Walking on the Edge: Towards SAT Solving Methods in Graph Aggregation

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Abstract

Graphs are important data structures and can be used to represent relationships between elements in multiple application domains. In the field of computational social choice, graph aggregation involves computing a unified output graph that serves as a well-balanced compromise over multiple input graphs. An example of such task is in voting theory, in which each agent casts a graph of ordered preferences over multiple alternatives, and the aggregation rule determines a collective preference order that satisfies certain normative properties (axioms). Finding satisfactory aggregation rules is a central challenge in various applications of computational social choice. This paper introduces representations and methods with the aim of verifying Arrovian-style theorems for graph aggregation using SAT solving.

1 Introduction

Graph data structures are often used to represent relations between objects. They are particularity useful in capturing and illustrating complex relations in an interpretable manner, and are used in fields as broad as technology, mathematics, natural and social sciences, and fine arts [Zawiślak and Rysiński, 2022]. The task of computing collective output graph from multiple input graphs is known as *graph aggregation*. Graph aggregation is a complex operation with applications in multiple fields, such as voting theory, clustering data, and argumentation theory [Endriss and Grandi, 2017].

One relevant application of graph aggregation in computational social choice is to model preferences in the context of voting rules. For example, each agent encodes their preferences in an individual graph, such that each vertex represents an alternative that can be elected and a directed edge from vertex x to y means that alternative x is preferred over alternative y. Aggregating the individual graphs of all agents leads to a collective graph representing the socially preferred alternatives in order, for instance, to elect one or multiple winning alternatives.

SAT solving is a computational technique used to automatically find models satisfying a specific set of constraints. These are written with propositional logic formulas and encoded in conjunctive normal form (CNF), that is, a conjunction of disjunctions of propositions.

Our paper investigates the potential use of SAT solving algorithms, namely PySat's Glucose3 [Ignatiev *et al.*, 2018], in graph aggregation. We make the following contributions:

- We implement an extensive PySAT-based graph aggregation framework (code found at https://github.com/satchitchatterji/TowardsGraphAggregationSAT/), providing many graph properties and axioms that can be applied to various graph aggregation problems.
- We test our implementation by attempting a SAT-based proof for Arrow's theorem for graph aggregation, as defined by [Endriss and Grandi, 2017].
- We show that Arrow's theorem for graphs in the case of weak orders and strict linear orders are overspecified by [Endriss and Grandi, 2017] and provide alternative formulations.
- We implement a complete Glucose3-compatible "explanation" framework which can be used to display graph CNFs in an easily interpretable form.

2 Framework

We first introduce basic graph notations that will be used throughout definitions and logical formulas, and describe a number of useful properties that a graph may hold.

2.1 Graph Notation

We define a directed graph $G = \langle V, E \rangle$ as a finite set of m vertices $V = \{0, ..., m-1\}$ and a finite set of directed edges $E \subseteq V \times V$. An edge $e_{x,y} \in E$ from vertex x to vertex y, is denoted as xEy.

Let $N = \{0, ..., n-1\}$ be the set of n agents (or individuals). Each agent $i \in N$ submits a graph, represented as an edge set $E_i \in \mathcal{G}$. The set of all rational[1] $\mathcal{G} \in 2^{V \times V}$ graphs that the agent can submit is referred to as the graph domain. A profile $\boldsymbol{E}$ is defined as the collection of all individual graphs submitted such that $\boldsymbol{E} = (E_0, ...E_{n-1}) \in \mathcal{G}^n$. For each edge $e_{x,y}$, the set of individuals accepting $e_{x,y}$ under profile $\boldsymbol{E}$ is

[1] We call this set *rational* because all the graphs satisfy some properties with respect to collective rationality.

defined as $N_e^{\boldsymbol{E}} := \{i \in N \mid e_{x,y} \in E_i\}$. An aggregation function $F : \mathcal{G}^n \to \mathcal{G}$ maps any profile into a social collective graph $F(\boldsymbol{E})$.

2.2 Graph Properties

Rational graphs satisfy certain properties. Some first-order conditions for these properties are shown in Table 1, whose descriptions can be found in [Endriss and Grandi, 2017]. We translate these conditions into CNF (see Section 3.2) to select a subset of graphs that satisfies certain properties, which restricts the domain of individual graphs within a profile. This allows us to run SAT-solving for axioms on a small and relevant subset of graphs that fit a certain family profile, rather than considering all $2^{(m^2)}$ possible graphs, thereby reducing intractability.

