Unit - II :- [Discrete prob Distn]

2/3/21
2.20 to 4.30

Topics
(1) Conditional prob & Bayes theorem

(2) Random variable and properties

(3) Mathematical Expectation, Variance, co-variance

(4) Poisson Distn → properties

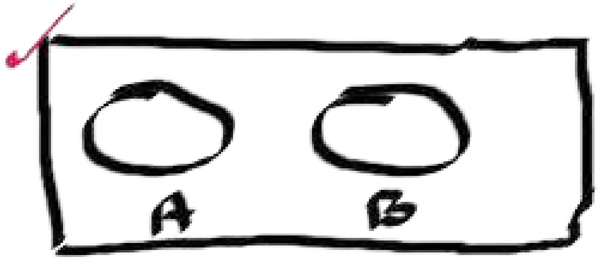
Probability:- [Chance]

$$P(E) = \frac{\text{favourable cases}}{\text{Exhaustive cases}} = \frac{m}{n}$$

$(m \le n)$

$$\left\{ \begin{array}{l} m = n \rightarrow \text{certain/sure} \\ m < n \rightarrow \text{possible} \end{array} \right\}$$

Types of Events:-

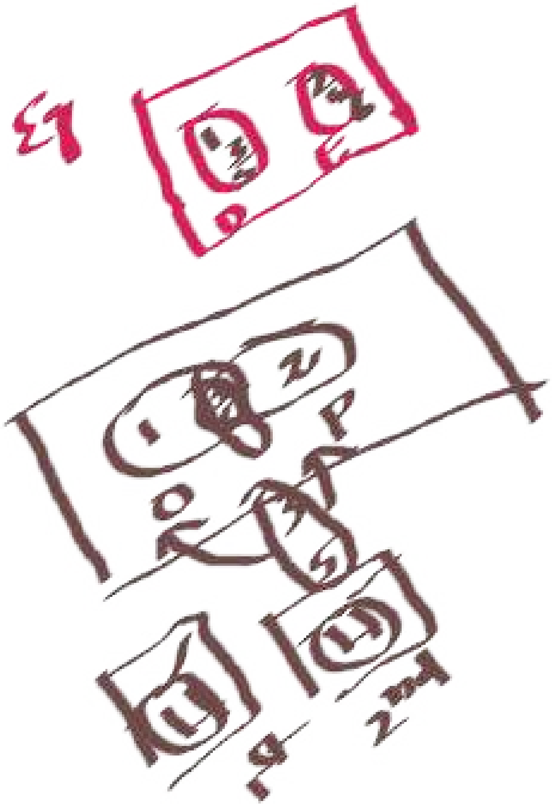


M.E/Disjoint

Dependent

Independent events

Eg



Addition Theorem.

If A & B two events

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

If A & B two M. Exclusive events

$$P(A\cup B) = P(A) + P(B)$$

$\left[\because \text{A \& B M.E} \quad P(A\cap B) = 0\right]$

If E_1, E_2, & E_3 three events

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - [P(E_1E_2) + P(E_2E_3) + P(E_3E_1)] + P(E_1E_2E_3)$$

(n-events)

$E_1, E_2, \ldots E_n$ are n-events

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i) - \sum_{i=1}^{n}\sum_{j=1}^{n} P(E_i \cap E_j) + \sum\sum\sum P(E_i \cap E_j \cap E_k)$$

$$+ \cdots - + (-1)^{n-1} P(E_1 \cap E_2 \cap \cdots \cap E_n)$$

Product Rule (Multiplication Theorem)

If A & B two indept events

$$P(A\cap B) = P(A)\cdot P(B) \qquad \left[\, P\left(\bigcap_{i=1}^{n} E_i\right) = \prod_{i=1}^{n} P(E_i) \right.$$

If A & B two dependent events → Conditional prob (Compound prob)

$$P(A\cap B) = P(A)\cdot P(B/A)$$

unknown

$$P(B\cap A) \leftarrow P(A\cap B) = P(B)\, P(A/B) \qquad \left[\, P(A/B) \neq P(B/A) \,\right]$$

known

If A, B & C are three dependent events

$$P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)$$

— x —

Prob -
A card is drawn from the pack of 52 cards, if it a diamond, find the prob of 'King' ?

Sol :- $P(K|D) = \frac{1}{13} = \left[\frac{P(D \cap K)}{P(K)}\right]$

A card is drawn from the pack of 52 card if it a King what is diamond

$\left[P(K/D) \neq P(D/K) \right]$

Sol :- $P(D/K) = \frac{P(D \cap K)}{P(K)} = \frac{1}{4}$

A family has two childrens, if there is a boy

find prob of Girl ?

$S : \{BB, BG, GB, GG\}$

1 2 3

Sol:- $P(B/G) = \frac{P(B \cap G)}{P(G)} = \frac{2}{3}$

$P(B/B) = \frac{P(B \cap B)}{P(B)} = \frac{1}{3}$

$P(B/1^{st}B) = \frac{P(B \cap 1^{st}B)}{P(1^{st}B)} = \frac{1}{2}$

$P(G/B) = \frac{P(G \cap B)}{P(B)}$

$= \frac{2}{3}$

3 coins are tossed, if 1st toss Head find the prob of consecutively 2 H's on 3-coins?

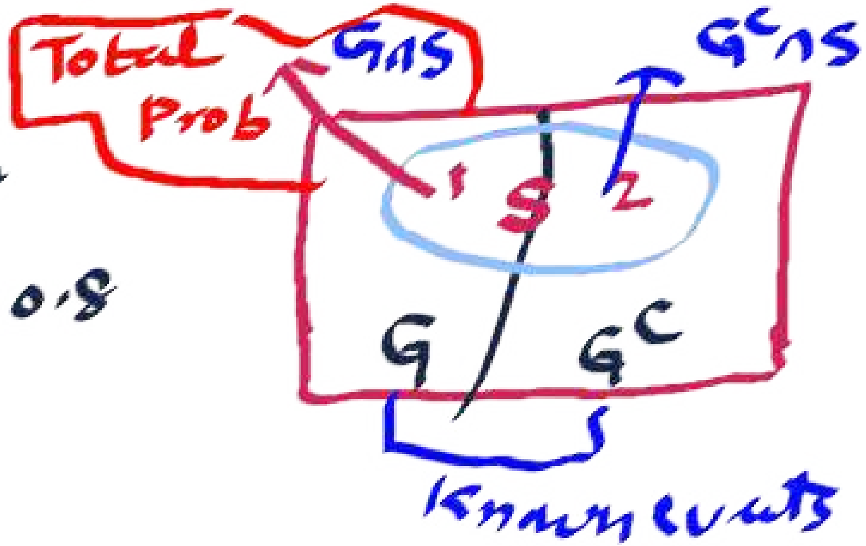
Sol: $P\left(\frac{\text{con } 2H\text{'s}}{1^{st} H}\right) = \dfrac{P(\text{con } 2H\text{'s} \cap 1^{st} H)}{P(1^{st} H)}$

$= \dfrac{2}{4} = \frac{1}{2}$

$\frac{3!}{3!} = 1$

HHH
HHT
HTH
THH ✗
TTH
HTT
THT
TTT

60% employees of the company are college Grads, of these 10% are sales dept, of the employees who didn't graduate from college are 80%, A person is selected at random find the prob that he is in Sales?



