Equation of circle

Excercise 10.4

1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of other chord.
   Solution:
   Consider the Circle of radius 1 and length of chord be 1.5

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, d = 1.5, r = 1 \tag{1}$$

   Consider the points on the circle

$$\mathbf{P} = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \tag{2}$$

$$\mathbf{Q} = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \tag{3}$$

$$\mathbf{R} = \begin{pmatrix} \cos\theta_3 \\ \sin\theta_3 \end{pmatrix} \tag{4}$$

$$\mathbf{S} = \begin{pmatrix} \cos\theta_4 \\ \sin\theta_4 \end{pmatrix} \tag{5}$$

   So, here

$$(\mathbf{P} - \mathbf{Q}) = \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{6}$$

$$\|\mathbf{P} - \mathbf{Q}\|^2 = d^2 \tag{7}$$

$$\implies \left((\cos\theta_1 - \cos\theta_2)^2\right) + \left((\sin\theta_1 - \sin\theta_2)^2\right) = d^2$$
$$\implies 2 - 2\left(\cos(\theta_1 - \theta_2)\right) = d^2$$
$$\implies 2(2\left(\sin^2(\tfrac{\theta_1-\theta_2}{2})\right) = d^2$$
$$\implies \left(\sin^2 \tfrac{\theta_1-\theta_2}{2}\right) = \frac{d^2}{4}$$
$$\implies \left(\sin \tfrac{\theta_1-\theta_2}{2}\right) = \frac{d}{2}$$
$$\implies \left(\tfrac{\theta_1-\theta_2}{2}\right) = \left(\sin^{-1}(0.75)\right)$$

$$\left(\theta_1 - \theta_2\right) = 97.1806 \tag{8}$$

Simillary we can say that

$$\left(\theta_3 - \theta_4\right) = 97.1806 \tag{9}$$

Let

$$\theta_1 = 30°, \theta_3 = 60° \tag{10}$$

here

$$\mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix} \tag{11}$$
$$\mathbf{Q} = \begin{pmatrix} 0.3878 \\ -0.9217 \end{pmatrix} \tag{12}$$
$$\mathbf{R} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{13}$$
$$\mathbf{S} = \begin{pmatrix} 0.7967 \\ -0.6043 \end{pmatrix} \tag{14}$$

Obtain the point of intersection of line **PQ**,**RS**

$$\mathbf{n_1}^\top \left(\mathbf{X} - \mathbf{P}\right) = 0 \tag{15}$$
$$\mathbf{n_2}^\top \left(\mathbf{X} - \mathbf{R}\right) = 0 \tag{16}$$
$$\mathbf{n_1} = \begin{pmatrix} 1.4217 \\ -0.47822 \end{pmatrix} \tag{17}$$
$$\mathbf{n_2} = \begin{pmatrix} 1.4703 \\ 0.2967 \end{pmatrix} \tag{18}$$

From the Result of python code , we got the point of intersection of (15) (16) is given as :

$$\mathbf{T} = \begin{pmatrix} 0.68341409 \\ -0.04288508 \end{pmatrix} \tag{19}$$

From the Result of python code , we get

$$\|\mathbf{P} - \mathbf{T}\| = 0.5727 \tag{20}$$
$$\|\mathbf{R} - \mathbf{T}\| = 0.9272 \tag{21}$$
$$\|\mathbf{Q} - \mathbf{T}\| = 0.9272 \tag{22}$$
$$\|\mathbf{S} - \mathbf{T}\| = 0.5727 \tag{23}$$

Hence, we proved that

$$\|\mathbf{P} - \mathbf{T}\| = \|\mathbf{S} - \mathbf{T}\| \tag{24}$$
$$\|\mathbf{Q} - \mathbf{T}\| = \|\mathbf{R} - \mathbf{T}\| \tag{25}$$

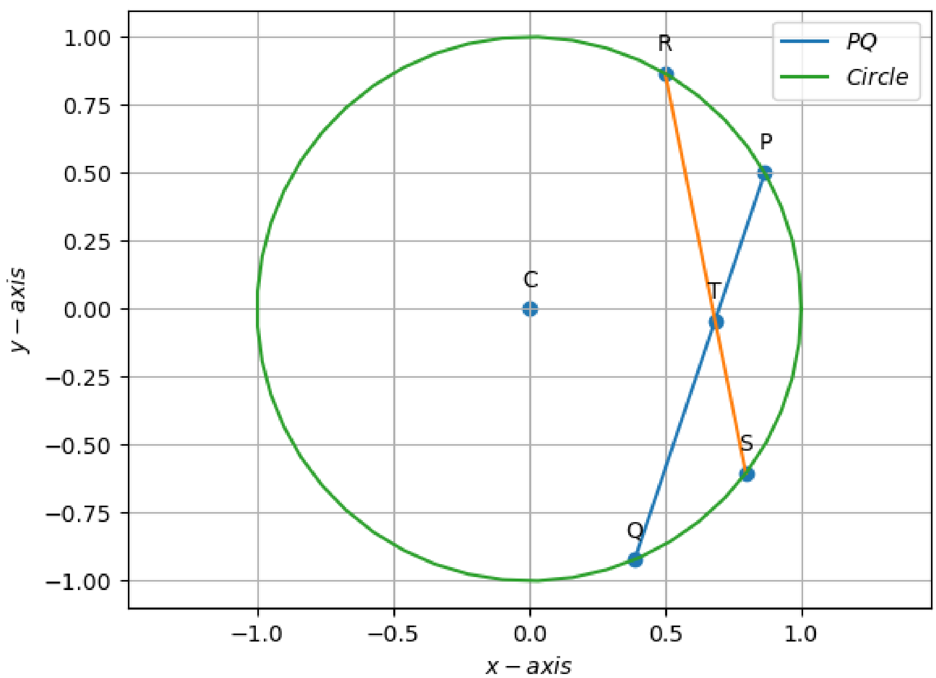
R
P
C
T
S
Q
PQ
Circle
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.0
−0.5
0.0
0.5
1.0
x − axis
y − axis

Figure 1