



Ungrouped data

Mean

→ Random data. Ex:- 5, 10, 12, 30, 60, 50.

if x_1, x_2, \dots, x_n , then avg of given data is $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$, where n is total number of quantities.

→ Variance :- $\sigma^2 = \text{variance} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$.

where \bar{x} is the mean of sample

x_i is variable, where $i = 1, 2, 3, \dots$

$\sigma = \text{Standard deviation} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$

Grouped data

here given data is arranged in ascending or descending order.

Ex:-

Marks Scored	fr
0-5	5
5-10	15
10-15	20

→ grouped data and discontinuity

Marks	f_i
1-5	10
6-10	15
11-15	20

→ When U.B of previous data = L.B of the present then the data is said to be continuous else discontinuous

→ We can add and sub some value from U.B and L.B resp to make data continuous

→ Average of grouped data

$$A.M = \bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum f_i}$$

$$\sum f_i = N, x_i = \text{midpoint} = \frac{U.B + L.B}{2}$$

$$\rightarrow \text{Variance} = \sigma^2 = \frac{1}{N} \left(\sum_{i=1}^n f_i (x_i - \bar{x})^2 \right)$$

$$\rightarrow S.D = \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2}$$

$$\rightarrow \text{Mean Deviation} = \frac{1}{N} \sum f_i |x_i - \bar{x}|$$

KARL PEARSONS COEFFICIENT OF SKEWNESS.

$$SKP = 3 \left(\frac{\text{mean} - \text{mode}}{\text{median}} \right) \text{ or } \frac{\text{mean} - \text{mode}}{\text{median}}$$

$$\text{mean} = \frac{\sum f_i x_i}{\sum f_i}, \quad \text{mean} = \bar{x} = A + \frac{\sum f d}{N} \times i$$

$$d = \left(\frac{m - A}{10} \right) \text{ where } m \text{ is midpoint.}$$

$$Q_2 = \text{Median}$$

→ coefficient of skewness by Bowley's method (SKB)

$$SKB = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

Q_3 = SKB is involved by quartile deviation

$$Q_1 = \frac{N}{4} = \frac{\sum f}{4} \rightarrow \text{Quartile deviation}$$

$$Q_2 = \frac{N}{2} = \frac{\sum f}{2}$$

→ two - " "

$$Q_3 = \frac{3N}{4} = \frac{3 \sum f}{4}$$

→ three - " "

$$Q_1 = L_1 + \left(\frac{\frac{N}{4} - P.C.F(m_1)}{f_1} \right) \times c \quad \left(\begin{array}{l} P.C.F = \text{Preceding} \\ \text{term of cumulative} \\ \text{frequency} \end{array} \right)$$

$f_1 \rightarrow \text{corresponding frequency.}$

$$Q_2 = L_2 + \left(\frac{\frac{N}{2} - P.C.F(m_2)}{f_2} \right) \times c$$

$f_2 \downarrow$
width of class interval.

$$Q_3 = L_3 + \left(\frac{\frac{3N}{4} - P.C.f(m_3)}{f_3} \right) \times C$$

L_1	$\frac{N}{4}$	f_1	m_1	cf $\rightarrow Q_1$
L_2	$\frac{N}{2}$	f_2	m_2	cf $\rightarrow Q_2$
L_3	$\frac{3N}{4}$	f_3	m_3	c.f $\rightarrow Q_3$

SKB negative indicates negative skewness.

→ curve fitting.

for a line normal equations are

$$\begin{aligned} \sum y &= na + b \sum x \rightarrow (1) \\ \sum xy &= a \sum x + b \sum x^2 \rightarrow (2) \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{normal equations}$$

a and b can be found out by solving equations (1) & (2)

x	y	x^2	xy

Ex:- Find Fit a Secondary eqn for given data.

Sol Let the Secondary equation can be written as $y = a + bx + cx^2$, where a, b, c are arbitrary constants which are determined by solving normal equations.

Normal equations:-

apply summations w.r.t to independent variable 'x' together eq I

$$\sum y = \sum a + b \sum x + c \sum x^2$$

$$\sum y = na + b \sum x + c \sum x^2 \text{ --- (1) } \rightarrow \text{Normal eq}$$

multiply independent variable 'x' together given eq I and take summation on both side

$$yx = ax + bx^2 + cx^3$$

$$\sum yx = a \sum x + b \sum x^2 + c \sum x^3 \text{ --- (2) } \rightarrow \text{Normal equation}$$

multiply x again and apply summation

$$\sum yx^2 = a \sum x^2 + b \sum x^3 + c \sum x^4 \text{ --- (3) } \rightarrow \text{Normal equation}$$

Now solve for a, b, c .

x	y	$\sum x^2$	x^3	x^4	xy	x^2y

other curves

④ $y = ab^x$, ⑤ $y = ae^{bx}$ ⑥ $y = ax^b$

→ fit an exponential function for the following data

$$y = ab^x$$

Since given equation is exponential, it reduces to linear form by applying log on both sides

$$\log y = \log a + x \log b \quad \text{--- (1)}$$

Put $\log y = Y$, $\log a = A$, $\log b = B$

then, the eq (1) becomes

$$Y = A + Bx \quad \text{--- (2)}$$

→ this is a linear equation in the form of a line

We know the normal equations for

above equation is $\sum_{i=1}^n Y = nA + B \sum_{i=1}^n x \quad \text{--- (2)}$

$$\sum_{i=1}^n Yx = A \sum_{i=1}^n x + B \sum_{i=1}^n x^2 \quad \text{--- (3)}$$

equations 2 and 3 and Normal equation

to eq (2) ∴ We can find A and B,

where $A = \log a$, $B = \log b$.

$$a = \text{antilog}(A), \quad b = \text{antilog}(B)$$

$$\log y = Y$$

Now substitute a and b in given eqn of curve, $y = ab^x$

$$(5) \quad y = ae^{bx}$$

$$\log y = \log a + bx \log e \quad \text{--- (1)}$$

$$\text{Now } \log y = Y$$

$$\log a = A$$

$$b \log e = B$$

$$\text{Now equation (1) becomes } Y = A + Bx \quad \text{--- (2)}$$

Now find A and B.

$$\text{Normal equations of eq (2) are } \sum_{i=1}^n Y = nA + B \sum_{i=1}^n x$$

$$\sum_{i=1}^n Yx + A \sum_{i=1}^n x + B \sum_{i=1}^n x^2$$

After finding A and B

$$a = \text{antilog}(A)$$

$$b = \text{antilog} \left(\frac{B}{\log e} \right)$$

$$\text{mode} = l + \frac{(f_k - f_{k-1}) \times c}{2f_k - f_{k-1} - f_{k+1}}$$

$f_k \rightarrow$ is the Mode frequency

$f_{k+1} \rightarrow$ frequency of next class

$f_{k-1} \rightarrow$ frequency of previous class

$l \rightarrow$ lower bound of mode class

Note: \rightarrow if $Q_1 + Q_3 = 2Q_2$ then $SKB = 0$
 \rightarrow if $Q_1 + Q_3 > 2Q_2 \rightarrow$ +ve skewness
 \rightarrow if $Q_1 + Q_3 < 2Q_2 \rightarrow$ -ve skewness
 \rightarrow if $Q_3 = Q_1 \Rightarrow SKB$ is infinity.

