

SMARTWIZ

GRADE 12 MATHEMATICS EXAM

MARKS: 150

TIME: 3 HOURS

SCHOOL _____

CLASS (eg. 4A) _____

SURNAME _____

NAME _____

MARKS	
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Instructions for Learners:

- Read all instructions carefully before you begin the exam.
- Write your full name and student number clearly on the answer sheet/book.
- Answer all questions unless otherwise instructed.
- Show all your work/calculations where necessary.
- Write neatly and clearly.
- Use only a blue or black pen. Do not use correction fluid or tape.
- Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of dishonesty will result in immediate disqualification from the exam.

This exam consists of Five pages, including the cover page.

QUESTION 1: ALGEBRAIC EXPRESSIONS & EQUATIONS [20 marks]

1.1 Solve for x :

$$x - 2x + 1 = 2x + 3x + 1 \quad \frac{x - 2}{x + 1} = \frac{2x + 3}{x + 1} \quad x + 1x - 2 = x + 12x + 3$$

1.2 Simplify:

$$2x^2 - 8x^2 - 4x + 4 \quad \frac{2x^2 - 8}{x^2 - 4x + 4} \quad x^2 - 4x + 4 \quad 2x^2 - 8$$

1.3 Solve for x :

$$\log_{f(0)} 5(x^2 - 4) = 2 \quad \log_5(x^2 - 4) = 2 \quad \log_5(x^2 - 4) = 2$$

QUESTION 2: FUNCTIONS AND GRAPHS [25 marks]

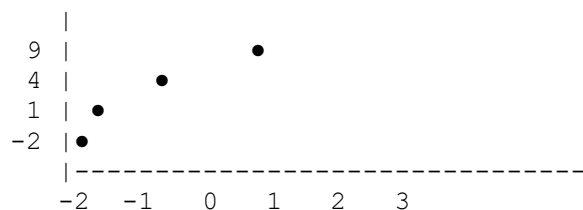
2.1 Given the piecewise function:

$$f(x) = \begin{cases} x + 3, & x \leq 1 \\ x^2, & x > 1 \end{cases} \quad f(x) = \begin{cases} x + 3, & x \leq 1 \\ x^2, & x > 1 \end{cases}$$

2.1.1 Calculate $f(1)$ and $f(2)$:

2.1.2 Sketch the graph from $x = -2$ to $x = 3$

Graph guide:



2.2 Given $f(x) = 3x - 2$, find $f^{-1}(x)$

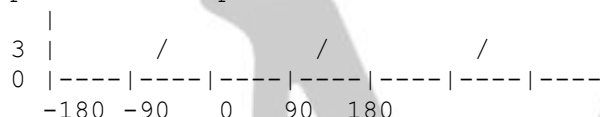
QUESTION 3: TRIGONOMETRY [25 marks]

3.1 Prove the identity:

$$\cos x \cdot \frac{1 - \sin x}{\cos x} + \cos x \cdot \frac{1 + \sin x}{\cos x} = 2 \cos x \cdot \frac{1}{\cos x} = 2$$

3.2 Sketch the graph of $y = \tan x$ for $x \in [-180^\circ, 180^\circ]$. Mark all asymptotes and intercepts.

Graph Sketch Template:



3.3 Solve for $\theta \in [0^\circ, 360^\circ]$:

$$\sin(2\theta) = \frac{\sqrt{3}}{2} \Rightarrow \sin(2\theta) = \frac{\sqrt{3}}{2}$$

QUESTION 4: CALCULUS [25 marks]

4.1 Given $f(x) = 5x^2$, find:

4.1.1 $f'(x)$

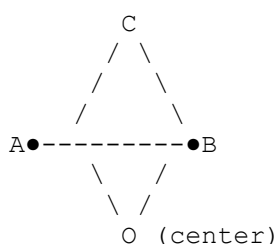
4.1.2 The gradient of the tangent at $x = 1$

4.2 Find the value of the definite integral:

$$\int_{-2}^2 (x^3 - x) dx$$

QUESTION 5: EUCLIDEAN GEOMETRY [30 marks]

In the figure, O is the center of the circle. AB is a diameter. C is a point on the circle. Let $\angle ACB = x$.



5.1 Name the theorem used to prove that $\angle ACB = 90^\circ$.

5.2 Prove that triangle ACB is a right-angled triangle

5.3 If $AB = 10$, find the area of triangle ACB .
(Use formula: $\frac{1}{2}ab \sin C$)

QUESTION 6: PROBABILITY & STATISTICS [25 marks]

6.1 A class contains 5 girls and 7 boys.

6.1.1 A learner is selected at random. Find the probability the learner is a girl.

6.1.2 Two learners are selected **without replacement**. Find the probability both are boys.

6.2 A dataset has values:
45, 50, 55, 60, 65, 70, 80

6.2.1 Find the **median**

6.2.2 Find the **range**

6.2.3 Find the **mean**

 **END OF PAPER**

TOTAL : 150

MYST PATHWORKS

MEMO

QUESTION 1: ALGEBRAIC EXPRESSIONS & EQUATIONS [20 marks]

1.1

$$x-2x+1=2x+3x+1 \Rightarrow x-2=2x+3 \Rightarrow -2-3=x \Rightarrow x=-5 \quad \frac{x-2}{x+1} = \frac{2x+3}{x+1} \Rightarrow x-2=2x+3 \Rightarrow -2-3=x \Rightarrow \boxed{x=-5} \quad x-2=x+1 \quad 2x+3 \Rightarrow x-2=2x+3 \Rightarrow -2-3=x \Rightarrow x=-5$$

1.2

$$2x^2-8x+4=2(x-2)(x-2)=2(x-2)(x+2)(x-2)=2(x+2)x-2 \quad (\text{for } x \neq 2) \quad \frac{2x^2-8}{x^2-4x+4} = \frac{2x(x-2)}{(x-2)^2} = \frac{2(x-2)}{(x-2)^2} = \frac{2}{x-2} \quad \text{quad } \text{for } x \neq 2$$

$$2x^2-8x+4=2(x-2)(x-2)=2(x-2)(x+2)(x-2)=2(x+2)x-2 \quad (\text{for } x \neq 2)$$

✓ **Final Answer:** $\frac{2(x+2)}{x-2}$

1.3

$$\log_5(x^2-4)=2 \Rightarrow x^2-4=5^2=25 \Rightarrow x^2=29 \Rightarrow x=\pm\sqrt{29} \quad \log_5(x^2-4)=2 \Rightarrow x^2-4=5^2=25 \Rightarrow x^2=29 \Rightarrow x=\pm\sqrt{29}$$

Domain restriction: $x^2-4>0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$ $x^2-4>0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

✓ **Final Answer:** $x=\pm\sqrt{29}$

QUESTION 2: FUNCTIONS AND GRAPHS [25 marks]

2.1.1

- $f(1)=1+3=4 \quad f(1)=1+3=\boxed{4}$
 - $f(2)=(2)^2=4 \quad f(2)=(2)^2=\boxed{4}$
-

2.1.2

Piecewise graph:

- Line from $x=-2$ to $x=1$, with slope 1 (i.e., $y = x + 3$)
- Parabola from $x=1$ to $x=3$, using $f(x)=x^2$

✓ Show a break at $x=1$, but values match.

2.2

$y=3x-2 \Rightarrow$ To find inverse: $x=3y-2 \Rightarrow 3y=x+2 \Rightarrow y=\frac{x+2}{3}$
 $y=3x-2 \Rightarrow$ To find inverse: $x=3y-2 \Rightarrow 3y=x+2 \Rightarrow y=\frac{x+2}{3}$

✓ **Final Answer:** $f^{-1}(x)=\frac{x+2}{3}$

QUESTION 3: TRIGONOMETRY [25 marks]

