CIM Chip Concept

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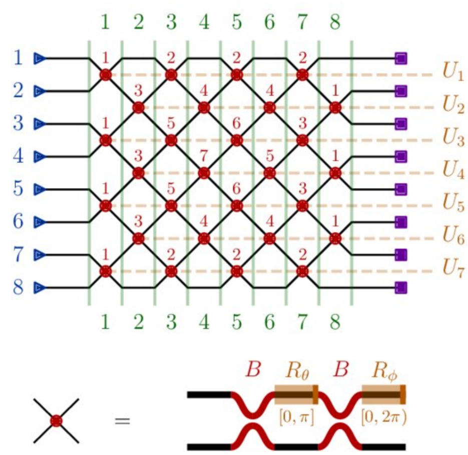
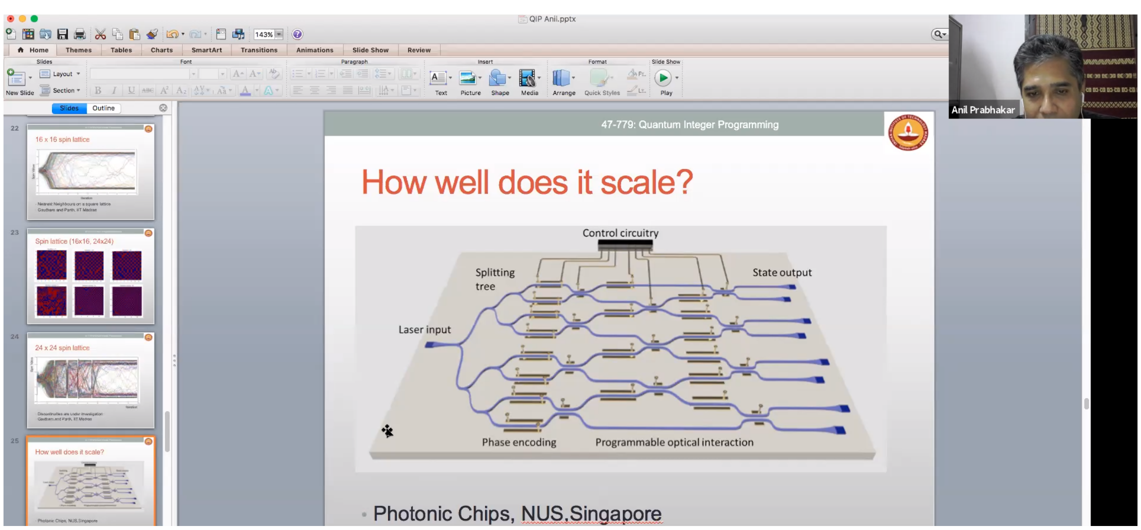
1 Background

On the side I was trying to work up on a chip scale implementation of the "Poor man's Ising Machine"[1]. There had already been some implementations of Optical Ising chip [2]-[5]. Also one of the crucial part of this is the matrix multiplication and almost all of the cited papers uses the Mach-Zehnder Interferometer(MZI) mesh for optical matrix multiplication.



MZI Grid for Matrix operation[8]

On one of your presentations last year at CMU for the Quantum Integer Program, you mentioned a Ising Photonic Chip design and from my guess the same MZI mesh is employed here for matrix multiplication here.



Design from your presentation

Given that the "Poor man's ising machine" is a time multiplexed system, a suitable approach for the matrix multiplication using photonics would be to use the approach given in [6], which I will explain in next section.

In "poor man's ising machine" the spin information is encoded in the intensity of the MZM since with proper added phase and offset the $cosine^2$ function bifurcates, but in this case the spin information will be encoded in the electric field amplitude of the MZM output since the output field amplitude of a dual drive MZM is a *cosine* function which also with proper phase bifurcates. More details on this in the next section.

