Unit II

Discrete probabilities and distributions

P(E) [probability of happening of an event] -

$$\frac{\text{favorable events}}{\text{Total events}}$$

Total probability = $P(E) + P(\bar{E})$

→ Axiometric probability → $P(A_i) \geq 0$

→ Statistical probability → $P(E) = 1$

→ $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

Bayes theorem:- if $E_1, E_2 \ldots E_n$ are mutually disjoint events with probability of E_i $P(E_i) \neq 0$ for $i = 1, 2, 3 \ldots$ then there is an or arbitary event which is a subset of Union of all E_i's is such that

$$P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

Proof :- $A \subseteq \bigcup_{i=1}^{n} E_i$ therefore extract A from Union of all Ai's

$$A \subseteq \bigcup_{i=1}^{n} E_i$$

$$A = A \cap \left(\bigcup_{i=1}^{n} E_i\right)$$

$$A = \bigcup_{i=1}^{n} (A \cap E_i) \text{ By demorgan's law}$$

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

By compound probability

$$P(A) = \sum_{i=1}^{n} P(E_i) P(A/E_i)$$

We know that compound probability or multiplicative prob is $P(A \cap E_i) = P(E_i) P(A/E_i)$

(or)

~~$P(E_i/A)$~~

$$P(A \cap E_i) = P(A) P(E_i/A)$$

$$\Rightarrow P(E_i/A) = \frac{P(A \cap E_i)}{P(A)}$$

$$\boxed{P(E_i/A) = \frac{P(E_i) P(A/E_i)}{\sum P(E_i) P(A/E_i)}}$$

Here A is common event for all E_i's

$P(A/E_i)$ is prior probability

$P(E_i/A)$ is called posterior probability

$$M.G.f = M_x(t) = E(e^{xt})$$

$$M_x(t) = \int_a^b e^{tx} f(x)\, dx$$

moment generating function

p.d.f

Properties of Probability density function.

a) $f(x) \geq 0$

b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Measures of central tendency abt origin.

$\mu_r' = \int_a^b x^r f(x)\,dx$.

put $r = 1, 2, 3, 4$ .. we get

$\mu_1' = \int_a^b x f(x)\,dx$ = mean

$\mu_2' = \int_a^b x^2 f(x)\,dx$ = ~~variance~~

$\mu_3' = \int_a^b x^3 f(x)\,dx$

$\mu_4' = \int_a^b x^4 f(x)\,dx$

μ_2 = varience $= \mu_2' - (\mu_1')^2$

third moment $\leftarrow \mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$\leftarrow \mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

4th moment

$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$?

$\beta_2 = \frac{\mu_4}{\mu_2^2} \cong 3 \rightarrow$ Normal distribution.

if $\beta_2 < 3$ then the distribution of curve ~~is~~ leads to Platikurtic

$\beta_2 > 3$ " " " " Leptokurtic

Q> find β_1 and ~~β_2 for given $f(x) = e^{-3}$~~.

$$\mu_r' = \int_0^\infty x^r e^{-x} dx$$

$$\int_0^\infty e^{-x} x^{(r+1)-1}$$

$$\mu_r' = \gamma(r+1) = r!$$

$$\mu_r' = r!$$

$$\gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$$

$$\gamma(n+1) = \int_0^\infty e^{-x} x^n dx$$

$$\gamma(n+1) = n!$$

Properties of P.m.f

i) $f(q_i) \geq 0$

ii) $\sum_{i=1}^{n} f(q_i) = 1$

iii) $f(q_i) = P(Q = q_i)$

Mathematical expectation :- (for discrete r.v)

Mean of the set of values.

Def :- Let x be a random variable ~~with~~ w.r.t. Probability mass function (Pmf) or probability density function exists mathematical expectation iff $E|x|$ exists.

$$E(x) = \sum_{i=1}^{\infty} x_i P(x_i)$$, where $x_i = X$ is called Random variable

for discrete r.v

$P(x_i) = P(x)$ is called probability function

$$E|x| = \sum_{i=1}^{\infty} |x_i P(x_i)| < \infty$$

$$= \sum_{i=1}^{\infty} |x_i| P(x_i) < \infty$$

Mathematical expectation for continuous Random variable

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx < \infty$$

$$E|x| = \int_{-\infty}^{\infty} |x f(x)| dx < \infty$$

$$= \int_{-\infty}^{\infty} |x| f(x) < \infty$$

Note :- $E(x)$ exists iff $E|x|$ exists.

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> if $\sum u_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots$

$\sum u_n = \sum_{i=1}^{\infty} \frac{1}{n}$ ∴ the series is divergent

P-series test :- $\sum u_n = \sum \frac{1}{n^P}$

i) if $P > 1$ → series is convergent

ii) if $P \leq 1$ → series is divergent

Lebintz test :- $\sum u_n = u_1 + u_2 + u_3 + \cdots$

if $u_1 > u_2 > u_3 \cdots$

It is decreasing one then as $n \to \infty$ $u_n \to 0$ then we can call the series is convergent.

