

Line Assignment

Srikanth

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Problem Statement - If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$

Solution

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \quad (1)$$

$$l_1^2 + m_1^2 + n_1^2 = 0 \quad (2)$$

$$l_2^2 + m_2^2 + n_2^2 = 0 \quad (3)$$

In order to prove the vector $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$ is perpendicular to both the vectors l_1, m_1, n_1 and l_2, m_2, n_2 . Let us consider the matrix \mathbf{P} .

$$\mathbf{P} = \begin{pmatrix} l_1 & l_2 & m_1n_2 - m_2n_1 \\ m_1 & m_2 & n_1l_2 - n_2l_1 \\ n_1 & n_2 & l_1m_2 - l_2m_1 \end{pmatrix}$$

$$\mathbf{P}^\top = \begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ m_1n_2 - m_2n_1 & n_1l_2 - n_2l_1 & l_1m_2 - l_2m_1 \end{pmatrix}$$

If the three vectors are mutually perpendicular then

$$\mathbf{P}^\top \mathbf{P} = \mathbf{I} \quad (4)$$

This is proved from the result of python and from the (1),(2) and (3).

$$\begin{pmatrix} l_1^2 + m_1^2 + n_1^2 & l_1l_2 + m_1m_2 + n_1n_2 & l_1(m_1n_2 - m_2n_1) + m_1(n_1l_2 - n_2l_1) + n_1(l_1m_2 - l_2m_1) \\ l_1l_2 + m_1m_2 + n_1n_2 & l_2^2 + m_2^2 + n_2^2 & l_2(m_1n_2 - m_2n_1) + m_2(n_1l_2 - n_2l_1) + n_2(l_1m_2 - l_2m_1) \\ l_1(m_1n_2 - m_2n_1) + m_1(n_1l_2 - n_2l_1) + n_1(l_1m_2 - l_2m_1) & l_2(m_1n_2 - m_2n_1) + m_2(n_1l_2 - n_2l_1) + n_2(l_1m_2 - l_2m_1) & (l_1m_2 - l_2m_1)^2 + (n_1l_2 - n_2l_1)^2 + (m_1n_2 - m_2n_1)^2 \end{pmatrix} \quad (5)$$

$$\mathbf{P}^\top \mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (6)$$

Hences, The three vectors are mutually perpendicular

So we proved that If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.