

Assignment: Isosceles triangle

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Problem: Check whether $(5, -2)$, $(6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.

Symbol	Values	Description
A	(6,4)	Vertex A
B	(7,-2)	Vertex B
C	(5,-2)	Vertex C
D	(6,-2)	Midpoint of AC
E	(13/2,1)	Midpoint of BC
F	(11/2,1)	Midpoint of AB

Solution: 1: Let the given points be $\mathbf{A}, \mathbf{B}, \mathbf{C}$ respectively. Then, the direction vectors of AB, BC and CA are

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \quad (1)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (3)$$

$$\mathbf{A} - \mathbf{B}^\top \mathbf{B} - \mathbf{C} = \begin{pmatrix} -1 & -6 \end{pmatrix} \begin{pmatrix} -1 \\ 6 \end{pmatrix} \quad (4)$$

$$= 37 \quad (5)$$

$$\mathbf{B} - \mathbf{C}^\top \mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (6)$$

$$= -2 \quad (7)$$

$$\mathbf{C} - \mathbf{A}^\top \mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -6 \end{pmatrix} \quad (8)$$

$$= -2 \quad (9)$$

$$\text{From the above equations,} \quad (10)$$

$$\mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C} \quad (11)$$

$$\angle BCA = \angle CAB \quad (12)$$

Thus, the triangle is isosceles triangle.

Method2: Let the given points be $\mathbf{E}, \mathbf{F}, \mathbf{D}$ respectively. Then, the direction vectors of BD, CF, AE are

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} 13/2 \\ 1 \end{pmatrix} \quad (13)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \quad (14)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 11/2 \\ 1 \end{pmatrix} \quad (15)$$

$$\mathbf{E} - \mathbf{A}^\top \mathbf{C} - \mathbf{B} = \begin{pmatrix} 3/2 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ -6 \end{pmatrix} \quad (16)$$

$$\neq 0 \quad (17)$$

$$\mathbf{F} - \mathbf{C}^\top \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3/2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix} \quad (18)$$

$$\neq 0 \quad (19)$$

$$\mathbf{D} - \mathbf{B}^\top \mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 & -6 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (20)$$

$$= 0 \quad (21)$$

$$\mathbf{B} - \mathbf{D} \perp \mathbf{A} - \mathbf{C} \quad (22)$$

$$\angle BCA = \angle CAB \quad (23)$$

1 Figure

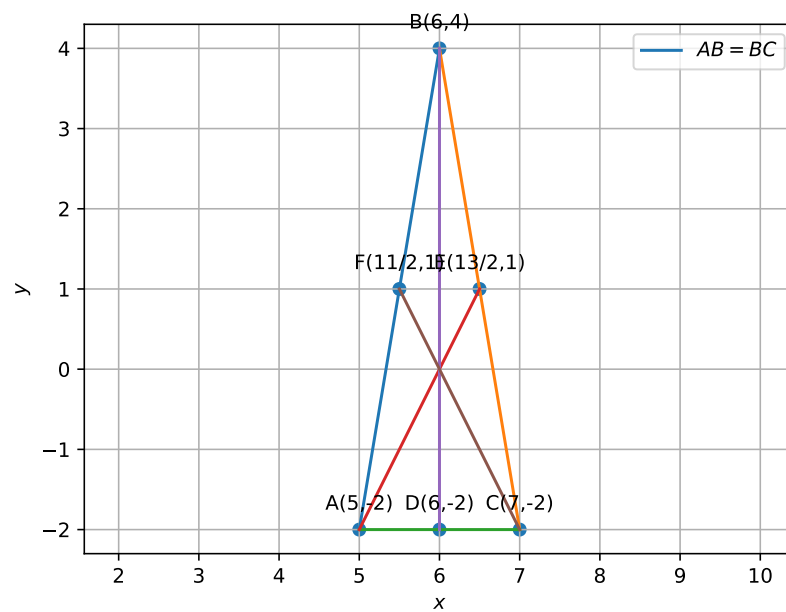


Figure 1: Isosceles triangle

<https://github.com/srikanth9515/FWC/tree/main/maths/vec1>