*Relative sizes of voxel-level rs-fMRI modules and their constituents*

# Explosive percolation and anti-fragmentation in fMRI network construction

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# Outline

- Explosive percolation
- Thresholded networks
- Anti-fragmentation
- Demonstration (with Jupyter notebook)

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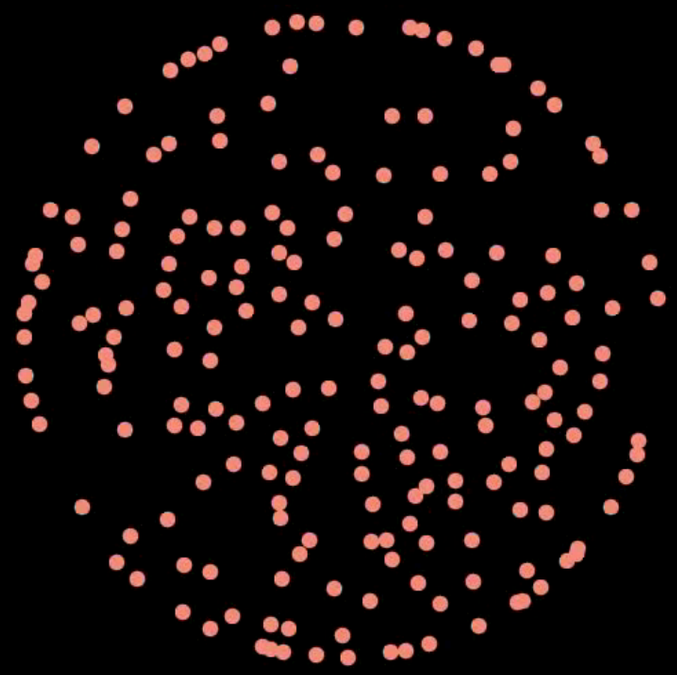
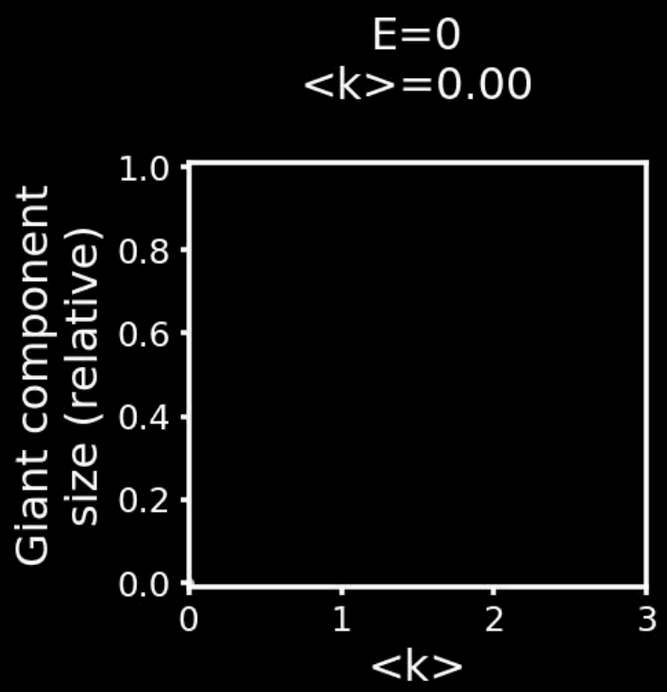
# Percolation

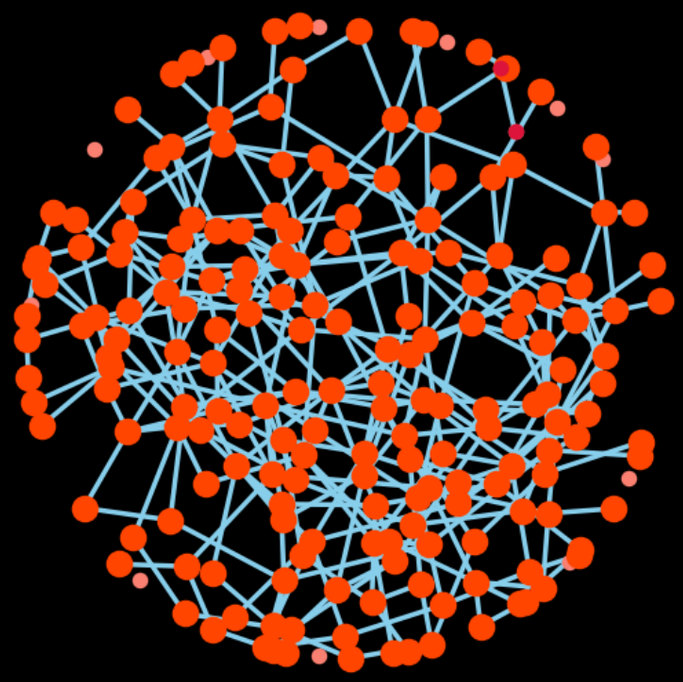
- In Erdős-Rényi random network model
  - Edges are added to isolated nodes
  - Randomly, one by one

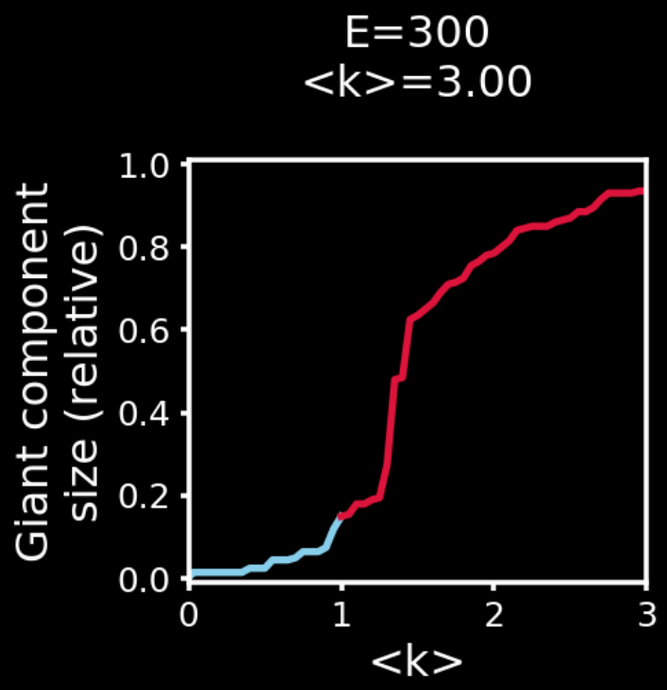




# Percolation

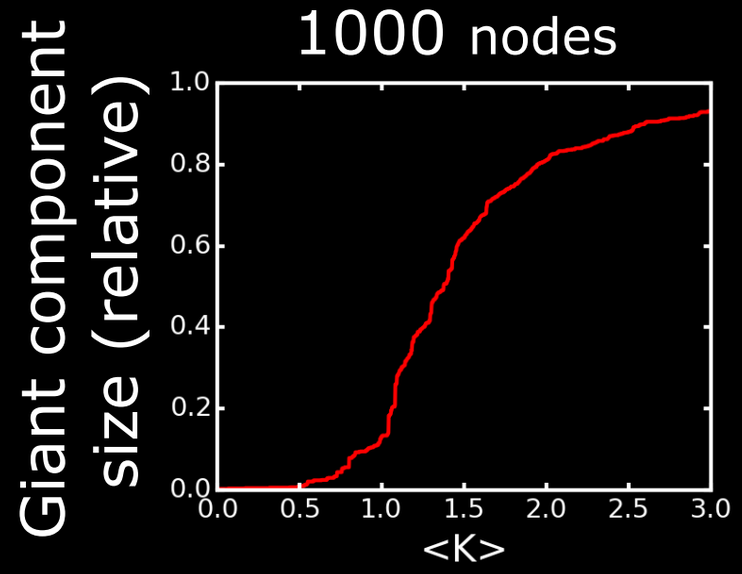
- Giant component becomes truly ***giant***
  - For <k> above 1
- Known as percolation

