Implementation of Open Source Software NavIC L1 Transmitter and Receiver

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***Abstract*—This paper introduces a receiver architecture for Navigation through Indian Constellation (NavIC) L1 Standard Positioning Service (SPS) signal used for civilian purposes. Details of the signal characteristics, various signal processing blocks of both transmitter and receiver as well as channel modeling are available in this work. The receiver performance is verifed by generating random navigation data at transmitter and recovering it using the architecture proposed in the paper.**

***Index Terms*—NavIC, SPS, GNSS**

I. Introduction

NavIC [2] is an independent regional navigation satellite system developed and maintained by Indian Space Research Organisation (ISRO). It provides accurate position and timing services to users in India as well as the region extending upto 1500 km from its boundary. NavIC provides two types of services, namely, SPS and Restricted Service (RS) with a position accuracy better than 20m over the primary service area and timing accuracy better than 50ns.

The current NavIC satellite constellation comprises of six operational navigation satellites supporting L5, L1 and S bands. Only one satellite has civilian L1 band (1575.42 MHz) transponder providing SPS service for low power receivers. ISRO has plans to launch more satellites with L1 band, in future. SPS signals from NavIC are interoperable with other GNSS systems like GPS and GLONASS etc.

The Government of India mandated all mobile device manufacturers to have support for navigation using NavIC in all the devices being used in India. Hence, there is a growing need for NavIC software implementations. [1] describes open source software implementation for Galileo signal, however, for Navic L1, no such implementation is available in the existing literature.

In this paper, we focus on the design and implementation of a Navic L1 transceiver for SPS services. This includes generating an SPS baseband signal, transmitting it through the channel and applying various algorithms at the receiver for recovering the original transmitted sequence.

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II. System Overview

The block diagram of the system is shown in Figure 1. Navigation data is randomly generated, subframes and master frames are created as per the frame structure described in subsequent sections. The transmitter module creates the required baseband signal as per the modulation scheme, with relevant channel encoding schemes, error correction and detection schemes. Channel modelling module adds various modelling parameters and AWGN noise to the baseband signals for different satellites, forming a composite signal. The receiver module receives the composite signal, processes it to extract the navigation data, that was originally sent.

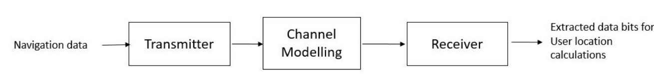

Fig. 1: System Overview

III. Transmitter Implementation

The NavIC transmitter is simulated to send baseband signal to the channel as shown in Fig 2.

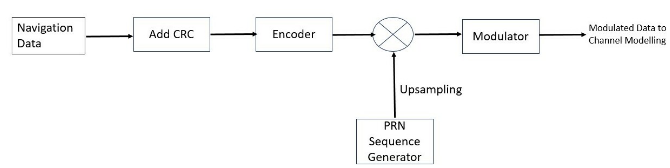

Fig. 2: Transmitter Block diagram

A. Modulation Scheme

The SPS signal is modulated using Synthesized Binary Offset Carrier (SBOC) modulation scheme [2], comprising of data signal and pilot signal.

B. Navigation Message structure

The NavIC L1 Master Frame [2] is of 1800 symbols long made of 3 subframes. Subframe 1 consists of 52 symbols, Subframe 2 is composed of 1200 symbols, and Subframe 3 is comprised of 548 symbols. The master frame and subframe [2] structures are shown in Figure 3 and Figure 4.

