Properties of vectors

1 12^{th} Maths - Exercise 10.4.1

1. Find $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ if $\overrightarrow{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\overrightarrow{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

2 Solution

Now,

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \tag{1}$$

$$\tag{2}$$

The cross product or vector product of $\mathbf{A}, \mathbf{B}$ is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \tag{3}$$

Hence

$$\begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} = \begin{vmatrix} -7 & -2 \\ 7 & 2 \end{vmatrix} = \Big(-14 + 14\Big) = 0 \tag{4}$$

$$\begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 7 & 2 \end{vmatrix} = \Big(2 - 21\Big) = -19 \tag{5}$$

$$\begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -7 & -2 \end{vmatrix} = \Big(-2 + 21\Big) = 19 \tag{6}$$

$$\tag{7}$$

which can be represented in matrix form as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 \\ -19 \\ 19 \end{pmatrix}. \tag{8}$$

Hence

$$\begin{aligned} \left|\mathbf{A} \times \mathbf{B}\right| &= \sqrt{0^2 + (-19^2) + 19^2} && (9) \\ &= \sqrt{0 + 361 + 361} && (10) \\ &= \sqrt{722} && (11) \\ &= 26.87 && (12) \end{aligned}$$