SMARTWIZ

GRADE 12 MATHEMATICS EXAM

MARKS: 150

MARKS	

TIME: 3 HOURS

SCHOOL _____

CLASS (eg. 4A) _____

SURNAME _____

NAME _____

Instructions for Learners:

• Read all instructions carefully before you begin the exam.

• Write your full name and student number clearly on the answer sheet/book.

• Answer all questions unless otherwise instructed.

• Show all your work/calculations where necessary.

• Write neatly and clearly.

• Use only a blue or black pen. Do not use correction fluid or tape.

• Electronic devices (calculators, cell phones, etc.) are not allowed unless explicitly permitted.

• Raise your hand if you have any questions.

• Do not talk to other learners during the exam.

• Any form of dishonesty will result in immediate disqualification from the exam.

This exam consists of Five pages, including the cover page.

QUESTION 1: NUMBER PATTERNS [20 marks]

1.1 Consider the sequence: 3; 7; 13; 21; 31; ...
1.1.1 Write down the 6th term of the sequence.

_____

_____

_____

1.1.2 Determine a general formula for the nth term.

_____

_____

_____

1.2 A certain geometric sequence has a first term of 5 and a common ratio of 3.
1.2.1 Write down the first 4 terms.

_____

_____

1.2.2 Calculate the 8th term.

_____

_____

_____

QUESTION 2: FUNCTIONS AND INEQUALITIES [25 marks]

2.1 Sketch the graph of $f(x) = \frac{2}{x}$ and show the asymptotes.

2.2 Solve for x:

$$x^2 - 5x + 6 < 0$$

2.3 Given the function $g(x) = -x^2 + 6x - 8$, find the coordinates of the:

- Vertex
- Axis of symmetry

QUESTION 3: TRIGONOMETRY [20 marks]

3.1 Simplify the expression:

$$\frac{\tan^2 x + 1}{\sec^2 x}$$

3.2 Given $\tan \theta = \frac{3}{4}$ and $\theta \in (0^\circ, 90^\circ)$, find $\sin \theta$ and $\cos \theta$.

3.3 Prove the identity:

$$\frac{1 - \cos^2 x}{\sin x} = \sin x$$

QUESTION 4: CALCULUS [25 marks]

4.1 Determine the derivative of:

$$f(x) = \frac{3x^2 - 2x + 1}{x}$$

4.2 Determine the equation of the tangent to the curve $y = x^3 - 4x$ at the point where $x = 2$.

4.3 Use first principles to find $f'(x)$ if $f(x) = x^2$.

QUESTION 5: GEOMETRY & MEASUREMENT [30 marks]

5.1 In the diagram below, $ABCD$ is a cyclic quadrilateral. $AB \parallel CD$. Angle $ABC = 78^\circ$.

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 A_____B
   /           /
  /           /
 D_____C
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5.1.1 Calculate angle DAB.

5.1.2 Give a reason why $ABCD$ is a cyclic quadrilateral.

5.2 The volume of a cylinder is given as 500 cm³. The radius is 5 cm. Calculate the height.

$$V = \pi r^2 h$$

QUESTION 6: DATA HANDLING & PROBABILITY [30 marks]

6.1 A class writes a test and the marks (out of 10) are:
3, 4, 4, 5, 5, 5, 6, 7, 7, 8, 9, 10

6.1.1 Draw a box-and-whisker plot for the data. *(Sketch on paper)*

6.1.2 Find the interquartile range (IQR).

_____

_____

6.2 A bag contains 5 red, 3 blue, and 2 green balls. A ball is chosen at random.
6.2.1 What is the probability that the ball is not green?

_____

_____

6.2.2 If two balls are drawn without replacement, what is the probability that both are red?

_____

_____

_____

END OF PAPER

MEMO

QUESTION 1: NUMBER PATTERNS [20 marks]

1.1.1
6th term: 3, 7, 13, 21, 31 → next difference = +11 →
6th term = 31 + 11 = 42

1.1.2
First differences: 4, 6, 8, 10 → second differences are constant (+2) → quadratic pattern
Let $T_n = an^2 + bn + c$
Form equations:

- $a(1)^2 + b(1) + c = 3$
- $a(2)^2 + b(2) + c = 7$
- $a(3)^2 + b(3) + c = 13$
  Solve to get: $a = 1, b = 1, c = 1$

✅ **General formula: $T_n = n^2 + n + 1$**

1.2.1
Geometric sequence: 5; 15; 45; 135

1.2.2

$T_8 = ar^{n-1} = 5 \cdot 3^7 = 5 \cdot 2187 = 10{,}935$

QUESTION 2: FUNCTIONS AND INEQUALITIES [25 marks]