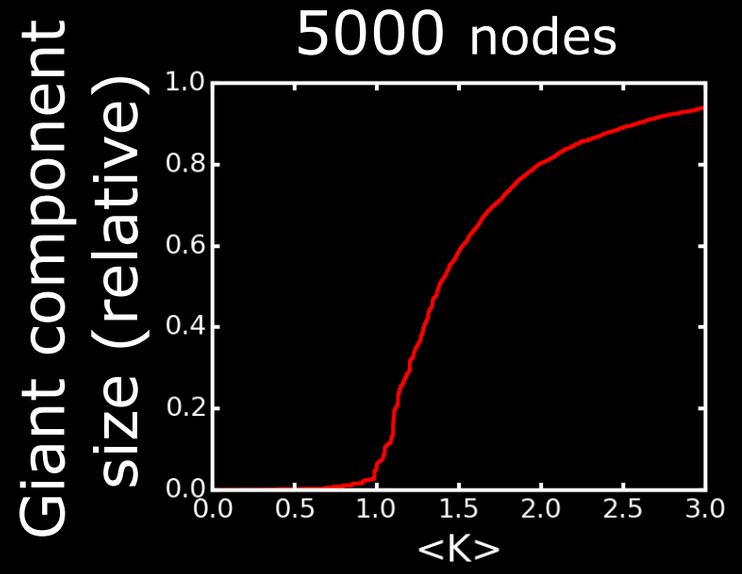


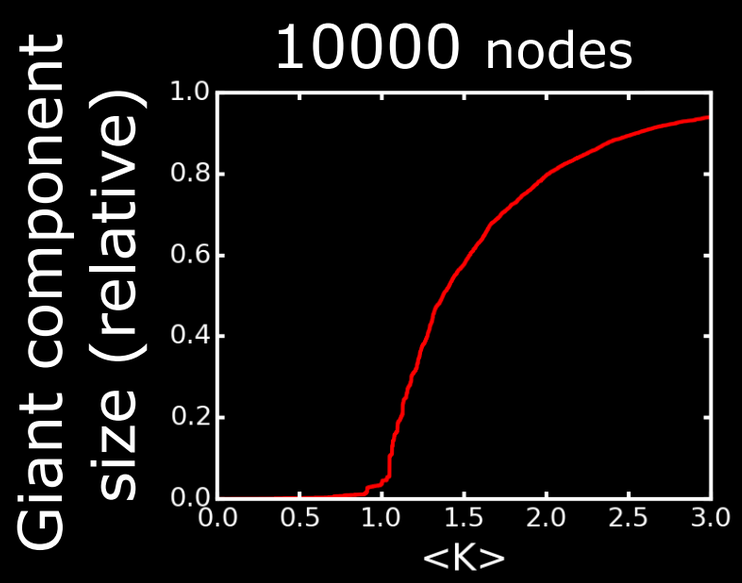


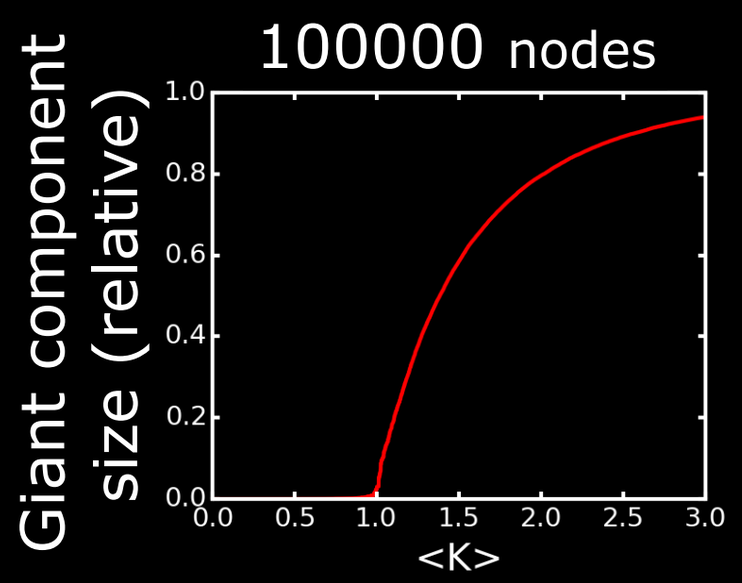
# Percolation

- More apparent for large networks



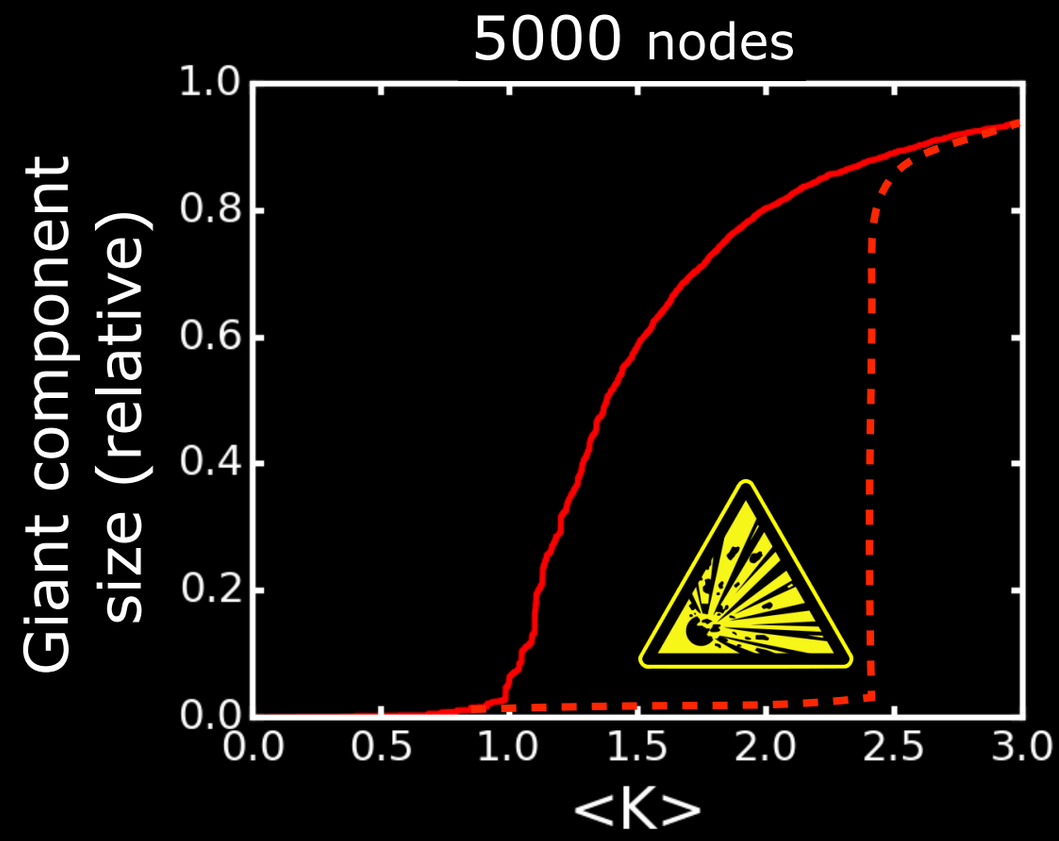



# Percolation

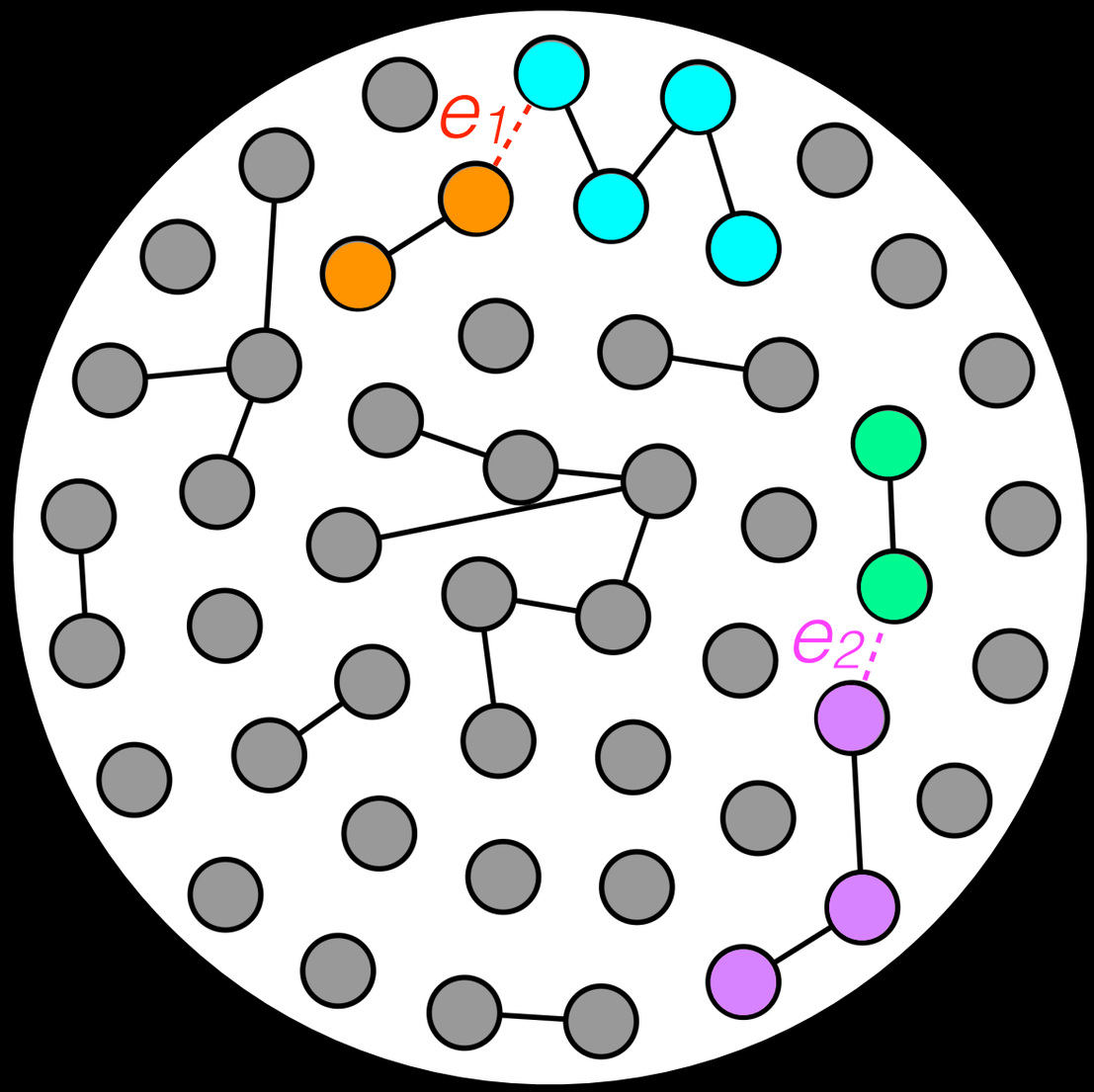
- Phase transition
  - Disconnected to connected
  - A gradual process
- Can it be abrupt?

  → ***Explosive percolation***

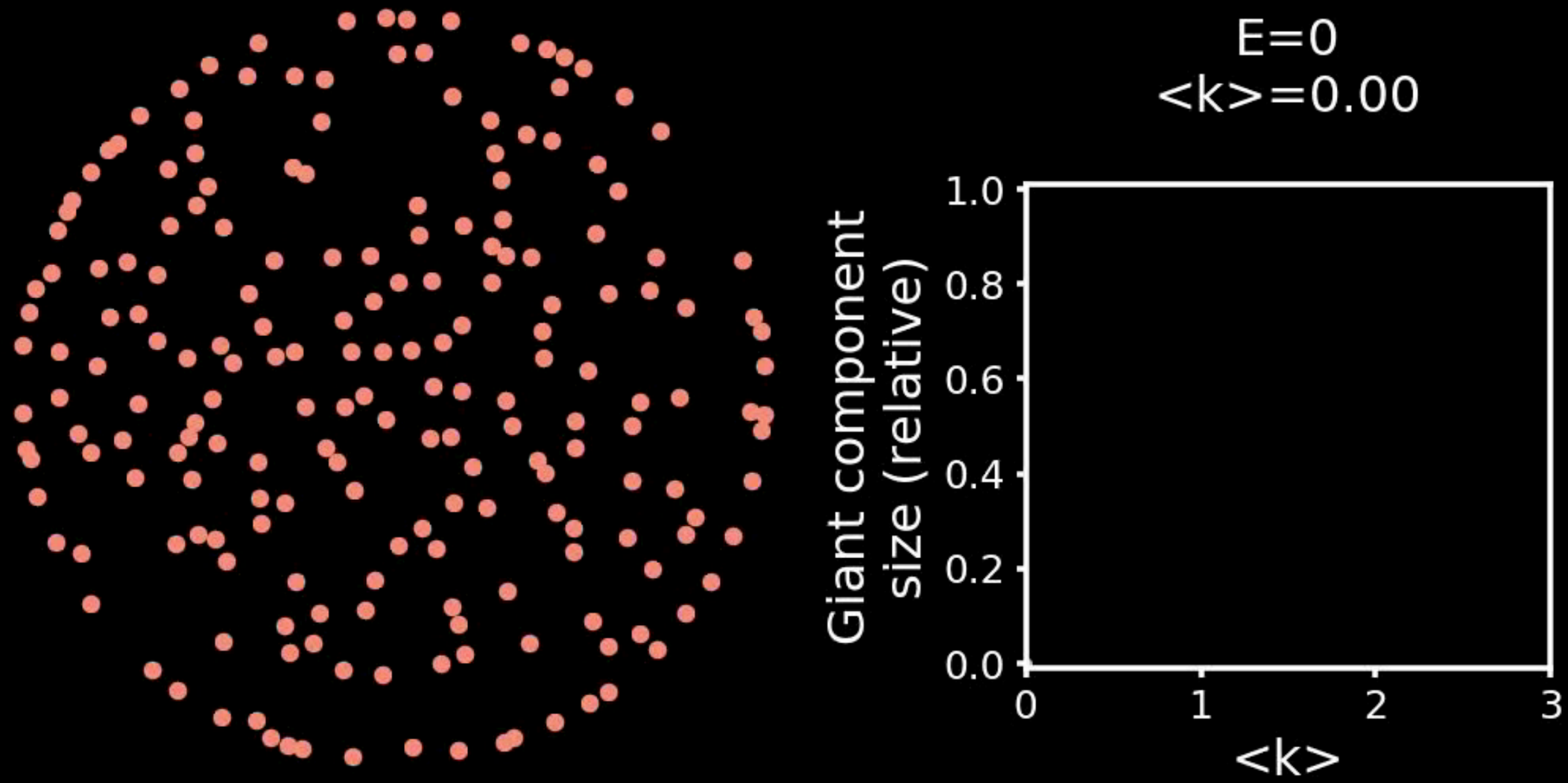


# Explosive percolation

- Introduce two random edges at a time
  - Keep the one resulting in smaller connected component
- Discard the other
- In this example,
  - We keep $e_2$, discard $e_1$



# Explosive percolation



# Outline

- Explosive percolation
- Thresholded networks
- Anti-fragmentation
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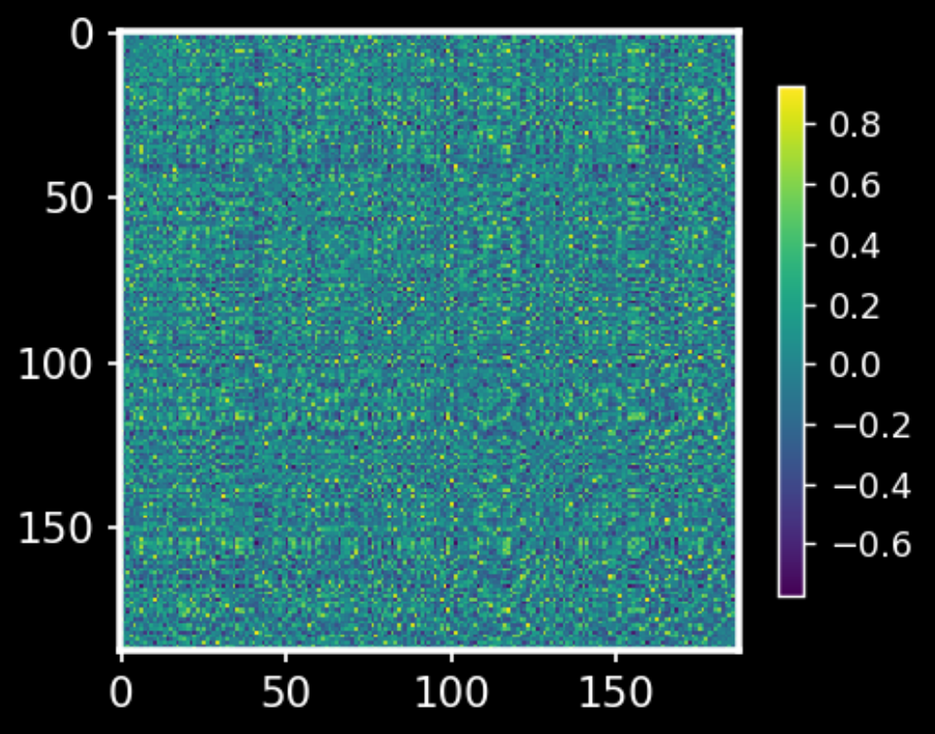
# Thresholded networks

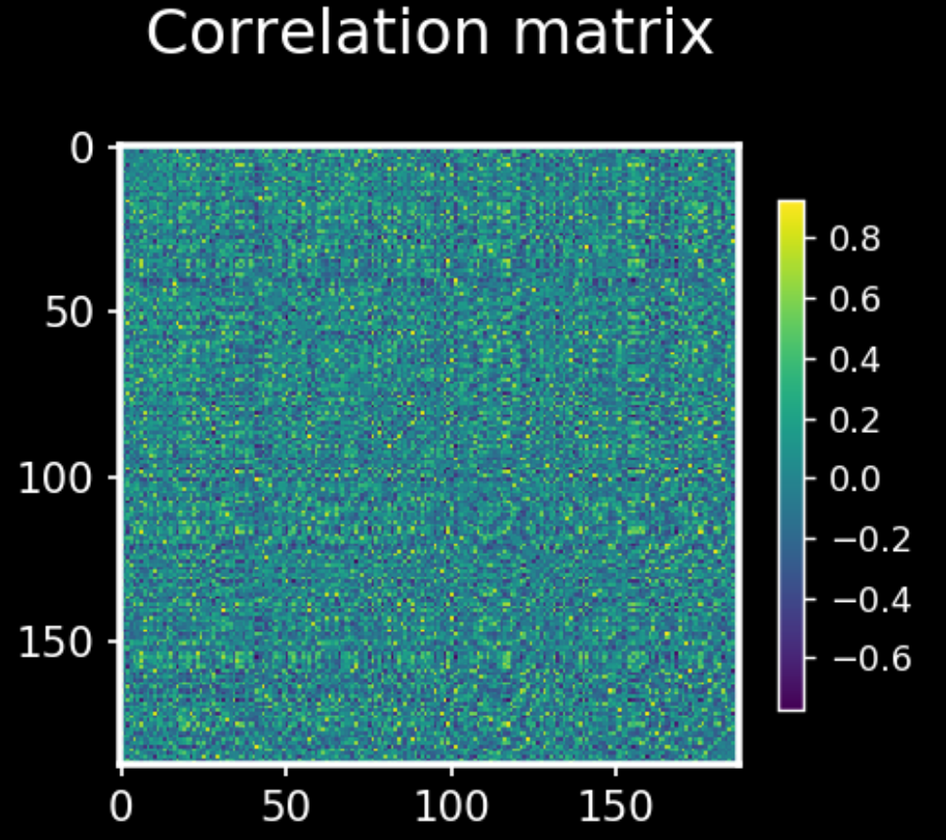
- Resulting from thresholding a (correlation) matrix
- Strong associations represented as edges
- Just a way to represent associations

