SMARTWIZ

GRADE11 MATHEMATICS EXAM

MARKS	

MARKS: 100

TIME: 2 HOURS

SCHOOL ____

CLASS (eg. 4A) ____________

SURNAME ____

NAME ____

Instructions for Learners:

- Read all instructions carefully before you begin the exam.
- Write your full name and student number clearly on the answer sheet/book.
- Answer all questions unless otherwise instructed.
- Show all your work/calculations where necessary.
- Write neatly and clearly.
- Use only a blue or black pen. Do not use correction fluid or tape.
- Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of cheating will result in immediate disqualification from the exam.

This exam consists of six pages, including the cover page.

SECTION A: ALGEBRA AND FUNCTIONS (40 marks)

1. Simplify the expression:

2x2−3x+5x+4x2+x−12x\frac{2x^2 - 3x + 5}{x} + \frac{4x^2 + x - 1}{2x}x2x2−3x+5+2x4x2+x−1

Answer:

2. Solve for xxx:

3x2−5x−2=03x^2 - 5x - 2 = 03x2−5x−2=0

Answer:

3. If f(x)=2x2−3x+4f(x) = 2x^2 - 3x + 4f(x)=2x2−3x+4, find:
a) f(2)f(2)f(2)

b) f(x+1)f(x+1)f(x+1)

4. Find the inverse function of:

f(x)=3x−25f(x) = \frac{3x - 2}{5}f(x)=53x−2

Answer:

SECTION B: TRIGONOMETRY (20 marks)

5. In triangle ABCABCABC, angle A=30∘A = 30^\circA=30∘, side BC=10 cmBC = 10\,cmBC=10cm. Find the length of side ABABAB if ABABAB is opposite angle CCC.
Answer:

6. Simplify:

sin2θ+cos2θ\sin^2 \theta + \cos^2 \thetasin2θ+cos2θ

Answer:

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7. Prove that:

1+tan2θ=sec2θ1 + \tan^2 \theta = \sec^2 \theta1+tan2θ=sec2θ

Answer:

SECTION C: DIFFERENTIATION AND CALCULUS (20 marks)

8. Differentiate the following functions with respect to xxx:
a) y=5x3−4x+7y = 5x^3 - 4x + 7y=5x3−4x+7

b) y=2x+3x2y = \frac{2}{x} + 3x^2y=x2+3x2

9. Find the stationary points of the function:

y=x3−6x2+9x+15y = x^3 - 6x^2 + 9x + 15y=x3−6x2+9x+15

Answer:

SECTION D: LOGARITHMS AND EXPONENTIALS (20 marks)

10. Solve for xxx:

log2(x+3)=4\log_2 (x + 3) = 4log2(x+3)=4

Answer:

11. Express as a single logarithm:

log a+2log b−12log c\log a + 2 \log b - \frac{1}{2} \log cloga+2logb−21logc

Answer:

12. If y=3xy = 3^xy=3x, find dydx\frac{dy}{dx}dxdy.
Answer:

END OF EXAM

MEMO

SECTION A: ALGEBRA AND FUNCTIONS

1. Simplify:

2x2−3x+5x+4x2+x−12x=2x2−3x+5x+4x2+x−12x\frac{2x^2 - 3x + 5}{x} + \frac{4x^2 + x - 1}{2x} = \frac{2x^2 - 3x + 5}{x} + \frac{4x^2 + x - 1}{2x}x2x2−3x+5+2x4x2+x−1=x2x2−3x+5+2x4x2+x−1

First, rewrite each term:

=2x2x−3xx+5x+4x22x+x2x−12x= \frac{2x^2}{x} - \frac{3x}{x} + \frac{5}{x} + \frac{4x^2}{2x} + \frac{x}{2x} - \frac{1}{2x}=x2x2−x3x+x5+2x4x2+2xx−2x1

Simplify terms:

=2x−3+5x+2x+12−12x= 2x - 3 + \frac{5}{x} + 2x + \frac{1}{2} - \frac{1}{2x}=2x−3+x5+2x+21−2x1

Combine like terms:

2x+2x=4x,−3+12=−2.52x + 2x = 4x, \quad -3 + \frac{1}{2} = -2.52x+2x=4x,−3+21=−2.5

So final answer:

4x−2.5+5x−12x=4x−2.5+(10−1)2x=4x−2.5+92x4x - 2.5 + \frac{5}{x} - \frac{1}{2x} = 4x - 2.5 + \frac{(10 - 1)}{2x} = 4x - 2.5 + \frac{9}{2x}4x−2.5+x5−2x1=4x−2.5+2x(10−1)=4x−2.5+2x9

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2. Solve:

3x2−5x−2=03x^2 - 5x - 2 = 03x2−5x−2=0

Using quadratic formula x=−b±b2−4ac2ax = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}x=2a−b±b2−4ac where a=3,b=−5,c=−2a=3, b=-5, c=-2a=3,b=−5,c=−2:

Discriminant:

(−5)2−4(3)(−2)=25+24=49(-5)^2 - 4(3)(-2) = 25 + 24 = 49(−5)2−4(3)(−2)=25+24=49

Roots:

x=5±76x = \frac{5 \pm 7}{6}x=65±7

So:

x=5+76=2orx=5−76=−13x = \frac{5 + 7}{6} = 2 \quad \text{or} \quad x = \frac{5 - 7}{6} = -\frac{1}{3}x=65+7=2orx=65−7=−31

3a.

f(2)=2(2)2−3(2)+4=2(4)−6+4=8−6+4=6f(2) = 2(2)^2 - 3(2) + 4 = 2(4) - 6 + 4 = 8 - 6 + 4 = 6f(2)=2(2)2−3(2)+4=2(4)−6+4=8−6+4=6

