Properties of Collinear (Assignment-3)

## 1 $10^{th}$ Maths - Chapter 7

This is Problem-2 from Exercise 7.3.2

## 2 Problem

In Each of the following find the value of k, for which the point of collinear (1)( (7, -2),(5, 1),(3, k) (2)(8,1),(k,-4),(2,-5).

## 3 Solution for problem 1

The input given

$$\boldsymbol{A} = \begin{pmatrix} \mathbf{7} \\ \mathbf{-2} \end{pmatrix} \tag{1}$$

$$\boldsymbol{B} = \begin{pmatrix} \mathbf{5} \\ \mathbf{1} \end{pmatrix} \tag{2}$$

$$\boldsymbol{C} = \begin{pmatrix} \mathbf{3} \\ \mathbf{4} \end{pmatrix} \tag{3}$$

$$\textbf{D=A-B} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{5}$$

$$\textbf{E=A-C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ k \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 4 \\ -2-k \end{pmatrix} \tag{7}$$

Now the matrix is

$$\textbf{F}=\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{E} \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \tag{9}$$

If points on a line are collinear, rank of matrix is " 1 "then the vectors are in linearlydependent. For 2 × 2 matrix Rank =1 means Determinant is 0.

Through pivoting,we obtain

$$= \begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \xrightarrow{R2=R2-R1} = \begin{pmatrix} -2-k & -4 \\ 4 & -2-k \end{pmatrix} \tag{11}$$

if the rank of the matrix is 1 means any one of the row must be zero.So, making the first element in the matrix to 0.

$$-2-k-2(-3) = 0 \tag{12}$$

$$-2-k-6 = 0 \tag{13}$$

$$k = 4 \tag{14}$$

Hence proved.

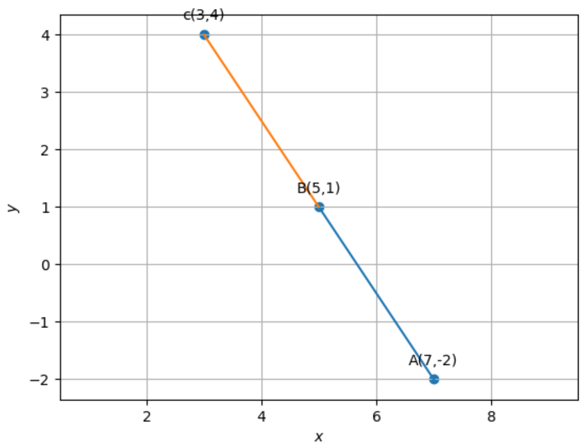


Figure 1:

## 4 Solution for problem 2

The input given

$$\boldsymbol{A} = \begin{pmatrix} \mathbf{8} \\ \mathbf{1} \end{pmatrix} \tag{15}$$

$$\boldsymbol{B} = \begin{pmatrix} \boldsymbol{k} \\ \mathbf{-4} \end{pmatrix} \tag{16}$$

$$\boldsymbol{C} = \begin{pmatrix} \mathbf{2} \\ \mathbf{-5} \end{pmatrix} \tag{17}$$

$$\textbf{D=A-B} = \begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} k \\ -4 \end{pmatrix} \tag{18}$$

$$= \begin{pmatrix} 8-k \\ 5 \end{pmatrix} \tag{19}$$

$$\textbf{E=A-C} = \begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{20}$$

$$= \begin{pmatrix} 6 \\ 6 \end{pmatrix} \tag{21}$$

Now the matrix is

$$\textbf{F=}\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{E} \end{pmatrix} \tag{22}$$

$$= \begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \tag{23}$$

If points on a line are collinear, rank of matrix is " 1 "then the vectors are in linearlydependent. For 2 $\times$ 2 matrix Rank =1 means Determinant is 0. hrough pivoting,we obtain

$$= \begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \tag{24}$$

$$= \begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \xrightarrow{R1=3R1-3R2} = \begin{pmatrix} 24-3k & 15 \\ 6 & 6 \end{pmatrix} \tag{25}$$

if the rank of the matrix is 1 means any one of the row must be zero.So, making the first element in the matrix to 0.

$$3(8-k) - 3(5) = 0 \tag{26}$$

$$24 - 3k - 15 = 0 \tag{27}$$

$$k = 3 \tag{28}$$

Hence proved.

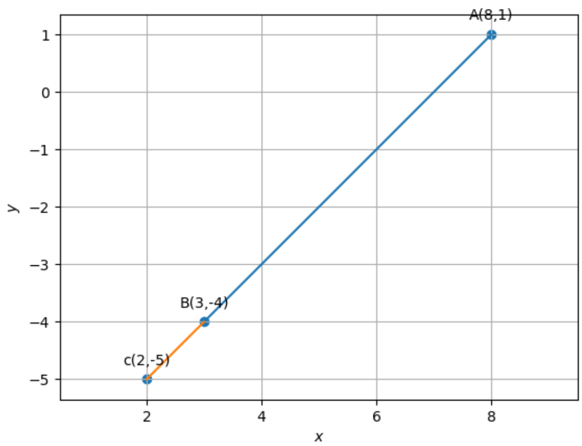


Figure 2: