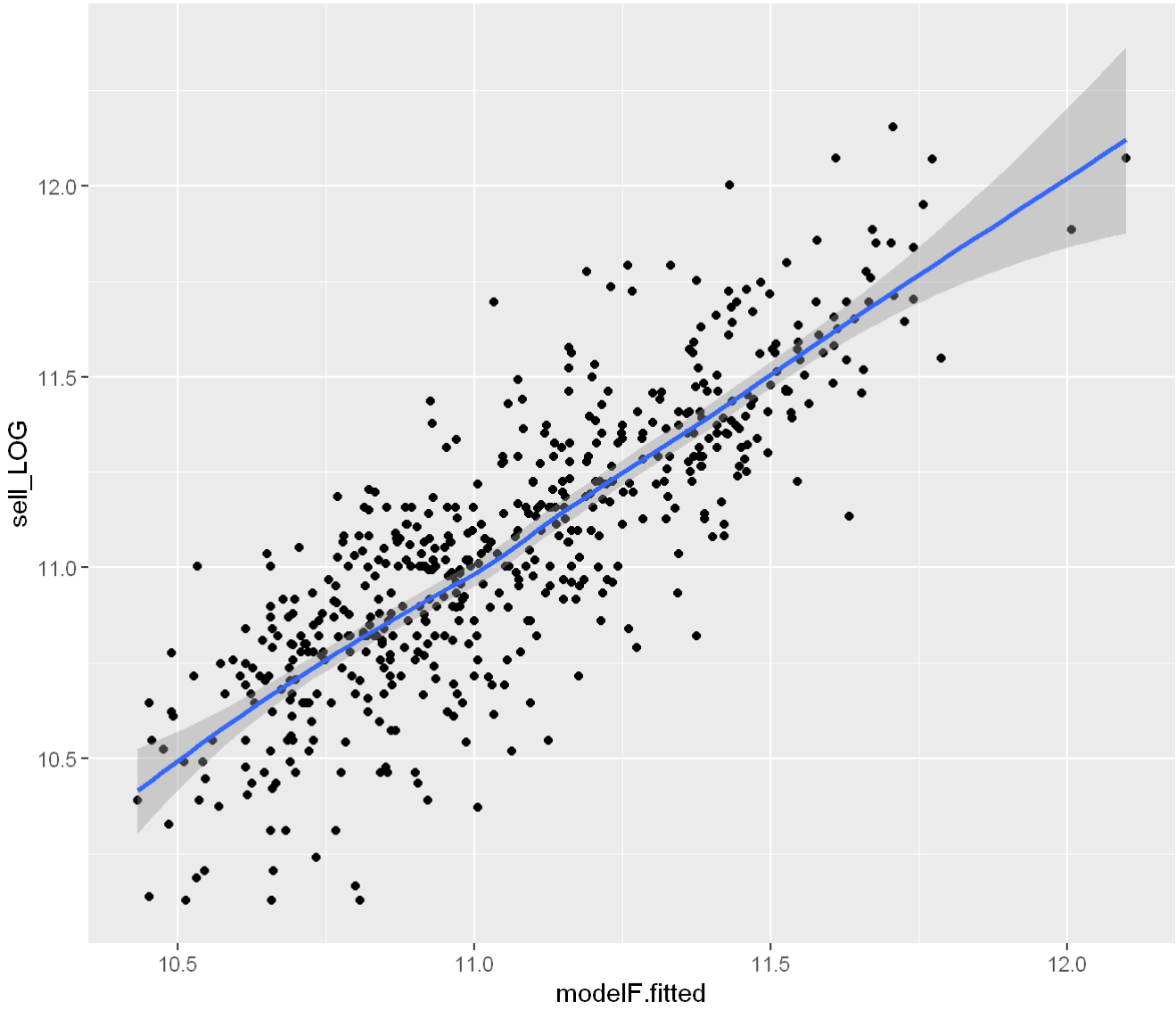
Mode	Test name	Test statistic	p-value	Test result
Sixth	Ramsey's RESET	F= **0.431**	0.5118	H_0: **Not rejected**
——-	—————	——————-	—————	—————
First	Jarque-Bera	JB= **247.62**	0	H_0: **Rejected**
Second	Jarque-Bera	JB= **8.443**	0.0147	H_0: **Rejected**
Third	Jarque-Bera	JB= **9.364**	0.0093	H_0: **Rejected**
Fourth	Jarque-Bera	JB= **8.203**	0.0165	H_0: **Rejected**
Fifth	Jarque-Bera	JB= **9.237**	0.0099	H_0: **Rejected**
Sixth	Jarque-Bera	JB= **10.348**	0.0057	H_0: **Rejected**

Conclusion:

> Both Ramsey's RESET and Jarque-Bera tests suggest that the sixth model is less linear and with less residuals normality than the models tested before (except from the first model).
>
> The Ramsey's RESET test suggests that the sixth model **might be correctly** specified, while the Jarque-Bera test suggests that it is still **NOT correctly** specified.
>
> This is also intuitively demonstrated by the sixth model real to fitted-values diagram shown at the next page (looks about the same or less like a linear relationship).

Real to fitted-values diagram

```
modelF.fitted <- fitted.values(modelF)

ggplot(dat, aes(x=modelF.fitted, y=sell_LOG)) +
    geom_point(shape=16) +
    geom_smooth()
```



(g) Endogeneity considerations

(g) One may argue that some of the explanatory variables are endogenous and that there may be omitted variables. For example, the 'condition' of the house in terms of how it is maintained is not a variable (and difficult to measure) but will affect the house price. It will also affect, or be reflected in, some of the other variables, such as whether the house has an air conditioning (which is mostly in newer houses). If the condition of the house is missing, will the effect of air conditioning on the (log of the) sale price be over- or underestimated? (For this question no computer calculations are required.)