2 Design

2.1 Matrix Multiplication Units

The matrix multiplication in this design will be the approach taken in [6]. A photonic component that can implements the Hadamard unitary is used here (basically a 2 port device that can implement the Hadamard unitary). Let x_{in}, y_{in} be the input amplitudes and x_{out},y_{out} be output amplitudes. So the matrix operation being done here will be:

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}(\frac{x+y}{\sqrt{2}})\\ (\frac{x-y}{\sqrt{2}})\end{bmatrix} \quad (1)$$

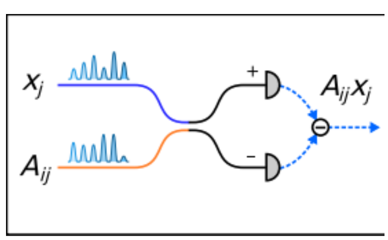
The output amplitudes will be sent to on-chip photo-detectors as show in the figure below an now the detected intensities would be proportional to $(\frac{x+y}{\sqrt{2}})(\frac{x+y}{\sqrt{2}})^*$ in the top detector and $(\frac{x-y}{\sqrt{2}})(\frac{x-y}{\sqrt{2}})^*$ in the bottom detector and then they are subtracted the result of which is $2 \times Re[xy^*]$. If x and y are real, then the value after subtraction will be 2xy.

If a sequence of modulated pulses are fed to the system, then the accumulated charges in the homodyne detectors will keep on accumulating until a readout is done which would lead it to discharge who's photo-current is proportional to dot product. The charge accumulated by the homodyne detector is given by:

$$Q_i = \frac{2\eta e}{\hbar\omega}\int \mathrm{Re}[E^{(\mathrm{in})}(t)^* E_i^{(\mathrm{wt})}(t)]dt \propto \sum_j A_{ij}x_j.$$

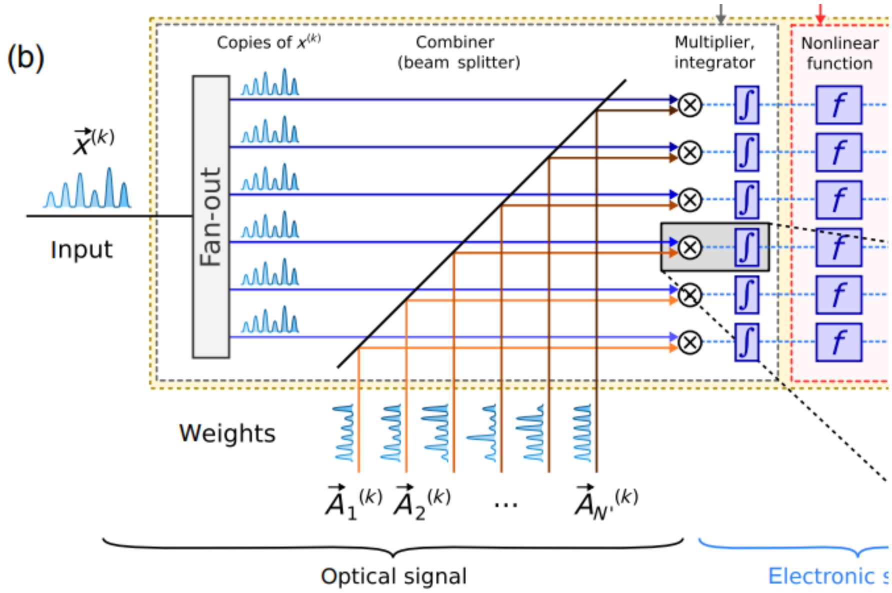
$E^{(in)}$ and $E^{(wt)}$ are the input fields, η is the detector efficiency.

More details on this is given in the section 1 of the supplementary [7] of [6]



A dot product from sequences of modulated pulses [6]

If we want to do a matrix-vector multiplication then either we can do a series of vector-vector dot products in a single unit or we can just use several units in parallel.



