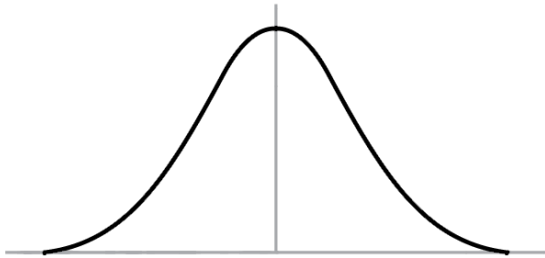


3.4 Normal Distribution Examples

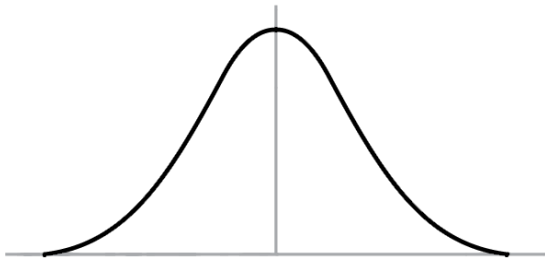
MBC 638

Data Analysis and Decision Making

Normal Distribution

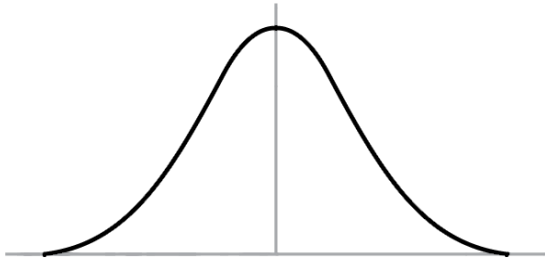


Normal Distribution



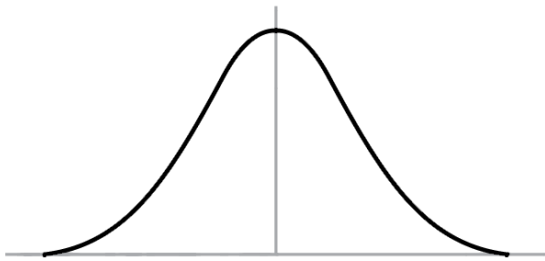
- Data: continuous

Normal Distribution



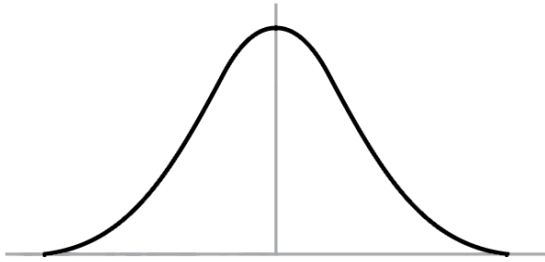
- Data: continuous
- Shape: bell curve

Normal Distribution



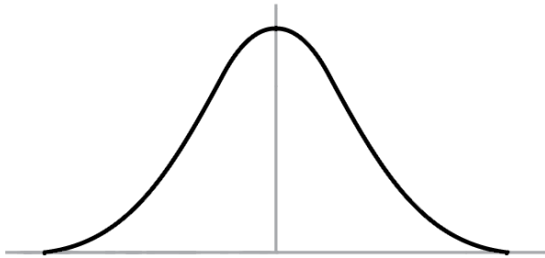
- Data: continuous
- Shape: bell curve
 - Total area under curve = 1

Normal Distribution



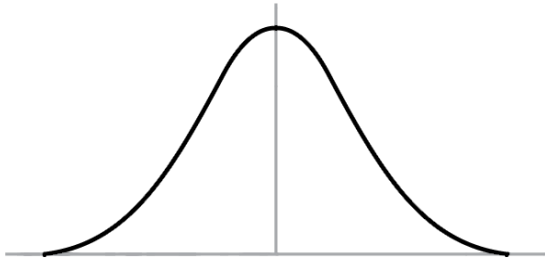
- Data: continuous
- Shape: bell curve
 - Total area under curve = 1
- Formula: $Z = \frac{x - \mu}{\sigma}$

Normal Distribution



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- Shape: bell curve
 - Total area under curve = 1
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- Mean: μ

Normal Distribution



- Data: continuous
- Shape: bell curve
 - Total area under curve = 1
- Formula: $Z = \frac{x - \mu}{\sigma}$
- Mean: μ
- Variance: standard deviation, or σ^2

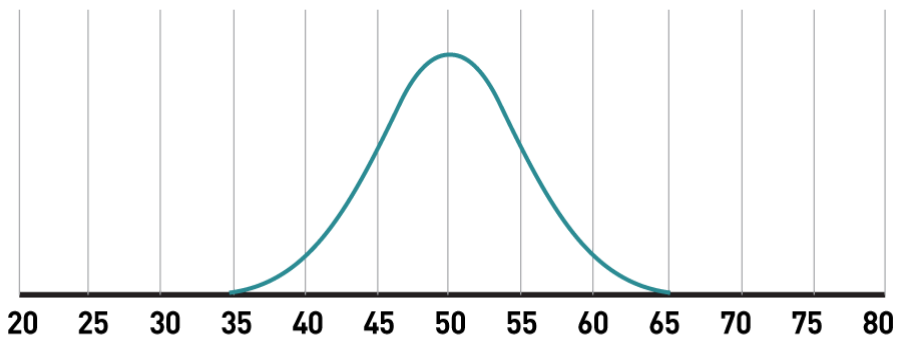
Example 1: Student Height



Example 1: Student Height

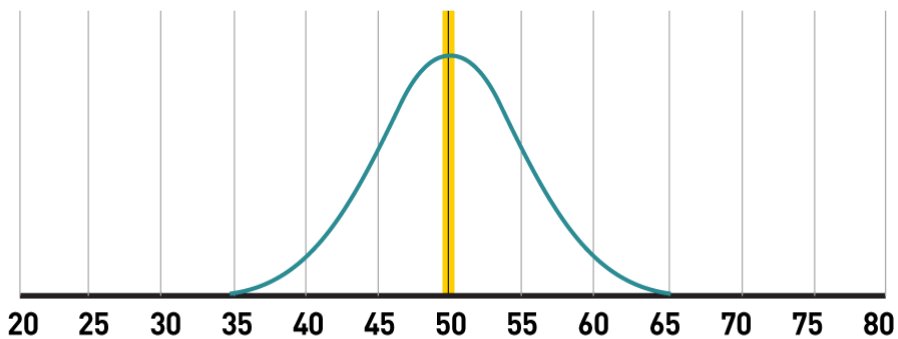
What is the probability of elementary students shorter than 5 feet?

Distribution of Elementary Student Heights (inches)



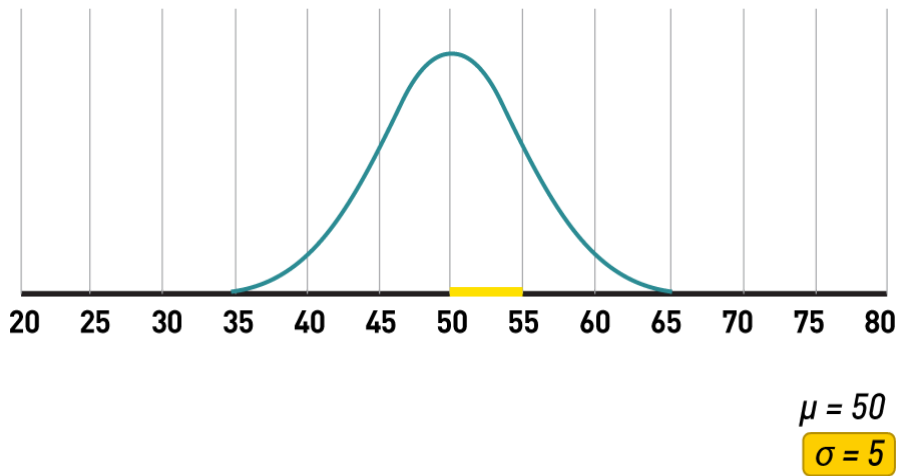
$$\mu = 50$$
$$\sigma = 5$$

Distribution of Elementary Student Heights (inches)



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$$\sigma = 5$$

Distribution of Elementary Student Heights (inches)



Example 1: Convert to Z-Value

- We need to convert these values to the standard normal.

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- We need to convert these values to the standard normal.
- In this case, we need a Z-score (our test statistic).

$$\circ Z = \frac{60-50}{5} = 2$$

- Use the Z-value to find the probability.

