1) parameter p based on a Bernoulli(p) sample of size n

Given:

$D = \{x1, x2..xn\}$, where m heads(1), n-m tails(0)

$prob(D|p) = \prod(N = 1 \text{ to } n)\ prob(xn|p) = \prod(N = 1 \text{ to } n)\ p^{xn} (1-p)^{(1-xn)}$

$\ln prob(D|p) = \sum(N = 1 \text{ to } n) \ln p(xn|p) = \sum(N = 1 \text{ to } n) \{xn \ln p + (1-xn) \ln(1-p)\}$

$p\ ML = (1/n) \sum(N = 1 \text{ to } n) xn$

$p\ ML = m/n$

2) parameter p based on a Binomial (N, p) sample of size n. Compute your estimators if the observed sample is (3, 6, 2, 0, 0, 3) and N = 10.

$D = \{x1, x2..xn\}$

$f(x1, x2..xn) = \prod(i = 1 \text{ to } n) \binom{N}{Xi}\ p^{xi} (1-p)^{N-xi}$

$\ln f(x1, x2..xn) = \sum(N = 1 \text{ to } n) \binom{N}{Xi} + \sum(N = 1 \text{ to } n) Xi \ln p + \sum(N = 1 \text{ to } n) (N-Xi) \ln(1-p)$

$\text{Partial } \frac{d}{dp} (\ln f(x1..xn)) = \sum(N = 1 \text{ to } n) xi/p - \sum(N = 1 \text{ to } n) (N-xi)/(1-p)$

$= (nX)/p - (nN - nX)/(1-p) = 0$ (X is the mean of input data(x1,x2..xn))

Solving this equation,

$(1-p)X = p(N-X) \Rightarrow p = X/N$

For given data and N

$X = (3+6+2+0+0+3)/6 = 14/6$

$pML = (14/6)/10$

$pML = 7/30$

3) parameters a and b based on a Uniform (a, b) sample of size n.

$f(x1, x2..xn) = \{ (1/(b-a))^n$ if $a \le x1 \ldots xn \le b$

Function is increasing in a and decreasing in b. It is maximized at the largest value of a and the smallest value of b where this density is not 0. These are

$aML = \min(xi)$ $bML = \max(xi)$

4) parameter μ based on a Normal(μ, σ^2) sample of size n with known variance σ 2 and unknown mean μ.

Joint density function is

$f(X1, . . . , Xn) = \prod i=1 \text{ to } n \quad 1/ (\sigma.\sqrt{2\pi}) \exp \{ -(Xi - \mu)^2/ 2\sigma^2\}$

taking log

$\ln f(X1.. Xn) = -n \ln(\sigma.\sqrt{2\pi}) - \sum i = 1 \text{ to } n -(Xi - \mu)^2/ 2\sigma^2$

$= -n \ln(\sigma.\sqrt{2\pi}) – (n \mu^2 – 2 \sum Xi \mu + \sum Xi^2)/2\sigma^2$

Is parabola in terms of μ and is max at **ML $\mu = \sum Xi/n$**

5) parameter σ based on a Normal(μ, σ 2) sample of size n with known mean μ and unknown variance σ 2 .

Joint density function is

$f(X1, . . . , Xn) = \prod i=1 \text{ to } n \quad 1/ (\sigma.\sqrt{2\pi}) \exp \{ -(Xi - \mu)^2/ 2\sigma^2\}$

partial differentiation with respect to σ

Partial $\frac{d}{dp}$ $(\ln f(x1..xn)) = - n/ \sigma + \sum (Xi – \mu)^2/ \sigma^3 = 0$

On taking root, we will get maximize the density so that

ML $\sigma = \sqrt{\sum(Xi- \mu)^2/n}$

6) parameters (μ, σ^2) based on a Normal(μ, σ^2) sample of size n with unknown mean μ and variance σ^2

Joint density function is

$f(X1, . . . , Xn) = \prod i=1 \text{ to } n \quad 1/ (\sigma.\sqrt{2\pi}) \exp \{ -(Xi - \mu)^2/ 2\sigma^2\}$

maximizing the function from 4^{th} question, ML $\mu = \sum Xi/n$ for all the values of σ

Then, substitute the obtained maximizer into ln f(X1, . . . , Xn) for the unknown value of μ and maximize the resulting function in terms of σ

ML $\sigma = \sqrt{(\sum(xi - X)^2)/n}$