Property	Definition
Reflexivity	$\forall x.xEx$
Irreflexivity	$\forall x.\neg(xEx)$
Symmetry	$\forall xy.(xEy \to yEx)$
Transitivity	$\forall xyz.(xEy \wedge yEz \to xEz)$
Completeness	$\forall xy.(x \neq y \to (xEy \vee yEx))$

Table 1: Graph properties defined in first-order logic used to restrict the domain of graphs.

2.3 Graph Families

We narrow our focus onto three families of graphs with different domains of application:

Graphs as weak orders

A weak order is an order relation on all the elements in a set, allowing an equivalence between two elements. Such graphs are *reflexive*, *transitive*, and *complete* [Endriss and Grandi, 2017]. They model the weak preference rankings in social choice theory. An example of graph is shown in Figure 1. For 3-vertex graph, this family contains 13 possible graphs.

Graphs as strict linear orders

A strict linear order is an order relation on all the elements in a set, forbidding an equivalence between two elements. They model strict linear preference rankings in social choice theory. Such graphs are *irreflexive*, *transitive*, and *complete* [Endriss and Grandi, 2017]. The first two properties enforce asymmetry upon the graph, preventing equivalences. An example of graph is shown in Figure 2. For 3-vertex graph, this family contains 6 possible graphs.

Graphs as equivalence relations

An equivalence relation is a relation where all the elements can be partitioned in some equivalence class, meaning that all the elements in one class are equivalent. All the elements must be in an equivalence class and the relation must be *reflexive*, *symmetric* and *transitive*. An example of graph is shown in Figure 3. For 3-vertex graph, this family contains 5 possible graphs.

This type of graph cannot represent preferences, because the equivalence classes are not ordered. However, they can represent clusters of classified data [Endriss and Grandi, 2017]. This shows that graph aggregation goes beyond preference aggregation.

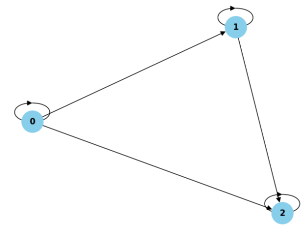

Figure 1: Example of a graph as a weak order.

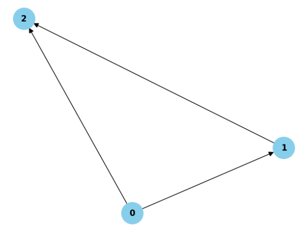

Figure 2: Example of a graph as a strict linear order.

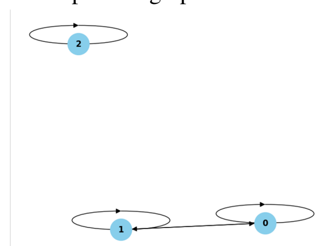

Figure 3: Example of a graph as an equivalence relation.

3 Propositional Logic Representations

SAT solving programs such as PySAT require meaningful representations of a problem in a propositional logical framework. In the following section, we discuss the representations implemented within our framework.

3.1 Literals

We introduce two types of propositional variables called literals that will be useful for our approach. The first is $e_{x,y}$, which indicates if there exists an edge between vertices x and y. This variable is useful for selecting a subset of graphs with relevant properties from the full graph domain. This allows us to consider only profiles that contain relevant individual graphs during SAT-solving, making the CNF shorter and easier to verify (see Section 3.2). We additionally use it to implement a rule called *collective rationality* (see Section 3.4).

Additionally, we introduce propositional variables of the form $p_{\boldsymbol{E},e_{x,y}}$, which is interpreted as *edge* $e_{x,y}$ *exists in the collective graph under profile* $\boldsymbol{E}$. These are used in the descriptions of the CNFs for axioms. These CNFs can be combined to prove theorem base-cases on subsets of graphs, allowing us to find profile-solution mappings that satisfy these axioms and correspond to satisfactory functions $F : \mathcal{G}^n \to \mathcal{G}$.

The remaining part of this section presents the CNFs of graph properties and aggregation rule axioms.