➔ No physical connections



# Thresholded networks

- Same threshold for the entire matrix
  - Known as ***hard thresholding***

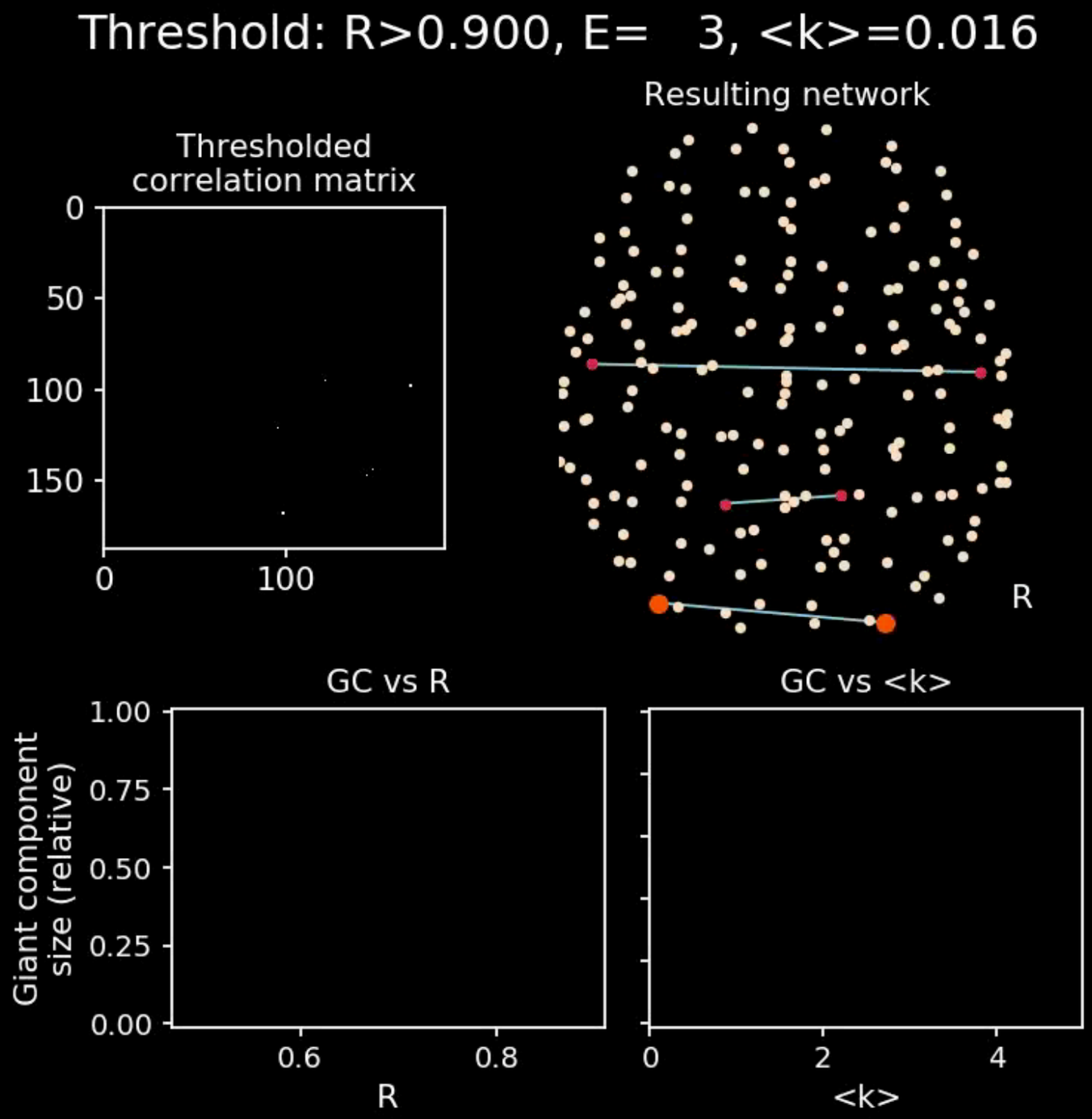


# Thresholded networks

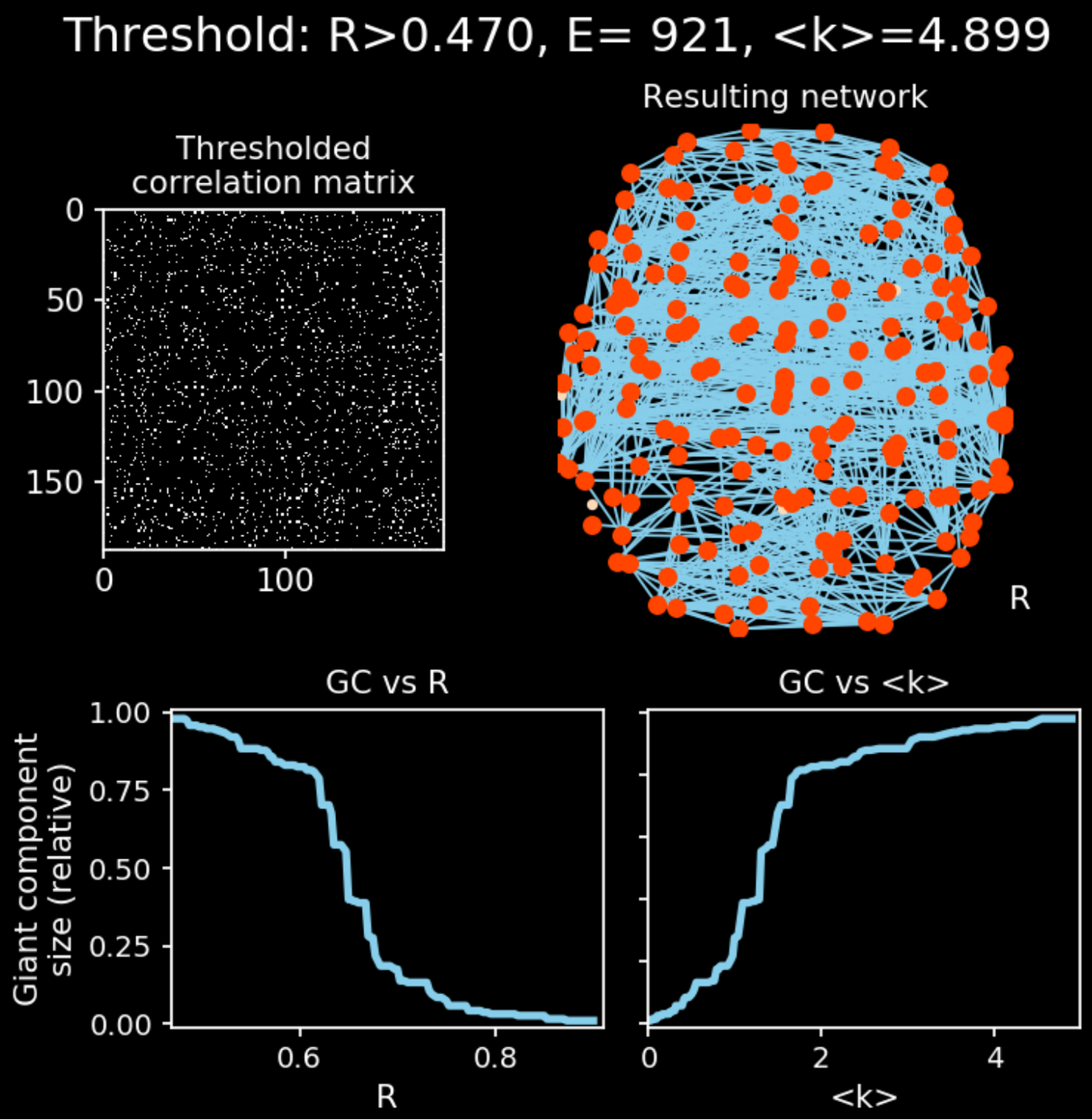
- Example: resting-state fMRI (rs-fMRI)
  - Correlation matrix of 188 ROI time courses
  - Threshold R is gradually lowered

  ➔ Giant component size recorded

# Thresholded networks



# Thresholded networks



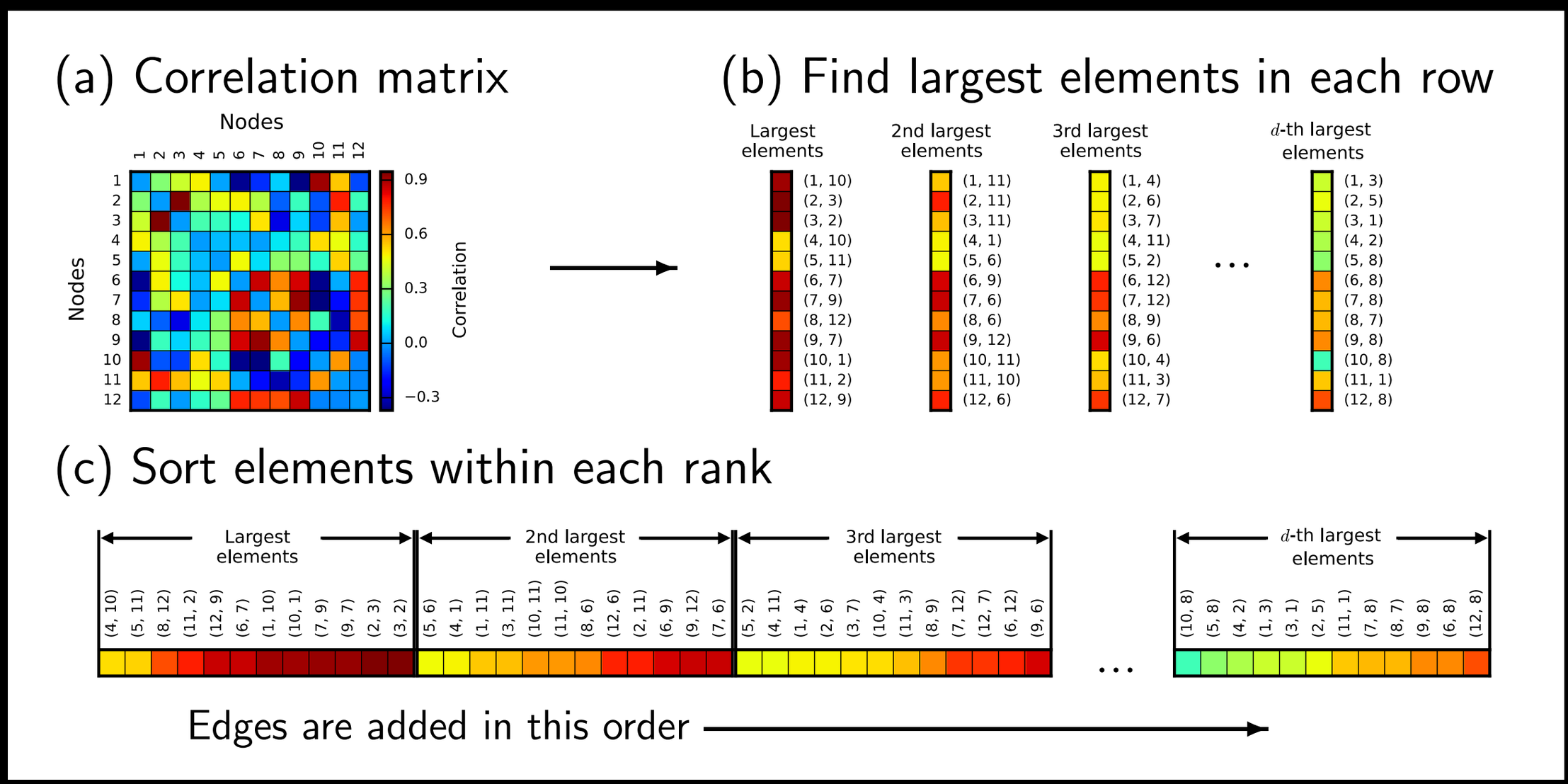

- Giant component grows gradually
- No apparent phase transition
- Any way to induce percolation?

