STRENGTH OF MATERIALS

(M–SCHEME)

N. IYANARAPPAN, M.E., M.I.S.T.E.

கல் பதிப்பகம்
வேலூர்- 632 011

(Strength of Materials)

First Edition : November 2003
Revised Edition : October 2008
Revised Edition : June 2016
Revised Edition : June 2019

Price : ₹ 168.00

For Contact :
99446 50380
96266 26747

Publisher :
KAL PATHIPPAGAM
Vellore – 632 011

Type setting :
Students' Media Computer Graphics
Vellore – 632 011.

PREFACE

This book on **Strength of Materials** has been written to cover the latest revised syllabus for the Polytechnic college students of III Semester Mechanical, Automobile, Production and Mechatronics Engineering.

While preparing this book, special care has been taken for solving the problems and presenting them neatly. Almost all the problems which are asked in the previous board examinations are solved in this book. I assure that the review questions and number of problems for practice added at the end of each chapter will be more helpful to the students while preparing for the examination.

I acknowledge my gratitude with thanks to **M/s. KAL PATHIPPAGAM** for their kind encouragement to bring out this book in time. The author would be very glad and thankful to receive any comments and constructive suggestions for the improvement of this book.

N. Iyanarappan
(iyanarvlr1976@gmail.com)

All the best...

32031 - STRENGTH OF MATERIALS

DETAILED SYLLABUS

Unit – I : STATICS OF PARTICLES

Introduction – Force - effects of a force - system of forces - resultant of force - Principle of transmissibility - parallelogram law of forces -triangular law - resultant of several forces acting on a particle - polygon law - resolution of a force into rectangular components – resultant of a system of forces acting on a particle using rectangular components - equilibrium of particles.

External and internal forces - moment of a force - Varignon's theorem - moment of a couple - equivalent couples - addition of couples - resolution of a force into a force and a couple - Free body diagram - Necessary and sufficient conditions for the equilibrium of rigid bodies in two dimension - Support reaction - types of support - removal of two dimensional supports - Simple problems only.

Friction : Introduction - Definition - Force of friction - Limiting friction - Static friction - Dynamic friction - Angle of friction - co-efficient of friction - Laws of static and dynamic friction.

Unit – II : DEFORMATION OF METALS

Mechanical properties of materials : Engineering materials – Ferrous and non-ferrous materials - Definition of mechanical properties - Alloying elements-effect of alloying element - Fatigue, fatigue strength, creep – temperature creep – cyclic loading and repeated loading – endurance limit.

Simple stresses and strains: Definition – Load, stress and strain – Classification of force systems – tensile, compressive and shear force systems – Behaviour of mild steel in tension up to rupture – Stress – Strain diagram – limit of proportionality – elastic limit – yield stress – breaking stress – Ultimate stress – percentage of elongation and percentage reduction in area – Hooke's law – Definition – Young's modulus - working stress, factor of safety, load factor, shear stress and shear strain - modulus of rigidity. Linear strain

– Deformation due to tension and compressive force – Simple problems in tension, compression and shear force.

Definition – Lateral strain – Poisson's ratio – volumetric strain – bulk modulus – volumetric strain of rectangular and circular bars – problems connecting linear, lateral and volumetric deformation – Elastic constants and their relationship - Problems on elastic constants - Definition – Composite bar – Problem in composite bars subjected to tension and compression – Temperature stresses and strains – Simple problems – Definition – strain energy – proof resilience – modulus of resilience – The expression for strain energy stored in a bar due to Axial load – Instantaneous stresses due to gradual, sudden, impact and shock loads – Problems computing instantaneous stress and deformation in gradual, sudden, impact and shock loadings.

