Relative sizes of voxel-level rs-fMRI modules and their constituents

Explosive percolation and anti-fragmentation in fMRI network construction

Satoru Hayasaka, Ph.D.
University of Texas at Austin

Explosive percolation and anti-fragmentation in fMRI network construction

Satoru Hayasaka, Ph.D.

University of Texas at Austin

Outline

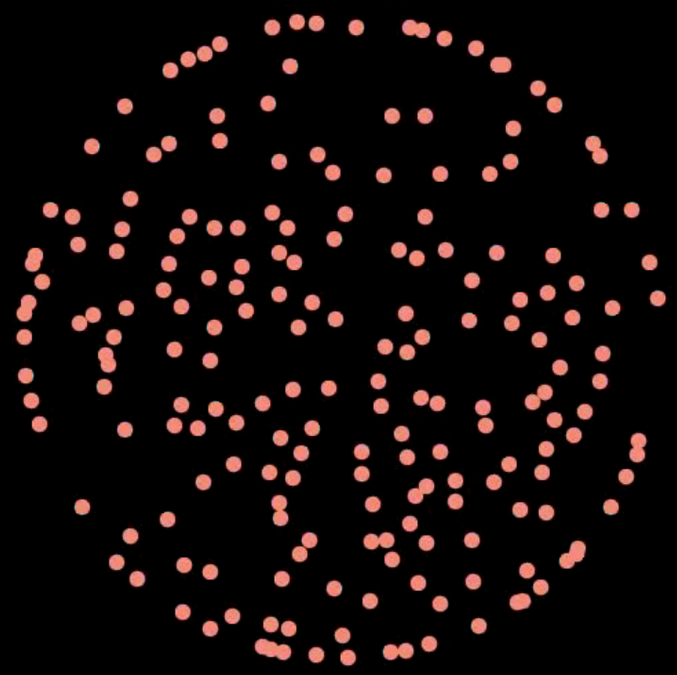
- Explosive percolation
- Thresholded networks
- Anti-fragmentation
- Demonstration (with Jupyter notebook)

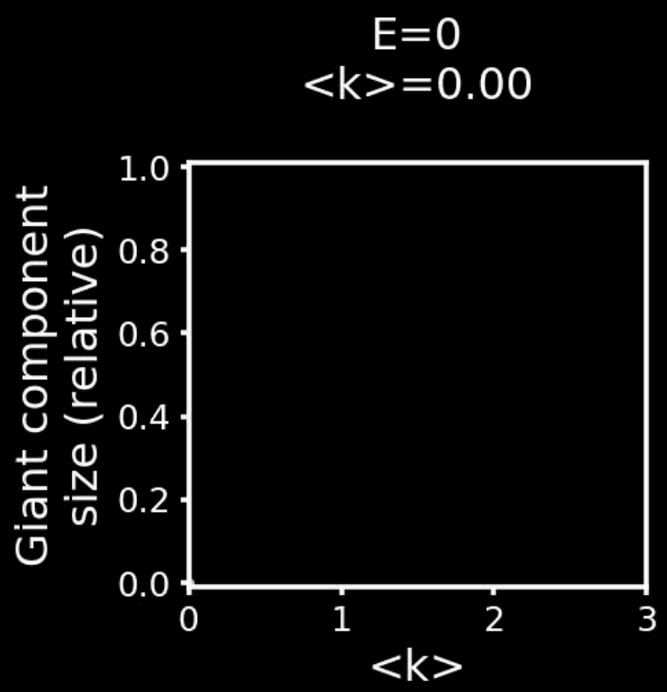
Outline

- Explosive percolation
- Thresholded networks
- Anti-fragmentation
- Demonstration (with Jupyter notebook)

Percolation

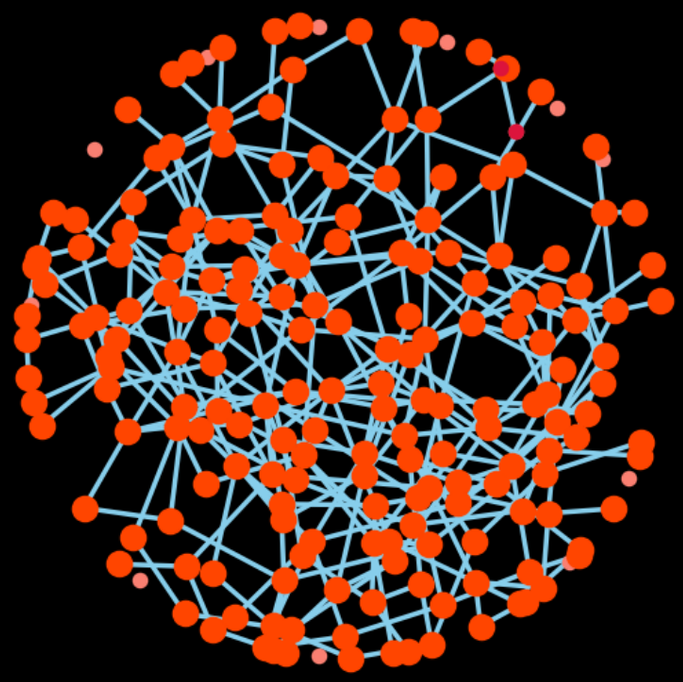
- In Erdős-Rényi random network model
  - Edges are added to isolated nodes
  - Randomly, one by one

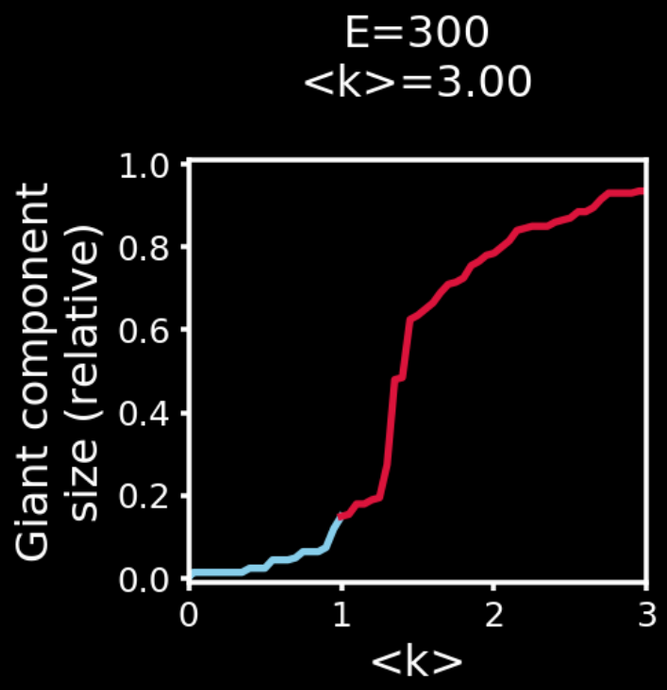




Percolation

- Giant component becomes truly ***giant***
  - For <k> above 1
- Known as percolation