The effect of the air conditioning ca variable on the logarithm of the sale price $LOG(sell)$ variable will be **overestimated**, because it is usually affected by the age (and therefore the condition) of houses both of which (logically) affect the house selling price positively.

So, the effect of the age and condition house properties (which are not available to our models as variables) is partially included in the air conditioning ca variable. And since that effect is expected to be positive on the house sale price (and its logarithm), it will increase the effect of the air-conditioning ca variable in our models (thus, its estimated effect is overestimated).

(h) Model predictive ability

(h) Finally we analyze the predictive ability of the model. Consider again the model where the log of the sale price of the house is the dependent variable and the explanatory variables are the log transformation of lot size, with all other explanatory variables in their original form (and no interaction effects). Estimate the parameters of the model using the first 400 observations. Make predictions on the log of the price and calculate the MAE for the other 146 observations. How good is the predictive power of the model (relative to the variability in the log of the price)?

Separating the data sample in two groups

```
# Separating the data sample in two groups
dat1 <- dat[which(dat$obs <= 400), ]
n1 <- nrow(dat1)
print(paste("Data group#1 has", n1, "entries."))
```

```
## [1] "Data group#1 has 400 entries."
```

```
summary(dat1)
```

```
##       obs             sell             lot             bdms
##  Min.   :  1.0   Min.   : 25000   Min.   : 1650   Min.   :1.00
##  1st Qu.:100.8   1st Qu.: 46150   1st Qu.: 3495   1st Qu.:2.00
##  Median :200.5   Median : 59250   Median : 4180   Median :3.00
##  Mean   :200.5   Mean   : 64977   Mean   : 4905   Mean   :2.95
##  3rd Qu.:300.2   3rd Qu.: 78000   3rd Qu.: 6000   3rd Qu.:3.00
##  Max.   :400.0   Max.   :190000   Max.   :16200   Max.   :6.00
##        fb              sty              drv              rec
##  Min.   :1.000   Min.   :1.000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:1.000   1st Qu.:1.000   1st Qu.:1.0000   1st Qu.:0.0000
##  Median :1.000   Median :2.000   Median :1.0000   Median :0.0000
##  Mean   :1.278   Mean   :1.718   Mean   :0.8125   Mean   :0.1625
##  3rd Qu.:1.000   3rd Qu.:2.000   3rd Qu.:1.0000   3rd Qu.:0.0000
##  Max.   :4.000   Max.   :4.000   Max.   :1.0000   Max.   :1.0000
##       ffin             ghw              ca              gar
##  Min.   :0.0000   Min.   :0.00   Min.   :0.000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.00   1st Qu.:0.000   1st Qu.:0.0000
##  Median :0.0000   Median :0.00   Median :0.000   Median :0.0000
##  Mean   :0.3475   Mean   :0.05   Mean   :0.285   Mean   :0.6925
##  3rd Qu.:1.0000   3rd Qu.:0.00   3rd Qu.:1.000   3rd Qu.:1.0000
##  Max.   :1.0000   Max.   :1.00   Max.   :1.000   Max.   :3.0000
##       reg           sell_LOG        lot_LOG
##  Min.   :0.000   Min.   :10.13   Min.   :7.409
##  1st Qu.:0.000   1st Qu.:10.74   1st Qu.:8.159
##  Median :0.000   Median :10.99   Median :8.338
##  Mean   :0.105   Mean   :11.01   Mean   :8.420
##  3rd Qu.:0.000   3rd Qu.:11.26   3rd Qu.:8.700
##  Max.   :1.000   Max.   :12.15   Max.   :9.693
```

```
dat2 <- dat[which(dat$obs > 400), ]
n2 <- nrow(dat2)
print(paste("Data group#2 has", n2, "entries."))
```

```
## [1] "Data group#2 has 146 entries."
```

```
summary(dat2)
```

```
##       obs             sell             lot             bdms
##  Min.   :401.0   Min.   : 31900   Min.   : 1950   Min.   :2.000
##  1st Qu.:437.2   1st Qu.: 60000   1st Qu.: 4678   1st Qu.:3.000
##  Median :473.5   Median : 72750   Median : 6000   Median :3.000
##  Mean   :473.5   Mean   : 76737   Mean   : 5821   Mean   :3.007
##  3rd Qu.:509.8   3rd Qu.: 91125   3rd Qu.: 6652   3rd Qu.:3.000
##  Max.   :546.0   Max.   :174500   Max.   :12944   Max.   :5.000
##        fb              sty              drv              rec
##  Min.   :1.000   Min.   :1.000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:1.000   1st Qu.:1.000   1st Qu.:1.0000   1st Qu.:0.0000
##  Median :1.000   Median :2.000   Median :1.0000   Median :0.0000
##  Mean   :1.308   Mean   :2.055   Mean   :0.9863   Mean   :0.2192
##  3rd Qu.:2.000   3rd Qu.:3.000   3rd Qu.:1.0000   3rd Qu.:0.0000
##  Max.   :2.000   Max.   :4.000   Max.   :1.0000   Max.   :1.0000
##       ffin             ghw                ca              gar
##  Min.   :0.0000   Min.   :0.00000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.00000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :0.00000   Median :0.0000   Median :0.0000
##  Mean   :0.3562   Mean   :0.03425   Mean   :0.4041   Mean   :0.6918
##  3rd Qu.:1.0000   3rd Qu.:0.00000   3rd Qu.:1.0000   3rd Qu.:1.0000
##  Max.   :1.0000   Max.   :1.00000   Max.   :1.0000   Max.   :3.0000
##       reg           sell_LOG        lot_LOG
##  Min.   :0.000   Min.   :10.37   Min.   :7.576
##  1st Qu.:0.000   1st Qu.:11.00   1st Qu.:8.451
##  Median :1.000   Median :11.19   Median :8.700
##  Mean   :0.589   Mean   :11.21   Mean   :8.595
##  3rd Qu.:1.000   3rd Qu.:11.42   3rd Qu.:8.803
##  Max.   :1.000   Max.   :12.07   Max.   :9.468
```

