

Equation of circle

Excercise 10.4

1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of other chord.

Solution:

Consider the Circle of radius 1 and length of chord be 1.5

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, d = 1.5, r = 1 \quad (1)$$

Consider the points on the circle

$$\mathbf{P} = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix} \quad (2)$$

$$\mathbf{Q} = \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix} \quad (3)$$

$$\mathbf{R} = \begin{pmatrix} \cos \theta_3 \\ \sin \theta_3 \end{pmatrix} \quad (4)$$

$$\mathbf{S} = \begin{pmatrix} \cos \theta_4 \\ \sin \theta_4 \end{pmatrix} \quad (5)$$

So, here

$$(\mathbf{P} - \mathbf{Q}) = \begin{pmatrix} \cos \theta_1 - \cos \theta_2 \\ \sin \theta_1 - \sin \theta_2 \end{pmatrix} \quad (6)$$

$$\|\mathbf{P} - \mathbf{Q}\|^2 = d^2 \quad (7)$$

$$\begin{aligned}
&\implies ((\cos \theta_1 - \cos \theta_2)^2) + ((\sin \theta_1 - \sin \theta_2)^2) = d^2 \\
&\implies 2 - 2(\cos(\theta_1 - \theta_2)) = d^2 \\
&\implies 2(2(\sin^2(\frac{\theta_1 - \theta_2}{2}))) = d^2 \\
&\implies (\sin^2 \frac{\theta_1 - \theta_2}{2}) = \frac{d^2}{4} \\
&\implies (\sin \frac{\theta_1 - \theta_2}{2}) = \frac{d}{2} \\
&\implies (\frac{\theta_1 - \theta_2}{2}) = (\sin^{-1}(0.75))
\end{aligned}$$

$$(\theta_1 - \theta_2) = 97.1806 \quad (8)$$

Simillary we can say that

$$(\theta_3 - \theta_4) = 97.1806 \quad (9)$$

Let

$$\theta_1 = 30^\circ, \theta_3 = 60^\circ \quad (10)$$

here

$$\mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix} \quad (11)$$

$$\mathbf{Q} = \begin{pmatrix} 0.3878 \\ -0.9217 \end{pmatrix} \quad (12)$$

$$\mathbf{R} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad (13)$$

$$\mathbf{S} = \begin{pmatrix} 0.7967 \\ -0.6043 \end{pmatrix} \quad (14)$$

Obtain the point of intersection of line **PQ,RS**

$$\mathbf{n}_1 \top (\mathbf{X} - \mathbf{P}) = 0 \quad (15)$$

$$\mathbf{n}_2 \top (\mathbf{X} - \mathbf{R}) = 0 \quad (16)$$

$$\mathbf{n}_1 = \begin{pmatrix} 1.4217 \\ -0.47822 \end{pmatrix} \quad (17)$$

$$\mathbf{n}_2 = \begin{pmatrix} 1.4703 \\ 0.2967 \end{pmatrix} \quad (18)$$

From the Result of python code , we got the point of intersection of (15) (16) is given as :

$$\mathbf{T} = \begin{pmatrix} 0.68341409 \\ -0.04288508 \end{pmatrix} \quad (19)$$

From the Result of python code , we get

$$\|\mathbf{P} - \mathbf{T}\| = 0.5727 \quad (20)$$

$$\|\mathbf{R} - \mathbf{T}\| = 0.9272 \quad (21)$$

$$\|\mathbf{Q} - \mathbf{T}\| = 0.9272 \quad (22)$$

$$\|\mathbf{S} - \mathbf{T}\| = 0.5727 \quad (23)$$

Hence, we proved that

$$\|\mathbf{P} - \mathbf{T}\| = \|\mathbf{S} - \mathbf{T}\| \quad (24)$$

$$\|\mathbf{Q} - \mathbf{T}\| = \|\mathbf{R} - \mathbf{T}\| \quad (25)$$

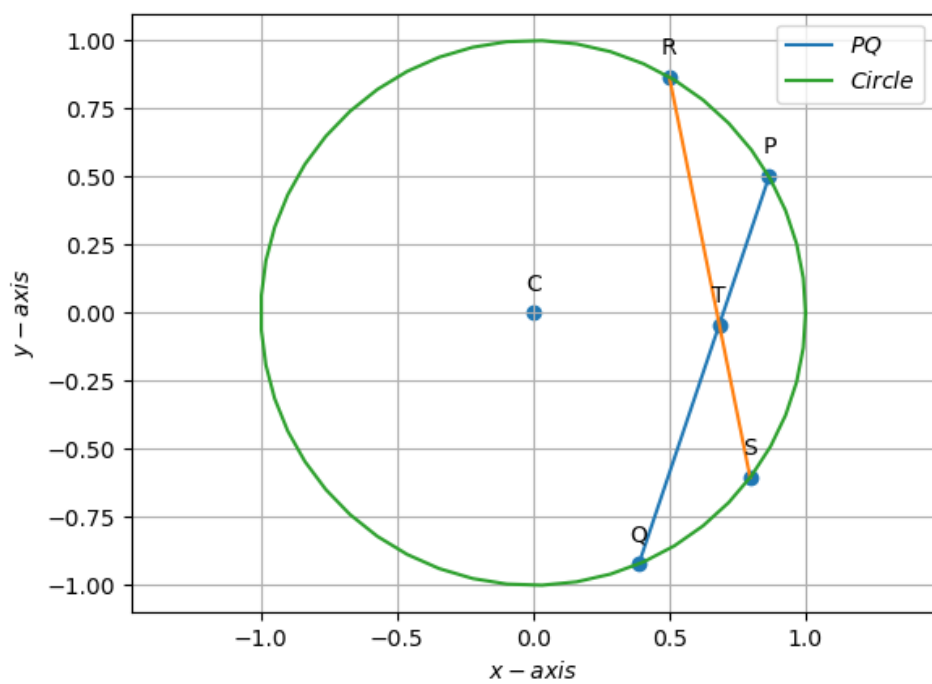


Figure 1