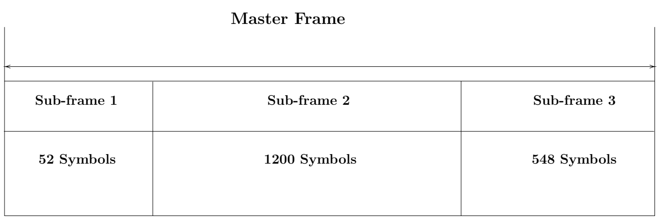

Fig. 3: Master Frame Structure

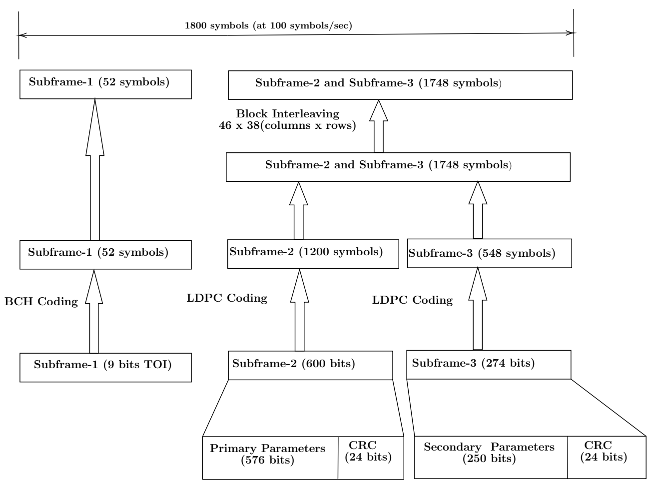

Fig. 4: NavIC L1 SPS Subframe Structure

C. PRN Codes

Each satellite is assigned a unique PRN ranging code number [2], termed as PRN ID. For each satellite, Data signal has one PRN code and Pilot signal has primary and secondary (overlay) PRN codes as shown in Figure 5 [2].

IV. Channel Modelling

The following parameters are modelled in the satellite communication channel [4]:

1) Doppler shift
2) Delay
3) Power scaling and
4) Thermal noise at the receiver

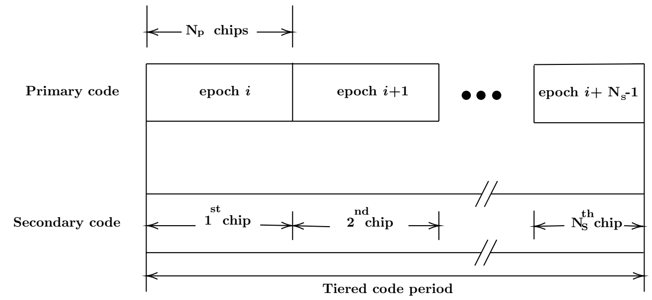

Fig. 5: Tiered code structure and timing relationship between primary and secondary codes

A. Doppler shift

The Doppler shift is introduced by muliplying the satellite signal with a complex exponential,

$$x_{Shift}\left[n\right] = x\left[n\right] e^{-2j\pi f_d n t_s} \tag{1}$$

where, $x_{Shift}\left[n\right]$ = Doppler shifted signal
$x\left[n\right]$ = Satellite signal
f_d = Doppler frequnecy applied
t_s = Sampling period

B. Delay

Since there is a finite distance between the satellite and the receiver, the signal at the reciever is a delayed version of the transmitted signal. This delay is given by

$$D_s = \frac{d}{c} f_s \tag{2}$$

where,
D_s = Total delay in samples
d = Distance between satellite and receiver
c = Speed of light
f_s = Sampling rate

The total delay on the satellite signal is modeled in two steps. First, to introduce the static delay, the samples are read from a queue whose size is the desired static delay length. To introduce the variable delay, the signal is passed throughan all-pass FIR filter with an almost constant phase response. Its coefficients are calculated using the delay value required.

C. Power Scaling

When a transmitting antenna transmits radio waves to a receiving antenna, the radio wave power received is given by,

$$P_r = P_t D_t D_r \left(\frac{1}{4\pi \left(f_c + f_{Shift}\right) D} \right)^2 \tag{3}$$

where,
P_r = Received power
P_t = Transmitted power
D_t = Directivity of transmitting antenna
D_r = Directivity of receiving antenna
D = Total delay in seconds

To scale the received signal as per the received power calculated,

$$x_{Scaled}\left[n\right] = \frac{\sqrt{P_r}}{\mathrm{rms}\left(x\left[n\right]\right)} x\left[n\right] \tag{4}$$

D. Thermal noise

The thermal noise power at the receiver is given by,

$$N_r = kTB \tag{5}$$

where,
N_r = Noise power in watts
k = Boltzmann's constant
T = Temperature in Kelvin
B = Bandwidth in Hz
AWGN (Additive White Guassian Noise) samples with zero mean and variance N_r are generated and added to the satellite signal to model thermal noise at receiver.

V. Receiver

The block diagram of the receiver is as shown in Figure 6.