Seventh model estimation

```
# Estimating third model.
modelH <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg,
             data = dat1)
print(modelH.summary <- summary(modelH))
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg, data = dat1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.66582 -0.13906  0.00796  0.14694  0.67596
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.67309    0.29240  26.241  < 2e-16 ***
## lot_LOG      0.31378    0.03615   8.680  < 2e-16 ***
## bdms         0.03787    0.01744   2.172 0.030469 *
## fb           0.15238    0.02469   6.170 1.71e-09 ***
## sty          0.08824    0.01819   4.850 1.79e-06 ***
## drv          0.08641    0.03141   2.751 0.006216 **
## rec          0.05465    0.03392   1.611 0.107975
## ffin         0.11471    0.02673   4.291 2.25e-05 ***
## ghw          0.19870    0.05301   3.748 0.000205 ***
## ca           0.17763    0.02724   6.521 2.17e-10 ***
## gar          0.05301    0.01480   3.583 0.000383 ***
## reg          0.15116    0.04215   3.586 0.000378 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2238 on 388 degrees of freedom
## Multiple R-squared:  0.6705, Adjusted R-squared:  0.6611
## F-statistic: 71.77 on 11 and 388 DF,  p-value: < 2.2e-16
```

The seventh model considered (based on the first data group only) is:

$$
\begin{aligned}
LOG(sell) = & + 7.673 \\
& + 0.314 \cdot LOG(lot) \\
& + 0.038 \cdot bdms \\
& + 0.152 \cdot fb \\
& + 0.088 \cdot sty \\
& + 0.086 \cdot drv \\
& + 0.055 \cdot rec \\
& + 0.115 \cdot ffin \\
& + 0.199 \cdot ghw \\
& + 0.178 \cdot ca \\
& + 0.053 \cdot gar \\
& + 0.151 \cdot reg \\
& + \epsilon
\end{aligned}
$$

Model characteristics: **R^2** = 0.6705 , **F-statistic**: 71.77 on 11 and 388 DF

> Notice the absence of all interaction variables as well as the absence of the lot variable itself (replaced by its logarithm $LOG(lot)$).

Model linearity testing

Ramsey's RESET (linearity) testing

```
# Model Ramsey's RESET testing.
modelH.RESET <- resettest(modelH, power = 2, type = "fitted",
                          data = dat1)
print(modelH.RESET)
```

```
##
##  RESET test
##
## data:  modelH
## RESET = 0.03955, df1 = 1, df2 = 387, p-value = 0.8425
```

Conclusion:

> With a statistic of **~0.04** and a p-value of **~0.8425**, the Ramsey's RESET test suggests that the seventh model **might be correctly** specified (H_0 of correct/linear specification **NOT rejected**, at the 5% level of significance).
>
> It also suggests that this is not the best model constructed so far, as the fourth model had scored an even lower statistic and a higher p-value than this; this seventh model scored the second best Ramsey's RESET test score.

Jarque-Bera (residuals normality) testing

```
# Model Jarque-Bera testing.
modelH.JB <- jarqueberaTest(modelH.summary$residuals)
print(modelH.JB)
```

```
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##   STATISTIC:
##     X-squared: 0.6976
##   P VALUE:
##     Asymptotic p Value: 0.7055
##
## Description:
##  Sun Jun 05 00:13:26 2016 by user: Yiannis
```

Conclusion:

With a statistic of **~0.698** and a p-value of **~0.7055**, the Jarque-Bera test suggests that the model residuals are **normally distributed**; therefore the model is considered **correctly** specified.

This Jarque-Bera test result is the best scored so far, and indicates a sufficient residuals normality.

Testing summary

Seventh model linearity testing results (comparison chart):

Mode	Test name	Test statistic	p-value	Test result
First	Ramsey's RESET	F= **26.986**	0	H_0: **Rejected**
Second	Ramsey's RESET	F= **0.27**	0.6033	H_0: **Not rejected**
Third	Ramsey's RESET	F= **0.068**	0.7948	H_0: **Not rejected**
Fourth	Ramsey's RESET	F= **0.012**	0.9144	H_0: **Not rejected**
Fifth	Ramsey's RESET	F= **0.046**	0.8308	H_0: **Not rejected**
Sixth	Ramsey's RESET	F= **0.431**	0.5118	H_0: **Not rejected**
Seventh	Ramsey's RESET	F= **0.04**	0.8425	H_0: **Not rejected**
——-	———————	——————————-	———————	———————
First	Jarque-Bera	JB= **247.62**	0	H_0: **Rejected**
Second	Jarque-Bera	JB= **8.443**	0.0147	H_0: **Rejected**
Third	Jarque-Bera	JB= **9.364**	0.0093	H_0: **Rejected**
Fourth	Jarque-Bera	JB= **8.203**	0.0165	H_0: **Rejected**
Fifth	Jarque-Bera	JB= **9.237**	0.0099	H_0: **Rejected**
Sixth	Jarque-Bera	JB= **10.348**	0.0057	H_0: **Rejected**
Seventh	Jarque-Bera	JB= **0.698**	0.7055	H_0: **Not rejected**

Conclusion:

