

Circles

1 11th Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. Find the centre of a circle passing through the points $(6, -6)$, $(3, -7)$ and $(3, 3)$.

Solution: The equation of the circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{x}^\top \mathbf{u} + f = 0 \quad (1)$$

where

$$\mathbf{u} = -\mathbf{c} \text{ and} \quad (2)$$

$$f = \|\mathbf{c}\|^2 - r^2 \quad (3)$$

Given points are

$$\mathbf{x}_1 = \begin{pmatrix} 6 \\ -6 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 3 \\ -7 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (4)$$

Substituting points from (4) into (1)

$$(6^2 + (-6)^2) + 2 \begin{pmatrix} 6 & -6 \end{pmatrix} \mathbf{u} + f = 0 \quad (5)$$

$$\implies 2 \begin{pmatrix} 6 & -6 \end{pmatrix} \mathbf{u} + f = -72 \quad (6)$$

$$(3^2 + (-7)^2) + 2 \begin{pmatrix} 3 & -7 \end{pmatrix} \mathbf{u} + f = 0 \quad (7)$$

$$\implies 2 \begin{pmatrix} 3 & -7 \end{pmatrix} \mathbf{u} + f = -58 \quad (8)$$

$$(3^2 + 3^2) + 2 \begin{pmatrix} 3 & 3 \end{pmatrix} \mathbf{u} + f = 0 \quad (9)$$

$$\implies 2 \begin{pmatrix} 3 & 3 \end{pmatrix} \mathbf{u} + f = -18 \quad (10)$$

Representing the above system of equations in matrix form

$$\begin{pmatrix} 6 & -14 & 1 \\ 12 & -12 & 1 \\ 6 & 6 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -58 \\ -72 \\ -18 \end{pmatrix} \quad (11)$$

The augmented matrix is expressed as

$$\left(\begin{array}{ccc|c} 6 & -14 & 1 & -58 \\ 12 & -12 & 1 & -72 \\ 6 & 6 & 1 & -18 \end{array} \right) \quad (12)$$

Performing sequence of row operations to transform into an Echelon form

$$\xleftrightarrow{R_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 6 & -14 & 1 & -58 \\ 0 & 16 & -1 & 44 \\ 6 & 6 & 1 & -18 \end{array} \right) \quad (13)$$

$$\xleftrightarrow{R_3 \rightarrow R_3 - R_1} \left(\begin{array}{ccc|c} 6 & -14 & 1 & -58 \\ 0 & 16 & -1 & 44 \\ 0 & 20 & 0 & 40 \end{array} \right) \quad (14)$$

$$\xleftrightarrow{R_3 \rightarrow R_3 - \frac{20}{16}R_2} \left(\begin{array}{ccc|c} 6 & -14 & 1 & -58 \\ 0 & 16 & -1 & 44 \\ 0 & 0 & \frac{20}{16} & -15 \end{array} \right) \quad (15)$$

$$\xleftrightarrow{\begin{array}{l} R_1 \rightarrow \frac{1}{6}R_1 \\ R_2 \rightarrow \frac{1}{16}R_2, R_3 \rightarrow \frac{16}{20}R_3 \end{array}} \left(\begin{array}{ccc|c} 1 & -\frac{14}{6} & \frac{1}{6} & -\frac{58}{6} \\ 0 & 1 & -\frac{1}{16} & \frac{44}{16} \\ 0 & 0 & 1 & -12 \end{array} \right) \quad (16)$$

$$\xleftrightarrow{\begin{array}{l} R_1 \rightarrow R_1 - \frac{1}{6}R_3 \\ R_2 \rightarrow R_2 + \frac{1}{16}R_3 \end{array}} \left(\begin{array}{ccc|c} 1 & -\frac{14}{6} & 0 & -\frac{46}{6} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -12 \end{array} \right) \quad (17)$$

$$\xleftrightarrow{R_1 \rightarrow R_1 + \frac{14}{6}R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -12 \end{array} \right) \quad (18)$$

$$\text{So, from (18)} \quad (19)$$

$$\mathbf{u} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad (20)$$

$$f = -12 \quad (21)$$

Since $\mathbf{u} = -\mathbf{c}$,

$$\mathbf{c} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (22)$$

$$(3) \implies r^2 = (3^2 + (-2)^2) + 12 \quad (23)$$

$$r = 5 \quad (24)$$

Therefore, the equation of the circle is

$$\|\mathbf{x}\|^2 + 2 \begin{pmatrix} -3 & 2 \end{pmatrix} \mathbf{x} - 12 = 0 \quad (25)$$

The relevant diagram is shown in Figure 1

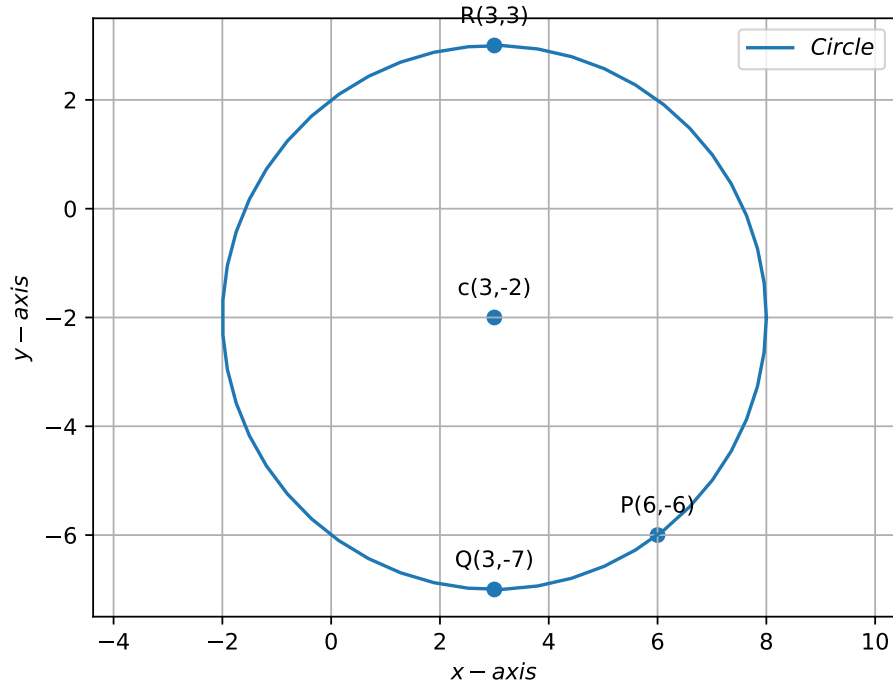


Figure 1