Example 1: Standard Normal Table

Z	0.00	0.01	0.02	0.03	0.04	0.05	...
⋮				⋮			
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	...
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	
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⋮			⋮				

97.72% of elementary students are shorter than 5 feet.

Example 2: Average Winter Temperature

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- Average winter temperature: 10°

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- Standard deviation: 5°

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What's the probability of a temperature between 3° and 18° ?

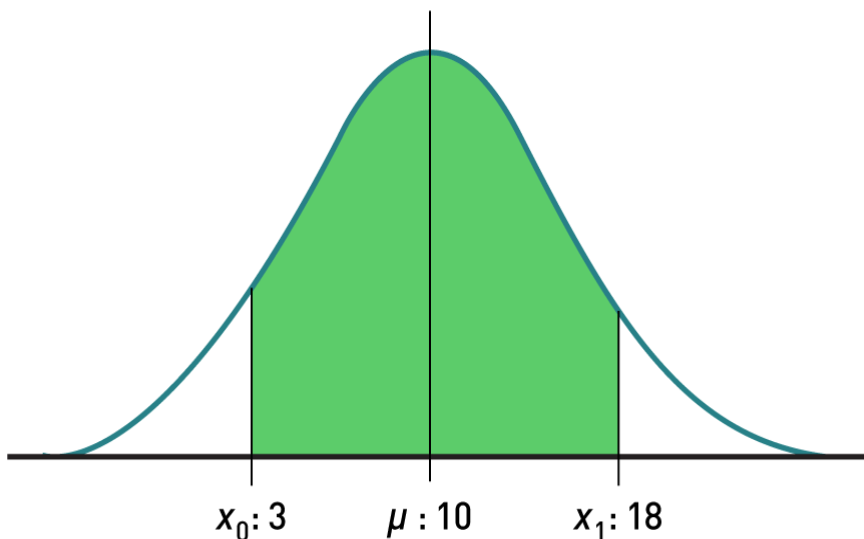
Example 2: Average Winter Temperature

- Average winter temperature: 10°
- Standard deviation: 5°

What's the probability of a temperature between 3° and 18° ?

- The area under the curve between 3° and 18°

Example 2: Average Winter Temperature



Example 2: Convert to Z-Values**Example 2: Convert to Z-Values**

- $Z = \frac{x - \mu}{\sigma}$

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- $Z = \frac{x - \mu}{\sigma}$
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Example 2: Convert to Z-Values