3.2 Graph Property CNFs

The first-order conditions on certain graph properties can be translated into CNF for generating a subset of relevant graphs as follows:

Reflexivity

$$\Gamma_{\text{ref}} := \bigwedge_{x\in V} e_{x,x}$$

Irreflexivity

$$\Gamma_{\text{irref}} := \bigwedge_{x\in V} \neg\left(e_{x,x}\right)$$

Symmetry

$$\Gamma_{\text{sym}} := \bigwedge_{x\in V} \left(\neg e_{x,y} \vee e_{y,x}\right)$$

Transitivity

$$\Gamma_{\text{trans}} := \bigwedge_{x\in V} \bigwedge_{y\in V} \bigwedge_{z\in V} \left(\neg e_{x,y} \vee \neg e_{y,z} \vee e_{x,z}\right)$$

Completeness

$$\Gamma_{\text{com}} := \bigwedge_{x\in V} \bigwedge_{y\in V|y\neq x} \left(e_{x,y} \vee e_{y,x}\right)$$

3.3 Graph Family CNFs

Often, we only wish to study subsets of graphs, for example, those that model weak preference orders. This has implications for the type of proofs we wish to examine (e.g. those with collective rationality, see Section 3.4). In these cases, we can use SAT solving to select graphs belonging to a relevant family. We defined the following CNFs on different graph families for the purpose of graph selection:

Weak Orders

$$\Gamma_{\text{weak}} := \Gamma_{\text{ref}} \wedge \Gamma_{\text{trans}} \wedge \Gamma_{\text{com}}$$

Strict Linear Orders

$$\Gamma_{\text{strict}} := \Gamma_{\text{irref}} \wedge \Gamma_{\text{trans}} \wedge \Gamma_{\text{com}}$$

Equivalence relations

$$\Gamma_{\text{equivalent}} := \Gamma_{\text{ref}} \wedge \Gamma_{\text{sym}} \wedge \Gamma_{\text{com}}$$

Selection process

We loop over all possible directed graphs $G \in 2^{V\times V}$, and represent them with a CNF sentence α_G consisting of $e_{x,y}$ or $\neg e_{x,y}$ clauses for all edges. We then combine this with the CNFs of the properties to form a propositional sentence φ_G. If φ_G is unsatisfiable, we discard the graph, else, we keep it. To find if a graph G satisfies the properties of a family CNF Γ_{fam}, we check if the following statement is satisfiable:

$$\varphi_G = \alpha_G \wedge \Gamma_{\text{fam}}$$

3.4 Aggregation Axiom CNFs

In social choice theory, an *axiom* is principle or rule that is used to evaluate normative properties of social choice or social welfare functions [Brandt *et al.*, 2016]. By extension, different aggregation rules may satisfy or violate certain axioms to varying degrees and may be considered better or worse depending on the desiderata of the system designer. Here, we discuss a few important axioms and discuss their CNF representations.

Non-dictatorship

An aggregation rule F is *dictatorial* if there exists a dictator $i \in N$ such that $e_{x,y} \in F(\boldsymbol{E}) \leftrightarrow e_{x,y} \in E_i$ for every edge $e_{x,y} \in E_i$. A rule F is thus *non-dictatorial* if a dictator does *not* exist. Non-dictatorship can be formulated into the CNF:

$$\gamma_{\text{nd}} := \bigwedge_{i\in N} \bigvee_{\boldsymbol{E}\in\mathcal{G}^n} \left(\left(\bigvee_{e_{x,y}\in E_i} \neg p_{\boldsymbol{E},e_{x,y}} \right) \vee \left(\bigvee_{e_{x,y}\in V\times V|e_{x,y}\notin E_i} p_{\boldsymbol{E},e_{x,y}} \right) \right)$$

Unanimity

An aggregation rule F is *unanimous* if whenever all individual graphs accept an edge $e_{x,y}$, the collective graph also accepts it. This is encoded into CNF form as:

$$\gamma_{\text{unan}} := \bigwedge_{\boldsymbol{E}\in\mathcal{G}^n} \left(\bigwedge_{e_{x,y}\in V\times V|\forall_{i\in N} e_{x,y}\in E_i} p_{\boldsymbol{E},e_{x,y}} \right)$$

Groundedness

An aggregation rule F is called *grounded* if for every edge in the collective graph, at least one agent accepts it. This axiom is encoded into CNF form as:

$$\gamma_{\text{ground}} := \bigwedge_{\boldsymbol{E}\in\mathcal{G}^n} \left(\bigwedge_{e_{x,y}\in V\times V|\forall_{i\in N} e_{x,y}\notin E_i} \neg p_{\boldsymbol{E},e_{x,y}} \right)$$

