SMARTWIZ

GRADE 12 MATHEMATICS EXAM

MARKS: 150	MARKS	
TIME: 3 HOURS		
SCHOOL		
CLASS (eg. 4A)		
SURNAME		
NAME		

Instructions for Learners:

- Read all instructions carefully before you begin the exam.
- Write your full name and student number clearly on the answer sheet/book.
- Answer all questions unless otherwise instructed.
- Show all your work/calculations where necessary.
- Write neatly and clearly.
- Use only a blue or black pen. Do not use correction fluid or tape.
- Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of dishonesty will result in immediate disqualification from the exam.

This exam consists of Five pages, including the cover page.

QUESTION 1: SEQUENCES & SERIES [20 marks]

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1 1	The sum of	t the tire	t nnn terms	ot an	arithmetic	Series 19	o ouven	hv
1.1	The sum o	i the int		or an	arrunnenc	SCITCS II	, given	U.y

Sn=n2(2a+(n-1)d)S $n = \frac{n}{2}(2a+(n-1)d)Sn=2n(2a+(n-1)d)$

Given that a=4a = 4a=4 and d=3d = 3d=3, find:

1.1.1 The 10th term

1.1.2 The sum of the first 15 terms

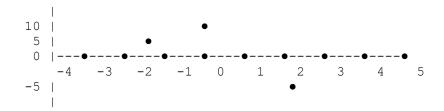
1.2 The sum to infinity of a geometric series is 24 and the first term is 18. Find the common ratio rrr.

 $S\infty=a1-rS \setminus infty = \frac{a}{1-r}S\infty=1-ra$

QUESTION 2: FUNCTIONS AND TRANSFORMATIONS [25 marks]

- 2.1 Given f(x)=x-1 $f(x) = \sqrt{x-1}$ f(x)=x-1, determine:
- 2.1.1 The domain of fff
- 2.1.2 The range of fff
- 2.2 Sketch the graphs of $f(x)=x^2f(x)=x^2f(x)=x^2$ and $g(x)=(x-3)^2-4$

Visual Guide (Rough Sketch Template):



Use this grid to indicate turning points, intercepts, and shifts.

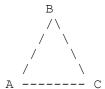
2.2.1 Describe the transformation from fff to ggg.

QUESTION 3: TRIGONOMETRY [20 marks]

3.1 Solve for xxx in the interval $0 \le x \le 360 \le 0 \le x \le 360 \le x$

 $2\cos[f_0]x=32\cos x = \sqrt{3}2\cos x=3$

3.2 In triangle ABCABCABC, AB=8AB = 8AB=8, AC=10AC=10AC=10AC=10, and angle A=60°A = 60°\circA=60°.



Use the **Cosine Rule** to find the length of side BCBCBC.

 $c2 = a2 + b2 - 2abcos[\underline{\mathit{fo}}]Cc^2 = a^2 + b^2 - 2ab \setminus cos \ Cc2 = a2 + b2 - 2abcosC$

QUESTION 4: CALCULUS [30 marks]

4.1 Given $f(x)=2x3-9x2+12xf(x)=2x^3-9x^2+12xf(x)=2x3-9x^2+12x$, determine:

4.1.1 f'(x)f'(x)f'(x)

4.1.2 The critical points of the function	4.1.2 T	he critical	points o	f the	function
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- 4.1.3 Determine the nature (min/max) of each turning point using the second derivative.
- 4.2 Calculate the area under the curve $y=x^2+2y=x^2+2y=x^2+2$ between x=0x=0 and x=3x=3x=3: $\int 0.03(x^2+2) \, dx \cdot \int 0.03(x^2+2) \, dx \cdot \int 0.03(x^2+2) \, dx$

QUESTION 5: ANALYTICAL GEOMETRY [25 marks]

5.1 In the diagram, the coordinates of AAA are (1, 2) and BBB are (5, 6). Find the length of AB.

Visual aid: Coordinate Grid



$$AB = (x2-x1)2 + (y2-y1)2AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}AB = (x2-x1)2 + (y2-y1)2AB = (x2-x1)2 + (y2-x1)2AB = (x2-x1)2AB = (x2-x1)$$

5.2 The line PQPQPQ has a gradient of -2 and passes through point P(3,4)P(3, 4)P(3,4). Find the equation of the line.

QUESTION 6: PROBABILITY & STATISTICS [30 marks]