2.1
Sketch of $f(x) = \frac{2}{x}$:

- Vertical asymptote at $x = 0$, horizontal asymptote at $y = 0$
- Shape: hyperbola

2.2

$x^2 - 5x + 6 < 0 \Rightarrow (x - 2)(x - 3) < 0 \Rightarrow x \in (2, 3)$

2.3
$g(x) = -x^2 + 6x - 8$

- Vertex: x=−b2a=−6−2=3x = \frac{-b}{2a} = \frac{-6}{-2} = 3x=2a−b=−2−6=3
  y=−32+6(3)−8=−9+18−8=1y = -3^2 + 6(3) - 8 = -9 + 18 - 8 = 1y=−32+6(3)−8=−9+18−8=1 →
  Vertex: **(3, 1)**
- Axis of symmetry: **x = 3**

QUESTION 3: TRIGONOMETRY [20 marks]

3.1

tan⁡fo2x+1sec⁡fo2x=sec⁡fo2xsec⁡fo2x=1\frac{\tan^2 x + 1}{\sec^2 x} = \frac{\sec^2 x}{\sec^2 x} = 1sec2xtan2x+1=sec2xsec2x=1

3.2
Given: tan⁡foθ=34⇒\tan \theta = \frac{3}{4} \Rightarrowtanθ=43⇒ Opp = 3, Adj = 4 → Hyp = 5

- sin⁡foθ=35,cos⁡foθ=45\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}sinθ=53,cosθ=54

3.3

1−cos⁡fo2xsin⁡fox=sin⁡fo2xsin⁡fox=sin⁡fox\frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin xsinx1−cos2x=sinxsin2x=sinx

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QUESTION 4: CALCULUS [25 marks]

4.1

f(x)=3x2−2x+1x=3x−2+1x⇒f′(x)=3−1x2f(x) = \frac{3x^2 - 2x + 1}{x} = 3x - 2 + \frac{1}{x} \Rightarrow f'(x) = 3 - \frac{1}{x^2}f(x)=x3x2−2x+1=3x−2+x1⇒f′(x)=3−x21

4.2
y=x3−4xy = x^3 - 4xy=x3−4x, x=2x = 2x=2

- y=8−8=0y = 8 - 8 = 0y=8−8=0 → Point: (2, 0)
- Derivative: y′=3x2−4y' = 3x^2 - 4y′=3x2−4, at x=2x = 2x=2:
  y′=3(4)−4=12−4=8y' = 3(4) - 4 = 12 - 4 = 8y′=3(4)−4=12−4=8
- Equation of tangent:
  y−0=8(x−2)⇒y=8x−16y - 0 = 8(x - 2) \Rightarrow y = 8x - 16y−0=8(x−2)⇒y=8x−16

4.3 (First Principles)

f(x)=x2⇒f′(x)=lim⁡foh→0(x+h)2−x2h=lim⁡foh→0x2+2xh+h2−x2h=lim⁡foh→02xh+h2h=lim⁡foh→0(2x+h)=2xf(x) = x^2 \Rightarrow f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} (2x + h) = 2xf(x)=x2⇒f′(x)=h→0limh(x+h)2−x2=h→0limhx2+2xh+h2−x2=h→0limh2xh+h2=h→0lim(2x+h)=2x

QUESTION 5: GEOMETRY & MEASUREMENT [30 marks]

5.1.1
$AB \parallel CD$, and in cyclic quadrilateral, alternate angles are equal:

- $\angle DAB = \angle BCD = 78^\circ$

5.1.2
Reason: All vertices lie on the circumference of a circle.

5.2
Volume:

$$500 = \pi \cdot 5^2 \cdot h \Rightarrow 500 = 25\pi h \Rightarrow h = \frac{500}{25\pi} \approx \frac{20}{\pi} \approx 6.37 \text{ cm}$$

QUESTION 6: DATA HANDLING & PROBABILITY [30 marks]

6.1.1
Marks in order: 3, 4, 4, 5, 5, 5, 6, 7, 7, 8, 9, 10

- Min = 3, Q1 = 4.5, Median = 5.5, Q3 = 7.5, Max = 10
  (Draw box-and-whisker accordingly)

6.1.2
IQR = Q3 - Q1 = 7.5 - 4.5 = **3**

6.2.1
Total balls = 10 → Not green = 8

$$P(\text{not green}) = \frac{8}{10} = 0.8$$

6.2.2

$$P(\text{both red}) = \frac{5}{10} \cdot \frac{4}{9} = \frac{20}{90} = \frac{2}{9}$$

TOTAL : 150