Independent of Irrelevant Edges

The outcome of the aggregation process should not be affected by the inclusion or exclusion of irrelevant individual edges. In other words, the independence condition requires that whether or not a given edge $e_{x,y}$ is accepted into the collective graph should only depend on how many graphs include $e_{x,y}$. Formally, an aggregation function F is called *independent of irrelevant edges* (IIE) if when $N^{\boldsymbol{E}}_{e_{x,y}} = N^{\boldsymbol{E}'}_{e_{x,y}}$ for some $e_{x,y} \in V \times V$ and $\boldsymbol{E}, \boldsymbol{E}' \in \mathcal{G}^n$, then F should either accept or reject $e_{x,y}$ in both corresponding collective graphs. This can be formulated into CNF form as:

$$\gamma_{\text{IIE}} := \bigwedge_{\boldsymbol{E}\in\mathcal{G}^n} \bigwedge_{\boldsymbol{E'}\in\mathcal{G}^n} \left(\bigwedge_{e_{x,y}\in V\times V|N^{\boldsymbol{E}}_{e_{x,y}}=N^{\boldsymbol{E'}}_{e_{x,y}}} \left(\neg p_{\boldsymbol{E},e_{x,y}} \vee p_{\boldsymbol{E'},e_{x,y}}\right) \right)$$

It is important to note that this axiom has the highest computational complexity out of all implemented axioms, meaning that the subset of graphs that that the agents can choose from is severely limited by available computational resources.

Collective Rationality
An aggregation rule F is *collectively rational* with respect to graph property Γ if $F(\boldsymbol{E})$ satisfies Γ whenever all of the individual graphs E_i in $\boldsymbol{E}$ do. This is expressed with the following CNF:

$$\gamma_{\mathrm{CR}(\Gamma)} := \bigwedge_{\boldsymbol{E} \in \mathcal{G}^n} \gamma_\Gamma(\boldsymbol{E})$$

Here, $\gamma_\Gamma(\boldsymbol{E})$ corresponds to a CNF written in terms of literals $p_{\boldsymbol{E},e_{x,y}}$, such that $p_{\boldsymbol{E},e_{x,y}}$ is true if and only if $e_{x,y}$ is true. The full CNF imposes the constraint that the output graph $F(\boldsymbol{E})$ must satisfy the property Γ for all possible $\boldsymbol{E}$.

Although this rationality requirement is not stated as an axiom in [Endriss and Grandi, 2017], it is useful to it as such in this application, and is combined with the axiom CNFs to find whether or not an aggregation rule can be found that satisfies certain CR constraints.

4 Impossibility Theorems

An important aspect of the axiomatic method in computational social choice is verifying whether or not sets of axioms can be consistently satisfied by a social choice or social welfare function. A famous example of the inconsistency of axioms in social choice is Arrow's theorem [Arrow, 1950]. Using SAT-solving, a common way of proving the non-coexistence of a set of axioms is to first prove a base-case (setting the number of agents and alternatives), and to inductively prove the universal case [Geist and Endriss, 2011].

4.1 Arrow's Theorem (Weak Orders)

Arrow's theorem was redefined in [Endriss and Grandi, 2017] within the domain of graph aggregation. The following section discusses this theorem in more detail within our proposed SAT-solving framework, and difficulties associated with it. First, we state Arrow's theorem for graphs:

Theorem 1 (Arrow's Theorem for Weak Orders)**.** *For graphs with $|V| \geq 3$, there exists no non-dictatorial, unanimous, grounded and IIE aggregation function that is collectively rational with respect to reflexivity, transitivity and completeness.*

This theorem would correspond to the following CNF, defined in terms of the axiom CNFs:

$$\gamma_{\mathrm{arrow}_1} := \gamma_{\mathrm{nd}} \wedge \gamma_{\mathrm{unan}} \wedge \gamma_{\mathrm{ground}} \wedge \gamma_{\mathrm{IIE}} \wedge \gamma_{\mathrm{CR}(\Gamma_{\mathrm{weak}})}$$

We now show that this definition of Arrow's theorem for graphs as **weak orders** is overspecified, as unanimity implies groundedness in this family of graphs.

