Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Ans: normal distribution = 45 and 8.0.

X = amount of time it takes to complete the repair on a customer's car

Pr(X > 50) = 1 - Pr(X ≤ 50)

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability of the service manager will not meet his demand will be = 100-73.4 = 26.6%

The answer is B 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.
    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. normal distribution = 38 and = 6

X = number of employees

Probabilty of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X ≤ 44)

Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345%

Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%

the probability of number of employees between 38-44 years of age =

Pr(X<44)-0.5=84.1345-0.5= 34.1345%

More employees at the processing center are older than 44 than between 38 and 44" is TRUE

Probabilty of employees less than age of 30 = Pr(X<30)

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

the number of employees with probability 0.912 of them being under age

30 = 0.0912*400=36.48(or 36 employees)

Therefore the statement B of the question is also TRUE

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans.We know ∼ N(μ1, σ1^2), and Y ∼ N(μ2, σ2^2) are two independent random variables
Then X + Y ∼ N(μ1 + μ2, σ1^2 + σ2^2) and X − Y ∼ N(μ1 − μ2, σ1^2 + σ2^2)
Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y
Then Z ∼ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2)
Therefore
2X1~ N(2 u,4 σ^2)
X1+X2 ~ N(μ + μ, σ^2 + σ^2) ~ N(2 u, 2σ^2)
2X1-(X1+X2) = N(4μ,6 σ^2)

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

Ans. As we need to find the values of a and b, symmetric about the mean, the probability of random variable value between a and b is 0.99
the Probability outside the a and b area is 0.01
The Probability towards left from a = -0.005
The Probability towards right from b = +0.005
we need to calculate X, By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.
Z=(X- μ) / σ
For Probability 0.005 the Z Value is -2.57
Z * σ + μ = X
Z(-0.005)*20+100 = -(-2.57)*20+100 = 151.4
Z(+0.005)*20+100 = (-2.57)*20+100 = 48.6
The correct answer is D 48.5, 151.5

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    stats.norm.interval(0.95,540,315)
    (-77.38865513011706, 1157.388655130117)

B. Specify the 5th percentile of profit (in Rupees) for the company

To find the 5th Percentile, we use the $X=\mu + Z\sigma$; from z table, 5 percentile = -1.64

X= 540+(-1.64)*(315)

X= 23.4

C. Which of the two divisions has a larger probability of making a loss in a given year?

Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

0.0477903522728147

Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

0.040059156863817086

Probability of Division 1 making a loss in a given year is more than Division 2.