$\boxed{\lim_{n \to \infty} u_n = 0}$ → convergent

$\boxed{\text{if } \lim_{n \to \infty} u_n \neq 0}$ → divergent

mathematical expectation of d.r.v case :-

d.r.v ↓ discrete random variable

$E(x) = \sum_{i=1}^{\infty} x_i P(x_i)$

iff E|x| exists.

Ex:- verify $E(x)$ for given infinite series exists or not for following problems.

Let $x_i = (-1)^{i+1}$ where $i = 1, 2, 3 - -$

$P(x_i) = \frac{1}{i}$ $\quad P(x_i) = \frac{1}{i}, \ i = 1, 2, 3 - - \infty$

$$E(x_i) = \sum_{i=1}^{\infty} x_i P(x_i) = \sum_{i=1}^{\infty} (-1)^{i+1} \frac{1}{i} \quad , i = 1, 2, 3 - - -$$

$$E(x) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} - -$$

the above series is called alternate series.

$$\Sigma u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - - - -$$

according to Lebenitz test

$$1 > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} - -$$

$$\Rightarrow u_n = \frac{1}{n}$$

$$\lim_{n \to \infty} u_n = \frac{1}{\infty} = 0 \rightarrow \text{convergent} - (1)$$

$$|\Sigma u_n| = E|x| = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - -$$

$$E|x| = |\Sigma u_n| = \sum_{i=1}^{\infty} \frac{1}{n}$$

According to p-series test $\sum_{i=1}^{\infty} \frac{1}{n^p}$ if $p \leq 1$ diverged, $p > 1$ convergent

$\therefore E|x|$ is divergent. $-(2)$

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from Eq ① & ② , ① is convergent } ② is divergent }

So mathematical expectation doesnot exist.

if both $E(x)$ and $E|x|$ are convergent then mathematical expectation exists.

Leibnitz Test for Alternating Series

Let $\sum_{n=1}^{\infty} (-1)^{n-1} u_n$, $u_n > 0$, for all n, be the alternating series, then the series is convergent iff

i) $u_{n+1} \le u_n$, for all n

ii) $\lim_{n \to \infty} u_n = 0$

Properties of mathematical expectations:-

1) $E(ax) = aE(x)$

2) $E(ax+b) = aE(x)+b$

3) $\boxed{E(a) = a}$

4) $E(x+y) = E(x)+E(y)$

5) $E(xy) = E(x)E(y)$

] We can prove this results by continuous random variables.

① $V(x) = E\{x - E(x)\}^2$

↓ (or)

varience $= E(x^2) - \{E(x)\}^2$

(or)

$= E(x^2) - (\bar{x})^2$

varience : $V(x)$

② $V(ax) = a^2 V(x)$

③ $V(b) = 0$

④ $V(ax+b) = a^2 V(x)$

Proofs :-

i) $E(a) = a$

Proof :- We know by defenition of mathematical expectation $E(x) = \sum_{i=1}^{\infty} x_i P(x_i)$

substitute $x = a$

$$E(a) = \sum_{i=1}^{\infty} a P(x_i)$$

$$= a \sum_{i=1}^{\infty} P(x_i)$$

(use the property of Probabity mass function)

$$\boxed{E(a) = a(1) = a}$$

2) $E(ax) = aE(x)$

Proof:- We know by definition of Mathema[tical]

expectation

$$E(x) = \sum_{i=1}^{\infty} x_i P(x_i)$$

Substitute $x = ax$

$$E(ax) = \sum_{i=1}^{\infty} (ax) P(x_i)$$

$$= a \sum_{i=1}^{\infty} x_i P(x_i)$$

$$= a \underbrace{\left(\sum_{i=1}^{\infty} x_i P(x_i) \right)}_{E(x)}$$

$$\Rightarrow E(ax) = aE(x)$$

3) P.T $E(ax+b) = aE(x)+b$

Proof:- We know by defenition of Mathematical

Expectation

$$E(x) = \sum_{i=1}^{\infty} x_i P(x_i)$$

Now Substitute $x = ax_i + b$

$$E(ax+b) = \sum_{i=1}^{\infty} (ax_i + b) P(x_i)$$

$$= \sum_{i=1}^{\infty} a x_i P(x_i) + b \sum_{i=1}^{\infty} P(x_i)$$

$E(ax+b) = aE(x) + b(1)$

$\Rightarrow E(ax+b) = aE(x) + b$

[By property of ~~mo~~ probability mass distribution $\sum_{i=1}^{\infty} P(x_i) = 1$]

$\therefore E(ax+b) = aE(x) + b.$

4 $\quad V(x) = E\{(x - E(x))^2\}$

$E\{x^2 + \{E(x)\}^2 - 2xE(x)\}$

$[E(x) = \bar{x}]$ E

$E[x^2 + (\bar{x})^2 - 2x\bar{x}]$

$\Rightarrow E(x^2) + E((\bar{x})^2) - E(2x\bar{x})$

$E(x^2) + (\bar{x})^2 - 2\bar{x}E(x)$

$E(K) = K$ $\quad E(x^2) + (\bar{x})^2 - 2(\bar{x})^2$

$\Rightarrow E(x^2) - (\bar{x})^2$

$\therefore V(x) = E((x - E(x))^2)$

(or)