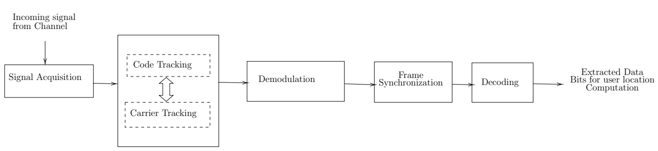

Fig. 6: The Block Level Architecture for Receiver

The signal processing chain at the receiver is divided into five steps:

1) Acquisition
2) Tracking
   a) Carrier Tracking
   b) Code Tracking
3) Demodulation
4) Frame synchronisation
5) Channel decoding

A. Acquistion

1) Mathematical equations: A generic NavIC L1 SPS signal is defined by its complex baseband equivalent, $S(t)$. The digital signal at the input of an Acquisition block can be written as:

$$x_{IN}[n] = A(t)\hat{s}(t-\tau(t))e^{j(2\pi f_d(t)t+\Phi(t))}\Big|_{t=nT_s} + w(t)\Big|_{t=nT_s} \tag{6}$$

where f_d is Doppler shift frequency, τ is PRN Code delay, Φ is Phase shift and $w(t)$ is AWGN noise.
The composite SBOC modulated signal $S(t)$ [1], [2] is generated by quadrature multiplexing of data and pilot signals, as given below:

$$S(t) = [\alpha S_{p,a}(t) - \beta S_{p,b}(t)] + j[\gamma S_{d,a}(t) + \eta S_{d,b}(t)] \tag{7}$$

where $\alpha = \sqrt{\frac{6}{11}}$, $\beta = \sqrt{\frac{4}{110}}$, $\gamma = \sqrt{\frac{4}{11}}$ and $\eta = \sqrt{\frac{6}{110}}$

Pilot Signal:

$$S_{p,a}(t) = \sum_{i=-\infty}^{\infty} C_{p,s}\Big[|i|_{1800}\Big] \oplus \sum_{j=1}^{10230} C_{p,p}\Big[j\Big] \cdot \mathrm{rect}_{T_{c,p,p}}\left(t - iT_{c,p,s} - jT_{c,p,p}\right) \cdot sc_{p,a}(t,0) \tag{8}$$

$$S_{p,b}(t) = \sum_{i=-\infty}^{\infty} C_{p,s}\Big[|i|_{1800}\Big] \oplus \sum_{j=1}^{10230} C_{p,p}\Big[j\Big] \cdot \mathrm{rect}_{T_{c,p,p}}\left(t - iT_{c,p,s} - jT_{c,p,p}\right) \cdot sc_{p,b}(t,0) \tag{9}$$

where $C_{p,p}$ is pilot primary PRN code, $C_{p,s}$ is pilot secondary/overlay PRN code, $T_{c,p,p} = \frac{1}{1.023}\mu$s and $T_{c,p,s} = 10$ms. $|i|_L$ means i modulo L.

$S_{p,a}$ is sinBOC(1,1) component of pilot signal and $S_{p,b}$ is sinBOC(6,1) component of pilot signal.

The Binary NRZ sub-carrier is defined as:

$$sc_{p,x}(t,\varphi) = \mathrm{sgn}[\sin(2\pi f_{sc,x}t + \varphi)] \tag{10}$$

The subcarrier signals are sinBOC. Hence, the subcarrier phase $\varphi = 0$.

Data Signal:

$$S_{d,a}(t) = \sum_{i=-\infty}^{\infty} C_d\Big[|i|_{10230}\Big] \oplus d_d\Big[[i]_{10230}\Big] \cdot \mathrm{rect}_{T_{c,d}}\left(t - iT_{c,d}\right) \cdot sc_{d,a}(t,0) \tag{11}$$

where $T_{c,d} = \frac{1}{1.023}\mu$s, C_d is Data PRN code and $[i]_L$ means the integer part of $\frac{i}{L}$.