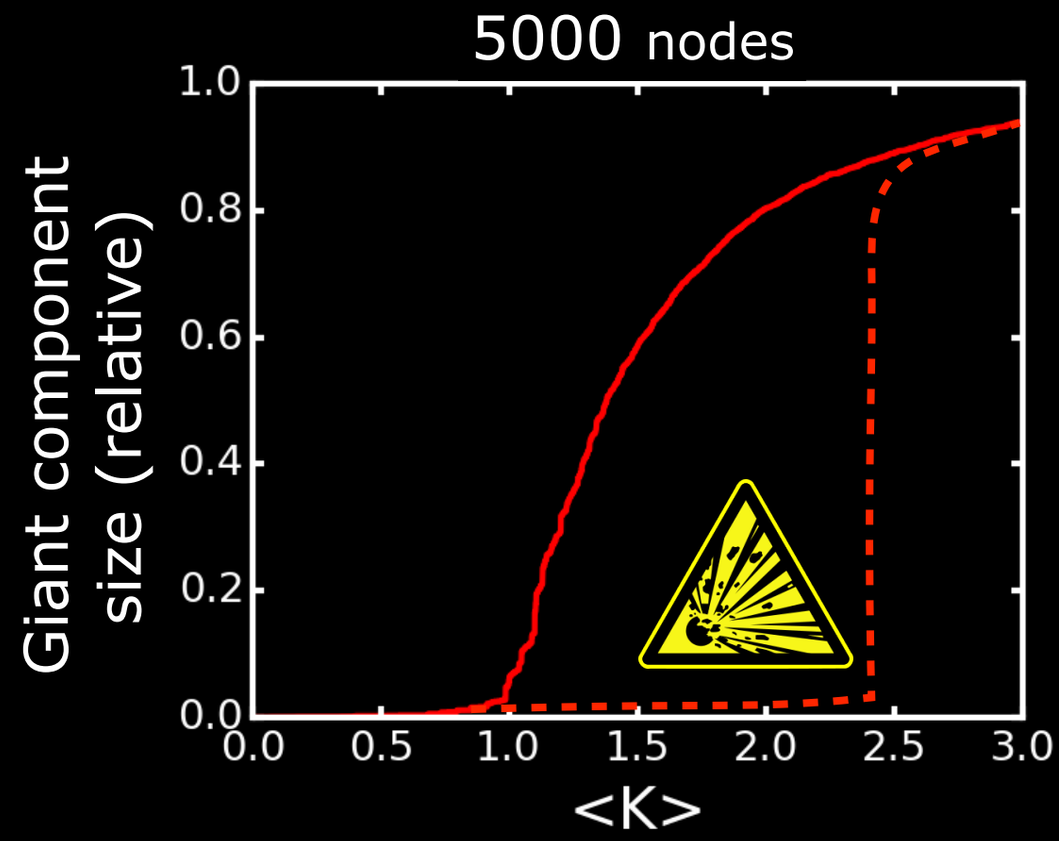




Percolation

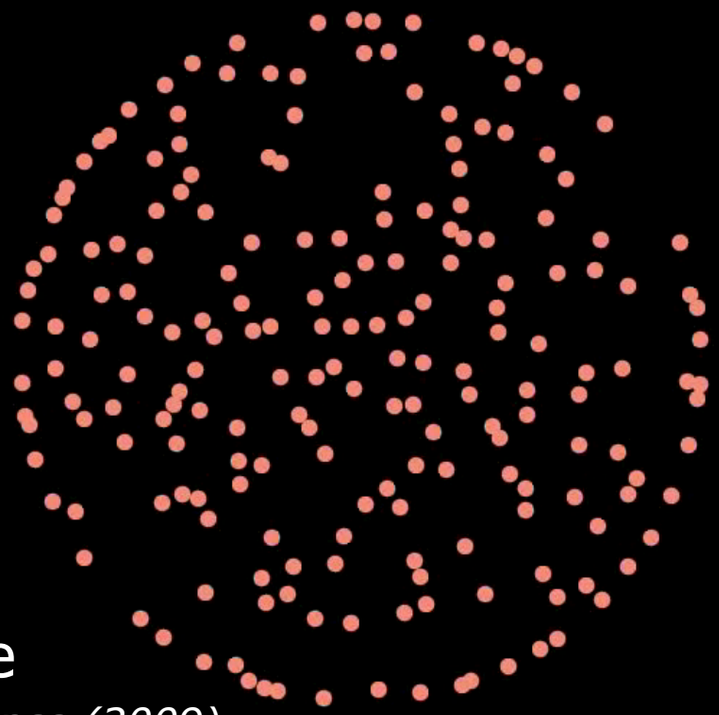
- Phase transition
  - Disconnected to connected
  - A gradual process
- Can it be abrupt?

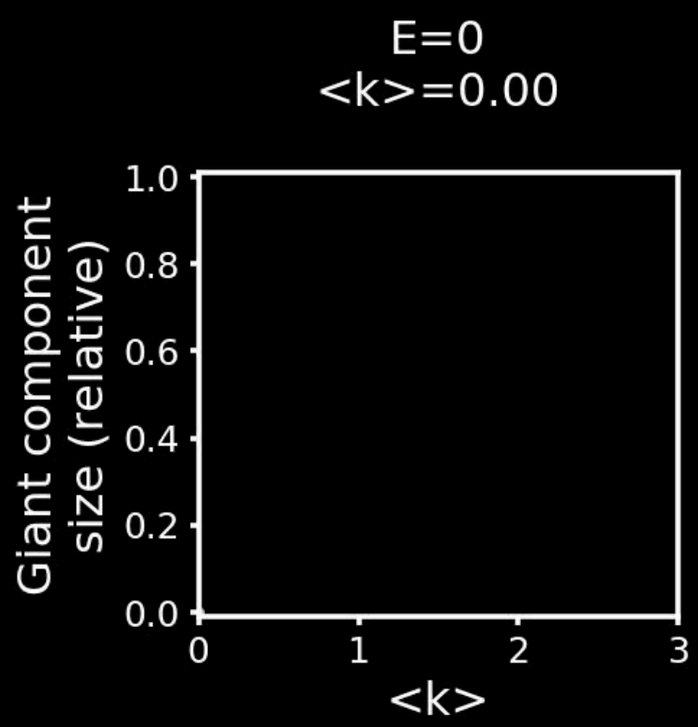
  → ***Explosive percolation***



Explosive percolation

- Keeping large components from forming
  - Selectively adding edges
- Delaying onset → more “explosive”





Explosive percolation by product rule

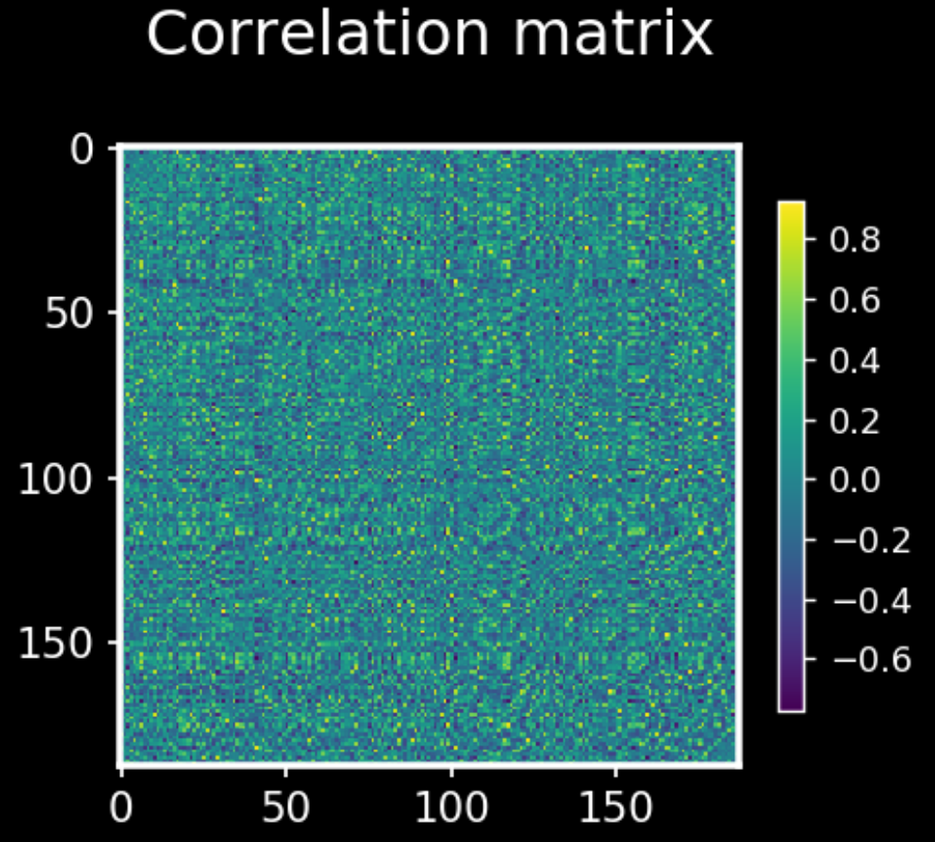
Achlioptas et al., Science (2009)

Outline

- Explosive percolation
- Thresholded networks
- Anti-fragmentation
- Demonstration (with Jupyter notebook)

