

SMARTWIZ

GRADE 10 MATHEMATICS EXAM

MARKS: 100

MARKS	

TIME: 2 hours

SCHOOL _____

CLASS (e.g. 4A) _____

SURNAME _____

NAME _____

Instructions for Learners:

- Read all the instructions carefully before you begin the exam.
- Write your name and learner number clearly on the answer sheet/booklet.
- Answer all the questions unless otherwise instructed.
- Show all your work/calculations where applicable.
- Write neatly and legibly.
- Use only blue or black ink. *Do not use correction fluid or tape.*
- No electronic devices (calculators, phones, etc.) are allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of cheating will lead to disqualification.

This test consists of 6 pages including the cover page.

SECTION A: ALGEBRA AND NUMBER PATTERNS (30 MARKS)

Question 1 (10 marks)

1.1 Simplify:

$$\frac{4x^3y^2 \times 8x^2y^{-1}}{3y} \times \frac{6x^{-1}y^3}{3y} \times \frac{8x^2y^{-1} \times 14x^3y^2}{6x^{-1}y^3}$$

..... (4)

1.2 Factorise completely:

$$8x^3 - 27$$

..... (3)

1.3 Solve for x:

$$\frac{2x+3}{4} = \frac{3x-1}{2}$$

..... (3)

Question 2 (10 marks)

2.1 Find the next 3 terms of the sequence and state the type:

$$3, 7, 13, 21, \dots$$

..... (4)

2.2 Write an expression for the n -th term of the sequence above. (3)

.....

2.3 Calculate the sum of the first 10 terms of the sequence. (3)

.....

Question 3 (10 marks)

3.1 Expand and simplify:

$$(2x-5)^2 - (x+1)(x-2)$$

..... (5)

3.2 Solve the quadratic equation by completing the square:

$$x^2 + 6x - 7 = 0 \quad x^2 + 6x - 7 = 0 \quad x^2 + 6x - 7 = 0$$

..... (5)

SECTION B: FUNCTIONS, GRAPHS AND COORDINATE GEOMETRY (30 MARKS)

Question 4 (15 marks)

4.1 Draw the graph of $y = 2x^2 - 4x + 1$ for $x = 0, 1, 2, 3$. Complete the table:

x	0	1	2	3
y				

..... (4)

4.2 Identify the vertex of the parabola. (3)

.....

4.3 Write down the equation of the axis of symmetry. (2)

.....

4.4 Determine the y-intercept. (2)

.....

4.5 Sketch the parabola using the points above. (4)

.....

Question 5 (15 marks)

5.1 Given points A(2,3) and B(-1,7), find:

a) The midpoint of AB. (3)

.....

b) The length of AB. (3)

.....

5.2 Determine the equation of the line passing through points A and B. (5)

.....

5.3 Find the equation of a line perpendicular to AB and passing through A. (4)

.....

SECTION C: TRIGONOMETRY AND GEOMETRY (40 MARKS)

Question 6 (20 marks)

6.1 In triangle PQR, $\angle P = 35^\circ$, $PQ = 10 \text{ cm}$, $PR = 15 \text{ cm}$.

a) Calculate the length QR using the cosine rule. (6)

.....

b) Calculate the area of the triangle using the sine rule. (4)

.....

6.2 A circle has a diameter of 20 cm.

a) Calculate the circumference of the circle. (3)

.....

b) Calculate the length of an arc subtending an angle of 45° at the center. (3)

.....

6.3 In a regular hexagon with side length 8 cm:

a) Calculate the perimeter. (2)

.....

b) Calculate the area. (2)

.....

Question 7 (20 marks)

7.1 A cylinder has radius 5 cm and height 12 cm.

a) Calculate the volume of the cylinder. (4)

.....

b) Calculate the total surface area of the cylinder. (5)

.....

7.2 Convert the following:

a) 4500 mm^3 to cm^3 . (2)

.....

b) 2.5 m^2 to cm^2 . (2)

.....

7.3 A rectangle has length 12 cm and width 7 cm.

a) Calculate its diagonal. (3)

.....

b) Calculate the angle between the diagonal and the length. (4)

.....

TOTAL: 100 MARKS



Answer: $x=54$
 $x = \frac{5}{4}x = 45$

Question 2

2.1 Sequence: 3, 7, 13, 21, ...

Differences: 4, 6, 8, so second differences are constant (2), so it's quadratic.

