# 3D Lines

## 1  JEE Maths - 65 C-1

This is Problem-30

1. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2$

   **Solution:** The given equation can be written as

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-0}{1} \tag{1}$$

$$\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0} \tag{2}$$

$$\implies \mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \tag{3}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} \tag{4}$$

$$\tag{5}$$

   This can be formulated as an unconstrained optimization problem as below:

$$\min_{\lambda_1, \lambda_2} \quad \|\mathbf{B} - \mathbf{A}\|^2 \tag{6}$$

Setting these parameters in cvxpy and solving, yields

$$\lambda_1 = 1.4, \lambda_2 = 0.969 \tag{7}$$

$$\mathbf{A} = \begin{pmatrix} 3.8 \\ 3.2 \\ 1.4 \end{pmatrix} \tag{8}$$

$$\mathbf{B} = \begin{pmatrix} 3.85 \\ 2.97 \\ 2 \end{pmatrix} \tag{9}$$

$$\|\mathbf{B} - \mathbf{A}\| = 0.6445 \text{ units} \tag{10}$$
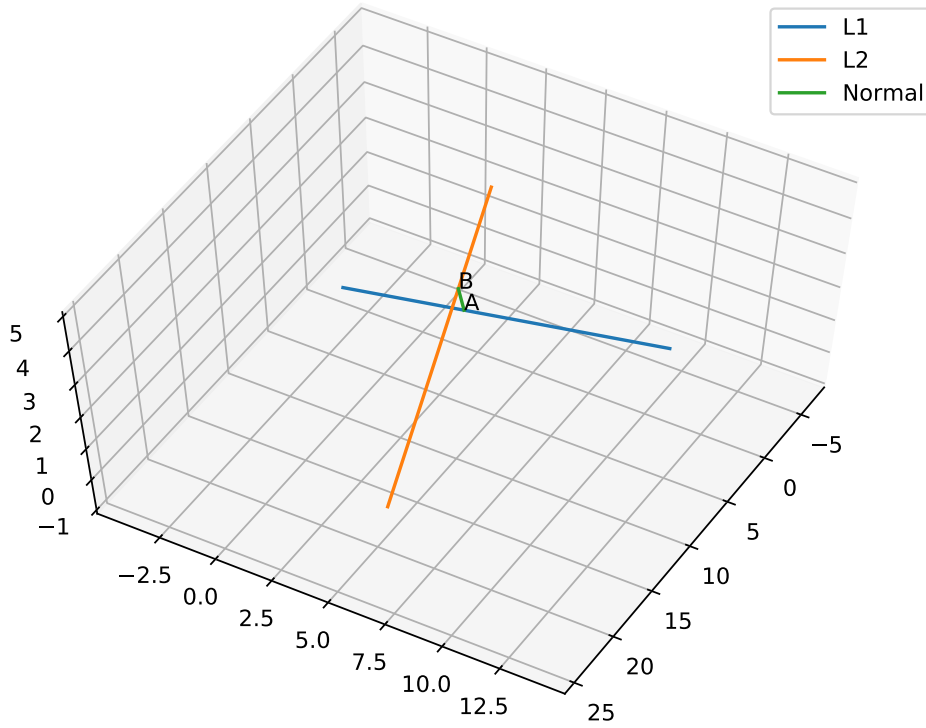
The relevant figure is shown in 1.



Figure 1