Sr

Unit - V :-

1. ANOVA → [Analysis of Variance]
   - One way classification
   - Two way classification

2. Time series Analysis
   - Components Time series
   - Moving averages
     - Semi
     - Three Years
     - Five Years

Def:- [ANOVA]

Treatments

t_1	t_2	t_3	t_4
x_1	y_1	z_1	s_1
x_2	y_2	z_2	s_2
x_3	y_3	z_3	s_3
x_4	y_4		s_4
x_5	y_5		s_5
	y_6		s_6
			s_7

Def. "The main technique adopted for the analysis and interpretation of the data collected from an experiment is the analysis of variance technique which essentially consists of partitioning the total variation into components ascribable to different source of variation due to the controlled factors and error;"

One way classification :-

Model :- $y_{ij} = \mu + t_i + e_i$ → error

y_{ij} → Total variation in the data

μ → fixed mean

t_i → individual component

Mathematical Formulae :

✓ Correction factor $[c \cdot f] = \dfrac{G^2}{n}$

G : Grand Total

n : no of observations

✓ Sum of Squares of Total (SST) $= \sum\sum y_{ij}^2 - cf$

✓ Sum of Squares Treatments (SSTr) (columns) $= \dfrac{\sum y_{i\cdot}^2}{r} - cf$

✓ Sum of squares Error [SSE] $= SST - SST_r$

Note :- All sum of squares must be positive.

ANOVA TABLE for one way classification:-

Source of Variation (SV)	degrees of freedom (d.f)	Sum of Squares (S.S)	Mean sum of Squares (MSS)	Fcal	Ftable
Treatments (columns)	$K-1$	$S.S T_r$	$M_{Tr} = \frac{SST_r}{K-1}$	$F = \frac{M_{Tr}}{ME}$ $(M_{Tr} > M_E)$ (or) $F = \frac{ME}{MT_r}$	$F(K-1, n-K)$ or $F(n-K, K-1)$
Error	$n-1-(K-1) = n-K$	SSE	$M_E = \frac{SSE}{n-K}$		
Total	$n-1$	SST	—		

Inference:- Since Fcal < Ftab at different d.f in different levels our Ho will be accepted otherwise rejected.

Test the total variations within the data using ANOVA at 5%

LOS:

T_1	T_2	T_3	T_4
15	1	3	1
8	4	0	7
11	5	5	9
4	6	1	5
3	7		4
6			3
			1

T_1 : 15 8 11 4 3 6

T_2 : 1 4 5 6 7

T_3 : 3 0 5 1

T_4 : 1 7 9 5 4 3 1

no of observation

$T_1 = 6$
$T_2 = 5$
$T_3 = 4$
$T_4 = 7$
$\underline{\underline{22}}$

Sol:-

$\Sigma T_1 = 47$
$\Sigma T_2 = 23$
$\Sigma T_3 = 9$
$\Sigma T_4 = 30$

$G.T = \underline{109}$

1. Correction factor $= \frac{(G.T)^2}{n} = \frac{(109)^2}{22} = 540.0454$
(C.f)

2. $SST = \sum y_{ij}^2 - C.f$ [individual observation square and sum of all observations]

$= 815 - 540.0454$

$= 274.9546$

3. $S.S.T_r = \frac{\sum y_{i.}^2}{r} - C.f$

$= \left[\frac{(\sum T_1)^2}{C_1} + \frac{(\sum T_2)^2}{C_2} + \frac{(\sum T_3)^2}{C_3} + \frac{(\sum T_4)^2}{C_4} \right] - C.f$

$= \left[\frac{(47)^2}{6} + \frac{(23)^2}{5} + \frac{(9)^2}{4} + \frac{(30)^2}{7} \right] - 540.0454$

