

EE604A - DIGITAL IMAGE PROCESSING ASSIGNMENT

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Image Sources

Sunset (high.jpg) : <https://www.flickr.com/photos/mediaflex/4190084346>

Carving (low.jpg) : <https://www.flickr.com/photos/30440933@N06/2847993403>

Dog (small.jpg) : https://res.cloudinary.com/rover-com/image/upload/a_exif,c_fill,f_jpg,fl_progressive,g_face:center,h_100,q_80,w_100/remote/images/pets/4NpPz08N/50e4a023d9/original.jpg

Solution 1

(a)

Code has been written in two MATLAB® files :

- Q1\lloyd_max_quantizer.m : The file containing the function
- Q1\Q1.m : The script to perform the required tasks.

(b)

The 4 representation levels are :

The corresponding transition levels are :

(c)

(d)

(e)

Solution 2

A C++ function, EE604A::histogram_matching(), for histogram matching of cv::Mat images has been developed. The code required for the matching function is present in :

- Q2\src\histogram_matching.cxx
- Q2\src\histogram_matching.h

A small program which uses the histogram matching function is present in Q2\src\Q2.cxx

In order to run the code, the following steps need to be followed :

1. Enter Q2\src.

2. run build.sh (OpenCV needs to be installed and a version of g++ supporting c++14 needs to be present)
3. Execute Q2 (the binary file) with the reference and target relative/absolute image paths. Eg. [./Q2 ../../images/high.jpg ../../images/low.jpg] or [./Q2 ../../images/low.jpg ../../images/high.jpg] (On Ubuntu)
4. The program can be closed by Ctrl+C on the terminal or by pressing 'q' when one of the image windows is open.

REMARK : A simple plot of the the histograms can be obtained by toggling

Solution 3

Solution 4

We are assuming that $\eta_i(x_1, y_1)$ and $\eta_j(x_2, y_2)$ are independent if $i \neq j$ or $x_1 \neq x_2$ or $y_1 \neq y_2$.

$$\therefore \text{ if } i \neq j, E[\eta_i(x, y)\eta_j(x, y)] = 0 \quad (1)$$

$$\text{We have, } g_i(x, y) = f_i(x, y) + \eta_i(x, y)$$

$$\text{Also, } f_i(x, y) = f(x, y)$$

$$\begin{aligned} \text{Now, } \hat{g}(x, y) &= \frac{1}{K} \sum_{i=1}^K g_i(x, y) \\ &= \frac{1}{K} K f(x, y) + \frac{1}{K} \sum_{i=1}^K \eta_i(x, y) \\ &= f(x, y) + \frac{1}{K} \sum_{i=1}^K \eta_i(x, y) \end{aligned}$$

$$\begin{aligned} \text{Now, noise variance, } E[(\hat{g}(x, y) - f(x, y))^2] &= E[(\frac{1}{K} \sum_{i=1}^K \eta_i(x, y))^2] \\ &= \frac{1}{K^2} \sum_{i=1}^K \sum_{j=1}^K E[\eta_i(x, y)\eta_j(x, y)] \\ &= \frac{1}{K^2} \sum_{i=1}^K E[\eta_i(x, y)^2] \quad \text{from (1)} \\ &= \frac{1}{K^2} \sum_{i=1}^K \sigma^2 \quad \because E[\eta_i(x, y)^2] = \sigma^2 \\ &= \frac{\sigma^2}{K} \end{aligned}$$

q.e.d

Solution 5

Let $f(x, y, z) : \mathbb{R}^3 \rightarrow \mathbb{R}$. Let $[u \ v \ w]^T$ be the position of $[x \ y \ z]^T$ in a rotated coordinate frame. The relationship between $[u \ v \ w]^T$ and $[x \ y \ z]^T$ is given by :

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = R \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (2)$$

$$\text{where, } R = [R_1 \ R_2 \ R_3] \quad (3)$$

$$= \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \quad (4)$$

$$\text{Known properties of } R, |R_1| = 1 \quad (5)$$

$$|R_2| = 1 \quad (6)$$

$$R_1^T R_2 = 0 \quad (7)$$

$$R_3 = R_1 \times R_2 \quad (8)$$

$$\implies |R_3| = 1 \quad (9)$$

$$R_1^T R_3 = R_2^T R_3 = 0 \quad (10)$$

$$R^{-1} = R^T \quad (11)$$

$$\text{We can also write, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \text{from (2), (9)} \quad (12)$$

$$\text{Now, } \nabla^2 f(u, v, w) = \frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} \quad (13)$$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial u} \quad (14)$$

