Equation of Unit Vector

$1 \quad 12^{th} \text{ Maths}$ - Chapter 11

This is Problem-2 from Exercise 3.2

1. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$

2 Solution

Normal vector to the plane is

$$\mathbf{c} = \begin{pmatrix} 3 \\ 5 \\ -6 \end{pmatrix} \tag{1}$$

unit vector

$$\hat{\mathbf{c}} = \frac{1}{||\mathbf{c}||} \mathbf{c} \tag{2}$$

$$||\mathbf{c}|| = \sqrt{c^{\top}c} \tag{3}$$

$$= \sqrt{\begin{pmatrix} 3 & 5 & -6 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ -6 \end{pmatrix}} \tag{4}$$

$$=\sqrt{70}\tag{5}$$

$$\hat{\mathbf{c}} = \frac{1}{\sqrt{70}} \begin{pmatrix} 3\\5\\-6 \end{pmatrix} \tag{6}$$

The vector equation of a plane Perpedicular to vector $3\hat{i}+5\hat{j}-6\hat{k}$ and at a distance of 7 units from origin is

$$\mathbf{r} \top \mathbf{n} = d \tag{7}$$

(8)

Where,

$$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{9}$$

$$\hat{\mathbf{n}} = \frac{1}{\sqrt{70}} \begin{pmatrix} 3\\5\\-6 \end{pmatrix} \tag{10}$$

$$d = 7 \tag{11}$$

(12)

Hences,

$$\begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ -6 \end{pmatrix} = \sqrt{70} \times 7 \tag{13}$$