EE604A - Digital Image Processing Assignment

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Image Sources

Sunset (high.jpg) : https://www.flickr.com/photos/mediaflex/4190084346
Carving (low.jpg) : https://www.flickr.com/photos/30440933@N06/2847993403
Dog (small.jpg) : https://res.cloudinary.com/rover-com/image/upload/a_exif,c_fill,f_jpg,fl_progressive,g_face:center,h_100,q_80,w_100/remote/images/pets/4NpPzO8N/50e4a023d9/original.jpg

Solution 1

(a)

Code has been written in two MATLAB ® files

Solution 2

Solution 3

Solution 4

We are assuming that $\eta_i(x_1, y_1)$ *and* $\eta_j(x_2, y_2)$ are independent if i $\neq j$ *or* $x_1 \neq x_2$ *or* $y_1 \neq y_w$.

$$\therefore\ if\ i \neq j,\ E[\eta_i(x,y)\eta_j(x,y)] = 0 \tag{1}$$

$$
\begin{aligned}
We\ have,\ g_i(x,y) &= f_i(x,y) + \eta_i(x,y) \\
Also,\ f_i(x,y) &= f(x,y) \\
Now,\ \hat{g}(x,y) &= \frac{1}{K}\Sigma_{i=1}^{K} g_i(x,y) \\
&= \frac{1}{K} K f(x,y) + \frac{1}{K}\Sigma_{i=1}^{K}\eta_i(x,y) \\
&= f(x,y) + \frac{1}{K}\Sigma_{i=1}^{K}\eta_i(x,y) \\
Now,\ noise\ variance,\ E[(g(\hat{x,}y) - f(x,y))^2] &= E[(\frac{1}{K}\Sigma_{i=1}^{K}\eta_i(x,y))^2] \\
&= \frac{1}{K^2}\Sigma_{i=1}^{K}\Sigma_{j=1}^{K} E[\eta_i(x,y)\eta_j(x,y)] \\
&= \frac{1}{K^2}\Sigma_{i=1}^{K} E[\eta_i(x,y)^2] && from\ (1) \\
&= \frac{1}{K^2}\Sigma_{i=1}^{K}\sigma^2 && \because\ E[\eta_i(x,y)^2] = \sigma^2 \\
&= \frac{\sigma^2}{K} \\
q.e.d
\end{aligned}
$$

Solution 5

Let $f(x, y, z) : \Re^3 \rightarrow \Re$. Let $[u\ v\ w]^T$ be the position of $[x\ y\ z]^T$ in a rotated cooridnate frame. The relationship between $[u\ v\ w]^T$ and $[x\ y\ z]^T$ is given by :

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = R \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{2}$$

$$where,\ R = [R_1\ R_2\ R_3] \tag{3}$$

$$= \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \tag{4}$$

$$Known\, properties\, of\ R,\ |R_1| = 1 \tag{5}$$

$$|R_2| = 1 \tag{6}$$

$$R_1^T R_2 = 0 \tag{7}$$

$$R_3 = R_1 \times R_2 \tag{8}$$

$$\implies\ |R_3| = 1 \tag{9}$$

$$R_1^T R_3 = R_2^T R_3 = 0 \tag{10}$$

$$R^{-1} = R^T \tag{11}$$

$$We\, can\, also\, write,\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad from\ (2),\ (9) \tag{12}$$

$$Now,\ \nabla^2 f(u, v, w) = \frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} \tag{13}$$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial u} \tag{14}$$

$$= R_{11}\frac{\partial f}{\partial x} + R_{12}\frac{\partial f}{\partial y} + R_{13}\frac{\partial f}{\partial z} \qquad from\ (4), (10) \tag{15}$$

$$Similarily,\ \frac{\partial f}{\partial v} = R_{21}\frac{\partial f}{\partial x} + R_{22}\frac{\partial f}{\partial y} + R_{23}\frac{\partial f}{\partial z} \tag{16}$$

