

Parallel Lines

11th Maths - Chapter 10

This is Problem-6 from Exercise 10.3

1. Find the distance between parallel lines

(i) $15x+8y-34=0$ and $15x+8y+31=0$

(ii) $l(x+y)+p=0$ and $l(x+y)-r=0$

2. solution for problem 1

Given line is

$$15x + 8y - 34 = 0 \text{ and } 15x + 8y + 31 = 0 \quad (1)$$

this equation can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c_1 \quad (2)$$

$$\mathbf{n}^\top \mathbf{x} = c_2 \quad (3)$$

$$\text{where } \mathbf{n} = \begin{pmatrix} 15 \\ 8 \end{pmatrix}, c_1 = -34, c_2 = 31 \quad (4)$$

$$\begin{pmatrix} 15 & 8 \end{pmatrix} \mathbf{x} = -34 \quad (5)$$

$$\begin{pmatrix} 15 & 8 \end{pmatrix} \mathbf{x} = 31 \quad (6)$$

$$(7)$$

distance between parallel lines

$$d = \frac{|c_1 - c_2|}{\|\mathbf{n}\|} \quad (8)$$

$$= \frac{|-34 - 31|}{\sqrt{289}} \quad (9)$$

$$= \frac{65}{17} \quad (10)$$

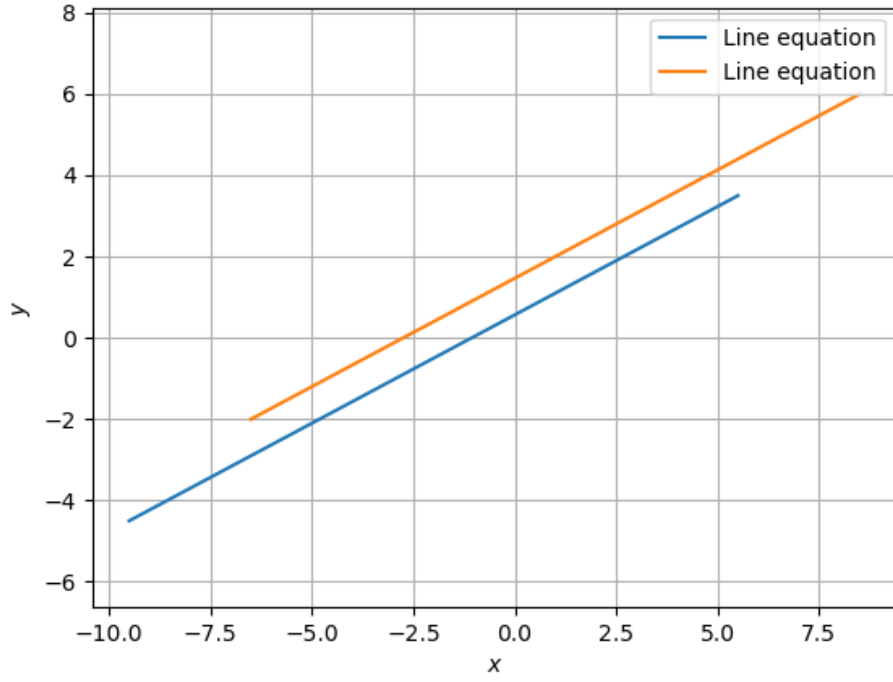


Figure 1

3. solution for problem 2

Given line is

$$l(x + y) + p = 0 \text{ and } l(x + y) - r = 0 \quad (11)$$

this equation can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c_1 \quad (12)$$

$$\mathbf{n}^\top \mathbf{x} = c_2 \quad (13)$$

$$\text{where } \mathbf{n} = \begin{pmatrix} l \\ l \end{pmatrix}, c_1 = p, c_2 = -r \quad (14)$$

$$= \begin{pmatrix} l & l \end{pmatrix} \mathbf{x} = p \quad (15)$$

$$= \begin{pmatrix} l & l \end{pmatrix} \mathbf{x} = -r \quad (16)$$

distance between parallel lines

$$d = \frac{|c_1 - c_2|}{\|\mathbf{n}\|} \quad (17)$$

$$= \frac{|p + r|}{l\sqrt{2}} \quad (18)$$

The distance between parallel lines is shown in figure 2 with normal vector as

$$\mathbf{n} = \begin{pmatrix} 8 \\ 8 \end{pmatrix} \text{ and } c_1 = 16, c_2 = -16$$

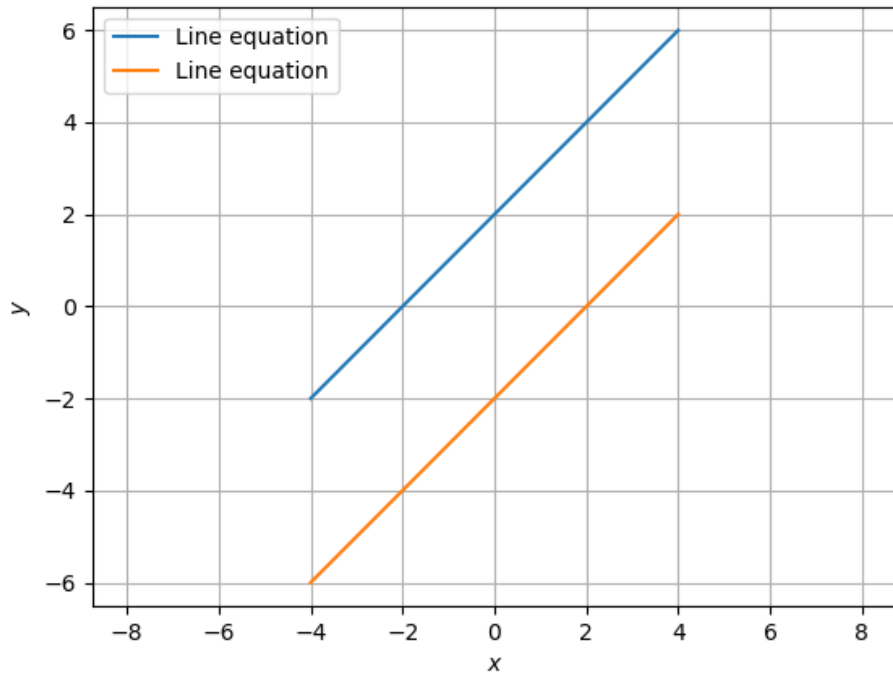


Figure 2