Next differences: 10, 12

Next terms:

$$21 + 10 = 31$$

$$31 + 12 = 43$$

$$43 + 14 = 57$$

Answer: 31, 43, 57

2.2 Find n -th term:

The sequence fits: $T_n = an^2 + bn + c$

Using terms:

$$T_1 = 3, T_2 = 7, T_3 = 13$$

Set up equations:

$$a(1)^2 + b(1) + c = 3 \Rightarrow a + b + c = 3$$

$$a(2)^2 + b(2) + c = 7 \Rightarrow 4a + 2b + c = 7$$

$$a(3)^2 + b(3) + c = 13 \Rightarrow 9a + 3b + c = 13$$

Subtract first from second:

$$(4a - a) + (2b - b) + (c - c) = 7 - 3 \Rightarrow 3a + b = 4$$

Subtract second from third:

$$(9a - 4a) + (3b - 2b) + (c - c) = 13 - 7 \Rightarrow 5a + b = 6$$

Subtract the two:

$$(5a + b) - (3a + b) = 6 - 4 \Rightarrow 2a = 2 \Rightarrow a = 1$$

Use $3a + b = 4$:

$$3(1) + b = 4 \Rightarrow b = 1$$

Use $a + b + c = 3$:

$$1 + 1 + c = 3 \Rightarrow c = 1$$

Answer: $T_n = n^2 + n + 1$

2.3 Sum of first 10 terms:

$$S_n = \sum_{n=1}^{10} (n^2 + n + 1) = \sum_{n=1}^{10} n^2 + \sum_{n=1}^{10} n + \sum_{n=1}^{10} 1$$

Use formulas:

$$\begin{aligned} \sum_{n=1}^n n^2 &= \frac{n(n+1)(2n+1)}{6} = \frac{10 \times 11 \times 21}{6} = 385 \\ \sum_{n=1}^n n &= \frac{n(n+1)}{2} = \frac{10 \times 11}{2} = 55 \\ \sum_{n=1}^n 1 &= n = 10 \end{aligned}$$

So,

$$S_{10} = 385 + 55 + 10 = 450$$

Answer: 450

Question 3

3.1

$$\begin{aligned} (2x-5)^2 - (x+1)(x-2) &= (4x^2 - 20x + 25) - (x^2 - x - 2) = 4x^2 - 20x + 25 - x^2 + x + 2 = 3x^2 - 19x + 27 \\ (2x-5)^2 - (x+1)(x-2) &= (4x^2 - 20x + 25) - (x^2 - x - 2) = 4x^2 - 20x + 25 - x^2 + x + 2 = 3x^2 - 19x + 27 \end{aligned}$$

Answer: $3x^2 - 19x + 27$

3.2 Complete the square:

$$x^2 + 6x - 7 = 0 \Rightarrow x^2 + 6x = 7 \Rightarrow x^2 + 6x + 9 = 7 + 9 \Rightarrow (x+3)^2 = 16$$

Add $(\frac{6}{2})^2 = 9$ to both sides:

$$x^2 + 6x + 9 = 7 + 9 \Rightarrow (x+3)^2 = 16$$

Take square root:

$$x+3 = \pm 4 \Rightarrow x = -3 \pm 4$$

So,

$$x = -3 \pm 4x = -3 \pm 4$$

Solutions:

$$x = 1 \text{ or } x = -7 \quad \text{or} \quad x = -7 \text{ or } x = 1$$

SECTION B

Question 4

4.1 Calculate yyy:

$$y = 2x^2 - 4x + 1$$

xxx 0	1	2	3
yyy 1	(2 - 4 + 1)	= -1	(8 - 8 + 1) = 1
			(18 - 12 + 1) = 7

4.2 Vertex: Use $x = -\frac{b}{2a} = -\frac{4}{2} = -2$, then $y = -1$

Answer: Vertex is (1, -1)

4.3 Axis of symmetry: $x = 1$

4.4 y-intercept: $x = 0 \Rightarrow y = 1$

Question 5

5.1

a) Midpoint:

$$\left(\frac{2 + (-1)}{2}, \frac{3 + 7}{2} \right) = \left(\frac{1}{2}, 5 \right)$$

b) Length AB:

$$\sqrt{(2 - (-1))^2 + (3 - 7)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

