

Equation of Line

1 11th Maths - Chapter 10

This is Problem-12 from Exercise 10.2

1. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $(2, 3)$.

Solution: Let $\mathbf{P}(a, 0)$, and $\mathbf{Q}(0, a)$ be the 2 points on x and y-axes respectively having a as the intercept on both the axes. We know that the the direction vector \mathbf{m} of the line joining two points \mathbf{P}, \mathbf{Q} is given by

$$\mathbf{m} = \mathbf{P} - \mathbf{Q} \quad (1)$$

$$= \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ a \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2)$$

$$(3)$$

\Rightarrow The normal vector \mathbf{n} to the line is given as

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (4)$$

The equation of a line with normal vector \mathbf{n} and passing through a point $\mathbf{A}(2, 3)$ is given by

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \quad (5)$$

$$(1 \ 1) \left(\mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) = 0 \quad (6)$$

$$(1 \ 1) \mathbf{x} - 5 = 0 \quad (7)$$

$$(1 \ 1) \mathbf{x} = 5 \quad (8)$$

The line segment is as shown in Figure 1

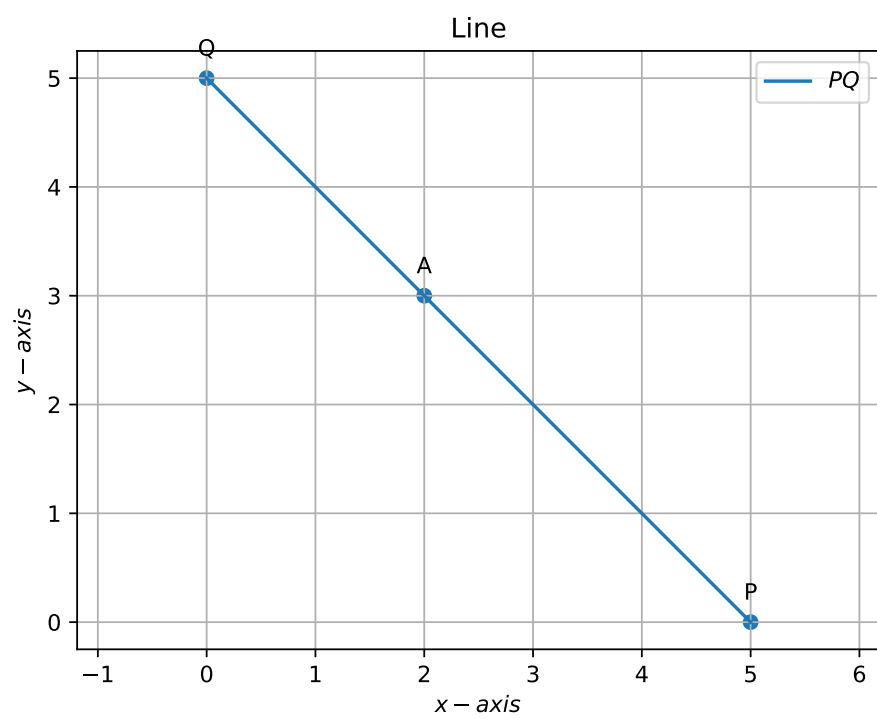


Figure 1