Vector Dot Product

1   12th Maths - Chapter 10

This is Problem-9 from Exercise 10.3

1. Find $\|\mathbf{x}\|$, if for a unit vector $\mathbf{a}$, $(\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} + \mathbf{a}) = 12$.
   Solution:

$$(\mathbf{x} - \mathbf{a})^{\top} (\mathbf{x} + \mathbf{a}) = 12$$
$$\mathbf{x}^{\top}\mathbf{x} - \mathbf{a}^{\top}\mathbf{x} + \mathbf{x}^{\top}\mathbf{a} - \mathbf{a}^{\top}\mathbf{a} = 12$$
$$\implies \|\mathbf{x}\|^2 - \|\mathbf{a}\|^2 = 12$$
$$\|\mathbf{x}\|^2 - 1 = 12$$
$$\|\mathbf{x}\|^2 = 13$$
$$\|\mathbf{x}\| = \sqrt{13}$$