

Properties of Collinear

1 10th Maths - Chapter 7

This is Problem-2 from Exercise 7.3.2

1. In each of the following find the value of 'k', for which the points are collinear.
2. (7, -2), (5, 1), (3, k)
3. (8, 1), (k, -4), (2, -5).

4. Solution for problem : 1

$$\mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ k \end{pmatrix} \quad (1)$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad (2)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ k \end{pmatrix} \right) = \begin{pmatrix} 4 \\ -2-k \end{pmatrix} \quad (3)$$

If points on a line are collinear, rank of matrix is "1" then the vectors are linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \xrightarrow{R_2=R_2-2R_1} \begin{pmatrix} 2 & -3 \\ 0 & -k+4 \end{pmatrix} \quad (6)$$

If the rank of the matrix has to be 1, then $-k+4=0$.

$$\begin{aligned} (-k+4) &= 0 \\ (k &= 4) \end{aligned} \quad (7)$$

5. Solution for problem :2

$$\mathbf{A} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} k \\ -4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \quad (8)$$

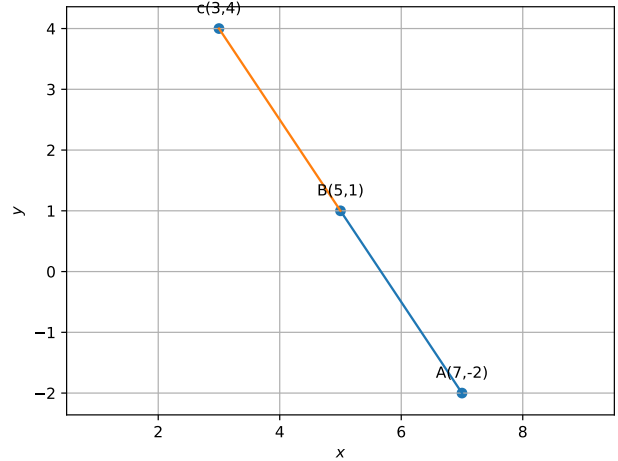


Figure 1

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} k \\ -4 \end{pmatrix} \right) = \begin{pmatrix} 8-k \\ 5 \end{pmatrix} \quad (9)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right) = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \quad (10)$$

If points on a line are collinear, rank of matrix is "1" then the vectors are linearly dependent. For 2×2 matrix Rank = 1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \quad (11)$$

$$\begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \quad (12)$$

$$\begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \xrightarrow{R_1=3R_1-3R_2} \begin{pmatrix} 24-3k & 15 \\ 6 & 6 \end{pmatrix} \quad (13)$$

$$\xrightarrow{R_2=5R_2-2R_1} \begin{pmatrix} 24-3k & 15 \\ 0 & 0 \end{pmatrix} \quad (14)$$

If the rank of the matrix is 1, it means any one of the row must be zero. So, making the first element in the matrix equal to 0.

$$\begin{aligned} ((5(6) - 2(24 - 3k)) &= 0 \\ (30 - 48 + 6k) &= 0 \\ (k &= 3) \end{aligned} \quad (15)$$

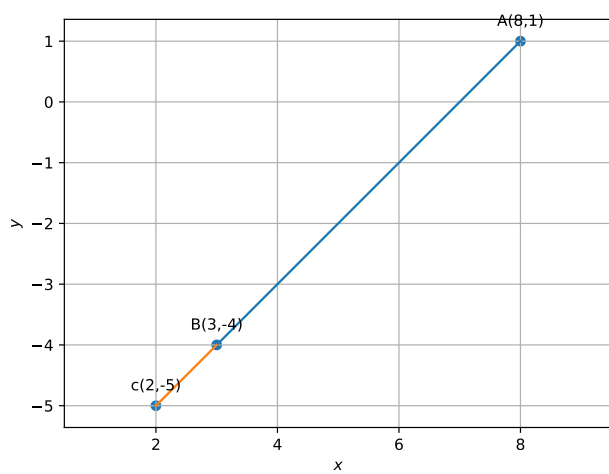


Figure 2