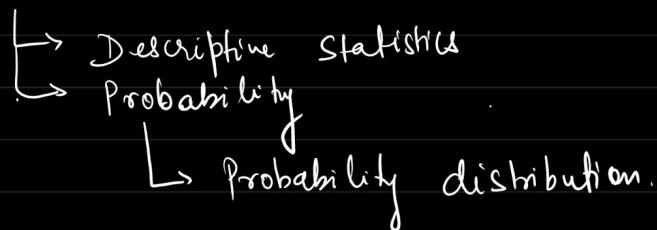


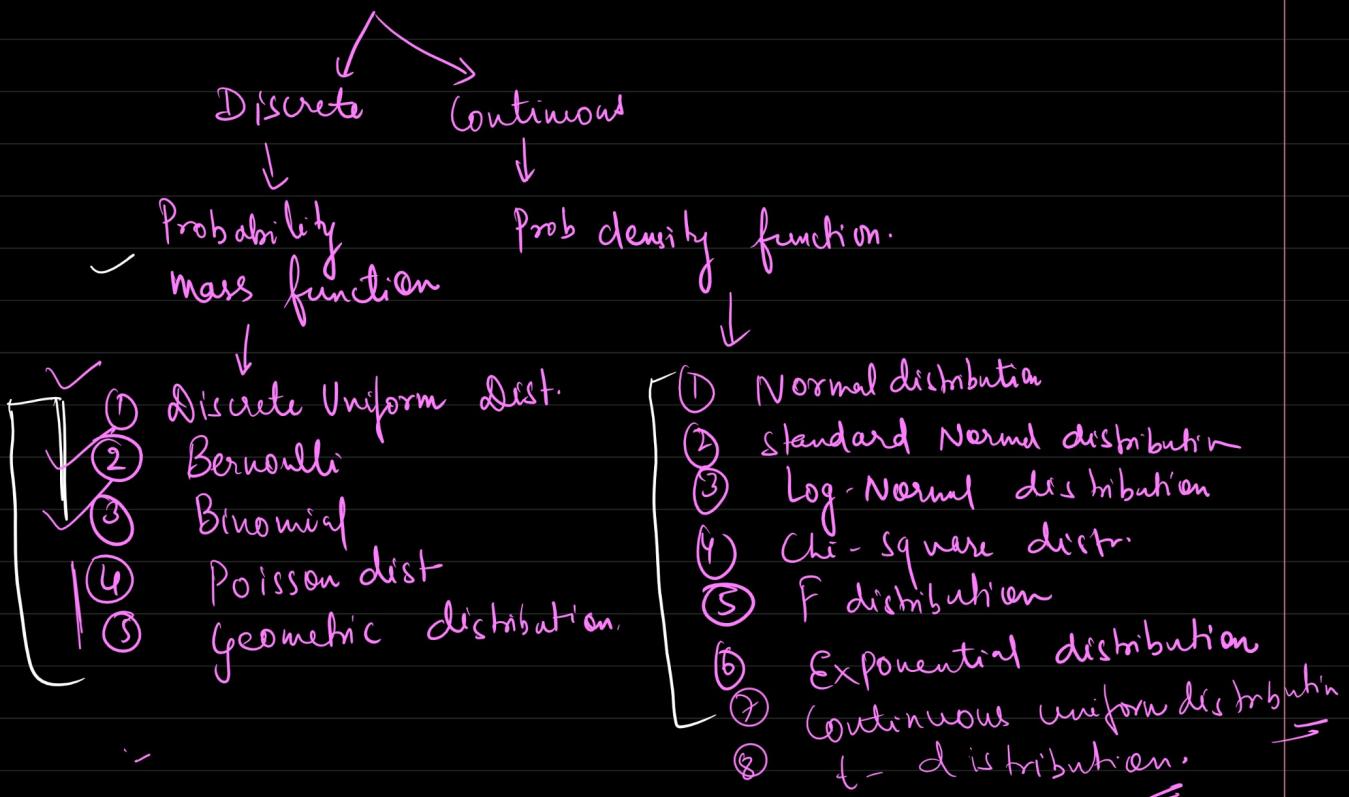
Till now



Agenda

→ probability distribution.

random variable



✓ Uniform < Discrete
Continuous.