$= [368.166 + 105.8 + 20.25 + 128.5714] - 540.0454$

$SST_r = 622.7874 - 540.0484$

$SST_r = 82.7426$

4. $SSE = SST - SST_r$

$= 274.9546 - 82.7426$

$= 192.2120$

ANOVA TABLE:-

S.V	d.f	S.S	M.S.S		F_{cal}	F_{tab}
Treatment (column)	$K-1 = 4-1 = 3$	82.7426	$M_{Tr} = \frac{82.7426}{3}$	$= 27.5808$	$F_{cal} = \frac{M_{Tr}}{ME} = \frac{27.5808}{10.6784}$	$F_{(3,18)} = 3.16$
Error	$n-K = 22-4 = 18$	192.2120	$M_E = \frac{192.2120}{18}$	$= 10.6784$	$= 2.5828$	
Total	$n-1 = 22-1 = 21$	274.9546				

Inference:- Since $F_{cal} < F_{tab}$ at $(3, 18)$ d.f in 0.05 LOS our H_0 will be accepted.

i.e there is no significancy differency b/w the treatments.

— x —

Test the total Variation within factors at 5% LOS

Sv
18/5/2021
IT3
2.20 to 4.30 PM

F_1	F_2	F_3	F_4
3	4	8	3
5	2	11	6
9	0	14	9
6	1	12	8

Sol:-

$\Sigma F_1 = 23$

$\Sigma F_2 = 7$

$\Sigma F_3 = 45$

$\Sigma F_4 = 26$

$\overline{101}$

$[r_1 = r_2 = r_3 = r_4 = 4] \Rightarrow n = 16$

$$\text{correction factor} = \frac{G^2}{n} = \frac{(101)^2}{16} = 637.5625$$

every observation square then sum

$$\text{Sum of squares Total (SST)} = \Sigma\Sigma y_{ij}^2 - C.f$$

$$= 887 - 637.5625$$

$$SST = 249.4375$$

Sum of Squares of Treatment (SSTr) $= \frac{\sum y_{i.}^2}{r} - c.f$

$$= \left[\frac{(\Sigma F_1)^2}{r_1} + \frac{(\Sigma F_2)^2}{r_2} + \frac{(\Sigma F_3)^2}{r_3} + \frac{(\Sigma F_4)^2}{r_4}\right] - c.f$$

$$= \left[\frac{(23)^2}{4} + \frac{7^2}{4} + \frac{(45)^2}{4} + \frac{(26)^2}{4}\right] - 637.5625$$

$$= 819.75 - 637.5625$$

$$\boxed{SSTr = 182.1875}$$

Sum of Squares of Error (SSE) $= SST - SSTr$

$$= 249.4375 - 182.1875$$

$$= 67.2500$$

ANOVA TABLE [one way]

S.V	d.f	S.S	M.S.S	F cal	F tab
Treatments (column)	$K-1 = 4-1 = 3$	182.1875	$M_{Tr} = \frac{182.1875}{3} = 60.7291$	$F_{cal} = \frac{M_{Tr}}{M_E} = \frac{60.7291}{5.6041} = 10.836$	$F(3, 12) = 3.49$
Error	$n-K = 16-4 = 12$	67.25	$M_E = \frac{67.25}{12} = 5.6041$		
Total	$n-1$ $16-1 = 15$	249.4375			

Inference:- Since $F_{cal} > F_{tab}$ at (3, 12) d.f in 0.05 LOS our H_0 will be rejected.
i.e There is a significant differences within the factors.

Two way ANOVA:-

	T_1	T_2	T_3	T_4
B_1	y_{11}	y_{12}	y_{13}	y_{14}
B_2	y_{21}	y_{22}	y_{23}	y_{24}
B_3	y_{31}	y_{22}	y_{33}	y_{34}

Formulae:-

Correction factor $(C.f) = \frac{G^2}{n}$

Sum of squares of Total $(SST) = \sum\sum y_{ij}^2 - C.f$

Sum of squares of Treatment (columns) $(SSTr) = \frac{\sum y_{i.}^2}{r} - C.f$

Sum of squares of Blocks (rows) $(SSB) = \frac{\sum y_{.j}^2}{s} - C.f$

Sum of squares of Error $(SSE) = SST - SSTr - SSB$

ANOVA (Two way Tables)

S.V	d.f	S.S	MSS	Fcal	Ftab.
Treatment (columns)	$K-1$	$SSTr$	$M_{Tr}=\frac{SSTr}{K-1}$	$F_{Tr}=\frac{M_{Tr}}{ME}$	$F(K-1,\ (K-1)(t-1))$
Blocks (Rows)	$t-1$	SSB	$M_B=\frac{SSB}{t-1}$	$F_B=\frac{M_B}{ME}$	$F(t-1,\ (K-1)(t-1))$
Error	$(K-1)(t-1)$	SSE	$M_E=\frac{SSE}{(K-1)(t-1)}$		
Total	$n-1$ $(Kt-1)$	SST			

Inference: (Treat) Fcal < Ftab at diff d.f in diff levels Ho will be accepted. otherwise rejected.