Multiple parallel units for matrix-vector multiplication[6]

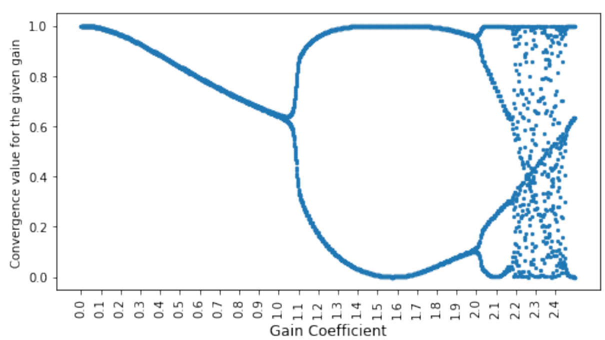
I find this method of opto-electronic matrix multiplication suitable for our time multiplexed system. In our case one of the input will be from the J matrix and the other input will be from the Ising spins.

2.2 Cosine Bifurcation

We know that in the "Poor man's Ising machine" the spin information are encoded in the intensity of the light coming out of the MZM who's output power is given by the equation below:

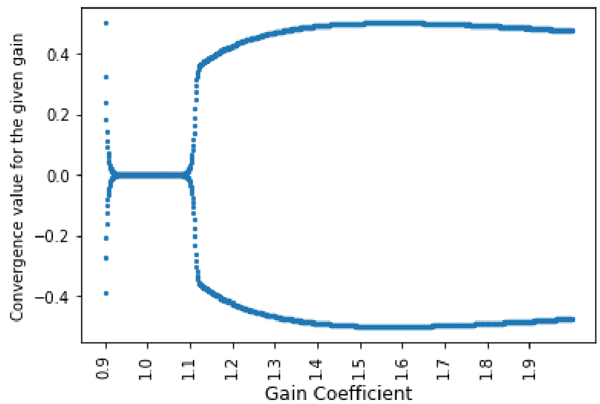
$$P_{out} = P_{in} \cos^2 \phi \tag{2}$$

And $cosine^2$ function bifurcates for $x_{t+1} = \cos^2(gain * x_t)$:



Bifurcation of $cosine^2$ function

With the appropriate phase and offset, we get a symmetric pitchfork bifurcation about x-axis, $x_{t+1} = \cos^2(gain * x_t - \frac{\pi}{4}) - 0.5$ which was used in the case of "Poor man's Ising machine" :



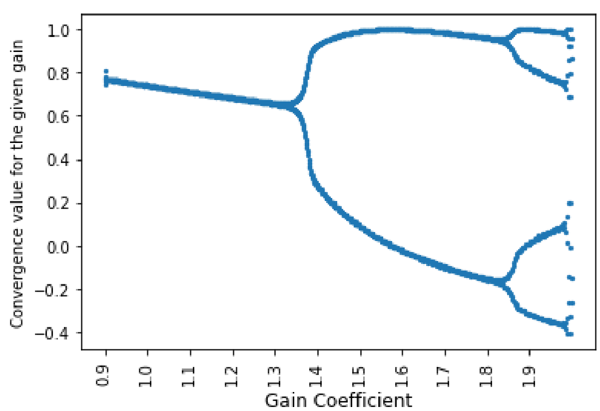
Symmetric bifurcation of $x_{t+1} = \cos^2(gain * x_t - \frac{\pi}{4}) - 0.5$, and the values converge near 0.5 and -0.5 for appropriate gain coefficients