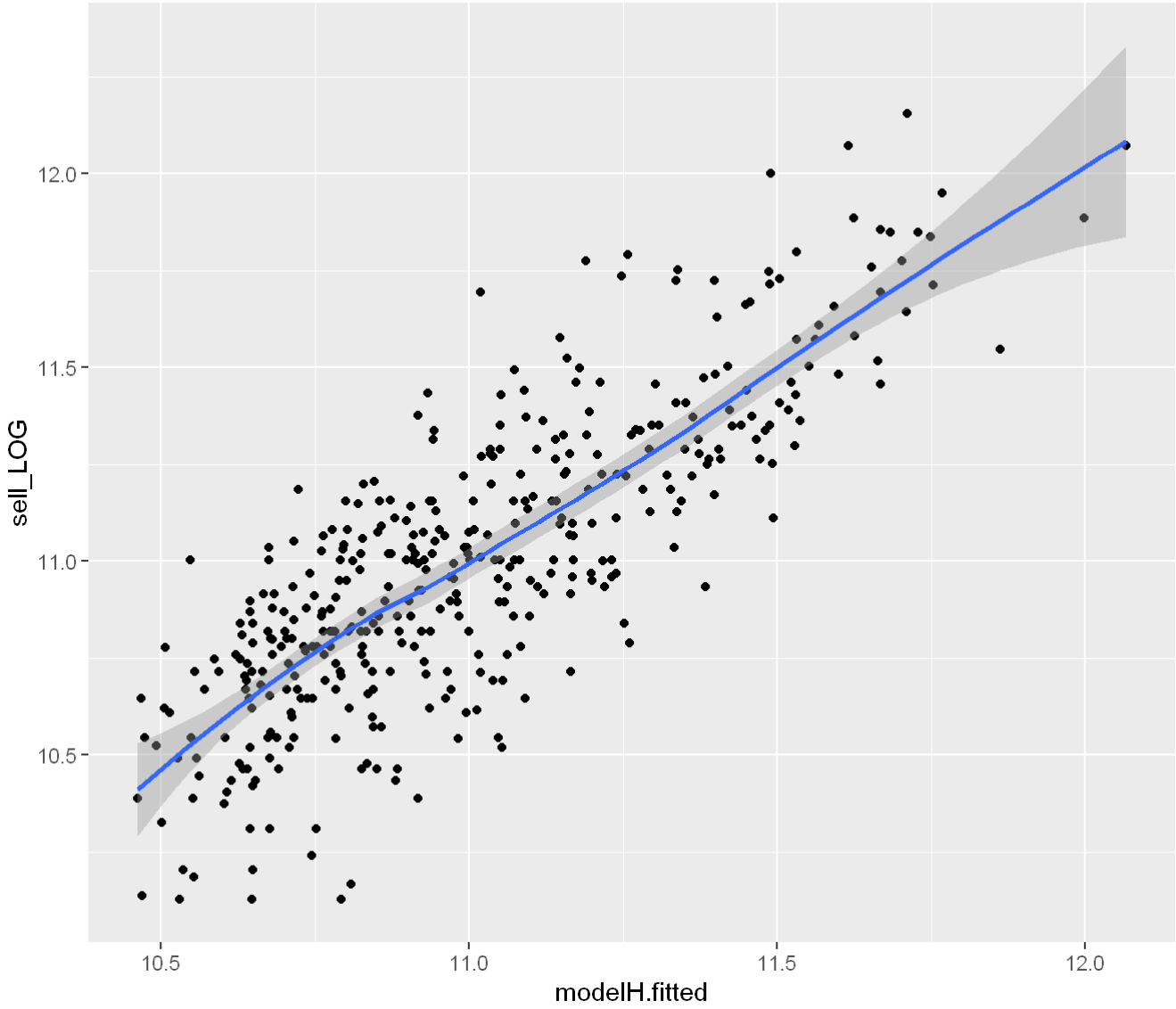
Both Ramsey's RESET and Jarque-Bera tests suggest that the seventh model is sufficiently linear and with good residuals normality.

Both Ramsey's RESET and Jarque-Bera tests suggest that the seventh model **might be correctly** specified.

This is also intuitively demonstrated by the seventh model real to fitted-values diagram shown at the next page (looks about the same or more like a linear relationship).