(Block) Fcal < Ftab at different d.f in diff levels Ho will be accepted otherwis rejected.

Test the total variation between the cells at 5% LOS.

	α	β	γ
A	4	1	8
B	3	2	3
C	2	–	2
D	0	8	1

Sol:-

(C.T) $\Sigma\alpha = 9$ (R.T) $\Sigma A = 13$ [4]   no of observation

$\Sigma\beta = 11$ $\Sigma B = 8$ [3] $\alpha = 4$

$\Sigma\gamma = 14$ $\Sigma C = 4$ [2] $\beta = 3$

$\Sigma D = 9$ [3] $\gamma = 4$

$\boxed{34 = 34 = G}$ 11

Correction factor $\left.\right\}$ $C.f = \frac{G^2}{n}$

$= \frac{(34)^2}{11}$

$Cf = 105.0909$

Sum Square of Total (SST) $\left.\right\}$ =

$= \Sigma\Sigma y_{ij}^2 - C.f$

$= 176 - 105.0909$

$SST = 70.9091$

Sum of Squares of Treatments (Columns) $\left.\right\} = \frac{\sum y_{i.}^2}{r} - c.f$

$= \left[\frac{(9)^2}{4} + \frac{(11)^2}{3} + \frac{(14)^2}{4}\right] - c.f$

$= [109.5833] - 105.0909$

$\boxed{SS_{Tr} = 4.4924}$ //

Sum of Squares of Blocks (rows) SSB $\left.\right\} = \frac{\sum y_{.j}^2}{s} - c.f$

$= \left[\frac{13^2}{3} + \frac{8^2}{3} + \frac{4^2}{2} + \frac{9^2}{3}\right] - 105.0909$

$= 112.6666 - 105.0909$

$\boxed{SSB = 7.5757}$ //

Sum of square Error (SSE) $= SST - SST_r - SSB$

$= 70.9091 - 4.4924 - 7.5757$

$SSE = 58.8418$ //

ANOVA Table (Two way)

S.V	df	SS	MSS	Fcal	Ftab
Treatment	$k-1 = 3-1 = 2$	$SST_r = 4.4924$	$M_{Tr} = \frac{SST_r}{k-1} = \frac{4.4924}{2} = 2.2462$	$F_{cal,Tr} = \frac{ME}{M_{Tr}} = \frac{9.8069}{2.2462}$, $\boxed{F_{Tr} = 4.3659}$	$F(6,2) = 19.33$
Blocks	$t-1 = 4-1 = 3$	$SSB = 7.5757$	$M_B = \frac{SSB}{t-1} = \frac{7.5757}{3} = 2.5252$	$F_{B,cal} = \frac{ME}{MB}$, $\boxed{F_B = 3.8836}$	$F(6,3) = 8.94$
Error	$(k-1)(t-1) = 2 \times 3 = 6$	$SSE = 58.8418$	$M_E = \frac{SSE}{(k-1)(t-1)} = \frac{58.8418}{6} = 9.8069$		
Total	$kt-1 = 12-1 = 11$	$SST = 70.9091$			

Inferences :-

(Treatment | column)

Since $F_{cal} < F_{tab}$ at (6,2) d.f in 0.05 LOS our H_0 will be accepted. There is no significant differences with in the columns.

(Block | Rows):

Since $F_{cal} < F_{tab}$ at (6,3) d.f in 0.05 LOS our H_0 will be accepted. There is no significant difference with in the rows.

Test the total Variation b/w the cells at 5% LOS?

