SMARTWIZ

GRADE11 MATHEMATICS EXAM

MARKS	

MARKS: 100

TIME: 2 HOURS

SCHOOL ____

CLASS (eg. 4A) ____________

SURNAME ____

NAME ____

Instructions for Learners:

- Read all instructions carefully before you begin the exam.
- Write your full name and student number clearly on the answer sheet/book.
- Answer all questions unless otherwise instructed.
- Show all your work/calculations where necessary.
- Write neatly and clearly.
- Use only a blue or black pen. Do not use correction fluid or tape.
- Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.
- Raise your hand if you have any questions.
- Do not talk to other learners during the exam.
- Any form of cheating will result in immediate disqualification from the exam.

This exam consists of six pages, including the cover page.

SECTION A: ALGEBRA AND FUNCTIONS (35 marks)

1. Factor completely:

4x3−27y34x^3 - 27y^34x3−27y3

Answer:

2. Solve for xxx:

2x+5x−1=3\frac{2x + 5}{x - 1} = 3x−12x+5=3

Answer:

3. Given the function f(x)=2x+1x−3f(x) = \frac{2x+1}{x-3}f(x)=x−32x+1, find:
a) f(4)f(4)f(4)

b) The domain of fff

4. Simplify:

(3x2y−1)3×(2x−1y2)2(3x^2 y^{-1})^3 \times (2x^{-1} y^2)^2(3x2y−1)3×(2x−1y2)2

Answer:

SECTION B: TRIGONOMETRY (25 marks)

5. Given sinA=35\sin A = \frac{3}{5}sinA=53 and angle AAA is acute, find cosA\cos AcosA and tanA\tan AtanA.
Answer:

6. Prove that:

1−cos2θsin2θ=tanθ\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \thetasin2θ1−cos2θ=tanθ

Answer:

7. In triangle XYZXYZXYZ, side XY=8cmXY = 8cmXY=8cm, side YZ=6cmYZ = 6cmYZ=6cm, and angle Y=60∘Y = 60^\circY=60∘. Find the length of side XZXZXZ using the cosine rule.
Answer:

SECTION C: DIFFERENTIATION AND CALCULUS (20 marks)

8. Differentiate:
a) y=x3+1y = \sqrt{x^3 + 1}y=x3+1

b) y=5xx2+1y = \frac{5x}{x^2 + 1}y=x2+15x

9. Find the coordinates of the turning points of:

y=2x3−9x2+12x+1y = 2x^3 - 9x^2 + 12x + 1y=2x3−9x2+12x+1

Answer:

SECTION D: LOGARITHMS AND EXPONENTIALS (20 marks)

10. Solve for xxx:

e2x=7e^{2x} = 7e2x=7

Answer:

11. Express as a single logarithm:

3log x−12log (x+1)+log 43 \log x - \frac{1}{2} \log (x + 1) + \log 43logx−21log(x+1)+log4

Answer:

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12. If y=log 5(2x+1)y = \log_5 (2x + 1)y=log5(2x+1), find dydx\frac{dy}{dx}dxdy.
Answer:

END OF EXAM

TOTAL : 100

MEMO

SECTION A: ALGEBRA AND FUNCTIONS

1. Factor completely:

4x3−27y34x^3 - 27y^34x3−27y3

This is a difference of cubes:

a3−b3=(a−b)(a2+ab+b2)a^3 - b^3 = (a - b)(a^2 + ab + b^2)a3−b3=(a−b)(a2+ab+b2)

Where a=4x33=43·xa = \sqrt[3]{4x^3} = \sqrt[3]{4} \cdot xa=34x3=34·x, but since 4 is not a perfect cube, better to write as:

4x3−27y3=(2x)3−(3y)3=(2x−3y)(4x2+6xy+9y2)4x^3 - 27y^3 = (2x)^3 - (3y)^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)4x3−27y3=(2x)3−(3y)3=(2x−3y)(4x2+6xy+9y2)

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2. Solve for xxx:

2x+5x−1=3\frac{2x + 5}{x - 1} = 3x−12x+5=3

Multiply both sides by x−1x-1x−1:

2x+5=3(x−1)2x + 5 = 3(x - 1)2x+5=3(x−1)

Expand right side:

2x+5=3x−32x + 5 = 3x - 32x+5=3x−3

Bring all terms to one side:

2x+5−3x+3=0 ⟹ −x+8=02x + 5 - 3x + 3 = 0 \implies -x + 8 = 02x+5−3x+3=0⟹−x+8=0

Solve for xxx:

x=8x = 8x=8

3a.

f(4)=2(4)+14−3=8+11=9f(4) = \frac{2(4) + 1}{4 - 3} = \frac{8 + 1}{1} = 9f(4)=4−32(4)+1=18+1=9

3b. Domain of f(x)=2x+1x−3f(x) = \frac{2x+1}{x-3}f(x)=x−32x+1
Denominator cannot be zero:

x−3≠0 ⟹ x≠3x - 3 \neq 0 \implies x \neq 3x−3□=0⟹x□=3

Domain: all real numbers except x=3x=3x=3.