3b.

f(x+1)=2(x+1)2−3(x+1)+4=2(x2+2x+1)−3x−3+4=2x2+4x+2−3x+1=2x2+x+3f(x+1) = 2(x+1)^2 - 3(x+1) + 4 = 2(x^2 + 2x + 1) - 3x - 3 + 4 = 2x^2 + 4x + 2 - 3x + 1 = 2x^2 + x + 3f(x+1)=2(x+1)2−3(x+1)+4=2(x2+2x+1)−3x−3+4=2x2+4x+2−3x+1=2x2+x+3

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4. Inverse of

f(x)=3x−25f(x) = \frac{3x - 2}{5}f(x)=53x−2

Let y=3x−25y = \frac{3x - 2}{5}y=53x−2, solve for xxx:

5y=3x−2 ⟹ 3x=5y+2 ⟹ x=5y+235y = 3x - 2 \implies 3x = 5y + 2 \implies x = \frac{5y + 2}{3}5y=3x−2⟹3x=5y+2⟹x=35y+2

Swap xxx and yyy:

f−1(x)=5x+23f^{-1}(x) = \frac{5x + 2}{3}f−1(x)=35x+2

SECTION B: TRIGONOMETRY

5. Given A=30∘A = 30^\circA=30∘, side BC=10cmBC=10cmBC=10cm
Assuming right triangle with angle AAA, find ABABAB opposite CCC (some info missing, but assuming):

If side opposite angle AAA is BCBCBC, then:

AB=BC×tan30∘=10×33=1033≈5.77cmAB = BC \times \tan 30^\circ = 10 \times \frac{\sqrt{3}}{3} = \frac{10\sqrt{3}}{3} \approx 5.77cmAB=BC×tan30∘=10×33=3103≈5.77cm

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6. Simplify:

sin2θ+cos2θ=1\sin^2 \theta + \cos^2 \theta = 1sin2θ+cos2θ=1

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7. Prove:

1+tan2θ=sec2θ1 + \tan^2 \theta = \sec^2 \theta1+tan2θ=sec2θ

Using definitions:

tanθ=sinθcosθ,secθ=1cosθ\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sec \theta = \frac{1}{\cos \theta}tanθ=cosθsinθ,secθ=cosθ1

Start with RHS:

sec2θ=1cos2θ\sec^2 \theta = \frac{1}{\cos^2 \theta}sec2θ=cos2θ1

LHS:

1+tan2θ=1+sin2θcos2θ=cos2θ+sin2θcos2θ=1cos2θ=sec2θ1 + \tan^2 \theta = 1 + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta1+tan2θ=1+cos2θsin2θ=cos2θcos2θ+sin2θ=cos2θ1=sec2θ

Q.E.D.

SECTION C: DIFFERENTIATION AND CALCULUS

8a.

y=5x3−4x+7 ⟹ dydx=15x2−4y = 5x^3 - 4x + 7 \implies \frac{dy}{dx} = 15x^2 - 4y=5x3−4x+7⟹dxdy=15x2−4

8b.

y=2x+3x2=2x−1+3x2 ⟹ dydx=−2x−2+6x=−2x2+6xy = \frac{2}{x} + 3x^2 = 2x^{-1} + 3x^2 \implies \frac{dy}{dx} = -2x^{-2} + 6x = -\frac{2}{x^2} + 6xy=x2+3x2=2x−1+3x2⟹dxdy=−2x−2+6x=−x22+6x

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9. Stationary points of

y=x3−6x2+9x+15y = x^3 - 6x^2 + 9x + 15y=x3−6x2+9x+15

Find dydx\frac{dy}{dx}dxdy:

dydx=3x2−12x+9\frac{dy}{dx} = 3x^2 - 12x + 9dxdy=3x2−12x+9

Set to zero:

3x2−12x+9=0 ⟹ x2−4x+3=03x^2 - 12x + 9 = 0 \implies x^2 - 4x + 3 = 03x2−12x+9=0⟹x2−4x+3=0

Factor:

(x−3)(x−1)=0 ⟹ x=1,3(x-3)(x-1) = 0 \implies x = 1, 3(x−3)(x−1)=0⟹x=1,3

Find yyy values:

y(1)=1−6+9+15=19y(1) = 1 - 6 + 9 + 15 = 19y(1)=1−6+9+15=19 y(3)=27−54+27+15=15y(3) = 27 - 54 + 27 + 15 = 15y(3)=27−54+27+15=15

Stationary points at:

(1,19)and(3,15)(1, 19) \quad \text{and} \quad (3, 15)(1,19)and(3,15)

SECTION D: LOGARITHMS AND EXPONENTIALS

10. Solve:

log2(x+3)=4 ⟹ x+3=24=16 ⟹ x=13\log_2 (x + 3) = 4 \implies x + 3 = 2^4 = 16 \implies x = 13log2(x+3)=4⟹x+3=24=16⟹x=13

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11. Express as a single logarithm:

loga+2logb−12logc=loga+logb2−logc1/2=log(ab2c)\log a + 2 \log b - \frac{1}{2} \log c = \log a + \log b^2 - \log c^{1/2} = \log \left(\frac{a b^2}{\sqrt{c}} \right)loga+2logb−21 logc=loga+logb2−logc1/2=log(cab2)

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12. If y=3xy = 3^xy=3x, find dydx\frac{dy}{dx}dxdy:

dydx=3xln3\frac{dy}{dx} = 3^x \ln 3dxdy=3xln3

TOTAL : 100