	α	β	γ	δ	R.T
A	3	5	7	2	17
B	11	14	17	16	58
C	9	8	7	5	29
D	3	1	2	0	06
C.T	26	28	33	23	$\boxed{110}$

Sol:- $n = 16$: $G = 110$ $k = 4$ $t = 4$

$$c.f = \frac{G^2}{n} = \frac{(110)^2}{16} = 756.25$$

Sum of square Total $= \Sigma\Sigma y_{ij}^2 - c.f$

SST $= 1182 - 756.25$

$SST = 1182 - 756.25$

$\therefore SST = 425.75$ //

$$SST_r = \frac{\Sigma y_{i.}^2}{r} - c.f$$

(SSTe)

$$= \left[\frac{(26)^2}{4} + \frac{(28)^2}{4} + \frac{(33)^2}{4} + \frac{(23)^2}{4}\right] - c.f$$

$$= 769.58 - 756.25$$

$SST_r = 13.25$ //

$$S.S.B = \frac{\Sigma y._j^2}{s} - C.f$$

$$= \left[\frac{(17)^2}{4} + \frac{(58)^2}{4} + \frac{(29)^2}{4} + \frac{(6)^2}{4}\right] - C.f$$

$$= 1132.5 - 756.25$$

$$SSB = 376.25$$

$$S.S.E = SST - SST_r - SSB$$

$$= 425.75 - 13.25 - 376.25$$

$$SSE = 36.25$$

ANOVA TABLE [Two way]

S.V	d.f	S.S	MSS	Fcal	Ftab
Treatment (Col)	$k-1 = 4-1 = 3$	$SSTr = 13.25$	$M_{Tr} = \frac{SSTr}{k-1} = \frac{13.25}{3} = 4.4167$	$F_{cal(Tr)} = \frac{M_{Tr}}{ME} = \frac{4.4167}{4.0277} = 1.0965$	$F_{tab}(3,9) = 3.86$
Block (Row)	$t-1 = 4-1 = 3$	$SSB = 376.25$	$M_B = \frac{SSB}{t-1} = \frac{376.25}{3} = 125.4167$	$F_B = \frac{M_B}{ME} = \frac{125.4167}{4.0277} = 31.1385$	$F(3,9) = 3.86$
Error	$(k-1)(t-1) = 9$	$SSE = 36.25$	$ME = \frac{SSE}{9} = \frac{36.25}{9} = 4.0277$		
Total	$kt-1 = 16-1 = 15$	$SST = 425.75$			

Inference (Treat) } since $F_{cal} < F_{tal}$ at $(3, 9)$ d.f in 0.05 LOS

our H_0 will be accepted. There is no significant

difference within the treatment/columns.

(Blocks) ; since $F_{cal} > F_{tab}$ at $(3, 9)$ df in 0.05

LOS our H_0 will be rejected: There is a significant

different within the Blocks / Rows.

