

Area of a Traingle

1 10th Maths - Chapter 7

This is Problem-1 from Exercise 7.3

1. Find the area of the triangle whose vertices are :

- (a) $(2, 3), (-1, 0), (2, -4)$

Solution: Refer figure 1

The area of the triangle with vertices **A, B, C** is given by

$$\frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| \quad (1)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (2)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix} \quad (3)$$

The value of the cross product of two vectors is given by

$$|\mathbf{M}| = \begin{vmatrix} \mathbf{A} & \mathbf{B} \end{vmatrix} \quad (4)$$

$$= \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \quad (5)$$

Therefore, (1) equals

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 3 & 0 \\ 3 & 7 \end{vmatrix} \quad (6)$$

$$= \frac{1}{2} (21) \quad (7)$$

$$= 10.5 \text{ Sq units} \quad (8)$$

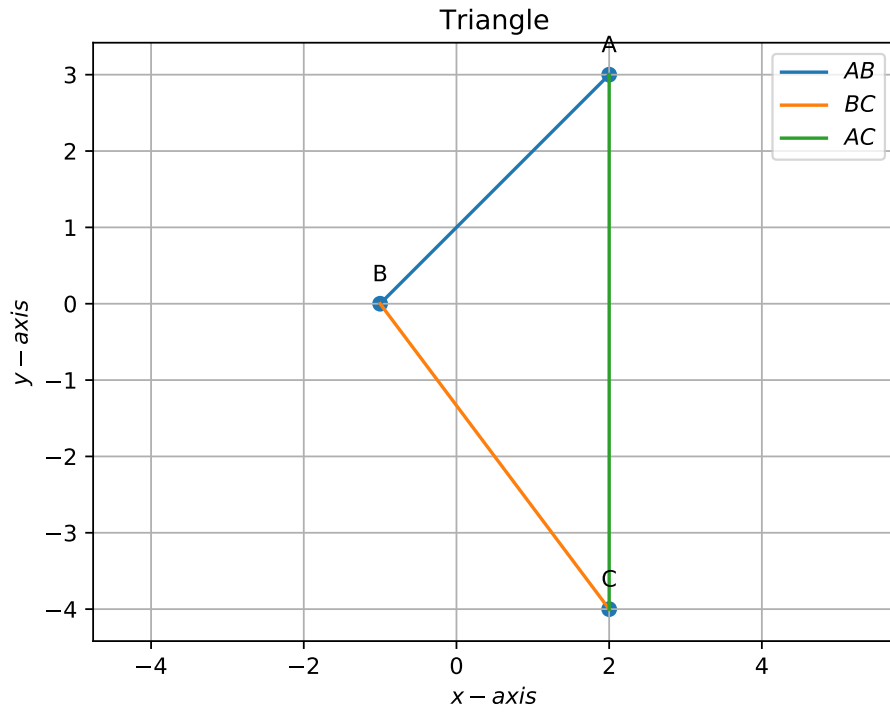


Figure 1

(b) $(-5, -1), (3, -5), (5, 2)$

Solution: Refer figure 2

The area of the triangle with vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ is given by

$$\frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| \quad (9)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -5 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \end{pmatrix} \quad (10)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -5 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -10 \\ -3 \end{pmatrix} \quad (11)$$

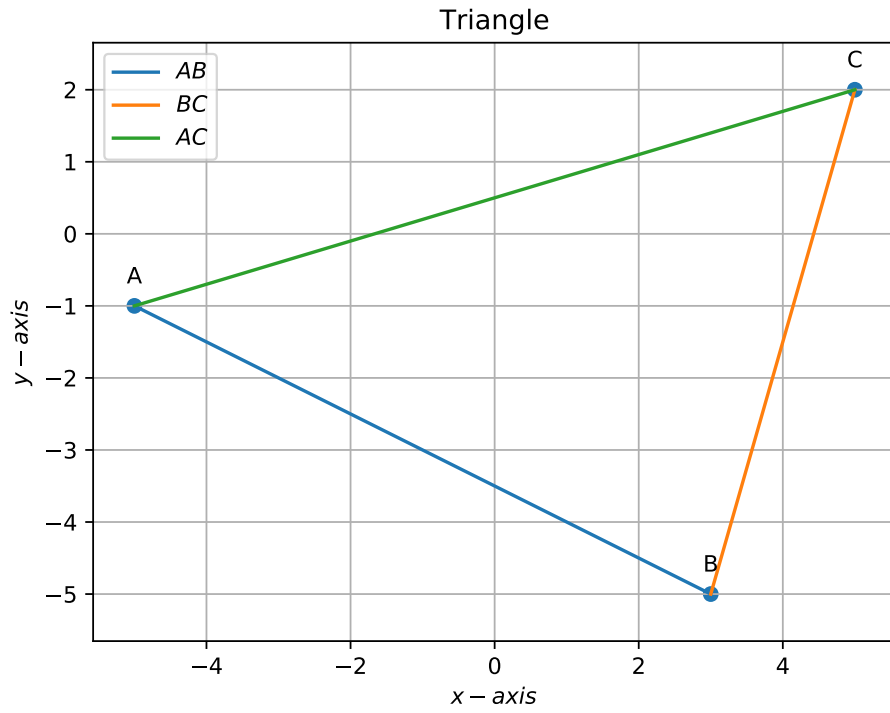


Figure 2

The value of the cross product of two vectors is given by

$$|\mathbf{M}| = |\mathbf{A} \quad \mathbf{B}| \quad (12)$$

$$= \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \quad (13)$$

Therefore, (9) equals

$$\text{Area} = \frac{1}{2} \begin{vmatrix} -8 & -10 \\ 4 & -3 \end{vmatrix} \quad (14)$$

$$= \frac{1}{2} (24 + 40) \quad (15)$$

$$= \frac{1}{2} (64) \quad (16)$$

$$= 32 \text{ Sq units} \quad (17)$$