Sol:- $P(G) = 0.6, \quad P(G^c) = 0.4$

$$P(S/G) = 0.1, \quad P(\bar{S}/G^c) = 0.8$$

$$P(S) = P(G \cap S) + P(G^c \cap S)$$

$$= P(G)\,P(S/G) + P(G^c)\,P(S/G^c)$$

$$= 0.6 \times 0.1 + 0.8 \times 0.4$$

$$= 0.06 + 0.32$$

$$\boxed{P(S) = 0.38 \text{ or } 38\%}$$

Imp: **State & prove Baye's Theorem:-**

Statement:- If $E_1, E_2, \ldots E_n$ are M.E ($P(E_i) \neq 0$) such that 'A' is a arbitrary event ($P(A) > 0$) which is subset of "$\bigcup_{i=1}^{n} E_i$" then

Total prob $\leftarrow$ $P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)$

Reverse prob $\leftarrow$ $P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$

Proof :-

Since

$$A \subset \bigcup_{i=1}^{n} E_i$$

$$A = \left(A \cap \bigcup_{i=1}^{n} E_i\right)$$

$$A = \bigcup_{i=1}^{n} (A \cap E_i) \longrightarrow (1)$$

Take prob on both sides of eq ①

$$P(A) = P\left(\bigcup_{i=1}^{n} (A \cap E_i)\right)$$

$$= \sum_{i=1}^{n} P(A \cap E_i)$$

$$\boxed{P(A) = \sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

→ Total prob

[$A \cap E_i$ are M.E]

[$P(\cup E_i) = \sum P(E_i)$]

E_i's are distinct

$$P(E_i \cap A) = P(A)\,P(E_i/A)$$

$$\boxed{P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}}$$

$\Downarrow$

Reverse prob

$$\left[\,P(E_1/A) + P(E_2/A) + \cdots + P(E_n/A) = 1\,\right]$$

Prob:- 9/3/21

If $P(H) = 2/3$, $P(T) = 1/3$, if H occurs select number from $\{1, \dots 20\}$, T occurs select $\{1, \dots 10\}$, if the selected number is prime no, what is Prob that it is in H? 2-20 to 4-30

Sol:- $P(H) = 2/3$, $P(T) = 1/3$

$P(P/H) = 8/20 = 4/10$

$P(P/T) = 4/10$

$\{1, 2, \dots 20\}$

P: 2, 3, 5, 7, 11, 13, 17, 19

$\{1, 2, 3, \dots 10\}$

P: 2, 3, 5, 7

$$P(P) = P(H \cap P) + P(T \cap P)$$

$$= P(H) \cdot P(P/H) + P(T) \cdot P(P/T)$$

$$= \left(2/3 \times 4/10\right) + 1/3 \times 4/10$$

$$\left[P(P) = 12/30\right] \to \text{Total prob}$$

$$P(H/P) = \frac{P(H \cap P)}{P(P)} = \frac{P(H) \cdot P(P/H)}{P(P)} = \frac{\left(2/3 \times 4/10\right)}{12/30} = \frac{8}{12}$$

Reverse prob

$$P(T/P) = \frac{P(T \cap P)}{P(P)} = \frac{P(T) \cdot P(P/T)}{P(P)} = \frac{1/3 \times 4/10}{12/36}$$

$$= 4/12$$

$$\left[\frac{8}{12} + \frac{4}{12} = 1 \right]$$

(12) In answering a on a multiple choice Test
a student either knows the answer (or) guesses the
answer, let 'p' be the prob student knows the answer
and '1-p' be the prob student guesses the answer.
if student guesses will be correct with prob '1/5'.
What is the conditional prob that student knows
the answer question given that his answer is
correct ?

Sol:-

$P(K) = p.$ $P(G) = 1-p$

$P(C|K) = 1.$ $P(C/G) = 1/5$

$P(C) = P(K \cap C) + P(G \cap C)$

$= P(K) \cdot P(C|K) + P(G) \cdot P(C|G)$

$= p \times 1 + (1-p) \times 1/5$

$= \frac{4p+1}{5}$

$$P(K/C) = \frac{P(K \cap C)}{P(C)}$$

$$= \frac{P(K) \cdot P(C|K)}{P(C)}$$

$$= \frac{p \times 1}{\frac{4p+1}{5}}$$

$$P(K/C) = \frac{5p}{4p+1}$$

(3)

Color Marbles	Bags A	B	C
B	2	3	1
R	3	1	2
G	1	2	3
	6	6	6

A bag is drawn at random, two marbles are selected from it, they are found be (1 R, 1 G)

(find the prob, the selected marbles are happened from Bag 'C'?)

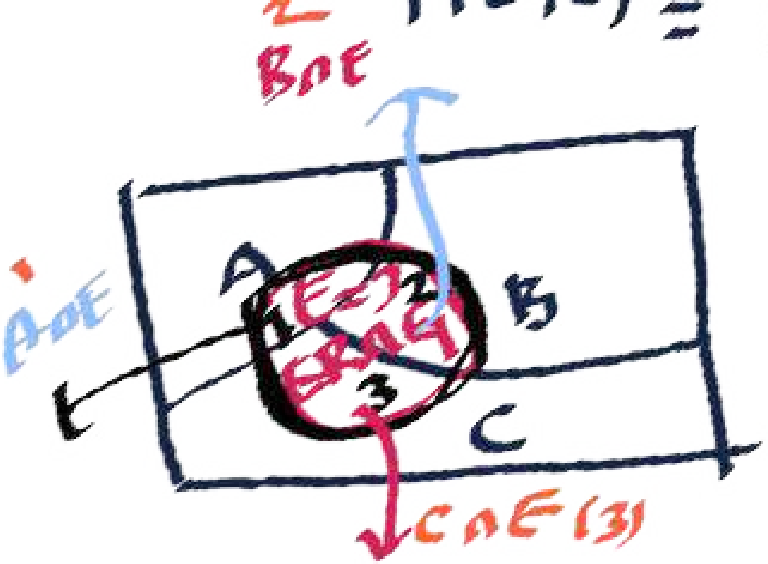
$P(A) = 1/3 = P(B) = P(C)$

E: Getting $1R \cap 1G$

$$P(E/A) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3\times 1}{15} = 3/15 \quad \left[2\left(\frac{{}^3C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^5C_1}\right)\right]$$

$$P(E/B) = \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{1\times 2}{15} = 2/15$$

$$P(E/C) = \frac{{}^2C_1 \times {}^3C_1}{{}^6C_2} = \frac{2\times 3}{15} = 6/15$$



$$P(E) = P(A\cap E) + P(B\cap E) + P(C\cap E)$$

$$= P(A)P(E/A) + P(B)P(E/B) + P(C)P(E/C)$$

$$= 1/3 \times 3/15 + 1/3 \times 2/15 + 1/3 \times 6/15$$

$$= \frac{3+2+6}{45} = 11/45$$

$$P(C|E) = \frac{P(C \cap E)}{P(E)} = \frac{P(C) \cdot P(E|C)}{P(E)} = \frac{\frac{1}{3} \times \frac{6}{\cancel{115}}}{\frac{11}{\cancel{345}}}$$

$$\boxed{P(C|E) = \frac{6}{11}}$$

— x —

Random variable | Expectation | Variance | co-Variance

Random variable (r.v)

$X \in \mathbb{R}(-\infty, +\infty)$

eg:- Two coins $S: \{HH, HT, TH, TT\}$

(1) (count heads) $\longrightarrow \{\underbrace{2 \quad 1 \quad 1 \quad 0}_{\text{r.v's}}\}$

(2) die: $S = \{1, \underbrace{2, 3, 4, 5, 6}_{\text{r.v}}\}$

$\underline{X} \sim (\text{r.v})$

$$\left\{\begin{array}{c} -3 \\ -1/6 \\ 0 \\ 4 \\ 6.8 \\ 2 \end{array}\right\}$$

$$\left[\underset{\substack{\Downarrow \\ \text{r v}}}{X} : \underset{\substack{\Downarrow \\ \text{Sample} \\ \text{Space}}}{S} \Rightarrow \underset{\text{real values}}{\mathbb{R}} \right]$$

"It means that Connecting the outcomes with real values of the Random experiment is known as random vaiable"

R.V $\rightarrow$ Univariate random vaible.

