Excercise 11.1

Q4.Find the equation of the circle with centre $(1, 1)$ and radius $\sqrt{2}$.
Solution: Given

$$\mathbf{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } r = \sqrt{2} \tag{1}$$

We know the equation of the circle is given as

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

So, here

$$\mathbf{u} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{3}$$

$$f = \|\mathbf{u}\|^2 - r^2 \tag{4}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} - 2 \tag{5}$$

$$= -\frac{3}{2} \tag{6}$$

Thus ,the equation of circle is obtained as

$$\|\mathbf{x}\|^2 + \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - \frac{3}{2} = 0 \tag{7}$$
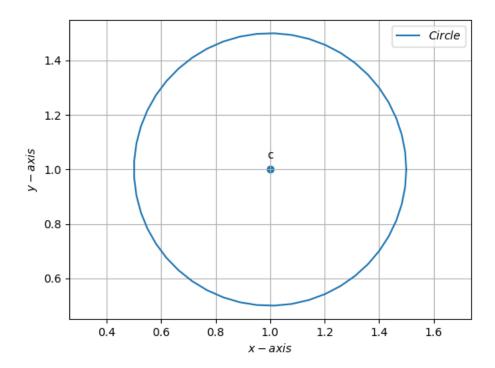
Figure 1: