

3D Lines

1 JEE Maths - 65 C-1

This is Problem-30

1. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2$

Solution: The given equation can be written as

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-0}{1} \quad (1)$$

$$\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0} \quad (2)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad (3)$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} \quad (4)$$

$$(5)$$

This can be formulated as an optimization problem as below:

$$\min_{\lambda_1, \lambda_2} \|\mathbf{B} - \mathbf{A}\|^2 \quad (6)$$

$$\text{s.t. } \lambda_1 \geq 0, \lambda_2 \geq 0 \quad (7)$$

Setting these parameters in cvxpy and solving, yield

$$\lambda_1 = 1.4, \lambda_2 = 0.969 \quad (8)$$

$$\mathbf{A} = \begin{pmatrix} 3.8 \\ 3.2 \\ 1.4 \end{pmatrix} \quad (9)$$

$$\mathbf{B} = \begin{pmatrix} 3.85 \\ 2.97 \\ 2 \end{pmatrix} \quad (10)$$

$$\|\mathbf{B} - \mathbf{A}\| = 0.6445 \text{ units} \quad (11)$$

The relevant figure is shown in 1.

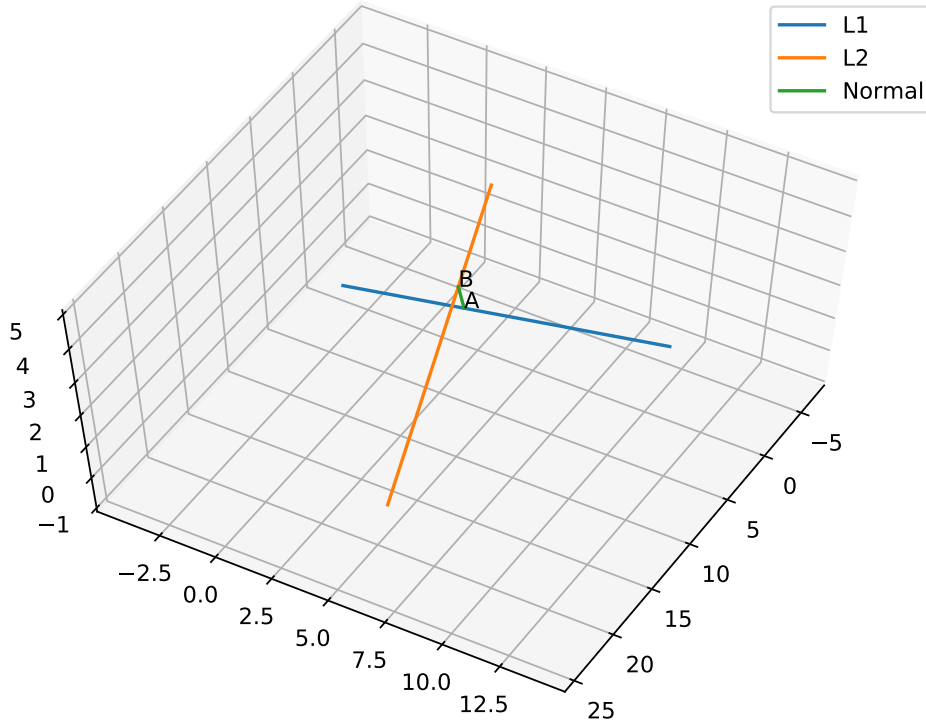


Figure 1