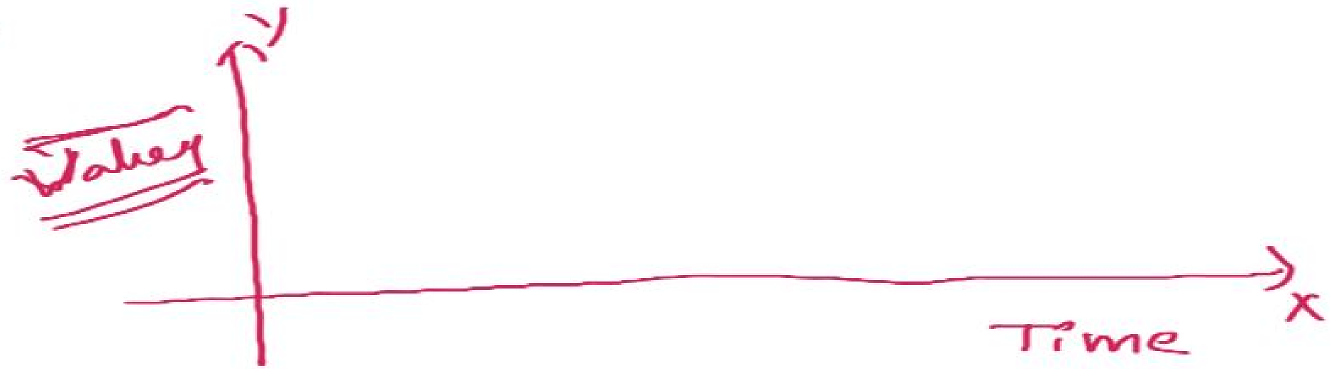
— X —

Time series:-

Method for analysing Trend:-

1. Semi Average Method
2. Moving Averages Method — 3 Years / 5 Years

Graph:-



Time series Graph

Method · I :- Semi Averages Method :-

$n = 8$	GA	GB	
	4	4	
$n = 9$	GA	GB	(middle year – skip)
	4	4	

Divided the data into two equal parts. If is odd no skip middle year, and make a data into two equal parts and then take the average of each part, that gives the average trend.

Apply the semi average method and analyse trend construct the graph.

Year	Value	Year	Value
1990	176	1997	180
1991	135	1998	190
1992	140	1999	151
1993	147	2000	172
1994	152	2001	140
1995	150		
1996	161		

Group I

Group

Cal:

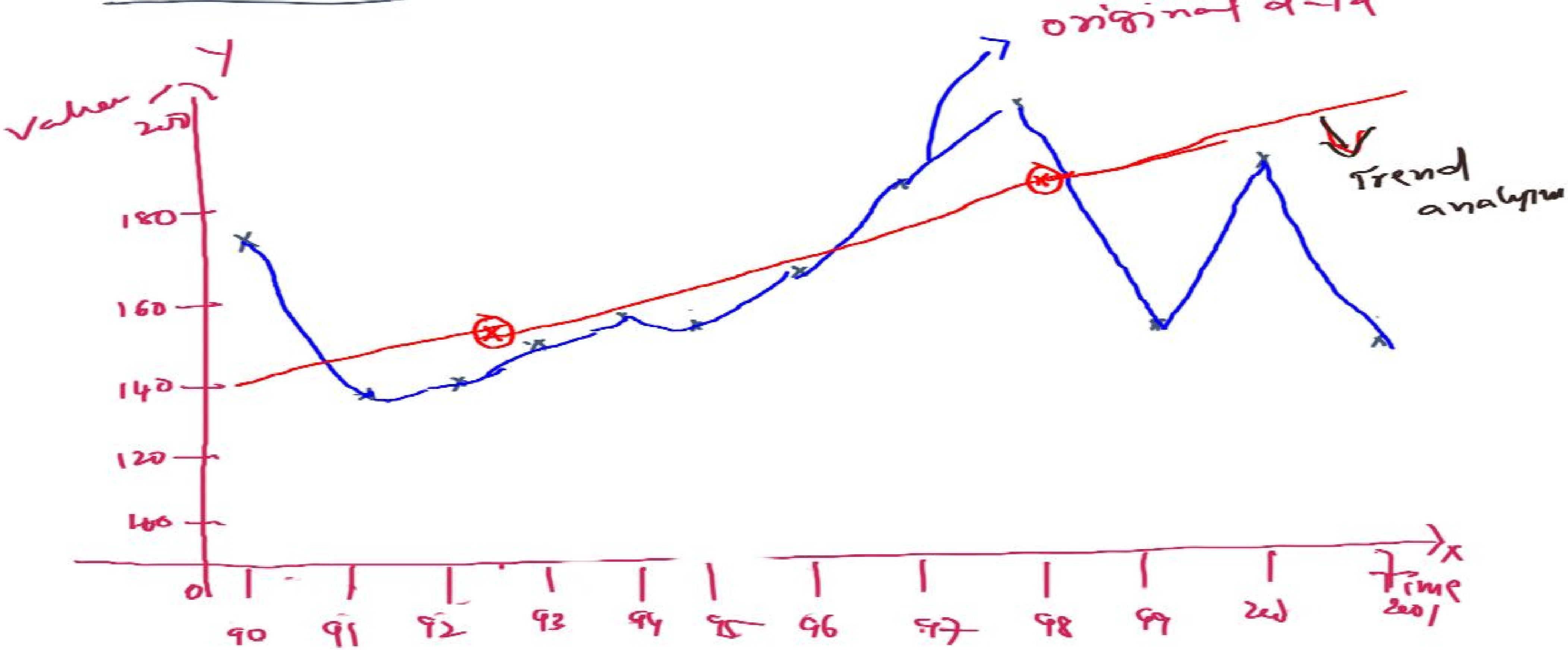
Group I	Value	
1990	176	
91	135	
92	140	900
93	147	
94	152	
95	150	