Thresholded networks

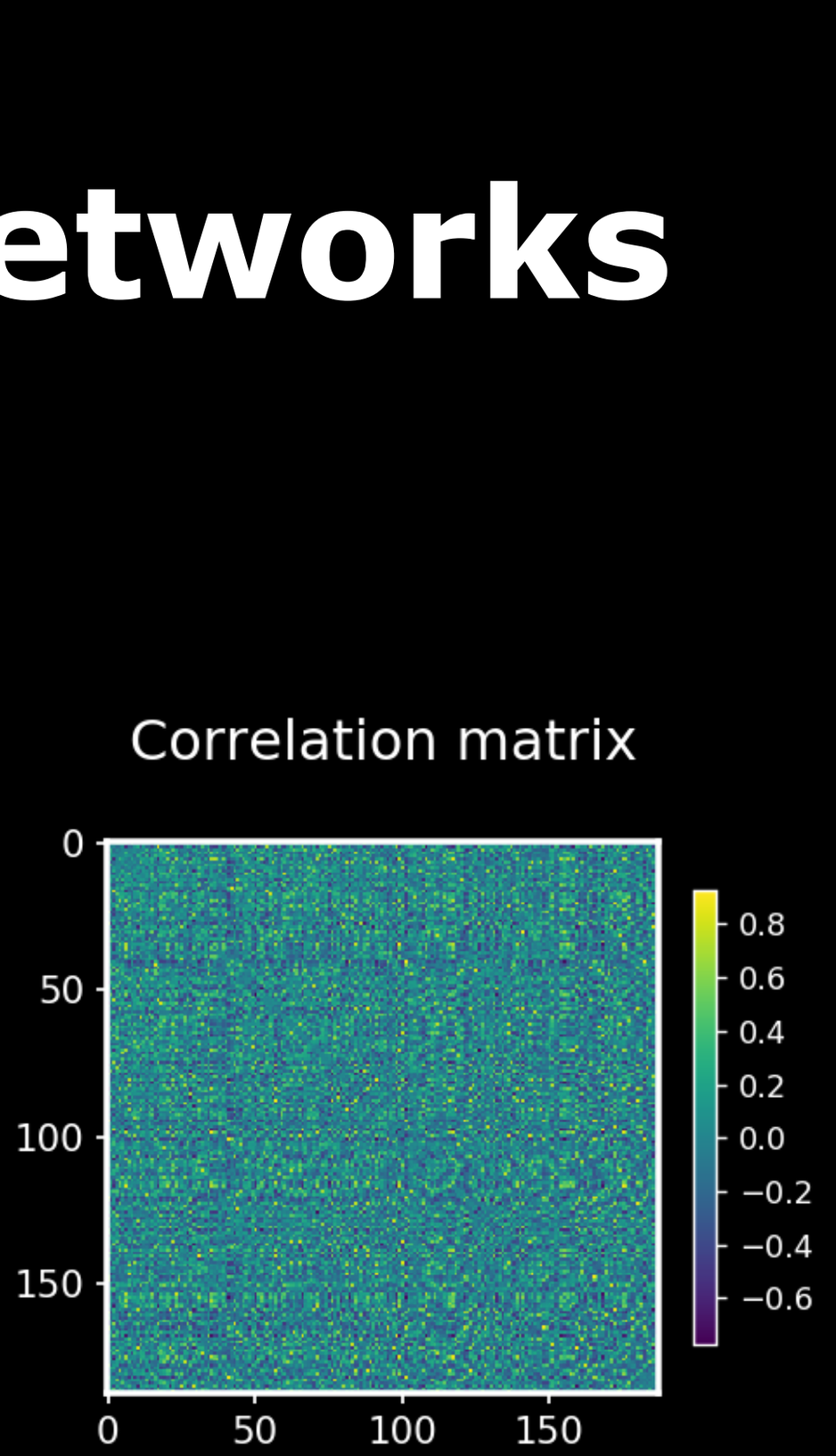
- Formed by thresholding a (correlation) matrix
  - Strong associations represented as edges



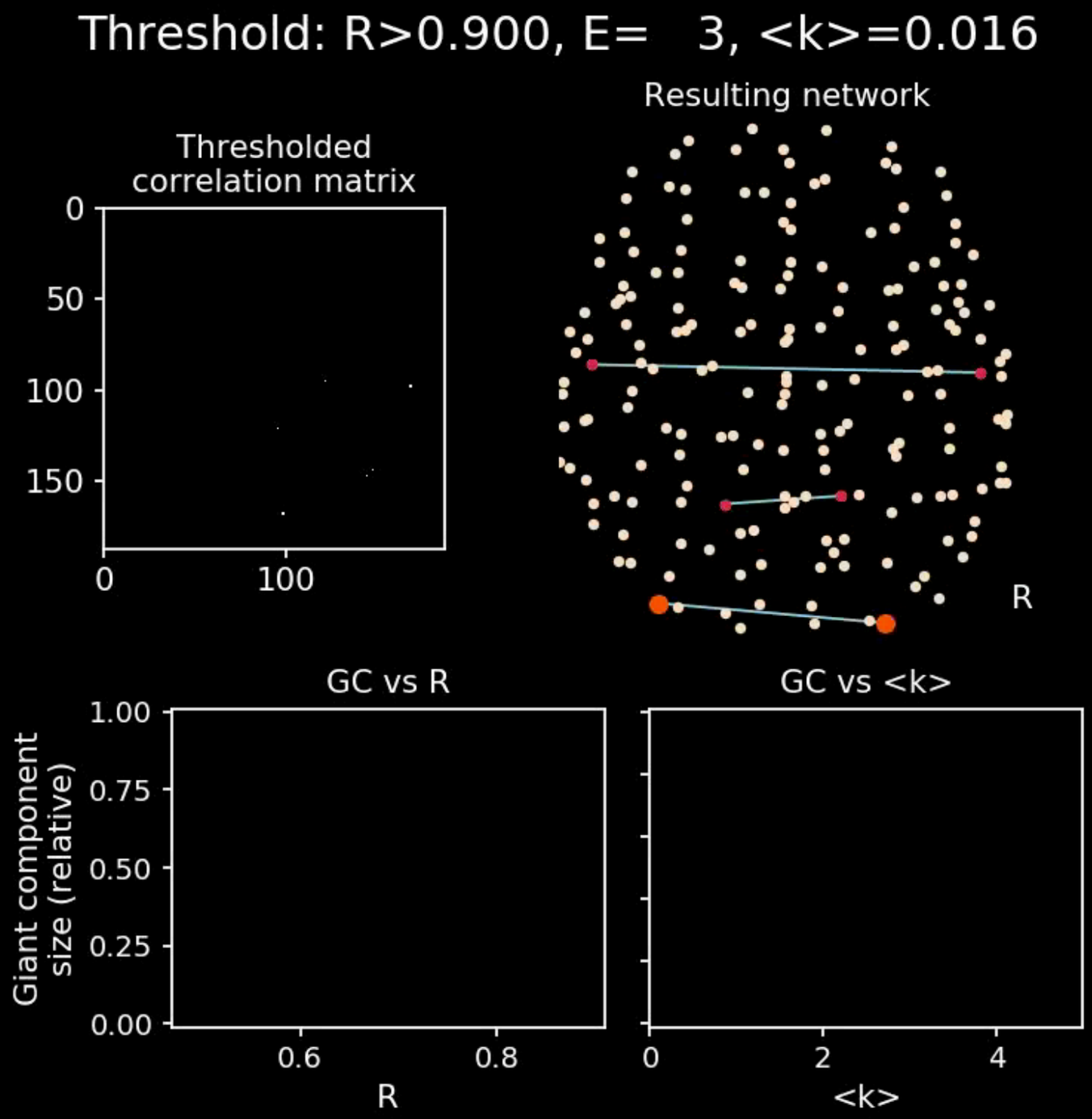
Thresholded networks

Conventional approach:

