

Properties of Collinear

1 10th Maths - Chapter 7

This is Problem-2 from Exercise 7.3.2

1. In each of the following find the value of 'k', for which the points are collinear.

1. (7, -2), (5, 1), (3, k)
2. (8, 1), (k, -4), (2, -5).

1. Solution for problem : 1

$$\mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 3 \\ k \end{pmatrix} \quad (1)$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad (2)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ K \end{pmatrix} \right) = \begin{pmatrix} 4 \\ 12 - K \end{pmatrix} \quad (3)$$

If points on a line are collinear, rank of matrix is " 1 " then the vectors are linearly dependent. For 2×2 matrix Rank =1 means Determinant is 0.

Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} 2 & -3 \\ 4 & -2-k \end{pmatrix} \xleftrightarrow{R_2=R_2-2R_1} \begin{pmatrix} -2-k & -4 \\ 4 & -2-k \end{pmatrix} \quad (6)$$

if the rank of the matrix is 1 means any one of the row must be zero. So, making the first element in the matrix to 0.

$$\begin{aligned} (-2-k-2(-3)) &= 0 \\ (-2-k-6) &= 0 \\ (k &= 4) \end{aligned} \quad (7)$$

Hence proved.

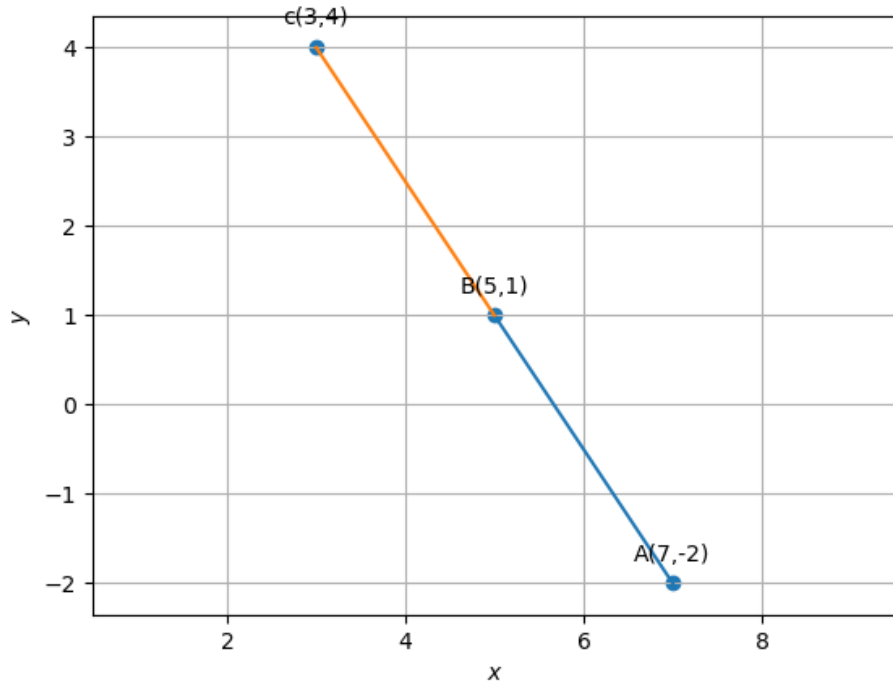


Figure 1

1. Solution for problem :2

$$\mathbf{A} = \begin{pmatrix} 8 \\ 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} k \\ -4 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \quad (8)$$

$$\mathbf{D} = (\mathbf{A} - \mathbf{B}) = \left(\begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} k \\ -4 \end{pmatrix} \right) = \begin{pmatrix} 8-k \\ 5 \end{pmatrix} \quad (9)$$

$$\mathbf{E} = (\mathbf{A} - \mathbf{C}) = \left(\begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right) = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \quad (10)$$

If points on a line are collinear, rank of matrix is " 1 " then the vectors are linearly dependent. For 2×2 matrix Rank =1 means Determinant is 0. Through pivoting, we obtain

$$\mathbf{F} = \begin{pmatrix} \mathbf{D} \\ \mathbf{E} \end{pmatrix} \quad (11)$$

$$\begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \quad (12)$$

$$\begin{pmatrix} 8-k & 5 \\ 6 & 6 \end{pmatrix} \xleftrightarrow{R_1=3R_1-3R_2} \begin{pmatrix} 24-3k & 15 \\ 6 & 6 \end{pmatrix} \quad (13)$$

if the rank of the matrix is 1 means any one of the row must be zero. So, making the first element in the matrix to 0.

$$\begin{aligned} (3(8-k) - 3(5)) &= 0 \\ (24 - 3k - 15) &= 0 \\ (k &= 3) \end{aligned} \quad (14)$$

Hence proved.

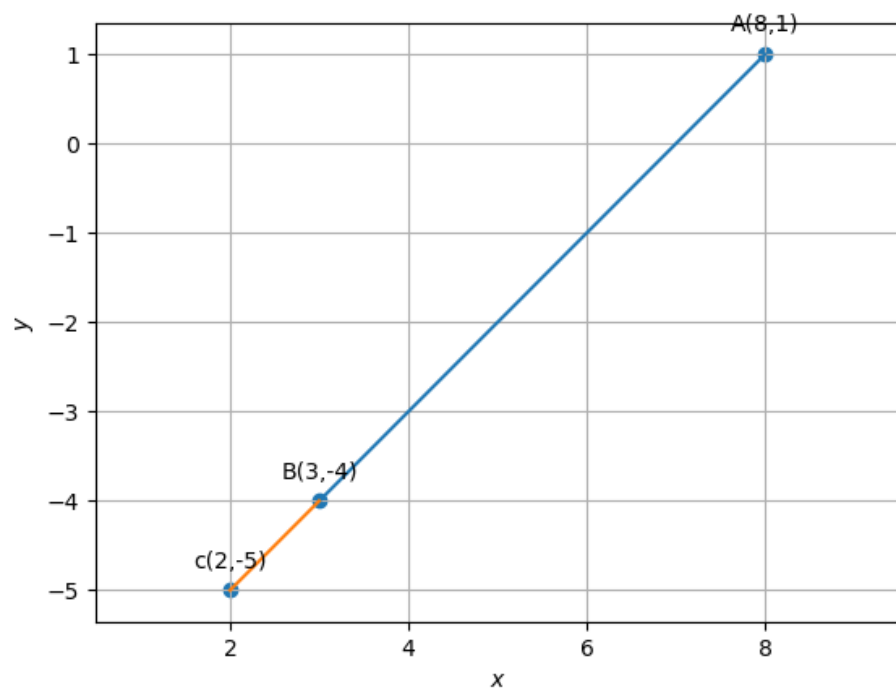


Figure 2