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4. Simplify:

(3x2y−1)3×(2x−1y2)2(3x^2 y^{-1})^3 \times (2x^{-1} y^2)^2(3x2y−1)3×(2x−1y2)2

Expand powers:

=33x6y−3×22x−2y4=27x6y−3×4x−2y4= 3^3 x^{6} y^{-3} \times 2^2 x^{-2} y^{4} = 27 x^6 y^{-3} \times 4 x^{-2} y^{4}=33x6y−3×22x−2y4=27x6y−3×4x−2y4

Multiply coefficients and like bases:

=27×4×x6+(−2)×y−3+4=108x4y1=108x4y= 27 \times 4 \times x^{6 + (-2)} \times y^{-3 + 4} = 108 x^4 y^1 = 108 x^4 y=27×4×x6+(−2)×y−3+4=108x4y1=108x4y

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SECTION B: TRIGONOMETRY

5. Given sinA=35\sin A = \frac{3}{5}sinA=53, acute AAA:

Use Pythagoras to find cosA\cos AcosA:

cosA=1−sin2A=1−(35)2=1−925=1625=45\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}cosA=1−sin2A =1−(53)2=1−259=2516=54

Calculate tanA\tan AtanA:

tanA=sinAcosA=3/54/5=34\tan A = \frac{\sin A}{\cos A} = \frac{3/5}{4/5} = \frac{3}{4}tanA=cosAsinA=4/53/5=43

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6. Prove:

1−cos2θsin2θ=tanθ\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \thetasin2θ1−cos2θ=tanθ

Recall:

cos2θ=1−2sin2θ,sin2θ=2sinθcosθ\cos 2\theta = 1 - 2 \sin^2 \theta, \quad \sin 2\theta = 2 \sin \theta \cos \thetacos2θ=1−2sin2θ,sin2θ=2sinθcosθ

LHS:

$$\frac{1 - (1 - 2 \sin^2 \theta)}{2 \sin \theta \cos \theta} = \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

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7. Use cosine rule in $\triangle XYZ$:

$$XZ^2 = XY^2 + YZ^2 - 2(XY)(YZ) \cos Y$$

Given:

$$XY = 8, \quad YZ = 6, \quad \angle Y = 60^\circ$$

Calculate:

$$XZ^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \cos 60^\circ = 64 + 36 - 96 \times 0.5 = 100 - 48 = 52$$

So:

$$XZ = \sqrt{52} = 2 \sqrt{13} \approx 7.21 \text{ cm}$$

SECTION C: DIFFERENTIATION AND CALCULUS

8a.

$$y = \sqrt{x^3 + 1} = (x^3 + 1)^{1/2}$$

Using chain rule:

$$\frac{dy}{dx} = \frac{1}{2}(x^3 + 1)^{-1/2} \times 3x^2 = \frac{3x^2}{2 \sqrt{x^3 + 1}}$$

8b.

$$y = \frac{5x}{x^2 + 1}$$

Use quotient rule:

$$\frac{dy}{dx} = \frac{(5)(x^2 + 1) - 5x (2x)}{(x^2 + 1)^2} = \frac{5x^2 + 5 - 10x^2}{(x^2 + 1)^2} = \frac{-5x^2 + 5}{(x^2 + 1)^2} = \frac{5(1 - x^2)}{(x^2 + 1)^2}$$

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9. Find turning points of:

y=2x3−9x2+12x+1y = 2x^3 - 9x^2 + 12x + 1y=2x3−9x2+12x+1

Differentiate:

dydx=6x2−18x+12\frac{dy}{dx} = 6x^2 - 18x + 12dxdy=6x2−18x+12

Set derivative zero:

6x2−18x+12=0 ⟹ x2−3x+2=06x^2 - 18x + 12 = 0 \implies x^2 - 3x + 2 = 06x2−18x+12=0⟹x2−3x+2=0

Factor:

(x−1)(x−2)=0 ⟹ x=1,2(x-1)(x-2) = 0 \implies x=1, 2(x−1)(x−2)=0⟹x=1,2

Find yyy values:

y(1)=2−9+12+1=6y(1) = 2 - 9 + 12 + 1 = 6y(1)=2−9+12+1=6 y(2)=16−36+24+1=5y(2) = 16 - 36 + 24 + 1 = 5y(2)=16−36+24+1=5

Turning points at:

(1,6),(2,5)(1,6), (2,5)(1,6),(2,5)

SECTION D: LOGARITHMS AND EXPONENTIALS

10. Solve:

e2x=7e^{2x} = 7e2x=7

Take natural log:

2x=ln7 ⟹ x=ln722x = \ln 7 \implies x = \frac{\ln 7}{2}2x=ln7⟹x=2ln7

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11. Express as a single logarithm:

3logx−12log(x+1)+log43 \log x - \frac{1}{2} \log (x + 1) + \log 43logx−21log(x+1)+log4

Rewrite powers:

=logx3−log(x+1)1/2+log4=log(4x3x+1)= \log x^3 - \log (x+1)^{1/2} + \log 4 = \log \left(\frac{4 x^3}{\sqrt{x+1}} \right)=logx3−log(x+1)1/2+log4=log(x+14x3)

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12. If

y=log⁡5(2x+1)y = \log_5 (2x + 1)y=log5(2x+1)

Use chain rule and base change formula:

dydx=1(2x+1)ln⁡5×2=2(2x+1)ln⁡5\frac{dy}{dx} = \frac{1}{(2x + 1) \ln 5} \times 2 = \frac{2}{(2x + 1) \ln 5}dxdy=(2x+1)ln51×2=(2x+1)ln52

TOTAL : 100