# Area of a Traingle

## 1 $10^{th}$ Maths - Chapter 7

This is Problem-1 from Exercise 7.3

1. Find the area of the triangle whose vertices are :

   (a) $(2,3), (-1,0), (2,-4)$
   **Solution:** Refer figure 1
   The area of the triangle with vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ is given by

$$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C})\right\| \tag{1}$$

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{2}$$

$$\mathbf{A}-\mathbf{C} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix} \tag{3}$$

   The value of the cross product of two vectors is given by

$$\left|\mathbf{M}\right| = \left|\mathbf{A} \quad \mathbf{B}\right| \tag{4}$$

$$= \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 \tag{5}$$

   Therefore, (1) equals

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 3 & 0 \\ 3 & 7 \end{vmatrix} \tag{6}$$

$$= \frac{1}{2}(21) \tag{7}$$

$$= 10.5 \text{ Sq units} \tag{8}$$

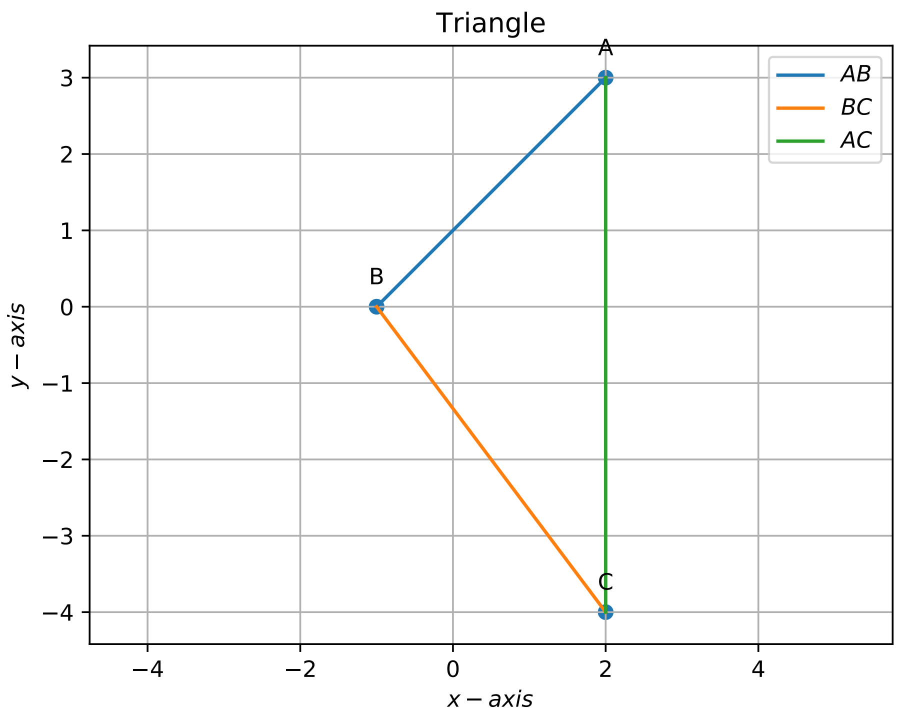


Figure 1

(b) $(-5,-1),(3,-5),(5,2)$
**Solution:** Refer figure 2
The area of the triangle with vertices $\mathbf{A},\mathbf{B},\mathbf{C}$ is given by

$$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C})\right\| \tag{9}$$

$$\mathbf{A}-\mathbf{B}=\begin{pmatrix}-5\\-1\end{pmatrix}-\begin{pmatrix}3\\-5\end{pmatrix}=\begin{pmatrix}-8\\4\end{pmatrix} \tag{10}$$

$$\mathbf{A}-\mathbf{C}=\begin{pmatrix}-5\\-1\end{pmatrix}-\begin{pmatrix}5\\2\end{pmatrix}=\begin{pmatrix}-10\\-3\end{pmatrix} \tag{11}$$

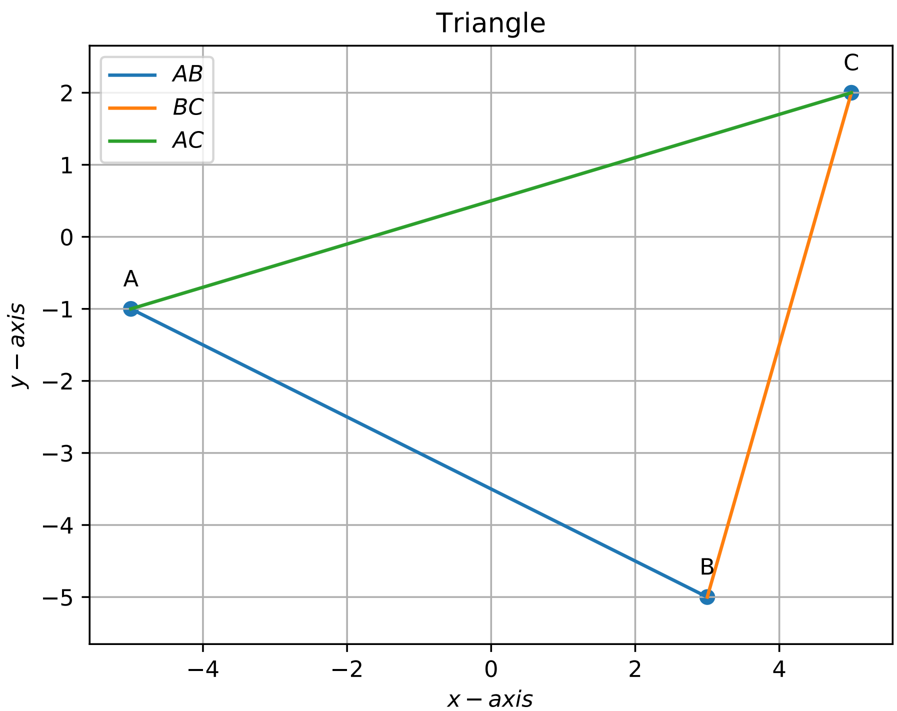


Figure 2

The value of the cross product of two vectors is given by

$$\left|\mathbf{M}\right| = \left|\mathbf{A} \quad \mathbf{B}\right| \tag{12}$$

$$= \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \tag{13}$$

Therefore, (9) equals

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -8 & -10 \\ 4 & -3 \end{vmatrix} \tag{14}$$
$$= \frac{1}{2}(24+40) \tag{15}$$
$$= \frac{1}{2}(64) \tag{16}$$
$$= 32 \text{ Sq units} \tag{17}$$

This is Problem-8 in Exercise 7.4

2. ABCD is a rectangle formed by the points $\mathbf{A}(-1,-1), \mathbf{B}(-1,4), \mathbf{C}(5,4)$ and $\mathbf{D}(5,-1)$. $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ are the mid-points of $\mathbf{AB}, \mathbf{BC}, \mathbf{CD}$ and $\mathbf{DA}$ respectively.
Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.
**Solution:** Refer figure 3

$$\mathbf{P} = \frac{1}{2}(\mathbf{A}+\mathbf{B}) = \frac{1}{2}\left(\begin{pmatrix}-1\\-1\end{pmatrix} + \begin{pmatrix}-1\\4\end{pmatrix}\right) = \begin{pmatrix}-1\\1.5\end{pmatrix} \tag{18}$$
$$\mathbf{Q} = \frac{1}{2}(\mathbf{B}+\mathbf{C}) = \frac{1}{2}\left(\begin{pmatrix}-1\\4\end{pmatrix} + \begin{pmatrix}5\\4\end{pmatrix}\right) = \begin{pmatrix}2\\4\end{pmatrix} \tag{19}$$
$$\mathbf{R} = \frac{1}{2}(\mathbf{C}+\mathbf{D}) = \frac{1}{2}\left(\begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}5\\-1\end{pmatrix}\right) = \begin{pmatrix}5\\1.5\end{pmatrix} \tag{20}$$
$$\mathbf{S} = \frac{1}{2}(\mathbf{D}+\mathbf{A}) = \frac{1}{2}\left(\begin{pmatrix}5\\-1\end{pmatrix} + \begin{pmatrix}-1\\-1\end{pmatrix}\right) = \begin{pmatrix}2\\-1\end{pmatrix} \tag{21}$$
$$\tag{22}$$