① Discrete Uniform distribution ($U(a, b)$)

→ Discrete

→ A uniform distribution refers to a type of probability distribution in which outcomes are equally likely.

→ In discrete uniform dist. the outcomes are discrete in nature.

→ The outcomes are independent.

e.g. rolling a dice

[dice] $\rightarrow 1, 2, 3, 4, 5, 6$

e.g. tossing a coin

1st - $3 \rightarrow \frac{1}{6}$

e.g. picking up card from well shuffled deck of card.

2nd - $\rightarrow \frac{1}{6}$

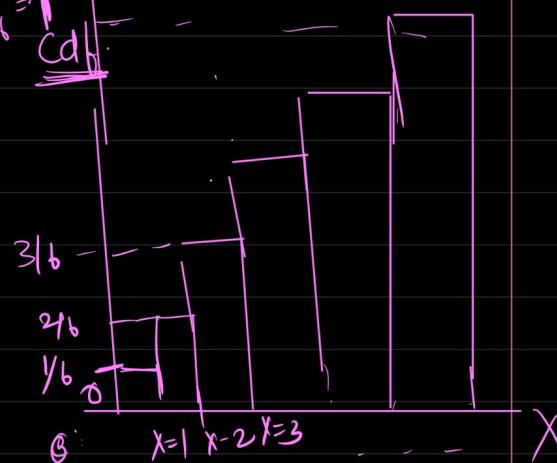
prob

$p(x)$



$$\rightarrow p_{mf} = \frac{1}{n}, n = b - a + 1 \\ = 6 - 1 + 1 \\ = 6.$$

$\frac{1}{6}$ ~~Card~~



throwing a dice $\rightarrow 1, 2, 3, 4, 5, 6$

mean of discrete Uniform distribution $= \frac{a+b}{2}$

Variance of " " " " $= \frac{n^2-1}{12}$

mean = sum of nos divided by total nos



Expected value \rightarrow In the long run avg values of repetitions of experiments.

For U.D $\rightarrow 1, 2, 3, 4, 5, 6$ (dice)

$$\text{Mean} = \frac{1+2+3+4+5+6}{6} = 3.5$$

$\sum_{i=1}^6 x_i \cdot p(x_i)$	x	$p(x)$	$x \cdot p(x)$
0.17	1	$\frac{1}{6}$	$1 \times \frac{1}{6} = 0.167 \times 1$
0.33	2	$\frac{1}{6}$	$2 \times \frac{1}{6} = 0.167 \times 2$
0.50	3	$\frac{1}{6}$	$3 \times \frac{1}{6} = 0.167 \times 3$
0.67	4	$\frac{1}{6}$	$4 \times \frac{1}{6} = 0.167 \times 4$
0.83	5	$\frac{1}{6}$	$5 \times \frac{1}{6} = 0.167 \times 5$
1	6	$\frac{1}{6}$	$6 \times \frac{1}{6} = 0.167 \times 6$

mean & EV

$$\sum_{i=1}^6 x_i \cdot p(x_i) \Rightarrow 3.5$$

$$E(x)/\mu = \sum_{i=1}^n x_i p(x_i)$$

$$\checkmark \text{Var}(x), \mu(a, b) = \frac{n^2 - 1}{12}$$

$$\boxed{\text{Var}(x) = E(x^2) - (E(x))^2}$$

$$\underline{E(x)} \rightarrow \sum_{i=1}^n x_i p(x)$$

$$= \sum_{i=1}^n x_i \cdot \frac{1}{N} = \frac{1}{N} \left(\underbrace{1+2+3+\dots+N}_{\text{Sum of } N \text{ natural no.}} \right)$$

