

# EE604A - DIGITAL IMAGE PROCESSING ASSIGNMENT

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## Image Sources

Sunset (high.jpg) : <https://www.flickr.com/photos/mediaflex/4190084346>

Carving (low.jpg) : <https://www.flickr.com/photos/30440933@N06/2847993403>

Dog (small.jpg) : [https://res.cloudinary.com/rover-com/image/upload/a\\_exif,c\\_fill,f\\_jpg,f1\\_progressive,g\\_face:center,h\\_100,q\\_80,w\\_100/remote/images/pets/4NpPz08N/50e4a023d9/original.jpg](https://res.cloudinary.com/rover-com/image/upload/a_exif,c_fill,f_jpg,f1_progressive,g_face:center,h_100,q_80,w_100/remote/images/pets/4NpPz08N/50e4a023d9/original.jpg)

divyat.jpg : Own work. Taken with permission.

## Solution 1

(a)

Code has been written in two MATLAB® files :

- Q1\lloyd\_max\_quantizer.m : The file containing the function.
- Q1\Q1.m : The script to perform the required tasks.

(b)

The 4 representation levels are :

The corresponding transition levels are :

(c)

(d)

(e)

## Solution 2

A C++ function, EE604A::histogram\_matching(), for histogram matching of cv::Mat images has been developed. The code required for the matching function is present in :

- Q2\src\histogram\_matching.cxx
- Q2\src\histogram\_matching.h

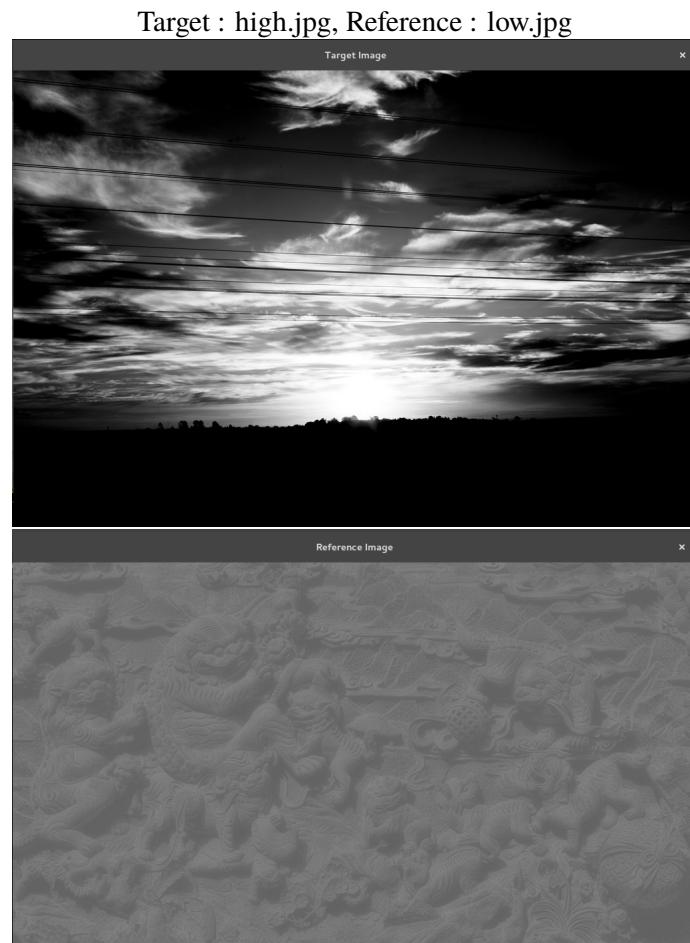
A small program which uses the histogram matching function is present in Q2\src\Q2.cxx

In order to build and run the code, the following steps need to be followed :

1. Enter Q2\src.
2. run build.sh (OpenCV needs to installed and a version of g++ supporting c++14 needs to be present)
3. Execute Q2 (the binary file) with the reference and target relative/absolute image paths. Eg. [./Q2 ..../images/high.jpg ..../images/low.jpg] or [./Q2 ..../images/low.jpg ..../images/high.jpg] (On Ubuntu)
4. The program can be closed by Ctrl+C on the terminal or by pressing 'q' when one of the image windows is open.