But the matrix multiplication method discussed in the previous section only multiplies the amplitudes of the fields, and the output amplitudes of MZM is:

$$E_{out} = E_{in} \cos\phi \tag{3}$$

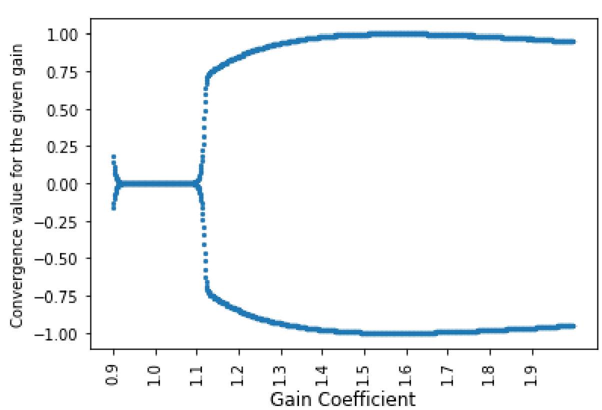
As you can see that this is a bit problematic here because we need a $cosine^2$ function for bifurcation. A trivial approach would be to measure the output power from the MZM with a detector and and use that signal to modulate the amplitude of another source which will then be fed in to the matrix multiplication unit which would be a tedious.

As it turns out that even *cosine* function bifurcates $x_{t+1} = \cos(gain * x_t)$:



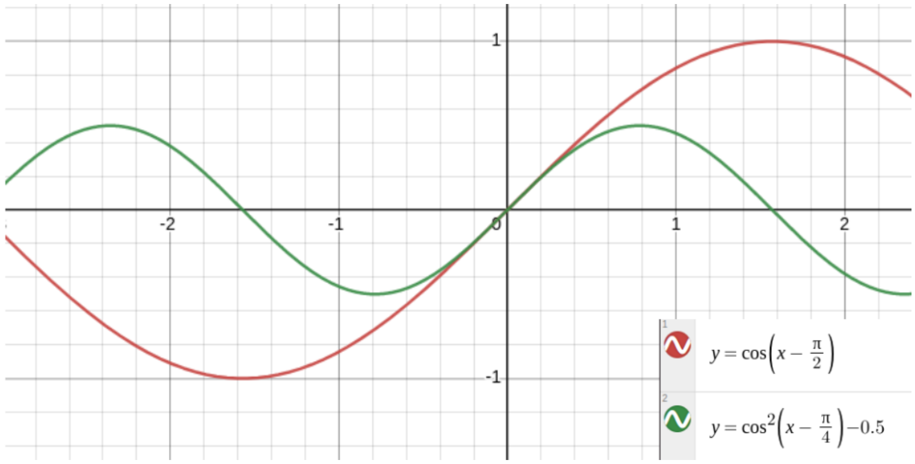
cosine bifurcation

With appropriate phase we see a symmetric pitchfork bifurcation, $x_{t+1} = \cos(gain * x_t - \frac{\pi}{2})$:



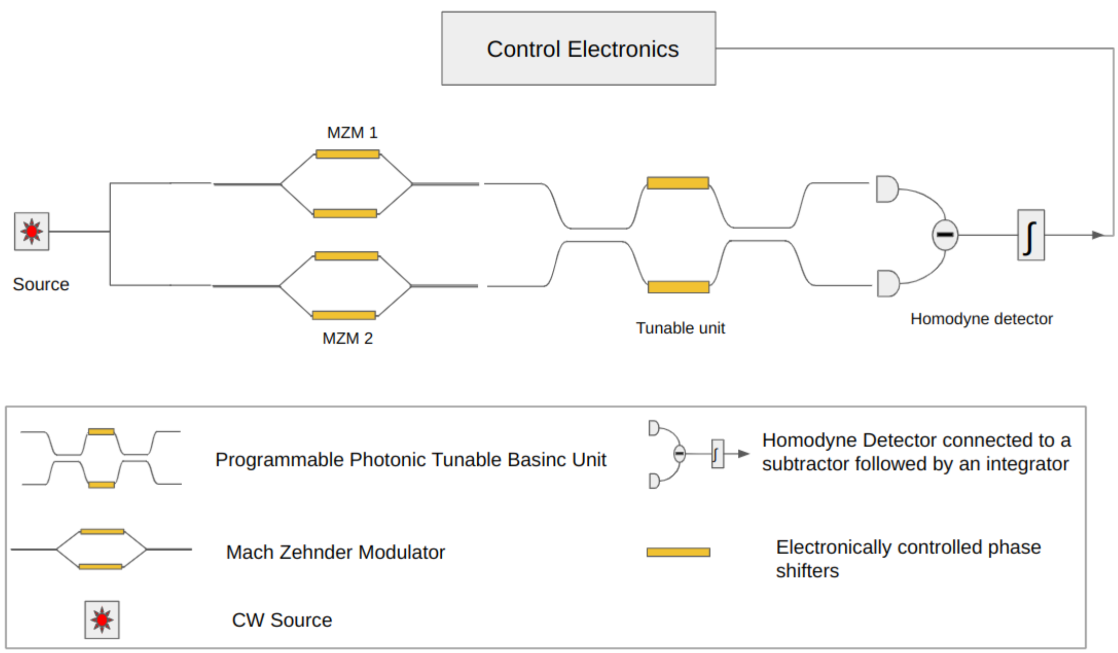
Symmetric bifurcation of $x_{t+1} = \cos(gain * x_t - \frac{\pi}{2})$, and the values converge near 1 and -1 for appropriate gain coefficients

