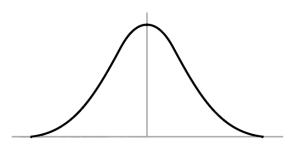
3.4 Normal Distribution Examples

MBC 638

Data Analysis and Decision Making

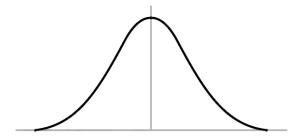
2 of 50

Normal Distribution



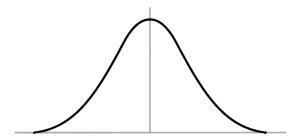
3 of 50

Normal Distribution



• Data: continuous

Normal Distribution

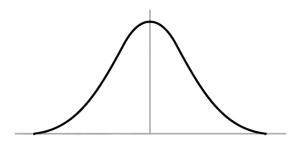


Data: continuous

• Shape: bell curve

5 of 50

Normal Distribution

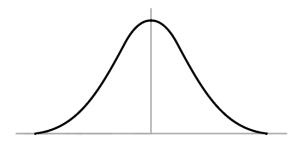


Data: continuous

• Shape: bell curve

∘ Total area under curve = 1

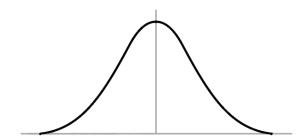
Normal Distribution



- Data: continuous
- Shape: bell curve
 - ∘ Total area under curve = 1
- Formula: $Z = \frac{x \mu}{\sigma}$

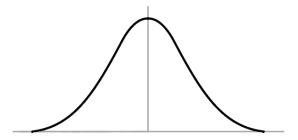
7 of 50

Normal Distribution



- Data: continuous
- Shape: bell curve
 - ∘ Total area under curve = 1
- Formula: $Z = \frac{x \mu}{\sigma}$
- Mean: μ

Normal Distribution



- Data: continuous
- Shape: bell curve
 - ∘ Total area under curve = 1
- Formula: $Z = \frac{x \mu}{\sigma}$
- Mean: μ
- Variance: standard deviation, or σ^2

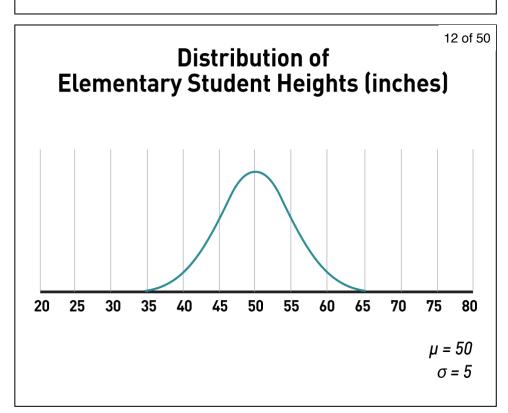
9 of 50

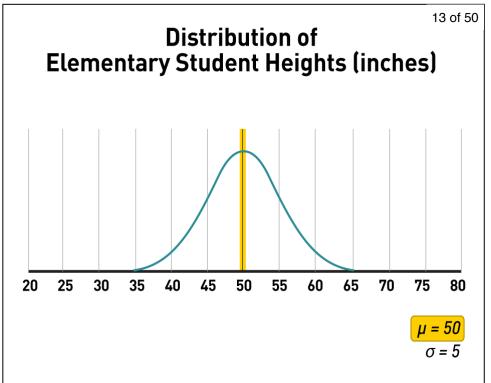
Example 1: Student Height

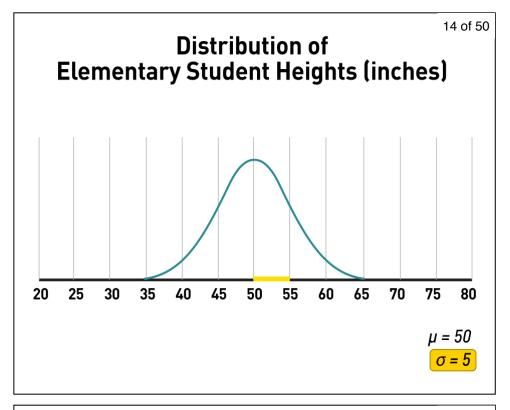


Example 1: Student Height

What is the probability of elementary students shorter than 5 feet?







15 of 50

Example 1: Convert to Z-Value

 We need to convert these values to the standard normal.

16 of 50

Example 1: Convert to Z-Value

- We need to convert these values to the standard normal.
- In this case, we need a Z-score (our test statistic).

Example 1: Convert to Z-Value

- We need to convert these values to the standard normal.
- In this case, we need a Z-score (our test statistic).

$$\circ Z = \frac{60-50}{5} = 2$$

18 of 50

Example 1: Convert to Z-Value

- We need to convert these values to the standard normal.
- In this case, we need a Z-score (our test statistic).

$$\circ Z = \frac{60-50}{5} = 2$$

• Use the Z-value to find the probability.