REMARK : A simple plot of the the histograms can be obtained by toggling the third argument of the function EE604A::histogram\_matching() to true.

## Results





Target : low.jpg, Reference : high.jpg



### Solution 3

Bilinear interpolation in a 8-connect neighborhood.

### Solution 4

We are assuming that  $\eta_i(x_1, y_1)$  and  $\eta_j(x_2, y_2)$  are independent if  $i \neq j$  or  $x_1 \neq x_2$  or  $y_1 \neq y_2$ .

$$\therefore \text{if } i \neq j, E[\eta_i(x, y)\eta_j(x, y)] = 0 \quad (1)$$

We have,  $g_i(x, y) = f_i(x, y) + \eta_i(x, y)$

Also,  $f_i(x, y) = f(x, y)$

$$\begin{aligned} \text{Now, } \hat{g}(x, y) &= \frac{1}{K} \sum_{i=1}^K g_i(x, y) \\ &= \frac{1}{K} K f(x, y) + \frac{1}{K} \sum_{i=1}^K \eta_i(x, y) \\ &= f(x, y) + \frac{1}{K} \sum_{i=1}^K \eta_i(x, y) \end{aligned}$$

$$\begin{aligned} \text{Now, noise variance, } E[(\hat{g}(x, y) - f(x, y))^2] &= E[(\frac{1}{K} \sum_{i=1}^K \eta_i(x, y))^2] \\ &= \frac{1}{K^2} \sum_{i=1}^K \sum_{j=1}^K E[\eta_i(x, y)\eta_j(x, y)] \\ &= \frac{1}{K^2} \sum_{i=1}^K E[\eta_i(x, y)^2] \quad \text{from (1)} \\ &= \frac{1}{K^2} \sum_{i=1}^K \sigma^2 \\ &= \frac{\sigma^2}{K} \end{aligned}$$

q.e.d

## Solution 5

Let  $f(x, y, z) : \mathbb{R}^3 \rightarrow \mathbb{R}$ . Let  $[u \ v \ w]^T$  be the position of  $[x \ y \ z]^T$  in a rotated coordinate frame. The relationship between  $[u \ v \ w]^T$  and  $[x \ y \ z]^T$  is given by :

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = R \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (2)$$

$$\text{where, } R = [R_1 \ R_2 \ R_3] \quad (3)$$

$$= \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \quad (4)$$

$$\text{Known properties of } R, |R_1| = 1 \quad (5)$$

$$|R_2| = 1 \quad (6)$$

$$R_1^T R_2 = 0 \quad (7)$$

$$R_3 = R_1 \times R_2 \quad (8)$$

$$\implies |R_3| = 1 \quad (9)$$

$$R_1^T R_3 = R_2^T R_3 = 0 \quad (10)$$

$$R^{-1} = R^T \quad (11)$$

$$\text{We can also write, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \text{from (2), (9)} \quad (12)$$

$$\text{Now, } \nabla^2 f(u, v, w) = \frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} \quad (13)$$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial u} \quad (14)$$

$$= R_{11} \frac{\partial f}{\partial x} + R_{12} \frac{\partial f}{\partial y} + R_{13} \frac{\partial f}{\partial z} \quad \text{from (4), (10)} \quad (15)$$

$$\text{Similarly, } \frac{\partial f}{\partial v} = R_{21} \frac{\partial f}{\partial x} + R_{22} \frac{\partial f}{\partial y} + R_{23} \frac{\partial f}{\partial z} \quad (16)$$

$$\frac{\partial f}{\partial w} = R_{31} \frac{\partial f}{\partial x} + R_{32} \frac{\partial f}{\partial y} + R_{33} \frac{\partial f}{\partial z} \quad (17)$$

