

SMARTWIZ

GRADE11 MATHEMATICS EXAM

MARKS: 100

TIME: 2 HOURS

SCHOOL _____

CLASS (eg. 4A) _____

SURNAME _____

NAME _____

MARKS	
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Instructions for Learners:

- Read all instructions carefully before you begin the exam.
- Write your full name and student number clearly on the answer sheet/book.
- Answer all questions unless otherwise instructed.
- Show all your work/calculations where necessary.
- Write neatly and clearly.
- Use only a blue or black pen. Do not use correction fluid or tape.
- Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of cheating will result in immediate disqualification from the exam.

This exam consists of six pages, including the cover page.

SECTION A: MATRICES AND VECTORS (30 marks)

1. Given matrices

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 5 \\ 7 & -2 \end{bmatrix}$$

Calculate:

a) $A + B$

b) AB

2. Find the determinant of:

$$C = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$$

3. If $\vec{u} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{v} = -\hat{i} + 4\hat{j} + 2\hat{k}$, find:

a) $\vec{u} + \vec{v}$

b) The dot product $\vec{u} \cdot \vec{v}$

SECTION B: SEQUENCES AND SERIES (20 marks)

4. The n th term of an arithmetic sequence is given by:

$$T_n = 7n - 3$$

Find:

a) The first term T_1

b) The 15th term T_{15}

c) The sum of the first 15 terms S_{15}

5. Find the sum to infinity of the geometric series:

$$2+1+12+14+\cdots 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots 2+1+21+41+\cdots$$

SECTION C: PROBABILITY AND STATISTICS (25 marks)

6. A bag contains 4 red, 5 green, and 6 blue balls. One ball is drawn at random. Find the probability that the ball is:

a) Red

b) Not green

7. Two dice are rolled. Find the probability that:

a) The sum is 7

b) Both dice show the same number

8. The mean of the data set $\{5, 7, x, 10, 12\}$ is 9. Find the value of x .

SECTION D: COORDINATE GEOMETRY (25 marks)

9. Find the equation of the line passing through points $P(2, 3)$ and $Q(-1, 5)$ in:

a) Slope-intercept form $y = mx + c$

b) General form $Ax + By + C = 0$

10. Find the coordinates of the midpoint of the line segment joining points $A(4, -2)$ and $B(-6, 8)$.

11. Find the distance between the points $M(1,2)$ and $N(7,-4)$.

END OF EXAM

TOTAL : 100



MEMO

SECTION A: MATRICES AND VECTORS

1a. Calculate $A+B$ and $BA+B$:

$$A+B = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 7 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 2+0 & 3+7 \\ 0+5 & 5-2 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 5 & 3 \end{bmatrix}$$

$$BA+B = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 7 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 2(0)+3(5) & 2(7)+3(-2) \\ 0(0)+5(5) & 0(7)+5(-2) \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ 25 & -10 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 17 & 11 \\ 25 & -5 \end{bmatrix}$$

1b. Calculate AB and BA :

$$AB = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 7 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 2(0)+3(5) & 2(7)+3(-2) \\ 0(0)+5(5) & 0(7)+5(-2) \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ 25 & -10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 7 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 2(0)+3(5) & 2(7)+3(-2) \\ 0(0)+5(5) & 0(7)+5(-2) \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ 25 & -10 \end{bmatrix}$$

Multiply rows by columns:

- Element (1,1): $2 \times 0 + 3 \times 5 = 0 + 15 = 15$
- Element (1,2): $2 \times 7 + 3 \times (-2) = 14 - 6 = 8$
- Element (2,1): $0 \times 0 + 5 \times 5 = 0 + 25 = 25$
- Element (2,2): $0 \times 7 + 5 \times (-2) = 0 - 10 = -10$

So:

$$AB = \begin{bmatrix} 15 & 8 \\ 25 & -10 \end{bmatrix}$$

2. Determinant of C :

$$\det(C) = \begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix} = (4)(2) - (-3)(1) = 8 + 3 = 11$$

