

Properties of lines

1 12th Maths - Chapter 11

This is Problem-2 from Exercise 11.4

1. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.

2 Solution

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \quad (1)$$

$$l_1^2 + m_1^2 + n_1^2 = 0 \quad (2)$$

$$l_2^2 + m_2^2 + n_2^2 = 0 \quad (3)$$

In order to prove the vector $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$ is perpendicular to both the vectors l_1, m_1, n_1 and l_2, m_2, n_2 . Let us consider the matrix \mathbf{P} .

$$\mathbf{P} = \begin{pmatrix} l_1 & l_2 & m_1n_2 - m_2n_1 \\ m_1 & m_2 & n_1l_2 - n_2l_1 \\ n_1 & n_2 & l_1m_2 - l_2m_1 \end{pmatrix}$$

$$\mathbf{P}^\top = \begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ m_1n_2 - m_2n_1 & n_1l_2 - n_2l_1 & l_1m_2 - l_2m_1 \end{pmatrix}$$

If the three vectors are mutually perpendicular then

$$P^{\top} P = I \quad (4)$$

This is proved from the result of python and from the (1),(2) and (3).
where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (5)$$

Hences, The three vectors are mutually perpendicular

So we proved that If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both these are $m_1 n_2 - m_2 n_1$, $n_1 l_2 - n_2 l_1$, $l_1 m_2 - l_2 m_1$.

<https://github.com/srikanth9515/FWC/blob/main/matrices/lines/12.11.4.2/code/cosin.py>