Lemma 2. *In the aggregation of weak orders, groundedness and IIE implies unanimity.*

Proof. Assume aggregation rule F follows groundedness, IIE and CR with respect to reflexivity, transitivity and completeness. Assume a subset of profiles where everyone accepts $e_{x,y}$, such that $N^{\boldsymbol{E}}_{e_{x,y}} = N$ for all profiles in the subset. IIE would dictate that across all profiles, the collective graph must exclusively either include or exclude $e_{x,y}$. If $e_{x,y}$ loses, it must be the case due to completeness that $e_{y,x}$ wins across all these profiles. There is at least one profile where $e_{x,y}$ is accepted across all agents, and $e_{y,x}$ is not. However, as stated before, $e_{y,x}$ must win regardless – this breaks the axiom of groundedness in this case. Thus, $e_{x,y}$ must win instead. IIE then dictates that it must win in all profiles where the accepting coalition is the same, in this case, defined as where it is accepted by everyone. This means that $e_{x,y} \in F(\boldsymbol{E})$ when it is accepted by all voters in $\boldsymbol{E}$. Thus, F is unanimous. □

Therefore, we may reformulate Theorem 1 to only include groundedness and exclude unanimity. Thus we need to show the unsatisfiability of the following CNF to prove Arrow's theorem:

$$\gamma_{\mathrm{arrow}_1} \setminus \gamma_{\mathrm{unan}} = \gamma_{\mathrm{nd}} \wedge \gamma_{\mathrm{ground}} \wedge \gamma_{\mathrm{IIE}} \wedge \gamma_{\mathrm{CR}(\Gamma_{\mathrm{weak}})}$$

Base case
In our implementation, we work with a base case of 3 agents, each submitting preference orders as a 3-vertex graph. By automatically selecting graphs with certain properties from the total graph domain (Section 3.2), we found 13 graphs that are representative of weak preference orders.

Results
As expected, we achieve unsatisfiability on $\gamma_{\mathrm{arrow}_1}$. Table 2 shows the satisfiabilities of CNFs formed by excluding certain axioms from the full theorem. The following (minimal) set of axioms result in unsatisfiability, and as described previously, is expected to be unsatisfiable:

$$\gamma_{\mathrm{nd}} \wedge \gamma_{\mathrm{ground}} \wedge \gamma_{\mathrm{IIE}} \wedge \gamma_{\mathrm{CR}(\Gamma_{\mathrm{weak}})}$$

Removing any further axioms from this CNF results in satisfiability. Hence, the base case of Theorem 1 is proved. □

CNF	Satisfiability
$\gamma_{\mathrm{arrow}_1}$	Unsatisfiable
$\gamma_{\mathrm{arrow}_1} \setminus \gamma_{\mathrm{nd}}$	Satisfiable
$\gamma_{\mathrm{arrow}_1} \setminus \gamma_{\mathrm{ground}}$	Satisfiable
$\gamma_{\mathrm{arrow}_1} \setminus \gamma_{\mathrm{unan}}$	Unsatisfiable
$\gamma_{\mathrm{arrow}_1} \setminus \gamma_{\mathrm{IIE}}$	Satisfiable
$\gamma_{\mathrm{arrow}_1} \setminus \gamma_{\mathrm{CR}(\Gamma_{\mathrm{weak}})}$	Satisfiable

Table 2: Effects of removing axioms from $\gamma_{\mathrm{arrow}_1}$ for weak orders.

4.2 Arrow's Theorem (Strict Linear Orders)

We expect Arrow's Theorem would still hold for strict linear orders, which can be done if we enforce irreflexivity instead of reflexivity [Endriss and Grandi, 2017]. This leads to an amended theorem:

Theorem 3 (Arrow's Theorem for Strict Linear Orders)**.** *For graphs with $|V| \geq 3$, there exists no non-dictatorial, unanimous, grounded and IIE aggregation function that is collectively rational with respect to irreflexivity, transitivity and completeness.*

Theorem 3 is modelled using the following CNF:

$$\gamma_{\mathrm{arrow}_2} := \gamma_{\mathrm{nd}} \wedge \gamma_{\mathrm{unan}} \wedge \gamma_{\mathrm{ground}} \wedge \gamma_{\mathrm{IIE}} \wedge \gamma_{\mathrm{CR}(\Gamma_{\mathrm{strict}})}$$

However we note that in the case of strict linear orders, unanimity implies groundedness and vice versa and that Theorem 3 is overspecified. To prove this, we start with the following proposition:

Proposition 4. *Assume an aggregation function F that is CR with Γ_{strict}. In a strict linear order graph with the set of edges E, for any two vertices x and y,*

$$e_{x,y} \in E \rightarrow e_{y,x} \notin E \tag{1}$$
$$e_{x,y} \notin E \rightarrow e_{y,x} \in E \tag{2}$$

Proof. Irreflexivity and transitivity are properties within Γ_{strict}, and this implies asymmetry. Implication 1 follows from asymmetry of strict linear orders, and implication 2 follows from completeness. □

Lemma 5. *In the aggregation of strict linear orders, unanimity implies groundedness and vice versa.*

