

SMARTWIZ

GRADE 10 MATHEMATICS EXAM

MARKS: 100

MARKS	

TIME: 2 hours

SCHOOL _____

CLASS (e.g. 4A) _____

SURNAME _____

NAME _____

Instructions for Learners:

- Read all the instructions carefully before you begin the exam.
- Write your name and learner number clearly on the answer sheet/booklet.
- Answer all the questions unless otherwise instructed.
- Show all your work/calculations where applicable.
- Write neatly and legibly.
- Use only blue or black ink. *Do not use correction fluid or tape.*
- No electronic devices (calculators, phones, etc.) are allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of cheating will lead to disqualification.

This test consists of 6 pages including the cover page.

SECTION A: ALGEBRA AND EQUATIONS (30 MARKS)

Question 1 (15 marks)

1.1 Simplify:

$$\frac{3x^2y - 36x - 1y^2 \times 4x^3y - 12y^4}{6x - 1y^2} \times \frac{3x^2y^{-3}}{6x^{-1}y^2} \times \frac{4x^3y^{-1}}{2y^4}$$

..... (5)

1.2 Solve for x:

$$\frac{2x - 1}{3} + \frac{x + 4}{2} = 5$$

..... (5)

1.3 Solve the quadratic equation by completing the square:

$$x^2 - 6x + 5 = 0$$

..... (5)

Question 2 (15 marks)

2.1 Factorise completely:

$$4x^2 - 25$$

..... (3)

2.2 Expand and simplify:

$$(2x - 3)(x + 5) - (x - 1)^2$$

..... (6)

2.3 Solve the inequality and represent on a number line:

$$3x+7 \geq 2x+10 \quad 3x+7 \geq 2x+10$$

..... (6)

SECTION B: FUNCTIONS AND GRAPHS (30 MARKS)

Question 3 (15 marks)

3.1 Complete the table for $y=2x^2-3x+1$ $y = 2x^2 - 3x + 1$ $y=2x^2-3x+1$:

x	-1	0	1	2	3
y					

..... (5)

3.2 Sketch the graph of $y=2x^2-3x+1$ $y = 2x^2 - 3x + 1$ $y=2x^2-3x+1$ using the table. Label the vertex and intercepts. (5)

.....

.....

.....

3.3 Determine the coordinates of the vertex of the parabola. (5)

.....

.....

Question 4 (15 marks)

4.1 Given the linear function $y=-4x+7$ $y = -4x + 7$ $y=-4x+7$:

a) Find the gradient and y-intercept. (3)

.....

b) Calculate the value of y when $x=2$ $x = 2$ $x=2$. (2)

.....

4.2 Find the equation of the line that passes through the points $A(1,5)$ and $B(3,-1)$. (5)

.....

4.3 Determine the point(s) of intersection between $y = x^2 - 1$ and $y = 2x + 3$. (5)

.....

SECTION C: GEOMETRY AND MEASUREMENT (40 MARKS)

Question 5 (20 marks)

5.1 A triangle has sides $a = 8 \text{ cm}$, $b = 15 \text{ cm}$, and the included angle $\theta = 60^\circ$.

a) Calculate the length of side c using the cosine rule. (6)

.....

b) Calculate the area of the triangle. (4)

.....

5.2 In circle O , the radius is 14 cm.

a) Calculate the circumference. (3)

.....

b) Calculate the length of an arc subtended by an angle of 120° . (3)

.....

c) Calculate the area of the sector formed by the 120° angle. (4)

.....

Question 6 (20 marks)

6.1 A cylinder has a radius of 5 cm and height 12 cm.

a) Calculate the volume of the cylinder. (4)

.....

b) Calculate the total surface area of the cylinder. (6)

.....

6.2 Convert the following:

a) 3.2 kilometers to meters. (2)

.....

b) 4500 milliliters to liters. (2)

.....

c) 120 square meters to square centimeters. (3)

.....

d) 7.5 hectares to square meters. (3)

.....

TOTAL: 100 MARKS

MEMO

SECTION A: ALGEBRA AND EQUATIONS (30 MARKS)

Question 1

1.1 Simplify:

$$\frac{3x^2y - 36x - 1y^2}{4x^3y - 12y^4} = \frac{?}{6x - 1y} \cdot \frac{2x^2y - 3}{2y^4} = ?$$

Step 1: Simplify each fraction first:

$$\begin{aligned} \frac{3x^2y - 36x - 1y^2}{4x^3y - 12y^4} &= \frac{3x^2y - 36x - 1y^2}{4x^3y - 12y^4} = \frac{3x^2y - 36x - 1y^2}{4x^3y - 12y^4} \\ &= \frac{3x^2y - 36x - 1y^2}{4x^3y - 12y^4} = \frac{3x^2y - 36x - 1y^2}{4x^3y - 12y^4} \end{aligned}$$

