Properties of Triangles

1 10th Maths - Chapter 7

This is Problem-3 from Exercise 7.1

1. Determine if the points $(1,5), (2,3),$ and $(-2,-11)$ are collinear.
   Solution: We know that points $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are collinear, if

$$\text{rank} \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} < 2 \tag{1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} \qquad = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{2}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -2 \\ -11 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} \qquad = \begin{pmatrix} -3 \\ -16 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ -2 & -16 \end{pmatrix} \tag{4}$$

   It is quite obvious that the above matrix mentioned in equation 4 has non zero determinant value implying that it is a full rank matrix with rank equal to 2.
   Hence from equation 1, it can be inferred that the points are not collinear.
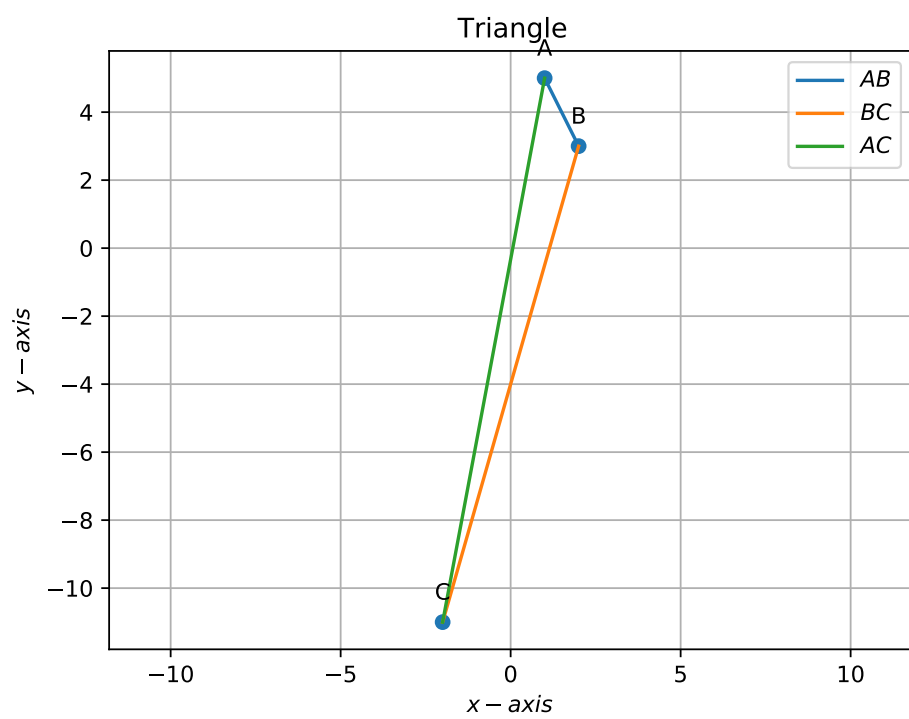   Refer Figure 1.

Figure 1