Properties of lines

1 12^{th} Maths - Chapter 11

This is Problem-2 from Exercise 11.4

1. If l_1,m_1,n_1 and l_2,m_2,n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.

2 Solution

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \tag{1}$$

$$l_1^2 + m_1^2 + n_1^2 = 0 \tag{2}$$

$$l_2^2 + m_2^2 + n_2^2 = 0 \tag{3}$$

In order to proove the vector $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$ is perpendicular to both the vectors l_1,m_1,n_1 and l_2,m_2,n_2. Let us consider the matrix **P**.

$$\mathbf{P}= \begin{pmatrix} l_1 & l_2 & m_1n_2 - m_2n_1 \\ m_1 & m_2 & n_1l_2 - n_2l_1 \\ n_1 & n_2 & l_1m_2 - l_2m_1 \end{pmatrix}$$

$$\mathbf{P}^\top = \begin{pmatrix} l_1 & m_1 & n_1 & & \\ l_2 & m_2 & n_2 & & \\ m_1n_2 - m_2n_1 & m_2 & n_1l_2 - n_2l_1 & n_2 & l_1m_2 - l_2m_1 \end{pmatrix}$$

If the three vectors are mutually perpendicular then

$$P^\top P = I \tag{4}$$

This is prooved from the result of python and from the (1),(2) and (3). where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

Hences,The three vectors are mutually perpendicular
So we prooved that If l_1,m_1,n_1 and l_2,m_2,n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.

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https://github.com/ahilan22/fwc-2/tree/main/probability/
assignment/codes/12-13-4-6.py
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