Intercepts Lines

11^{th} Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6,resspectively.

1 Solution

Let the x intercept be a and the y intercept be b ,Then

$$\big(a+b\big) = 1 \tag{1}$$
$$\big(ab\big) = -6 \tag{2}$$

upon simplifying (1) and (2)

$$\mathbf{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{3}$$
$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{4}$$
$$= \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{5}$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} 0r \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{6}$$

case 1 $\implies$ The normal vector $\mathbf{n}$ to the line is given as

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{8}$$

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{9}$$

$$\begin{pmatrix} -2 & 3 \end{pmatrix} \mathbf{x} = 6 \tag{10}$$

or

case 2 $\implies$ The normal vector $\mathbf{n}$ to the line is given as

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} -3 \\ -2 \end{pmatrix} \tag{12}$$

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{B}) = 0 \tag{13}$$

$$\begin{pmatrix} -3 & -2 \end{pmatrix} \mathbf{x} = 6 \tag{14}$$

The equation of a line with normal vector $\mathbf{n}$ and passing through a point $\mathbf{A}$ is given by $(-2, 3)\mathbf{x} = 6$ and for $\mathbf{B}$ is $(-3, -2)\mathbf{x} = 6$

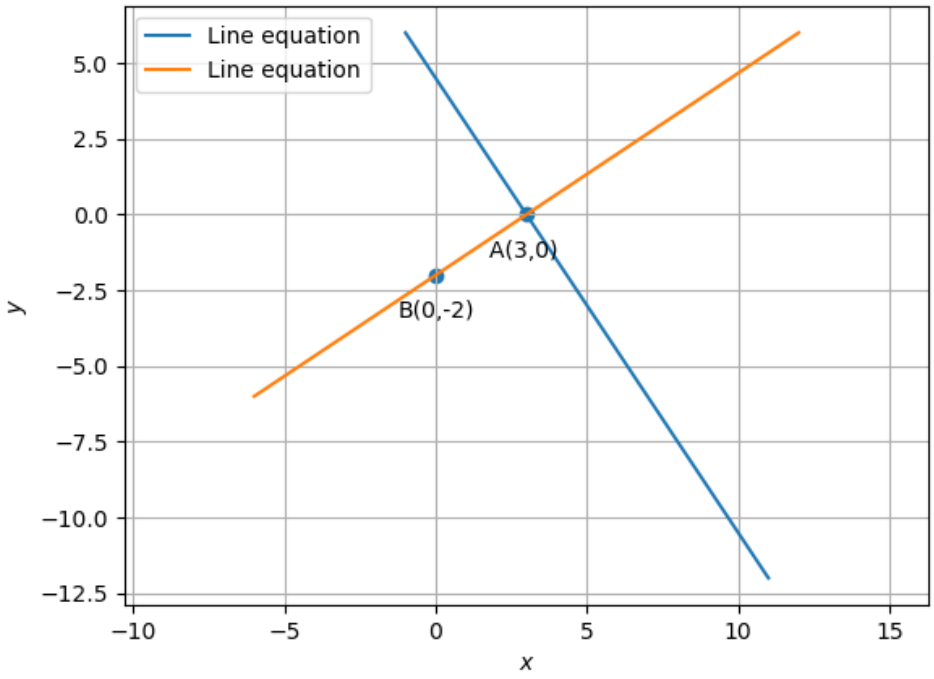
Line equation
Line equation
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−12.5
y
−10
−5
0
5
10
15
x
A(3,0)
B(0,-2)

Figure 1