- Same threshold for the entire matrix
  - Known as ***hard thresholding***



Thresholded networks



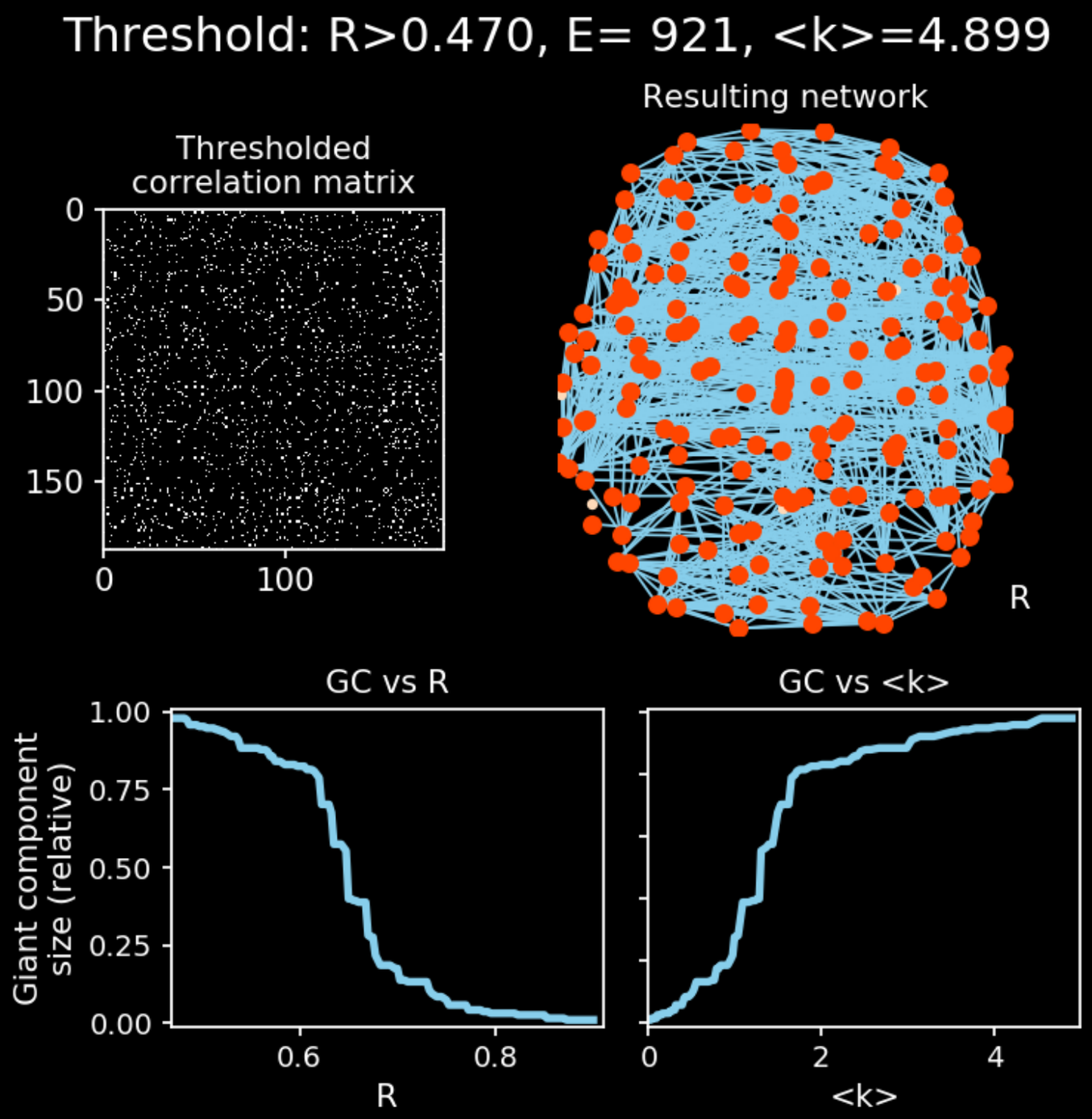
Resting state fMRI (rs-fMRI) network

with 188 nodes (ROIs)

Gradual lowering of threshold

➔ How GC size evolves

Thresholded networks



Resting state fMRI (rs-fMRI) network

with 188 nodes (ROIs)

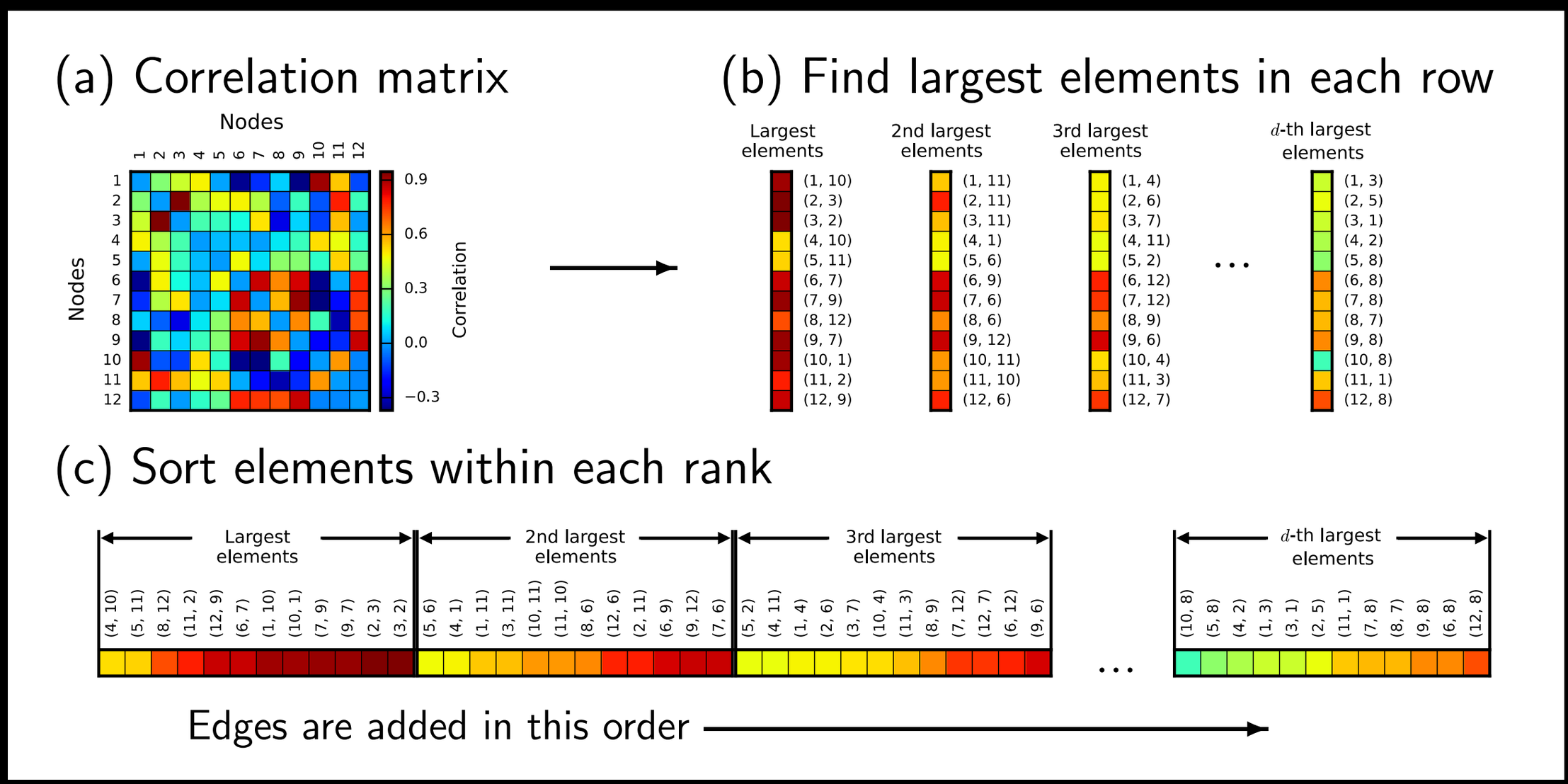
- Giant component grows incrementally
- No apparent phase transition
- Any way to induce percolation?