The interplexed component $S_{d,b}(t)$ is given by:

$$S_{d,b}(t) = \sum_{i=-\infty}^{\infty} C_d\Big[|i|_{10230}\Big] \oplus d_d\Big[[i]_{10230}\Big] \cdot \mathrm{rect}_{T_{c,d}}\left(t - iT_{c,d}\right) \cdot sc_{d,b}(t,0) \tag{12}$$

The above equation can also be represented as

$$S_{d,b}(t) = S_{p,a}(t) \cdot S_{p,b}(t) \cdot S_{d,a}(t) \tag{13}$$

The Binary NRZ sub-carrier is defined as:

$$sc_{d,x}(t,\varphi) = \mathrm{sgn}[\sin(2\pi f_{sc,x}t + \varphi)] \tag{14}$$

The subcarrier signals are sinBOC. Hence, the subcarrier phase $\phi = 0$.

$f_{sc,a}$ is Sub-carrier frequency of $sc_{p,a}$ and $sc_{d,a}$ sub-carriers and equal to 1.023 MHz. $f_{sc,b}$ is Sub-carrier frequency of $sc_{p,b}$ and $sc_{d,b}$ sub-carriers and equal to 6.138 MHz.

Ranging code C_d, defined in (11) and (12), includes only primary code of data channel.

2) Implementation of PCPS Acquisition: The role of the acquisition block is to check the presence/absence of signals coming from a given satellite. In the case of signal being present, it should provide coarse estimations of the Code delay ($\hat{\tau}$) and the Carrier Doppler shift ($\hat{f}_d$), yet accurate enough to initialize the carrier and code tracking loops.

The Parallel Code Phase Search (PCPS) algorithm [1], [3] is used in Acquisition block and is shown in Figure 7. ML estimates of $\hat{f}_d$ and $\hat{\tau}$ are obtained by maximizing the objective function

$$\hat{f}_{d_{ML}}, \hat{\tau}_{ML} = \max_{f_d, \tau}\{|R_{xd}(f_D, \tau)|^2\} \tag{15}$$

where

$$R_{xd}(f_d, \tau) = \frac{1}{N}\sum_{n=0}^{N-1} x_{IN}[n] \cdot C[nT_s - \tau] \cdot e^{-j2\pi f_d n T_s} \tag{16}$$

$x_{IN}[n]$ is complex vector of incoming I and Q samples, T_s is the sampling period , N is number of samples (40,000 for this simulation) and $C[n]$ is locally generated code, defined as

$$C[n] = G_{p,p}[n] \cdot sc_{p,a}(nT_s, 0) \cdot \mathrm{rect}_{T_{c,p,p}}(t - nT_{c,p,p}) \tag{17}$$

where $G_{p,p}$ is upsampled and BPSK modulated pilot primary PRN code ($C_{p,p}$) for a given satellite.

Using a 2-dimensional search, maximization mentioned in 15 is computed as per the PCPS algorithm. The frequency bin having maximum signal power is determined as $f_{d_{acq}}$. The index at which the peak power is present in that frequency bin is termed as τ_{acq}. These values are passed to the Tracking module.

If the Input signal power is less than a threshold, then the satellite is considered to be non-visible. Under low SNR conditions, samples from 2-3 successive coherent integration periods can be added to have a better acquisition sensitivity.

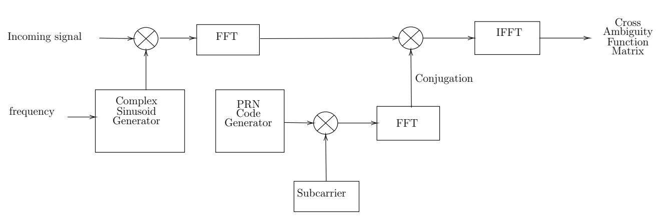

Fig. 7: PCPS algorithm flow

B. Tracking

The role of tracking block [6] is to refine coarse estimations and follow signal synchronization parameters: code phase, Doppler shift and carrier phase and extract the baseband signal. It runs continuously for specified simulation period , taking 10ms samples for each iteration. In this simulation, it runs for 3600 times (k=1,2,....,3600). Input samples are skipped by an amount equal to τ_{acq} and fed to tracking module.

It performs the following 3 functions to decipher the baseband signal from the incoming signal as shown in figure 8. The tracking algorithm specified in [1] is used in this work.