$$\begin{aligned}\therefore \frac{\partial^2 f}{\partial^2 u} &= R_{11} \left( \frac{\partial^2 f}{\partial^2 x} \frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial y \partial x} \frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial z \partial x} \frac{\partial z}{\partial u} \right) \\ &\quad + R_{12} \left( \frac{\partial^2 f}{\partial x \partial y} \frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial^2 y} \frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial z \partial x} \frac{\partial z}{\partial u} \right) \\ &\quad + R_{13} \left( \frac{\partial^2 f}{\partial x \partial z} \frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial y \partial z} \frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial^2 z} \frac{\partial z}{\partial u} \right) \quad \text{from (13) and chain rule} \\ &\quad \quad \quad (18)\end{aligned}$$

$$\begin{aligned}&= R_{11}^2 \frac{\partial^2 f}{\partial^2 x} + R_{11} R_{12} \frac{\partial^2 f}{\partial y \partial x} + R_{11} R_{13} \frac{\partial^2 f}{\partial z \partial x} \\ &\quad + R_{12} R_{11} \frac{\partial^2 f}{\partial x \partial y} + R_{12}^2 \frac{\partial^2 f}{\partial^2 y} + R_{12} R_{13} \frac{\partial^2 f}{\partial z \partial y} \\ &\quad + R_{13} R_{11} \frac{\partial^2 f}{\partial x \partial z} + R_{13} R_{12} \frac{\partial^2 f}{\partial y \partial z} + R_{13}^2 \frac{\partial^2 f}{\partial^2 z} \quad \text{from (4), (10)} \\ &\quad \quad \quad (19)\end{aligned}$$

$$\begin{aligned}\text{Similarly, } \frac{\partial^2 f}{\partial^2 v} &= R_{21}^2 \frac{\partial^2 f}{\partial^2 x} + R_{21} R_{22} \frac{\partial^2 f}{\partial y \partial x} + R_{21} R_{23} \frac{\partial^2 f}{\partial z \partial x} \\ &\quad + R_{22} R_{21} \frac{\partial^2 f}{\partial x \partial y} + R_{22}^2 \frac{\partial^2 f}{\partial^2 y} + R_{22} R_{23} \frac{\partial^2 f}{\partial z \partial y} \\ &\quad + R_{23} R_{21} \frac{\partial^2 f}{\partial x \partial z} + R_{23} R_{22} \frac{\partial^2 f}{\partial y \partial z} + R_{23}^2 \frac{\partial^2 f}{\partial^2 z} \quad (20)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 f}{\partial^2 w} &= R_{31}^2 \frac{\partial^2 f}{\partial^2 x} + R_{31} R_{32} \frac{\partial^2 f}{\partial y \partial x} + R_{31} R_{33} \frac{\partial^2 f}{\partial z \partial x} \\ &\quad + R_{32} R_{31} \frac{\partial^2 f}{\partial x \partial y} + R_{32}^2 \frac{\partial^2 f}{\partial^2 y} + R_{32} R_{33} \frac{\partial^2 f}{\partial z \partial y} \\ &\quad + R_{33} R_{31} \frac{\partial^2 f}{\partial x \partial z} + R_{33} R_{32} \frac{\partial^2 f}{\partial y \partial z} + R_{33}^2 \frac{\partial^2 f}{\partial^2 z} \quad (21)\end{aligned}$$

On performing (17) + (18) + (19), we get using (5), (6), (7), (9) and (10)

$$\frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} = \frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} \quad (22)$$

$$\implies \nabla^2 f(u, v, w) = \frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} \quad (23)$$

$$= \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y} + \frac{\partial^2 f}{\partial^2 z} \quad \text{from (22)} \quad (24)$$

$$= \nabla^2 f(x, y, z) \quad (25)$$

*q.e.d*

## Solution 6

A C++ file contains the code required to perform the required task.