$E(x^2) - (\bar{x})^2$

(5) $V(ax+b) = a^2 V(x)$

Proof By definition we know $V(x) = E[\{x - E(x)\}^2]$

$$V(ax+b) = E[\{ax+b - E(ax+b)\}^2]$$

$$= E[\{ax+\cancel{b} - aE(x) - \cancel{b}\}^2]$$

$$E[\{ax - aE(x)\}^2]$$

$$a^2 E[(x-E(x))^2]$$

$$a^2 V(x)$$

$\therefore$ $V(ax+b) = a^2 V(x)$

(6) $V(b) = 0$

We know by def $V(x) = E[\{x - E(x)\}^2]$

$$V(a) = E[\{a - E(a)\}^2]$$

$[\because E(a) = a]$ $V(a) = E[\{a - a\}^2]$

$$V(a) = E(0)$$

$$V(a) = 0$$

$\therefore$ $V(b) = 0$

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Covariance

$cov(x,y)$:- the relation b/w 2 random variables x and y is denoted as $cov(x,y)$

$$cov(x,y) = E\{\underbrace{(\underbrace{x}_{r.v}-\underbrace{E(x)}_{\substack{M.E \\ \text{mean of r.v } x}})(y-\underbrace{E(y)}_{\text{Mean of r.v } y})}\}$$

$$= E\{xy - xE(y) - E(x)y + E(x)E(y)\}$$

$$= E(xy) - E(x)E(y) - \cancel{E(x)E(y)} + \cancel{E(x)E(y)}$$

$$E(xy) - E(x)E(y)$$

$$\therefore Cov(x,y) = E(xy) - E(x)E(y)$$

Note :- if x & y are independent r.v we can write $E(xy) = E(x)E(y)$

$$\Rightarrow Cov(x,y) = E(x)E(y) - E(x)E(y)$$

$$= 0$$

↳ It indicates ~~there is~~ x and y are independent variables.

Note: if x and y r.v data provided by some industry

Cal. $Cov(x, y) = ?$

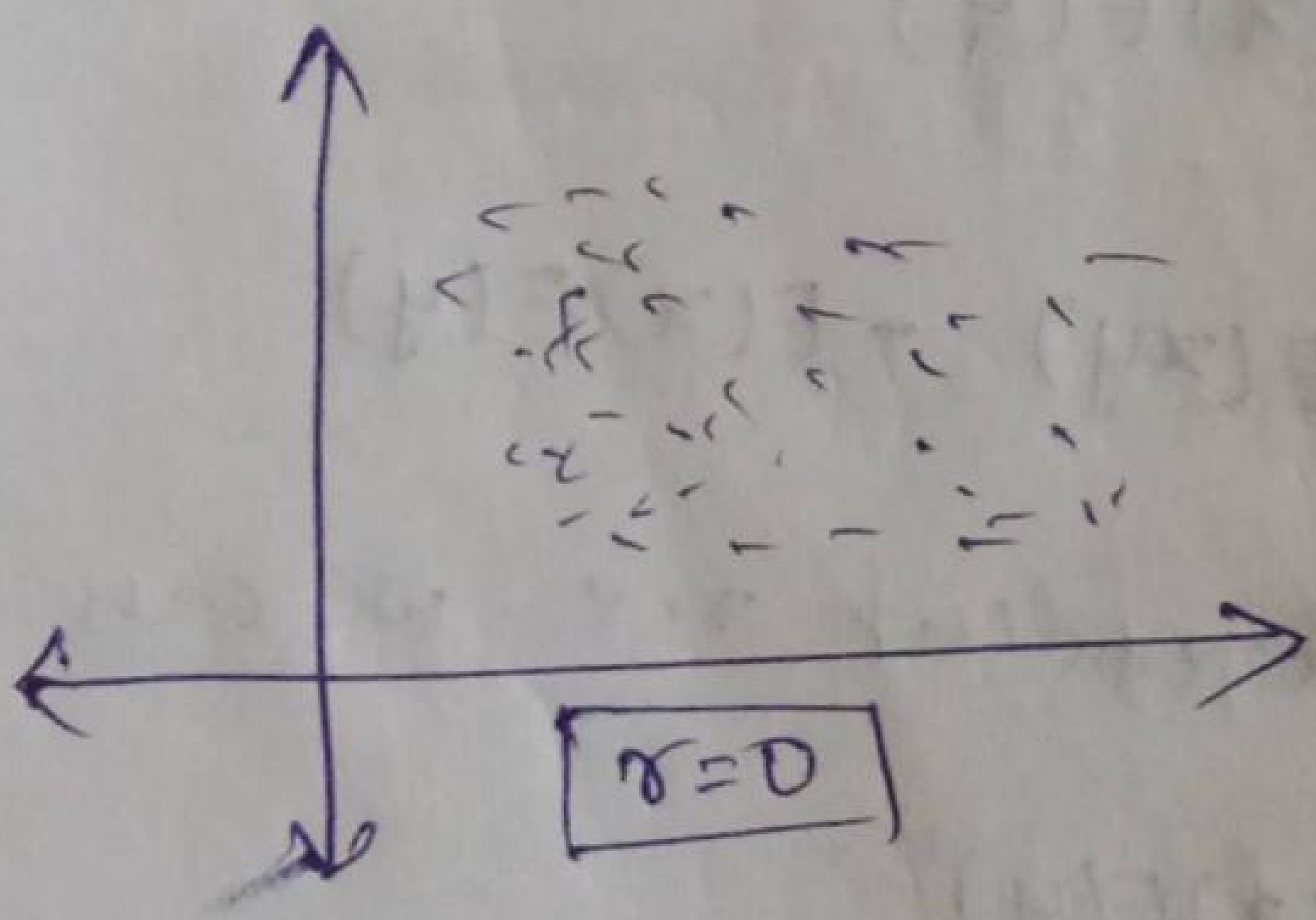
σ_x (S.D) $= ?$

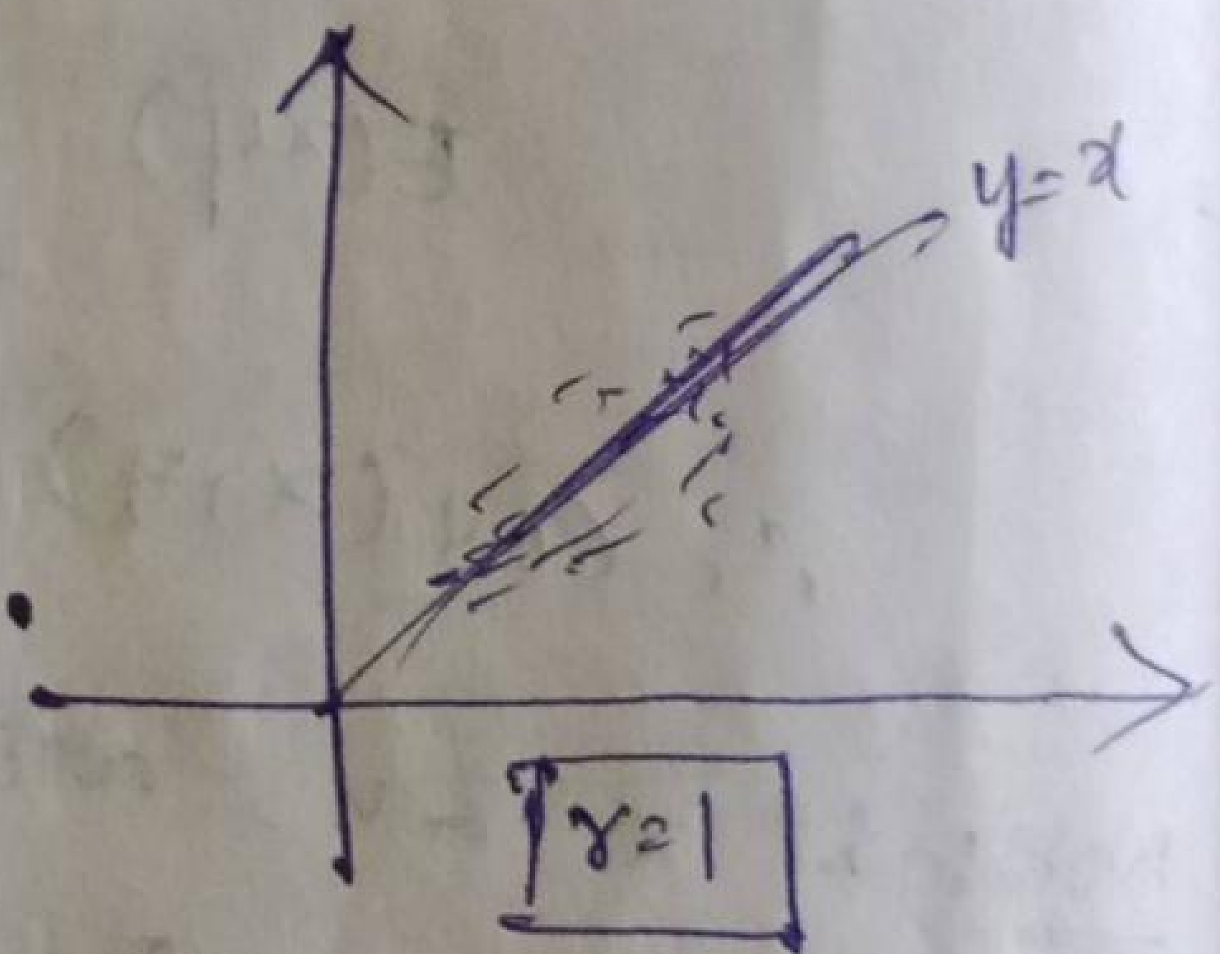
σ_y (S.D) $= ?$

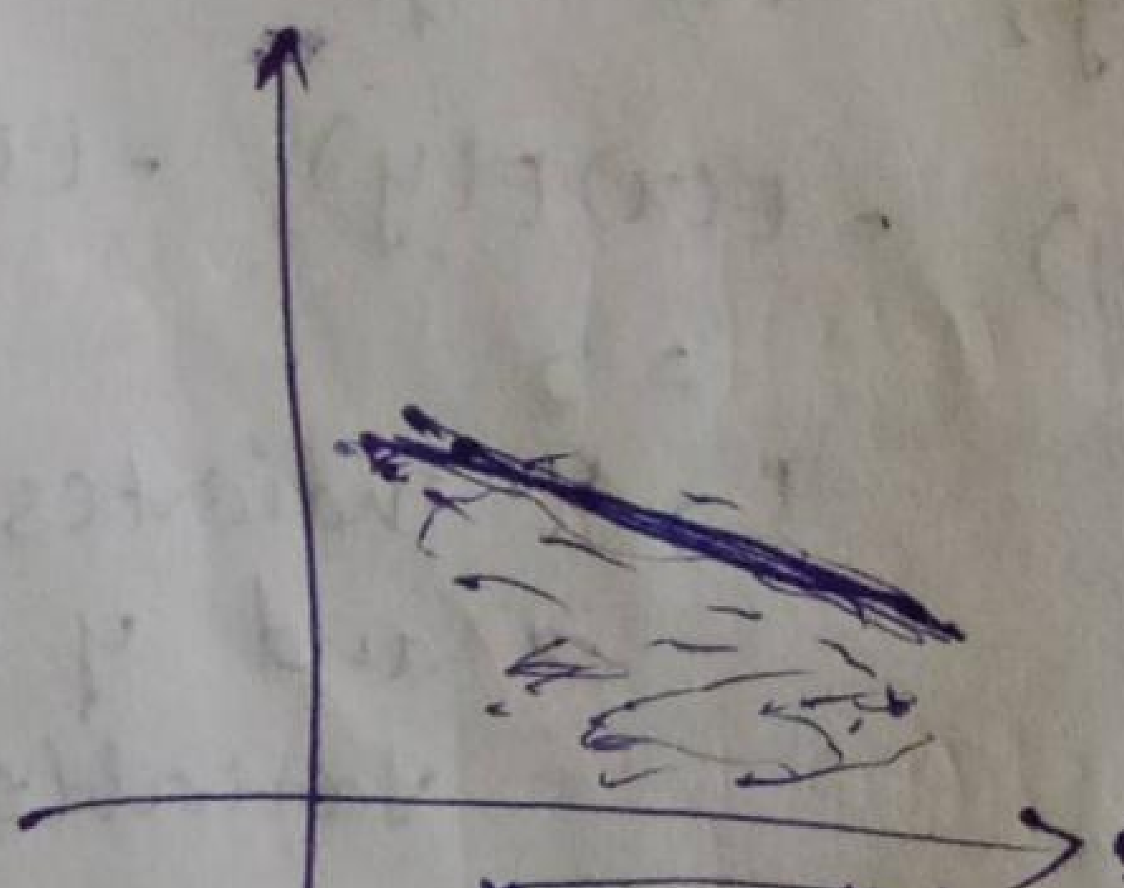
rank Corelation $\boxed{r(x, y) = \dfrac{Cov(x, y)}{\sigma_x \cdot \sigma_y}}$

$$\frac{E(xy) - E(x)E(y)}{\sigma_x \cdot \sigma_y}$$

Geometric veiw of $r(x, y)$ is given below







$\boxed{r = -1}$ (inversly proportional)