Group – II		
1996	161	
97	180	
98	190	984
99	141	
2000	172	
2001	140	

Using Semi Average Method

	Year	Total Value	Average
Group I	1990		
	1991		
	1992		
	1993 1992.5	900	$\frac{900}{6} = 150$
	1994		
	1995		
Group II	1996		
	1997		
	1998		
	1999 1998.5	984	$\frac{984}{6} = 164$
	2000		
	2001		

Time Series Graph



Apply semi average method for analyzing the data:

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
Value	148	156	160	150	170	176	187	185	110

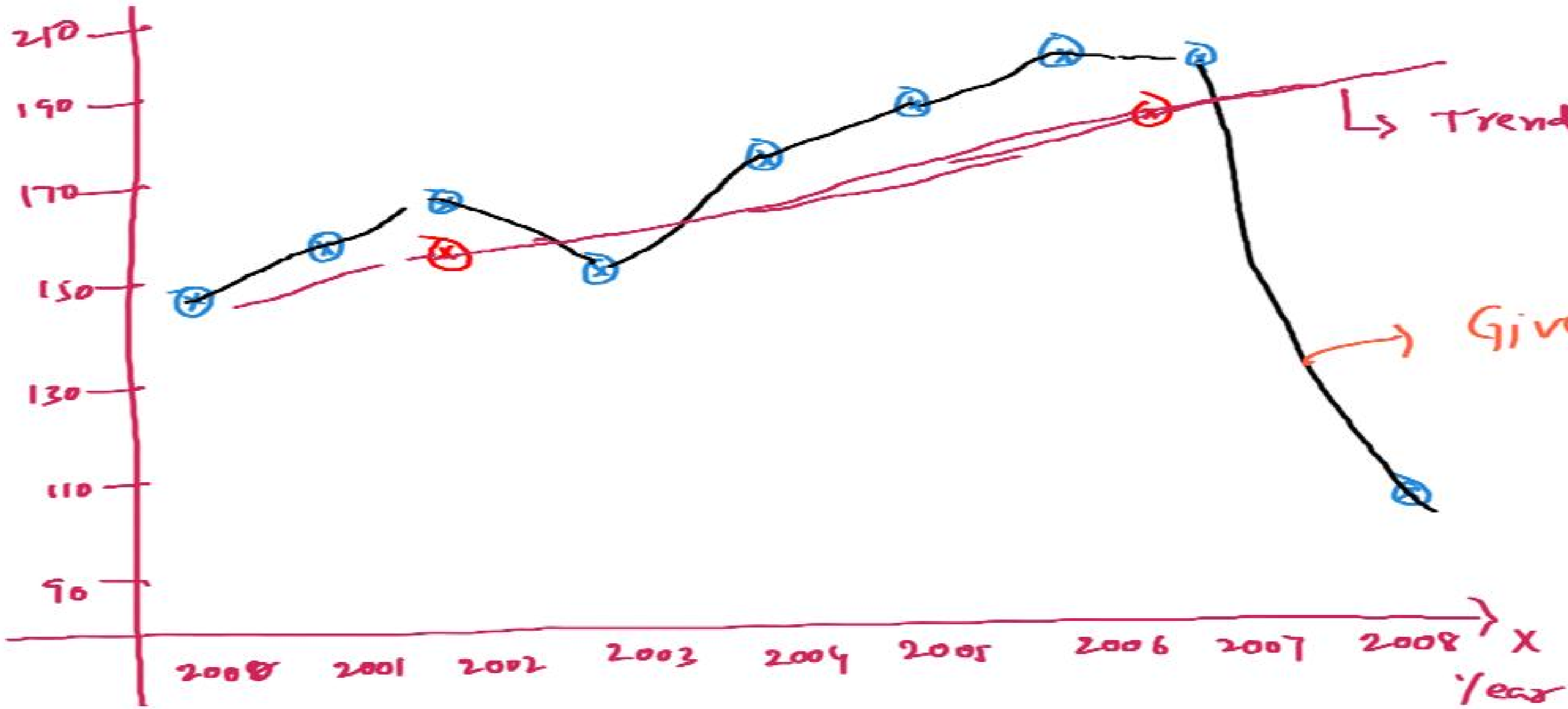
Sol: Given sequence is odd sequence: We should skip the middle of year for the given data: {2004 ↓ skiped}