19 of 50

Example 1: Standard Normal Table

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | |
|-----|--------|--------|--------|--------|--------|--------|--|
| ÷ | | | | : | | | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | |
| : | | | | : | | | |

| | | | | | | | 20 of 50 | | |
|----------------------------------|--------|---------|--------|---------|--------|--------|----------|--|--|
| Fxa | mnle 1 | · Stand | ard No | rmal Ta | hle | | | | |
| Example 1: Standard Normal Table | | | | | | | | | |
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | | | |
| ÷ | | | | : | | | | | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | | | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | | | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | | | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | | | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | | | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | | | |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | | | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | | | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | | | |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | | | |
| ; | | | | : | | | | | |

| | | | | | | | 21 of 50 | | |
|----------------------------------|--------|---------|---------|---------|--------|--------|----------|--|--|
| Evo | mnla 1 | . Stand | ard Nai | rmal Ta | hlo | | | | |
| Example 1: Standard Normal Table | | | | | | | | | |
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | | | |
| ÷ | | | | : | | | | | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | | | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | | | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | | | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | | | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | | | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | | | |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | | | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | | | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | | | |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | | | |
| ÷ | | | | : | | | | | |
| | | | | | | | | | |

| | | | | | | | 22 of 50 | | |
|----------------------------------|--------|---------|--------|---------|--------|--------|----------|--|--|
| Fxa | mnle 1 | · Stand | ard No | rmal Ta | hle | | | | |
| Example 1: Standard Normal Table | | | | | | | | | |
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | | | |
| ÷ | | | | : | | | | | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | | | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | | | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | | | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | | | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | | | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | | | |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | | | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | | | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | | | |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | | | |
| ÷ | | | | : | | | | | |

23 of 50 **Example 1: Standard Normal Table** Z 0.00 0.02 0.03 0.01 0.04 0.05 : 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9452 1.6 0.9463 0.9474 0.9484 0.9495 0.9505 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9641 0.9649 0.9671 1.8 0.9656 0.9664 0.9678 0.9713 1.9 0.9719 0.9726 0.9732 0.9738 0.9744 0.9772 0.9788 0.9798 2.0 0.9778 0.9783 0.9793 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 :

| | | | | | | | 04 - 150 |
|---------------|--------|---------|---------|-------------|--------|--------|----------|
| | | | | | | | 24 of 50 |
| Exa | mple 1 | : Stand | ard Noi | rmal Ta | ble | | |
| Z | 0.00 | 0.01 | 0.00 | 0.02 | 0.04 | 0.05 | |
| 2 : | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | ••• |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | ••• |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | |

:

25 of 50 **Example 1: Standard Normal Table** Z 0.00 0.01 0.02 0.03 0.04 0.05 : 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9452 1.6 0.9463 0.9474 0.9484 0.9495 0.9505 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9641 0.9649 0.9664 0.9671 1.8 0.9656 0.9678 0.9713 1.9 0.9719 0.9726 0.9732 0.9738 0.9744 0.9772 0.9788 0.9793 0.9798 2.0 0.9778 0.9783 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 2.4 0.9918 0.9925 0.9920 0.9922 0.9927 0.9929 :

| | | | | | | | 26 of 50 | | |
|----------------------------------|--------|---------|--------|---------|--------|--------|----------|--|--|
| Exa | mple 1 | : Stand | ard No | rmal Ta | ble | | | | |
| Example 1: Standard Normal Table | | | | | | | | | |
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | | | |
| ÷ | | | | : | | | | | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | | | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | | | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | | | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | | | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | | | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | ••• | | |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | | | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | | | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | | | |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | | | |
| ÷ | | | | : | | | | | |
| | | | | | | | | | |

27 of 50

97.72% of elementary students are shorter than 5 feet.

28 of 50

Example 2: Average Winter Temperature

29 of 50

Example 2: Average Winter Temperature

• Average winter temperature: 10°

30 of 50

Example 2: Average Winter Temperature

• Average winter temperature: 10°

Standard deviation: 5°

Example 2: Average Winter Temperature

Average winter temperature: 10°

• Standard deviation: 5°

What's the probability of a temperature between 3° and 18°?

32 of 50

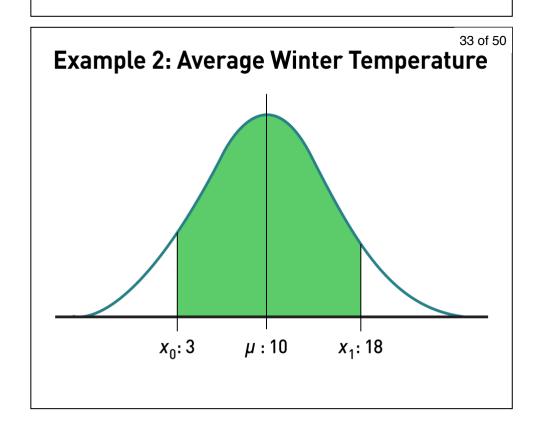
Example 2: Average Winter Temperature

Average winter temperature: 10°

Standard deviation: 5°

What's the probability of a temperature between 3° and 18°?