We can also simply see the graph of $cos(gain * x_t - \frac{\pi}{2})$ and $\cos^2(gain * x_t - \frac{\pi}{4}) - 0.5$ to see that graphs are similar:



So now since the *cosine* function bifurcate as well we can use the amplitudes to encode our spin values instead of intensity, so that the matrix multiplication can be done using the circuit described in the previous section.

2.3 Circuit Model



Circuit Schematic I made

The figure above shows the circuit and the components of the Photonic Ising chip that I propose. There is a CW source which provides the input to the MZMs. The MZMs are controlled electronically with the control circuitry to modulate the light. The output amplitudes of MZM_1 represent the spins and the output amplitudes of MZM_2 represents the rows of the J matrix. An important ting to note here is that the output amplitudes of the MZMs will always be in range [-1,1] for an input amplitude of 1 and the elements of J will not always be in that range. So, for MZM_2 a multiplicative factor must be introduced to make the range bigger than the maximum value of elements of J and this will remain fixed throughout the computation and has to be done digitally, :$[-1, 1] \rightarrow [-a, a]$ where 'a' is the factor.
The output amplitudes of the MZMs are given as input to the programmable photonic unit what can implement any unitary matrix. In this case the phase shifters will be set such that the tunable photonic element implements the Hadamard unitary $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

The outputs from the tunable element are fed into the on-chip balanced homodyne detector which calculates the dot product. When we have all the dot products done (i.e the matrix multiplication split apart), we will update the spin values and multiply the updated spin values with the J matrix.

The MZM will be modulated at a frequency of 10 GHz, and the homodyne readouts will be done at a frequency of $(\frac{MZM modulation frequency}{Number of spins})$

2.4 Simulated Results

For now I have done the simulation using python assuming ideal photo-detectors. I am also making a simulation model using Lumerical INTERCONNECT to get a more realistic simulation using realistic device parameters(in progress).

```
class tunable_unit():
  '''

  ------\       /------ theta_1 ------\      /------
         \_____/                       \____/
          _____                         ____
         /     \                       /    \
  ------/       \------ theta_2 ------/      \------


  Arguements:
  the phases neede to get a Hadamard matrix
  the 2 input amplitudes

  return:
  the 2 output amplitudes

  2*self.V_pi+(self.V_pi/np.pi)*(theta1)

  '''
```

The python class for the tunable element

```
class MZI_dual_drive():

  '''
          /------ theta_1 ------\
   ______/                       \______
         \                       /
          \------ theta_2 ------/


  For getting real amplitudes, the top arm should be given +theta (V_theta)
  and the bottom arm should be given -theta (2*V_pi-V_theta)....
  the above calculation will be done in the electronic part


  Arguements:
  modulating frequency
  voltage for top shifter
  voltage for botton shifter
  loss in dB

  return:
  modulated amplitude , dtype=complex
  '''
```