Types of r.v's :-

IR

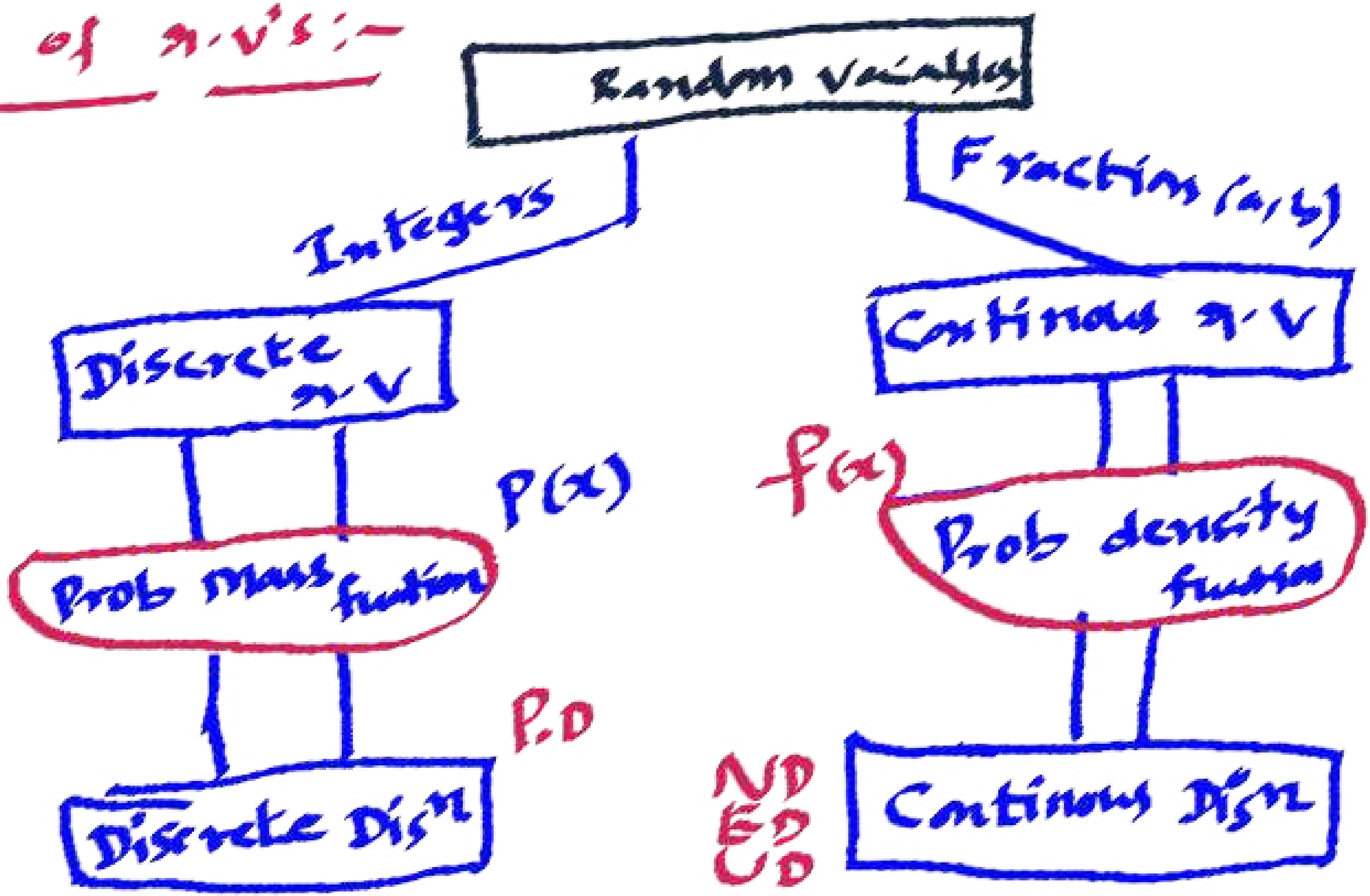
IR

Rch

Prob Distn

$\begin{Bmatrix} H & T \\ 1 & 0 \end{Bmatrix}$ r.v

$\frac{1}{2}$ $\frac{1}{2}$



P.m.f

$p(x)$

$P(x=0)$

$P(x=1)$

$(\times P(x=1.5) \times = 0)$

$P(x=2)$

$\vdots$

$P(x=n)$

$\sum_{1}^{2}$ $\quad$ $\int_{1}^{2}$

Discrete r.v

Continuous r.v

Pdf

$f(x)$

$f(x) \in (a, b)$

$P(x < 2.5) = \int_{-\infty}^{2.5}$

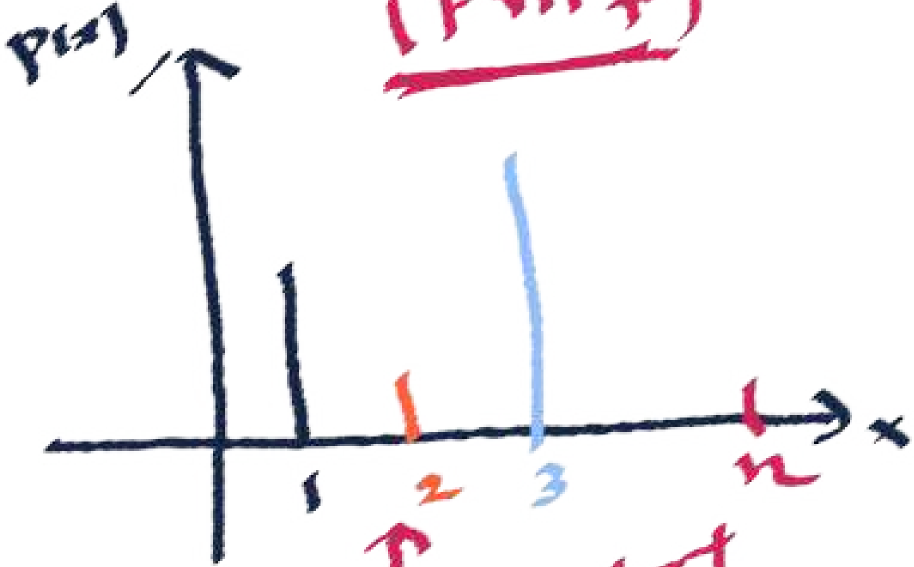
$P(x > 2.5) = \int_{2.5}^{\infty}$

$(\times P(x = 2.5) \times = 0)$

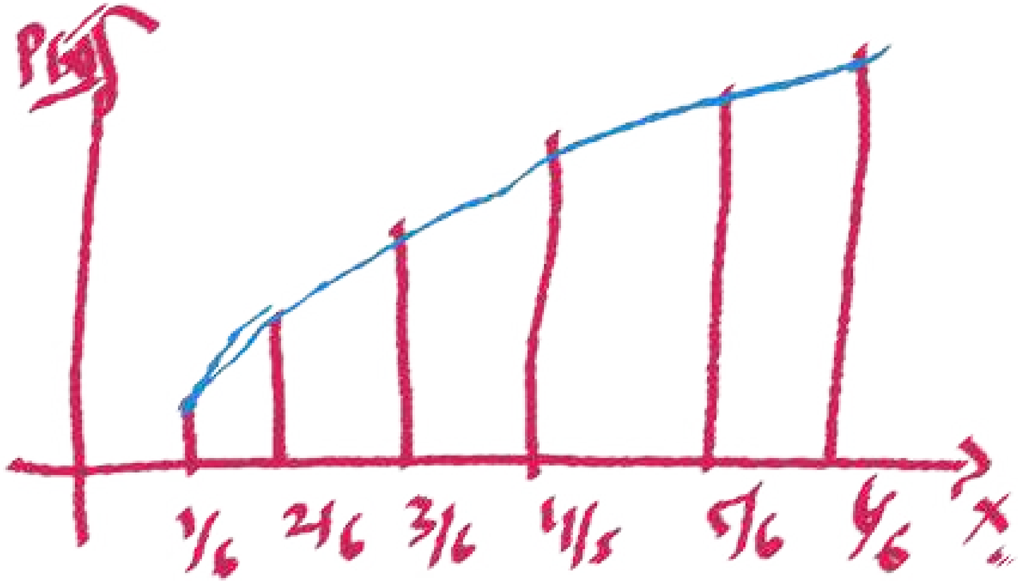
$= \int_{2.5}^{2.5} = 0$

Geometrically

(Pmf)

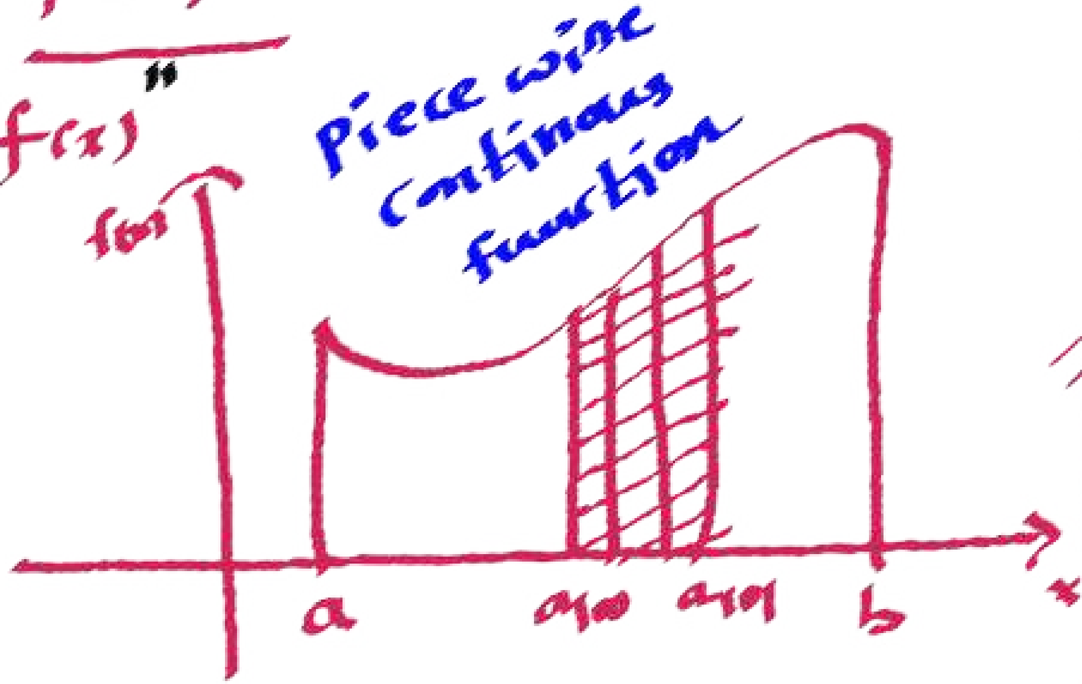


step function



Pdf :-

"f(x)"



