Properties of Circle

1 9^{th} Maths - Chapter 10

This is Problem-2 from Exercise 10.4

1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of other chord.

Solution:
Consider the Circle of radius 1 and length of chord be 1.5

Symbol	**Value**	**Description**
$\mathbf{C}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Circle point
d	1.5	Length of Chord
r	1	Radius
$\theta_{\mathbf{1}}$	$30°$	$-$
$\theta_{\mathbf{2}}$	$60°$	$-$

Table 1

$$\mathbf{P} = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \tag{1}$$

$$\mathbf{R} = \begin{pmatrix} \cos\theta_3 \\ \sin\theta_3 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} \cos\theta_4 \\ \sin\theta_4 \end{pmatrix} \tag{2}$$

So, here

$$(\mathbf{P} - \mathbf{Q}) = \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{3}$$

$$\|\mathbf{P} - \mathbf{Q}\|^2 = d^2 \tag{4}$$

$$\implies \left((\cos\theta_1 - \cos\theta_2)^2\right) + \left((\sin\theta_1 - \sin\theta_2)^2\right) = d^2$$
$$\implies 2 - 2\left(\cos(\theta_1 - \theta_2)\right) = d^2$$
$$\implies 2(2\left(\sin^2(\tfrac{\theta_1-\theta_2}{2})\right) = d^2$$
$$\implies \sin^2(\frac{\theta_1 - \theta_2}{2}) = \frac{d^2}{4}$$
$$\implies \sin\frac{\theta_1 - \theta_2}{2} = \frac{d}{2}$$
$$\implies \left(\tfrac{\theta_1-\theta_2}{2}\right) = \sin^{-1}(\frac{d}{2})$$

$$\left(\theta_1 - \theta_2\right) = 97.1806 \tag{5}$$

Simillary we can say that

$$\left(\theta_3 - \theta_4\right) = 97.1806 \tag{6}$$

Let

$$\theta_1 = 30^\circ, \theta_3 = 60^\circ \tag{7}$$
$$\theta_2 = -67.1806, \theta_4 = -37.1806 \tag{8}$$

Obtain the point of intersection of line **PQ**,**RS**

$$\mathbf{n_1}^\top(\mathbf{X} - \mathbf{P}) = 0 \tag{9}$$
$$\mathbf{n_2}^\top(\mathbf{X} - \mathbf{R}) = 0 \tag{10}$$
$$\mathbf{Omat} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{11}$$
$$\mathbf{n_1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ \sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{12}$$
$$= \begin{pmatrix} \sin\theta_1 - \sin\theta_2 \\ \cos\theta_2 - \cos\theta_1 \end{pmatrix} \tag{13}$$
$$\mathbf{n_2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta_3 - \cos\theta_4 \\ \sin\theta_3 - \sin\theta_4 \end{pmatrix} \tag{14}$$
$$= \begin{pmatrix} \sin\theta_3 - \sin\theta_4 \\ \cos\theta_4 - \cos\theta_3 \end{pmatrix} \tag{15}$$

From the Result of python code by sunstituting $\theta_1, \theta_2, \theta_3, \theta_4$ in point $\mathbf{T}$ then we got the point of intersection as :

$$\begin{pmatrix} \sin\theta_2 - \sin\theta_1 & \cos\theta_2 - \cos\theta_1 \\ \sin\theta_4 - \sin\theta_3 & \cos\theta_4 - \cos\theta_3 \end{pmatrix} \mathbf{X} = \begin{pmatrix} \cos\theta_1(\sin\theta_2 - \sin\theta_1) - \sin\theta_1(\cos\theta_2 - \cos\theta_1) \\ \cos\theta_3(\sin\theta_4 - \sin\theta_3) - \sin\theta_3(\cos\theta_4 - \cos\theta_2) \end{pmatrix} \tag{16}$$

$$\mathbf{T} = \begin{pmatrix} \frac{\cos\theta_4 - \cos\theta_2(\sin\theta 2\cos\theta_1 - \sin\theta_1\cos\theta_2)}{(\cos\theta_4 - \cos\theta_2)(\sin\theta_2 - \sin\theta_1) - (\cos\theta_2 - \cos\theta_1)(\sin\theta_4 - \sin\theta_3)} \\ \sin\theta_1 + \frac{\sin\theta_2 - \sin\theta_1}{\cos\theta_2 - \cos\theta_1}\left(\frac{\cos\theta_4 - \cos\theta_2(\sin\theta 2\cos\theta_1 - \sin\theta_1\cos\theta_2)}{(\cos\theta_4 - \cos\theta_2)(\sin\theta_2 - \sin\theta_1) - (\cos\theta_2 - \cos\theta_1)(\sin\theta_4 - \sin\theta_3)}\right) - \cos\theta_1) \end{pmatrix} \tag{17}$$

$$\mathbf{T} = \begin{pmatrix} 0.68341409 \\ -0.04288508 \end{pmatrix} \tag{18}$$