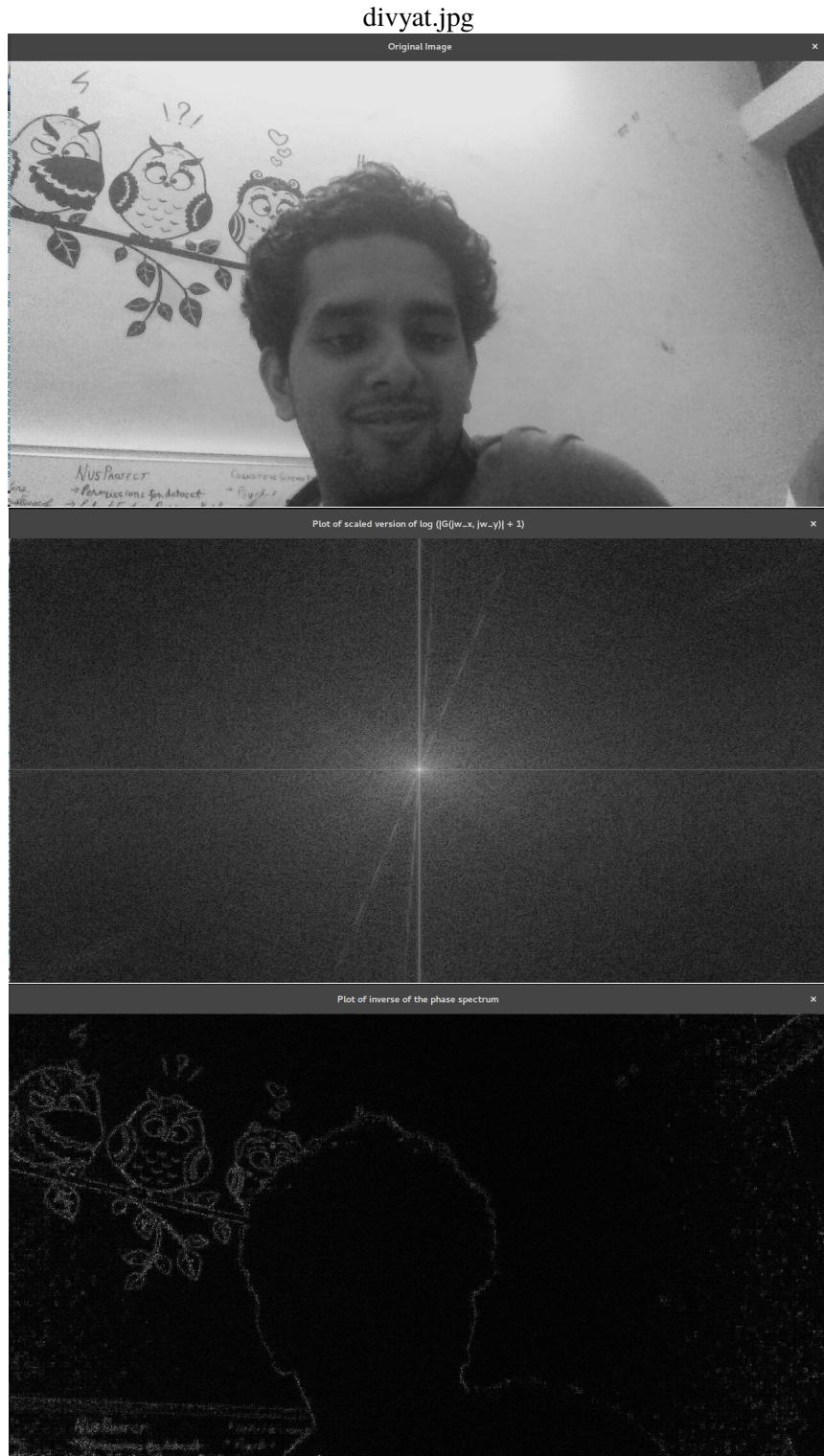
In order to build and run the code, the following steps need to be followed :

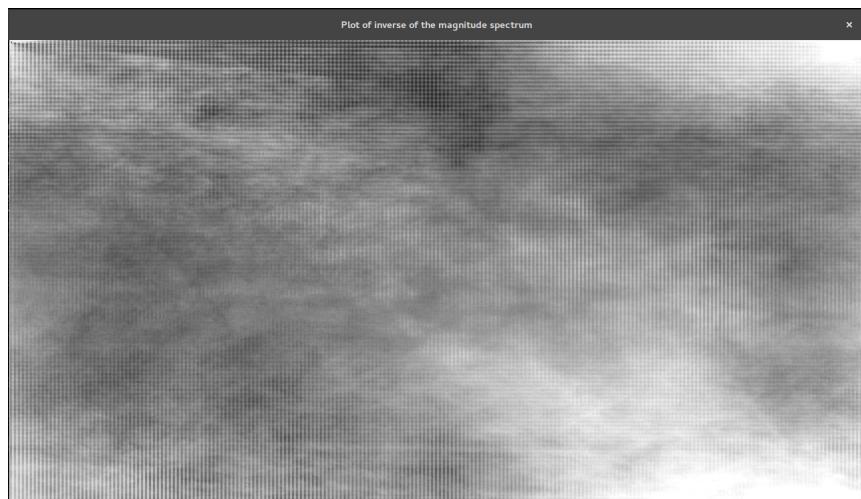
1. Enter Q6\src.
2. run build.sh (OpenCV needs to be installed and a version of g++ supporting c++14 needs to be present)
3. Execute Q6 (the binary file) with the relative/absolute image path. Eg. [./Q6 ../../images/high.jpg] or [./Q2 ../../images/low.jpg ../../images/high.jpg] (On Ubuntu)

4. The program can be closed by Ctrl+C on the terminal or by pressing ‘q’ when one of the image windows is open.

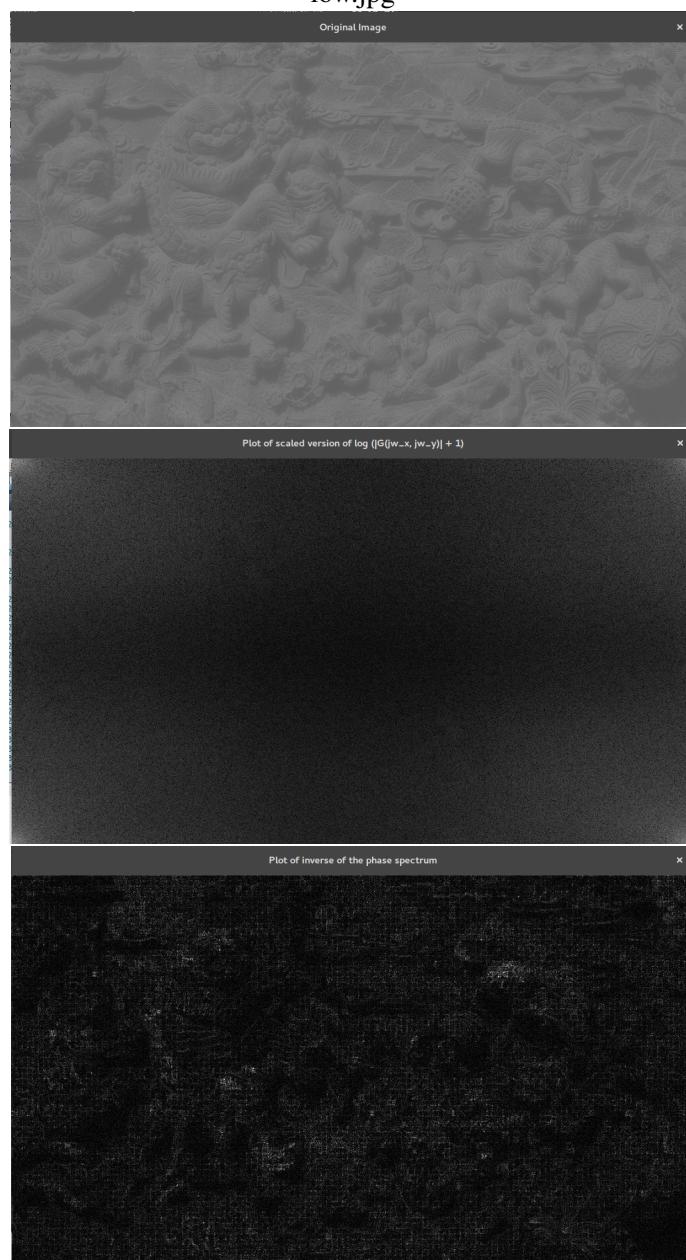
REMARK : A simple plot of the histograms can be obtained by toggling the third argument of the function EE604A::histogram\_matching() to true.