# Node-wise thresholding

- Apply a separate threshold
- To each row of the correlation matrix
- Top *d* highest correlation per row ➔ edges

*Ruan et al., BMC Systems Bio (2010)*
*Foti et al., PLoS ONE (2011)*
*Hayasaka, Physica A (2016)*

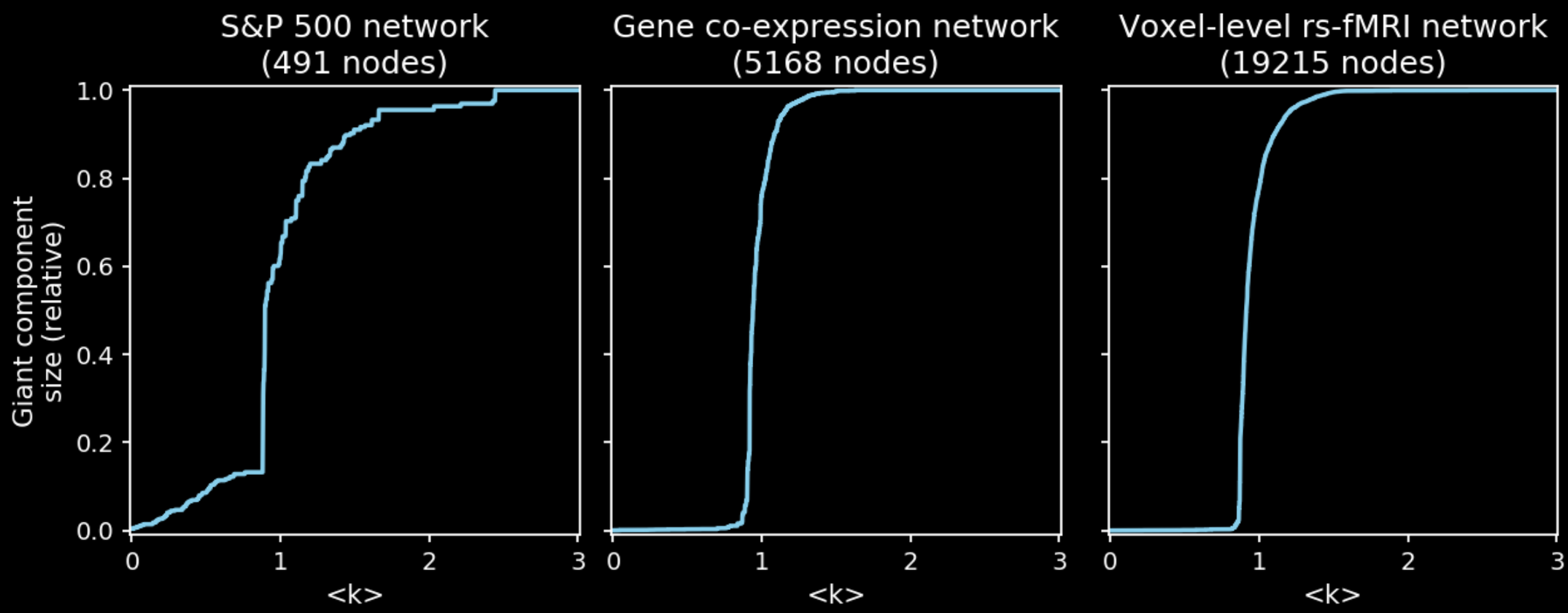
# Node-wise thresholding




*Hayasaka, Physica A (2016)*

# Node-wise thresholding

Evolution of giant component sizes in node-wise thresholding networks




*Hayasaka, Physica A (2016)*

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# Fragmented networks

- Hard thresholding
  - No percolation → does not reach connected state
    → Unless a large number of edges are added
- Lack of phase transition
  - Disconnected components
  - Concentration of edges

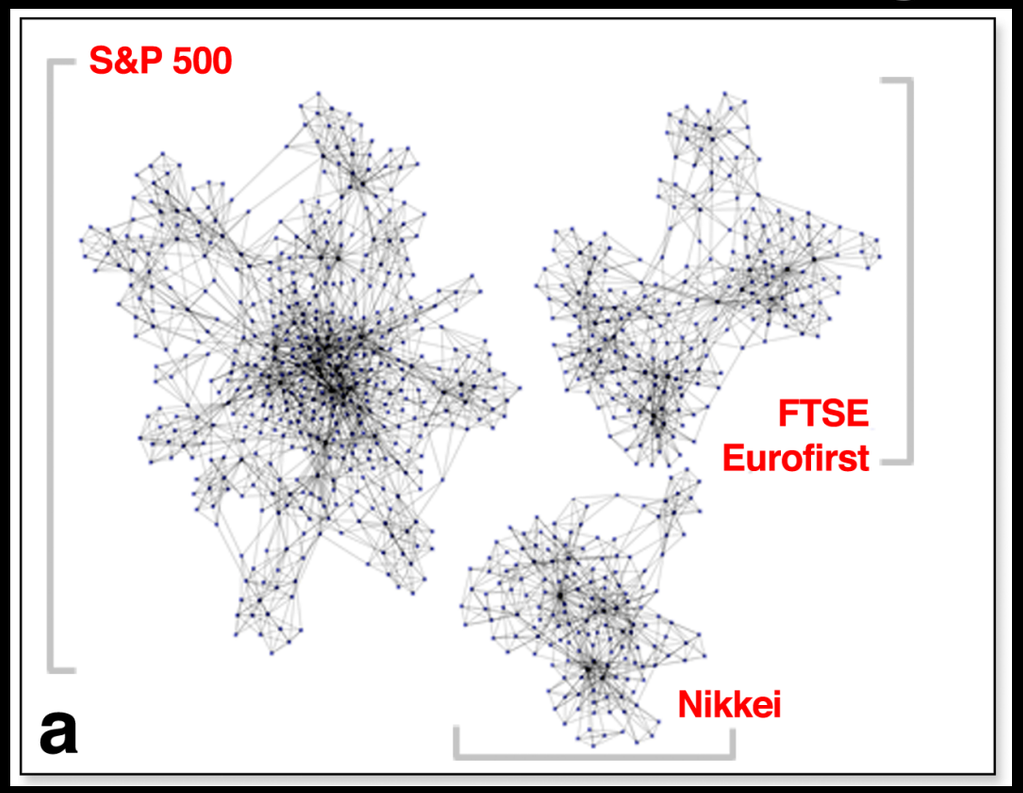
# Anti-fragmentation

- Node-wise thresholding
  - Explosive percolation ➔ connected state
    ➔ Even with a small number of edges
  - Virtually no disconnected component
  - No excessive concentration of edges