$$= \frac{1}{N} \left(\cancel{N} \frac{(N+1)}{2} \right) = \frac{N+1}{2}$$

$$\begin{aligned} E(x^2) &= \sum_{i=1}^n x_i^2 \cdot p(x) \\ &= \sum_{i=1}^n x_i^2 \cdot \frac{1}{N} \Rightarrow \frac{1}{N} \left(1^2 + 2^2 + 3^2 + \dots + N^2 \right) \\ &= \frac{1}{N} \left(\cancel{N} \frac{(N+1)(2N+1)}{6} \right) \\ &= \frac{(N+1)(2N+1)}{6} \end{aligned}$$

$$\begin{aligned} \text{Var}(x) &= E(x^2) - (E(x))^2 \\ &= \frac{(N+1)(2N+1)}{6} - \left(\frac{N+1}{2} \right)^2 \\ &= \frac{2N^2 + 3N + 1}{6} - \frac{N^2 + 2N + 1}{4} \end{aligned}$$

$$\text{Var}(x) \hat{=} \frac{N^2 - 1}{12}$$

$$\sigma = \sqrt{\frac{N^2 - 1}{12}}$$

$$\Rightarrow \text{Var}(x) = \frac{N^2 - 1}{12} = \frac{6^2 - 1}{12} = \frac{36 - 1}{12} \Rightarrow \frac{35}{12} = \frac{3}{2}$$

pre-requisite

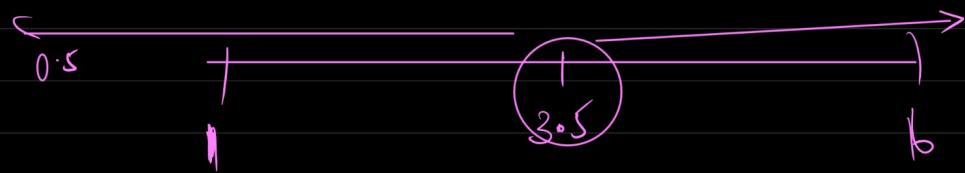
$$\text{Sum of first } n \text{ natural no.} = \frac{N(N+1)}{2}$$

$$\begin{aligned} 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \\ = 55 \leftarrow \frac{10(10+1)}{2} = 10 \times 11 \end{aligned}$$

$$\begin{aligned} * \text{ Sum of first } N \text{ square no.} \\ = \sum x^2 \end{aligned}$$

$$\begin{aligned} &= \frac{1^2 + 2^2 + 3^2}{6} \\ &= \frac{N(N+1)(2N+1)}{6} \end{aligned}$$

No derive
 Var



* Continuous Uniform distribution

* That has infinite no of values in a specified range.

→ r.v is continuous.

→ rectangular distⁿ.

- A perfect random no generator.
- Prob of guessing exact time at any moment
- Waiting time at a bus stop.
- Temp variation in a day.

Notation :- $U(a, b)$

Parameter $- \infty < a < b < \infty$



Area under curve

→ Probability value = 1

Total area in qdf gives you the prob.

