Properties of Circle

1 9^{th} Maths - Chapter 10

This is Problem-2 from Exercise 10.4

1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of other chord.

   Solution: : Consider the circle of radius 1 and length of chord be 1.5

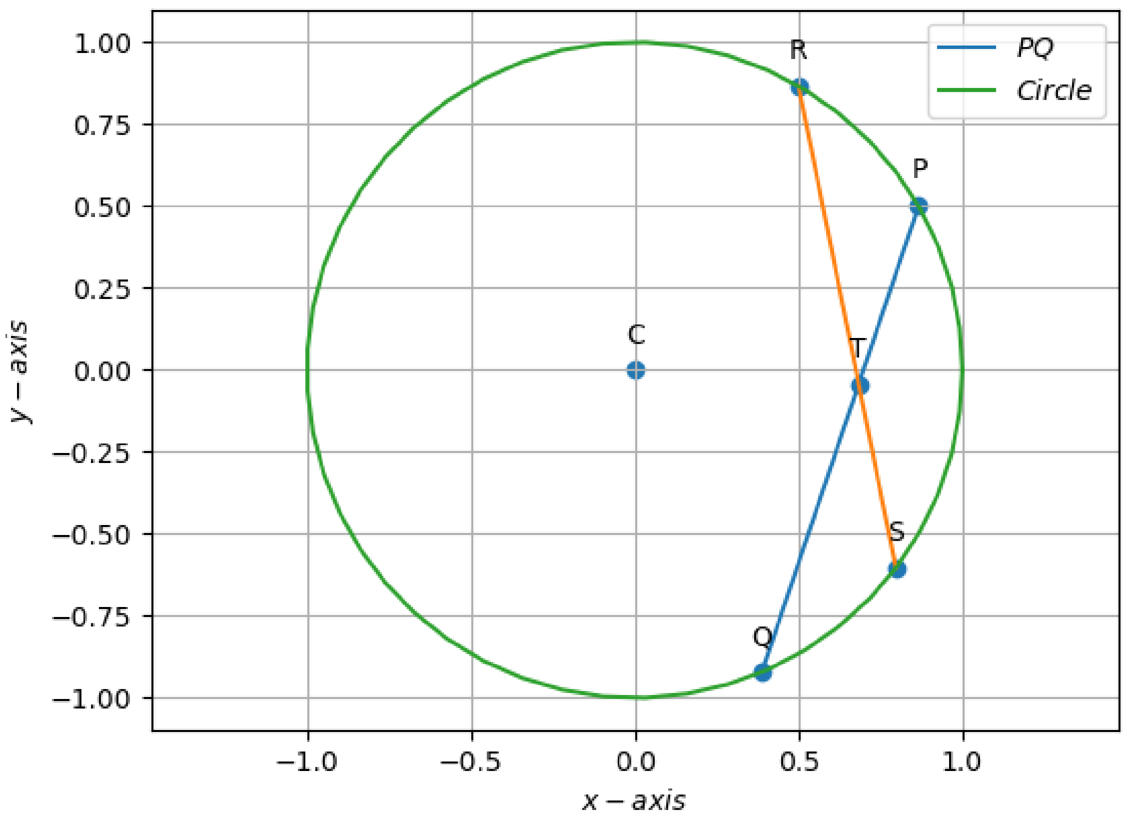

Figure 1: Two equal chords intersecting in a circle

Symbol	Value	Description
$\mathbf{C}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Circle point
d	1.5	Length of Chord
r	1	Radius
$\boldsymbol{\theta_1}$	$30°$	–
$\boldsymbol{\theta_3}$	$60°$	–
$\boldsymbol{\theta_2}$	$-67.1806°$	–
$\boldsymbol{\theta_4}$	$37.1806°$	–

Table 1

Construction

Consider

$$\mathbf{P} = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \tag{1}$$

$$\mathbf{Q} = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \tag{2}$$

$$\mathbf{R} = \begin{pmatrix} \cos\theta_3 \\ \sin\theta_3 \end{pmatrix} \tag{3}$$

$$\mathbf{S} = \begin{pmatrix} \cos\theta_4 \\ \sin\theta_4 \end{pmatrix} \tag{4}$$