Node-wise thresholding

- Apply a separate threshold
- To each row of the correlation matrix
- Top *d* highest correlation per row ➔ edges

Ruan et al., BMC Systems Bio (2010)
Foti et al., PLoS ONE (2011)
Hayasaka, Physica A (2016)

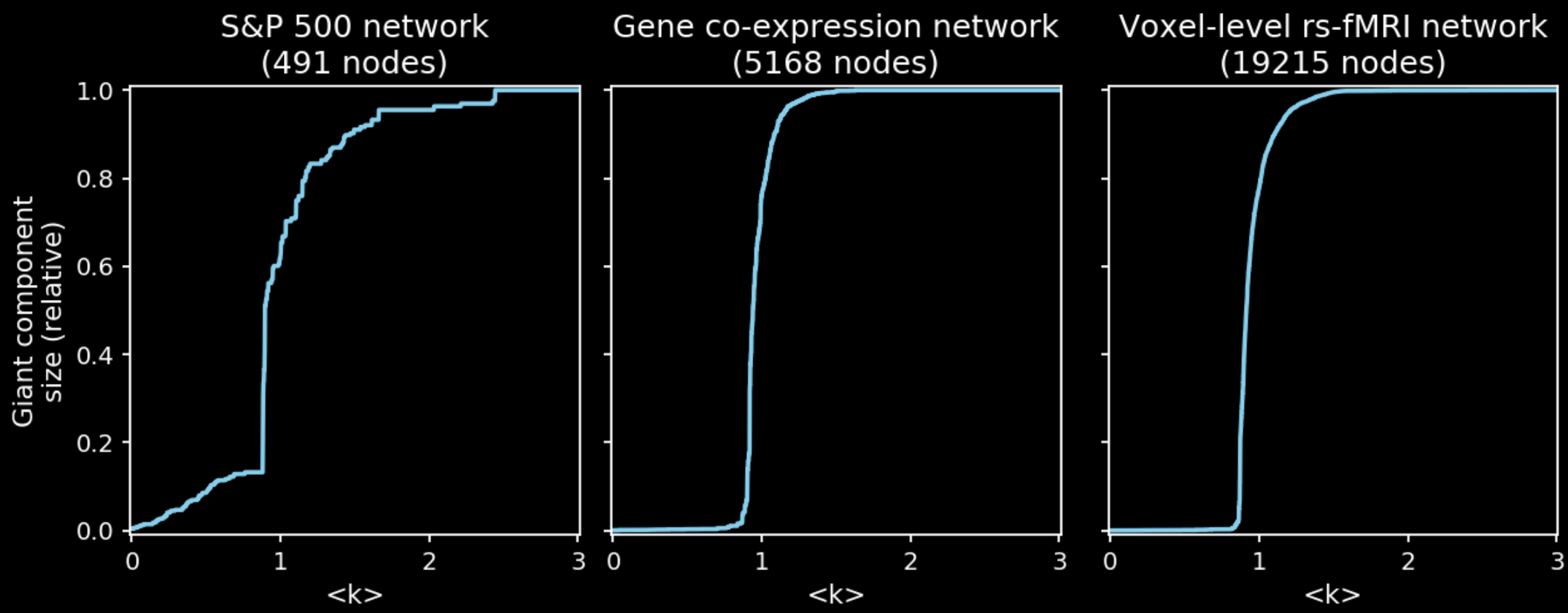
Node-wise thresholding



Hayasaka, Physica A (2016)

Node-wise thresholding

Evolution of giant component sizes in node-wise thresholding networks



Hayasaka, Physica A (2016)

Outline

- Explosive percolation
- Thresholded networks
- Anti-fragmentation
- Demonstration (with Jupyter notebook)

Fragmented networks

- Hard thresholding
  - No percolation → do not reach connected state
    → Unless a large number of edges are added
- Lack of phase transition
  - Disconnected components

Anti-fragmentation

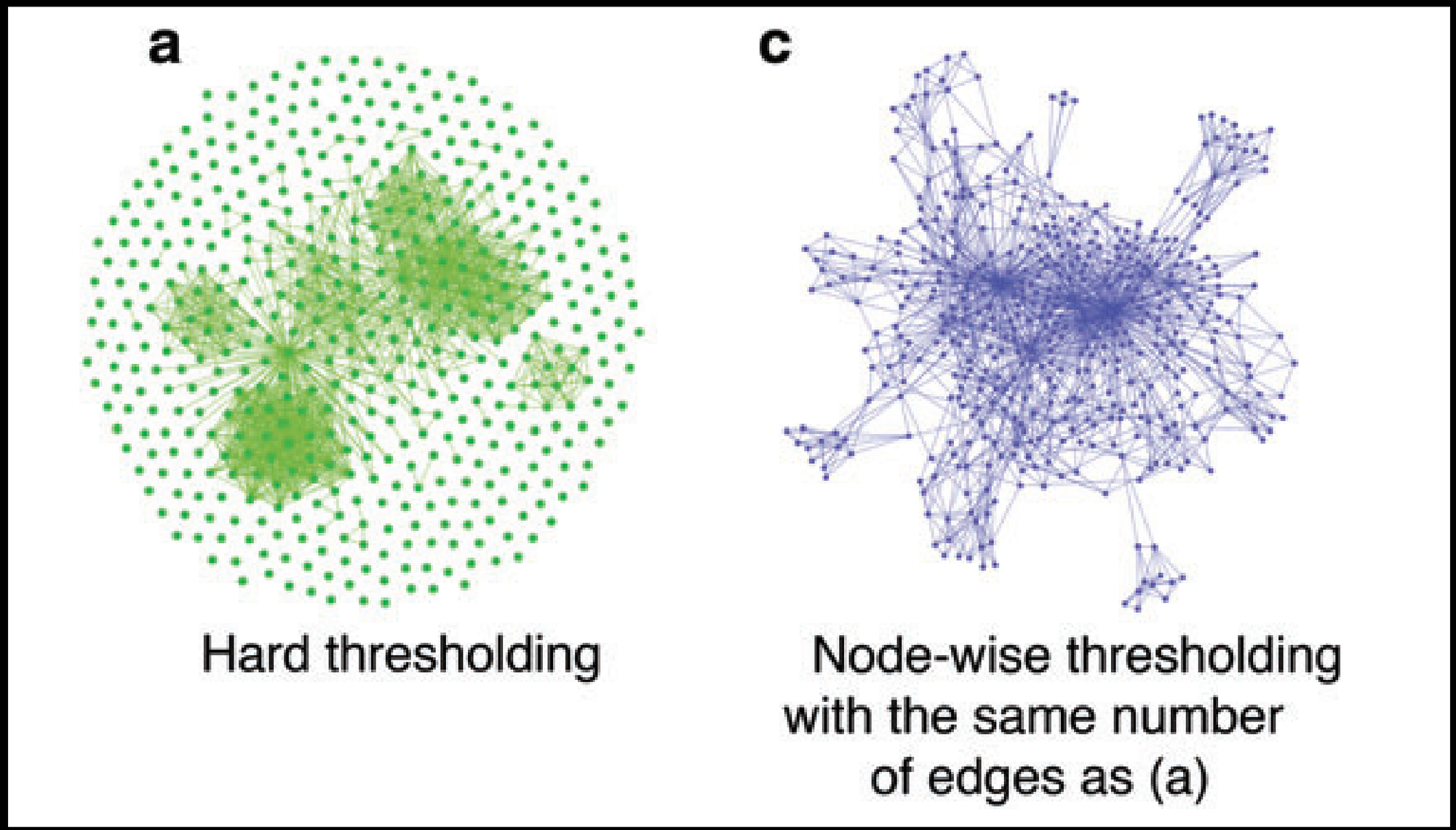
- Node-wise thresholding
- Explosive percolation → connected state