Proof. ($\rightarrow$) Assume aggregation rule F follows unanimity and is CR with respect to irreflexivity, transitivity and completeness. Assume for some profile of strict linear orders $\boldsymbol{E}$, $e_{x,y}$ is absent in all agents' graphs. Due to implication 2, $e_{y,x}$ must be present in all graphs. Since F is unanimous, $e_{y,x}$ also exists in the collective graph $F(\boldsymbol{E})$. Since the collective graph is also strict linear, implication 1 means $e_{x,y}$ is absent in $F(\boldsymbol{E})$. Thus, F satisfies groundedness. □

Proof. ($\leftarrow$) Assume F follows groundedness and is CR with respect to irreflexivity, transitivity and completeness. Assume for some profile of strict linear orders $\boldsymbol{E}$, $e_{x,y}$ is present for all agents. This implies, from implication 1, that $e_{y,x}$ is absent in all agents' graphs. Due to groundedness, $e_{y,x}$ must be absent in $F(\boldsymbol{E})$. Moreover, due to implication 2, $e_{x,y}$ must be present in $F(\boldsymbol{E})$. Thus, F satisfies unanimity. □

Proof. ($\leftrightarrow$) Follows from Proof ($\rightarrow$) and Proof ($\leftarrow$). □

Thus, we see that Theorem 3 is overspecified, and that it can be rewritten to exclusively include only unanimity or groundedness. Thus, what we need to show is the unsatisfiability of the one (or both) of following two CNFs:

$$\gamma_{\text{arrow}_2} \setminus \gamma_{\text{ground}} = \gamma_{\text{nd}} \wedge \gamma_{\text{unan}} \wedge \gamma_{\text{IIE}} \wedge \gamma_{\text{CR}(\Gamma_{\text{strict}})}$$
$$\gamma_{\text{arrow}_2} \setminus \gamma_{\text{unan}} = \gamma_{\text{nd}} \wedge \gamma_{\text{ground}} \wedge \gamma_{\text{IIE}} \wedge \gamma_{\text{CR}(\Gamma_{\text{strict}})}$$

Base case

Using the same base case as in Section 4.1 of 3 agents and 3 vertices, there are only 6 graphs that satisfy the above properties, reducing the number of profiles from $13^3 = 2,197$ (13 graphs of weak orders) to $6^3 = 216$ (6 graphs of strict linear order). This makes the overall CNF significantly more tractable than for weak orders.

Results

CNF	Satisfiability
γ_{arrow_2}	Unsatisfiable
$\gamma_{\text{arrow}_2} \setminus \gamma_{\text{nd}}$	Satisfiable
$\gamma_{\text{arrow}_2} \setminus \gamma_{\text{ground}}$	Unsatisfiable
$\gamma_{\text{arrow}_2} \setminus \gamma_{\text{unan}}$	Unsatisfiable
$\gamma_{\text{arrow}_2} \setminus \gamma_{\text{IIE}}$	Satisfiable
$\gamma_{\text{arrow}_2} \setminus \gamma_{\text{CR}(\Gamma_{\text{strict}})}$	Satisfiable

Table 3: Effects of removing specific axioms from γ_{arrow_2} for strict linear orders

Table 3 shows the satisfiability of γ_{arrow_2} as well as those CNFs formed by removing one axiom from it at a time. The unsatisfiability of these CNFs is expected and all smaller sets of axioms are satisfiable. Hence, the base case of Theorem 3 is proved. □

5 Discussion

We have provided ample framework to explore the use of SAT solvers in graph aggregation. Several useful tools have been provided in our code base to facilitate future graph aggregation projects that may utilize SAT-solving. Additionally, we proved the base cases for Theorems 1 and 3. We suggest a number of future directions of work in this area:

- Develop an inductive proof to universal agents and vertices taking advantage of the base case unsatisfiabilities of Theorem 1 and Theorem 3.
- Generalize Arrovian theorems onto other graph families such as equivalence relations. These are important for fields such as data clustering.
- Develop other types of axioms (e.g. liberalism) and impossibility theorems (e.g. Sen's theorem, Gilbert-Satterthwaite theorem) to extend into the graph aggregation domain, taking inspiration from preference aggregation as a starting point.
- If these methods are intended to be used in real-world scenarios such as traffic or social network modelling, more tractable representations and better SAT solving methods must be used. Our implementation also tests and provides translation mechanisms to CDCL and DPLL solvers, in addition to PySAT's Glucose implementation.

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