

# SMARTWIZ

## GRADE 12 MATHEMATICS EXAM

MARKS: 150

TIME: 3 HOURS

SCHOOL \_\_\_\_\_

CLASS (eg. 4A) \_\_\_\_\_

SURNAME \_\_\_\_\_

NAME \_\_\_\_\_

MARKS	
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### Instructions for Learners:

- Read all instructions carefully before you begin the exam.
- Write your full name and student number clearly on the answer sheet/book.
- Answer all questions unless otherwise instructed.
- Show all your work/calculations where necessary.
- Write neatly and clearly.
- Use only a blue or black pen. Do not use correction fluid or tape.
- Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of dishonesty will result in immediate disqualification from the exam.

**This exam consists of Five pages, including the cover page.**

## SECTION A: ALGEBRA & NUMBER PATTERNS [30 marks]

### 1.1 Solve for xxx:

$$x^2 - 5x + 6 = 0 \quad x^2 - 5x + 6 = 0 \quad x^2 - 5x + 6 = 0$$


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### 1.2 Solve for xxx:

$$3x + 1 = 81 \quad 3^{x+1} = 81 \quad 3x + 1 = 81$$


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### 1.3 Given the quadratic sequence:

$$T_n = n^2 + 2n - 3 \quad T_n = n^2 + 2n - 3 \quad T_n = n^2 + 2n - 3$$

1.3.1 Find the 4th term.

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1.3.2 Determine a general formula for the **first difference**.

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## SECTION B: FUNCTIONS AND INVERSES [25 marks]

2.1 Let  $f(x) = -x^2 + 4$

2.1.1 State the range of  $f(x)$ .

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2.1.2 Find the values of  $x$  where  $f(x) = 0$ .

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2.1.3 Sketch the graph of  $f(x)$  clearly indicating intercepts.

Graph Sketch:



**2.2 Given  $g(x)=2x-3$ , determine:**

2.2.1 The inverse function  $g^{-1}(x)$

2.2.2 Sketch the graphs of  $g(x)$  and  $g^{-1}(x)$  on the same set of axes

## SECTION C: TRIGONOMETRY [20 marks]

**3.1 Solve for  $x \in [0^\circ, 360^\circ]$ :**

$$2\sin x - 1 = 0 \quad \sin x - \frac{1}{2} = 0 \quad \sin x = \frac{1}{2}$$

**3.2 A man observes a tower from two points, A and B, 40 m apart. The angles of elevation from A and B to the top of the tower are  $30^\circ$  and  $45^\circ$ , respectively.**

Calculate the height of the tower using trigonometry.  
(Use a sketch)

Visual layout:



## SECTION D: CALCULUS [30 marks]

**4.1 Differentiate:**

4.1.1  $f(x)=4x^3-3x^2+2x-7$   $f'(x) = 4x^3 - 3x^2 + 2x - 7$

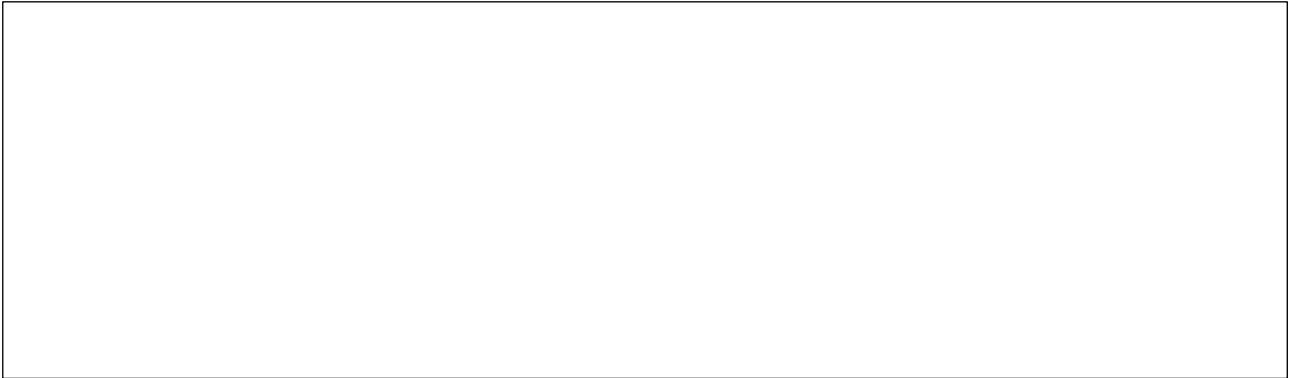
4.1.2  $h(x)=\frac{3x+2}{x}$   $h'(x)=\frac{3x+2}{x^2}$

**4.2 Determine the equation of the tangent to the curve**

$$y=x^3-6x+2$$

at the point where  $x=2$ .