## Results





low.jpg





## Observations

### Reconstruction using only phase spectrum

From the above the tests, it can be inferred that performing reconstruction using only phase information, edges are being preserved and we are losing all grey-levels of homogenous regions. From, the spectrum plots, we can infer that the lower frequency components tend to have greater values than the higher frequency ones in general images. Thus, when we remove the magnitude information, the lower frequency components are getting more suppressed than the higher frequency components, i.e. the operation is similar to a high-pass filter. This premise explains the above images obtained using only the phase spectrum.

### Reconstruction using only magnitude spectrum

The reconstruction using only magnitude spectrum appears to only contain information regarding the prominent greylevels present in the image.

## Solution 7

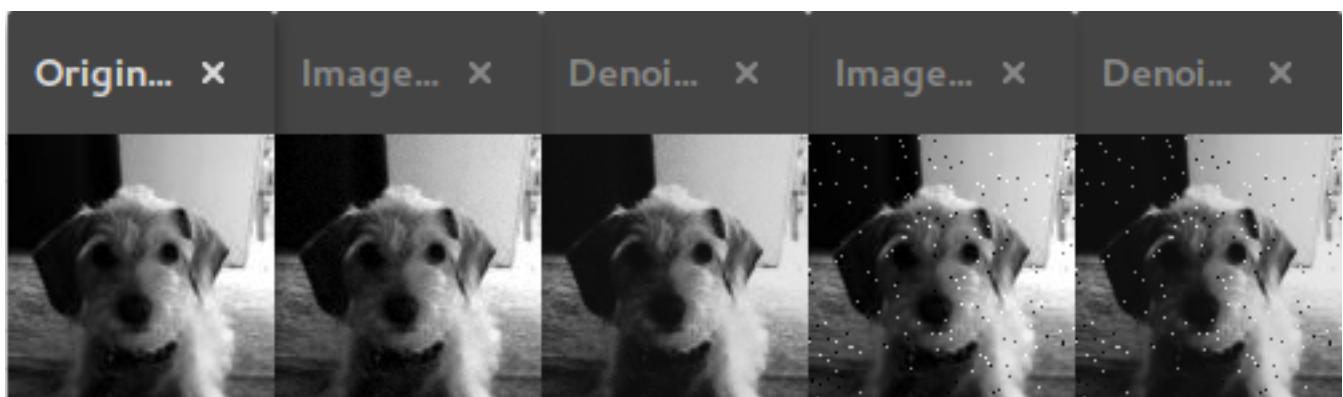


Figure 1: The images from left to right are as follows : (a) The original image (small.jpg), (b) The images with added zero-mean gaussian noise with  $\sigma^2 = 0.1$ , (c) The reconstructed image [b], (d) The image with impulse noise, (e) The reconstructed image [d]