1) Carrier and code wipeoff
2) Pre-detection integration
3) Baseband signal processing

1) Carrier and code wipeoff: **Carrier wipeoff:** Referring to the Figure 8, first, the incoming signal is stripped off the carrier by a local replica carrier. The replica carrier signals are synthesized by the carrier numerically controlled oscillator (NCO). In closed loop operation, the carrier NCO is controlled by the carrier tracking loop in the receiver processor. Local carrier for n = 1...N_k is given as:

$$lc[n] = e^{-j(2\pi \hat{f}_{d_{k-1}} nT_s + mod(\hat{\phi}_{k-1}, 2\pi))} \tag{18}$$

Code wipeoff: ACF of BOC signal contains 2 local maximas, apart from having a global maxima at 0^{th} chip delay. To detect these maximas, the received signal is correlated with Very Early(VE), Early(E), Prompt(P), Late(L) and Very Late(VL) local replica codes (plus code Doppler) synthesized by a multi-delay sampler. In the closed loop operation, the code NCO is controlled by the code tracking loop in the receiver processor. E and L are typically separated in phase by 0.3 chips. VE and VL are separated by 1.2 chips.

The prompt replica code phase is aligned with the incoming satellite code phase producing maximum correlation if it is tracking the incoming satellite code phase. Any misalignment in the replica code phase with respect to the incoming code phase produces a difference in the vector magnitudes of the VE,E,L and VL correlated outputs so that the amount and direction of the phase change can be detected and corrected by the code tracking loop.

Pilot channel Local primary pilot PRN code reference $s_p[n]$ for n = 0,1,..., N_k, is given as:

$$s_p[n] = G_{p,p}\left[round\left(n + (\psi_k + \epsilon)\frac{N}{f_{chip}}\right)\right] \tag{19}$$

$$P_{p_k} = \frac{1}{N_k}\sum_{n=0}^{N_k-1} x_{IN}[n] s_p[n] lc[n]; \epsilon = 0 \tag{20}$$

Similarly, $VE_{p_k}, E_{p_k}, L_{p_k}$ and VL_{p_k} are generated with $\epsilon = -0.6, -0.15, 0.15, 0.6$ respectively.

Data channel Local data PRN code reference $s_d[n]$ for n = 0,1,..., N_k, is given as:

$$s_d[n] = D\left[round\left(n + (\psi_k)\frac{N}{f_{chip}}\right)\right] \tag{21}$$

where $D[n]$ is given as

$$D[n] = D_d[n] \cdot sc_{d,a}(nT_s, 0) \cdot \mathrm{rect}_{T_{c,d}}(t - nT_{c,d}) \tag{22}$$

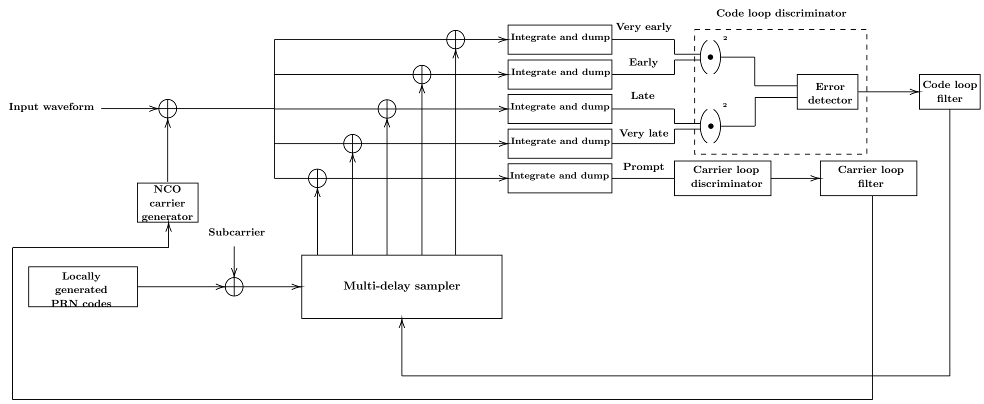

Fig. 8: Tracking block diagram

and $D_d[n]$ is upsampled and BPSK modulated Data PRN Code (C_d). Only P correlator is used for Data channel.

$$P_{d_k} = \frac{1}{N_k} \sum_{n=0}^{N_k-1} x_{IN}[n] s_d[n] lc[n] \tag{23}$$

2) Pre-detection and integration: Above correlator outputs are integrated and dumped to produce VE,E,P,L,VL versions of the Pilot channel and P version for Data channel.