$$= R_{11} \frac{\partial f}{\partial x} + R_{12} \frac{\partial f}{\partial y} + R_{13} \frac{\partial f}{\partial z} \quad \text{from (4), (10)} \quad (15)$$

$$\text{Similarly, } \frac{\partial f}{\partial v} = R_{21} \frac{\partial f}{\partial x} + R_{22} \frac{\partial f}{\partial y} + R_{23} \frac{\partial f}{\partial z} \quad (16)$$

$$\frac{\partial f}{\partial w} = R_{31} \frac{\partial f}{\partial x} + R_{32} \frac{\partial f}{\partial y} + R_{33} \frac{\partial f}{\partial z} \quad (17)$$

$$\begin{aligned}
\therefore \frac{\partial^2 f}{\partial^2 u} &= R_{11} \left(\frac{\partial^2 f}{\partial^2 x} \frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial y \partial x} \frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial z \partial x} \frac{\partial z}{\partial u} \right) \\
&+ R_{12} \left(\frac{\partial^2 f}{\partial x \partial y} \frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial^2 y} \frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial z \partial y} \frac{\partial z}{\partial u} \right) \\
&+ R_{13} \left(\frac{\partial^2 f}{\partial x \partial z} \frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial y \partial z} \frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial^2 z} \frac{\partial z}{\partial u} \right) \quad \text{from (13) and chain rule}
\end{aligned} \tag{18}$$

$$\begin{aligned}
&= R_{11}^2 \frac{\partial^2 f}{\partial^2 x} + R_{11} R_{12} \frac{\partial^2 f}{\partial y \partial x} + R_{11} R_{13} \frac{\partial^2 f}{\partial z \partial x} \\
&+ R_{12} R_{11} \frac{\partial^2 f}{\partial x \partial y} + R_{12}^2 \frac{\partial^2 f}{\partial^2 y} + R_{12} R_{13} \frac{\partial^2 f}{\partial z \partial y} \\
&+ R_{13} R_{11} \frac{\partial^2 f}{\partial x \partial z} + R_{13} R_{12} \frac{\partial^2 f}{\partial y \partial z} + R_{13}^2 \frac{\partial^2 f}{\partial^2 z} \quad \text{from (4), (10)}
\end{aligned} \tag{19}$$

$$\begin{aligned}
\text{Similarly, } \frac{\partial^2 f}{\partial^2 v} &= R_{21}^2 \frac{\partial^2 f}{\partial^2 x} + R_{21} R_{22} \frac{\partial^2 f}{\partial y \partial x} + R_{21} R_{23} \frac{\partial^2 f}{\partial z \partial x} \\
&+ R_{22} R_{21} \frac{\partial^2 f}{\partial x \partial y} + R_{22}^2 \frac{\partial^2 f}{\partial^2 y} + R_{22} R_{23} \frac{\partial^2 f}{\partial z \partial y} \\
&+ R_{23} R_{21} \frac{\partial^2 f}{\partial x \partial z} + R_{23} R_{22} \frac{\partial^2 f}{\partial y \partial z} + R_{23}^2 \frac{\partial^2 f}{\partial^2 z}
\end{aligned} \tag{20}$$

$$\begin{aligned}
\frac{\partial^2 f}{\partial^2 w} &= R_{31}^2 \frac{\partial^2 f}{\partial^2 x} + R_{31} R_{32} \frac{\partial^2 f}{\partial y \partial x} + R_{31} R_{33} \frac{\partial^2 f}{\partial z \partial x} \\
&+ R_{32} R_{31} \frac{\partial^2 f}{\partial x \partial y} + R_{32}^2 \frac{\partial^2 f}{\partial^2 y} + R_{32} R_{33} \frac{\partial^2 f}{\partial z \partial y} \\
&+ R_{33} R_{31} \frac{\partial^2 f}{\partial x \partial z} + R_{33} R_{32} \frac{\partial^2 f}{\partial y \partial z} + R_{33}^2 \frac{\partial^2 f}{\partial^2 z}
\end{aligned} \tag{21}$$

On performing (17) + (18) + (19), we get using (5), (6), (7), (9) and (10)

$$\frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y} + \frac{\partial^2 f}{\partial^2 z} \tag{22}$$

$$\implies \nabla^2 f(u, v, w) = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y} + \frac{\partial^2 f}{\partial^2 z} \tag{23}$$

$$= \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y} + \frac{\partial^2 f}{\partial^2 z} \quad \text{from (22)} \tag{24}$$

$$= \nabla^2 f(x, y, z) \tag{25}$$

q.e.d

Solution 6