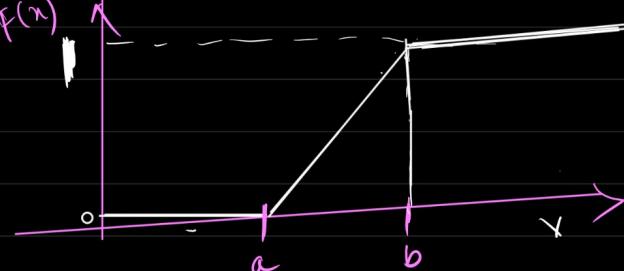
Area of rectangle = $b - a \times f(x)$

$$1 = b - a \times f(x)$$

$$f(x) = \frac{1}{b-a}$$

Cdf

$\rightarrow F(x)$



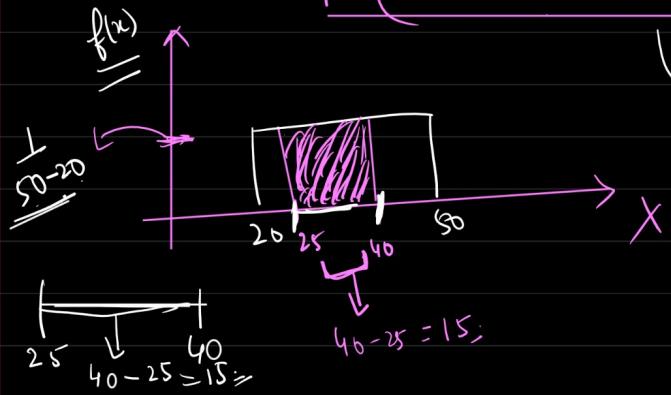
$$\underline{\text{Cdf}} = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & x \in (a, b) \\ 1 & x > b \end{cases}$$

$$\checkmark \text{ The mean of Cont Uniform dist} = \frac{1}{2}(a+b)$$

$\therefore \quad \quad \quad \therefore \quad \quad \quad \therefore = \frac{1}{2}(b-a)$

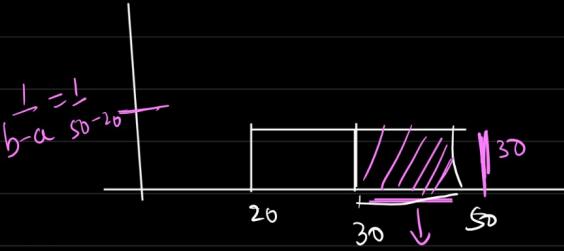
Q. The number of items sold at a shop daily is uniformly distributed with max and min item sold $50 \geq 20$ respectively. What is prob of daily sales to fall between $25 \leq 40$?

$$\rightarrow P(25 \leq x \leq 40) = ??$$



$$\begin{aligned} \frac{1}{b-a} & \times 15 \\ = \frac{1}{30} \times 15 & = 0.5 \end{aligned}$$

$$P(X \geq 30)$$



$$20 \times \frac{1}{30} = 0.66$$

Q. The amount of time for pizza delivery is uniformly distributed b/w 15 \leq 60 mins - what is standard deviation of the amount of time it takes for pizza to be delivered?

$$\rightarrow \text{Var}(x) = \frac{1}{12} (b-a)^2$$

$$= \frac{1}{12} (60-15)^2 = \frac{45^2}{12} = 168.7 \text{ cm.}$$

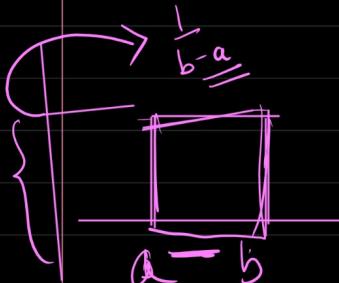


$$\sigma = \sqrt{\text{Var}} = \sqrt{168.75} \approx 13 \text{ min}$$

* for a continuous random var prob density fn ($f(x)$)

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

$\leq x \cdot p(x)$
↓
discrete



$$= \int_a^b x f(x) dx = \int_a^b x \cdot \frac{1}{b-a} dx = \frac{1}{b-a} \int_a^b x dx.$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b$$

$$= \frac{1}{2(b-a)} [x^2]_a^b$$

$$= \frac{1}{2(b-a)} (b^2 - a^2)$$

$$= \frac{1}{2(b-a)} (b+a)(b-a) \frac{b^2 - a^2}{(b-a)} = \frac{a+b}{2}$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_a^b x^2 f(x) dx = \int_a^b x^2 \cdot \frac{1}{b-a} dx$$