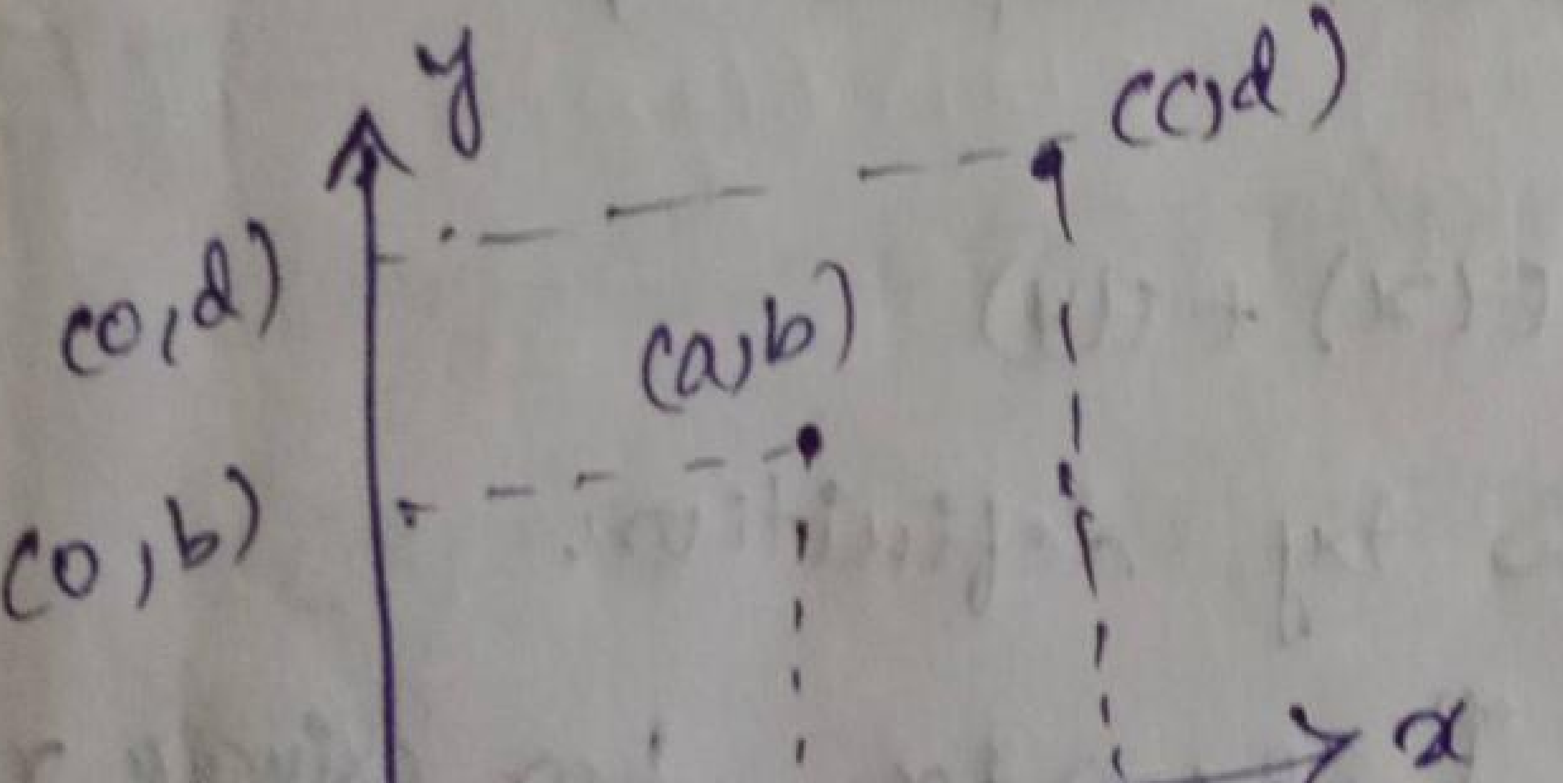
$-1 \le r(x,y) \le 1$

Mathematical expectation for continuous random variable:-

$E(x) = \int_{-\infty}^{\infty} x f_x(x)\, dx$ provided the integral must be Absolute convergent

Joint probability :- if x and y are 2 random variables with Prob fns $f_{xy}(x,y)$ is defined as ~~$f_{xy}(x,y)$~~ =

$f_{xy}(x,y) = \left\{ x \in X,\ y \in Y : f_{xy}(x,y) \in \text{2dim D geometry} \right\}$



$f_{xy}(x,y)$ is p.d.f of Joint variable (x,y) w.r.t x and y

$$\boxed{① \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)\, dx\, dy = 1}$$

$$\boxed{② \ f_{xy}(x,y) \ge 0.}$$

if x is random variable then p.d.f $f_x(x)$

if x, y " " " $f_{xy}(x,y)$

if x, y, z " " " $f_{xyz}(x,y,z) \to$ Joint

" x, y, z, t " " " $f_{xyzt}(x,y,z,t) \to$ "

if $x_1, x_2 \ldots x_n$ " " $f_{x_1, x_2 \ldots x_n}(x_1, x_2 \ldots) \to$ "

Note:- $f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dy$

$f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dx$

$\to$ marginal probability density function w.r.t x and y respectively.

P.T $E(x+y) = E(x) + E(y)$

Proof:- we know by definition

$$E(x) = \int_{-\infty}^{\infty} x f_x(x)\,dx \quad \text{for single r.v}$$

$$E(x+y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y) f_{xy}(x,y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{xy}(x,y)\,dx\,dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f_{xy}(x,y)\,dx\,dy$$

→ P.d.f

→ Joint P.d.f

$$E(x+y) = \int \underbrace{\int x f_{xy}(x,y)\,dx\,dy}_{\substack{\Downarrow \\ f_x(x) = ? \\ \text{[integrate w.r.t } x]}} + \int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{\infty} y f_{xy}(x,y)\,dx\,dy}_{\substack{\Downarrow \\ \text{extract } f_y(y) \\ \text{[integrate w.r.t } y]}}$$

$$E(x+y) = \int_{-\infty}^{\infty} x\,dx \int_{-\infty}^{\infty} f_{xy}(x,y)\,dy + \int_{-\infty}^{\infty} y\,dy \int_{-\infty}^{\infty} f_{xy}(x,y)\,dx$$

$$= \int_{-\infty}^{\infty} x\,dx\,(f_x(x)) + \int_{-\infty}^{\infty} y\,dy\,(f_y(y))$$

$$E(x+y) = \int_{-\infty}^{\infty} x f_x(x)\,dx + \int_{-\infty}^{\infty} y f_y(y)\,dy$$

$$E(x+y) = E(x) + E(y)$$

$||^{ly}$, $E(x+y+z) = E(x) + E(y) + E(z)$

$$E(x_1 + x_2 + \cdots + x_n) = E(x_1) + E(x_2) + \cdots + E(x_n)$$

② P.T $E(xy) = E(x) \cdot E(y)$, x, y are independent.

