Parallel Lines

11^{th} Maths - Chapter 10

This is Problem-3 from Exercise 10.4

1. Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6,resspectively.

1 Solution

Let the x intercept be a and the y intercept be b ,Then

$$\big(a+b\big) = 1 \tag{1}$$

$$\big(ab\big) = -6 \tag{2}$$

upon simplifying (1) and (2)

$$\mathbf{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{3}$$

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{5}$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} 0r \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{6}$$

$\implies$ The normal vector $\mathbf{n}$ to the line is given as

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{8}$$

The equation of a line with normal vector $\mathbf{n}$ and passing through a point $\mathbf{A}$ is given by

$$\mathbf{n}^{\top}\left(\mathbf{x}-\mathbf{A}\right)=0 \tag{9}$$

$$\begin{pmatrix}-2 & 3\end{pmatrix}\mathbf{x}=6 \tag{10}$$

Or

$$\mathbf{n}=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}-2\\ 3\end{pmatrix} \tag{11}$$

$$=\begin{pmatrix}-3\\ -2\end{pmatrix} \tag{12}$$

The equation of a line with normal vector $\mathbf{n}$ and passing through a point $\mathbf{B}$ is given by

$$\begin{pmatrix}-3 & -2\end{pmatrix}\mathbf{x}=6 \tag{13}$$

The line segment is as shown in Figure **??**.

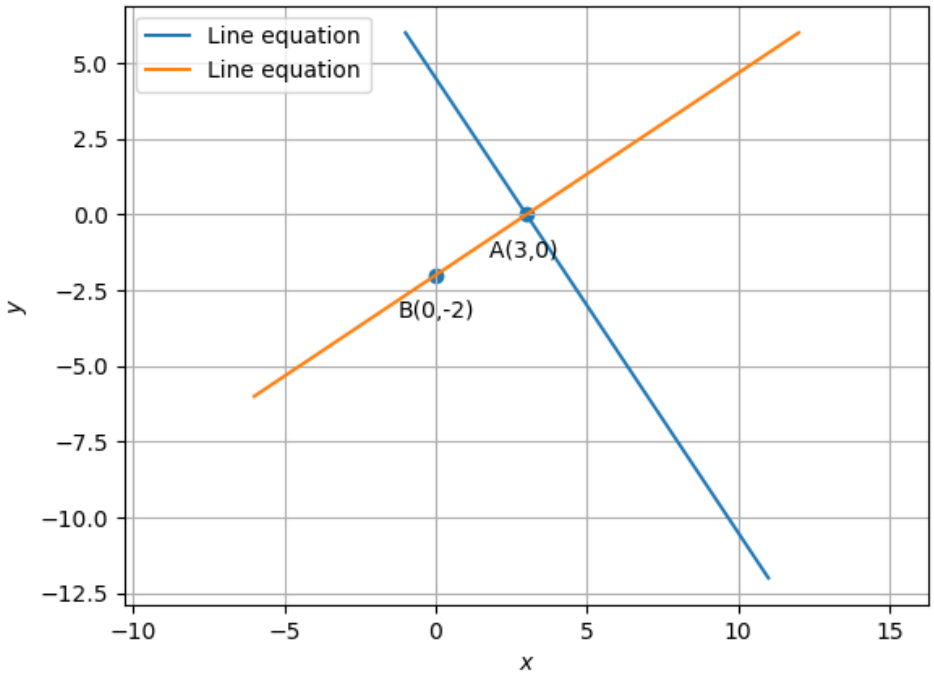
Line equation
Line equation
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−12.5
y
−10
−5
0
5
10
15
x
A(3,0)
B(0,-2)

Figure 1