**4.3 Sketch the graph of the derivative of a cubic function that has turning points at  $x = -1$  and  $x = 3$ . Label axes and shape.**




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## **SECTION E: GEOMETRY & MEASUREMENT [25 marks]**

**5.1 In a circle,  $O$  is the centre. Chords  $AB$  and  $CD$  intersect at point  $E$ . Prove:**

$$AE \cdot EB = CE \cdot ED$$

(Use the intersecting chords theorem.)

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**5.2 A cone has a radius of 3 cm and a height of 8 cm. Calculate:**

5.2.1 The volume

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (3)^2 (8) = 31\pi \text{ cm}^3$$

5.2.2 The slant height (use Pythagoras)

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## **SECTION F: DATA HANDLING & PROBABILITY [20 marks]**

**6.1 The ages of learners in a club are:**

13, 14, 14, 15, 15, 15, 16, 17, 18, 18

6.1.1 Determine the **mean** and **mode**.

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6.1.2 Calculate the **standard deviation** (use calculator/formula).

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**6.2 A die is rolled.**

6.2.1 What is the probability of getting a prime number?

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6.2.2 What is the probability of **not** getting an even number?

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**END OF PAPER**

**TOTAL : 150**

## MEMO

**SECTION A: ALGEBRA & NUMBER PATTERNS [30 marks]****1.1**

$$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x=2 \text{ or } x=3 \quad x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x=2 \text{ or } x=3$$


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**1.2**

$$3x+1=81 \Rightarrow 3x+1=3^4 \Rightarrow x+1=4 \Rightarrow x=3 \quad 3^{x+1} = 81 \Rightarrow 3^{x+1} = 3^4 \Rightarrow x+1=4 \Rightarrow x=3$$


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**1.3**

$$\text{Given } T_n = n^2 + 2n - 3$$

**1.3.1**

$$T_4 = (4)^2 + 2(4) - 3 = 16 + 8 - 3 = 21 \quad T_4 = (4)^2 + 2(4) - 3 = 16 + 8 - 3 = \boxed{21}$$

**1.3.2**

First difference:

$$T_n = n^2 + 2n - 3 \Rightarrow \text{Let's find a few terms: } T_1 = 0, T_2 = 5, T_3 = 12, T_4 = 21 \Rightarrow \text{1st differences: } 5, 7, 9 \Rightarrow \text{1st diff} = 2n + 1$$


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**SECTION B: FUNCTIONS AND INVERSES [25 marks]**

$$2.1 \quad f(x) = -x^2 + 4$$

**2.1.1**

Parabola opens downward. Maximum value at vertex (0, 4)

$$\text{Range: } y \in (-\infty, 4] \quad \boxed{\text{Range: } y \in (-\infty, 4]}$$

**2.1.2**

$$f(x)=0 \Rightarrow -x^2+4=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2 \quad f(x)=0 \Rightarrow -x^2+4=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2$$

### 2.1.3

Sketch shows parabola with:

- Vertex at (0, 4)
- x-intercepts at  $x=-2$  and  $x=2$
- y-intercept at (0, 4)

✓ Graph shape:  $\cap$

## 2.2 $g(x)=2x-3$

### 2.2.1

Find inverse:

$$y=2x-3 \Rightarrow x=2y-3 \Rightarrow 2y=x+3 \Rightarrow y=\frac{x+3}{2} \quad y=2x-3 \Rightarrow x=2y-3 \Rightarrow 2y=x+3 \Rightarrow y=\frac{x+3}{2}$$

### 2.2.2

- Straight lines
- $g(x)$ : slope 2, y-int = -3
- $g^{-1}(x)$ : slope 0.5, y-int = 1.5
- Reflect across  $y=x$

## SECTION C: TRIGONOMETRY [20 marks]

### 3.1

$$2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = 30^\circ, 150^\circ \quad 2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = 30^\circ, 150^\circ$$