Proof:- We know ~~that~~ by definition of M.E

$$E(x) = \int_{-\infty}^{\infty} x f_x(x) dx$$ for single r.v

$$E(xy) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (xy) f_{xy}(x,y) \, dx dy.$$

Since x and y are independent variables

we have

$$E(xy) = \int_{-\infty}^{\infty} x f_x(x) dx \int_{-\infty}^{\infty} y f_y(y) dy$$

$$\boxed{E(x,y) = E(x) E(y)}$$

lly, $E(x_1 \cdot x_2 \cdots x_n) = E(x_1) E(x_2) \cdots E(x_n)$

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Poission distribution

Poission distribution is a limitingcase of Binomial distribution under following Condition

(i) ~~$x \to$ finite value~~

(ii) P : Probability of const sucess

(iii) Q : Probability of failure

$n \to$ the number of trials is indefenitely large

(iv) $np = \lambda$ prob of sucess , $P = \frac{\lambda}{n}$, $q = 1 - P$
$1 - \frac{\lambda}{n}$

n → no. of trails

λ → is called parameter or mean of the random variable.

Def:- a random variable 'x' is said to follow poission distribution for all non negative values and its prob mass fn is given by

$$P(x, \lambda) = P(X = x) = P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad \text{Where } x = 0, 1, 2, -$$

λ = parameter or mean of random variable

'x' is a random variable

How to cal mean and Variance of Poission distribution.

→ by using measures of central tendency abt orig

$$\mu_r' = \sum_{i=1}^{\infty} x^r P(x)$$

→ by using moment generating fns

→ Commulent generating functions

= x =

We know that measures of central tendency about origin is denoted as

$$\mu_r' = \sum_{r=1}^{\infty} x^r P(x)$$

↓ r.v

↳ prob mass function

$$\mu_1' = \sum_{r=1}^{\infty} x P(x) = \sum_{r=1}^{\infty} \frac{x e^{-\lambda} \lambda^x}{x!}$$

$$\mu_1' = \sum_{x=1}^{\infty} x P(x) = \sum_{r=1}^{\infty} \frac{x e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{x e^{-\lambda} \lambda^{x-1} \lambda}{x(x-1)!}$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^{x-1} \lambda}{(x-1)!}$$

$$e^{-\lambda} \lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$e^{-\lambda}\lambda\left(1+\frac{\lambda}{1}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots\right)$$

$$e^{-\lambda}\lambda(e^{\lambda})$$

$$= \lambda$$

$$\therefore \boxed{\mu_1' = \lambda}$$

$$\mu_2' = \sum_{x=0}^{\infty} x^2 P(x)$$

$$= \sum_{x=0}^{\infty} \{x(x-1)+x\} P(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) P(x) + \sum_{x=0}^{\infty} x P(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^x}{x!} + \{E(x)\}$$

$$= \sum_{x=0}^{\infty} \cancel{x(x-1)} \frac{e^{-\lambda}\lambda^x}{\cancel{x(x-1)}(x-2)!}$$

$$\lambda^2 \cdot e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}$$

$$e^{-\lambda}\cdot\lambda^2 \cancel{\sum_{x=0}^{\infty}} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}$$

$$e^{-\lambda}\cdot\lambda^2\left(1+\frac{\lambda}{1}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots\right)$$

$$= e^{-\lambda}\lambda^2(e^{\lambda}) = \lambda^2$$

$\boxed{\cancel{\mu_2' = \lambda^2}}$

$\boxed{\mu_2' = \lambda^2 + E(x)} = \boxed{\mu_2' = \lambda^2 + \lambda}$

Varience $\mu_2 = \mu_2' - \mu_1'^2$

$= \lambda^2 + \lambda - \lambda^2 = \lambda$

$\Rightarrow$ mean $= \lambda$

varience $= \lambda$

$\mu_3' = \sum_{x=0}^{\infty} x^3 P(x) = \sum_{x=0}^{\infty} \{x(x-1)(x-2)\} P(x)$

add and sub some values to get x^3.

Define moment generating function $M_x(t)$ (MGF)

$M_x(t) = \sum_{x=0}^{\infty} e^{tx} P(x)$

$\sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda} \lambda^x}{x!}$ $x = 0, 1, 2, 3 \ldots$

$= \sum_{x=0}^{\infty} \frac{e^{-\lambda} (e^t \lambda)^x}{x!}$ for $x = 0, 1, 2 \ldots$

$e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t \lambda)^x}{x!}$

$$M_x(t) = e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t\lambda)^x}{x!}$$

$$e^{-\lambda}\left\{1+\frac{(e^t\lambda)}{1!}+\frac{(e^t\lambda)^2}{2!}+\frac{(e^t\lambda)^3}{3!}+\cdots\right\}$$