$$= \frac{1}{b-a} \int_a^b x^2 dx = \frac{1}{b-a} \left[\frac{x^3}{3} \right]_a^b$$

$$= \frac{1}{3(b-a)} [b^3 - a^3]$$

$$= \frac{1}{3(b-a)} (b^3 - a^3)$$

$$b^3 - a^3 = (b-a)(b^2 + ab + a^2)$$

$$= \frac{1}{3(b-a)} (b-a)(b^2 + ab + a^2)$$

$$= \frac{b^2 + ab + a^2}{3}$$



$$E(X^2) \rightarrow (\underbrace{E(x)}_3)^2$$

$$\left(\frac{b^2 + ab + a^2}{3} \right) - \left(\frac{a+b}{2} \right)^2$$

$$= \overbrace{b^2 + ab + a^2}^3 - \frac{a^2 + b^2 + 2ab}{4}$$

$$\frac{4b^2 + 4ab + 4a^2 - 3a^2 - 6ab - 3b^2}{12} = \frac{b^2 - 2ab + a^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

distribution
 ↘
 pmf pdf
 → discrete → continuous
 $\stackrel{\text{UD}}{=} \quad \stackrel{\text{VD}}{=}$

two possible outcomes \rightarrow binary

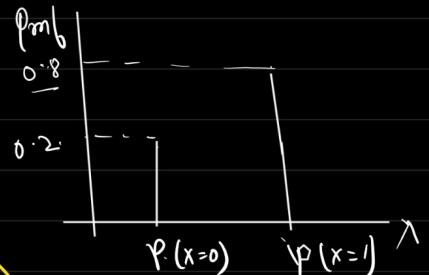
② Bernoulli distribution:

A discrete prob distribution of a random variable which takes only two possible outcomes typically labelled as success (1) and failure (0).

* To model a problem statement with only two possible outcomes

e.g. To toss a coin. (Head or tail)

$$\begin{cases} P(X=0) = 0.5 = p \\ P(X=1) = 1 - 0.5 = q \end{cases}$$



$$pmf \Rightarrow P(X=k) = \begin{cases} p & \text{if } k=1 \\ 1-p & \text{if } k=0 \end{cases}$$

$$\Rightarrow P(X=k) = p^k (1-p)^{1-k}$$

① if $k=1$
 $p^1 (1-p)^{1-1}$
 $p^1 (1-p)^0$
 p

* Conditions of Bernoulli distⁿ

$$\begin{aligned} \textcircled{2} \text{ if } k = 0 \\ p^0 (1-p)^{1-0} \\ = 1-p \end{aligned}$$

(1) No of trial should be finite.

(2) Each trial should be independent

(3) only two possible outcome

(4) Prob of each output should be same in every trial.

Examples — tossing a coin

→ prob of getting 3 and not 3 while throwing a dice.

→ Pass/Fail, Fraud/not F

Classification

diabetes | not diabetes

transaction amt	cont of trans	loans	salary	Fraud / not fraud
1500	2	5	3	1
—	—	—	—	0
—	—	—	—	0
..				
0				1
0				1
0				0
0				0
1				1

Bernoulli dist
↓

# studies	marks in internal exam	# test paper sale	pass/fail.
=	=	=	=

* Mean and Variance of Bernoulli distribution

$$\frac{1}{p}$$

$$p(1-p)$$

$$\begin{aligned} \textcircled{1} \text{ Mean } E(X) &= \sum_{x=0}^k x \cdot p(x) \\ &= x=1 + x=0 \\ &= 1 \times 0.6 + 0 \times 0.4 = 0.6 \rightarrow p \end{aligned}$$

$$(2) \text{ Variance } E(X^2) - E(X)^2$$

$$E(X^2) = \sum x^2 p(x) \rightarrow 1^2 \cdot p + 0 \cdot (1-p)$$

$$= \underline{\underline{p}}$$

$$E(X) - (\bar{x})^2$$

$$0 - (p)^2 = p(1-p)$$

\Rightarrow Proud

* Binomial distribution

\downarrow
Bi \rightarrow 2 \rightarrow two outcomes.