The python class for the MZM

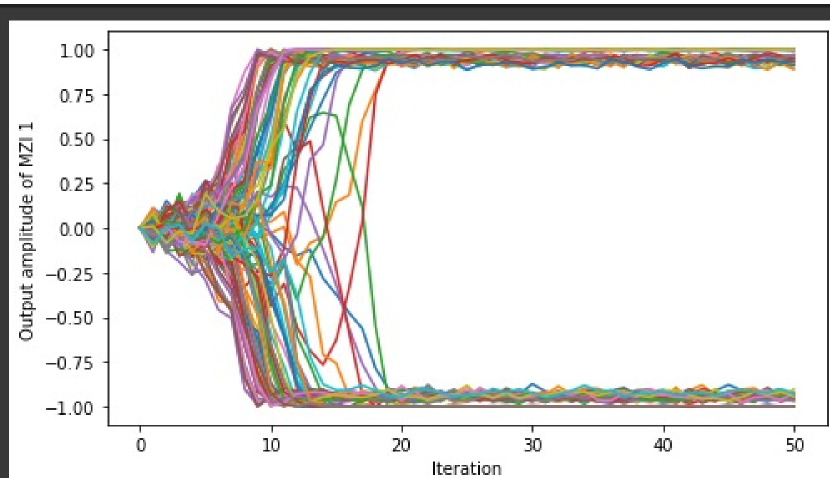
```python
class homodyne_detector_batch(): ## will return one x value
  '''

  D------|
         |
         |_________
          _________(-)--------[Integrator]--->
         |
         |
  D------|



  Arguements:
  the 2 input amplitudes

  Return:
  The product*2*factor

  '''
```

The python class for the homodyne detector unit

```python
class electronics_control():

  '''
  Arguements:
  Input from the homodyne detectors

  Stores the J matrix

  Return:
  eletrical signal to control the modulator:
  mzi_1_v1, mzi_1_v2,mzi_2_v1, mzi_2_v2

  vectors basically,
  '''
```

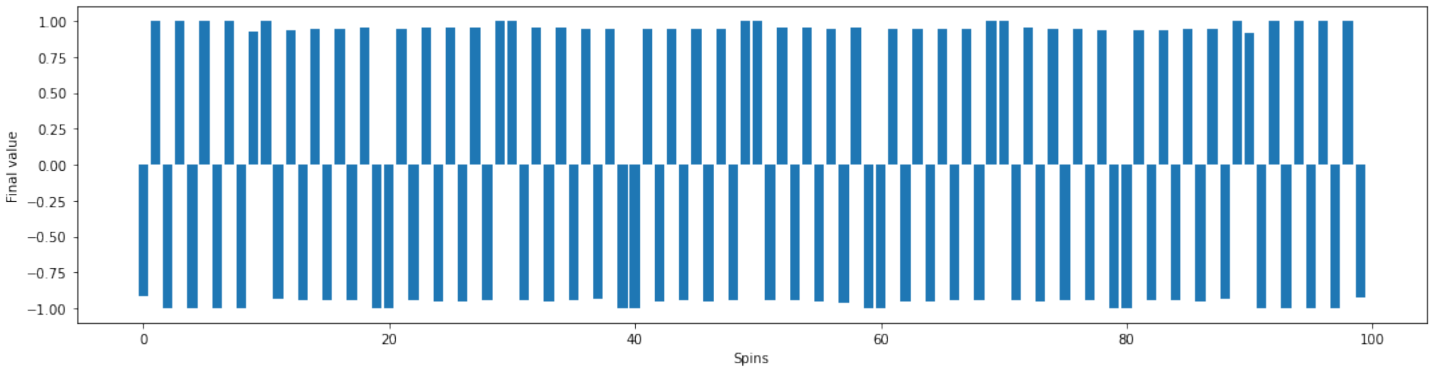
The python class for the electronic control

