

Assignment: Isosceles triangle

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Problem: Check whether $(5, -2)$, $(6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.

Symbol	Values	Description
A	$(6,4)$	Vertex A
B	$(7,-2)$	Vertex B
C	$(5,-2)$	Vertex C
D	$(6,-2)$	Midpoint of AC
E	$(13/2,1)$	Midpoint of BC
F	$(11/2,1)$	Midpoint of AB

Solution: 1: Let the given points be **A**, **B**, **C** respectively. Then, the direction vectors of AB , BC and CA are

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \quad (1)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (3)$$

$$\begin{aligned} \mathbf{A} - \mathbf{B}^\top \mathbf{B} - \mathbf{C} &= \begin{pmatrix} -1 & -6 \end{pmatrix} \begin{pmatrix} -1 \\ 6 \end{pmatrix} \\ &= 37 \end{aligned} \quad (4)$$

$$\begin{aligned} \mathbf{B} - \mathbf{C}^\top \mathbf{C} - \mathbf{A} &= \begin{pmatrix} -1 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \\ &= -2 \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{C} - \mathbf{A}^\top \mathbf{A} - \mathbf{B} &= \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -6 \end{pmatrix} \\ &= -2 \end{aligned} \quad (6)$$

$$\text{From the above equations, } \mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C} \quad (7)$$

$$\angle BCA = \angle CAB \quad (8)$$

Thus, the triangle is isosceles triangle.

Method2: Let the given points be $\mathbf{E}, \mathbf{F}, \mathbf{D}$ respectively. Then, the direction vectors of BD, CF, AE are

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} 13/2 \\ 1 \end{pmatrix} \quad (12)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \quad (13)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 11/2 \\ 1 \end{pmatrix} \quad (14)$$

$$\mathbf{E} - \mathbf{A}^\top \mathbf{C} - \mathbf{B} = (3/2 \quad 3) \begin{pmatrix} -1 \\ -6 \end{pmatrix} \quad (15)$$

$$\neq 0 \quad (16)$$

$$\mathbf{F} - \mathbf{C}^\top \mathbf{B} - \mathbf{A} = (3/2 \quad 3) \begin{pmatrix} 1 \\ 6 \end{pmatrix} \quad (17)$$

$$\neq 0 \quad (18)$$

$$\mathbf{D} - \mathbf{B}^\top \mathbf{C} - \mathbf{A} = (0 \quad -6) \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (19)$$

$$= 0 \quad (20)$$

$$\mathbf{B} - \mathbf{D} \perp \mathbf{A} - \mathbf{C} \quad (21)$$

$$\angle BCA = \angle CAB \quad (22)$$

1 Figure

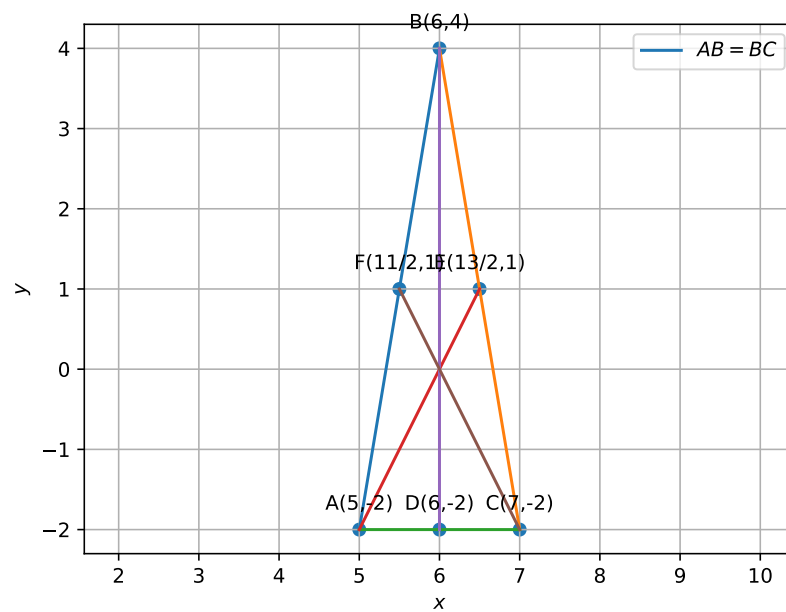


Figure 1: Isosceles triangle

<https://github.com/srikanth9515/FWC/tree/main/maths/vec1>