Step 2: Multiply:

$$\begin{aligned} (12x^3y - 5)(2x^3y - 5) &= 1 \times x^3 + 3 \times y - 5 + (-5) = x^6y - 10 \\ (12x^3y - 5)(2x^3y - 5) &= 1 \times x^3 + 3 \times y - 5 + (-5) = x^6y - 10 \end{aligned}$$

Final answer:

$$x^6y - 10 = x^6y - 10$$

1.2 Solve for xxx:

$$2x - 13 + x + 42 = 5 \Rightarrow \frac{2x - 1}{3} + \frac{x + 4}{2} = 5 \Rightarrow 2x - 1 + 2x + 4 = 5$$

Multiply both sides by 6 (LCM of 3 and 2):

$$\begin{aligned} 2(2x - 1) + 3(x + 4) &= 30 \Rightarrow 2(2x - 1) + 3(x + 4) = 30 \\ 4x - 2 + 3x + 12 &= 30 \Rightarrow 7x + 10 = 30 \Rightarrow 7x = 20 \Rightarrow x = \frac{20}{7} \end{aligned}$$

1.3 Solve by completing the square:

$$x^2 - 6x + 5 = 0 \Rightarrow x^2 - 6x + 5 = 0$$

Step 1: Move constant to right:

$$x^2 - 6x = -5 \Rightarrow x^2 - 6x = -5$$

Step 2: Half coefficient of xxx, square it:

$$(-6)^2 = (-3)^2 = 9 \left(\frac{-6}{2} \right)^2 = (-3)^2 = 9 \quad (2-6)^2 = (-3)^2 = 9$$

Add 9 both sides:

$$x^2 - 6x + 9 = -5 + 9 \quad x^2 - 6x + 9 = -5 + 9 \quad (x-3)^2 = 4 \quad (x-3)^2 = 4$$

Step 3: Take square roots:

$$x-3 = \pm 2 \quad x-3 = \pm 2 \quad x = 3 \pm 2 \quad x = 3 \pm 2 \quad x = 5 \text{ or } x = 1 \quad \text{or } x = 1 \quad \text{or } x = 5$$

Question 2

2.1 Factorise:

$$4x^2 - 25 = (2x-5)(2x+5) \quad 4x^2 - 25 = (2x-5)(2x+5)$$

2.2 Expand and simplify:

$$(2x-3)(x+5) - (x-1)^2 \quad (2x-3)(x+5) - (x-1)^2$$

First expand:

$$(2x)(x) + (2x)(5) - 3(x) - 3(5) - (x^2 - 2x + 1) \quad (2x)(x) + (2x)(5) - 3(x) - 3(5) - (x^2 - 2x + 1) \quad 2x^2 + 10x - 3x - 15 - x^2 + 2x - 1 \quad 2x^2 + 10x - 3x - 15 - x^2 + 2x - 1$$

Simplify terms:

$$(2x^2 - x^2) + (10x - 3x + 2x) + (-15 - 1) = x^2 + 9x - 16 \quad (2x^2 - x^2) + (10x - 3x + 2x) + (-15 - 1) = x^2 + 9x - 16$$

2.3 Solve inequality:

$$3x + 7 \geq 2x + 10 \quad 3x + 7 \geq 2x + 10$$

Subtract $2x$ from both sides:

$$x + 7 \geq 10 \quad x + 7 \geq 10$$

Subtract 7:

$$x \geq 3 \quad x \geq 3$$

Number line: All values $x \geq 3$, including 3.

SECTION B: FUNCTIONS AND GRAPHS (30 MARKS)

Question 3

3.1 Complete the table $y=2x^2-3x+1$ $y = 2x^2 - 3x + 1$ $y=2x^2-3x+1$:

xxx	-1	0	1	2	3
yyy	6	1	0	3	10

Calculations:

- $x=-1: y=2(1)-3(-1)+1=2+3+1=6$ $x=-1: y=2(1)-3(-1)+1=2+3+1=6$
- $x=0: y=0-0+1=1$ $x=0: y=0-0+1=1$ $x=0: y=0-0+1=1$
- $x=1: y=2-3+1=0$ $x=1: y=2-3+1=0$ $x=1: y=2-3+1=0$
- $x=2: y=8-6+1=3$ $x=2: y=8-6+1=3$ $x=2: y=8-6+1=3$
- $x=3: y=18-9+1=10$ $x=3: y=18-9+1=10$ $x=3: y=18-9+1=10$

3.2 Graph plotting based on table.

(Students expected to draw and label points, vertex approx at $x=\frac{3}{4}$ $x=\frac{3}{4}$, $y\approx-0.125$ $y\approx-0.125$).

