Area of a Traingle

1 10^{th} Maths - Chapter 7

This is Problem-1 from Exercise 7.3

1. Find the area of the triangle whose vertices are :

   (a) $(2,3),(-1,0),(2,-4)$

   Solution: The area of the triangle with vertices $\mathbf{A},\mathbf{B},\mathbf{C}$ is given by

   $$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C})\right\| = \frac{1}{2}\left\|\mathbf{A}\times\mathbf{B}+\mathbf{B}\times\mathbf{C}+\mathbf{C}\times\mathbf{A}\right\| \quad (1)$$

   The value of the cross product of two vectors is given by

   $$\left|\mathbf{M}\right| = \left|\mathbf{A} \quad \mathbf{B}\right| \quad (2)$$

   $$= \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 \quad (3)$$

   Therefore, (1) equals

   $$\text{Area} = \frac{1}{2}\left(a_1b_2 - a_2b_1 + b_1c_2 - b_2c_1 + c_1a_2 - c_2a_1\right) \quad (4)$$

   $$= \frac{1}{2}\left(2\times 0 - 3\times(-1) + (-1)\times(-4) - 0\times 2 + 2\times 3 - (-4)\times 2\right) \quad (5)$$

   $$= \frac{1}{2}\left(0+3+4-0+6+8\right) \quad (6)$$

   $$= \frac{1}{2}\left(21\right) \quad (7)$$

   $$= 10.5 \text{ Sq units} \quad (8)$$

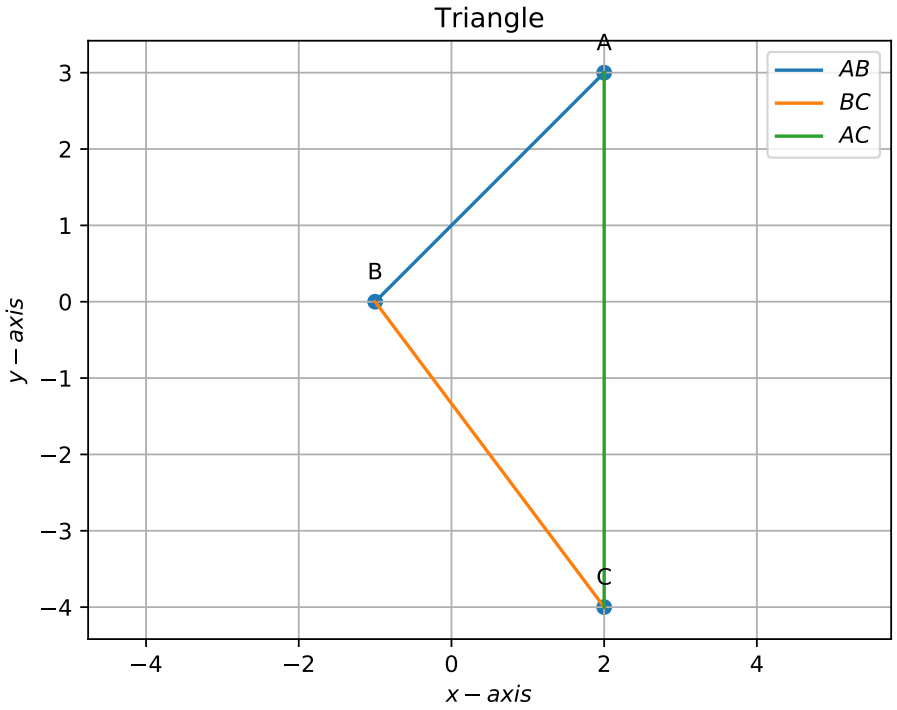

Figure 1

(b) $(-5,-1),(3,-5),(5,2)$

Solution: The area of the triangle with vertices $\mathbf{A},\mathbf{B},\mathbf{C}$ is given by

$$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C})\right\| = \frac{1}{2}\left\|\mathbf{A}\times\mathbf{B}+\mathbf{B}\times\mathbf{C}+\mathbf{C}\times\mathbf{A}\right\| \quad (9)$$

The value of the cross product of two vectors is given by

$$\left|\mathbf{M}\right| = \begin{vmatrix}\mathbf{A} & \mathbf{B}\end{vmatrix} \quad (10)$$

$$= \begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix} = a_1b_2 - a_2b_1 \quad (11)$$

Therefore, (9) equals

$$\text{Area} = \frac{1}{2}\left(a_1b_2 - a_2b_1 + b_1c_2 - b_2c_1 + c_1a_2 - c_2a_1\right) \tag{12}$$

$$= \frac{1}{2}\left((-5)\times(-5) - (-1)\times 3 + 3\times 2 - (-5)\times 5 + (-1)\times 5 - (-5)\times 2\right) \tag{13}$$

$$= \frac{1}{2}\left(25 + 3 + 6 + 25 - 5 + 10\right) \tag{14}$$

$$= \frac{1}{2}\left(64\right) \tag{15}$$

$$= 32 \text{ Sq units} \tag{16}$$

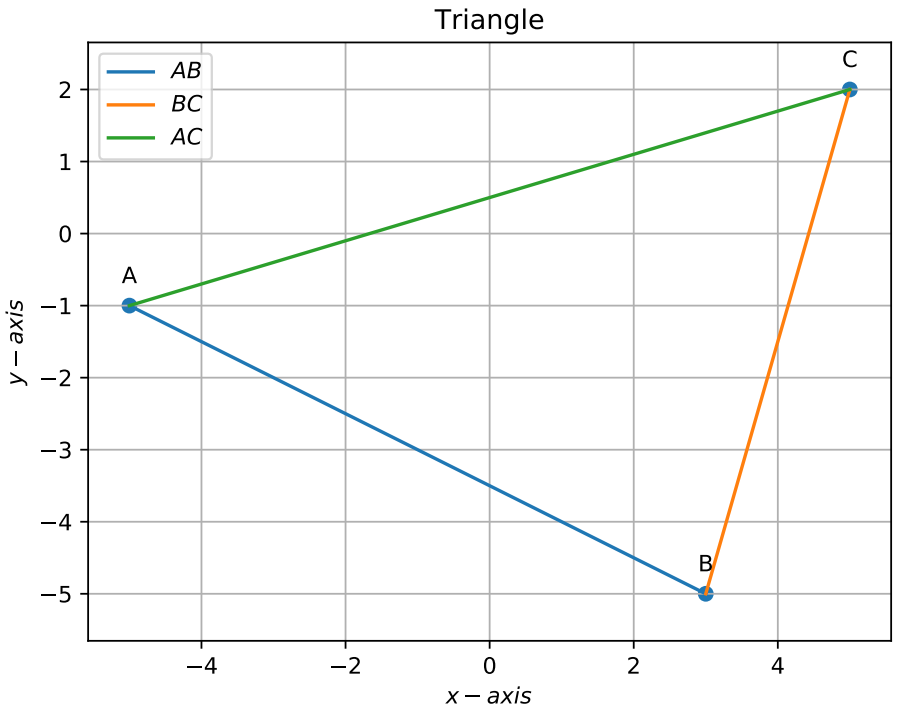

Figure 2