5.2 Gradient mmm:

$$m = \frac{7 - 3}{-1 - 2} = \frac{4}{-3} = -\frac{4}{3}$$

Equation using point-slope:

$$y-3=-43(x-2) \quad y-3=-\frac{4}{3}(x-2) \quad y-3=-34(x-2)$$

Or in slope-intercept:

$$y=-43x+83+3=-43x+173 \quad y=-\frac{4}{3}x+\frac{8}{3}+3=-\frac{4}{3}x+\frac{17}{3} \quad y=-34x+38+3=-34x+317$$

5.3 Gradient of perpendicular line:

$$m_{\perp}=34 \quad m_{\perp}=\frac{3}{4} \quad m_{\perp}=43$$

Equation through A(2,3) A(2,3) A(2,3):

$$y-3=34(x-2) \quad y-3=\frac{3}{4}(x-2) \quad y-3=43(x-2)$$

SECTION C

Question 6

6.1

a) Cosine Rule:

$$\begin{aligned} QR^2 &= PQ^2 + PR^2 - 2(PQ)(PR)\cos 35^\circ \\ QR^2 &= 10^2 + 15^2 - 2 \times 10 \times 15 \times \cos 35^\circ \\ &= 100 + 225 - 300 \times 0.8192 = 325 - 245.76 = 79.24 \\ QR &= \sqrt{79.24} \approx 8.9 \text{ cm} \end{aligned}$$

b) Area:

$$\begin{aligned} \text{Area} &= \frac{1}{2} PQ \times PR \times \sin 35^\circ \\ &= \frac{1}{2} \times 10 \times 15 \times 0.574 = 43.05 \text{ cm}^2 \end{aligned}$$

6.2

a) Circumference:

$$C = \pi d = \pi \times 20 = 62.83 \text{ cm} \quad C = \pi d = \pi \times 20 = 62.83 \text{ cm}$$

b) Arc length:

$$\begin{aligned} l &= \frac{45}{360} \times 62.83 = 7.85 \text{ cm} \\ l &= \frac{1}{8} \times 62.83 = 7.85 \text{ cm} \end{aligned}$$

6.3

a) Perimeter hexagon:

$$6 \times 8 = 48 \text{ cm} \quad 6 \times 8 = 48 \text{ cm} \quad 6 \times 8 = 48 \text{ cm}$$

b) Area hexagon:

$$\text{Area} = \frac{3\sqrt{3}}{2} s^2 = \frac{3 \times 1.732}{2} \times 8^2 = 2.598 \times 64 = 166.27 \text{ cm}^2$$

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Question 7

7.1

a) Volume cylinder:

$$V = \pi r^2 h = \pi \times 5^2 \times 12 = \pi \times 25 \times 12 = 942.48 \text{ cm}^3$$

$$V = \pi r^2 h = \pi \times 5^2 \times 12 = \pi \times 25 \times 12 = 942.48 \text{ cm}^3$$

b) Surface area cylinder:

$$SA = 2\pi r(r+h) = 2\pi \times 5(5+12) = 10\pi \times 17 = 534.07 \text{ cm}^2$$

$$SA = 2\pi r(r+h) = 2\pi \times 5(5+12) = 10\pi \times 17 = 534.07 \text{ cm}^2$$

7.2

a) $4500 \text{ mm}^3 = 4.5 \text{ cm}^3$
 $4500 \text{ mm}^3 = 4.5 \text{ cm}^3$ (since $1 \text{ cm}^3 = 1000 \text{ mm}^3$)b) $2.5 \text{ m}^2 = 2.5 \times 10,000 = 25,000 \text{ cm}^2$
 $2.5 \text{ m}^2 = 2.5 \times 10,000 = 25,000 \text{ cm}^2$ (since $1 \text{ m}^2 = 10,000 \text{ cm}^2$)

7.3

a) Diagonal rectangle:

$$d = \sqrt{12^2 + 7^2} = \sqrt{144 + 49} = \sqrt{193} = 13.89 \text{ cm}$$

$$d = \sqrt{12^2 + 7^2} = \sqrt{144 + 49} = \sqrt{193} = 13.89 \text{ cm}$$

b) Angle θ between diagonal and length:

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13.89} = 0.864$$

$$\theta = \cos^{-1}(0.864) = 30^\circ$$

TOTAL : 100