3) Baseband signal processing: This entails Carrier tracking and Code tracking using PLL, FLL and DLL.

PLL: Phase error is given by

$$\Delta\hat{\phi}_k = \tan^{-1}\left(\frac{P_{p_{I_k}}}{P_{p_{Q_k}}}\right) \tag{24}$$

FLL: The algorithm used in FLL discriminator is $\frac{\text{ATAN2}(cross,dot)}{t_2-t_1}$. The frequency error is given by

$$\Delta\hat{f}_{d_k} = \frac{\phi_2 - \phi_1}{t_2 - t_1} \tag{25}$$

Updated finer frequency estimate is given by

$$\hat{f}_{d_k} = f_{d_{acq}} + h_{FLL}(\Delta\hat{f}_{d_k}) \tag{26}$$

Updated phase estimate is given by

$$\hat{\phi}_k = \phi_{k-1} + 2\pi\hat{f}_{d_k} T_{int} + h_{PLL}(\Delta\hat{\phi}_k) \tag{27}$$

DLL: Code delay error is given using the following algorithm:

$$E_k = \sqrt{VE^2_{p_{I_k}} + VE^2_{p_{Q_k}} + E^2_{p_{I_k}} + E^2_{p_{Q_k}}} \tag{28}$$

$$L_k = \sqrt{VL^2_{p_{I_k}} + VL^2_{p_{Q_k}} + L^2_{p_{I_k}} + L^2_{p_{Q_k}}} \tag{29}$$

$$\Delta\hat{\tau}_k = \frac{1}{2}\frac{E_k - L_k}{E_k + L_k} \tag{30}$$

Updating the finer code delay estimate,

$$S = T_{int} f_s + \psi_k + h_{DLL}(\Delta\hat{\tau}_k) \times (\text{ samples per chip }) \\ N_{k+1} = round(S) \text{ and } \psi_{k+1} = S - N_{k+1} \tag{31}$$

When tracking loop is in **locked state**, $P_{d_{I_k}}$ component of Data channel will carry data symbols and $P_{p_{Q_k}}$ of Pilot channel will carry pilot overlay codes.

C. Demodulation

After the acquisition and tracking have been performed, for each satellite, $P_{d_{I_k}}$ and $P_{p_{Q_k}}$ are mapped using BPSK demodulation to recover the transmitted symbols. Thus, 3600 data symbols ($\hat{d}_s[n]$) and 3600 bits of Pilot overlay code ($\hat{C}_{p,s}[n]$) are obtained after demodulation process.

D. Frame synchronisation

The start of master frame for a given satellite is determined using correlation of received pilot overlay bits with locally generated pilot's overlay bits. The index k at which the maximum correlation output is present is considered as the start of master frame. The same frame index is used to decode the data symbols.

$$\hat{k}_{ML} = \max_k \{corr_{Cd}(k)^2\} \tag{32}$$

where

$$corr_{Cd}(k) = \sum_{n=0}^{N-1} \hat{d}_s[n] \cdot \hat{C}_{p,s}[n-k] \tag{33}$$

E. Decoding

Demodulated data is first separated into subframes using the frame index. Subframe 1 is decoded using Maximum-likelihood method (BCH decoding). Subframes 2 and 3 are deinterleaved and decoded using belief propagation (LDPC decoding) [7]. CRC is calculated to verify if there are any errors.

1) Process: The high-level diagram of the channel decoding process in NavIC L1 is shown in Figure 9.

VI. Simulation Results

A. Transmitter

The simulation of transmitter starts with generating Pilot PRN codes(primary and secondary) and Data PRN codes for 4 satellites and upsampling them with sampling frequency f_s

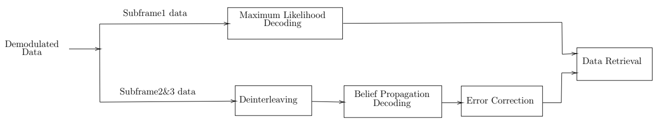

Fig. 9: The Block Level Architecture for Channel decoding

of 4MHz. Navigation bits for Master frames are randomly generated, CRC is padded and necessary encoding and interleaving are carried out to generate symbols for 3 subframes. 36 seconds of sample data is generated for each of the satellites(36×4M samples). Baseband signal for each of the satellites is generated as per the modulation scheme. For each of these signals, doppler shift (between -5000Hz to 5000Hz), code delay($0 - 10230$) and power scaling are applied. The composite signal is generated by adding all these 4 signals. AWGN noise (-15dB to 0dB) is added to this signal resulting in an SPS L1 signal. This simulated signal is transmitted (without carrier) through the channel.

