

Properties of Triangles

1 10th Maths - Chapter 7

This is Problem-3 from Exercise 7.1

1. Determine if the points $(1, 5)$, $(2, 3)$, and $(-2, -11)$ are collinear.

Solution: We know that points **A**, **B** and **C** are collinear, if

$$\text{rank} \begin{pmatrix} \mathbf{A}^\top \\ \mathbf{B}^\top \\ \mathbf{C}^\top \end{pmatrix} = 1 \quad (1)$$

$$\begin{pmatrix} \mathbf{A}^\top \\ \mathbf{B}^\top \\ \mathbf{C}^\top \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 2 & 3 \\ -2 & -11 \end{pmatrix} \quad (2)$$

Performing a sequence of row reduction operations

$$\begin{matrix} \xleftarrow{R_2 \rightarrow R_2 - 2R_1} \\ \xrightarrow{R_3 \rightarrow R_3 + 2R_1} \end{matrix} \begin{pmatrix} 1 & 5 \\ 0 & -7 \\ 0 & -1 \end{pmatrix} \quad (3)$$

$$\xleftarrow{R_3 \rightarrow R_3 - \frac{1}{7}R_2} \begin{pmatrix} 1 & 5 \\ 0 & -7 \\ 0 & 0 \end{pmatrix} \quad (4)$$

Since the matrix has only 2 pivot points, the rank of the matrix is 2. Therefore, referring to equation 1, the points are not collinear as the rank of the matrix is not equal to 1. Refer to the figure 1 depicting a triangle with points $(1, 5)$, $(2, 3)$, and $(-2, -11)$.

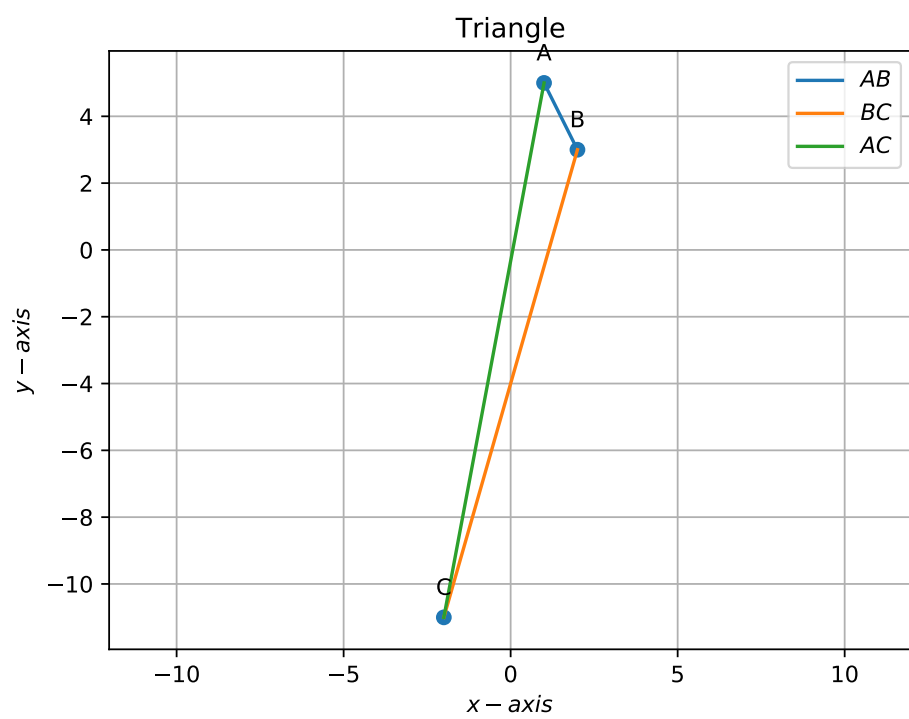


Figure 1