Vector Dot Product

$1 \quad 12^{th} \text{ Maths}$ - Chapter 10

This is Problem-9 from Exercise 10.3

1. Find $\|\mathbf{x}\|$, if for a unit vector \mathbf{a} , $(\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} + \mathbf{a}) = 12$. Solution:

$$(\mathbf{x} - \mathbf{a})^{\top} (\mathbf{x} + \mathbf{a}) = 12$$

$$\mathbf{x}.\mathbf{x} - \mathbf{x}.\mathbf{a} + \mathbf{a}.\mathbf{x} - \mathbf{a}.\mathbf{a} = 12$$

$$\|\mathbf{x}\|^{2} - \|\mathbf{a}\|^{2} = 12$$

$$\|\mathbf{x}\|^{2} - 1 = 12$$

$$\|\mathbf{x}\|^{2} = 13$$

$$\|\mathbf{x}\| = \sqrt{13}$$