Properties of vectors

1 12^{th} Maths - Chapter 11

This is Problem-2 from Exercise 11.4

1. If l_1,m_1,n_1 and l_2,m_2,n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.

2 Solution

Now,

$$\text{Let } \mathbf{A} = \begin{pmatrix} l_1 \\ m_1 \\ n_1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} l_2 \\ m_2 \\ n_2 \end{pmatrix} \tag{1}$$

$$\tag{2}$$

The cross product or vector product of $\mathbf{A}, \mathbf{B}$ is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \\ \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \tag{3}$$

Hence

$$\begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} = \begin{vmatrix} m_1 & n_1 \\ n_2 & n_2 \end{vmatrix} = \left(m_1n_2 - m_2n_1\right) \tag{4}$$

$$\begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} = \begin{vmatrix} n_1 & l_1 \\ n_2 & l_2 \end{vmatrix} = \left(n_1l_2 - n_2l_1\right) \tag{5}$$

$$\begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} = \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} = \left(l_1m_2 - l_2m_1\right) \tag{6}$$

which can be represented in matrix form as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} m_1 n_2 - m_2 n_1 \\ n_1 l_2 - n_2 l_1 \\ l_1 m_2 - l_2 m_1 \end{pmatrix} \tag{7}$$

Hence The given quation is that the direction cosines of the line perpendicular to both these are $m_1 n_2 - m_2 n_1$, $n_1 l_2 - n_2 l_1$, $l_1 m_2 - l_2 m_1$.