Cdf :- (F(x))

Cumulative distribution function

$$\boxed{\begin{aligned} \frac{d}{dx} F(x) &= f(x) \\ F(x) &= \int_{-\infty}^{x} f(x)\, dx \end{aligned}}$$

Properties of pmf:-

(i) $\sum_{x=0}^{n} p(x) = 1$

(2) It is a step function

(3) for fractional pmf does not exist

Properties of pdf :-

$\left[P(x=c) = 0 \right]$
$\downarrow$
CRV

(1) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

(2) It is also known as piecewise continous fuction

(3) $P(a \le x \le b) = P(a \le x < b) = P(a < x \le b) = P(a < x < b) =$

$= \int_a^b f(x)\,dx$

(4) Relation $\left[\frac{d}{dx} F(x) = f(x) \right] \longrightarrow \left[F(x) = \int_{-\infty}^{x} f(x)\,dx \right]$

Statistical Averages :-

$$\left[\bar{X} = \frac{\Sigma f x}{N} \right]$$

✓(1) Mathematical Expectation
✓(2) Variance and co-variance
✓(3) Moments

Mathematical Expectation :- [Mean / Average]

$$\boxed{E(x) = \sum_{x=0}^{n} x \cdot p(x) \quad x\text{-DRV} \\ = \int_{-\infty}^{\infty} x \cdot f(x)\, dx \quad x\text{-CRV}}$$

$$\left[\because \sum_{0}^{n} p(x) = 1 \qquad \int_{-\infty}^{\infty} f(x)\, dx = 1 \right]$$

Variance:- (var)

$$\boxed{\begin{aligned} Var(x) &= E(x - E(x))^2 \\ &= E(x^2) - (E(x))^2 \end{aligned}}$$

$$Var(x) = \sum_{0}^{n} x^2 \cdot p(x) - \left(\sum_{0}^{n} x \cdot p(x)\right)^2$$

$\boxed{(x - DRV)}$

$$= \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx - \left(\int_{-\infty}^{\infty} x \cdot f(x)\, dx\right)^2$$

$\boxed{(x - CRV)}$

Co-Variance [cov]

$$\boxed{\begin{aligned} Cov(x,y) &= E\big((x-E(x))(y-E(y))\big) \\ &= E(xy) - E(x)E(y) \end{aligned}}$$

Note:- If the random variables are independent $[Cov(x,y)=0]$ but converse of the statement is not true.

Moments (Movement]

I: If the r.v is moving about mean ($\bar{x}$ or $E(x)$) then these moments are known as "central moments"

In general, are denoted by $[\mu_1, \mu_2, \mu_3 \text{ \& } \mu_4]$

$\mu_1 = 0$ $\qquad$ μ_2 = Variance

Mathematically, central moments are defined as

$$\left[\mu_r = \frac{1}{N}\sum f_i(x_i-\bar{x})^r \quad \text{or} \quad \mu_r = \underset{r}{E}(x-E(x))^r\right]$$

II If r.v is moving about any value (A) then these moments are known as Non-central moments (or) Raw moments

In general, are denoted by $[\mu_1', \mu_2', \mu_3', \mu_4']$

μ_1' = Mean

Mathematically, defined as

$$\mu_r' = \frac{1}{N}\Sigma f_i(x_i - A)^r \text{ (or) } \mu_r' = E(x-A)^r$$

Relation b/w non central to central moments

$$\text{Variance} = \mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$\begin{bmatrix} \mu_3 \rightarrow \text{sk} \\ \mu_4 \rightarrow \text{ku} \end{bmatrix}$$

Skewness:-

$$\boxed{\begin{aligned} \beta_1 &= \frac{\mu_3^2}{\mu_2^3} \\ \gamma_1 &= \sqrt{\beta_1} \end{aligned}}$$

Note:-

(1) if $\mu_3 = 0 \Rightarrow \beta_1 = 0$ then the curve is symmetry.

(2) if $\mu_3 < 0 \Rightarrow \beta_1 = +ve$ then the curve is negatively skew

(3) if $\mu_3 > 0 \Rightarrow \beta_1 = +ve$ the curve is positive skewed

Kurtosis:-

$$\begin{cases} \beta_2 = \dfrac{\mu_4}{\mu_2^2} \\ \gamma_2 = \beta_2 - 3 \end{cases}$$

$\beta_2 = 3,\ \gamma_2 = 0 \Rightarrow$ Mesto Kurtic

$\beta_2 < 3,\ \gamma_2 < 0 \Rightarrow$ platy Kurtic

$\beta_2 > 3,\ \gamma_2 > 0 \Rightarrow$ Lepto Kurtic

Properties of Expectation:

(1) If 'x' r.v 'a' const $E(ax) = a \cdot E(x)$

Proof:- We know that $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

$$E(ax) = \int_{-\infty}^{\infty} a x f(x)\, dx$$

$$= a \left(\int_{-\infty}^{\infty} x f(x)\, dx \right)$$

$$= a \cdot E(x)$$

(2) Addition theorem of expectation :-

If x & y are r.v's $E(x+y) = E(x) + E(y)$

Proof:- $E(x) = \int x\, f(x)\, dx$; $E(y) = \int y\, f(y)\, dy$

$$E(x+y) = \int_x \int_y (x+y)\, f(x,y)\, dy\, dx$$

$$= \int_x \int_y x\, f(x,y)\, dy\, dx + \iint y\, f(x,y)\, dx\, dy$$

$$= \int_x x \underbrace{\left(\int_y f(x,y)\, dy\right)}_{f(x)} dx + \int_y y \underbrace{\left(\int_x f(x,y)\, dx\right)}_{f(y)} dy$$

$$= \left[\int_x x \cdot f(x)\,dx\right] + \left[\int_y y \cdot f(y)\,dy\right]$$

$$\boxed{E(x+y) = E(x) + E(y)}$$

Ily

$$E(x-y) = E(x) - E(y)$$

If a, b const's and, x, y are r.v's

$$E(ax \pm by) = aE(x) \pm bE(y)$$

(3) Multiplication Theorem:

If x and y are independent r.v's. iff

$$E(x\cdot y) = E(x)\cdot E(y)$$

$\left[\begin{array}{l} x, y \text{ I R.V's} \\ f(x,y) = f(x)\, f(y) \end{array}\right]$

Proof :-

$$E(x\cdot y) = \int_x \int_y (x\cdot y)\, f(x,y)\, dy\, dx$$

$$= \int_x \int_y (x\cdot y)\, f(x)\cdot f(y)\, dy\, dx$$

$$= \int_x x\cdot f(x)\, dx \cdot \int y\, f(y)\, dy$$

$$= E(x)\cdot E(y).$$

If x and y are dependent r.v's then

$$E(x \cdot y) = E(x) \cdot E(y/x)$$ conditional expectation

$$= E(y) \cdot E(x/y)$$

— x —

If $Y = ax+b$; a, b constants then

$E[Y] = a\,E[X] + b$ (or) $(\mu_y = a\mu_x + b)$

Proof :- Given $Y = ax+b$

$$E[Y] = \int_{-\infty}^{\infty} y\, f(y)\, dy$$

$$= \int_{x=-\infty}^{\infty} (ax+b)\, f(x)\, dx$$

$$= \int_{-\infty}^{\infty} ax\, f(x)\, dx + \int_{-\infty}^{\infty} b\, f(x)\, dx$$

$$= a\left(\int_{-\infty}^{\infty} x\, f(x)\,dx\right) + b\left(\underbrace{\int_{-\infty}^{\infty} f(x)\,dx}_{1}\right)$$

$$E(y) = a\,E(x) + b(1)$$

$$E(ax+b) = a\,E(x) + b$$

$$\boxed{\begin{aligned} &E[\text{const}] = \text{const} \\ &E[E[E(x)]] = \underbrace{E(x)}_{\text{const}} \end{aligned}}$$