Put $x = e^t\lambda$

$$e^{-\lambda}\left\{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right\}$$

$$= e^{-\lambda}\{e^x\}$$

$$= e^{-\lambda}(e^{e^t\lambda})$$

$$= e^{\lambda(e^t-1)}$$

Cummulent generating function: $K_x(t)$

$$K_x(t) = \log_e M_x(t)$$

$$= \log_e e^{\lambda(e^t-1)}$$

$$= \lambda(e^t-1)$$

$$K_x(t) = \lambda(e^t-1)$$

$$\lambda\left\{\left(1+t+\frac{t^2}{2!}+\cdots\frac{t^r}{r!}+\cdots\right)-1\right\}$$

$$K_x(t) = \lambda\left\{t+\frac{t^2}{2!}+\cdots\frac{t^r}{r!}+\cdots\right)$$

Where rth term of above expansion

K_r = coefficient of rth term of above expansion $\left(\frac{t^r}{r!}\right)$

$$\boxed{K_r(t) = \lambda\frac{t^r}{r!}}$$

$$\boxed{K_r = \lambda}$$

Put $r = 1,2,3\ldots$

$K_1 = \lambda, \; K_2 = \lambda, \; K_3 \ldots$

$K_1 = \mu_1' = \text{mean}$

$K_2 = \mu_2' = \text{varience}$

$K_3 = \mu_3' = \lambda$

$K_4 = \mu_4' = \lambda$

$$E(x) = \frac{d}{dt}(M_x(t))_{t=0} = \frac{d}{dt}(e^{-\lambda} e^{\lambda e^t})_{t=0}$$

$$= e^{-\lambda}(e^{\lambda e^t} \lambda e^t)_{t=0} = e^{-\lambda} e^{\lambda} \lambda$$

$$\boxed{\text{Mean} = E(x) = \lambda}$$

$$E(x^2) = \frac{d^2}{dt^2}(M_x(t))_{t=0} = e^{-\lambda} \frac{d}{dt}(e^{\lambda e^t} \lambda e^t)_{t=0}$$

$$e^{-\lambda}(e^{\lambda e^t}(\lambda e^t)^2 + e^{\lambda e^t} \lambda e^t)_{t=0}$$

$$e^{-\lambda}(e^{\lambda}\lambda^2 + e^{\lambda}\lambda) \cancel{t=0}$$

$$E(x^2) = \lambda^2 + \lambda$$

Varience $V(x) = E(x^2) - (E(x))^2$

$$= \lambda^2 + \lambda - \lambda^2$$

$$= \lambda$$

$K_1 = \mu_1' = \lambda$ = mean

$K_2 = \mu_2 = \lambda$ = varience

$K_3 = \mu_3 = \lambda$

$K_4 = \cancel{\mu_4 = \lambda} = \lambda$

$\boxed{\mu_3 = K_3} = \lambda \Rightarrow \cancel{\mu_3 = \mu_3'} - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$\boxed{\mu_4 = \lambda + 3\lambda^2} \Rightarrow \mu_4 = K_4 + 3K_2^2$

$$\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{\lambda^2}{\lambda^3} = \frac{1}{\lambda}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\lambda + 3\lambda^2}{\lambda^2} = \frac{1}{\lambda} + 3$$

$$\boxed{\begin{array}{l} \beta_1 = 1/\lambda \\ \beta_2 = 3 + \frac{1}{\lambda} \end{array}}$$

Problems:-

A manifacturer of cotton pins known that 5% of his product is deffective. If he sells pins in boxes of 100 and gurentee that not more than 10 pins will be defective. What is the probability that a box will fail to meet the guarentee of quality.

Ans the manifacuter claim is 5% of his product is defective. $5\% = 0.05 = P$

We know that $np = \lambda$

$$100 \times 0.05 = 5 = \lambda$$

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-5} 5^x}{x!}$$

$$P(X=x) = \frac{e^{-5} 5^x}{x!}, \quad x = 0, 1, 2, 3 \ldots$$

the probability of the box will fail to meet the guarentee of quality is

$$P(X>10) = 1 - \left(\sum_{x=0}^{10} \frac{e^{-5} 5^x}{x!} \right)$$

for $x = 0, 1, 2 \ldots 10$

of

Q> Six coins are tossed 6,400 times using the poision distribution, find the approx probability of getting 6 heads 'r' at a time.

Sol we know that probability of getting tail or head when tossing a coin is equi probable $\Rightarrow P(H) = P(T) = 1/2$

Now when 6 coins tossed at a time then $P(H) = \left(\frac{1}{2}\right)^6$

$\boxed{np = \lambda}$ $\Rightarrow$ where p is prob of getting heads at a time

$6400\left(\frac{1}{2}\right)^6 = \lambda = 100$.

Now you can cal probability of getting heads r at a time

$$P(x=r) = \frac{e^{-\lambda}\lambda^r}{r!} \text{ for } r = 0,1,2 \ldots$$

$$= \frac{e^{-100}(100)^r}{r!} \quad \text{put } r = 0,1,2 \ldots 6 \ldots$$

$$= \cancel{\sum_{r=0}^{6} \frac{e^{-100}(100)^r}{r!}}$$

Note:- the prob of getting 0 heads is r=0
" " " 1 " is r=1
" " " 2 " is r=2
" " " 3 " is r=3
" " " " r=4
⋮ " ⋮ " 100 "
⋮

$$P(x=x) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-100}(100)^r}{r!}$$

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