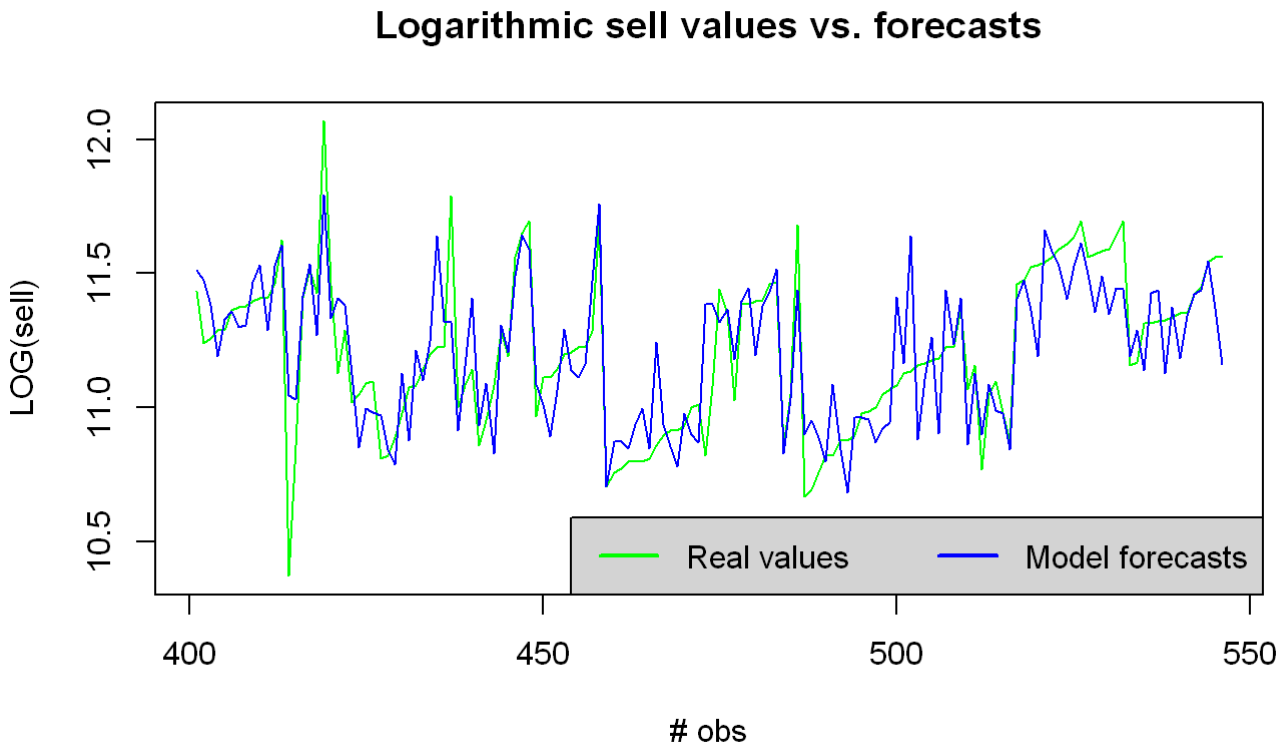
Real to fitted-values diagram

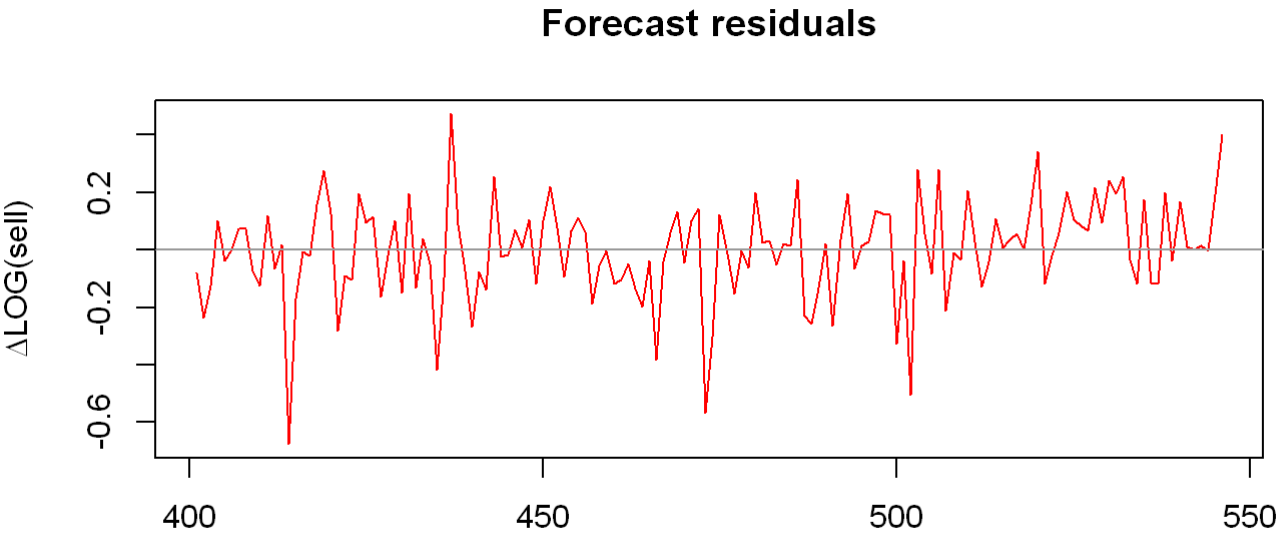
```
modelH.fitted <- fitted.values(modelH)

ggplot(dat1, aes(x=modelH.fitted, y=sell_LOG)) +
    geom_point(shape=16) +
    geom_smooth()
```



Model predictive ability

The seventh model, as estimated using the first data group, produced the following $\hat{LOG}(lot)$ values on the second data group:





LOG(sell) variance

Our sample $LOG(sell)$ dependent variable descriptive statistics are:

```
sell_LOG_mean <- mean(dat$sell_LOG)
sell_LOG_sd   <- sd(dat$sell_LOG)
```

```
## [1] 11.059
```

```
## [1] 0.372
```

MAE (Mean Absolute Error) index calculation

```
digits <- 3

n <- nrow(dat2)
resids_SUM <- sum(abs(dat2$residuals))
```

$$MAE = \frac{1}{n} \cdot \sum_{i=401}^{546} |\log sell - \log \hat{sell}| \qquad = \frac{18.665}{146} = 0.128 \qquad < \frac{0.372}{2}$$

Conclusion:

> Calculated MAE value of 0.128 is much less than the dependent variable standard deviation half, which leads to the conclusion that **the model has some/significant predictive ability**.

