9.5 Three Time Series Models

MBC 638

Data Analysis and Decision Making

Time Series Models

Time Series Models

1. First-order autoregressive model, a.k.a. AR(1)

Time Series Models

1. First-order autoregressive model, a.k.a. AR(1)
2. Moving average forecast model

Time Series Models

1. First-order autoregressive model, a.k.a. AR(1)
2. Moving average forecast model
3. Exponential smoothing model

Autoregressive Model: AR(1)

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series
- **First-order autoregressive model**

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series
- **First-order autoregressive model**
    - Linear regression equation: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series
- **First-order autoregressive model**
    - Linear regression equation: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$
    - y_t = output at time t

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series
- **First-order autoregressive model**
  - Linear regression equation: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$
  - y_t = output at time t
    - Example: March; $t = 3$

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series
- **First-order autoregressive model**
  - Linear regression equation: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$
  - y_t = output at time t
    - Example: March; $t = 3$
    - $y_3 = \beta_0 + \beta_1 y_{3-1} + \varepsilon_3$

Autoregressive Model: AR(1)

- Takes advantage of linear relationship between successive values of time series
- **First-order autoregressive model**
  - Linear regression equation: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$
  - y_t = output at time t
    - Example: March; $t = 3$
    - $y_3 = \beta_0 + \beta_1 y_{3-1} + \varepsilon_3$
    - I.e., predicting March data by using February data

Moving Average Model

Moving Average Model

A.k.a. "rolling average method"; smooths out short-term fluctuations

Moving Average Model

A.k.a. "rolling average method"; smooths out short-term fluctuations

- Uses average of last several values of time series to forecast next value; k = number of values in span

Moving Average Model

A.k.a. "rolling average method"; smooths out short-term fluctuations

- Uses average of last several values of time series to forecast next value; k = number of values in span
  - Example: Monthly data, span (k) = 3

Moving Average Model

A.k.a. "rolling average method"; smooths out short-term fluctuations

- Uses average of last several values of time series to forecast next value; k = number of values in span
    - Example: Monthly data, span (k) = 3
        - I.e., use average of values from January, February, and March to predict April value

Moving Average Model

A.k.a. "rolling average method"; smooths out short-term fluctuations

- Uses average of last several values of time series to forecast next value; k = number of values in span
    - Example: Monthly data, span (k) = 3
        - I.e., use average of values from January, February, and March to predict April value

- Can look back more than one time period

Moving Average Model

A.k.a. "rolling average method"; smooths out short-term fluctuations

- Uses average of last several values of time series to forecast next value; k = number of values in span
    - Example: Monthly data, span (k) = 3
        - I.e., use average of values from January, February, and March to predict April value

- Can look back more than one time period

- Disadvantage: If, say, n = 100 and k = 5, forecast overlooks 95% of available data

Exponential Smoothing Model

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally
- Forecasting equation: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally
- Forecasting equation: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
  - $\hat{y}_t$ = estimate of y at time period t

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally
- Forecasting equation: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
  - $\hat{y}_t$ = estimate of y at time period t
    - Example: t = February

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally
- Forecasting equation: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
  - $\hat{y}_t$ = estimate of y at time period t
    - Example: t = February
  - w = smoothing constant

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally
- Forecasting equation: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
  - $\hat{y}_t$ = estimate of y at time period t
    - Example: t = February
  - w = smoothing constant
    - Your choice, pick any number between 0 and 1

Exponential Smoothing Model

- Best suited for forecasting time series without seasonal variation
- Unlike moving average model, data values not all weighted equally
- Forecasting equation: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
  - $\hat{y}_t$ = estimate of y at time period t
    - Example: t = February
  - w = smoothing constant
    - Your choice, pick any number between 0 and 1
    - Example: w = 80%

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
- February forecast = $\hat{y}_t$

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
- February forecast = $\hat{y}_t$

- 80% of January value = $wy_{t-1} = 0.8\ y_{t-1}$

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
- February forecast = $\hat{y}_t$
- 80% of January value = $wy_{t-1} = 0.8\ y_{t-1}$

- 20% of January forecast = $(1 - w)\ \hat{y}_{t-1}$
  $= (1 - 0.8)\ \hat{y}_{t-1} = 0.2\ \hat{y}_{t-1}$

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
- February forecast = $\hat{y}_t$
- 80% of January value = $wy_{t-1} = 0.8\ y_{t-1}$
- 20% of January forecast = $(1 - w)\ \hat{y}_{t-1}$
  $= (1 - 0.8)\ \hat{y}_{t-1} = 0.2\ \hat{y}_{t-1}$

- January forecast = $\hat{y}_{t-1} = \hat{y}_{2-1}$

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
- February forecast = $\hat{y}_t$
- 80% of January value = $wy_{t-1} = 0.8\ y_{t-1}$
- 20% of January forecast = $(1 - w)\ \hat{y}_{t-1}$
  $= (1 - 0.8)\ \hat{y}_{t-1} = 0.2\ \hat{y}_{t-1}$
- January forecast = $\hat{y}_{t-1} = \hat{y}_{2-1}$
  - January forecast incorporates data from December and prior months

Exponential Smoothing Model: Example

- Formula: $\hat{y}_t = wy_{t-1} + (1 - w)\hat{y}_{t-1}$
- February forecast = $\hat{y}_t$
- 80% of January value = $wy_{t-1} = 0.8\ y_{t-1}$
- 20% of January forecast = $(1 - w)\ \hat{y}_{t-1}$
  $= (1 - 0.8)\ \hat{y}_{t-1} = 0.2\ \hat{y}_{t-1}$
- January forecast = $\hat{y}_{t-1} = \hat{y}_{2-1}$
  - January forecast incorporates data from December and prior months
- Final equation: $\hat{y}_2 = 0.8\ y_{2-1} + (1 - 0.8)\ \hat{y}_{2-1}$

Exponential Smoothing Notes

Exponential Smoothing Notes

- The smaller the *w*, the greater its smoothing effect

Exponential Smoothing Notes

- The smaller the *w*, the greater its smoothing effect
  - Smoothing constant always between 0 and 1

Exponential Smoothing Notes

- The smaller the *w*, the greater its smoothing effect
  - Smoothing constant always between 0 and 1
  - Larger smoothing constant → more fluctuation → closer to actual data

Exponential Smoothing Notes

- The smaller the *w*, the greater its smoothing effect
  - Smoothing constant always between 0 and 1
  - Larger smoothing constant → more fluctuation → closer to actual data
- Excel uses "damping constant": $(1 - w)$