### 3.2

Let height of tower =  $h$

From point A:

$$\tan(30^\circ) = \frac{h}{x} \Rightarrow h = x \cdot \tan(30^\circ) \quad \tan(30^\circ) = \frac{h}{x} \Rightarrow h = x \cdot \tan(30^\circ)$$

From point B:

$$\tan(45^\circ) = \frac{h}{40-x} \Rightarrow h = (40-x)\tan(45^\circ) = \frac{h}{40-x} \Rightarrow h = (40-x)\tan(45^\circ) = 40-x$$

Set equal:

$$x \cdot \tan(30^\circ) = 40-x \Rightarrow x(13) = 40-x \Rightarrow x+x(13) = 40 \Rightarrow x(1+13) = 40 \Rightarrow x \approx 19.05 \Rightarrow h = x \cdot \tan(30^\circ) \approx 11.0 \text{ m}$$

$$\tan(30^\circ) = \frac{h}{40-x} \Rightarrow x \cdot \tan(30^\circ) = 40-x \Rightarrow x \cdot \left(\frac{1}{\sqrt{3}}\right) = 40-x \Rightarrow x + x\left(\frac{1}{\sqrt{3}}\right) = 40 \Rightarrow x\left(1 + \frac{1}{\sqrt{3}}\right) = 40 \Rightarrow x \approx 19.05 \Rightarrow h = x \cdot \tan(30^\circ) \approx 11.0 \text{ m}$$

## SECTION D: CALCULUS [30 marks]

### 4.1

#### 4.1.1

$$f(x) = 4x^3 - 3x^2 + 2x - 7 \Rightarrow f'(x) = 12x^2 - 6x + 2 \Rightarrow f(x) = 4x^3 - 3x^2 + 2x - 7 \Rightarrow f'(x) = 12x^2 - 6x + 2$$

#### 4.1.2

$$h(x) = 3x + 2 = 3 + 2x \Rightarrow h'(x) = 0 - 2x^2 = -2x^2 \Rightarrow h(x) = \frac{3x+2}{x} = 3 + \frac{2}{x} \Rightarrow h'(x) = 0 - \frac{2}{x^2} = -\frac{2}{x^2}$$

### 4.2

Find derivative:

$$y = x^3 - 6x + 2 \Rightarrow y' = 3x^2 - 6 \Rightarrow \text{At } x = 2: y' = 3(2)^2 - 6 = 12 - 6 = 6 \Rightarrow y = 2^3 - 6(2) + 2 = 8 - 12 + 2 = -2$$

Tangent:  $(y + 2 = 6(x - 2)) \Rightarrow y = 6x - 14$

Derivative of cubic is a **quadratic** - Turns at  $(x = -1)$ ,  $(x = 3)$  - Parabola opens **upwards** ☒ Sketch: U-shape with x-intercepts at -1 and 3

**SECTION E: GEOMETRY & MEASUREMENT [25 marks]**

**5.1** Theorem: Intersecting chords theorem  $AE \cdot EB = CE \cdot ED$

**Proof Idea:** Use similar triangles formed by chords intersecting.

### 5.2

$$\text{Cone: } r=3r=3r=3, h=8h=8h=8$$



**5.2.1 Volume:**

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3)^2 (8) = 24\pi \text{ cm}^3 \approx 75.4 \text{ cm}^3$$

**5.2.2 Slant height:**

$$l = \sqrt{r^2 + h^2} = \sqrt{9 + 64} = \sqrt{73} \approx 8.54 \text{ cm}$$

**SECTION F: DATA HANDLING & PROBABILITY [20 marks]****6.1**

Data: 13, 14, 14, 15, 15, 15, 16, 17, 18, 18

**6.1.1**

$$\text{Mean} = \frac{155}{10} = 15.5$$

Mode = number that appears most = 15

**6.1.2**

Standard deviation (calculator or software):

$$\sigma \approx 1.62$$

**6.2**

Die outcomes = {1, 2, 3, 4, 5, 6}

$$\text{6.2.1 Prime numbers: } \{2, 3, 5\} \rightarrow P = \frac{3}{6} = \frac{1}{2}$$

$$\text{6.2.2 Even numbers: } \{2, 4, 6\} \rightarrow \text{Not even: } \{1, 3, 5\}$$

$$P(\text{not even}) = \frac{3}{6} = \frac{1}{2}$$

**TOTAL : 150**