B. Receiver

The symbol period of 10ms is considerd as Integration period at the receiver end. In the receiver module, Acquisition block reads 10ms of received samples (N= 40,000) and finds out visible satellites, $\hat{f}_d$ and $\hat{\tau}$ using PCPS algorithm. Frequency search band of -5000Hz to 5000Hz with a step size of 80Hz is used for this purpose. PCPS output shown in Figure 10 for a satellite, depicts received signal power for various doppler frequencies and code delay.

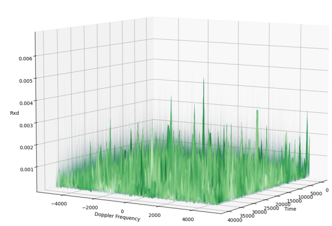

Fig. 10: Received signal power vs Frequency and Code delay

Acquisition results for 4 satellites are as shown below:

Acquisition results for **PRN ID 2**
Status:**True** Doppler:520 Code Delay : 301
Acquisition results for **PRN ID 3**
Status:**True** Doppler:1320 Code Delay : 588
Acquisition results for **PRN ID 4**
Status:**True** Doppler:3800 Code Delay : 426
Acquisition results for **PRN ID 6**
Status:**True** Doppler:4920 Code Delay : 313

The tracking loop runs for 3600 times (36×100 symbols/sec) to process received samples for each satellite. Tracking loop uses the following parameters:

1) Buffer size for power estimation = 10
2) $CN0_{min}$ = 25
3) Phase lock detector threshold = 0.85
4) Lock fail counter threshold = 25
5) Lock counter threshold = 20
6) PLL Noise Bandwidth = 18 Hz
7) DLL Noise Bandwidth = 2 Hz
8) SNR = -10db

Figure 11 shows tracking output for a satellite detailing Frequency, Phase and Code delay error and NCO output values.

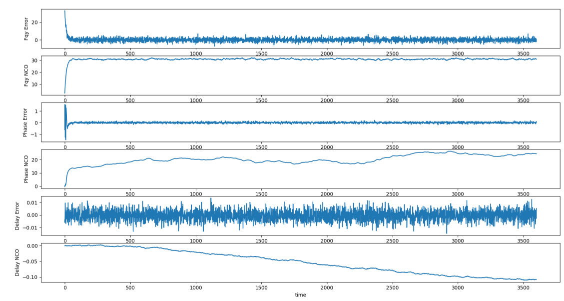

Fig. 11: Tracking output

Figure 12 shows, for a satellite, NAV bits transmitted and received for Subframes 1,2 and 3. For subframe 1, all 9 bits are shown while for subframes 2 and 3, first 24 bits are shown.

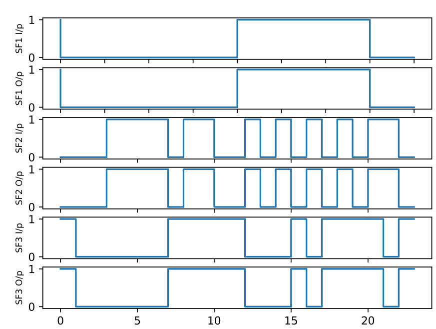

Fig. 12: NAV bits transmitted and received

VII. FUTURE WORK

The current work has focused on proper reception of NavIC L1 SPS baseband signal. In future work, the receiver should be fed with I and Q samples from a NavIC L1 simulator (ISRO had built a simulator) to calculate user location. Subsequently, when a minimum of 4 NavIC satellites with L1 band are available in constellation, the receiver has to be tested with real time data received from satellite.

VIII. CONCLUSIONS

In this paper, we presented open source implementation in Python of generating SPS NavIC L1 baseband signal, transmitting it through a modelled channel and processing the received data for accurate reception. Various signal processing blocks like Acquisition, tracking and decoding etc are discussed in detail. The open source code and documentation is available at https://github.com/satheeshsimha/navic-1/tree/main/L1.

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