Intercepts Lines

11^{th} Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6, respectively.

1 Solution

Let the x intercept be a and the y intercept be b, Then

$$(a+b) = 1 \tag{1}$$

$$(ab) = -6 \tag{2}$$

upon simplifying (1) and (2)

$$\mathbf{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{3}$$

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 3\\2 \end{pmatrix} \tag{5}$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} 0r \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{6}$$

 \implies The normal vector **n** to the line is given as

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} -2\\3 \end{pmatrix} \tag{8}$$

The equation of a line with normal vector \mathbf{n} and passing through a point \mathbf{A} is given by

$$\mathbf{n}^{\top} \left(\mathbf{x} - \mathbf{A} \right) = 0 \tag{9}$$

$$\begin{pmatrix} -2 & 3 \end{pmatrix} \mathbf{x} = 6 \tag{10}$$

Or

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} -3\\ -2 \end{pmatrix} \tag{12}$$

The equation of a line with normal vector \mathbf{n} and passing through a point \mathbf{B} is given by

$$\begin{pmatrix} -3 & -2 \end{pmatrix} \mathbf{x} = 6 \tag{13}$$

The line segment is as shown in Figure ??.

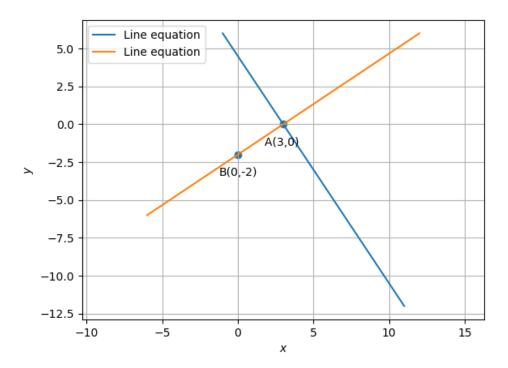


Figure 1