SMARTWIZ

GRADE11 MATHEMATICS EXAM

MARKS: 100

MARKS	

TIME: 2 HOURS

SCHOOL _____

CLASS (eg. 4A) _____

SURNAME _____

NAME _____

Instructions for Learners:

- Read all instructions carefully before you begin the exam.
- Write your full name and student number clearly on the answer sheet/book.
- Answer all questions unless otherwise instructed.
- Show all your work/calculations where necessary.
- Write neatly and clearly.
- Use only a blue or black pen. Do not use correction fluid or tape.
- Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of cheating will result in immediate disqualification from the exam.

This exam consists of six pages, including the cover page.

SECTION A: POLYNOMIALS AND FACTORING (20 marks)

1. Factor completely:
a) x3−27x^3 - 27x3−27

b) 2x2+5x−32x^2 + 5x - 32x2+5x−3

2. Divide 6x3+5x2−4x+76x^3 + 5x^2 - 4x + 76x3+5x2−4x+7 by 2x−12x - 12x−1 using long division. Write down the quotient and remainder.

SECTION B: LOGARITHMS AND EXPONENTIALS (20 marks)

3. Solve for xxx:
a) log⁡5(x−1)=2\log_5 (x - 1) = 2log5(x−1)=2

b) 4x+1=324^{x+1} = 324x+1=32

4. Express as a single logarithm:

log⁡381+2log⁡32−log⁡39\log_3 81 + 2 \log_3 2 - \log_3 9log381+2log32−log39

SECTION C: TRIGONOMETRY (20 marks)

5. In △PQR\triangle PQR△PQR, ∠Q=90∘\angle Q = 90^\circ∠Q=90∘, PQ=8PQ = 8PQ=8 cm, and PR=10PR = 10PR=10 cm. Find:

a) The length of QRQRQR.

b) sin⁡P\sin PsinP and cos⁡P\cos PcosP (give exact values).

6. Simplify:

sin⁡60∘cos⁡30∘tan⁡45∘\frac{\sin 60^\circ \cos 30^\circ}{\tan 45^\circ}tan45∘sin60∘cos30∘

SECTION D: MATRICES AND VECTORS (20 marks)

7. Given

A=[1−234],B=[05−12]A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 5 \\ -1 & 2 \end{bmatrix}A=[13−24],B=[0−152]

a) Find A−BA - BA−B.

b) Calculate the determinant of matrix AAA.

8. The vectors a⃗=4i^−3j^\vec{a} = 4\hat{i} - 3\hat{j}a=4i^−3j^ and b⃗=−2i^+5j^\vec{b} = -2\hat{i} + 5\hat{j}b=−2i^+5j^.

a) Find the magnitude of a⃗\vec{a}a.

b) Find the vector 2a⃗−3b⃗2\vec{a} - 3\vec{b}2a−3b.

SECTION E: DIFFERENTIATION AND APPLICATIONS (20 marks)

9. Differentiate the following:

a) y=5x4−3x2+7y = 5x^4 - 3x^2 + 7y=5x4−3x2+7

b) y=ln⁡(2x+1)y = \ln (2x + 1)y=ln(2x+1)

10. A particle moves along a line so that its displacement s(t)=4t3−9t2+6ts(t) = 4t^3 - 9t^2 + 6ts(t)=4t3−9t2+6t.

a) Find the velocity v(t)v(t)v(t).

b) Find the acceleration a(t)a(t)a(t).

SECTION F: COORDINATE GEOMETRY (20 marks)

11. Find the equation of the line passing through points A(3,−2)A(3, -2)A(3,−2) and B(−1,4)B(-1, 4)B(−1,4) in:

a) Gradient-intercept form y=mx+cy = mx + cy=mx+c

b) General form Ax+By+C=0Ax + By + C = 0Ax+By+C=0

12. Calculate the distance between points C(5,7)C(5, 7)C(5,7) and D(−2,−3)D(-2, -3)D(−2,−3).

END OF EXAM

MEMO

SECTION A: POLYNOMIALS AND FACTORING

1a. Factor x3−27x^3 - 27x3−27:

x3−33=(x−3)(x2+3x+9)x^3 - 3^3 = (x - 3)(x^2 + 3x + 9)x3−33=(x−3)(x2+3x+9)

