

# Conic Sections - Circle

## 1 10<sup>th</sup> Maths - Chapter 10

This is Problem-1 from Exercise 10.2

1. From a point  $\mathbf{Q}$ , the length of the tangent to a circle is  $24\text{cm}$  and the distance of  $\mathbf{Q}$  from the centre is  $25\text{cm}$ . Find the radius of the circle. Draw the circle and the tangents.

**Solution:** Let  $\mathbf{Q}$  be  $\mathbf{Q} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ . Let  $\mathbf{O}$  be the centre of the circle. Let  $\mathbf{R}_1$  and  $\mathbf{R}_2$  be the two points on the circle such that  $\mathbf{R}_1\mathbf{Q}$  and  $\mathbf{R}_2\mathbf{Q}$  are tangents to the circle from the point  $\mathbf{Q}$ . Given that,

$$\|\mathbf{OQ}\| = 25, \|\mathbf{R}_1\mathbf{Q}\| = \|\mathbf{R}_2\mathbf{Q}\| = 24 \quad (1)$$

$$\therefore \mathbf{c} = \mathbf{O} = \mathbf{O} \begin{pmatrix} 25 \\ 0 \end{pmatrix} \quad (2)$$

$$r = \|\mathbf{OR}_1\| = \sqrt{\|\mathbf{OQ}\|^2 - \|\mathbf{R}_1\mathbf{Q}\|^2} \quad (3)$$

$$= \sqrt{25^2 - 24^2} \quad (4)$$

$$= 7 \quad (5)$$

We have to find points  $\mathbf{R}_1$  and  $\mathbf{R}_2$ .

We know that the equation to the circle is given as

$$\|\mathbf{x}\|^2 + 2\mathbf{x}^\top \mathbf{u} + f = 0 \quad (6)$$

where

$$\mathbf{u} = -\mathbf{c} = - \begin{pmatrix} 25 \\ 0 \end{pmatrix} \text{ and} \quad (7)$$

$$f = \|\mathbf{c}\|^2 - r^2 = 576 \quad (8)$$

$$\boldsymbol{\Sigma} = (\mathbf{Q} + \mathbf{u})(\mathbf{Q} + \mathbf{u})^\top - (\|\mathbf{Q}\|^2 + 2\mathbf{u}^\top \mathbf{Q} + f) \mathbf{I} \quad (9)$$

$$= \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 25 \\ 0 \end{pmatrix} \right) \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 25 \\ 0 \end{pmatrix} \right)^\top - \left( 0 - 2 \begin{pmatrix} 25 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 576 \right) \mathbf{I} \quad (10)$$

$$= \begin{pmatrix} -25 \\ 0 \end{pmatrix} \begin{pmatrix} -25 & 0 \end{pmatrix} - (576) \mathbf{I} \quad (11)$$

$$= \begin{pmatrix} 625 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 576 & 0 \\ 0 & 576 \end{pmatrix} \quad (12)$$

$$= \begin{pmatrix} 49 & 0 \\ 0 & -576 \end{pmatrix} \quad (13)$$

From (13), we can deduce Eigen pairs as follow:

$$\lambda_1 = 49, \lambda_2 = -576 \quad (14)$$

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (15)$$

Then

$$\mathbf{n}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{|\lambda_1|} \\ \sqrt{|\lambda_2|} \end{pmatrix} = \begin{pmatrix} 7 \\ 24 \end{pmatrix} \quad (16)$$

$$\mathbf{n}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{|\lambda_1|} \\ -\sqrt{|\lambda_2|} \end{pmatrix} = \begin{pmatrix} 7 \\ -24 \end{pmatrix} \quad (17)$$

