University of Tehran
ECE

Social Network

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Homework 3
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Submission Guidelines and Policies

Submit your final file as a single **ZIP** file that includes the report in **PDF** format and all code files, and upload it to the **eLearn platform**. The name of the ZIP file should follow the pattern: `SN_HW#_StudentNumber`

- If you have any questions, contact the assignment TAs via **email**. Please avoid sending private messages on social media so that responses can remain organized and efficient.
- The length of the report is not a grading factor. For implementation questions, focus on providing clear explanations; clarity matters far more than word count.
- The procedure for submitting assignments is explained in detail separately in this file and in the Git workshop video. Before submitting your code, ensure that you have watched the entire workshop video.
- Every submission must include both the report and the corresponding code. Any code submitted without a report will receive **zero points**.
- In your assignment report, you must describe how you used these tools. Include details such as the tools you used, their specific applications, and any other relevant information.
- At the end of your report, you must include the link to the prompts used: (`The ChatGPT conversation link`).
- Assignments may be uploaded to eLearn for up to 7 days after the official deadline, but a **5% penalty per day** will be deducted from the grade for each late day. After 7 days, submissions will not be accepted.
- To verify your understanding of each assignment, there will be a brief 5–10 minute in-person or virtual review session. You will be selected for this session **once during the semester**. If there are discrepancies between your submitted report and your presentation, the chance of being chosen again will increase.
- **Plagiarism is strictly prohibited.** Any similarity in the report or code that indicates copying or if cheating occurs during exams, will result in a score of **0.25 for all students involved for the course.**

Contents

Theoretical Questions

Question 1: Optimizing Structural Balance in a Competitive Network

Consider a competitive setting involving six political actors

$$A, B, C, D, E, F,$$

whose relationships are modeled as a signed complete graph K_6. Figure 1 depicts the network of relationships among these actors.
Although the overall structure of the network exhibits a tendency toward a bipartite arrangement, the presence of certain incompatible relationships prevents the network from achieving strong structural balance.

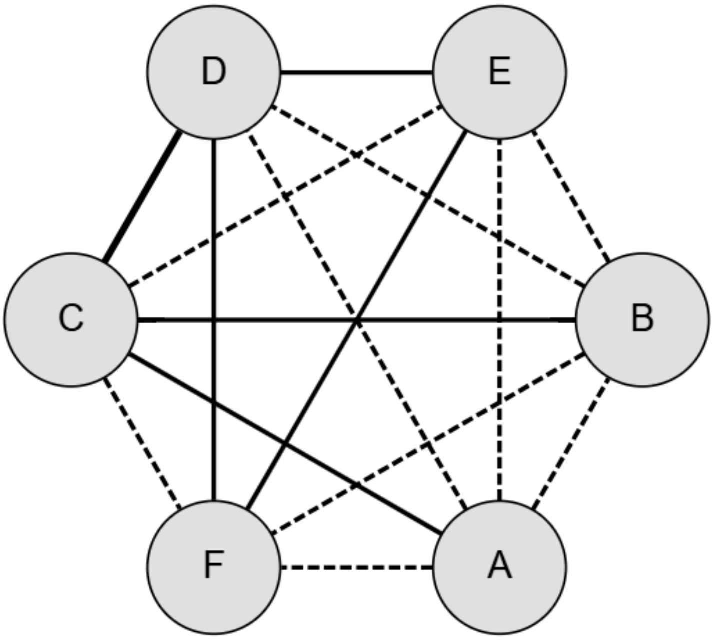

Figure 1: Signed complete graph K_6 representing relationships among six political actors. Solid edges indicate positive relations, while dashed edges indicate negative relations.

(a) Using a triadic (triangle-based) analysis, identify the structural patterns that are inconsistent with the definition of strong structural balance. In your answer, specify examples of problematic triangles and describe the sign configuration of their edges.

(b) The objective is to transform the network into a fully balanced two-faction structure, while minimizing the number of sign changes on edges.

  (i) Determine which edge or edges must change their sign in order for the network to achieve strong structural balance.

  (ii) After applying these changes, analyze whether the original bipartition of the nodes is preserved or whether a reorganization of the groups is required. If a reorganization occurs, specify the new partition and explain why it is structurally preferable.