We know that PQRS is a parallelogram. To know, if it is a rectangle, we need to ascertain whether any of the two adjacent sides are perpendicular. That means $(\mathbf{Q}-\mathbf{P})^T(\mathbf{R}-\mathbf{Q})$ should be equal to zero.

$$\mathbf{Q}-\mathbf{P} = \begin{pmatrix}2\\4\end{pmatrix} - \begin{pmatrix}-1\\1.5\end{pmatrix} = \begin{pmatrix}3\\2.5\end{pmatrix} \tag{23}$$
$$\mathbf{R}-\mathbf{Q} = \begin{pmatrix}5\\1.5\end{pmatrix} - \begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}3\\-2.5\end{pmatrix} \tag{24}$$
$$(\mathbf{Q}-\mathbf{P})^T(\mathbf{R}-\mathbf{Q}) = \begin{pmatrix}3 & 2.5\end{pmatrix}\begin{pmatrix}3\\-2.5\end{pmatrix} \neq 0 \tag{25}$$

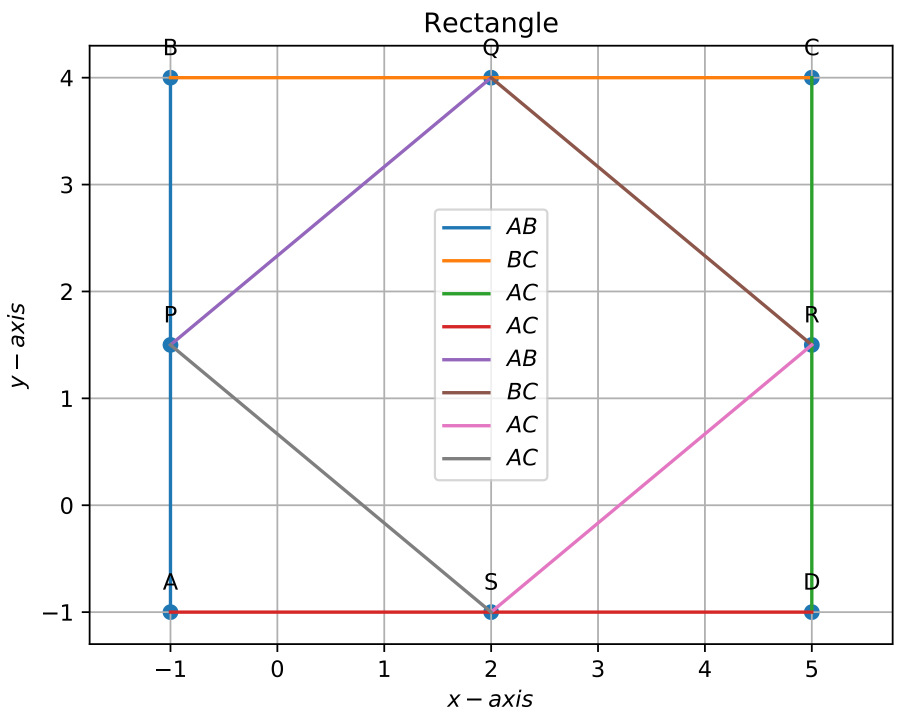


Figure 3

Therefore PQRS is not a rectangle. Let us check if it is a rhombus. For a rhombus, the diagonals bisect perpendicularly. That means $(\mathbf{R}-\mathbf{P})^T(\mathbf{S}-\mathbf{Q})$ should be equal to zero.

$$\mathbf{R}-\mathbf{P} = \begin{pmatrix}5\\1.5\end{pmatrix} - \begin{pmatrix}-1\\1.5\end{pmatrix} = \begin{pmatrix}6\\0\end{pmatrix} \tag{26}$$

$$\mathbf{S}-\mathbf{Q} = \begin{pmatrix}2\\-1\end{pmatrix} - \begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}0\\-5\end{pmatrix} \tag{27}$$

$$(\mathbf{R}-\mathbf{P})^T(\mathbf{S}-\mathbf{Q}) = \begin{pmatrix}6 & 0\end{pmatrix}\begin{pmatrix}0\\-5\end{pmatrix} = 0 \tag{28}$$

Therefore PQRS is a rhombus.