The area under the curve between 3° and 18°



Example 2: Convert to Z-Values

35 of 50

Example 2: Convert to Z-Values

•
$$Z = \frac{x - \mu}{\sigma}$$

36 of 50

Example 2: Convert to Z-Values

•
$$Z = \frac{x - \mu}{\sigma}$$

•
$$Z_0 = \frac{3-10}{5}$$

37 of 50

Example 2: Convert to Z-Values

•
$$Z = \frac{x-\mu}{\sigma}$$

•
$$Z_0 = \frac{3-10}{5} = -1.4$$

38 of 50

Example 2: Convert to Z-Values

•
$$Z = \frac{x - \mu}{\sigma}$$

•
$$Z_0 = \frac{3-10}{5} = -1.4$$

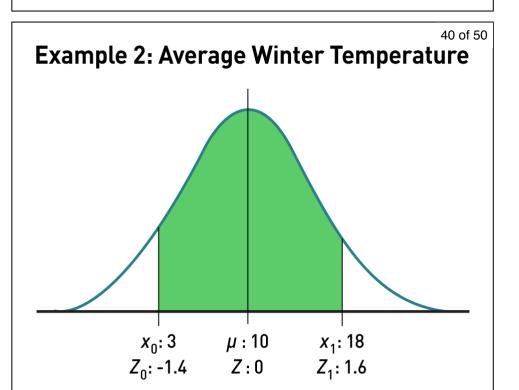
•
$$Z_1 = \frac{18-10}{5}$$

Example 2: Convert to Z-Values

•
$$Z = \frac{x - \mu}{\sigma}$$

•
$$Z_0 = \frac{3-10}{5} = -1.4$$

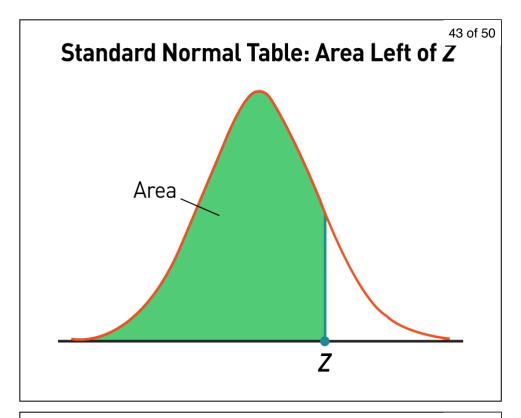
•
$$Z_1 = \frac{18-10}{5} = 1.6$$



| | | | | | | | _ |
|-----|--------|---------|--------|---------|--------|--------|----------|
| | | | | | | | 41 of 50 |
| Exa | mple 2 | : Stand | ard No | rmal Ta | ble | | |
| | • | | | | | | |
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | |
| : | | | | : | | | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | |
| | | | | | | | |

:

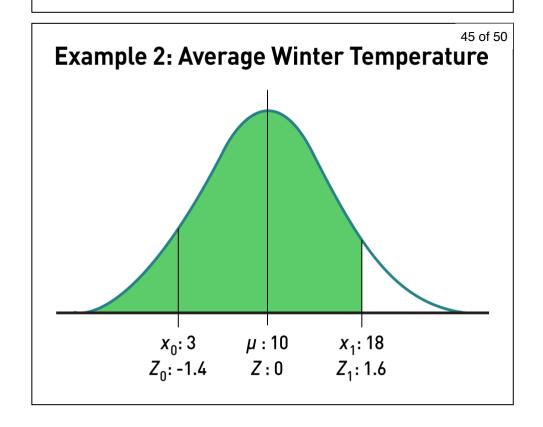
42 of 50 **Example 2: Standard Normal Table** Z 0.00 0.02 0.03 0.01 0.04 0.05 : 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9452 1.6 0.9463 0.9474 0.9484 0.9495 0.9505 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9641 0.9649 0.9671 1.8 0.9656 0.9664 0.9678 0.9713 1.9 0.9719 0.9726 0.9732 0.9738 0.9744 0.9772 0.9788 0.9798 2.0 0.9778 0.9783 0.9793 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 :



44 of 50

Example 2: Probabilities and Solution

• Probability left of Z₁ (18°): 0.9452



Example 2: Probabilities and Solution

Probability left of Z₁ (18°): 0.9452

47 of 50

Example 2: Probabilities and Solution

• Probability left of Z₁ (18°): 0.9452

48 of 50

Example 2: Probabilities and Solution

- Probability left of Z₁ (18°): 0.9452
- Probability left of Z₀ (3°): 0.0808

49 of 50

Example 2: Probabilities and Solution

- Probability left of Z₁ (18°): 0.9452
- Probability left of Z₀ (3°): 0.0808
- The space between Z₀ and Z₁ (3° and 18°): 0.9452 0.0808

Example 2: Probabilities and Solution

- Probability left of Z₁ (18°): 0.9452
- Probability left of Z₀ (3°): 0.0808
- The space between Z₀ and Z₁ (3° and 18°): 0.9452 0.0808
- Chance of weather between 3° and 18°: 0.8644, or 86.44%.