Verifying Array Manipulating Programs by Tiling

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2nd SAT+SMT School
Infosys Campus, Mysore

Motivating Example

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++) {
    if(!(i==0 || i==N-1)) {
      if (A[i] < THRESH) {
        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = THRESH;
    }
  }
  assert(for i in 0..N-1, A[i]>=THRESH);
}
```

Motivating Example

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++) {
    if(!(i==0 || i==N-1)) {
      if (A[i] < 5) {
        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = 5;
    }
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  assert(for k in 0..N-1, A[k]>=5);
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```

Initial array

0	1	2	3	4	5	6	7	Loop Counter
0	1	2	3	4	5	6	7	Indices
5	9	7	1	9	2	8	1	Cell Contents

$\neg\forall k . a[k] \geq 5$

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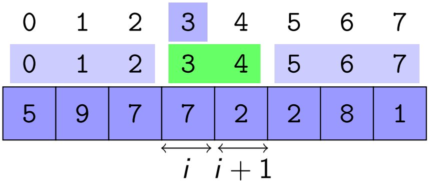

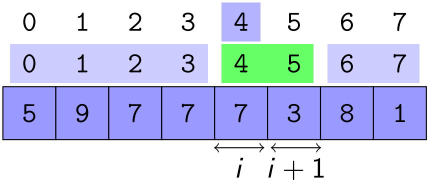

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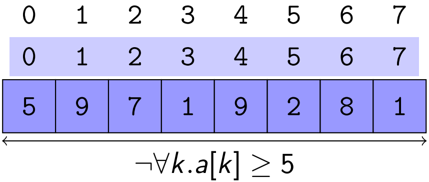

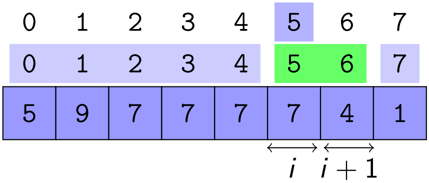

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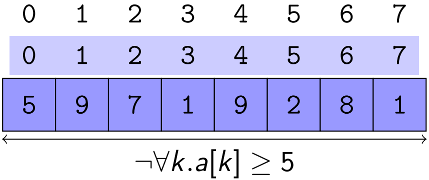

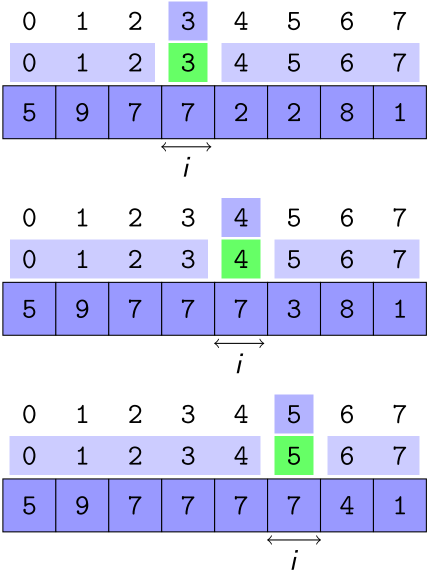

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$a[5] \not\geq 5$

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Finding the *right* tile is a challenge!

Proving Assertions using Tiles

If following conditions hold on the tile, we have proven the property

T1: Covers range

T2: Sliced post-condition holds inductively

T3: Non-interference across tiles

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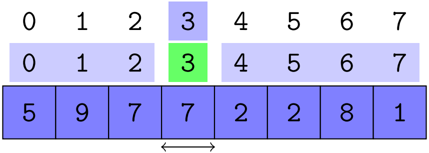

$$\mathsf{Tile}(i, j) := (j == i)$$

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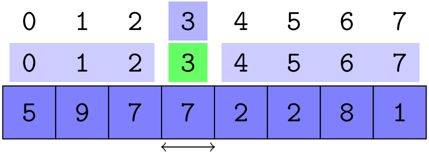

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- Validity of $\eta_1 \wedge \eta_2$ ensures T1
- Involves a quantifier alternation; can be handled by SMT solvers

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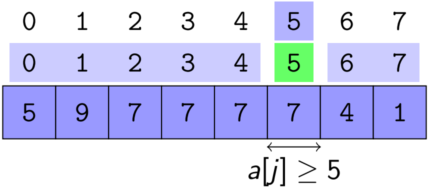

- Post $\triangleq \forall i (\Phi(i) \implies \Psi(\mathsf{A}, i))$
- Sliced post-condition for the i^{th} tile
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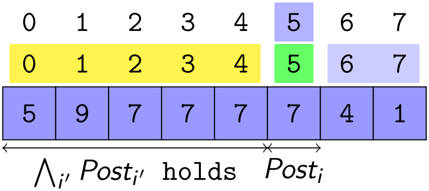

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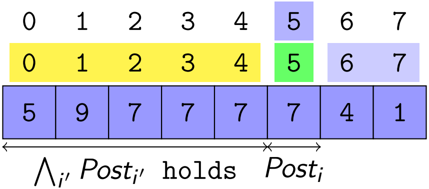

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- $\{\mathsf{Inv} \wedge \bigwedge_{i':0\leq i'<i} \mathsf{Post}_{i'}\}\ \mathrm{L_{body}}\ \{\mathsf{Inv} \wedge \mathsf{Post}_i\}$ must be valid

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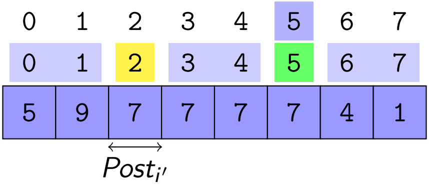

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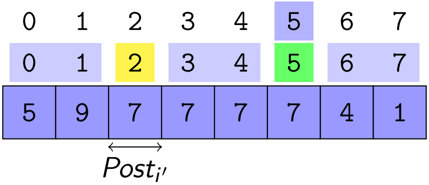

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Inductive Compositional Reasoning

- Inductive Reasoning
  - T2 Sliced post-condition holds for each iteration
- Compositional Reasoning
  - T3 Truth of sliced post-condition once established is not altered subsequently
  - T1 Tiles cover the entire range of array indices of interest

Sequentially Composed Loops