# Exhibit 1

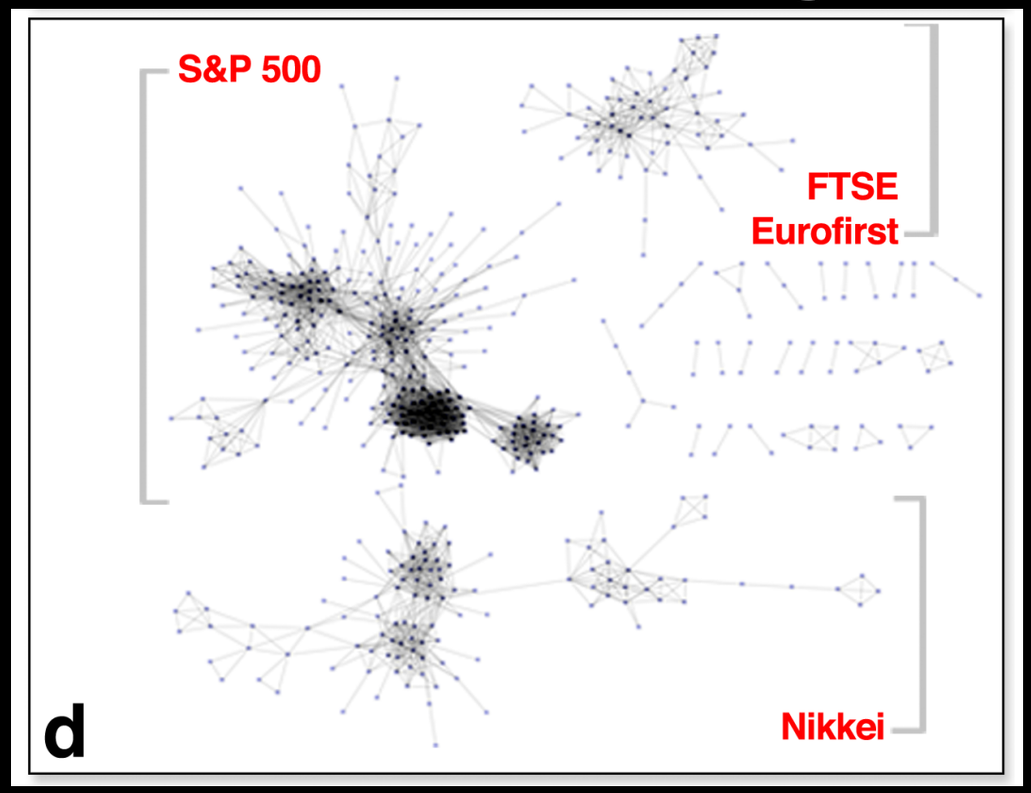
Stock market correlation network

Node-wise thresholding




Same number of edges

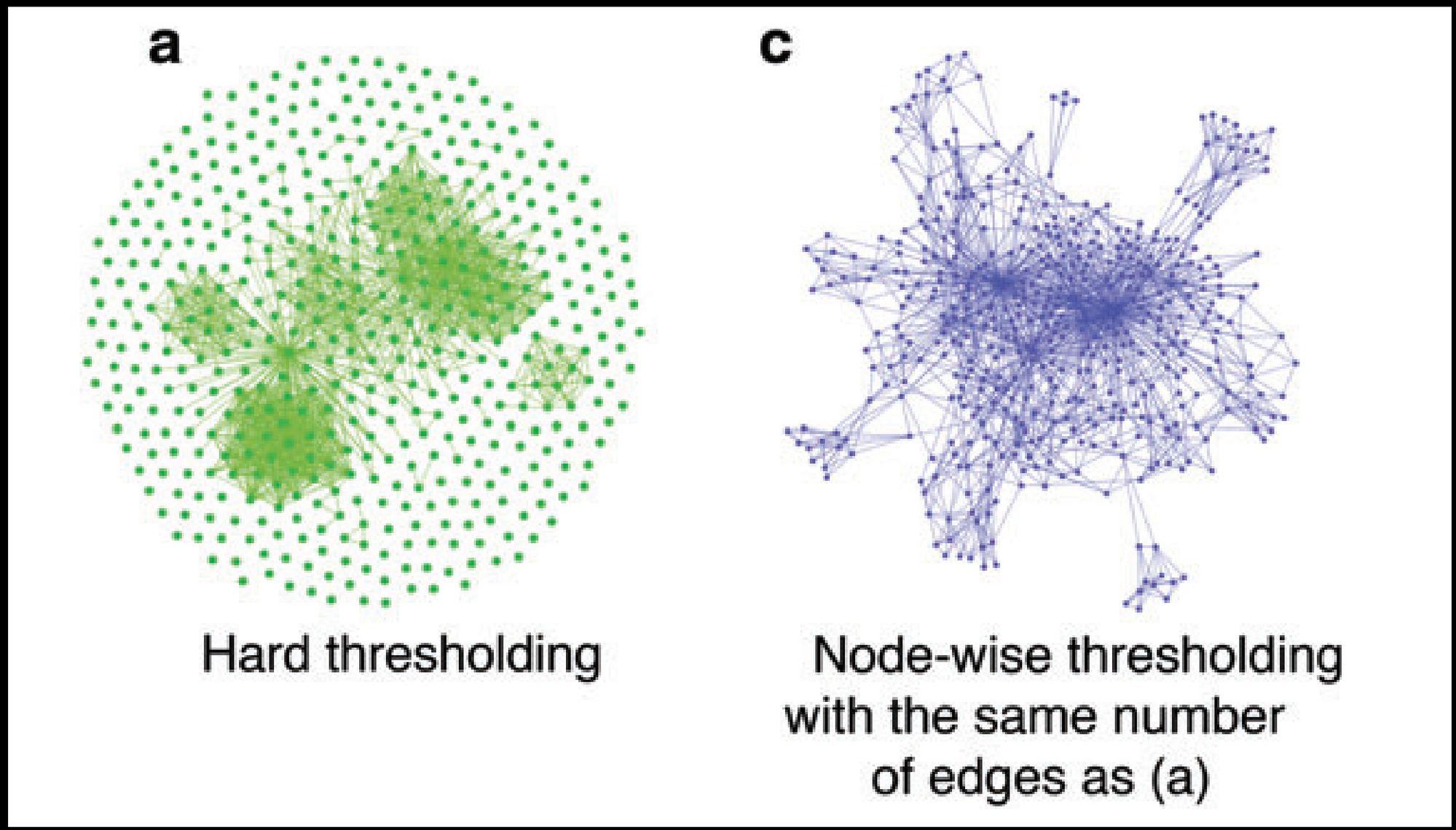
Hard thresholding




*Foti et al., PLoS ONE (2011)*

# Exhibit 2

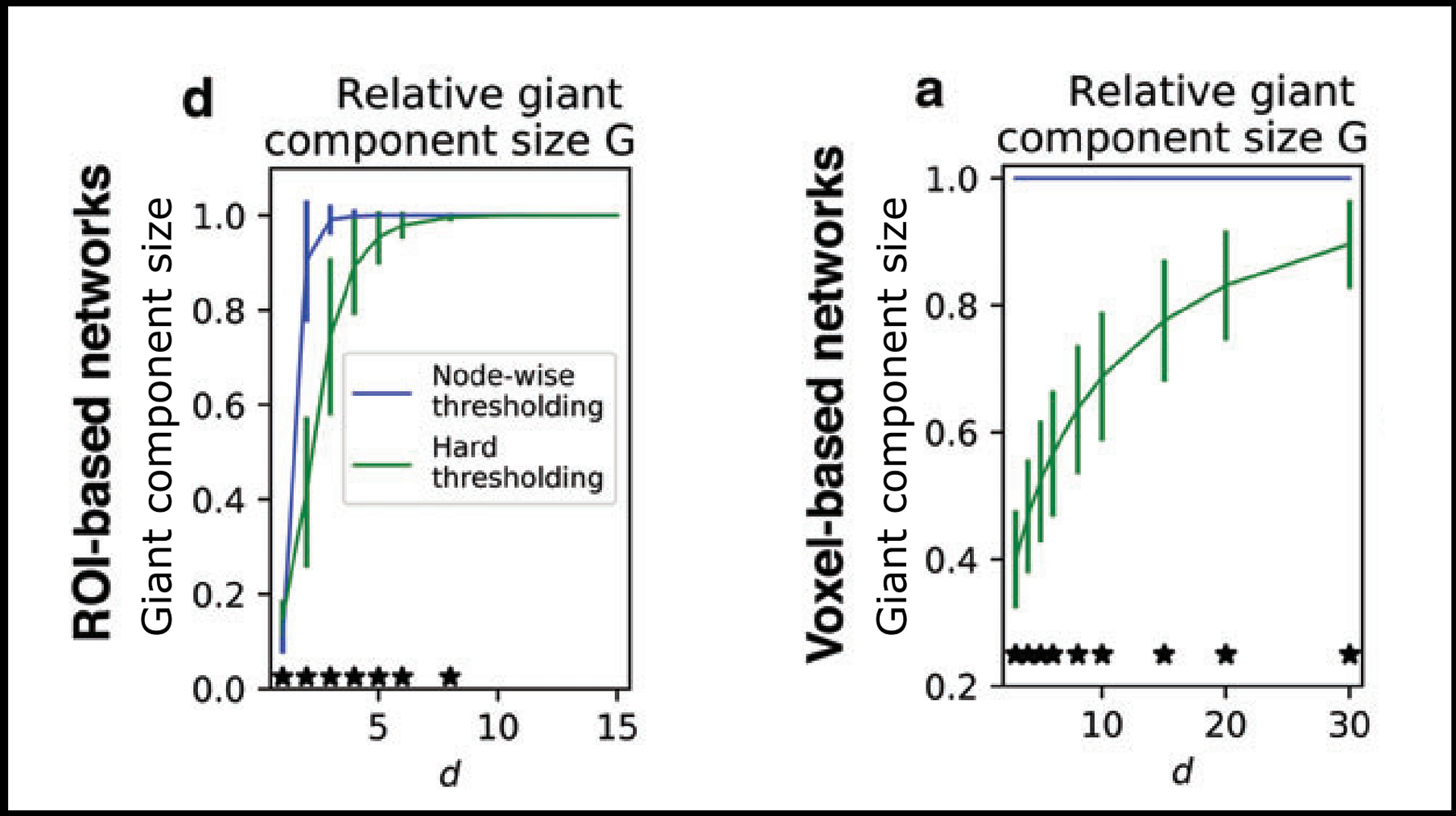
S&P 500 correlation network




*Hayasaka, Brain Connectivity (2017)*

# Exhibit 3

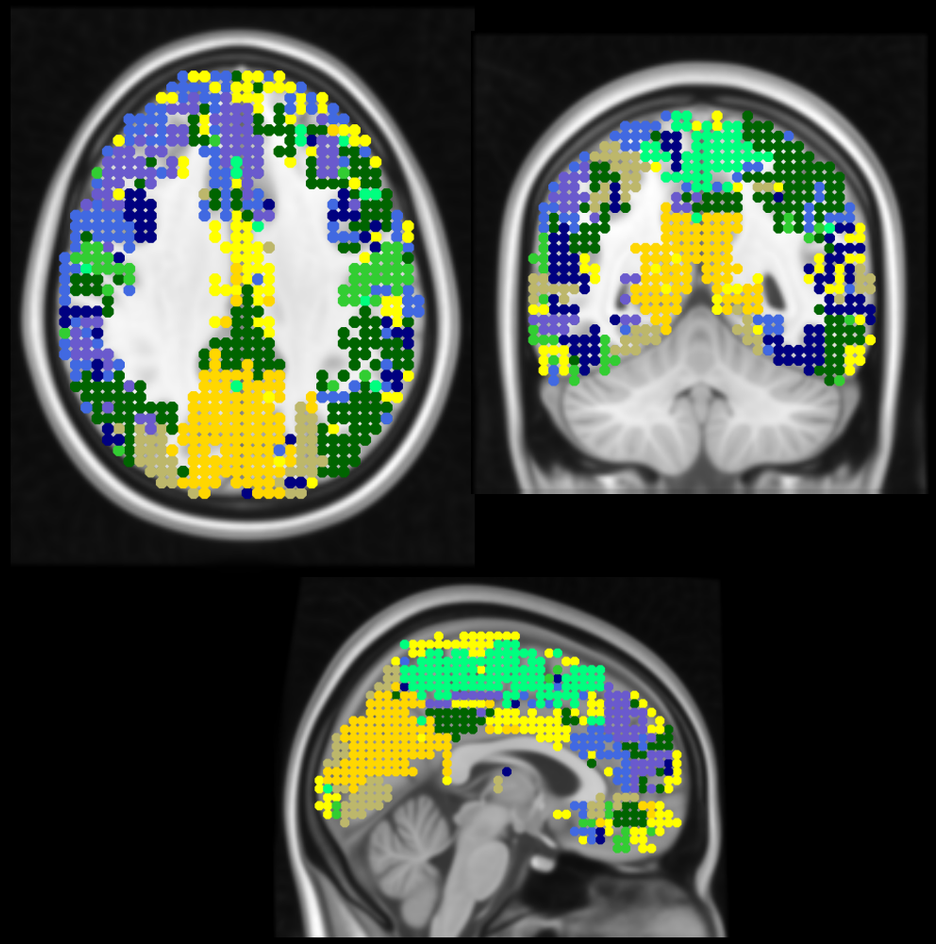
rs-fMRI networks from N=123 subjects



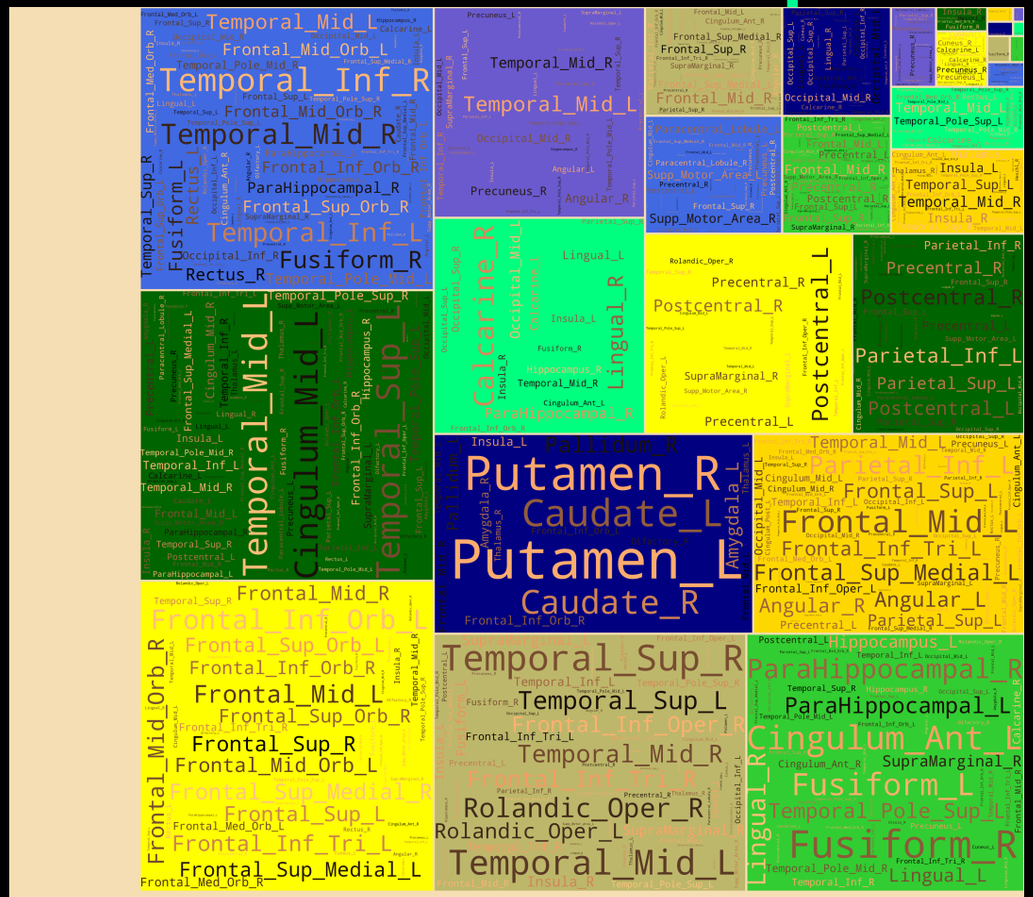