Appendix A

General-to-specific approach regressions run - section **(f)**

Results of regression round #1 (with all variables):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                  lot_LOG * bdms + lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * re
c +
                  lot_LOG * ffin + lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar + lot_LOG * re
g,
              data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * bdms + lot_LOG *
##     fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec + lot_LOG *
##     ffin + lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar + lot_LOG *
##     reg, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68306 -0.11612  0.00591  0.12486  0.65998
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.966499   1.070667   8.375 5.09e-16 ***
## lot_LOG       0.152685   0.128294   1.190   0.2345
## bdms          0.019075   0.326700   0.058   0.9535
## fb           -0.368234   0.429048  -0.858   0.3911
## sty           0.488885   0.309700   1.579   0.1150
## drv          -1.463371   0.717225  -2.040   0.0418 *
## rec           1.673992   0.655919   2.552   0.0110 *
## ffin         -0.031844   0.445543  -0.071   0.9430
## ghw          -0.505889   0.902733  -0.560   0.5754
## ca           -0.340276   0.496041  -0.686   0.4930
## gar           0.401941   0.258646   1.554   0.1208
## reg           0.118484   0.479856   0.247   0.8051
## lot_LOG:bdms  0.002070   0.038654   0.054   0.9573
## lot_LOG:fb    0.062037   0.050145   1.237   0.2166
## lot_LOG:sty  -0.046361   0.035942  -1.290   0.1977
## lot_LOG:drv   0.191542   0.087361   2.193   0.0288 *
## lot_LOG:rec  -0.188462   0.076373  -2.468   0.0139 *
## lot_LOG:ffin  0.015913   0.052851   0.301   0.7635
## lot_LOG:ghw   0.081135   0.106929   0.759   0.4483
## lot_LOG:ca    0.059549   0.058024   1.026   0.3052
## lot_LOG:gar  -0.041359   0.030142  -1.372   0.1706
## lot_LOG:reg   0.001515   0.055990   0.027   0.9784
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2095 on 524 degrees of freedom
## Multiple R-squared:  0.6951, Adjusted R-squared:  0.6829
## F-statistic: 56.89 on 21 and 524 DF,  p-value: < 2.2e-16
```

Results of regression round #2 (with $LOG(lot) \cdot reg$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * bdms + lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * re
c +
                lot_LOG * ffin + lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * bdms + lot_LOG *
##     fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec + lot_LOG *
##     ffin + lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68292 -0.11619  0.00573  0.12491  0.65976
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.96795    1.06831   8.394 4.37e-16 ***
## lot_LOG       0.15252    0.12802   1.191   0.2341
## bdms          0.01949    0.32603   0.060   0.9523
## fb           -0.36774    0.42824  -0.859   0.3909
## sty           0.48721    0.30316   1.607   0.1086
## drv          -1.46786    0.69713  -2.106   0.0357 *
## rec           1.67468    0.65480   2.558   0.0108 *
## ffin         -0.03494    0.43021  -0.081   0.9353
## ghw          -0.50427    0.89990  -0.560   0.5755
## ca           -0.33954    0.49483  -0.686   0.4929
## gar           0.40234    0.25797   1.560   0.1194
## reg           0.13145    0.02304   5.705 1.94e-08 ***
## lot_LOG:bdms  0.00202    0.03857   0.052   0.9582
## lot_LOG:fb    0.06198    0.05005   1.238   0.2161
## lot_LOG:sty  -0.04617    0.03518  -1.312   0.1900
## lot_LOG:drv   0.19207    0.08504   2.259   0.0243 *
## lot_LOG:rec  -0.18855    0.07623  -2.473   0.0137 *
## lot_LOG:ffin  0.01629    0.05098   0.319   0.7495
## lot_LOG:ghw   0.08094    0.10658   0.759   0.4479
## lot_LOG:ca    0.05946    0.05788   1.027   0.3047
## lot_LOG:gar  -0.04140    0.03007  -1.377   0.1691
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2093 on 525 degrees of freedom
## Multiple R-squared:  0.6951, Adjusted R-squared:  0.6835
## F-statistic: 59.85 on 20 and 525 DF,  p-value: < 2.2e-16
```

Results of regression round #3 (with $LOG(lot) \cdot bdms$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec +
                lot_LOG * ffin + lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * fb + lot_LOG * sty +
##     lot_LOG * drv + lot_LOG * rec + lot_LOG * ffin + lot_LOG *
##     ghw + lot_LOG * ca + lot_LOG * gar, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68301 -0.11617  0.00574  0.12490  0.66020
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.93486    0.86069  10.381  < 2e-16 ***
## lot_LOG       0.15647    0.10329   1.515   0.1304
## bdms          0.03655    0.01459   2.506   0.0125 *
## fb           -0.37745    0.38563  -0.979   0.3281
## sty           0.48200    0.28614   1.685   0.0927 .
## drv          -1.46253    0.68903  -2.123   0.0343 *
## rec           1.67592    0.65375   2.564   0.0106 *
## ffin         -0.03743    0.42717  -0.088   0.9302
## ghw          -0.50161    0.89761  -0.559   0.5765
## ca           -0.33869    0.49409  -0.685   0.4933
## gar           0.40103    0.25652   1.563   0.1186
## reg           0.13144    0.02302   5.710 1.89e-08 ***
## lot_LOG:fb    0.06313    0.04494   1.405   0.1607
## lot_LOG:sty  -0.04556    0.03321  -1.372   0.1706
## lot_LOG:drv   0.19143    0.08408   2.277   0.0232 *
## lot_LOG:rec  -0.18868    0.07612  -2.479   0.0135 *
## lot_LOG:ffin  0.01658    0.05062   0.328   0.7434
## lot_LOG:ghw   0.08062    0.10631   0.758   0.4486
## lot_LOG:ca    0.05935    0.05779   1.027   0.3049
## lot_LOG:gar  -0.04125    0.02989  -1.380   0.1682
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2091 on 526 degrees of freedom
## Multiple R-squared:  0.6951, Adjusted R-squared:  0.6841
## F-statistic: 63.12 on 19 and 526 DF,  p-value: < 2.2e-16
```

Results of regression round #4 (with $LOG(lot) \cdot ffin$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec +
                lot_LOG * ghw + lot_LOG * ca + lot_LOG * gar,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * fb + lot_LOG * sty +
##     lot_LOG * drv + lot_LOG * rec + lot_LOG * ghw + lot_LOG *
##     ca + lot_LOG * gar, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68181 -0.11724  0.00567  0.12594  0.65662
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.87651    0.84134  10.550  < 2e-16 ***
## lot_LOG      0.16359    0.10089   1.621   0.1055
## bdms         0.03655    0.01458   2.507   0.0125 *
## fb          -0.38191    0.38506  -0.992   0.3217
## sty          0.48851    0.28520   1.713   0.0873 .
## drv         -1.45022    0.68742  -2.110   0.0354 *
## rec          1.62140    0.63167   2.567   0.0105 *
## ffin         0.10232    0.02181   4.691 3.47e-06 ***
## ghw         -0.51600    0.89578  -0.576   0.5648
## ca          -0.35449    0.49131  -0.722   0.4709
## gar          0.40146    0.25629   1.566   0.1179
## reg          0.13227    0.02286   5.786 1.24e-08 ***
## lot_LOG:fb   0.06360    0.04487   1.417   0.1570
## lot_LOG:sty -0.04640    0.03308  -1.403   0.1613
## lot_LOG:drv  0.18991    0.08388   2.264   0.0240 *
## lot_LOG:rec -0.18218    0.07343  -2.481   0.0134 *
## lot_LOG:ghw  0.08250    0.10606   0.778   0.4370
## lot_LOG:ca   0.06123    0.05746   1.066   0.2871
## lot_LOG:gar -0.04129    0.02987  -1.383   0.1674
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2089 on 527 degrees of freedom
## Multiple R-squared:  0.695,  Adjusted R-squared:  0.6846
## F-statistic: 66.73 on 18 and 527 DF,  p-value: < 2.2e-16
```