Here is a simulated result for a 100 spin square grid coupling:

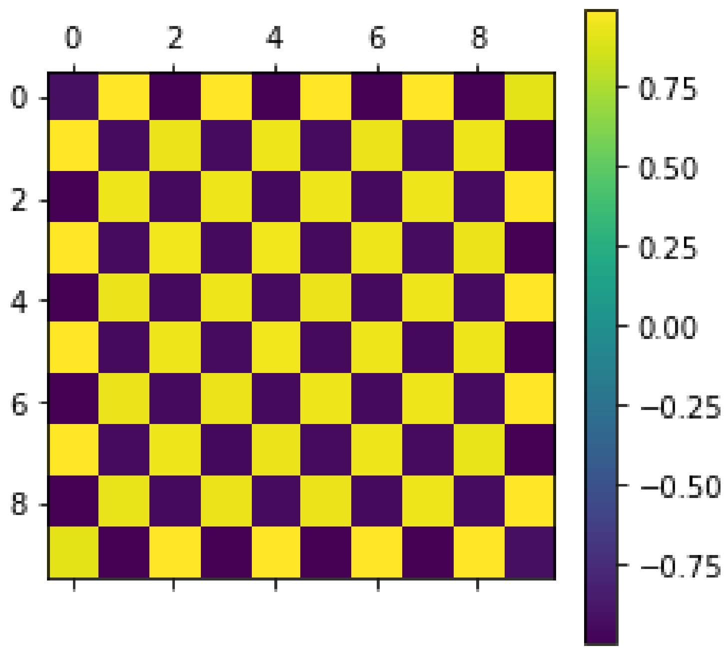



```
The final cut value is: 180.0
```