* Binomial distribution is in Bernoulli trial

$$\underline{p_m} = \underline{\underline{n}} \underline{\underline{C}_k} \underline{\underline{p^k (1-p)^{n-k}}}$$

$$\binom{n}{k} = \frac{n!}{n-k! k!} = \frac{!n}{!n-k !k}$$

For bernoulli
 \downarrow
 $p^k (1-p)^{n-k}$

$$\binom{5}{3} = \frac{5!}{3! 2!} = \frac{!5}{!3 !2}$$

$$5! = 1 \times 2 \times 3 \times 4 \times 5$$

$$3! = 1 \times 2 \times 3$$

$$\frac{5!}{3! 2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1}$$

1st toss 2nd toss 3rd toss 4th toss 5th toss 10th toss.

⇒ What is prob of getting 'K' heads out of n trials.

$n \rightarrow$ total no of trials

K - the no of events that you are interested in

Example With 3 tosses what is probability of getting exactly 2 heads ??

$$n \binom{n}{k} p^k (1-p)^{n-k}$$

$$n = 3, K = 2, p = 1/2$$

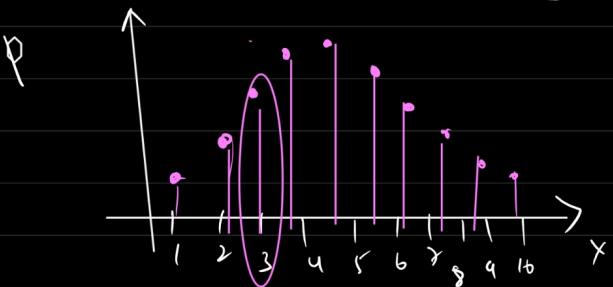
$$\begin{aligned} p(x=2) &= n \binom{n}{k} p^k (1-p)^{n-k} \\ &= 3 \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(1-\frac{1}{2}\right)^{3-2} = \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{(0.5)^2}{(0.5)} \\ &= \frac{3 \times 2}{2 \times 1} \cdot (0.5)^3 = 3 \times (0.5)^3 \\ &= 3 \times 0.125 \\ &= \underline{\underline{0.375}} \end{aligned}$$

Q. When you toss a coin 10 times, what is prob that you will get head exactly 3 times.

$$\rightarrow n = 10, k = 3 \quad p(x=3) = 10 \binom{10}{3} \cdot (0.5)^3 \cdot (0.5)^{10-3}$$

$$= \frac{10}{7} \cdot \frac{9}{8} \cdot \frac{8}{7} \cdot \frac{7}{6} \cdot \frac{6}{5} \cdot \frac{5}{4} \cdot \frac{4}{3} \cdot \frac{3}{2} \cdot (0.5)^{10} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{7 \times 6 \times 5 \times 4 \times 3 \times 2} \cdot (0.5)^{10}$$