The points of contact of a tangent on a circle from an external point is given by

$$\mathbf{q}_{ij} = \left( \pm r \frac{\mathbf{n}_j}{\|\mathbf{n}_j\|} - \mathbf{u} \right), \quad i, j = 1, 2 \quad (18)$$

$$\mathbf{q}_{i1} = \left( \pm r \frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} - \mathbf{u} \right) \quad (19)$$

$$= \left( \pm \frac{7}{25} \begin{pmatrix} 7 \\ 24 \end{pmatrix} + \begin{pmatrix} 25 \\ 0 \end{pmatrix} \right) \quad (20)$$

$$= \left( \pm \begin{pmatrix} \frac{49}{25} \\ \frac{25}{168} \end{pmatrix} + \begin{pmatrix} 25 \\ 0 \end{pmatrix} \right) \quad (21)$$

$$= \begin{pmatrix} \frac{674}{25} \\ \frac{25}{168} \end{pmatrix}, \begin{pmatrix} \frac{576}{25} \\ -\frac{25}{168} \end{pmatrix} \quad (22)$$

$$\mathbf{q}_{i2} = \left( \pm r \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|} - \mathbf{u} \right) \quad (23)$$

$$= \left( \pm \frac{7}{25} \begin{pmatrix} 7 \\ -24 \end{pmatrix} + \begin{pmatrix} 25 \\ 0 \end{pmatrix} \right) \quad (24)$$

$$= \left( \pm \begin{pmatrix} \frac{49}{25} \\ -\frac{168}{25} \end{pmatrix} + \begin{pmatrix} 25 \\ 0 \end{pmatrix} \right) \quad (25)$$

$$= \begin{pmatrix} \frac{674}{25} \\ -\frac{168}{25} \end{pmatrix}, \begin{pmatrix} \frac{576}{25} \\ \frac{168}{25} \end{pmatrix} \quad (26)$$

$$\therefore \mathbf{R}_1 = \mathbf{q}_{22} = \begin{pmatrix} \frac{576}{25} \\ \frac{168}{25} \end{pmatrix}, \mathbf{R}_2 = \mathbf{q}_{12} = \begin{pmatrix} \frac{576}{25} \\ -\frac{168}{25} \end{pmatrix} \quad (27)$$

The figure is as shown in 1

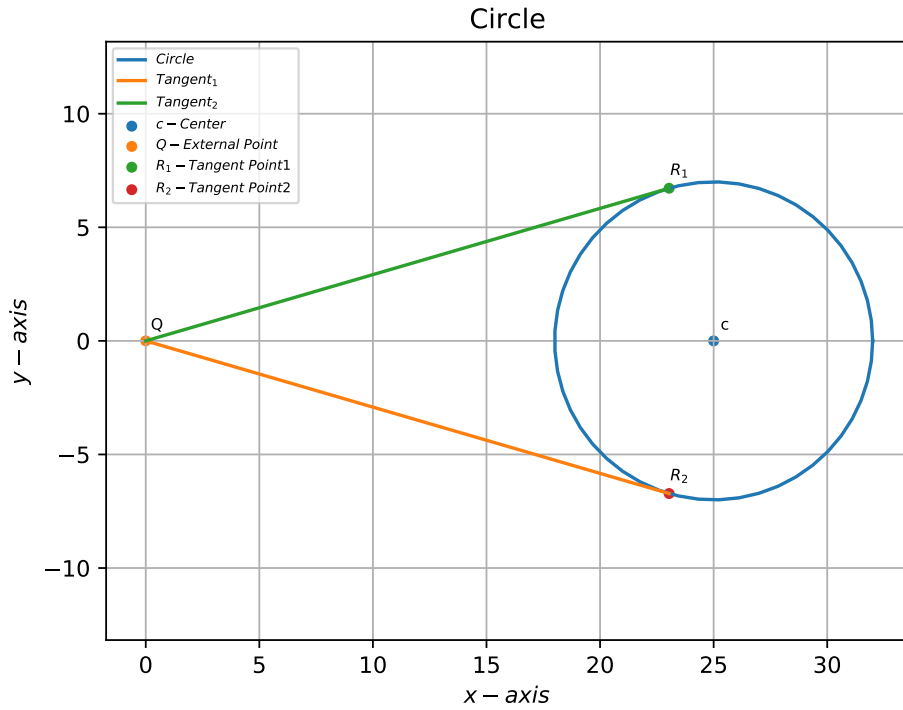


Figure 1