So, here

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{5}$$

$$\left\|\mathbf{P} - \mathbf{Q}\right\|^2 = d^2 \tag{6}$$

$$\implies \left(\cos\theta_1 - \cos\theta_2\right)^2 + \left(\sin\theta_1 - \sin\theta_2\right)^2 = d^2 \tag{7}$$

$$\implies 2 - 2\left(\cos(\theta_1 - \theta_2)\right) = d^2 \tag{8}$$

$$\implies 2(2\left(\sin^2(\tfrac{\theta_1-\theta_2}{2})\right) = d^2 \tag{9}$$

$$\implies \sin^2(\frac{\theta_1 - \theta_2}{2}) = \frac{d^2}{4} \tag{10}$$

$$\implies \sin\frac{\theta_1 - \theta_2}{2} = \frac{d}{2} \tag{11}$$

$$\implies \left(\tfrac{\theta_1-\theta_2}{2}\right) = \sin^{-1}(\frac{d}{2}) \tag{12}$$

Substituing the values from table (1) in (12) So we get

$$\left(\theta_1 - \theta_2\right) = 97.1806° \tag{13}$$

Simillary we can say that

$$\left(\theta_3 - \theta_4\right) = 97.1806° \tag{14}$$

Obtain the point of intersection of line **PQ**,**RS**

$$\mathbf{n_1}^\top(\mathbf{X}-\mathbf{P})=0 \tag{15}$$
$$\mathbf{n_2}^\top(\mathbf{X}-\mathbf{R})=0 \tag{16}$$
$$\mathbf{Omat}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} \tag{17}$$
$$\mathbf{n}_1=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\cos\theta_1-\cos\theta_2\\ \sin\theta_1-\sin\theta_2\end{pmatrix} \tag{18}$$
$$=\begin{pmatrix}\sin\theta_1-\sin\theta_2\\ \cos\theta_2-\cos\theta_1\end{pmatrix} \tag{19}$$
$$\mathbf{n}_2=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\cos\theta_3-\cos\theta_4\\ \sin\theta_3-\sin\theta_4\end{pmatrix} \tag{20}$$
$$=\begin{pmatrix}\sin\theta_3-\sin\theta_4\\ \cos\theta_4-\cos\theta_3\end{pmatrix} \tag{21}$$

From the Result of python code by substituting $\theta_1, \theta_2, \theta_3, \theta_4$ in point **T** then we get the point of intersection as :

$$\begin{pmatrix}\sin\theta_2-\sin\theta_1 & \cos\theta_2-\cos\theta_1\\ \sin\theta_4-\sin\theta_3 & \cos\theta_4-\cos\theta_3\end{pmatrix}\mathbf{X}=\begin{pmatrix}\cos\theta_1(\sin\theta_2-\sin\theta_1)-\sin\theta_1(\cos\theta_2-\cos\theta_1)\\ \cos\theta_3(\sin\theta_4-\sin\theta_3)-\sin\theta_3(\cos\theta_4-\cos\theta_2)\end{pmatrix} \tag{22}$$

$$\mathbf{T}=\begin{pmatrix}\frac{\cos\theta_4-\cos\theta_2(\sin\theta 2\cos\theta_1-\sin\theta_1\cos\theta_2)}{(\cos\theta_4-\cos\theta_2)(\sin\theta_2-\sin\theta_1)-(\cos\theta_2-\cos\theta_1)(\sin\theta_4-\sin\theta_3)}\\ \sin\theta_1+\frac{\sin\theta_2-\sin\theta_1}{\cos\theta_2-\cos\theta_1}(\frac{\cos\theta_4-\cos\theta_2(\sin\theta 2\cos\theta_1-\sin\theta_1\cos\theta_2)}{(\cos\theta_4-\cos\theta_2)(\sin\theta_2-\sin\theta_1)-(\cos\theta_2-\cos\theta_1)(\sin\theta_4-\sin\theta_3)})-\cos\theta_1)\end{pmatrix} \tag{23}$$

$$=\begin{pmatrix}0.68341409\\ -0.04288508\end{pmatrix} \tag{24}$$

From the Result of python code , we get

$$\|\mathbf{P}-\mathbf{T}\|=0.5727 \tag{25}$$
$$\|\mathbf{R}-\mathbf{T}\|=0.9272 \tag{26}$$
$$\|\mathbf{Q}-\mathbf{T}\|=0.9272 \tag{27}$$
$$\|\mathbf{S}-\mathbf{T}\|=0.5727 \tag{28}$$

Hence, we proved that

$$\|\mathbf{P}-\mathbf{T}\|=\|\mathbf{S}-\mathbf{T}\| \tag{29}$$
$$\|\mathbf{Q}-\mathbf{T}\|=\|\mathbf{R}-\mathbf{T}\| \tag{30}$$