Question 2: Graph Reduction in Large and Irregular Signed Networks

A signed graph $G = (V, E)$ with $|V| = 18$ nodes is shown in Figure 2. Solid edges represent positive relationships $(+)$, while dashed edges represent negative relationships $(-)$. Note that the graph is incomplete, and not all possible edges are present.

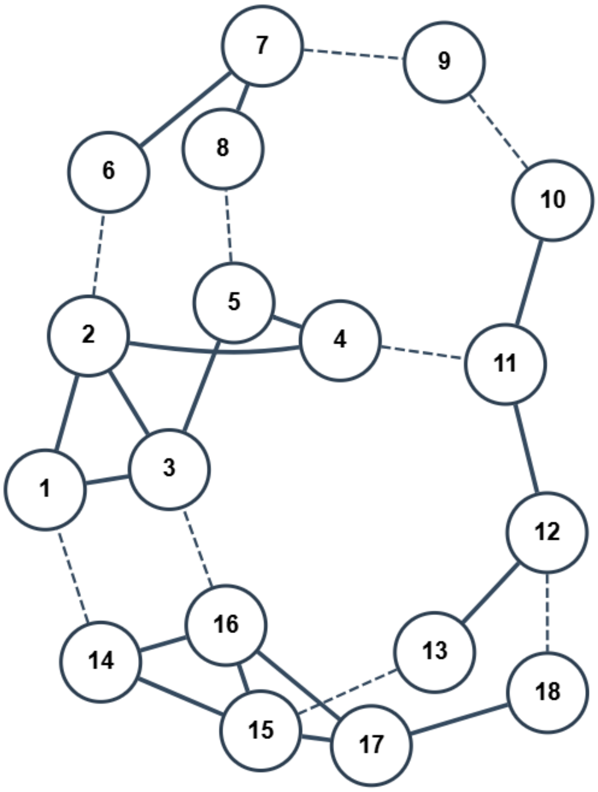

Figure 2: Signed network used in Question 2.

(a) Using the standard graph reduction algorithm based on positive connected components, construct the reduced graph G'. Clearly specify the vertices of G' (i.e., the supernodes), listing the original nodes contained in each supernode.

(b) By analyzing the structure of the reduced graph G', determine whether the original network G satisfies the property of strong structural balance. Provide your argument solely based on the examination of cycles present in the graph.

(c) The objective is to transform the reduced graph G' into a fully bipolar structure in accordance with the definition of strong structural balance, such that the supernodes are partitioned into exactly two opposing coalitions.
Determine the minimum number of edge sign changes (from negative to positive) required to achieve this configuration. Additionally, specify which edge or edges in the reduced graph G' must be modified in order to obtain the desired bipolar partition.

Question 3: Triadic Closure and Tie Stability in Social Networks

In a social network, the relationships among nodes are organized around a focal node A as follows:

- Node A has strong ties with nodes B and C.
- Node A has a tie of unspecified strength with node D.
- The edge (A, D) is a local bridge.
- There is no edge between nodes B and C.

(a) Analyze the structural configuration of node A with respect to the *Strong Triadic Closure (STC)* principle. Discuss under what changes to the network structure or to the strength of existing ties this configuration could become consistent with the STC principle.

(b) Assume that the network satisfies the Strong Triadic Closure principle. Using a logical argument and the definition of a local bridge, show that the edge (A, D) cannot be a strong tie.

Implementation Questions

Question 1: Balance, weak balance and unbalanced networks

1) **Sign prediction:**

   You are given a signed network that is known to be balanced (`Networks/Part_A/1/balance_graph.csv`). Some edge signs are known, others are unknown. Using the assumption of balance, infer the signs of the unknown edges.

2) **Balance test:**

   You are given several undirected signed networks (`Networks/Part_A/2`). Each edge is labeled either $+1$ (friendship) or -1 (enmity).

   i. Implement the super-node generation approach for testing structural balance:
      - Merge vertices connected by positive edges into super-nodes.
      - Construct the reduced graph of super-nodes.
      - Detect whether the reduced graph contains contradictions.

   ii. For each network, report whether it is balanced or unbalanced.

   iii. Output the super-node assignment for each vertex.