Results of regression round #5 (with $LOG(lot) \cdot ghw$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec +
                lot_LOG * ca + lot_LOG * gar,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * fb + lot_LOG * sty +
##     lot_LOG * drv + lot_LOG * rec + lot_LOG * ca + lot_LOG *
##     gar, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68122 -0.11898  0.00738  0.12611  0.65311
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.80857    0.83648  10.530  < 2e-16 ***
## lot_LOG      0.17159    0.10033   1.710   0.0878 .
## bdms         0.03661    0.01457   2.513   0.0123 *
## fb          -0.37636    0.38485  -0.978   0.3286
## sty          0.49092    0.28508   1.722   0.0857 .
## drv         -1.43262    0.68679  -2.086   0.0375 *
## rec          1.63058    0.63133   2.583   0.0101 *
## ffin         0.10361    0.02174   4.766 2.44e-06 ***
## ghw          0.17991    0.04391   4.098 4.83e-05 ***
## ca          -0.33972    0.49076  -0.692   0.4891
## gar          0.39730    0.25614   1.551   0.1215
## reg          0.13113    0.02281   5.750 1.51e-08 ***
## lot_LOG:fb   0.06302    0.04485   1.405   0.1606
## lot_LOG:sty -0.04669    0.03306  -1.412   0.1585
## lot_LOG:drv  0.18782    0.08380   2.241   0.0254 *
## lot_LOG:rec -0.18320    0.07339  -2.496   0.0129 *
## lot_LOG:ca   0.05932    0.05738   1.034   0.3017
## lot_LOG:gar -0.04085    0.02985  -1.368   0.1718
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2088 on 528 degrees of freedom
## Multiple R-squared:  0.6947, Adjusted R-squared:  0.6849
## F-statistic: 70.67 on 17 and 528 DF,  p-value: < 2.2e-16
```

Results of regression round #6 (with $LOG(lot) \cdot ca$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec +
                lot_LOG * gar,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * fb + lot_LOG * sty +
##     lot_LOG * drv + lot_LOG * rec + lot_LOG * gar, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.67934 -0.12004  0.00644  0.12660  0.64601
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.78218    0.83615  10.503  < 2e-16 ***
## lot_LOG      0.17484    0.10028   1.743   0.0818 .
## bdms         0.03523    0.01451   2.428   0.0155 *
## fb          -0.38030    0.38485  -0.988   0.3235
## sty          0.47196    0.28451   1.659   0.0977 .
## drv         -1.42861    0.68683  -2.080   0.0380 *
## rec          1.55669    0.62731   2.482   0.0134 *
## ffin         0.10341    0.02174   4.756 2.55e-06 ***
## ghw          0.17721    0.04383   4.043 6.06e-05 ***
## ca           0.16716    0.02121   7.880 1.87e-14 ***
## gar          0.33850    0.24976   1.355   0.1759
## reg          0.13298    0.02274   5.848 8.69e-09 ***
## lot_LOG:fb   0.06359    0.04485   1.418   0.1569
## lot_LOG:sty -0.04432    0.03299  -1.344   0.1797
## lot_LOG:drv  0.18733    0.08381   2.235   0.0258 *
## lot_LOG:rec -0.17463    0.07293  -2.395   0.0170 *
## lot_LOG:gar -0.03385    0.02908  -1.164   0.2448
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2088 on 529 degrees of freedom
## Multiple R-squared:  0.6941, Adjusted R-squared:  0.6848
## F-statistic: 75.01 on 16 and 529 DF,  p-value: < 2.2e-16
```