From the Result of python code , we get

$$\|\mathbf{P} - \mathbf{T}\| = 0.5727 \tag{19}$$
$$\|\mathbf{R} - \mathbf{T}\| = 0.9272 \tag{20}$$
$$\|\mathbf{Q} - \mathbf{T}\| = 0.9272 \tag{21}$$
$$\|\mathbf{S} - \mathbf{T}\| = 0.5727 \tag{22}$$

Hence, we proved that

$$\|\mathbf{P} - \mathbf{T}\| = \|\mathbf{S} - \mathbf{T}\| \tag{23}$$
$$\|\mathbf{Q} - \mathbf{T}\| = \|\mathbf{R} - \mathbf{T}\| \tag{24}$$

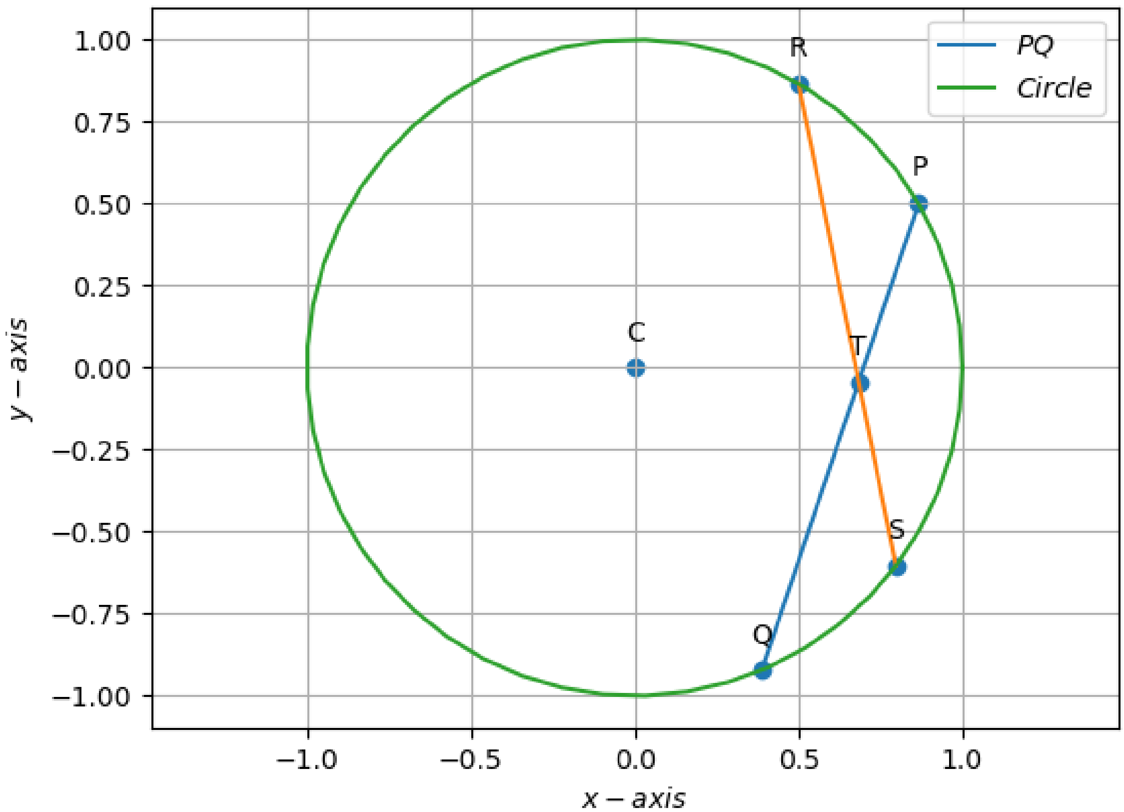

Figure 1: Two equal chords intersecting in a circle