3a. Vector sum $\vec{u} + \vec{v}$:

$$\vec{u} + \vec{v} = (3-1)\hat{i} + (-2+4)\hat{j} + (1+2)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

3b. Dot product $\vec{u} \cdot \vec{v}$:

$$\vec{u} \cdot \vec{v} = (3)(-1) + (-2)(4) + (1)(2) = -3 - 8 + 2 = -9$$

SECTION B: SEQUENCES AND SERIES

4a. First term T_1 :

$$T_1 = 7(1) - 3 = 7 - 3 = 4$$

4b. Fifteenth term T_{15} :

$$T_{15} = 7(15) - 3 = 105 - 3 = 102$$

4c. Sum of first 15 terms S_{15} :

$$S_n = \frac{n}{2}(T_1 + T_n) = \frac{15}{2}(4 + 102) = \frac{15}{2} \times 106 = 15 \times 53 = 795$$

5. Sum to infinity of the geometric series:

$$2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$$

First term: $a = 2$

Common ratio:

$$r = \frac{1}{2} \div 1 = \frac{1}{2}$$

Since $|r| < 1$, sum to infinity is:

$$S_{\infty} = \frac{a}{1 - r} = \frac{2}{1 - \frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$$

SECTION C: PROBABILITY AND STATISTICS**6a. Probability of red ball:**

$$\text{Total balls} = 4 + 5 + 6 = 15$$

$$P(\text{red}) = \frac{4}{15}$$

6b. Probability not green:

$$P(\text{not green}) = 1 - P(\text{green}) = 1 - \frac{5}{15} = \frac{10}{15} = \frac{2}{3}$$

7a. Probability sum is 7 when rolling two dice:

Possible sums of 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \rightarrow 6 outcomes

$$\text{Total outcomes: } 6 \times 6 = 36$$

$$P(\text{sum}=7)=\frac{6}{36}=\frac{1}{6} \quad P(\text{sum}=7)=\frac{1}{6}$$

7b. Probability both dice show same number:

Possible doubles: (1,1), (2,2), ..., (6,6) → 6 outcomes

$$P(\text{same number})=\frac{6}{36}=\frac{1}{6} \quad P(\text{same number})=\frac{1}{6}$$

8. Find xxx with mean 9:

Mean:

$$\frac{5+7+x+10+12}{5}=9 \implies 5+7+x+10+12=9 \times 5=45$$

Sum:

$$5+7+x+10+12=9 \times 5=45 \implies x=45-5-7-10-12=11$$

Calculate xxx:

$$34+x=45 \implies x=45-34=11$$

SECTION D: COORDINATE GEOMETRY

9a. Equation of line through P(2,3) and Q(-1,5):

Slope:

$$m=\frac{5-3}{-1-2}=\frac{2}{-3}=-\frac{2}{3}$$

Use point-slope form with point P(2,3):

$$y-3=-\frac{2}{3}(x-2)$$

Simplify:

$$y-3=-\frac{2}{3}x+\frac{4}{3} \implies y=-\frac{2}{3}x+\frac{4}{3}+3=-\frac{2}{3}x+\frac{13}{3}$$

9b. General form:

Multiply both sides by 3:

$$3y=-2x+13$$

Bring all terms to one side:

$$2x+3y-13=0 \quad 2x + 3y - 13 = 0 \quad 2x+3y-13=0$$

10. Midpoint of A(4,-2)A(4, -2)A(4,-2) and B(-6,8)B(-6, 8)B(-6,8):

$$(4+(-6)/2, -2+8/2) = (-1, 3) \quad \left(\frac{4 + (-6)}{2}, \frac{-2 + 8}{2} \right) = \left(\frac{-2}{2}, \frac{6}{2} \right) = (-1, 3)$$

11. Distance between M(1,2)M(1, 2)M(1,2) and N(7,-4)N(7, -4)N(7,-4):

$$d = \sqrt{(7-1)^2 + (-4-2)^2} = \sqrt{6^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2} \approx 8.49$$

END OF MEMO

TOTAL : 100

MYST PATHWORKS