Group-I: 2000, 2001, 2002, 2003

Group-II: 2005, 2006, 2007, 2008

Year	Total value	Average
G_I 2000		
2001 } 2001.5	614	$\frac{614}{4} = 153.5$
2002		
2003		
2004	skiped	———
G_{II} 2005		
2006 } 2006.5	652	$\frac{652}{4} = 163$
2007		
2008		

Time Series Graph



W
19/5/21
I T_3

Moving Averages Method..

	Total	Average
a_1 — g_1 (a_1, a_2, a_3)	$a_1+a_2+a_3$	$\frac{a_1+a_2+a_3}{3}$
a_2 — g_2 (a_2, a_3, a_4)	$a_1+a_2+a_3$	
a_3 — g_3 (a_3, a_4, a_5)	$a_2+a_3+a_4$	
a_4 — g_4 (a_4, a_5, a_6)	$a_3+a_4+a_5$	
a_5 — g_5 (a_5, a_6, a_7)	$a_4+a_5+a_6$	
a_6 — g_6 (a_6, a_7, a_8)	$a_5+a_6+a_7$	
a_7 — g_7 (a_7, a_8, a_9)	$a_6+a_7+a_8$	
a_8 — g_8 (a_8, a_9, a_{10})	$a_8+a_9+a_{10}$	
a_9		
a_{10}		

Apply the 3 years Moving Averages Method for analysing the trend and construct the graph:

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Prod	175	180	210	260	300	210	218	220	210	205	195	199	206