3.3 Vertex formula:

$$x_v = -\frac{b}{2a} = -\frac{-3}{2 \times 2} = \frac{3}{4} = 0.75$$

$$y_v = 2(0.75)^2 - 3(0.75) + 1 = 2(0.5625) - 2.25 + 1 = 1.125 - 2.25 + 1 = -0.125$$

Vertex: $(0.75, -0.125)$

Question 4

4.1 Given $y=-4x+7$ $y = -4x + 7$ $y=-4x+7$

a) Gradient = -4 , y-intercept = 7

b) $x=2$

$$y = -4(2) + 7 = -8 + 7 = -1$$

4.2 Equation of line through points A(1,5)A(1,5)A(1,5) and B(3,-1)B(3,-1)B(3,-1):

Slope:

$$m = \frac{-1 - 5}{3 - 1} = \frac{-6}{2} = -3$$

Use point-slope form:

$$y - 5 = -3(x - 1) \quad y - 5 = -3x + 3 \quad y = -3x + 8$$

4.3 Intersection between:

$$y = x^2 - 1$$

and

$$y = 2x + 3$$

Set equal:

$$x^2 - 1 = 2x + 3 \quad x^2 - 2x - 4 = 0$$

Use quadratic formula:

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$$

Approximate:

$$x_1 = 1 + 2.236 = 3.236, x_2 = 1 - 2.236 = -1.236$$

Find y:

$$y_1 = 2(3.236) + 3 = 6.472 + 3 = 9.472$$

$$y_2 = 2(-1.236) + 3 = -2.472 + 3 = 0.528$$

Points of intersection:

$$(3.236, 9.472), (-1.236, 0.528)$$

SECTION C: GEOMETRY AND MEASUREMENT (40 MARKS)

Question 5

5.1 Triangle with $a=8$, $b=15$, $\theta=60^\circ$:

a) Cosine rule for side c :

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

$$c^2 = 8^2 + 15^2 - 2(8)(15) \cos 60^\circ = 64 + 225 - 240 \times 0.5 = 289 - 120 = 169$$

$$c = \sqrt{169} = 13 \text{ cm}$$

b) Area:

$$\text{Area} = \frac{1}{2} ab \sin \theta$$

$$\text{Area} = \frac{1}{2} \times 8 \times 15 \times \sin 60^\circ = 60 \times \frac{\sqrt{3}}{2} = 30\sqrt{3} \approx 30 \times 1.732 = 51.96 \text{ cm}^2$$

5.2 Circle with radius 14 cm:

a) Circumference:

$$C = 2\pi r = 2 \times 3.14 \times 14 = 87.92 \text{ cm}$$

b) Length of arc (120°):

$$\text{Arc length} = \frac{120}{360} \times 2\pi r = \frac{1}{3} \times 87.92 = 29.31 \text{ cm}$$

c) Area of sector:

$$\text{Sector area} = \frac{120}{360} \times \pi r^2 = \frac{1}{3} \times 3.14 \times 14^2 = \frac{1}{3} \times 3.14 \times 196 = 205.3 \text{ cm}^2$$

Question 6

6.1 Cylinder: radius $r=5$ cm, height $h=12$ cm.

a) Volume:

$$V = \pi r^2 h = 3.14 \times 25 \times 12 = 942 \text{ cm}^3$$

$$V = \pi r^2 h = 3.14 \times 25 \times 12 = 942 \text{ cm}^3$$

b) Total surface area:

$$A = 2\pi r(r+h) = 2 \times 3.14 \times 5 \times (5+12) = 31.4 \times 17 = 533.8 \text{ cm}^2$$

$$A = 2\pi r(r+h) = 2 \times 3.14 \times 5 \times (5+12) = 31.4 \times 17 = 533.8 \text{ cm}^2$$

6.2 Unit conversions:

a) $3.2 \text{ km} = 3.2 \times 1000 = 3200 \text{ m}$
 $3.2 \text{ km} = 3.2 \times 1000 = 3200 \text{ m}$

b) $4500 \text{ ml} = 4500 \div 1000 = 4.5 \text{ L}$
 $4500 \text{ ml} = 4500 \div 1000 = 4.5 \text{ L}$

c) $120 \text{ m}^2 = 120 \times 10,000 = 1,200,000 \text{ cm}^2$
 $120 \text{ m}^2 = 120 \times 10,000 = 1,200,000 \text{ cm}^2$

d) $7.5 \text{ hectares} = 7.5 \times 10,000 = 75,000 \text{ m}^2$
 $7.5 \text{ hectares} = 7.5 \times 10,000 = 75,000 \text{ m}^2$

END OF MEMO