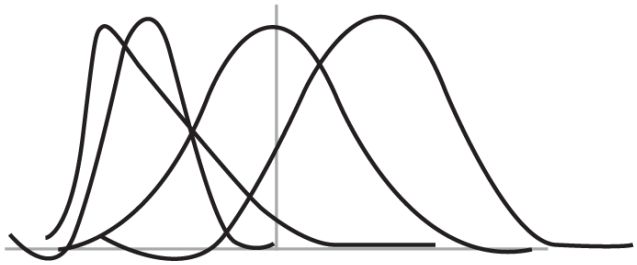
3.5 Binomial Distribution Examples

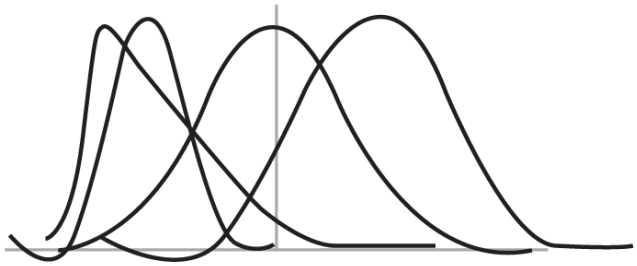
MBC 638

Data Analysis and Decision Making

Binomial Distribution

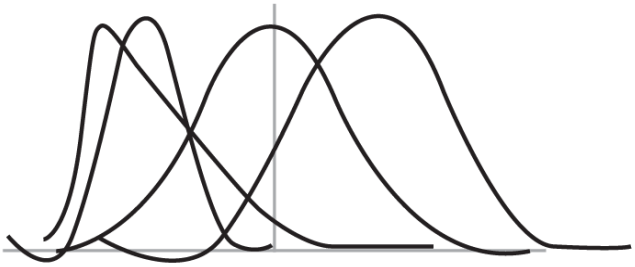


Binomial Distribution



- Shape: dependent on *n* and *p*

Binomial Distribution



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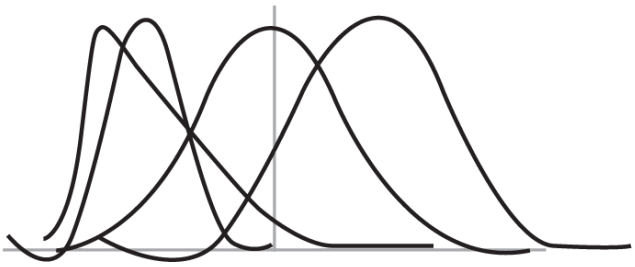
Binomial Distribution



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  - n = number of trials or samples
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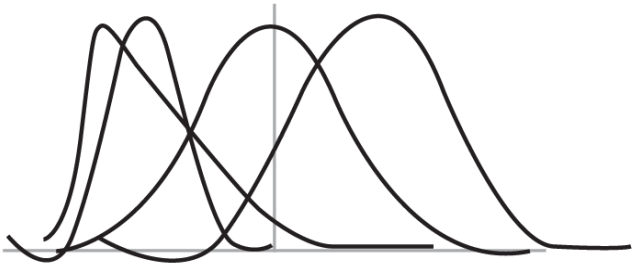

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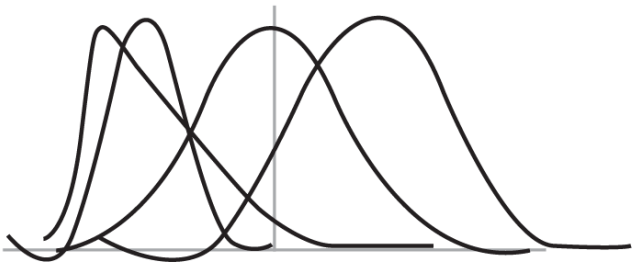

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- Variance: $\sigma^2 = n \times p(1 - p)$
- Data: discrete; only two possible outcomes

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4	0	0.6561	0.5220	0.4096		0.1296	0.0915	0.0625
	1	0.2916	0.3685	0.4096		0.3456	0.2995	0.2500
	2	0.0486	0.0975	0.1536		0.3456	0.3675	0.3750
	3	0.0036	0.0115	0.0256		0.1536	0.2005	0.2500
	4	0.0001	0.0005	0.0016		0.0256	0.0410	0.0625
5	0	0.5905	0.4437	0.3277	…	0.0778	0.0503	0.0312
	1	0.3280	0.3915	0.4096		0.2592	0.2059	0.1562
	2	0.0729	0.1382	0.2048		0.3456	0.3369	0.3125
	3	0.0081	0.0244	0.0512		0.2304	0.2757	0.3125
	4	0.0004	0.0022	0.0064		0.0768	0.1128	0.1562
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The probability of answering three questions correctly is 0.3125.

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3. Probability of success is equivalent for all trials.