Properties of Variance :-

$$\left[\begin{aligned} V(x) &= E(x - E(x))^2 \\ V(x) &= E(x^2) - (E(x))^2 \end{aligned}\right]$$

(1) If 'x' is a r.v and 'a' const

$$V(ax) = a^2 \cdot V(x)$$

<u>Prof</u> :- know that $V(x) = E(x^2) - (E(x))^2$

$$\begin{aligned} V(ax) &= E(a^2x^2) - (E(ax))^2 \\ &= a^2E(x^2) - (a \cdot E(x))^2 \\ &= a^2 E(x^2) - a^2 \cdot (E(x))^2 \\ &= a^2(E(x^2) - (E(x))^2) = a^2 \cdot V(x) \end{aligned}$$

$\therefore V(ax) = a^2 \cdot V(x)$

$$\left[V(-y) = (-1)^2 V(y) = V(y)\right]$$

[Variance of variable data set is always positive]

— x —

(2) If x and y are independent r.v's then

$$V(x+y) = V(x) + V(y)$$

Proof:- put $Z = x + y$ ———— ①

$$E(Z) = E(x) + E(y) \text{ ———— ②}$$

① − ②

$$E\left((z - E(z))\right)^2 = E\left[(x - E(x)) + (y - E(y))\right]^2$$

$$E(z - E(z))^2 = E\left[(x - E(x))^2 + (y - E(y))^2 + 2(x - E(x))(y - E(y))\right]$$

$$= E((x - E(x))^2) + E((y - E(y))^2) + 2\,\boxed{E(x - E(x))(y - E(y))}$$

$\to 0$ ($\because$ x & y IRV's)

$$\boxed{\begin{aligned} V(z) &= V(x) + V(y) \\ V(x+y) &= V(x) + V(y) \end{aligned}}$$

Ily

$$\left[\begin{array}{l} V(x-y) = V(x) + V(y) \\ V(ax-by) = a^2V(x) + b^2V(y) \\ V(x/a - y/b) = \frac{1}{a^2}V(x) + \frac{1}{b^2}V(y) \end{array}\right]$$

Important Results

—x—

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(3) If x and y are dept r.v's

$$V(x+y) = V(x) + V(y) + 2\,cov(x, y)$$

Proof:- put $z = x + y$ ——— ①

$E(z) = E(x) + E(y) \longrightarrow$ ②

① − ②

$$E(z - E(z))^2 = E[(x - E(x)) + (y - E(y))]^2$$

$$= E[(x - E(x))^2 + (y - E(y))^2 + 2(x - E(x))(y - E(y))]$$

$$= E(x - E(x))^2 + E(y - E(y))^2 + 2E(x - E(x))(y - E(y))$$

$$V(z) = E(x - E(x))^2 + E(y - E(y))^2 + 2E(x - E(x))(y - E(y))$$

$$\boxed{V(x+y) = V(x) + V(y) + 2\,cov(x, y)}$$

Hly

If X and Y are dept r.v's

$$V(X-Y) = V(X) + V(Y) - 2\,\mathrm{cov}(X,Y)$$

—x—

** (4) If $Y = aX + b$: a, b const's then $V(Y) = a^2 \cdot V(X)$

Proof:

$$Y = aX + b \longrightarrow (1)$$

$$E(Y) = aE(X) + b \longrightarrow (2)$$

$$(1) - (2)$$

$$Y - E(Y) = a(X - E(X))$$

Square on both sides, then take expectation

$$E(Y - E(Y))^2 = E(a(x - E(x))^2$$

$$V(y) = a^2 \, E(x - E(x))^2$$

$$\boxed{V(ax+b) = a^2 \cdot V(x)}$$

$$\boxed{V[\text{const}] = 0}$$

*

$E[\text{const}] = \text{const}$

$V[\text{const}] = 0$

$V[\text{variable set}] = +ve$

Problems:-

(1) If 'x' is d.r.v and its pmf is

x	1	2	3	4	5	6	7
$p(x)$	K	$3K$	$2K$	K	$2K$	K	$3K$

find
(i) K (ii) $E(x)$ (iii) $V(x)$
(iv) $P(X>2.5 / X \leq 6.5)$

Sol:- Since $\sum_{x=0}^{n} p(x) = 1 \Rightarrow \sum_{x=1}^{7} p(x) = 1$

$13K = 1$

$\boxed{K = 1/13}$ → (Positive real value)

(ii) $$E(x) = \sum_{x=1}^{7} x \cdot P(x)$$

$$= 1 \cdot P(1) + 2 \cdot P(2) + 3 \cdot P(3) + 4 \cdot P(4) + 5 \cdot P(5) + 6 \cdot P(6) + 7 \cdot P(7)$$

$$= 1 \cdot k + 2 \cdot 3k + 3 \cdot 2k + 4 \cdot k + 5 \cdot 2k + 6 \cdot k + 7 \cdot 3k$$

$$= k[1 + 6 + 6 + 4 + 10 + 6 + 21) = 54k$$

$$= \frac{54}{13} = 4.15$$

$$\left[\therefore E(x) = 4.15\right]$$

(iii) $V(x) = E(x^2) - (E(x))^2 \Rightarrow \mu_2 = \mu_2' - (\mu_1')^2$

Now $E(x^2) = \sum_{x=1}^{7} x^2 \cdot p(x)$

$= 1^2 \cdot k + 2^2 \cdot 3k + 3^2 \cdot 2k + 4^2 \cdot k + 5^2 \cdot 2k + 6^2 \cdot k + 7^2 \cdot 3k$

$= k(1 + 12 + 18 + 16 + 50 + 36 + 147)$

$= \underline{280} k = 280 \cdot 1/13 = \frac{280}{13} = 21.53$

$V(x) = 21.53 - (4.15)^2$

$\boxed{V(x) = 4.3}$ //

(iv) $P(x > 2.5 / x \leq 6.5) =$

$$\left[P(A/B) = \frac{P(A \cap B)}{P(B)} \right]$$

$$= \frac{P(x > 2.5 \cap x \leq 6.5)}{P(x \leq 6.5)}$$

$$= \frac{P(2.5 < x \leq 6.5)}{P(x \leq 6.5)}$$

$$= \frac{P(3 \leq x \leq 6)}{P(x \leq 6)} = \frac{6k}{10k} = 6/10 = 3/5 = 0.6$$

(2) A die is rolled find exp & Var for face values on die ?

Sol:-

X r.v	1	2	3	4	5	6
p(x)	1/6	1/6	1/6	1/6	1/6	1/6

$$\text{Mean} = E(x) = \sum_{x=1}^{6} x \cdot p(x)$$

$$= 1 \cdot 1/6 + 2 \cdot 1/6 + 3 \cdot 1/6 + 4 \cdot 1/6 + 5 \cdot 1/6 + 6 \cdot 1/6$$

$$= 1/6\left[1 + 2 + 3 + 4 + 5 + 6\right] = \frac{21}{6} = 7/2 = 3.5$$

$$V(x) = \mu_2 = E(x^2) - (E(x))^2$$

Now $E(x^2) = \sum_{x=1}^{6} x^2 \cdot p(x)$

$$= 1^2 \cdot p(1) + 2^2 \cdot p(2) + 3^2 \cdot p(3) + 4^2 \cdot p(4) + 5^2 \cdot p(5) + 6^2 \cdot p(6)$$

$$= \tfrac{1}{6}[1 + 4 + 9 + 16 + 25 + 36]$$

$$E(x^2) = \frac{91}{6}$$

$$\therefore Var(x) = E(x^2) - (E(x))^2$$

$$= \frac{91}{6} - (7/2)^2 = \frac{35}{12} = 2.91$$

Note:- The Expectation and Variance for sum of the number on the dice are

$$E(x) = \frac{7n}{2}$$

$$V(x) = \frac{35}{12}n$$

Here 'n' is no of dice

5 dice are rolled find exp & Var for the sum of the no's on the die

Sol:- $n = 5$

$$E(x) = \frac{7}{2} \times 5 = 35/2$$

$$V(x) = 35/12 \times 5 = \frac{175}{12}$$

④ If x is discrete r.v and p.m.f is

x	1	2	3	4	5	6	7	8	9
$p(x)$	K	$4K$	$3K$	K	$2K$	$3K$	K	K	K

(i) find K (ii) $P(X \geq 6.5)$ (iii) $P(x \leq 8.5)$ (iv) $P(X>5/x \leq 8)$

$P(x \geq 6.5) =$

Sol:- (i) $\sum_{1}^{9} p(x) = 1 \Rightarrow 17K = 1 \Rightarrow K = 1/17$

(ii) $P(x \geq 6.5) = P(x=7) + P(x=8) + P(x=9)$

$= 3K = 3/17$

(iii) $P(x \leq 8.5) = 1 - P(x > 8.5)$

$$= 1 - P(x = 9)$$

$$= 1 - \tfrac{1}{17} = \tfrac{16}{17} //$$

(iv) $P(x > 5 / x \leq 8) =$

$$= \frac{P(x > 5 \cap x \leq 8)}{P(x \leq 8)} = \frac{P(x=6) + P(x=7) + P(x=8)}{P(x \leq 8)}$$

