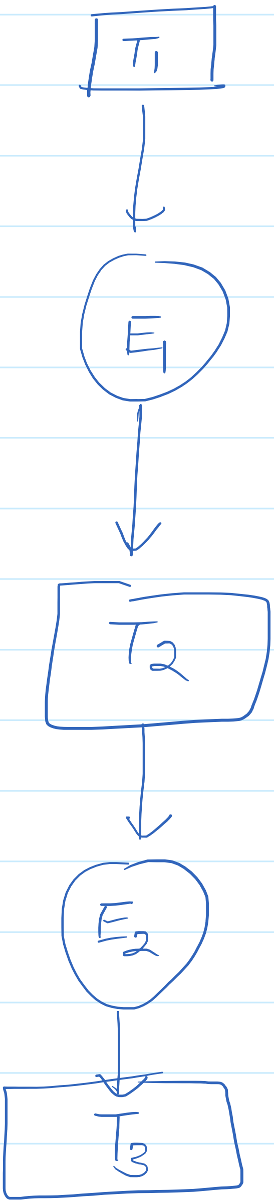
Assignment 10

31 March 2022 22:02

(2.)



Engines are connected in series. The sink temprature of first engine is source temprature of second engine.

The efficiency of both engines are same.

$$\eta_1 = \eta_2$$

$$\Rightarrow 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$$

$$T_1 = 1300\,K, \quad T_2 = 300\,K$$

$$\Rightarrow 1 - \frac{T_2}{1300K} = 1 - \frac{300}{T_2}$$

$$T_2 = \sqrt{1300 \times 300}$$

$$\boxed{T_2 = 624.5\,K}$$

(3.) Boltzmann entropy equation derivation

3. Boltzmann entropy equation derivation

We can write entropy 'S' as a function of thermodynamic probability (w)

$$S = f(w)$$

state function

$$S = S_1 + S_2 \quad \text{(individual states } 1 \rightarrow 2\text{)}$$

Similarly $\quad w = w_1 . w_2$

$$f(w) = f(w_1 . w_2)$$

$$S = S_1 + S_2$$

$$= f(w_1 . w_2)$$

$$f(w_1) + f(w_2) = f(w_1 . w_2)$$

differentiating wrt w_1 keeping w_2 constant

$$f'(w_1) + 0 = w_2 f'(w_1 . w_2)$$

again differentiating wrt w_2 keeping

ω_1 constant

$$0 = f'(\omega_1\omega_2) + \omega_2\omega_1 f''(\omega_1\omega_2)$$

$$0 = f'(w) + w f''(w)$$

consider $P = f'(w)$ $\quad \frac{dP}{dw} = f''(w)$

On substituting

$$P + w \cdot \frac{dP}{dw} = 0$$

$$P\,dw + w\,dP = 0$$

$$d(Pw) = 0$$

$$Pw = k \quad (\text{constant})$$

$$f'(w) = \frac{d}{dw} f(w) = P$$

$$\therefore \quad w \times \frac{d\,f(w)}{dw} = k$$

$$\int df(\omega) = \int \frac{k \cdot d\omega}{\omega}$$

on integrating;

$$f(\omega) = k \ln(\omega) + c$$

k = boltzmann constant

from Planck's claim $c = 0$

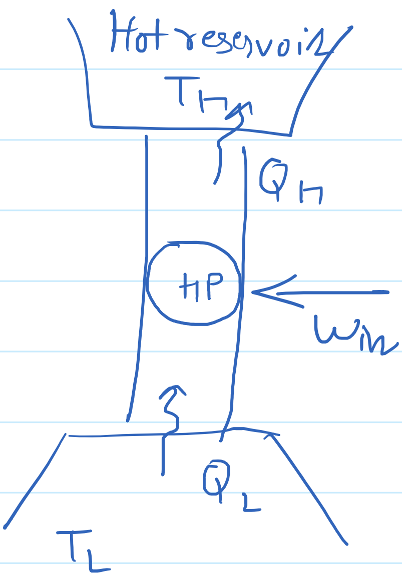
$$f(\omega) = k \ln(\omega)$$

$$\therefore \quad S = f(\omega) = k \ln(\omega)$$

$$\boxed{S = k \ln(\omega)}$$

which is the boltzmann entropy equation.

1.



$$(COP)_{HP} = \frac{\dot{Q}_h}{\dot{W}_{in}} \quad \text{--- (1)}$$

from 1st law,

$$-\dot{W}_{in} = -\dot{Q}_h + \dot{Q}_L$$

T_L $\quad \dot{Q}_L$

cold reservoir

$$\dot{W}_{in} = \dot{Q}_H - \dot{Q}_L$$

$$\therefore (COP)_{HP} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L}$$

also, $$(COP)_{HP} = \frac{T_H}{T_H - T_L}$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{T_H}{T_H - T_L}$$

$\dot{Q}_H = 300\,kW$, $T_H = 297\,K$, $T_L = 280\,K$

$$\frac{300}{300 - \dot{Q}_L} = \frac{297}{297 - 280}$$

$$\frac{300}{300 - \dot{Q}_L} = \frac{297}{17}$$

$$\frac{300 - \dot{Q}_L}{300} = \frac{17}{297}$$

$$300 - \dot{Q}_L = 300 \times \frac{17}{297}$$

$$300 - \dot{Q}_L = 5100/297$$

$$300 - \dot{Q}_L = 7.17$$

$$\dot{Q}_L = 282.83 kW$$

Change in entropy of the high temprature reservoir

$$\Delta S_H = \frac{\dot{Q}_H}{T_H}$$

$$\Delta S_H = \frac{300}{297} = 1.01 \ kW/K$$

$\therefore$ entropy change of high temprature reservoir is

$$\boxed{\Delta S_H = 1.01 kW/K}$$

for low temprature reservoir;

$$\Delta S_L = \frac{\dot{Q}_L}{T_L}$$

$$\Delta S_L = \frac{-282.83}{280}$$

$$\Delta S_L = -1.01 \, KW/K$$

$\therefore$ Change in entropy for low temp. reservoir

$$\boxed{\Delta S_L = -1.01 \, KW/K}$$