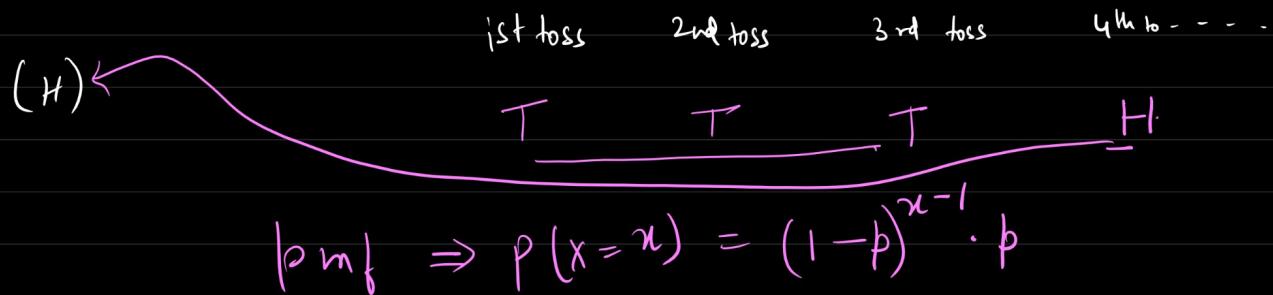
prob



$\text{mean} = np$ $\text{variance} = np(1-p)$ npq
--

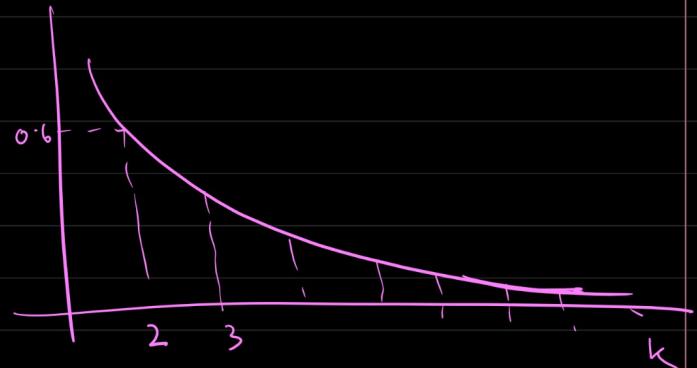
* geometric distribution

It represents the probability of getting first success after having a consecutive no of failure (no of trials)



$$\text{mean} = \frac{1}{p}$$

$$\text{Variance} \rightarrow \frac{1-p}{p^2}$$



* Poisson distribution

→ The Poisson distribution is a discrete prob distn that describes the no of events that occur within a fixed interval of time or space given a known average rate of occurrence.

* No of events occurring in a fixed time interval.

e.g. No of calls received by a customer care every hour:

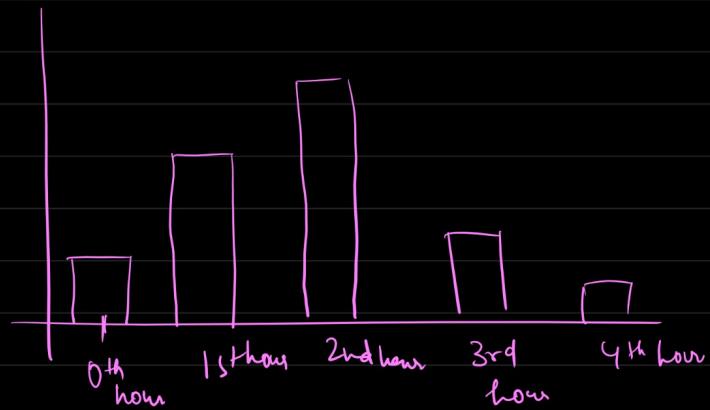
Expected no of calls every hour is $\underline{\underline{h}} = 100$

e.g. No. of people going to temple

Pmf.

e.g. No. of people going to hospital / airport every hour

e.g. No. of emails every hour



$$Pmf = P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

λ - average of events every interval

$e = 2.718$

$\lambda = 10$

Prob of a person visiting at 5th hour.

$$P(X=5) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{(2.718)^{-10} \cdot 10^5}{5!}$$

Q The average no. of customers entering a store in an hour is 5. What is prob exactly 3 customers will enter the store next hour.

$\lambda = 5$

$P(X=3)$

$$\frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{e^{-5} \cdot 5^3}{3!}$$

$$= \frac{(2.718)^{-5} \times 125}{6}$$

$$= \frac{0.00674}{6} \times 125 \approx 0.14$$

$$\left\{ \begin{array}{l} \text{Mean} - \frac{\lambda x t}{\lambda x t} \rightarrow \text{time} \\ \text{Varian} - \lambda x t \end{array} \right.$$