

Properties of Circle

1 9th Maths - Chapter 10

This is Problem-2 from Exercise 10.4

1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of other chord.

Solution:

Consider the Circle of radius 1 and length of chord be 1.5

Symbol	Value	Description
C	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Circle point
d	1.5	Length of Chord
r	1	Radius
θ_1	30°	—
θ_2	60°	—

Table 1

$$\mathbf{P} = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix} \quad (1)$$

$$\mathbf{R} = \begin{pmatrix} \cos \theta_3 \\ \sin \theta_3 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} \cos \theta_4 \\ \sin \theta_4 \end{pmatrix} \quad (2)$$

So, here

$$(\mathbf{P} - \mathbf{Q}) = \begin{pmatrix} \cos \theta_1 - \cos \theta_2 \\ \sin \theta_1 - \sin \theta_2 \end{pmatrix} \quad (3)$$

$$\|\mathbf{P} - \mathbf{Q}\|^2 = d^2 \quad (4)$$

$$\begin{aligned}
&\implies ((\cos \theta_1 - \cos \theta_2)^2) + ((\sin \theta_1 - \sin \theta_2)^2) = d^2 \\
&\implies 2 - 2(\cos(\theta_1 - \theta_2)) = d^2 \\
&\implies 2(2(\sin^2(\frac{\theta_1 - \theta_2}{2}))) = d^2 \\
&\implies \sin^2(\frac{\theta_1 - \theta_2}{2}) = \frac{d^2}{4} \\
&\implies \sin \frac{\theta_1 - \theta_2}{2} = \frac{d}{2} \\
&\implies (\frac{\theta_1 - \theta_2}{2}) = \sin^{-1}(\frac{d}{2})
\end{aligned}$$

$$(\theta_1 - \theta_2) = 97.1806 \quad (5)$$

Simillary we can say that

$$(\theta_3 - \theta_4) = 97.1806 \quad (6)$$

Let

$$\theta_1 = 30^\circ, \theta_3 = 60^\circ \quad (7)$$

$$\theta_2 = -67.1806, \theta_4 = -37.1806 \quad (8)$$

Obtain the point of intersection of line **PQ,RS**

$$\mathbf{n}_1^\top (\mathbf{X} - \mathbf{P}) = 0 \quad (9)$$

$$\mathbf{n}_2^\top (\mathbf{X} - \mathbf{R}) = 0 \quad (10)$$

$$\mathbf{Omat} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (11)$$

$$\mathbf{n}_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos \theta_1 - \cos \theta_2 \\ \sin \theta_1 - \sin \theta_2 \end{pmatrix} \quad (12)$$

$$= \begin{pmatrix} \sin \theta_1 - \sin \theta_2 \\ \cos \theta_2 - \cos \theta_1 \end{pmatrix} \quad (13)$$

$$\mathbf{n}_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos \theta_3 - \cos \theta_4 \\ \sin \theta_3 - \sin \theta_4 \end{pmatrix} \quad (14)$$

$$= \begin{pmatrix} \sin \theta_3 - \sin \theta_4 \\ \cos \theta_4 - \cos \theta_3 \end{pmatrix} \quad (15)$$

From the Result of python code by sunstituting $\theta_1, \theta_2, \theta_3, \theta_4$ in point \mathbf{T} then we got the point of intersection as :

$$\mathbf{T} = \begin{pmatrix} \sin \theta_2 - \sin \theta_1 & \cos \theta_2 - \cos \theta_1 \\ \sin \theta_4 - \sin \theta_3 & \cos \theta_4 - \cos \theta_3 \end{pmatrix} \mathbf{X} = \begin{pmatrix} \cos \theta_1 (\sin \theta_2 - \sin \theta_1) - \sin \theta_1 (\cos \theta_2 - \cos \theta_1) \\ \cos \theta_3 (\sin \theta_4 - \sin \theta_3) - \sin \theta_3 (\cos \theta_4 - \cos \theta_2) \end{pmatrix} \quad (16)$$