1b. Factor 2x2+5x−32x^2 + 5x - 32x2+5x−3:
Find two numbers that multiply to 2×−3=−62 \times -3 = -62×−3=−6 and add to 5: 6 and -1

2x2+6x−x−3=2x(x+3)−1(x+3)=(2x−1)(x+3)2x^2 + 6x - x - 3 = 2x(x + 3) - 1(x + 3) = (2x - 1)(x + 3)2x2+6x−x−3=2x(x+3)−1(x+3)=(2x−1)(x+3)

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2. Divide 6x3+5x2−4x+76x^3 + 5x^2 - 4x + 76x3+5x2−4x+7 by 2x−12x - 12x−1:

Long division:

- 6x3÷2x=3x26x^3 \div 2x = 3x^26x3÷2x=3x2
  Multiply: 3x2×(2x−1)=6x3−3x23x^2 \times (2x - 1) = 6x^3 - 3x^23x2×(2x−1)=6x3−3x2
  Subtract: (5x2−(−3x2))=8x2(5x^2 - (-3x^2)) = 8x^2(5x2−(−3x2))=8x2
- 8x2÷2x=4x8x^2 \div 2x = 4x8x2÷2x=4x
  Multiply: 4x×(2x−1)=8x2−4x4x \times (2x - 1) = 8x^2 - 4x4x×(2x−1)=8x2−4x
  Subtract: (−4x−(−4x))=0(-4x - (-4x)) = 0(−4x−(−4x))=0
- Bring down +7
- 0÷2x=00 \div 2x = 00÷2x=0

Quotient: 3x2+4x3x^2 + 4x3x2+4x
Remainder: 7

SECTION B: LOGARITHMS AND EXPONENTIALS

3a. log⁡5(x−1)=2\log_5 (x - 1) = 2log5(x−1)=2
Rewrite in exponential form:

x−1=52=25 ⟹ x=26x - 1 = 5^2 = 25 \implies x = 26x−1=52=25⟹x=26

3b. 4x+1=324^{x+1} = 324x+1=32
Express 32 as power of 2: 32=2532 = 2^532=25
Rewrite:

(22)x+1=25 ⟹ 22(x+1)=25(2^2)^{x+1} = 2^5 \implies 2^{2(x+1)} = 2^5(22)x+1=25⟹22(x+1)=25

Equate exponents:

2(x+1)=5 ⟹ 2x+2=5 ⟹ 2x=3 ⟹ x=32=1.52(x + 1) = 5 \implies 2x + 2 = 5 \implies 2x = 3 \implies x = \frac{3}{2} = 1.52(x+1)=5⟹2x+2=5⟹2x=3⟹x=23=1.5

4. Simplify:

log381+2log32−log39=log381+log322−log39=log381+log34−log39\log_3 81 + 2 \log_3 2 - \log_3 9 = \log_3 81 + \log_3 2^2 - \log_3 9 = \log_3 81 + \log_3 4 - \log_3 9log381+2log32−log39=log381+log322−log39=log381+log34−log39

Combine:

=log381×49=log33249=log336= \log_3 \frac{81 \times 4}{9} = \log_3 \frac{324}{9} = \log_3 36=log3981×4=log39324=log336

SECTION C: TRIGONOMETRY

5a. PQ=8PQ = 8PQ=8, PR=10PR = 10PR=10, right angle at QQQ. Find QRQRQR:
Using Pythagoras:

QR=PR2−PQ2=102−82=100−64=36=6QR = \sqrt{PR^2 - PQ^2} = \sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6QR=PR2−PQ2=102−82=100−64=36=6

5b.

sinP=oppositehypotenuse=QRPR=610=35\sin P = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{6}{10} = \frac{3}{5}sinP=hypotenuseopposite=PRQR=106=53
cosP=adjacenthypotenuse=PQPR=810=45\cos P = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{8}{10} = \frac{4}{5}cosP=hypotenuseadjacent=PRPQ=108=54

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6. Simplify:

sin60∘cos30∘tan45∘\frac{\sin 60^\circ \cos 30^\circ}{\tan 45^\circ}tan45∘sin60∘cos30∘

Values:

sin60∘=32,cos30∘=32,tan45∘=1\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 45^\circ = 1sin60∘=23,cos30∘=23,tan45∘=1

Calculate numerator:

32×32=34\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}23×23=43

Divide by denominator 1:

34÷1=34\frac{3}{4} \div 1 = \frac{3}{4}43÷1=43

SECTION D: MATRICES AND VECTORS

7a.

A−B=[1−0−2−53−(−1)4−2]=[1−742]A - B = \begin{bmatrix} 1-0 & -2-5 \\ 3-(-1) & 4-2 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 4 & 2 \end{bmatrix}A−B=[1−03−(−1)−2−54−2]=[14−72]

7b.

det⁡(A)=(1)(4)−(3)(−2)=4+6=10\det(A) = (1)(4) - (3)(-2) = 4 + 6 = 10det(A)=(1)(4)−(3)(−2)=4+6=10

8a. Magnitude of a⃗=4i^−3j^\vec{a} = 4\hat{i} - 3\hat{j}a=4i^−3j^:

∣a⃗∣=42+(−3)2=16+9=25=5|\vec{a}| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5∣a∣=42+(−3)2 =16+9=25=5

8b.

2a⃗−3b⃗=2(4i^−3j^)−3(−2i^+5j^)=(8i^−6j^)−(−6i^+15j^)=(8+6)i^+(−6−15)j^=14i^−21j^2\vec{a} - 3\vec{b} = 2(4\hat{i} - 3\hat{j}) - 3(-2\hat{i} + 5\hat{j}) = (8\hat{i} - 6\hat{j}) - (-6\hat{i} + 15\hat{j}) = (8 + 6)\hat{i} + (-6 - 15)\hat{j} = 14\hat{i} - 21\hat{j}2a−3b=2(4i^−3j^)−3(−2i^+5j^)=(8i^−6j^)−(−6i^+15j^)=(8+6)i^+(−6−15)j^=14i^−21j^

SECTION E: DIFFERENTIATION AND APPLICATIONS

9a.

y=5x4−3x2+7 ⟹ dydx=20x3−6xy = 5x^4 - 3x^2 + 7 \implies \frac{dy}{dx} = 20x^3 - 6xy=5x4−3x2+7⟹dxdy=20x3−6x

9b.

y=ln⁡(2x+1) ⟹ dydx=22x+1y = \ln(2x + 1) \implies \frac{dy}{dx} = \frac{2}{2x + 1}y=ln(2x+1)⟹dxdy=2x+12

10a.

v(t)=dsdt=12t2−18t+6v(t) = \frac{ds}{dt} = 12t^2 - 18t + 6v(t)=dtds=12t2−18t+6

10b.

a(t)=d2sdt2=24t−18a(t) = \frac{d^2 s}{dt^2} = 24t - 18a(t)=dt2d2s=24t−18

SECTION F: COORDINATE GEOMETRY

11a. Find slope mmm:

m=4−(−2)−1−3=6−4=−32m = \frac{4 - (-2)}{-1 - 3} = \frac{6}{-4} = -\frac{3}{2}m=−1−34−(−2)=−46 =−23

Equation form:

y=mx+c ⟹ −2=−32(3)+c ⟹ −2=−92+c ⟹ c=−2+92=52y = mx + c \implies -2 = -\frac{3}{2}(3) + c \implies -2 = -\frac{9}{2} + c \implies c = -2 + \frac{9}{2} = \frac{5}{2}y=mx+c⟹−2=−23 (3)+c⟹−2=−29+c⟹c=−2+29=25

So,

y=−32x+52y = -\frac{3}{2} x + \frac{5}{2}y=−23x+25

11b. General form:

Multiply entire equation by 2:

2y=−3x+5 ⟹ 3x+2y−5=02y = -3x + 5 \implies 3x + 2y - 5 = 02y=−3x+5⟹3x+2y−5=0

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12. Distance between C(5,7)C(5,7)C(5,7) and D(−2,−3)D(-2,-3)D(−2,−3):

d=(5−(−2))2+(7−(−3))2=(7)2+(10)2=49+100=149≈12.21d = \sqrt{(5 - (-2))^2 + (7 - (-3))^2} = \sqrt{(7)^2 + (10)^2} = \sqrt{49 + 100} = \sqrt{149} \approx 12.21d=(5−(−2))2+(7−(−3))2=(7)2+(10)2=49+100=149 ≈12.21

END OF MEMO

TOTAL : 100