

## Instructions

- The homework is due on **Friday 4/29 at 5pm ET.**
- No extension will be provided, unless for serious documented reasons.
- **Start early!**
- Study the material taught in class, and feel free to do so in small groups, but the solutions should be a product of your own work.
- This is not a multiple choice homework; reasoning, and mathematical proofs are required before giving your final answer.
- $tr$  denotes the trace of a matrix.

## 1 Derivatives [55 points]

Compute the derivative  $\frac{df}{dx}$  for the following functions. It will be helpful to identify  $n, m$  where  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ , and the dimensions of the derivative first.

- (a) [5pts]  $f(x) = \frac{1}{1+e^{-x}}, x \in \mathbb{R}$
- (b) [5 pts]  $f(x) = \exp\left(-\frac{1}{2\sigma^2}(x - \mu)^2\right), x \in \mathbb{R}$
- (c) [5 pts]  $f(x) = \sin(x_1) \cos(x_2), x \in \mathbb{R}^2$ .
- (d) [5 pts]  $f(x) = xx^T, x \in \mathbb{R}^n$ .
- (e) [5 pts]  $f(x) = \sin(\log(x^T x)), x \in \mathbb{R}^n$ .
- (f) [10 pts]  $f(z) = \log(1 + z)$  where  $z = x^T x, x \in \mathbb{R}^n$
- (g) [10 pts]  $f(x) = tr(xx^T + \sigma^2 I), x \in \mathbb{R}^n$ .
- (h) [10 pts]  $f(x) = x^T A x$  where  $x \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}$ .

## 2 Matrix derivative [25 points]

Let  $A \in \mathbb{R}^{D \times E}, X \in \mathbb{R}^{E \times F}, B \in \mathbb{R}^{F \times D}$ . Compute  $\frac{dg}{dX}$  where  $g(X) = tr(AXB)$ .

## 3 SVD Practice [20 points]

Find the SVD of the following matrices without the use of computing devices/software.

- (a) [5 points]  $A = [1, 1]$ .

(b) [5 points]  $B = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}.$

(c) [5 points]  $C = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}.$

(d) [5 points]  $D = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}.$