  → Even with a small number of edges

- Virtually no disconnected component

Exhibit 1

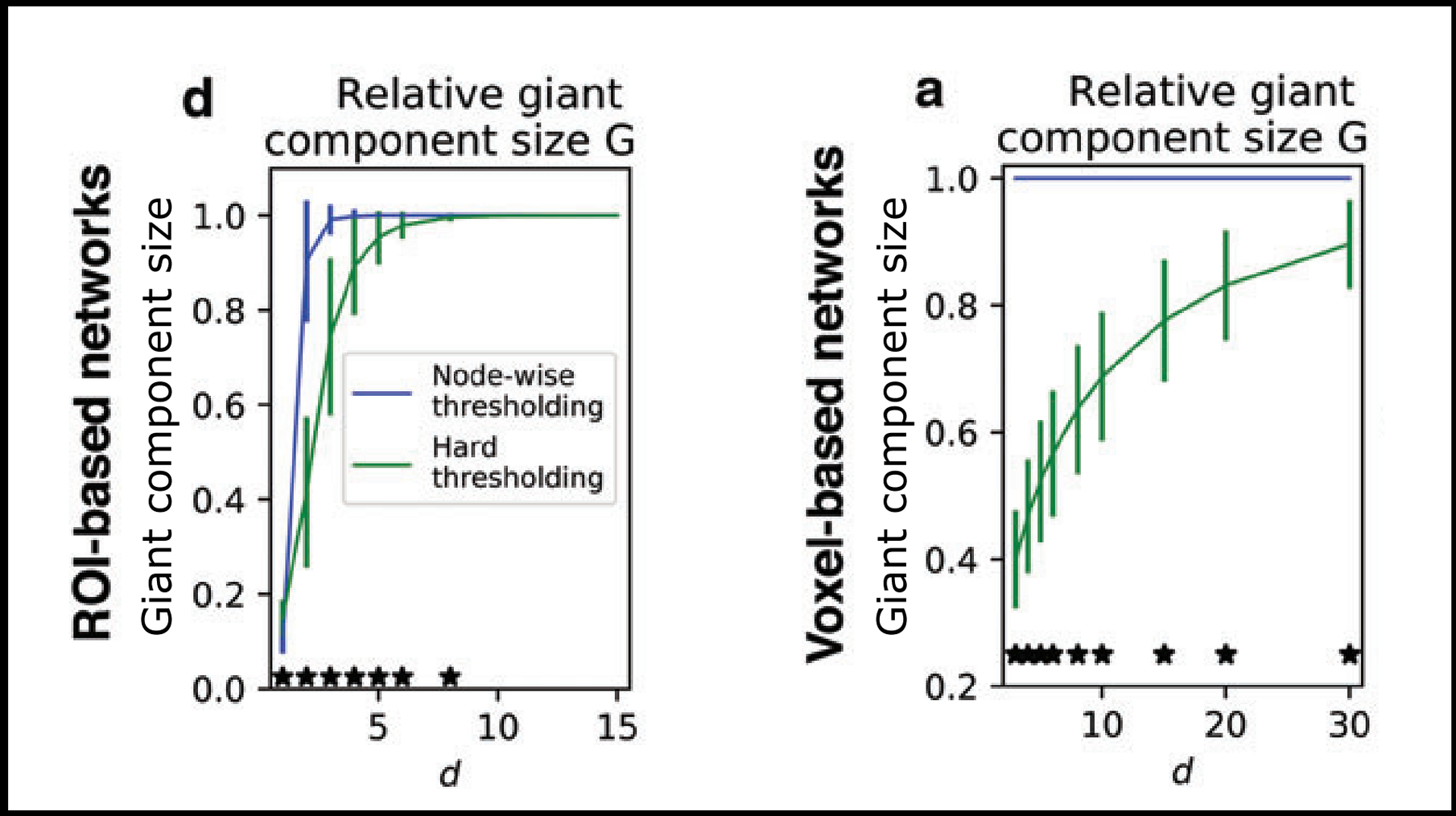
S&P 500 stock correlation network



Hayasaka, Brain Connectivity (2017)

Exhibit 2

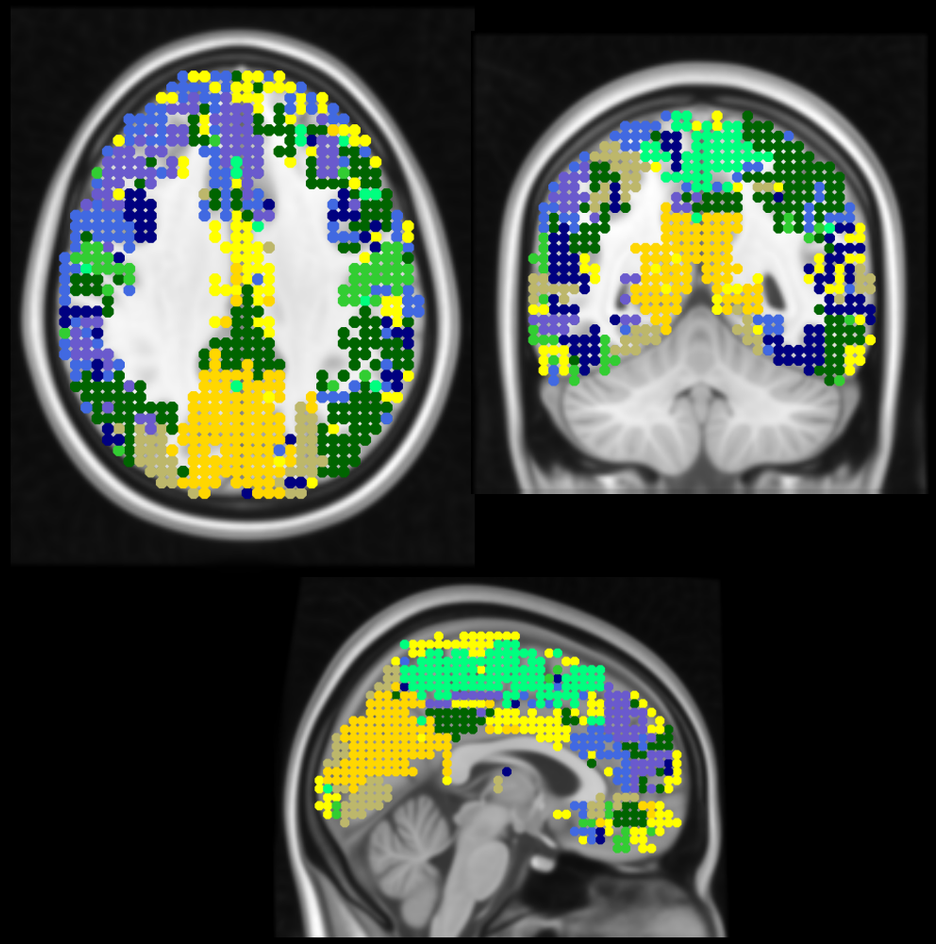
rs-fMRI networks from N=123 subjects



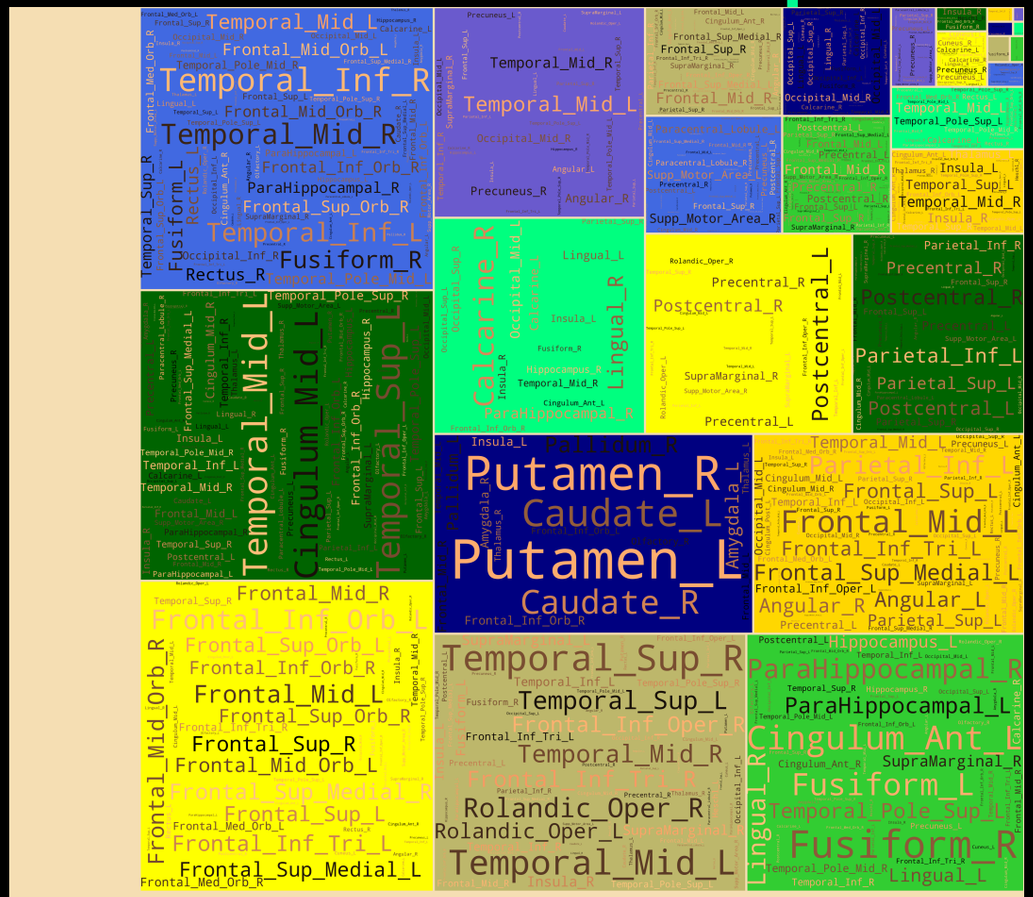
Hayasaka, Brain Connectivity (2017)