6.1 A bag contains 4 red, 3 blue, and 5 yellow balls.

6.1.1 Find the probability of drawing a blue or yellow ball.
6.1.2 Two balls are drawn without replacement. What is the probability that both are red?
6.2 The marks of 6 learners in a test are: 62, 67, 71, 74, 74, 85 6.2.1 Calculate the mean
6.2.2 Calculate the standard deviation (use calculator or formula): $\sigma = \ln \sum (x-x^-) 2 \cdot \sin \alpha = \sqrt{\frac{1}{n} \cdot \sin (x - \sqrt{x})^2} \sigma = n \cdot \sum (x-x^-) 2$

END OF PAPER

TOTAL : 150

MEMO

QUESTION 1: SEQUENCES & SERIES [20 marks]

1.1.1

10th term:

```
Tn=a+(n-1)d=4+(10-1)(3)=4+27=31T_n=a+(n-1)d=4+(10-1)(3)=4+27=\box{boxed}{31}Tn=a+(n-1)d=4+(10-1)(3)=4+27=31
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1.1.2

 $Sn=n2(2a+(n-1)d)=152(2\cdot 4+14\cdot 3)=152(8+42)=152\cdot 50=375\\S_n= \frac{n}{2}(2a+(n-1)d)=\frac{15}$

1.2

 $S\infty=a1-r\Rightarrow 24=181-r\Rightarrow 1-r=1824=34\Rightarrow r=1-34=14S_\inf\{y=\frac{a}{1-r} \times 1-r=1824=34\Rightarrow r=1-34=14S_\inf\{y=\frac{a}{1-r} \times 1-r=1824=34\Rightarrow r=1-34=14S_\inf\{y=\frac{a}{1-r} \times 1-r=1824=34\Rightarrow r=1-43=41$ \Rightarrow 1 - r = \frac{18}{24} = \frac{3}{4} \Rightarrow r = 1 - \frac{3}{4} = \boxed{\frac{1}{4}} S\infty=1-ra\Rightarrow 24=1-r18\Rightarrow 1-r=2418=43\Rightarrow 1-r=2418=41

QUESTION 2: FUNCTIONS AND TRANSFORMATIONS [25 marks]

2.1.1

 $f(x)=x-1 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \\ Domain: \ x \in [1,\infty) \\ f(x) = \sqrt{x-1} \\ Rightarrow \ x-1 \\ geq 0 \\ Rightarrow \ x \\ geq 1 \\ text\{Domain: \ \} \\ boxed\{x \in [1,\infty)\} \\ f(x)=x-1 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \\ Domain: \ x \in [1,\infty)$

2.1.2

Range: $x-1 \ge 0 \Rightarrow y \in [0,\infty) \setminus \{Range: \} \setminus \{x-1\} \setminus \{0\} \setminus \{x-1\} \setminus \{x-1\}$

2.2.1

Transformation:

- $f(x)=x2f(x) = x^2f(x)=x2$ is the original parabola
- $g(x)=(x-3)2-4g(x) = (x-3)^2 4g(x)=(x-3)2-4$: horizontal shift **right by 3**, vertical shift **down by**

✓ Transformation: Shift 3 units right, 4 units down

QUESTION 3: TRIGONOMETRY [20 marks]

3.1

 $2\cos[f_0]x=3\Rightarrow\cos[f_0]x=32\Rightarrow x=30\circ \text{ or } 330\circ2\setminus\cos x=\sqrt{3} \cdot \text{Rightarrow } \cos x=\frac{3}{3}\times 30^\circ \cos x=\frac{3}{3}\times$

3.2

Cosine Rule:

 $BC2=82+102-2(8)(10)\cos[60](60\circ)=64+100-160(0.5)=164-80=84\Rightarrow BC=84\approx9.17BC^2=8^2+10^2-2(8)(10)\cos(60\%circ)=64+100-160(0.5)=164-80=84\%circ BC=\sqrt{84} \Rightarrow C=84\%circ BC=\sqrt{84} \Rightarrow C=84\%circ BC=\sqrt{84} \Rightarrow C=84\%circ BC=\sqrt{84} \Rightarrow C=84\%circ BC=84\%circ BC=$

QUESTION 4: CALCULUS [30 marks]

4.1.1

 $f(x)=2x3-9x2+12x \Rightarrow f'(x)=6x2-18x+12f(x)=2x^3-9x^2+12x \setminus Rightarrow \ f'(x)= \setminus boxed \{6x^2-18x+12\} f(x)=2x3-9x2+12x \Rightarrow f'(x)=6x2-18x+12$

ST PATHWORKS

4.1.2

