SMARTWIZ

GRADE 12 MATHEMATICS EXAM

MARKS: 150

MARKS	

TIME: 3 HOURS

SCHOOL _____

CLASS (eg. 4A) _____

SURNAME _____

NAME _____

Instructions for Learners:

• Read all instructions carefully before you begin the exam.

• Write your full name and student number clearly on the answer sheet/book.

• Answer all questions unless otherwise instructed.

• Show all your work/calculations where necessary.

• Write neatly and clearly.

• Use only a blue or black pen. Do not use correction fluid or tape.

• Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.

• Raise your hand if you have any questions.

• Do not talk to other learners during the exam.

• Any form of dishonesty will result in immediate disqualification from the exam.

This exam consists of Five pages, including the cover page.

QUESTION 1: SEQUENCES & SERIES [20 marks]

1.1 The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Given that $a = 4$ and $d = 3$, find:

1.1.1 The 10th term

_____

_____

1.1.2 The sum of the first 15 terms

_____

_____

_____

1.2 The sum to infinity of a geometric series is 24 and the first term is 18.
Find the common ratio r.

$$S_\infty = \frac{a}{1 - r}$$

_____

_____

_____

QUESTION 2: FUNCTIONS AND TRANSFORMATIONS [25 marks]

2.1 Given $f(x) = \sqrt{x - 1}$, determine:

2.1.1 The domain of f
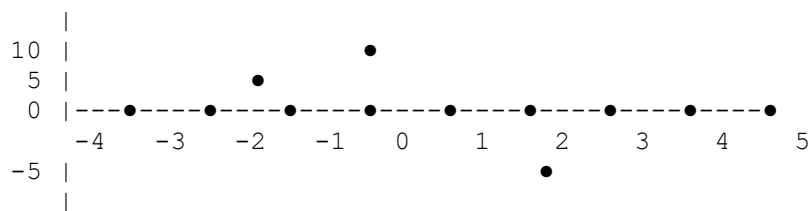
_____

_____

2.1.2 The range of f

_____

_____

_____

2.2 Sketch the graphs of $f(x) = x^2$ and $g(x) = (x - 3)^2 - 4$ on the same set of axes.

```
Visual Guide (Rough Sketch Template):
```

```
  |
10 |                    ●
 5 |          ●
 0 |---●----●----●----●----●----●----●----●----●
    -4   -3   -2   -1    0    1    2    3    4    5
-5 |                              ●
  |
```

Use this grid to indicate turning points, intercepts, and shifts.

2.2.1 Describe the transformation from fff to ggg.

_____

_____

_____

QUESTION 3: TRIGONOMETRY [20 marks]

3.1 Solve for xxx in the interval $0°≤x≤360°$ $0^\circ \leq x \leq 360^\circ$ $0°≤x≤360°$:

$2\cos x = 3$ $2\cos x = \sqrt{3}$ $2\cos x = 3$

_____

_____

3.2 In triangle ABCABCABC, AB=8AB = 8AB=8, AC=10AC = 10AC=10, and angle A=60°A = 60^\circ A=60°.

```
       B
      /\
     /  \
    /    \
   /      \
  A ------- C
```
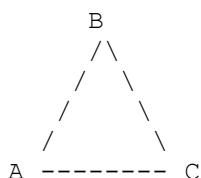
Use the **Cosine Rule** to find the length of side BCBCBC.

$c^2 = a^2 + b^2 - 2ab\cos C$ $c^2 = a^2 + b^2 - 2ab \cos C$ $c2=a2+b2−2abcosC$

_____

_____

_____

QUESTION 4: CALCULUS [30 marks]

4.1 Given $f(x)=2x^3 - 9x^2 + 12x$ $f(x) = 2x^3 - 9x^2 + 12x$ $f(x)=2x3−9x2+12x$, determine:

4.1.1 f'(x)f'(x)f'(x)

_____

4.1.2 The critical points of the function

4.1.3 Determine the nature (min/max) of each turning point using the second derivative.

4.2 Calculate the area under the curve $y = x^2 + 2$ between $x = 0$ and $x = 3$:

$$\int_0^3 (x^2 + 2)\, dx$$

QUESTION 5: ANALYTICAL GEOMETRY [25 marks]

5.1 In the diagram, the coordinates of A are (1, 2) and B are (5, 6).
Find the length of AB.

```
Visual aid: Coordinate Grid

      |
   6 -●------------------- B(5,6)
      |
   2 -●- A(1,2)
      |
      ------------------------------
      0    1    2    3    4    5    6
```

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

5.2 The line PQ has a gradient of -2 and passes through point $P(3, 4)$.
Find the equation of the line.

QUESTION 6: PROBABILITY & STATISTICS [30 marks]

6.1 A bag contains 4 red, 3 blue, and 5 yellow balls.

6.1.1 Find the probability of drawing a blue or yellow ball.

_____
_____

6.1.2 Two balls are drawn without replacement. What is the probability that both are red?

_____
_____
_____

6.2 The marks of 6 learners in a test are: 62, 67, 71, 74, 74, 85

6.2.1 Calculate the mean

_____
_____

6.2.2 Calculate the standard deviation (use calculator or formula):

$\sigma = \sqrt{\frac{1}{n} \sum (x - \bar{x})^2}$

_____
_____

END OF PAPER

TOTAL : 150

MEMO

QUESTION 1: SEQUENCES & SERIES [20 marks]

