

# Convex Optimization

## 1 12<sup>th</sup> Maths - Chapter 6

This is Problem-1(i) from Exercise 6.5

1. Determine whether the function  $f(x) = (2x - 1)^2 + 3$  is convex or not.

**Solution:** A single variable function  $f$  is said to be convex if

$$f[\lambda a + (1 - \lambda)b] \leq \lambda f(a) + (1 - \lambda)f(b), \quad (1)$$

for  $0 < \lambda < 1$ . The given function is

$$f(x) = (2x - 1)^2 + 3 \quad (2)$$

Consider two points  $a, b$  on x-axis.

$$a = 0 \quad (3)$$

$$b = 1.25 \quad (4)$$

$$\lambda = 0.45 \quad (5)$$

Substituting above values in Equation (1)

$$f[\lambda a + (1 - \lambda)b] = f[0.45(0) + (1 - 0.45)1.25] \quad (6)$$

$$= f[0.6875] = 3.141 \quad (7)$$

$$\lambda f(a) + (1 - \lambda)f(b) = 0.45f(0) + (1 - 0.45)f(1.25) \quad (8)$$

$$= 0.45(4) + (0.55)(5.25) = 4.6875 \quad (9)$$

$$(1) \implies 3.141 \leq 4.6875 \quad (10)$$

$\therefore$  The above inequality holds true, the given equation is convex. The figure is as shown in Fig1

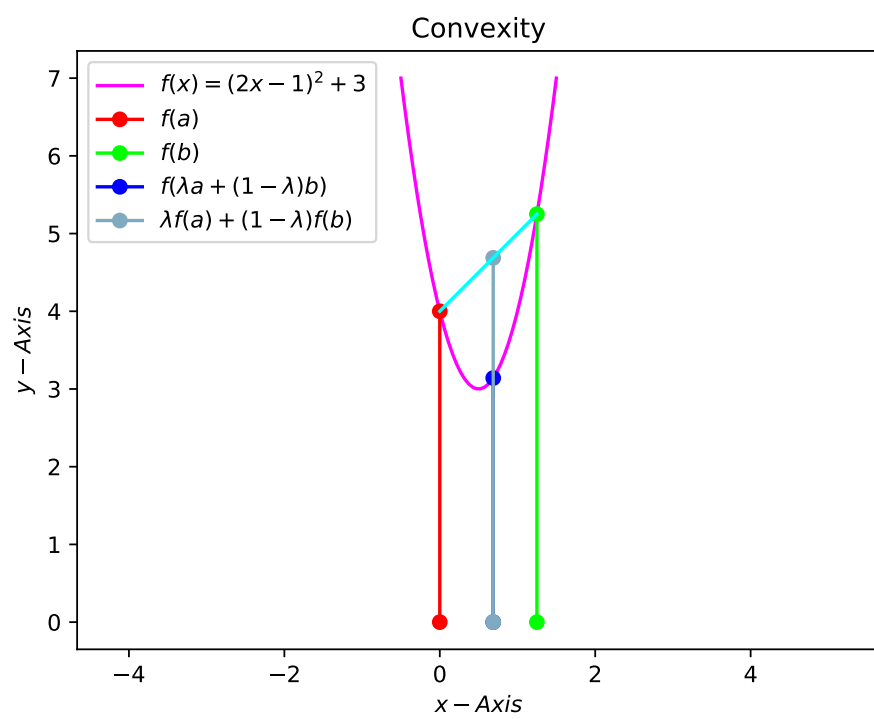


Figure 1