$$\frac{\partial f}{\partial w} = R_{31}\frac{\partial f}{\partial x} + R_{32}\frac{\partial f}{\partial y} + R_{33}\frac{\partial f}{\partial z} \tag{17}$$

$$
\begin{aligned}
\therefore\ \frac{\partial^2 f}{\partial^2 u} &= R_{11}(\frac{\partial^2 f}{\partial^2 x}\frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial y \partial x}\frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial z \partial x}\frac{\partial z}{\partial u}) \\
&+ R_{12}(\frac{\partial^2 f}{\partial x \partial y}\frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial^2 y}\frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial z \partial x}\frac{\partial z}{\partial u}) \\
&+ R_{13}(\frac{\partial^2 f}{\partial x \partial z}\frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial y \partial z}\frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial^2 z}\frac{\partial z}{\partial u}) \qquad from\ (13)\ and\ chain\ rule \qquad (18) \\
&= R_{11}^2\frac{\partial^2 f}{\partial^2 x} + R_{11}R_{12}\frac{\partial^2 f}{\partial y \partial x} + R_{11}R_{13}\frac{\partial^2 f}{\partial z \partial x} \\
&+ R_{12}R_{11}\frac{\partial^2 f}{\partial x \partial y} + R_{12}^2\frac{\partial^2 f}{\partial^2 y} + R_{12}R_{13}\frac{\partial^2 f}{\partial z \partial y} \\
&+ R_{13}R_{11}\frac{\partial^2 f}{\partial x \partial z} + R_{13}R_{12}\frac{\partial^2 f}{\partial y \partial z} + R_{13}^2\frac{\partial^2 f}{\partial^2 z} \qquad from\ (4), (10) \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
Similarily,\ \frac{\partial^2 f}{\partial^2 v} &= R_{21}^2\frac{\partial^2 f}{\partial^2 x} + R_{21}R_{22}\frac{\partial^2 f}{\partial y \partial x} + R_{21}R_{23}\frac{\partial^2 f}{\partial z \partial x} \\
&+ R_{22}R_{21}\frac{\partial^2 f}{\partial x \partial y} + R_{22}^2\frac{\partial^2 f}{\partial^2 y} + R_{22}R_{23}\frac{\partial^2 f}{\partial z \partial y} \\
&+ R_{23}R_{21}\frac{\partial^2 f}{\partial x \partial z} + R_{23}R_{22}\frac{\partial^2 f}{\partial y \partial z} + R_{23}^2\frac{\partial^2 f}{\partial^2 z} \qquad (20) \\
\frac{\partial^2 f}{\partial^2 w} &= R_{31}^2\frac{\partial^2 f}{\partial^2 x} + R_{31}R_{32}\frac{\partial^2 f}{\partial y \partial x} + R_{31}R_{33}\frac{\partial^2 f}{\partial z \partial x} \\
&+ R_{32}R_{31}\frac{\partial^2 f}{\partial x \partial y} + R_{32}^2\frac{\partial^2 f}{\partial^2 y} + R_{32}R_{33}\frac{\partial^2 f}{\partial z \partial y} \\
&+ R_{33}R_{31}\frac{\partial^2 f}{\partial x \partial z} + R_{33}R_{32}\frac{\partial^2 f}{\partial y \partial z} + R_{33}^2\frac{\partial^2 f}{\partial^2 z} \qquad (21)
\end{aligned}
$$

On performing $(17) + (18) + (19)$, we get using (5), (6), (7), (9) and (10)

$$
\begin{aligned}
\frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} &= \frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} \qquad (22) \\
\implies \nabla^2 f(u, v, w) &= \frac{\partial^2 f}{\partial^2 u} + \frac{\partial^2 f}{\partial^2 v} + \frac{\partial^2 f}{\partial^2 w} \qquad (23) \\
&= \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y} + \frac{\partial^2 f}{\partial^2 z} \qquad from\ (22) \qquad (24) \\
&= \nabla^2 f(x, y, z) \qquad (25)
\end{aligned}
$$

$q.e.d$

Solution 6