$$= \begin{pmatrix} \frac{\cos \theta_4 - \cos \theta_2 (\sin \theta_2 \cos \theta_1 - \sin \theta_1 \cos \theta_2)}{(\cos \theta_4 - \cos \theta_2)(\sin \theta_2 - \sin \theta_1) - (\cos \theta_2 - \cos \theta_1)(\sin \theta_4 - \sin \theta_3)} \\ \sin \theta_1 + \frac{\sin \theta_2 - \sin \theta_1}{\cos \theta_2 - \cos \theta_1} \left(\frac{\cos \theta_4 - \cos \theta_2 (\sin \theta_2 \cos \theta_1 - \sin \theta_1 \cos \theta_2)}{(\cos \theta_4 - \cos \theta_2)(\sin \theta_2 - \sin \theta_1) - (\cos \theta_2 - \cos \theta_1)(\sin \theta_4 - \sin \theta_3)} \right) - \cos \theta_1 \end{pmatrix} \quad (17)$$

$$= \begin{pmatrix} 0.68341409 \\ -0.04288508 \end{pmatrix} \quad (18)$$

From the Result of python code , we get

$$\|\mathbf{P} - \mathbf{T}\| = 0.5727 \quad (19)$$

$$\|\mathbf{R} - \mathbf{T}\| = 0.9272 \quad (20)$$

$$\|\mathbf{Q} - \mathbf{T}\| = 0.9272 \quad (21)$$

$$\|\mathbf{S} - \mathbf{T}\| = 0.5727 \quad (22)$$

Hence, we proved that

$$\|\mathbf{P} - \mathbf{T}\| = \|\mathbf{S} - \mathbf{T}\| \quad (23)$$

$$\|\mathbf{Q} - \mathbf{T}\| = \|\mathbf{R} - \mathbf{T}\| \quad (24)$$

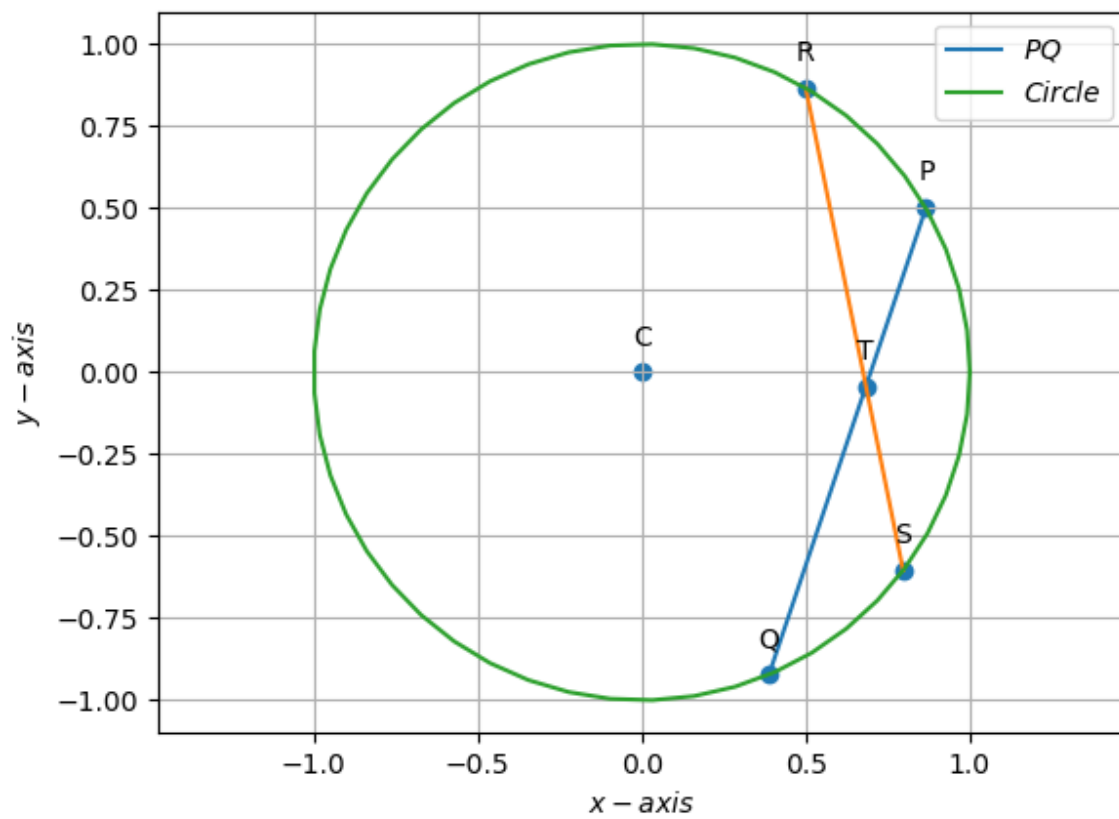


Figure 1: Two equal chords intersecting in a circle