Normal to a Parabola

1 12^{th} Maths - Chapter 6

This is Problem-22 from Exercise 6.6

1. Find the equation of the normal at the point $(1, 1)$ on the curve $2y + x^2 = 3$.

   Solution: The given equation can be written as

   $$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{1}$$

   where

   $$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

   $$\mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3}$$

   $$f = -3 \tag{4}$$

   The equation of normal to the parabola , at a given point $\mathbf{q}$ is given by

   $$(\mathbf{V}\mathbf{q} + \mathbf{u})^\top \mathbf{R}(\mathbf{x} - \mathbf{q}) = 0 \tag{5}$$

   where $\mathbf{R}$ is rotation matrix given by

   $$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

   Given

   $$\mathbf{q} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{7}$$

   $$\tag{8}$$

Substituting all the values in (5)

$$(5) \implies \left(\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right)^{\top}\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\left(\mathbf{x}-\begin{pmatrix}1\\1\end{pmatrix}\right)=0 \tag{9}$$

$$\implies \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\left(\mathbf{x}-\begin{pmatrix}1\\1\end{pmatrix}\right)=0 \tag{10}$$

$$\implies \begin{pmatrix}1 & -1\end{pmatrix}\left(\mathbf{x}-\begin{pmatrix}1\\1\end{pmatrix}\right)=0 \tag{11}$$

$$\implies \begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x}=0 \tag{12}$$

The relevant figure is shown in 1

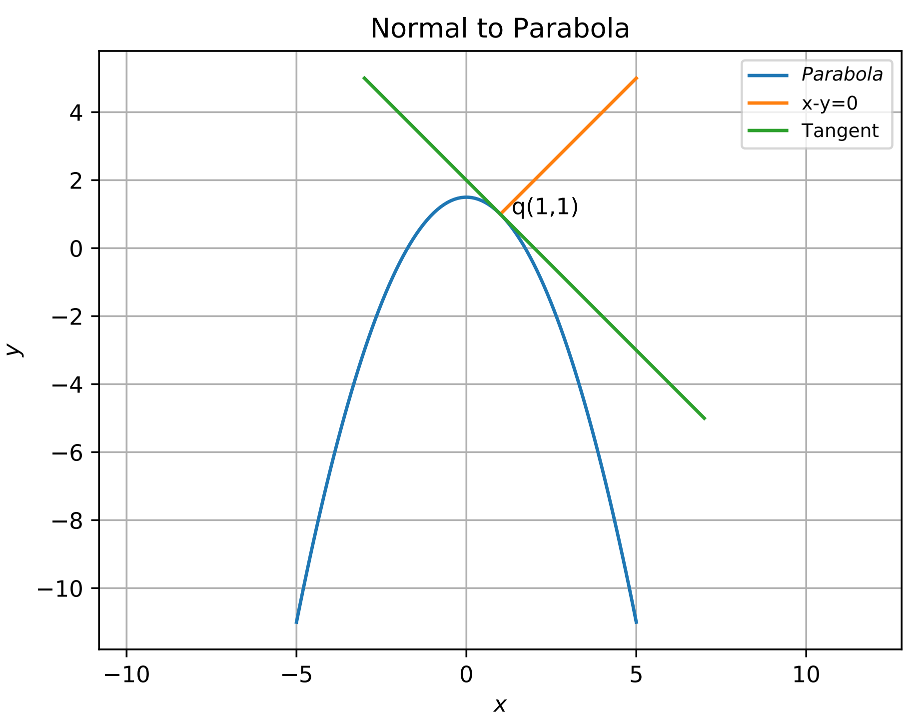

Figure 1