Modules from hard thresholding

Modular partition of voxel-level rs-fMRI network
(representative subject, formed at d=10)



Giant component

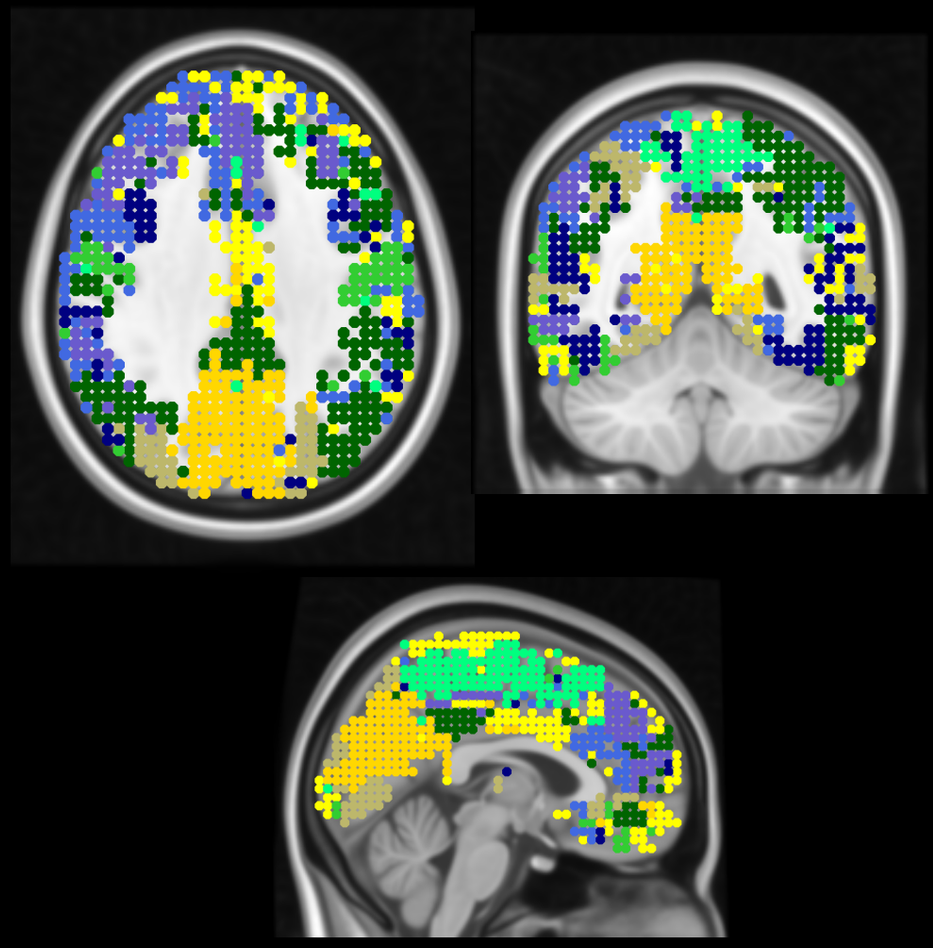


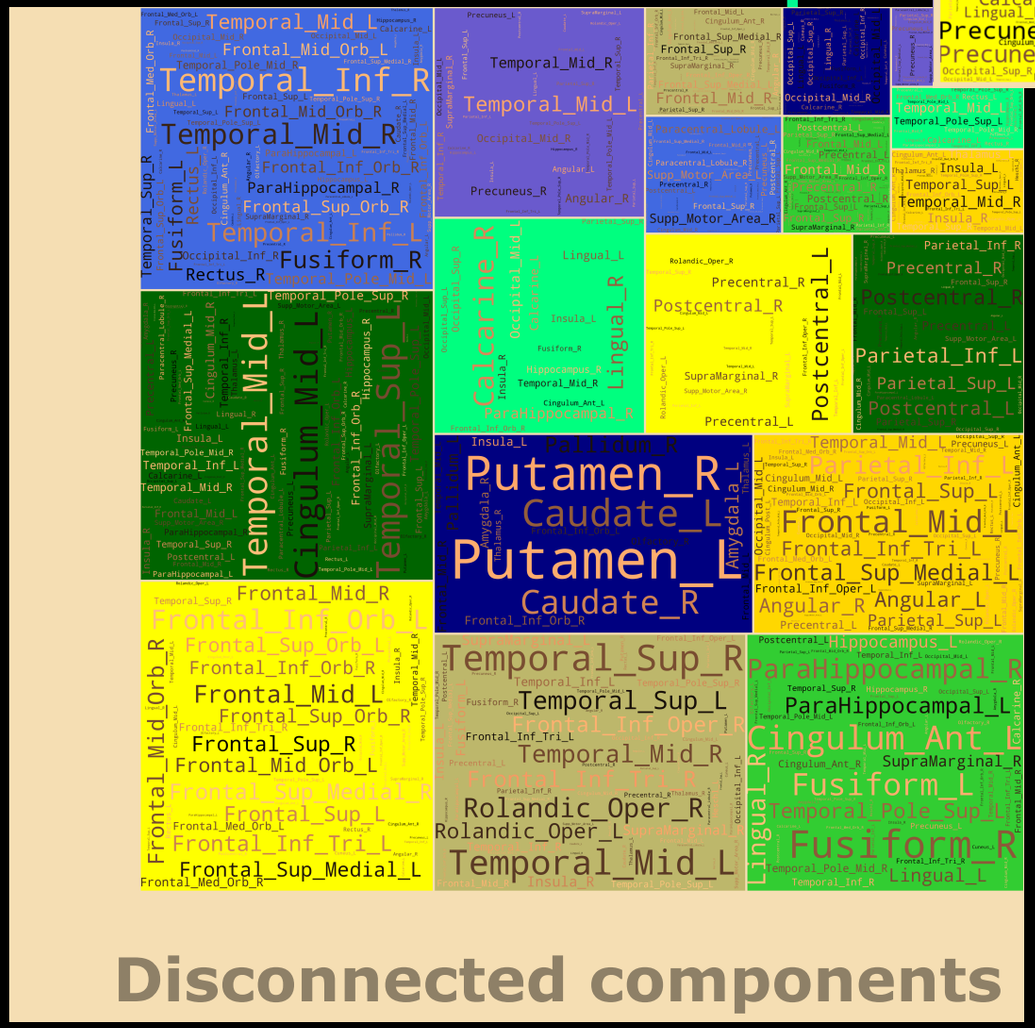
Disconnected components

Relative sizes of modules and their constituents

Modules from hard thresholding

Modular partition of voxel-level rs-fMRI network
(representative subject, formed at d=10)