$$= \frac{5k}{16k} = \tfrac{5}{16} //$$

(5) If 'x' CRV and its pdf is

$$f(x) = K(1+x) \quad 0<x<1$$

(i) K (ii) $E(x)$ (iii) $Var(x)$ (iv) $F(x)$

Sol: - Since $\int_0^1 f(x)\,dx = 1$

$$\int_0^1 K(1+x)\,dx = 1$$

$$K\int_0^1 (1+x)\,dx = 1$$

$$K\left[x + \frac{x^2}{2}\right]_0^1 = 1$$

$$K\left[1 + \frac{1}{2} - 0\right] = 1$$

$$K\left[\frac{3}{2}\right] = 1$$

$$\boxed{K = \frac{2}{3}}$$

(ii) $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$= \int_0^1 x\ f(x)\, dx$

$= \frac{2}{3}\int_0^1 x \cdot (1+x)\, dx$

$= \frac{2}{3}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_0^1$

$= \frac{2}{3}\left[\frac{1}{2} + \frac{1}{3} - 0\right]$

$= \frac{2}{3}\left(\frac{5}{6}\right) = \frac{10}{18}$ //

$\left[\therefore E(x) = \frac{10}{18}\right]$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_0^1 x^2 f(x)\,dx$$

$$= 2/3 \int_0^1 x^2 (1+x)\,dx$$

$$= 2/3 \int_0^1 (x^2 + x^3)\,dx$$

$$= 2/3 \left[x^3/3 + x^4/4 \right]_0^1$$

$$= 2/3 \left[1/3 + 1/4 - 0 \right]$$

$$= 2/3 \left[\frac{7}{12} \right] = \frac{14}{36} =$$

$$V(x) = \frac{14}{36} - \left(\frac{10}{18} \right)^2$$

$$= \frac{13}{162} //$$

(iv) cdF $F(x)$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

$$\boxed{\begin{array}{l} F(\min) = 0 \\ F(\max) = 1 \end{array}}$$

$$= \int_{0}^{x} f(x)\,dx$$

$$= \frac{2}{3}\int_{0}^{x} (1+x)\,dx$$

$$= \frac{2}{3}\left[x + x^2/2\right]_{0}^{x}$$

$$\therefore F(x) = \frac{2}{3}\left[\frac{2x+x^2}{2}\right] = \frac{1}{3}\left[2x+x^2\right]$$

⑥ If x is c.r.v and pdf is

$$f(x) = k \cdot x^2 \cdot e^{-x^2}$$
$x > 0$

find (i) k (ii) $E(x)$ (iii) $V(x)$

Sol.- $\int_0^\infty f(x)\,dx = 1 \Rightarrow \int_0^\infty k \cdot x^2 \cdot e^{-x^2}\,dx = 1$

Put $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

$$dx = \frac{dt}{2\sqrt{t}}$$

$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx$ $n > 0$

$\Gamma n+1 = n!$

$\Gamma 1/2 = \sqrt{\pi}$ $\Gamma 0 =$ undefined

$\Gamma n+1 = n\,\Gamma n$
$= n(n-1)\,\Gamma n-1$

$k\int_0^\infty t\, e^{-t} \frac{dt}{2\sqrt{t}} = 1$

$\frac{k}{2}\left[\int_0^\infty t^{1/2} e^{-t}\,dt\right] = 1$ $\quad 3/2 - 1$

$\frac{k}{2}\,\Gamma 3/2 = 1$

$\frac{k}{2}\left[\frac{1}{2}\,\Gamma 1/2\right] = 1$

$\frac{k}{4}\sqrt{\pi} = 1$

$\left[k = \frac{4}{\sqrt{\pi}}\right]$ +ve

(ii) $E(x) = \int_0^\infty x \cdot f(x)\, dx$

$= \frac{4}{\sqrt{\pi}} \int_0^\infty x \cdot x^2\, e^{-x^2}\, dx$

$= \frac{\cancel{4}^{2}}{\sqrt{\pi}} \int_0^\infty t \cdot e^{-t}\, \frac{dt}{\cancel{2}}$

$= \frac{2}{\sqrt{\pi}} \int_0^\infty t^{\overset{2-1}{1}} e^{-t}\, dt$

$= \frac{2}{\sqrt{\pi}} \Gamma 2 = \frac{2}{\sqrt{\pi}}$

$\therefore \boxed{E(x) = \frac{2}{\sqrt{\pi}}}$

(iii) $V(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \int_0^\infty x^2 \cdot f(x)\,dx$$

$$= \frac{4}{\sqrt{\pi}} \int_0^\infty x^2 \cdot x^2 \cdot e^{-x^2}\,dx$$

$$= \frac{4}{\sqrt{\pi}} \int_0^\infty \cancel{t} \cdot t \cdot e^{-t} \frac{dt}{2\sqrt{t}}$$

$$= \frac{\cancel{4}^{2}}{\sqrt{\pi}} \int_0^\infty t^{3/2} e^{-t}\,dt \qquad 5/2 - 1$$

$$= \frac{2}{\sqrt{\pi}} \Gamma 5/2$$

$$= \frac{\cancel{2}}{\cancel{\sqrt{\pi}}} 3/\cancel{2} \times \frac{1}{2} \cancel{\sqrt{\pi}}$$

$$E(x^2) = 3/2$$

$$\therefore V(x) = 3/2 - \left(\frac{2}{\sqrt{\pi}}\right)^2$$

$$= 3/2 - \frac{4}{3.14}$$

$$\boxed{V(x) = 0.22}$$

(7) If $f(x) = kx^2$

$0 < x < 2$

(i) k (ii) $P(x \geq 0.5)$ (iii) $P(0.5 \leq x \leq 1.5)$ (iv) $P(x \leq 1.2)$

Sol:- (i) $\int_0^2 f(x)\,dx = 1 \Rightarrow k\int_0^2 x^2\,dx = 1 \Rightarrow k \left.\frac{x^3}{3}\right]_0^2 = 1$

$\Rightarrow k\left[\frac{8}{3}\right] = 1$

$\Rightarrow \boxed{k = 3/8}$

(ii) $P(x \overset{\text{min}}{\geq} 0.5) = \int_{0.5}^{2} f(x)\,dx$

$= \frac{3}{8}\int_{0.5}^{2} x^2\,dx$

$$= \cancel{3}/8 \left[x^3/\cancel{3} \right]_{0.5}^{2}$$

$$P(x \geq 0.5) = \frac{1}{8}\left[x^3 - (0.5)^3 \right]$$

$$= \frac{7.8}{8} = 0.984$$

(ii) $P(0.5 \leq x \leq 1.5) =$

$$= \int_{0.5}^{1.5} f(x)\,dx$$

$$= \frac{3}{8}\int_{0.5}^{1.5} x^2\,dx$$

$$= \cancel{3}/8 \; x^3/\cancel{3} \Big]_{0.5}^{1.5}$$

$$= 1/8 \left[(1.5)^3 - (0.5)^3 \right]$$

$$= 0.4$$

(iv) $P(x \overset{\nearrow \text{ma}}{\leq} 1.2) = \int_0^{1.2} f(x)\,dx$

$= \frac{3}{8}\int_0^{1.2} x^2\,dx$

$= \frac{3}{8}\left[\frac{x^3}{3}\right]_0^{1.2}$

$= \frac{1}{8}\left[(1.2)^3 - 0\right]$

$= 0.216$ //

— x —

Poisson Distribution:-

Def:- If x is a discrete r.v defined in the interval $[0, \infty)$ with the parameter $(\lambda > 0)$, the probability mass function is defined as

$$P(x;\lambda>0) = P(x) = \begin{cases} \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!} & \lambda > 0 \\ & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Conditions:-

(i) $n \to \infty$

(ii) either probability success is very small (or) failures are very large. ($p \to 0$ or $q \to 1$)

(iii) $np = \lambda \Rightarrow p = \lambda/n$

Theorem :-

Prove that poisson distribution is limiting form of Binomial distribution ?