- $Z = \frac{x - \mu}{\sigma}$
- $Z_0 = \frac{3 - 10}{5} = -1.4$

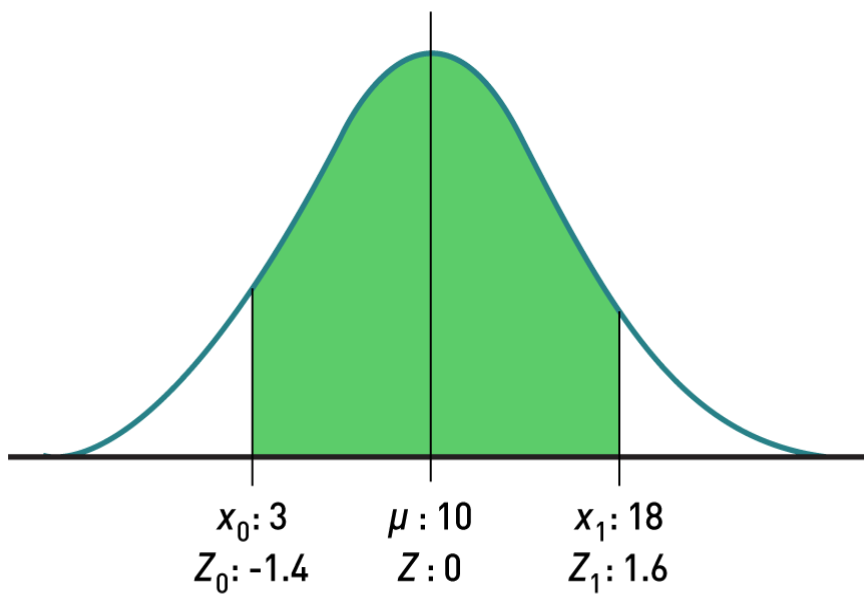
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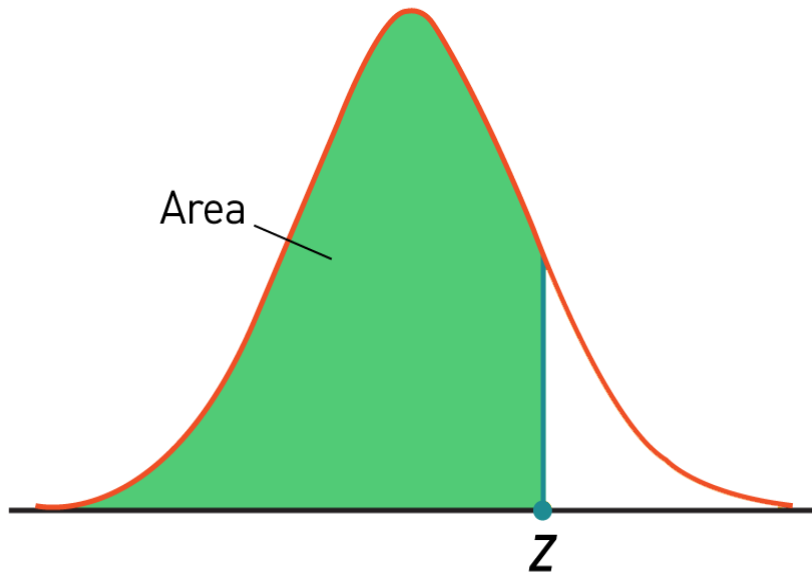
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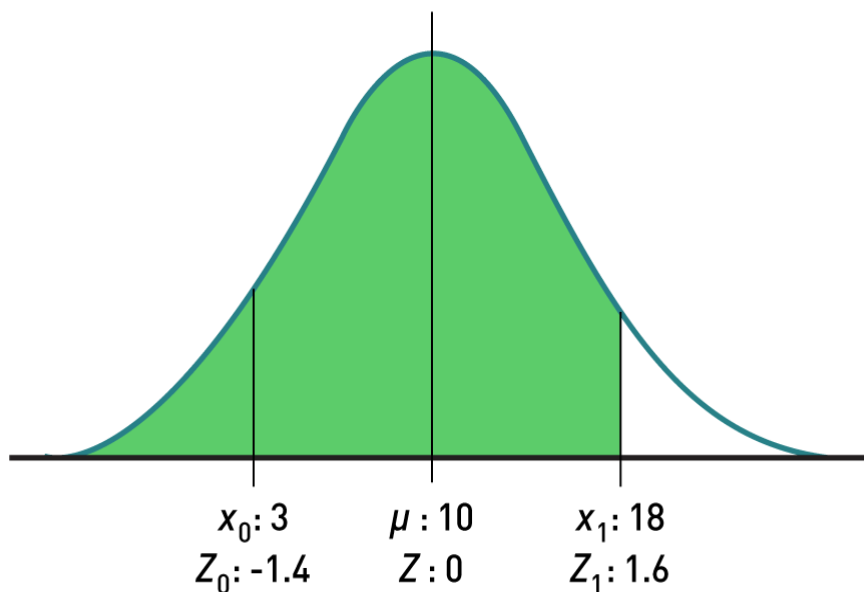
Standard Normal Table: Area Left of Z



Example 2: Probabilities and Solution

- Probability left of Z_1 (18°): 0.9452

Example 2: Average Winter Temperature



Example 2: Probabilities and Solution

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Example 2: Probabilities and Solution

- Probability left of Z_1 (18°): 0.9452

Example 2: Probabilities and Solution

- Probability left of Z_1 (18°): 0.9452
- Probability left of Z_0 (3°): 0.0808

Example 2: Probabilities and Solution

- Probability left of Z_1 (18°): 0.9452
- Probability left of Z_0 (3°): 0.0808
- The space between Z_0 and Z_1 (3° and 18°): $0.9452 - 0.0808$

Example 2: Probabilities and Solution

- Probability left of Z_1 (18°): 0.9452
- Probability left of Z_0 (3°): 0.0808
- The space between Z_0 and Z_1 (3° and 18°): $0.9452 - 0.0808$
- Chance of weather between 3° and 18° : 0.8644, or 86.44%.