*Hayasaka, Brain Connectivity (2017)*

# Modules from hard thresholding

Modular partition of voxel-level rs-fMRI network
(representative subject, formed at d=10)



**Giant component**




**Disconnected components**

*Relative sizes of modules and their constituents*

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Modular partition of voxel-level rs-fMRI network
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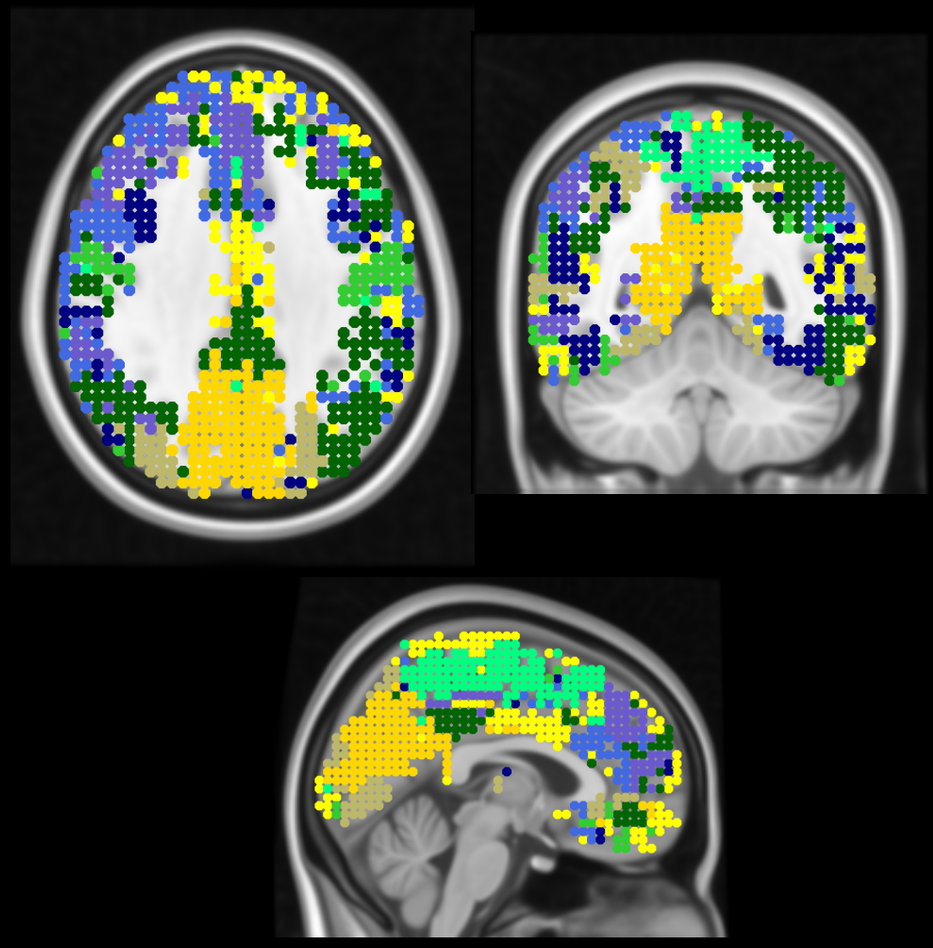
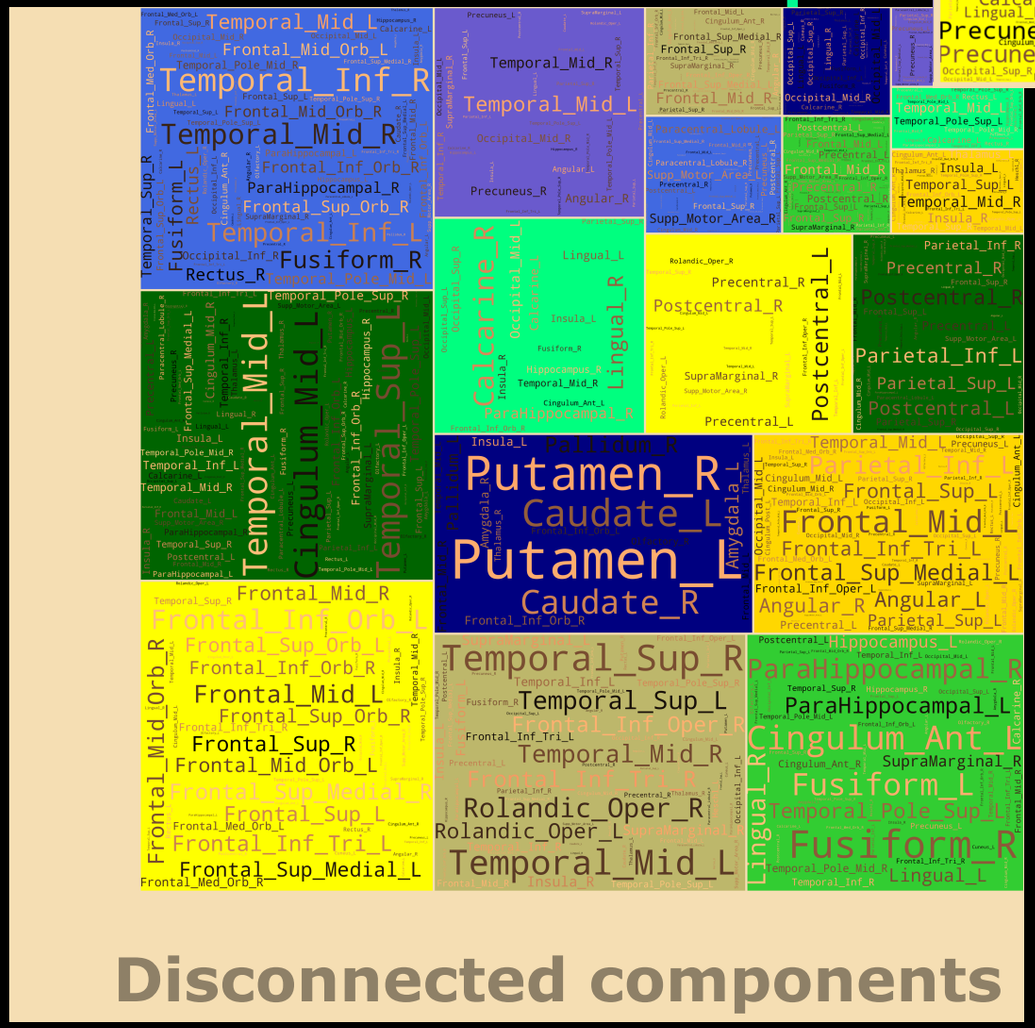


