

Properties of vectors

1 12th Maths - Exercise 10.4.1

1. Find $\left| \vec{a} \times \vec{b} \right|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

2 Solution

Now,

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \quad (1)$$

(2)

The cross product or vector product of \mathbf{A}, \mathbf{B} is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \left| \begin{matrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \\ \mathbf{A}_{12} & \mathbf{B}_{12} \end{matrix} \right| \end{pmatrix} \quad (3)$$

Hence

$$\left| \begin{matrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{matrix} \right| = \left| \begin{matrix} -7 & -2 \\ 7 & 2 \end{matrix} \right| = \begin{pmatrix} -14 \\ 14 \end{pmatrix} \quad (4)$$

$$\left| \begin{matrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{matrix} \right| = \left| \begin{matrix} 1 & 3 \\ 7 & 2 \end{matrix} \right| = \begin{pmatrix} 2 \\ -25 \end{pmatrix} \quad (5)$$

$$\left| \begin{matrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{matrix} \right| = \left| \begin{matrix} 1 & 3 \\ -7 & -2 \end{matrix} \right| = \begin{pmatrix} -2 \\ -21 \end{pmatrix} \quad (6)$$

(7)

Substituting the values

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} -14 & 14 \\ 2 & -25 \\ -2 & -21 \end{pmatrix} \quad (8)$$

Since

$$\mathbf{A} \times \mathbf{B} = \mathbf{0}, \quad (9)$$

$$\begin{pmatrix} -14 & 14 \\ 2 & -25 \\ -2 & -21 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (10)$$

which can be represented in matrix form as

$$\begin{pmatrix} -14 & 14 \\ 2 & -25 \\ -2 & -21 \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ -19 \\ 19 \end{pmatrix}. \quad (11)$$

Hence

$$\left| \vec{a} \times \vec{b} \right| = \sqrt{0^2 + 19^2 + 19^2} \quad (12)$$

$$= \sqrt{722} \quad (13)$$