Maxcut result



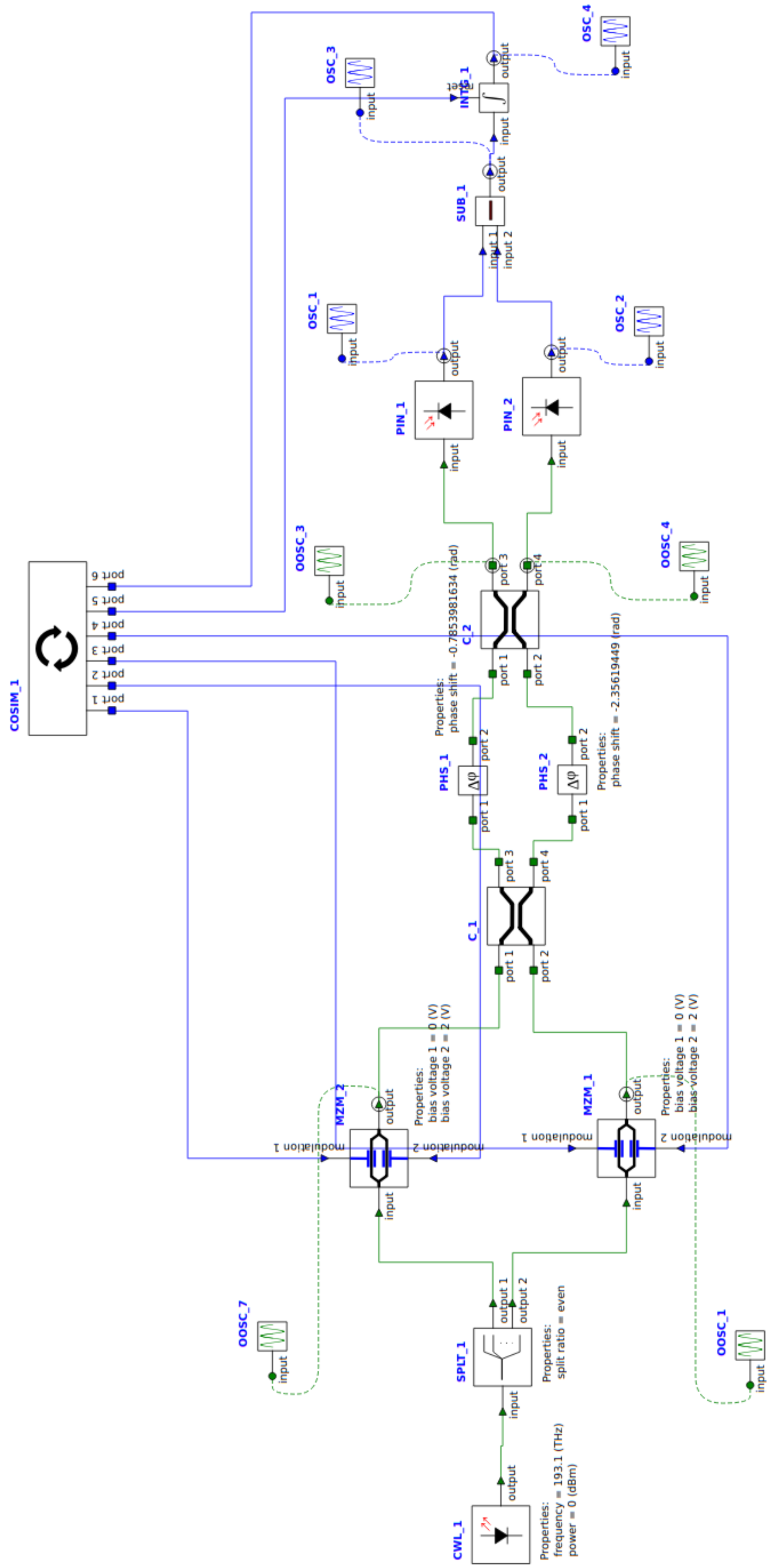
Final Spin status



Checkered board pattern of the final result

I have put up all the simulation files and codes in my github repository[9].

Below is the Lumerical Circuit for the chip design. There is a 'COSIM' module which acts like the control electronics for this photonic circuit. The MZM is modulated with a frequency of 10 GHz.(Will give update on this simulation soon)



Lumerical INTECONNECT circuit model

References

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[3] Roques-Carmes, C., Shen, Y., Zanoci, C. et al. Heuristic recurrent algorithms for photonic Ising machines. Nat Commun 11, 249 (2020). https://doi.org/10.1038/s41467-019-14096-z

[4] Mihika Prabhu, Charles Roques-Carmes, Yichen Shen, Nicholas Harris, Li Jing, Jacques Carolan, Ryan Hamerly, Tom Baehr-Jones, Michael Hochberg, Vladimir Čeperić, John D. Joannopoulos, Dirk R. Englund, and Marin Soljačić, "Accelerating recurrent Ising machines in photonic integrated circuits," Optica 7, 551-558 (2020)

[5] Tezak, Nikolas A., Thomas Van Vaerenbergh, Jason S. Pelc, Gabriel Joe Mendoza, David Kielpinski, Hideo Mabuchi and Raymond G. Beausoleil. "Integrated Coherent Ising Machines Based on Self-Phase Modulation in Microring Resonators." IEEE Journal of Selected Topics in Quantum Electronics 26 (2020): 1-15.

[6] R. Hamerly, L. Bernstein, A. Sludds, M. Soljacic and D. Englund, "Large-scale optical neural networks based on photoelectric multiplication", Phys. Rev. X, vol. 9, no. 2, May 2019.

[7] Large-Scale Optical Neural Networks Based on Photoelectric Multiplication-supplementary

[8] S. Pai, B. Bartlett, O. Solgaard, and D. A. B. Miller, Matrix Optimization on Universal Unitary Photonic Devices, Phys. Rev. Appl. 11, 064044 (2019).

[9] My Photonic Ising Github Repository