Properties of Collinear

1 10^{th} Maths - Chapter 7

This is Problem-2 from Exercise 7.3.2

1. In each of the following find the value of 'k', for which the points are collinear.

1. (7, –2), (5, 1), (3, k)

2. (8, 1), (k, – 4), (2, –5).

1. Solution for problem : 1

$$\mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 3 \\ k \end{pmatrix} \tag{1}$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{2}$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ K \end{pmatrix} \right) = \begin{pmatrix} 4 \\ 12 - K \end{pmatrix} \tag{3}$$

If points on a line are collinear, rank of matrix is " 1 "then the vectors are in linearlydependent. For 2×2 matrix Rank =1 means Determinant is 0. Through pivoting,we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \xleftrightarrow{R_2=R_2-2R_1} \begin{pmatrix} -2-k & -4 \\ 4 & -2-k \end{pmatrix} \tag{6}$$

$$\xleftrightarrow{R_2=3R_2+2R_1} \begin{pmatrix} -2-k & -4 \\ 0 & -2-k \end{pmatrix} \tag{7}$$

$$\xleftrightarrow{R_2=3R_2+2R_1} \begin{pmatrix} -2-k & -4 \\ 0 & 0 \end{pmatrix} \tag{8}$$

if the rank of the matrix is 1 means any one of the row must be zero.So, making the first element in the matrix to 0.

$$\begin{aligned} (2(-2-k) - 3(-4)) &= 0 \\ (-4 - 2k + 12) &= 0 \\ (k = 4) \end{aligned} \tag{9}$$

Hence proved

1. Solution for problem :2

$$\mathbf{A} = \begin{pmatrix} 8 \\ 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} k \\ -4 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{10}$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} k \\ -4 \end{pmatrix} \right) = \begin{pmatrix} 8-k \\ 5 \end{pmatrix} \tag{11}$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right) = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \tag{12}$$

If points on a line are collinear, rank of matrix is " 1 "then the vectors are in linearlydependent. For 2×2 matrix Rank =1 means Determinant is 0. Through pivoting,we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \tag{13}$$

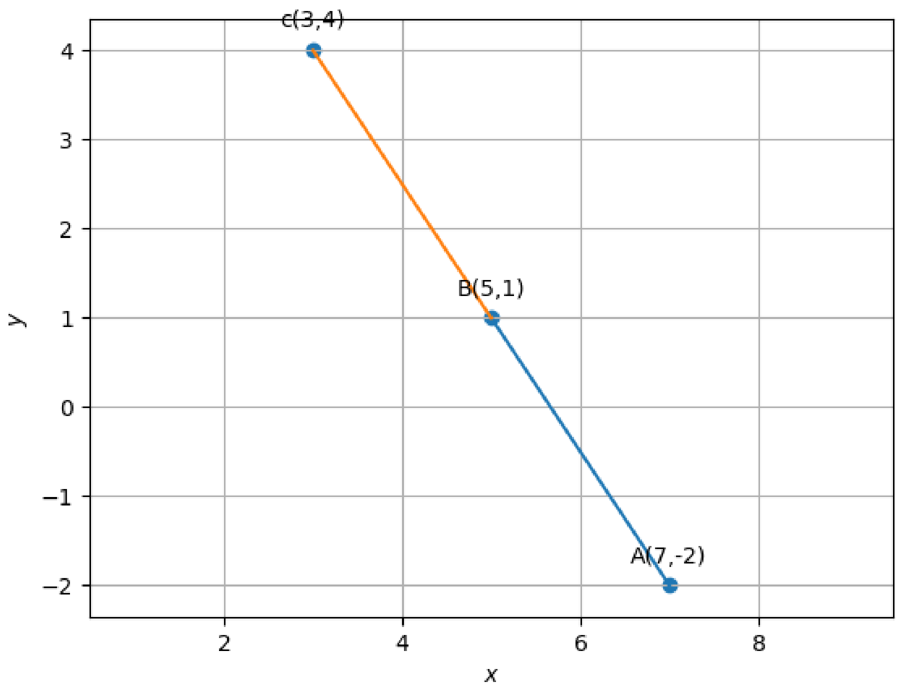

Figure 1

$$\begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \xleftrightarrow{R_1=3R_1-3R_2} \begin{pmatrix} 24-3k & 15 \\ 6 & 6 \end{pmatrix} \tag{15}$$

$$\xleftrightarrow{R_2=5R_2-2R_1} \begin{pmatrix} 24-3k & 15 \\ 0 & 0 \end{pmatrix} \tag{16}$$

if the rank of the matrix is 1 means any one of the row must be zero.So, making the first element in the matrix to 0.

$$\begin{aligned} ((5(6)-2(24-3k)) = 0 \\ (30-48+6k) = 0 \\ (k=3) \end{aligned} \tag{17}$$

Hence proved.

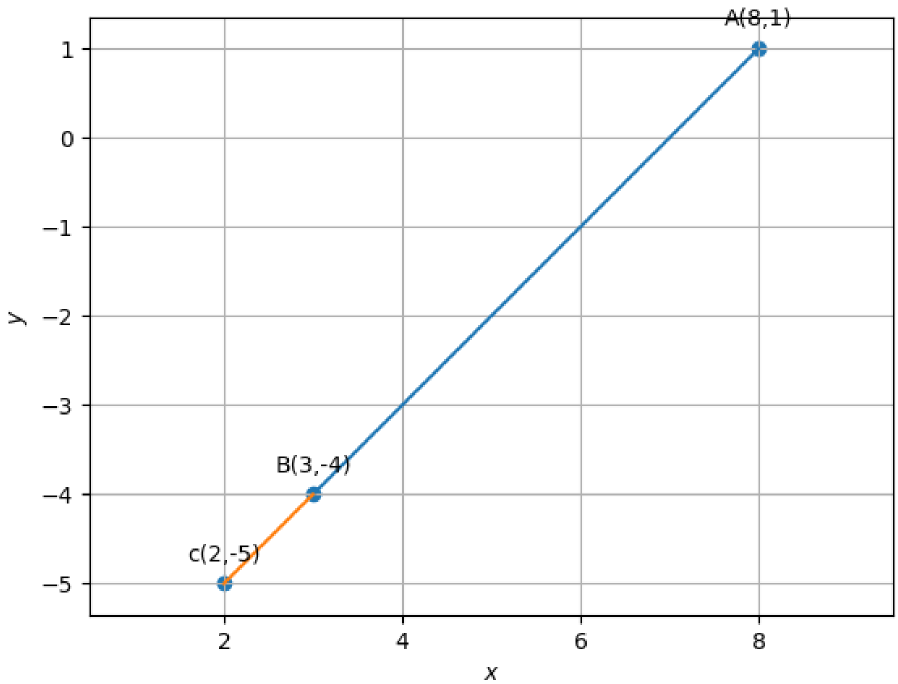

Figure 2