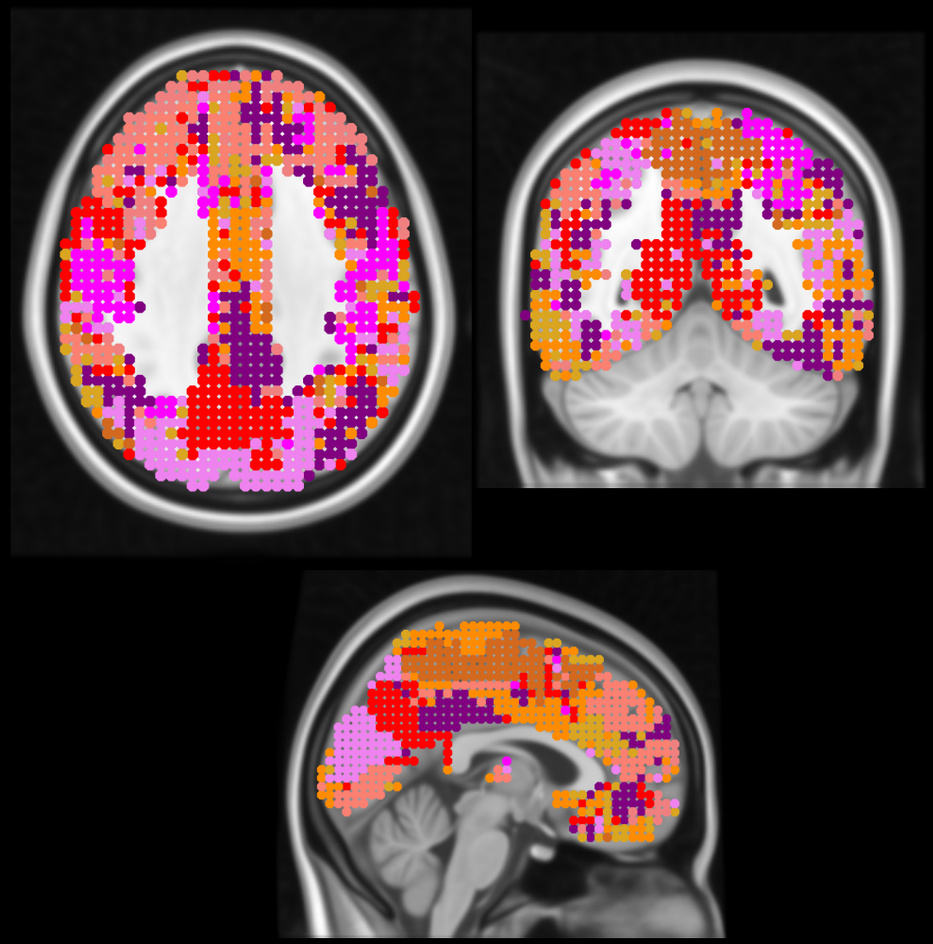


Disconnected components

Relative sizes of modules and their constituents

Modules from node-wise thresholding

Modular partition of voxel-level rs-fMRI network
(representative subject, formed at d=10)

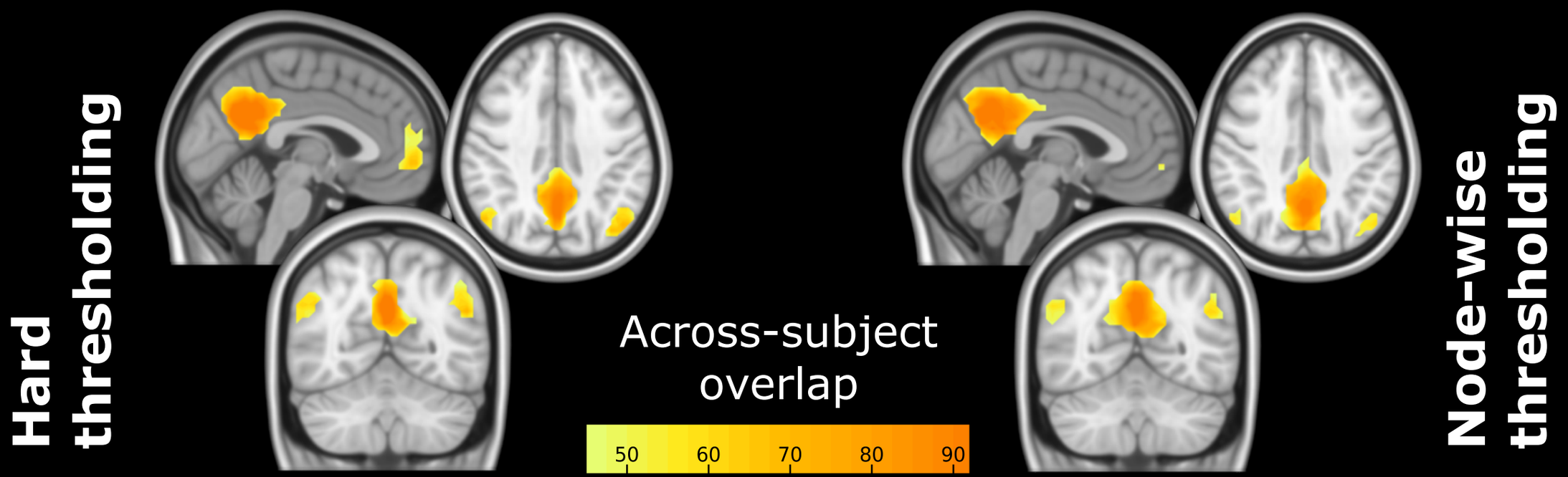


Relative sizes of modules and their constituents

Consistency of modules

Overlap of modules across N=123 subjects
voxel-level rs-fMRI network
(formed at d=10)

Default mode network module

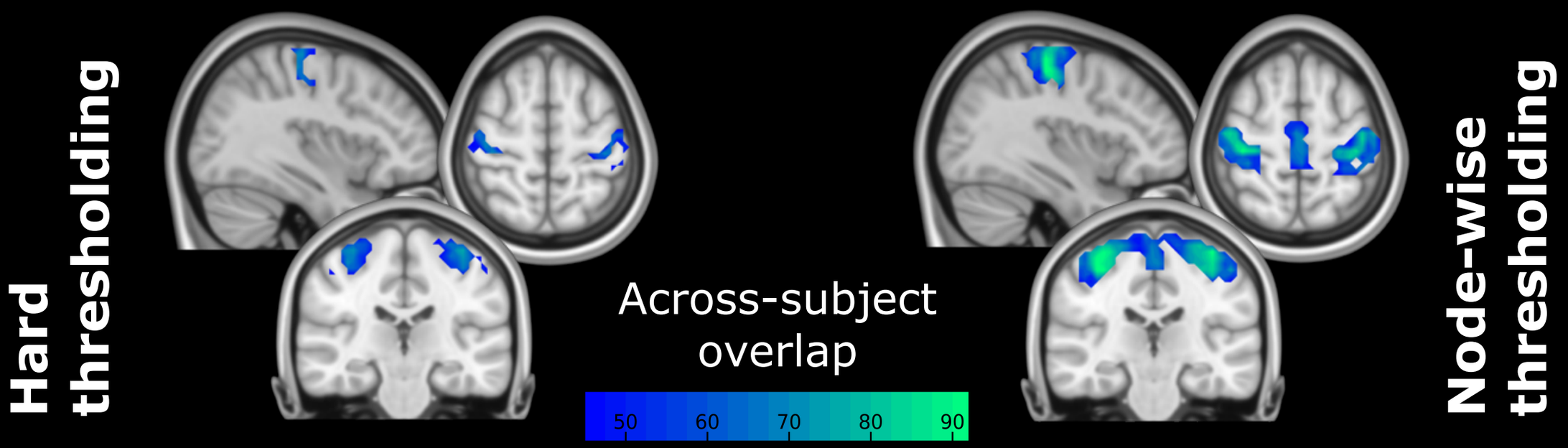


Hayasaka, OHBM 2018 Poster

Consistency of modules

Overlap of modules across N=123 subjects
voxel-level rs-fMRI network
(formed at d=10)

Sensory-motor module



Hayasaka, OHBM 2018 Poster

Take home messages

Hard thresholding	Node-wise thresholding
Gradual growth, no percolation	Percolation during formation
Some disconnected components	No disconnected components
Possible micro modules	No micro modules
Limited consistency in some modules	Consistent modules

Outline

- Explosive percolation
- Thresholded networks
- Anti-fragmentation
- Demonstration (with Jupyter notebook)

GitHub Repo

- Codes, data, figures, animations, slides (PDF), and note

sathayas/NetNeurosci2019

- Link from
  - Web: sathayas.github.io
  - Twitter: @sathayas42