$(q + p = 1$
$q = 1 - p)$

Proof :- We know that Binomial pmf is

$$B(x; n, p) = ({}^{n}C_{x})\, p^{x} q^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} p^{x} q^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} p^{x} (1-p)^{n-x}$$

$$= \frac{n(n-1)(n-2)\cdots\cancel{(n-x)!}}{\cancel{(n-x)!}\; x!} \left(\lambda/n\right)^x \left(1-\lambda/n\right)^{n-x}$$

$$= \frac{n(n-1)(n-2)\cdots(n-(x+1))}{x!\; n^x} \lambda^x \frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}$$

divided nr with n

$$= \frac{1(1-1/n)(1-2/n)\cdots(1-\frac{x+1}{n}).\cancel{n^x}}{x!\quad \cancel{n^x}} \cdot \lambda^x \frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}$$

$$\underset{n\to\infty}{Lt}\, B(x;n,p) = \underset{n\to\infty}{Lt}\left(\frac{1(1-1/n)(1-2/n)\cdots\left(1-\frac{(x+1)}{n}\right)}{x!}\,\lambda^x\,\frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}\right)$$

$$= \frac{\lambda^x}{x!}\,\underbrace{(1-0)(1-0)\cdots(1-0)}_{1}\;\underset{n\to\infty}{Lt}\left[\frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}\right]$$

$$\boxed{\begin{aligned}&\underset{n\to\infty}{Lt}(1-\lambda/n)^n = e^{-\lambda}\\ &\underset{n\to\infty}{Lt}(1-\lambda/n)^x = 1\end{aligned}}$$

$$P(x) = \frac{\lambda^x}{x!}\,\frac{e^{-\lambda}}{1}$$

$$\therefore \boxed{P(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}}$$

Moment Generating fuction (MGF)

$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} \cdots$ 17/3/21 1.10 to 3.30

$$\begin{aligned} M_x(t) &= E[e^{tx}] \\ &= \sum e^{tx} p(x) \\ &= \int e^{tx} f(x)\, dx \end{aligned}$$

$$\begin{aligned} M_x(t) &= E[e^{tx}] \\ &= \sum_{x=0}^{\infty} e^{tx} \cdot p(x) \\ &= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\lambda} \cdot \lambda^x}{x!} \\ &= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!} \end{aligned}$$

$$\begin{aligned} &= e^{-\lambda}\left[1 + \frac{(\lambda e^t)^1}{1!} + \frac{(\lambda e^t)^2}{2!} + \frac{(\lambda e^t)^3}{3!} + \cdots\right] \\ &= e^{-\lambda}\left[e^{\lambda e^t}\right] \\ &= e^{\lambda e^t - \lambda} = e^{\lambda(e^t - 1)} \end{aligned}$$

$$\therefore \boxed{M_x(t) = e^{\lambda(e^t - 1)}}$$

Derive Expectation (μ_1') and Variance (μ_2):

Relation

$$\boxed{\mu_r' = E(x^r) = \frac{d^r}{dt^r} M_x(t)\Big]_{t=0}}$$

r^{th} non central moment

Put $r = 1$

$$E(x) = \mu_1' = \frac{d}{dt} M_x(t)\Big]_{t=0}$$

$$= \frac{d}{dt}\left(e^{\lambda(e^t-1)}\right]$$

$$= \frac{d}{dt}\left[e^{\lambda e^t}\, e^{-\lambda}\right]$$

$$= e^{-\lambda}\frac{d}{dt}\left(e^{\lambda e^t}\right]$$

$$= e^{-\lambda}\, e^{\lambda e^t}\cdot \lambda e^t\Big]_{t=0}$$

$$= \cancel{e^{-\lambda}}\, \cancel{e^{\lambda}}\cdot \lambda \cdot 1$$

$$\boxed{E(x) = \mu_1' = \lambda} = \text{Parameter}$$

$\checkmark Var(x) = \mu_2 = \mu_2' - (\mu_1')^2$

Now $\mu_2' = E(x^2) = \frac{d^2}{dt^2} M_x(t) \Big]_{t=0}$

$= \frac{d}{dt}\left(\frac{d}{dt} M_x(t)\right)$

$= \frac{d}{dt}\left[e^{-\lambda} \cdot \lambda e^t \, e^{\lambda e^t} \right]$

$= \lambda e^{-\lambda} \frac{d}{dt}\left[e^t \, e^{\lambda e^t} \right]$

$= \lambda e^{-\lambda}\Big[e^t \, e^{\lambda e^t} \lambda e^t + e^{\lambda e^t} e^t \Big]_{t=0}$

$= \lambda e^{-\lambda}\Big[1 \cdot e^{\lambda} \cdot \lambda \cdot 1 + 1 \cdot e^{\lambda} \Big]$

$= \lambda^2 + \lambda$

$\mu_2' = E(x^2) = \lambda^2 + \lambda$

$Var(x) = (\cancel{\lambda^2} + \lambda) - \cancel{\lambda^2}$

$= \lambda$

[$\therefore$ In poisson dis^n

$$E(x) = V(x) = \lambda = \text{parameter}$$]

Cumulate Generating function (CGF)

[we know that

$$M_x(t) = e^{\lambda(e^t-1)}$$]

$$K_x(t) = \log_e M_x(t)$$

$$= \log_e\left(e^{\lambda(e^t-1)}\right)$$

$$= \lambda(e^t-1) \cdot \cancel{\log_e e}^{\,1}$$

$$= \lambda(e^t-1)$$

$$K_X(t) = \lambda(e^t - 1)$$

$$= \lambda\left(\cancel{1} + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots \cancel{-1}\right)$$

$$\boxed{K_X(t) = \lambda\left(\frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right)}$$

$$E(X) = \mu_1' = k_1 = \text{coeff } \frac{t}{1!}$$

$$\boxed{E(X) = \mu_1' = \lambda}$$

$$V(X) = \mu_2 = k_2 = \text{coeff } \frac{t^2}{2!}$$

$$\boxed{\mu_2 = \lambda}$$

$$\mu_3 = k_3 = \text{coeff } \frac{t^3}{3!}$$

$$\boxed{\mu_3 = \lambda}$$

$$k_4 = \text{coefficient } \frac{t^4}{4!}$$

$$k_4 = \lambda$$

$$\boxed{\begin{aligned}\mu_4 &= k_4 + 3k_2^2 \\ &= \lambda + 3\lambda^2\end{aligned}}$$

Coefficient of Skewness:-

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \; ; \quad \gamma_1 = \sqrt{\beta_1}$$

$$\beta_1 = \frac{\lambda^2}{\lambda^3} = \frac{1}{\lambda}$$

$$\gamma_1 = \sqrt{\beta_1} = \sqrt{1/\lambda} = \frac{1}{\sqrt{\lambda}}$$

Poisson probability curve is **Positively skewed** ($\mu_3 > 0$)

Coefficient of Kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \; ; \quad \gamma_2 = \beta_2 - 3$$

