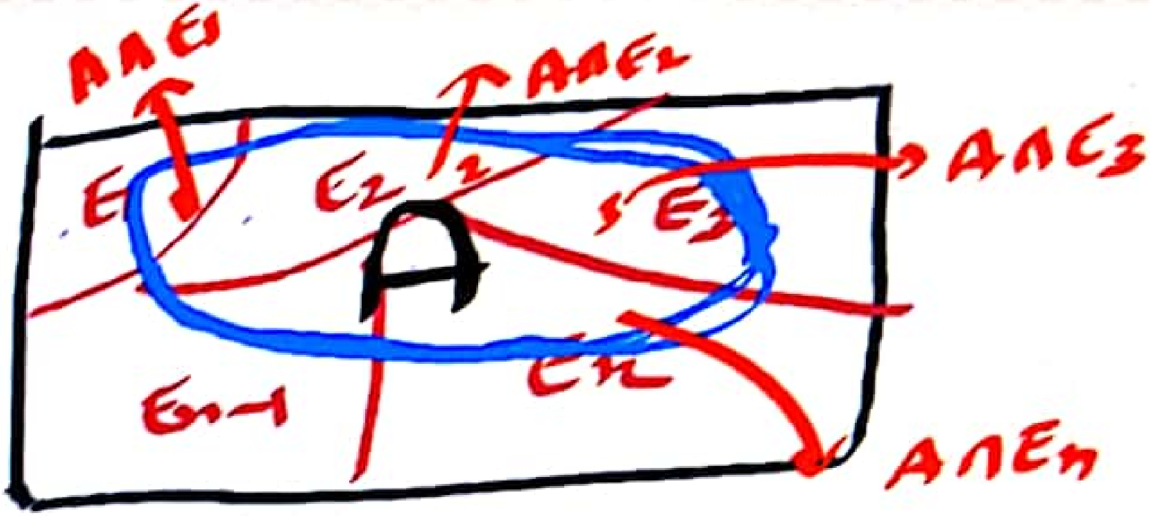
Imp: State & prove Baye's Theorem:-

Statement:- If $E_1, E_2, \ldots E_n$ are M.E ($P(E_i) \neq 0$) such that 'A' is a arbitrary event ($P(A) > 0$) which is subset of "$\bigcup_{i=1}^{n} E_i$" then

Total prob $\leftarrow$ $$P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)$$

Reverse prob $\leftarrow$ $$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

Proff :-



Since

$$A \subset \bigcup_{i=1}^{n} E_i$$

$$A = \left(A \cap \bigcup_{i=1}^{n} E_i\right)$$

$$A = \bigcup_{i=1}^{n} (A \cap E_i) \longrightarrow (1)$$

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Take prob on both sides of eqn ①

$$P(A) = P\left(\bigcup_{i=1}^{n} (A \cap E_i)\right)$$

$$= \sum_{i=1}^{n} P(A \cap E_i)$$

$$\boxed{P(A) = \sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

→ Total prob

[$A \cap E_i$ are M.E]

[$P(\bigcup E_i) = \sum P(E_i)$]

E_i's are disjoi

$$P(E_i \cap A) = P(A)\, P(E_i/A)$$

$$\boxed{P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}}$$

$\Downarrow$

Reverse prob

$$\left[P(E_1|A) + P(E_2|A) + \cdots + P(E_n|A) = 1 \right]$$

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