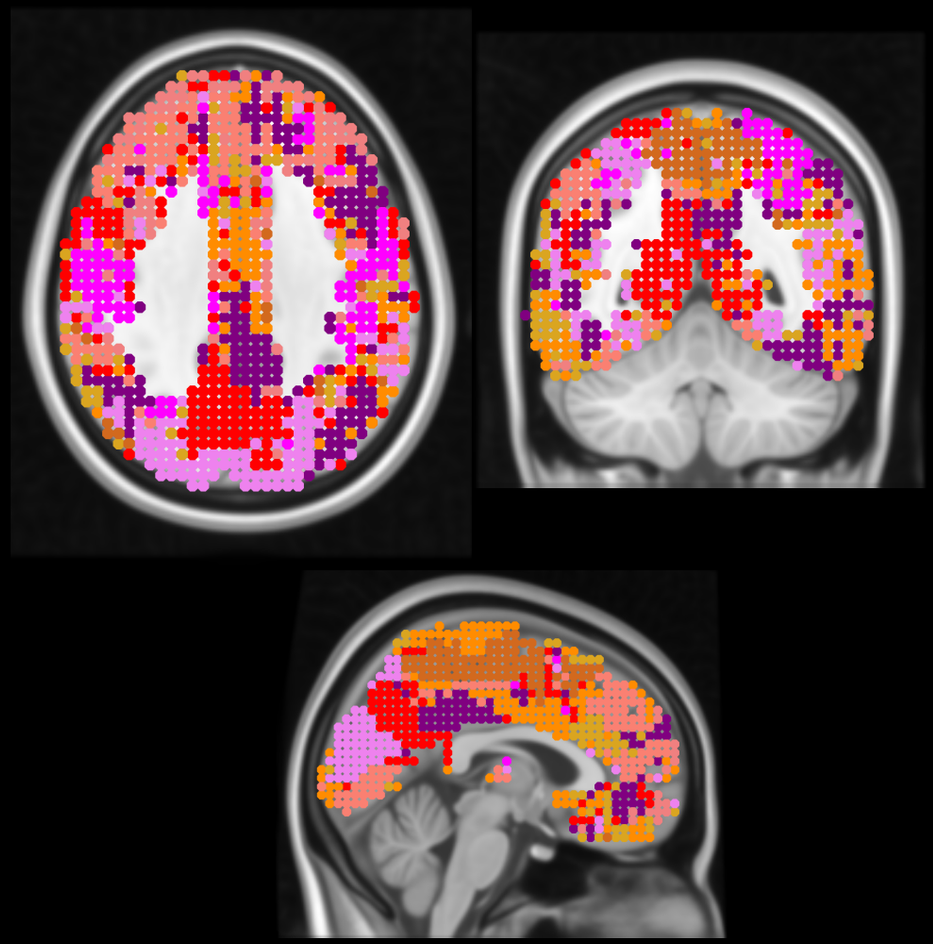



**Disconnected components**

*Relative sizes of modules and their constituents*

# Modules from node-wise thresholding

Modular partition of voxel-level rs-fMRI network
(representative subject, formed at d=10)

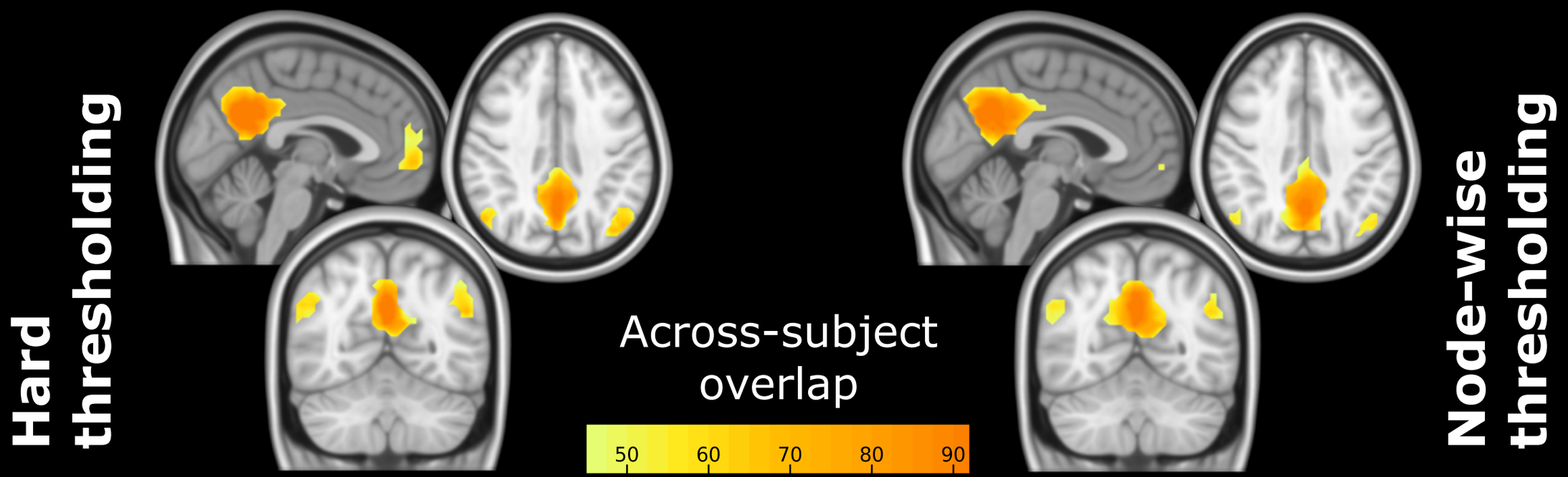



*Relative sizes of modules and their constituents*

# Consistency of modules

Overlap of modules across N=123 subjects
voxel-level rs-fMRI network
(formed at d=10)

Default mode network module

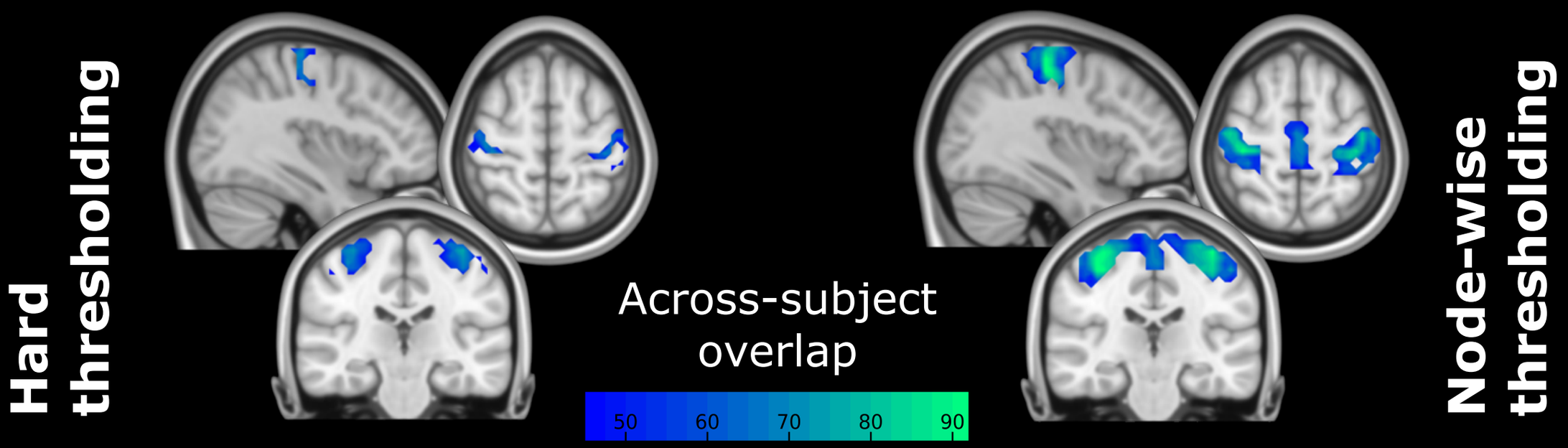



*Hayasaka, OHBM 2018 Poster*

# Consistency of modules

Overlap of modules across N=123 subjects
voxel-level rs-fMRI network
(formed at d=10)

Sensory-motor module




*Hayasaka, OHBM 2018 Poster*

# Take home messages

| Hard thresholding | Node-wise thresholding |
| --- | --- |
| Gradual growth, no percolation | Percolation during formation |
| Some disconnected components | No disconnected components |
| Possible micro modules | No micro modules |
| Limited consistency in some modules | Consistent modules |

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- Demonstration (with Jupyter notebook)

# GitHub Repo

- Codes, data, some figures, slides (PDF), and notes

sathayas/NetNeurosci2019

- Link from
  - Web: sathayas.github.io
  - Twitter: @sathayas42