1.1.1
10th term:

$T_n = a + (n-1)d = 4 + (10-1)(3) = 4 + 27 = \boxed{31}$ Tn=a+(n−1)d=4+(10−1)(3)=4+27=31

1.1.2

$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{15}{2}(2 \cdot 4 + 14 \cdot 3) = \frac{15}{2}(8 + 42) = \frac{15}{2} \cdot 50 = \boxed{375}$ Sn=2n(2a+(n−1)d)=215(2·4+14·3)=215(8+42)=215·50=375

1.2

$S_\infty = \frac{a}{1 - r} \Rightarrow 24 = \frac{18}{1 - r} \Rightarrow 1 - r = \frac{18}{24} = \frac{3}{4} \Rightarrow r = 1 - \frac{3}{4} = \boxed{\frac{1}{4}}$ S∞=1−ra⇒24=1−r18⇒1−r=2418=43⇒r=1−43=41

QUESTION 2: FUNCTIONS AND TRANSFORMATIONS [25 marks]

2.1.1

$f(x) = \sqrt{x - 1} \Rightarrow x - 1 \geq 0 \Rightarrow x \geq 1 \text{Domain: } \boxed{x \in [1, \infty)}$ f(x)=x−1⇒x−1≥0⇒x≥1Domain: x∈[1,∞)

2.1.2

Range: $\sqrt{x - 1} \geq 0 \Rightarrow \boxed{y \in [0, \infty)}$ Range: x−1≥0⇒y∈[0,∞)

2.2.1
Transformation:

- $f(x) = x^2$ f(x)=x2 is the original parabola
- $g(x) = (x - 3)^2 - 4$ g(x)=(x−3)2−4: horizontal shift **right by 3**, vertical shift **down by 4**

✅ **Transformation: Shift 3 units right, 4 units down**

QUESTION 3: TRIGONOMETRY [20 marks]

3.1

$$2\cos x = \sqrt{3} \Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \boxed{30^\circ \text{ or } 330^\circ}$$

3.2
Cosine Rule:

$$BC^2 = 8^2 + 10^2 - 2(8)(10)\cos(60^\circ) = 64 + 100 - 160(0.5) = 164 - 80 = 84 \Rightarrow BC = \sqrt{84} \approx \boxed{9.17}$$

QUESTION 4: CALCULUS [30 marks]

4.1.1

$$f(x) = 2x^3 - 9x^2 + 12x \Rightarrow f'(x) = \boxed{6x^2 - 18x + 12}$$

4.1.2
Set derivative to zero:

$$6x^2 - 18x + 12 = 0 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = 1 \text{ or } x = 2$$

✅ Critical points: $x = 1, 2$

4.1.3
Second derivative:

$$f''(x) = 12x - 18 \quad f''(1) = -6 \Rightarrow \text{Max at } x = 1 \quad f''(2) = 6 \Rightarrow \text{Min at } x = 2$$

4.2

$$\int_0^3 (x^2 + 2)\, dx = \left[\frac{x^3}{3} + 2x \right]_0^3 = \left(\frac{27}{3} + 6 \right) - 0 = 9 + 6 = \boxed{15 \text{ units}^2}$$

QUESTION 5: ANALYTICAL GEOMETRY [25 marks]

5.1

$$AB = \sqrt{(5 - 1)^2 + (6 - 2)^2} = \sqrt{4^2 + 4^2} = \sqrt{16 + 16} = \sqrt{32} = \boxed{5.66}$$

5.2
Equation of a line:

$$y - y_1 = m(x - x_1) \Rightarrow y - 4 = -2(x - 3) \Rightarrow y = -2x + 6 + 4 = \boxed{y = -2x + 10}$$

QUESTION 6: PROBABILITY & STATISTICS [30 marks]

6.1.1
Blue or yellow = $3 + 5 = 8$ out of 12

$$P(\text{blue or yellow}) = \frac{8}{12} = \boxed{\frac{2}{3}}$$

6.1.2
Red balls: 4

$$P(\text{red then red}) = \frac{4}{12} \cdot \frac{3}{11} = \frac{12}{132} = \boxed{\frac{1}{11}}$$

6.2.1
Mean:

$$\bar{x} = \frac{62 + 67 + 71 + 74 + 74 + 85}{6} = \frac{433}{6} = \boxed{72.17}$$

6.2.2
Use calculator or formula:
Deviations from mean:

- $(62 - 72.17)^2 = 103.43$
- $(67 - 72.17)^2 = 26.75$
- $(71 - 72.17)^2 = 1.37$
- $(74 - 72.17)^2 = 3.37$ (x2)
- $(85 - 72.17)^2 = 164.74$

Sum = approx 302.03

$$\sigma = \sqrt{\frac{302.03}{6}} \approx \boxed{7.09}$$

TOTAL : 150