```
void copynswap(int N)
{
 int i, tmp;
 int a[], b[], acopy[];

 for (i = 0; i < N; i++) {
   acopy[i] = a[i];
 }

 for (i = 0; i < N; i++) {
   tmp = a[i];
   a[i] = b[i];
   b[i] = tmp;
 }

 for (i = 0; i < N; i++) {
   assert(b[i] == acopy[i]);
 }
}
```

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Mid-conditions

- Invariants between sequentially composed loops
- Hard to generate precise invariants
- Identify *candidate* mid-conditions using annotation assistants

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- $\forall i(a[i] = acopy[i])$
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Mid-conditions

- Invariants between sequentially composed loops
- Hard to generate precise invariants
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- *Prove* them using Tiling

Candidate mid-conditions

- $\forall i(a[i] = acopy[i])$
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Proved mid-conditions

- $\forall i(a[i] = acopy[i])$

Tiler Tool Diagram

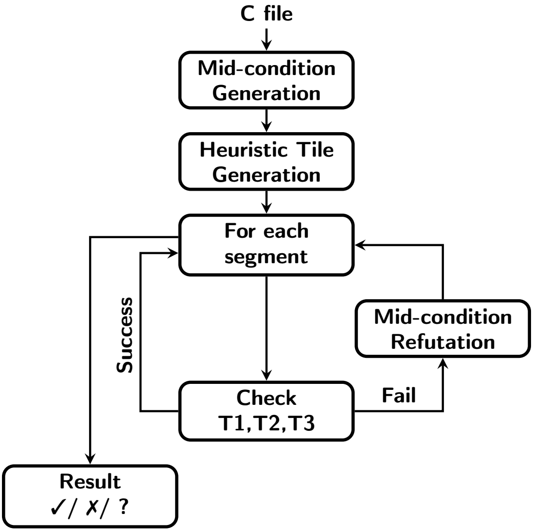

Figure : Tiler Tool Diagram

Tiler Benchmarking

- 60 benchmarks from industry and academia
- Performance compared with tools
  - SMACK+Corral - Bounded model checker
  - Booster - Acceleration based verification for arrays
  - Vaphor - Distinguished cell abstraction for arrays
- Memory limit - 1GB
- Time limit - 900s

Tiler in Action

Benchmark	#L	**Tiler**	S+C	Booster	Vaphor
cpynrev.c	2	✓3.8	†	✓3.1	✓5.4
cpynswp.c	2	✓4.2	†	✓12.4	✓1.38
cpynswp2.c	3	✓10.2	†	✓198	✓7.2*
maxinarr.c	1	✓0.51	†	✓0.01	✓0.11
mininarr.c	1	✓0.53	†	✓0.02	✓0.13
poly1.c	1	TO	†	✓15.7	TO
poly2.c	2	? 6.44	†	? 19.5	TO
tcpy.c	1	? 0.65	†	TO	✓25.1
rew.c	1	✓0.48	†	✓0.01	TO
skipped.c	1	✓1.24	†	TO	TO
rewrev.c	1	✓0.39	†	TO	TO
pr4.c	1	✓0.68	†	TO	TO
pr5.c	1	✓1.32	†	TO	TO
pnr4.c	1	✓0.86	†	TO	TO
pnr5.c	1	✓1.98	†	TO	TO
mbpr4.c	4	✓12.75	†	TO	TO