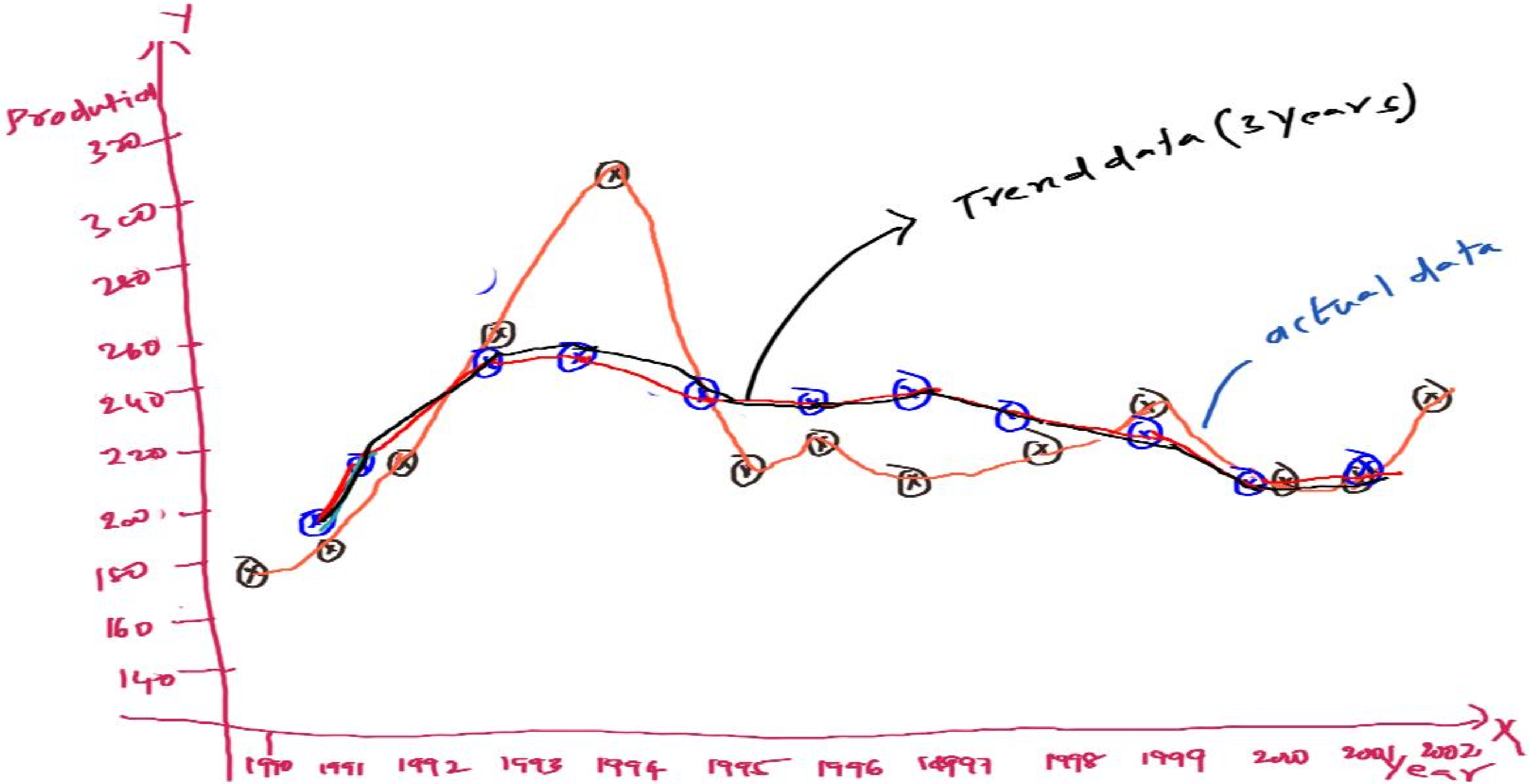
Sol:- 3 years Moving Average Method

(1) Group 3 years runne as a one set

(2) Find total of prod (value) of each group

(3) Average of the production then construct graph
b/w Year vs actual data
Year vs Average production value

Year	Production	Group	Total Production	Averrge
1990	175			
1991	180	G(90-92) 1	565	565/3 = 188.33
1992	210	G(91-93) 2	650	650/3 = 216.666
1993	260	G(92-94) 3	770	770/3 = 256.666
1994	300	G(93-95) 4	770	770/3 = 256.666
1995	210	G(94-96) 5	728	728/3 = 242.666
1996	218	G(95-97) 6	628	628/3 = 209.333
1997	200	G(96-98) 7	628	628/3 = 209.333
1998	210	G(97-99) 8	615	615/3 = 205 33
1999	205	G(98-00) 9	610	610/3 = 203.333
2000	195	G(99-01) 10	599	599/3 = 199.666
2001	199	G(2000-02) 11	600	600/3 = 200
2002	205			



Y
Production
320
300
280
260
240
220
200
180
160
140
Trend data (3 years)
actual data
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
X
Year

Analyze following Time series data with 5-years Moving Averages Method and also construct the graph ?

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Produ	260	310	280	320	360	380	400	420	350	350	410	440	410	450	410

Procedure:-

(1) consider 5 conse years as a one group/set

(2) Find total of each group, and write the total value at the middle of the year.

(3) Find Average of each group and write value at the middle year

(4) construct graph b/w
[Year vs production
and
Year Vs 5 year Average]

Year	Prod	Grop/set 5-year	Total of 5year prod	Average
1995	260			
1996	310			
1997	280	G(95-99)	1530	1530/5 = 306
1998	320	G(96-00)	1650	1650/5 : 330
1999	360	G(97-01)	1740	1740/5 : 348
2000	380	G(98-02)	1880	1880/5 = 376
2001	400	G(99-03)	1910	1910/5 = 382
2002	420	G(00-04)	1900	1900/5 = 380
2003	350	G(01-05)	1930	1930/5 = 386
2004	350	G(02-06)	1970	1970/5 = 394
2005	410	G(03-07)	2000	2000/5 = 400
2006	440	G(04-08)	2100	2100/5 = 420
2007	450	G(05-09)	2160	2160/5 = 432
2008	450			
2009	410			

Time series Graph:-