3) **Clusterability:**

   You are given several weakly balanced signed networks (`Networks/Part_A/3`). A weakly balanced network contains positive edges appear within clusters and negative edges appear between clusters.

   For each network:

   - Detect clusters and report the total number of the nodes in each cluster. You should explain the method you used here.
   - Visualize each network with cluster structure.

4) **Line Index:**

   You are given a signed network with 4 clusters (`Networks/Part_A/4/network_line_index.csv`). Node assignments are not provided.

   i. **Random clustering**
      - Assign each node randomly to one of the 4 clusters.
      - Compute the Line Index using the formula:

$$\text{Line Index} = \alpha \cdot P + (1 - \alpha) \cdot N$$

      Where:

$$P = \text{Number of the positive edges between clusters}$$

$$N = \text{Number of the negative edges within clusters}$$

$$\alpha = 0.5 \text{ is a given weighting factor}$$

ii. **Heuristic/Creative clustering**
   - Implement an algorithm to improve the clustering and reduce the Line Index.
   - You may move nodes between clusters, swap nodes, or design your own heuristic.
   - Compute the Line Index after applying your heuristic.

5) **Transitivity:**

   You are given a directed network (`Networks/Part_A/5/network_transivity.csv`). The triad involving nodes i, j and k is transitive iff: whenever $(i \to j)$ and $(j \to k)$, then $(i \to k)$.

   i. For every pair of edges $(i \to j)$ and $(j \to k)$, check whether $(i \to k)$ exists.
   ii. Count the number of transitive triples and missing edges.
   iii. Compute the transitivity ratio:
   $$\text{Transitivity ratio} = \frac{\text{(\# of transitive triples)}}{\text{(\# of all possible triples)}}$$
   iv. Add the minimum number of the edges needed to make the network transitive.
   v. Report the edges you added and the total number of the edges

Question 2: Evolution of a School Social Network

You are given a simulated social network of a school with 120 students. The network evolves over 4 time steps: Day 1, Day 30, Day 60, and Day 90.
The data for this question are located in:

`Networks/Part_B`

Your task is to analyze how connections and personal attributes evolve, and to use social network concepts — triadic closure, focal closure, and membership closure — to explain the changes in the network over time. The main goal of this question is to inspect and analyze how these closure mechanisms relate to **smoking behavior**.
This is an analytical question: you must interpret patterns in the network and justify your answers using the data. If the network is too large for detailed analysis, you may analyze a smaller subgraph, but you must clearly explain your choice.
Visualization of the network is **not required**, but clear and informative visualizations will receive **extra points**.

Data provided

1. **Connections data (CSV)**
   a) Example: `connections_day_1.csv`
   b) Each row: a connection between two students (`student_id_1, student_id_2`)
2. **Properties / Attributes data (CSV)**

You are provided these data frames for Day 1, Day 30, Day 60, and Day 90.

Feature	Description
id	Student identifier (0–119)
gender	Binary: 0 = Girl, 1 = Boy
age	14–18 years
studies	1–5, how much the student studies
plays_football	Binary: 0 = No, 1 = Yes
watches_movies	Binary: 0 = No, 1 = Yes
club	Binary: 0 = No, 1 = Yes (participates in extracurricular clubs)
smokes	Binary: 0 = No, 1 = Yes
class_number	1–4, each class has 30 students

Table 1: Description of student attributes in the school social network

Tasks

- Inspect and analyze triadic closure, focal closure, and membership closure over time, with a focus on smokers.
- Identify triadic closure events (friend of a friend becomes friend).
- Analyze membership closure patterns (connections within the same class).
- Analyze focal closure patterns related to smoking, and infer the reasons for the emergence of new smokers in terms of network closure mechanisms.
- Compare smokers and non-smokers:
  - Feature distributions.
  - Behavioral and attribute evolution over time.
- Compute degree centrality, identify the top 5 most central students at the end of evolution, and analyze their role in the network.

Scoring and Guidance

- Analytical insight and quality of interpretation.
- Sensitivity to subtle patterns, especially among smokers.
- Correct use of closure concepts.
- Completeness of analysis.
- Visualization quality (extra credit).

Note

- You must use the provided CSV data for all time steps.
- Focus on analytical understanding, not just computation.
- Clearly reference triadic, focal, and membership closure in your answers.