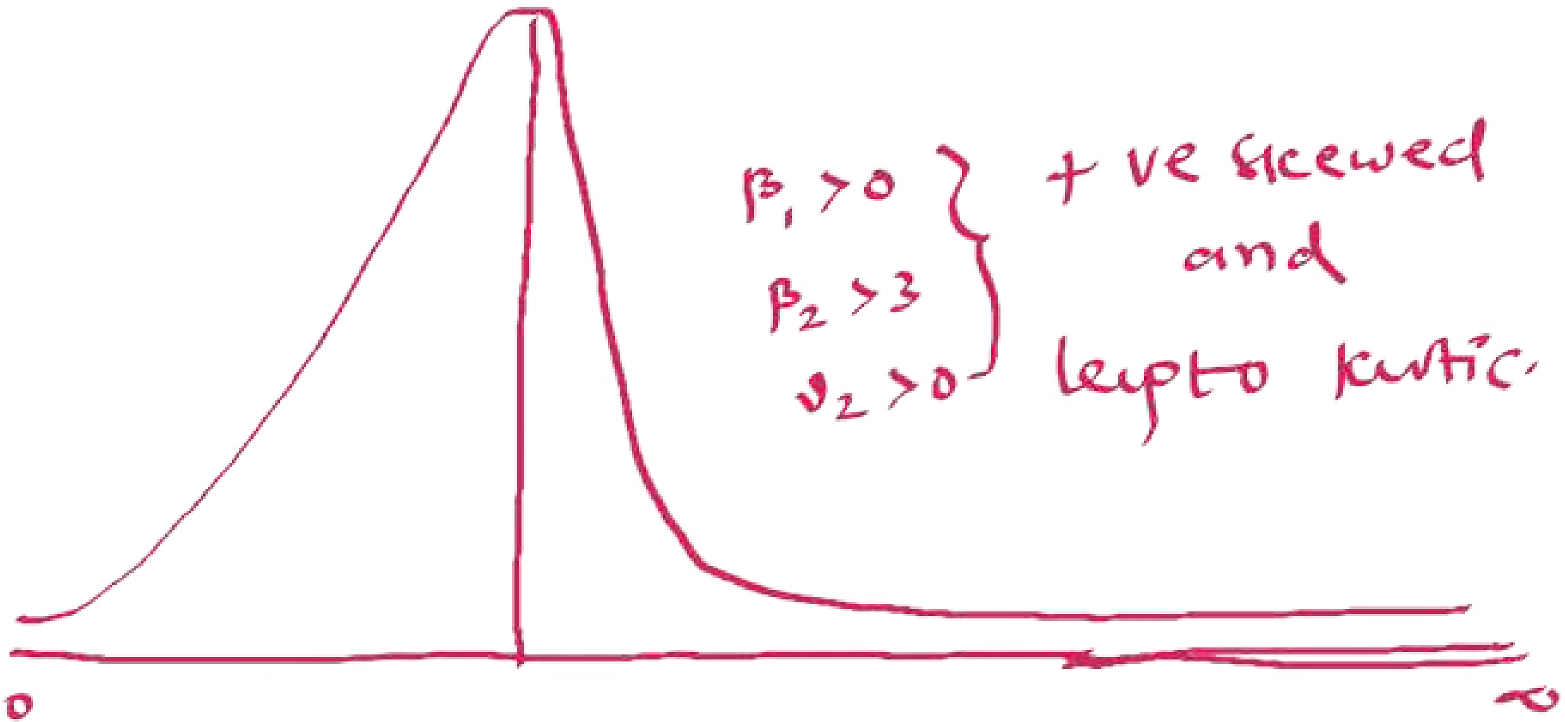
$$\beta_2 = \frac{3\lambda^2 + \lambda}{\lambda^2} = 3 + \frac{1}{\lambda} > 3$$

$$\gamma_2 = \beta_2 - 3 = \cancel{3} + \frac{1}{\lambda} - \cancel{3}$$

$$= 1/\lambda > 0$$

$\beta_2 > 3, \; \gamma_2 > 0$

Lepto Kurtic



Poisson probability curve

Problems:-

(1) The prob of defect item is 3×10^{-5}, there are 30,000 items find the probability that

(i) P(no defective) (ii) P(exactly 2 defectives)

(iii) P(at most 2 defectives) (iv) P(at least 2 defectives)

Sol:- Given that $n = 30,000$, $p = 3\times10^{-5}$

$$\lambda = np$$
$$= 3\times10^{4}\times3\times10^{-5}$$
$$= 9\times10^{-1}$$
$$\boxed{\lambda = 0.9}$$

We know that Prob mass function of poisson dis^n

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \qquad [\lambda = 0.9]$$

(i) $P(\text{no defective}) = P(X=0) = \dfrac{e^{-0.9}\,\cancelto{1}{(0.9)^0}}{\cancelto{1}{0!}} = e^{-0.9}$

$= 0.4065$

(ii) $P(\text{exactly 2 defectives}) = P(X=2) = \dfrac{e^{-0.9}(0.9)^2}{2!} = 0.1646$

(iii) $P(\text{atmost 2 defectives}) = P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-0.9} + \dfrac{e^{-0.9}(0.9)^1}{1!} + \dfrac{e^{-0.9}(0.9)^2}{2!}$

$= 0.9365$

(iv) $P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x = 0) + P(x = 1)]$

$= 1 - \left[e^{-0.9} + \frac{e^{-0.9} (0.9)^1}{1!} \right]$

$= 0.2276$ //

(2): An average 8 calls are received during an hour. find the probability for half an hour

(i) P(no call is received) (ii) P(exactly one call recived)

(iii) P(atleast one call received)

Sol:-

$60'$	λ 8
$1'$	$8/60$
$30'$	$4\ \cancel{8}/\cancel{60}_{2} \times \cancel{30} = 4$

[$\lambda = 4$ for half an hour]

(1) $P(\text{no call is received}) = P(x=0) = \dfrac{e^{-4} \cdot 4^{0}}{0!} = e^{-4} = 0.0183$

(2) $P(\text{exactly one call}) = P(x=1) = \dfrac{e^{-4} \cdot 4^{1}}{1!} = 4e^{-4} = 0.0732$

(3) $P(\text{atleast one call}) = P(x \geq 1) = 1 - P(x=0)$

$= 1 - e^{-4} = 1 - 0.0183 = 0.9817$

(3) If 'x' poisson r.v given that

$P(x=2) = P(x=3)$ find

(i) $E(x)$; $V(x)$ (ii) $E(2x+5)$ (iii) $V(2x-6)$

(iv) $P(x \leq 1.5)$?

Sol:- Given that

$$P(x=2) = P(x=3)$$

$$\frac{\cancel{e^{-\lambda}} \cdot \cancel{\lambda^2}}{\cancel{2!}} = \frac{\cancel{e^{-\lambda}} \cdot \lambda^{\cancel{3}}}{3\cancel{!}}$$

$$\boxed{\lambda = 3}$$

$$E(x) = 3$$

$$V(x) = 3$$

(ii) $$E(2x+5) = 2E(x) + 5$$
$$= 2(3) + 5$$
$$= 11$$

(iii) $$V(2x-6) = V(2x) + V(6)$$
$$= 4V(x) + 0$$
$$= 4(3) = 12$$

(iv) $$P(X \leq 1.5) = P(X \leq 1)$$
$$= P(x=0) + P(x=1)$$
$$= e^{-3} + e^{-3} \cdot 3^{1}/1! = e^{-3} + 3e^{-3} = 4e^{-3} = 0.1988$$

Fitting of poisson Dis^n :-

(1) Fit a poisson dis^n for the following freq data ?

x	0	1	2	3	4	5	6	7	8	9
f	1500	1450	1300	1150	875	650	510	400	250	65

Sol :- Table I (mean of data)

x	f	fx

x	0	1	2	3	4	5	6	7	8	9
f	1500	1450	1300	1150	875	650	510	400	250	65
$f.x$	0	1450	2600	3450	3500	3250	3060	2800	2000	585

$$\text{Mean} = \bar{x} = \frac{\Sigma f x}{N} = \frac{22695}{8150} = 2.78 \simeq \underline{\underline{2.8}}$$

$\Sigma f = \underline{8150}$

$\Sigma f.x = 22695$

$$E(x) = \bar{x} = \lambda$$

$$\boxed{\lambda = 2.8}$$

Task ②:- Fitting Poisson distribution:

$\lambda = 2.8$

x	f	$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	$E(x) = N \cdot P(x)$ Experimental Values
0		$P(0) = \frac{e^{-2.8}\cdot(2.8)^0}{0!} = 0.0608$	$E(0) = 8150 \times 0.0608 = 495.52$
1		$P(1) = \frac{e^{-2.8}(2.8)^1}{1!} = 0.1702$	$E(1) = 1387.30$
2		$P(2) = \frac{e^{-2.8}(2.8)^2}{2!} = \frac{P(1)\cdot\lambda}{2} = 0.2383$	$E(2) = 1942.14$
3		$P(3) = \frac{P(2)\cdot\lambda}{3} = 0.2224$	$E(3) = 1812.57$
4		$P(4) = P(3)\cdot\lambda/4 = 0.1556$	$E(4) = 1268.14$
5		$P(5) = P(4)\cdot\lambda/5 = 0.0872$	$E(5) = 709.05$
6		$P(6) = P(5)\cdot\lambda/6 = 0.0406$	$E(6) = 326$

7	$P(7) = P(6)\cdot\lambda/7 = 0.0162$
8	$P(8) = P(7)\cdot\lambda/8 = 0.0056$
9	$P(9) = P(8)\cdot\lambda/8 = 0.0017$

$E(7) = 132.03$

$E(8) = 45.64$

$E(9) = 14.32$ $= 8139.46$

fitting Poisson dis^n for the following freq's data

x	0	1	2	3	4	5	6	7	8	9	10
f	1150	940	852	761	601	490	346	210	250	110	47