Results of regression round #7 (with $LOG(lot) \cdot gar$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * fb + lot_LOG * sty + lot_LOG * drv + lot_LOG * rec,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * fb + lot_LOG * sty +
##     lot_LOG * drv + lot_LOG * rec, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68420 -0.12071  0.00669  0.12322  0.64513
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.77393    0.83640  10.490  < 2e-16 ***
## lot_LOG      0.17584    0.10031   1.753   0.0802 .
## bdms         0.03530    0.01451   2.432   0.0153 *
## fb          -0.34021    0.38344  -0.887   0.3753
## sty          0.46819    0.28459   1.645   0.1005
## drv         -1.23688    0.66702  -1.854   0.0642 .
## rec          1.51405    0.62645   2.417   0.0160 *
## ffin         0.10279    0.02174   4.727 2.92e-06 ***
## ghw          0.18002    0.04378   4.112 4.55e-05 ***
## ca           0.16697    0.02122   7.869 2.02e-14 ***
## gar          0.04802    0.01143   4.200 3.13e-05 ***
## reg          0.12990    0.02259   5.750 1.51e-08 ***
## lot_LOG:fb   0.05903    0.04469   1.321   0.1872
## lot_LOG:sty -0.04392    0.03300  -1.331   0.1837
## lot_LOG:drv  0.16448    0.08150   2.018   0.0441 *
## lot_LOG:rec -0.16943    0.07281  -2.327   0.0203 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2089 on 530 degrees of freedom
## Multiple R-squared:  0.6933, Adjusted R-squared:  0.6846
## F-statistic: 79.87 on 15 and 530 DF,  p-value: < 2.2e-16
```

Results of regression round #8 (with $LOG(lot) \cdot fb$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * sty + lot_LOG * drv + lot_LOG * rec,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * sty + lot_LOG * drv +
##     lot_LOG * rec, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68209 -0.11831  0.00758  0.12350  0.63856
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.29846    0.75549  10.984  < 2e-16 ***
## lot_LOG      0.23088    0.09131   2.529   0.0117 *
## bdms         0.03623    0.01451   2.497   0.0128 *
## fb           0.16549    0.02061   8.030 6.30e-15 ***
## sty          0.38420    0.27758   1.384   0.1669
## drv         -1.25462    0.66735  -1.880   0.0607 .
## rec          1.47254    0.62610   2.352   0.0190 *
## ffin         0.10042    0.02168   4.631 4.58e-06 ***
## ghw          0.18093    0.04381   4.130 4.21e-05 ***
## ca           0.16623    0.02123   7.831 2.64e-14 ***
## gar          0.04751    0.01143   4.155 3.79e-05 ***
## reg          0.13126    0.02258   5.812 1.06e-08 ***
## lot_LOG:sty -0.03402    0.03216  -1.058   0.2906
## lot_LOG:drv  0.16690    0.08154   2.047   0.0412 *
## lot_LOG:rec -0.16467    0.07278  -2.263   0.0241 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2091 on 531 degrees of freedom
## Multiple R-squared:  0.6923, Adjusted R-squared:  0.6842
## F-statistic: 85.33 on 14 and 531 DF,  p-value: < 2.2e-16
```

Results of regression round #9 (with $LOG(lot) \cdot sty$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * drv + lot_LOG * rec,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * drv + lot_LOG * rec,
##     data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.67934 -0.12225  0.00849  0.12259  0.65051
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.74189    0.62863  13.906  < 2e-16 ***
## lot_LOG      0.17906    0.07707   2.323  0.02053 *
## bdms         0.03881    0.01430   2.714  0.00686 **
## fb           0.16145    0.02025   7.971 9.62e-15 ***
## sty          0.09083    0.01254   7.242 1.56e-12 ***
## drv         -1.18996    0.66462  -1.790  0.07395 .
## rec          1.50253    0.62553   2.402  0.01665 *
## ffin         0.10276    0.02157   4.763 2.46e-06 ***
## ghw          0.18448    0.04368   4.223 2.83e-05 ***
## ca           0.16526    0.02121   7.792 3.48e-14 ***
## gar          0.04690    0.01142   4.107 4.65e-05 ***
## reg          0.13260    0.02255   5.880 7.24e-09 ***
## lot_LOG:drv  0.15943    0.08124   1.962  0.05024 .
## lot_LOG:rec -0.16826    0.07270  -2.314  0.02103 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2091 on 532 degrees of freedom
## Multiple R-squared:  0.6916, Adjusted R-squared:  0.6841
## F-statistic: 91.79 on 13 and 532 DF,  p-value: < 2.2e-16
```

Results of regression round #10 (with $LOG(lot) \cdot drv$ variable removed):

```
model <- lm(sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec + ffin + ghw + ca + gar + reg +
                lot_LOG * rec,
            data = dat)
summary(model)
```

```
##
## Call:
## lm(formula = sell_LOG ~ lot_LOG + bdms + fb + sty + drv + rec +
##     ffin + ghw + ca + gar + reg + lot_LOG * rec, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.68111 -0.12208  0.00593  0.12731  0.66275
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.59071    0.22656  33.505  < 2e-16 ***
## lot_LOG      0.32024    0.02770  11.562  < 2e-16 ***
## bdms         0.03842    0.01434   2.680   0.0076 **
## fb           0.16318    0.02029   8.043 5.71e-15 ***
## sty          0.09080    0.01258   7.220 1.80e-12 ***
## drv          0.11312    0.02815   4.018 6.72e-05 ***
## rec          1.44313    0.62646   2.304   0.0216 *
## ffin         0.10450    0.02161   4.835 1.74e-06 ***
## ghw          0.18429    0.04380   4.208 3.03e-05 ***
## ca           0.16593    0.02126   7.804 3.19e-14 ***
## gar          0.04810    0.01144   4.206 3.05e-05 ***
## reg          0.13373    0.02260   5.917 5.89e-09 ***
## lot_LOG:rec -0.16112    0.07281  -2.213   0.0273 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2096 on 533 degrees of freedom
## Multiple R-squared:  0.6894, Adjusted R-squared:  0.6824
## F-statistic: 98.59 on 12 and 533 DF,  p-value: < 2.2e-16
```

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