Parallel Lines

11^{th} Maths - Chapter 10

This is Problem-6 from Exercise 10.3

1. Find the distance between parallel lines

   (i) 15x+8y-34=0 and 15x+8y+31=0
   (ii) l(x+y)+p=0 and l(x+y)-r=0

2. solution for problem 1
   Given line is

$$15x + 8y - 34 = 0 \text{ and } 15x + 8y + 31 = 0 \tag{1}$$

   this equation can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c_1 \tag{2}$$
$$\mathbf{n}^\top \mathbf{x} = c_2 \tag{3}$$

$$\text{where } \mathbf{n} = \begin{pmatrix} 15 \\ 8 \end{pmatrix}, c_1 = -34, c_2 = 31 \tag{4}$$
$$\begin{pmatrix} 15 & 8 \end{pmatrix} \mathbf{x} = -34 \tag{5}$$
$$\begin{pmatrix} 15 & 8 \end{pmatrix} \mathbf{x} = 31 \tag{6}$$
$$\tag{7}$$

   distance between parallel lines

$$d = \frac{|c_1 - c_2|}{\|n\|} \tag{8}$$
$$= \frac{|-34 - 31|}{\sqrt{289}} \tag{9}$$
$$= \frac{65}{17} \tag{10}$$

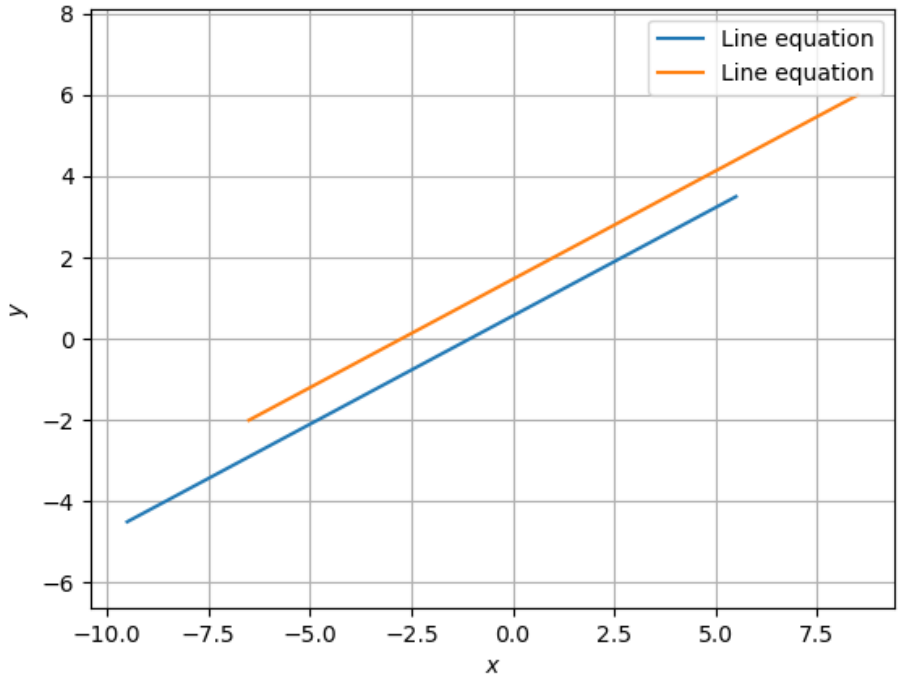

Figure 1

3. solution for problem 2
   Given line is

$$l(x+y)+p=0 \text{ and } l(x+y)-r=0 \tag{11}$$

this equation can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c_1 \tag{12}$$

$$\mathbf{n}^\top \mathbf{x} = c_2 \tag{13}$$

$$\text{where } \mathbf{n} = \begin{pmatrix} l \\ l \end{pmatrix}, c_1 = p, c_2 = -r \tag{14}$$

$$= \begin{pmatrix} l & l \end{pmatrix} \mathbf{x} = p \tag{15}$$

$$= \begin{pmatrix} l & l \end{pmatrix} \mathbf{x} = -r \tag{16}$$

distance between parallel lines

$$d = \frac{|p - (-r)|}{\sqrt{l^2}} \tag{17}$$

$$= \frac{|p + r|}{l\sqrt{2}} \tag{18}$$

The distance between parallel lines is shown in figure (2) with normal vector as

$$\mathbf{n} = \begin{pmatrix} 8 \\ 8 \end{pmatrix} \text{ and } c_1 = 16, c_2 = -16$$

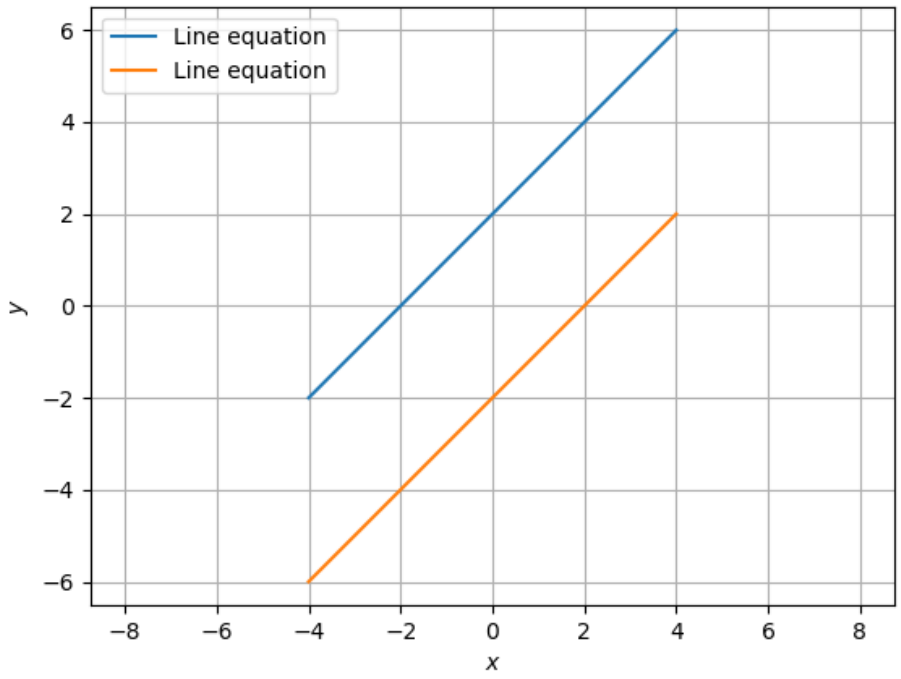

Figure 2