3.1

$$\begin{aligned} \cos x - \sin x + \cos x + \sin x &= \cos x(1 + \sin x) + \cos x(1 - \sin x) \\ &= \cos x(1 + \sin x + 1 - \sin x) = 2\cos x \\ \frac{\cos x}{1 - \sin x} + \frac{\cos x}{1 + \sin x} &= \frac{\cos x(1 + \sin x) + \cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{2\cos x}{1 - \sin^2 x} = \frac{2\cos x}{\cos^2 x} = \frac{2}{\cos x} \end{aligned}$$

✓ Proven

3.2

Sketch:

- Vertical asymptotes at $x = \pm 90^\circ$
- Intercepts at $x = 0^\circ, 180^\circ$

✓ Curve rises steeply through 0 and repeats every 180°

3.3

$$\begin{aligned} \sin(2\theta) = \frac{1}{2} &\Rightarrow 2\theta = 30^\circ, 150^\circ + 360^\circ k \Rightarrow \theta = 15^\circ, 75^\circ \\ \text{Also include: } 2\theta = 30^\circ + 360^\circ &\Rightarrow \theta = 15^\circ + 180^\circ \\ \sin(2\theta) = \frac{\sqrt{3}}{2} &\Rightarrow 2\theta = 60^\circ, 120^\circ + 360^\circ k \Rightarrow \theta = 30^\circ, 60^\circ \\ \text{Also include: } 2\theta = 60^\circ + 360^\circ &\Rightarrow \theta = 30^\circ + 180^\circ \end{aligned}$$

✓ Final Answers: $30^\circ, 60^\circ, 210^\circ, 240^\circ$ $\boxed{30^\circ, 60^\circ, 210^\circ, 240^\circ}$

QUESTION 4: CALCULUS [25 marks]

4.1.1

$$f(x) = 5x^{-2} \Rightarrow f'(x) = -10x^{-3} = -\frac{10}{x^3} \quad f(x) = 5x^{-2} \Rightarrow f'(x) = -10x^{-3} = \boxed{-\frac{10}{x^3}}$$

4.1.2

$$f'(1) = -\frac{10}{1^3} = -10 \quad f(1) = -\frac{10}{1^3} = \boxed{-10} \quad f'(1) = -10$$

4.2

$$\int_{-2}^2 (x^3 - x) dx = 0 \quad \text{Odd function over symmetric interval} \Rightarrow \int_{-2}^2 (x^3 - x) dx = 0$$

QUESTION 5: EUCLIDEAN GEOMETRY [30 marks]

5.1

✓ **Theorem:** Angle in a semicircle is 90°

"The angle subtended by a diameter at the circumference is a right angle."

5.2

Triangle ACB is right-angled since $\angle ACB = 90^\circ$

5.3

If $AB = 10$ and $\angle ACB = 90^\circ$, assume $AC = BC = 5$ (isosceles right triangle) $\text{Area} = \frac{1}{2}ab \sin C = \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin(90^\circ) = 12.5 \text{ units}^2$

If $AB = 10$ and $\angle ACB = 90^\circ$, assume $AC = BC = 5$ (isosceles right triangle) $\text{Area} = \frac{1}{2}ab \sin C = \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin(90^\circ) = \frac{25}{2} = \boxed{12.5} \text{ units}^2$

QUESTION 6: PROBABILITY & STATISTICS [25 marks]

6.1.1

Total learners = 12

$$P(\text{girl}) = \frac{5}{12} \quad P(\text{girl}) = \frac{5}{12}$$

6.1.2

$$P(\text{both boys}) = \frac{7}{12} \cdot \frac{6}{11} = \frac{7}{22} \quad P(\text{both boys}) = \frac{7}{22}$$

6.2.1

Sorted data: 45, 50, 55, 60, 65, 70, 80

Middle value = **60**

✓ Median = **60**

6.2.2

$$\text{Range} = \text{Max} - \text{Min} = 80 - 45 = 35$$

6.2.3

Mean =

$$\frac{45 + 50 + 55 + 60 + 65 + 70 + 80}{7} = \frac{425}{7} = 60.71$$

TOTAL : 150