mbpr5.c	5	✓18.08	†	TO	TO
nr4.c	1-1	✓2.43*	†	TO	TO
nr5.c	1-1	✓2.90*	†	TO	TO
copy9u.c	9	✗0.16	✗4.48	✗0.44	✗30.8
skippedu.c	1	✗0.81	✗2.94	✗0.02	TO

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copy9u.c	9	✗0.16	✗4.48	✗0.44	✗30.8
skippedu.c	1	✗0.81	✗2.94	✗0.02	TO

Tiler in Action

Benchmark	#L	**Tiler**	S+C	Booster	Vaphor
cpynrev.c	2	✓3.8	†	✓3.1	✓5.4
cpynswp.c	2	✓4.2	†	✓12.4	✓1.38
cpynswp2.c	3	✓10.2	†	✓198	✓7.2*
maxinarr.c	1	✓0.51	†	✓0.01	✓0.11
mininarr.c	1	✓0.53	†	✓0.02	✓0.13
poly1.c	1	TO	†	✓15.7	TO
poly2.c	2	? 6.44	†	? 19.5	TO
tcpy.c	1	? 0.65	†	TO	✓25.1
rew.c	1	✓0.48	†	✓0.01	TO
skipped.c	1	✓1.24	†	TO	TO
rewrev.c	1	✓0.39	†	TO	TO
pr4.c	1	✓0.68	†	TO	TO
pr5.c	1	✓1.32	†	TO	TO
pnr4.c	1	✓0.86	†	TO	TO
pnr5.c	1	✓1.98	†	TO	TO
mbpr4.c	4	✓12.75	†	TO	TO
mbpr5.c	5	✓18.08	†	TO	TO
nr4.c	1-1	✓2.43*	†	TO	TO
nr5.c	1-1	✓2.90*	†	TO	TO
copy9u.c	9	✗0.16	✗4.48	✗0.44	✗30.8
skippedu.c	1	✗0.81	✗2.94	✗0.02	TO

Tiles in Benchmarks

- **Reverse** the contents of the array
  - $\text{Tile}(i,j) := j == N - i - 1$
- A **bunch** of indices updated in a loop
  - $\text{Tile}(i,j) := 2 * i - 2 \leq j < 2 * i$
  - $\text{Tile}(i,j) := 3 * i - 3 \leq j < 3 * i$
  - $\text{Tile}(i,j) := 4 * i - 4 \leq j < 4 * i$
- **Adjacent** indices to the counter
  - $\text{Tile}(i,j) := j == i - 1$
  - $\text{Tile}(i,j) := j == i + 1$
- Most **common** tile in array processing loops
  - $\text{Tile}(i,j) := j == i$

Conclusion and Future Work

- Presented a novel verification technique that
  - proves universally quantified assertions over arrays
  - decomposes reasoning about arrays using *tiles*
  - is property driven, compositional and efficient
- Future directions
  - Automated synthesis of *tiles*
  - Combining the strengths of Booster, Vaphor and Tiler
  - Integration of other candidate invariant generators like Houdini
  - Verify sorting by tiling

Thank you