Set derivative to zero:

 $6x2-18x+12=0 \Rightarrow x2-3x+2=0 \Rightarrow x=1$ or $x=26x^2-18x+12=0$ \Rightarrow $x^2-3x+2=0$ \Rightarrow $x=1 \neq 0$ \Righta

 \checkmark Critical points: x=1,2x=1,2x=1,2

4.1.3

Second derivative:

 $f''(x)=12x-18f''(1)=-6 \Rightarrow Max \text{ at } x=1f''(2)=6 \Rightarrow Min \text{ at } x=2f''(x)=12x-18 \text{ f''}(1)=-6 \land Rightarrow \land x=1f''(2)=6 \Rightarrow Min \text{ at } x=1f''(2)=6 \Rightarrow Min \text{ at } x=2f''(x)=12x-18f''(1)=-6 \Rightarrow Max \text{ at } x=1f''(2)=6 \Rightarrow Min \text{ at } x=2f''(x)=12x-18f''(1)=-6 \Rightarrow Max \text{ at } x=1f''(2)=6 \Rightarrow Min \text{ at } x=2f''(x)=12x-18f''(x)$

4.2

 $\int 03(x2+2)dx = [x33+2x]03 = (273+6)-0=9+6=15 \text{ units 2} \\ \text{long} 0^3 = \left\{ \frac{x^3}{3} + 6 \right\} - 0=9+6=15 \text{ units 2} \\ \text{long} 0^3 = \left\{ \frac{27}{3} + 6 \right\} - 0=9+6=15 \text{ units 2} \\ +2x \\ \text{long} 0^3 = \frac{27}{3} + 6 \\ \text{long} 0^3 = \frac{27}{3} + 6$

QUESTION 5: ANALYTICAL GEOMETRY [25 marks]

5.1

$$AB = (5-1)2 + (6-2)2 = 42 + 42 = 16 + 16 = 32 = 5.66AB = \sqrt{(5-1)^2 + (6-2)^2} = \sqrt{4^2 + 4^2} = \sqrt{(5-1)^2 + (6-2)^2} = \sqrt{32} = \sqrt{32} = \sqrt{5.66}AB = (5-1)^2 + (6-2)^2 = 42 + 42 = 16 + 16 = 32 = 5.66$$

5.2

Equation of a line:

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y-y1=m(x-x1) \Rightarrow y-4=-2(x-3) \Rightarrow y=-2x+6+4=y=-2x+10y - y_1 = m(x - x_1) \ \text{Rightarrow } y - 4 = -2(x - 3) \ \text{Rightarrow } y = -2x + 6 + 4 = \ boxed \\ y = -2x + 10 \\ y-y1=m(x-x1) \Rightarrow y-4=-2(x-3) \Rightarrow y=-2x+6+4=y=-2x+10
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QUESTION 6: PROBABILITY & STATISTICS [30 marks]

6.1.1

Blue or yellow = 3+5=83+5=83+5=8 out of 12

 $P(blue \ or \ yellow)=812=23P(\ text\{blue \ or \ yellow\})=\ frac\{8\}\{12\}=\ boxed\{\ frac\{2\}\{3\}\}P(blue \ or \ yellow)=128=32$

6.1.2

Red balls: 4

 $P(\text{red then red}) = 412 \cdot 311 = 12132 = 111P(\text{text}\{\text{red then red}\}) = \frac{4}{12} \cdot \frac{3}{11} = \frac{12}{132} = \frac{11}{11} P(\text{red then red}) = 124 \cdot 113 = 13212 = 111$

6.2.1

Mean:

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x=62+67+71+74+74+856=4336=72.17 \{x\} = \frac{62+67+71+74+74+85}{6} = \frac{433}{6} = \frac{72.17}{x=662+67+71+74+74+85=6433=72.17}
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6.2.2

Use calculator or formula:

Deviations from mean:

- (62-72.17)2=103.43(62 72.17)² = 103.43(62-72.17)2=103.43
- $(67-72.17)2=26.75(67-72.17)^2=26.75(67-72.17)2=26.75$
- $(71-72.17)2=1.37(71-72.17)^2=1.37(71-72.17)2=1.37$
- $(74-72.17)2=3.37(74-72.17)^2=3.37(74-72.17)2=3.37(x2)$
- $(85-72.17)2=164.74(85-72.17)^2=164.74(85-72.17)2=164.74$

Sum = approx 302.03

 $\sigma=302.036\approx7.09 \text{ sigma} = \sqrt{\frac{302.03}{6}} \operatorname{approx} \operatorname{boxed}{7.09} \sigma=6302.03\approx7.09$

TOTAL: 150

