Assignment: Isosceles triangle

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Problem: Check whether$(5, -2), (6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle. **Solution:** 1: Let the given points be $\mathbf{A}, \mathbf{B}, \mathbf{C}$ respectively. Then,

Symbol	Values	Description
A	(6,4)	Vertex A
B	(7,-2)	Vertex B
C	(5,-2)	Vertex C
D	(6,-2)	Midpoint of AC
E	(13/2,1)	Midpoint of BC
F	(11/2,1)	Midpoint of AB

the direction vectors of AB, BC and CA are

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \tag{1}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{A} - \mathbf{B}^\top \mathbf{B} - \mathbf{C} = \begin{pmatrix} -1 & -6 \end{pmatrix} \begin{pmatrix} -1 \\ 6 \end{pmatrix} \qquad = 37 \tag{4}$$

$$\mathbf{B} - \mathbf{C}^\top \mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{5}$$

$$= -2 \tag{6}$$

$$\mathbf{C} - \mathbf{A}^\top \mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -6 \end{pmatrix} \tag{7}$$

$$= -2 \tag{8}$$

$$From the above equations \mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C} \tag{9}$$

$$\angle BCA = \angle CAB \tag{10}$$

Thus, the triangle is isosceles triangle. **Method2**: Let the given points be

$\mathbf{E}, \mathbf{F}, \mathbf{D}$ respectively. Then, the direction vectors of BD, CF, AE are

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} 13/2 \\ 1 \end{pmatrix} \tag{11}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \tag{12}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 11/2 \\ 1 \end{pmatrix} \tag{13}$$

$$\mathbf{E} - \mathbf{A}^\top \mathbf{C} - \mathbf{B} = \begin{pmatrix} 3/2 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ -6 \end{pmatrix} \tag{14}$$

$$\neq 0 \tag{15}$$

$$\mathbf{F} - \mathbf{C}^\top \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3/2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix} \tag{16}$$

$$\neq 0 \tag{17}$$

$$\mathbf{D} - \mathbf{B}^\top \mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 & -6 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{18}$$

$$= 0 \tag{19}$$

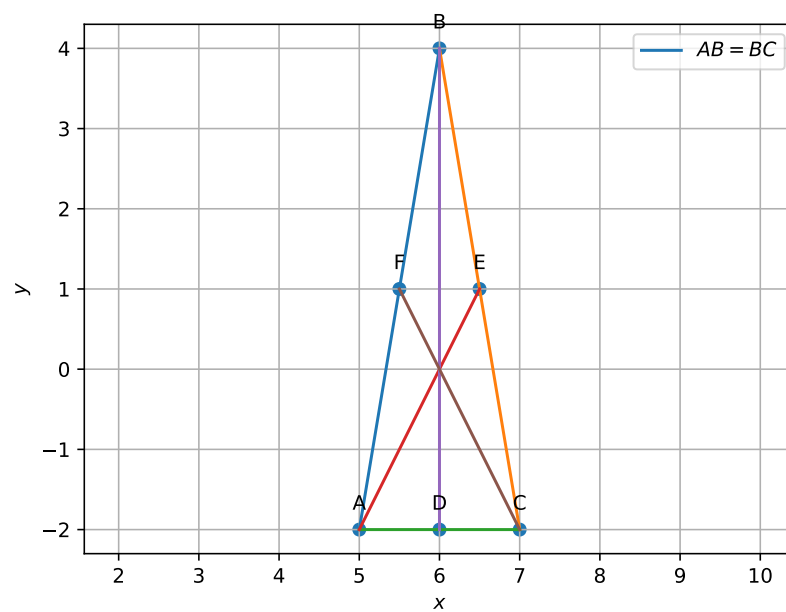$$\mathbf{B} - \mathbf{D} \perp \mathbf{A} - \mathbf{C} \tag{20}$$

$$\angle BCA = \angle CAB \tag{21}$$

1 Figure

Figure 1: Isosceles triangle

https://github.com/srikanth9515/FWC/tree/main/maths/vec1