Unit – III : GEOMETRICAL PROPERTIES OF SECTIONS AND THIN SHELLS

Properties of sections : Definition – center of gravity and centroid - position of centroids of plane geometrical figures such as rectangle, triangle, circle and trapezium-problems to determine the centroid of angle, channel, T and I sections only - Definition-centroidal axis-Axis of symmetry. Moment of Inertia – Statement of parallel axis theorem and perpendicular axis theorem. Moment of Inertia of lamina of rectangle, circle, triangle, I and channel sections-Definition-Polar moment of Inertia-radius of gyration – Problems computing moment of inertia and radius of gyration for angle, T, Channel and I sections.

Thin Shells : Definition – Thin and thick cylindrical shell – Failure of thin cylindrical shell subjected to internal pressure – Derivation of Hoop and longitudinal stress causes in a thin cylindrical shell subjected to internal pressure – simple problems – change in dimensions of a thin cylindrical shell subjected to internal pressure – problems – Derivation of tensile stress induced in a thin spherical shell subjected to internal pressure – simple problems – change in diameter and volume of a thin spherical shell due to internal pressure – problems.

Unit – IV: SF AND BM DIAGRAMS OF BEAMS AND THEORY OF BENDING

Classification of beams – Definition – shear force and Bending moment – sign conventions for shear force and bending moment – types of loadings – Relationship between load, force and bending moment at a section – shear force diagram and bending moment diagram of cantilever and simply supported beam subjected to point load and uniformly distributed load (udl) – Determination of Maximum bending moment in cantilever beam and simply supported beam when they are subjected to point load and uniformly distributed load.

Theory of simple bending – Assumptions – Neutral axis – bending stress distribution – moment of resistance – bending equation $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ – Definition – section modulus – rectangular and circular sections – strength of beam – simple problems involving flexural formula for cantilever and simple supported beam.

Unit – V: THEORY OF TORSION AND SPRINGS

Theory of torsion – Assumptions – torsion equation – strength of solid and hollow shafts – power transmitted – Definition – Polar modulus – Torsional rigidity – strength and stiffness of shafts – comparison of hollow and solid shafts in weight and strength considerations – Advantages of hollow shafts over solid shafts – Problems.

Types of springs – Laminated and coiled springs and applications – Types of coiled springs – Difference between open and closely coiled helical springs – closely coiled helical spring subjected to an axial load – problems to determine shear stress, deflection, stiffness and resilience of closed coiled helical springs.

Contents

SYMBOLS USED IN THIS BOOK

Symbol	*Explanation*
$1/m$	Poisson's ratio
$\theta, \alpha, \beta, \gamma$	Angles
μ	Coefficient of friction
ϕ	Angle of friction
α	Co-efficient of linear expansion
δ	Deflection, Change in dimension
C	Modulus of rigidity
E	Young's modulus
F	Force
I	Moment of inertia
J	Polar moment of inertia
K	Bulk modulus, Radius of gyration
M	Moment
N	Speed of shaft
P	Axial load, Power transmitted
R	Resultant of force, Suppport reaction
T	Temperature, Torque
U	Strain energy
V	Volume
W	Load, Weight, Workdone
$\bar{X}$	X- coordinate of centroid
$\bar{Y}$	Y- coordinate of centroid
Z	Section modulus, Polar modulus

Symbol	*Explanation*
a, A	Area
b	Width
d	Depth
d, D	Diameter
d_1	Outside diameter
d_2	Inside diameter
e	Strain
f	Stress
f_1	Circumferential or hoop stress
f_2	Longitudinal stress
f_s	Shear stress
g	Acceleration due to gravity
h	Height
l, L	Length
m	Mass
n	Number of coils in spring
p	Pressure
r, R	Radius
s	Stiffness of spring
t	Thickness
v	Velocity
w	*udl*
y	Distance of extreme layer from N.A

Unit – I

Chapter 1. STATICS OF PARTICLES

1.1 Introduction

The branch of science which deals with the study of a body when the body is at rest, is known as *statics*. The body at rest or moves along a straight line with uniform velocity is said to be in equilibrium. For analysing the equilibrium of bodies, knowledge of forces, resultants, moments and couples is essential. The following are the important terms related with statics of particles.

Matter : Matter is the substance which any object is composed of. The substance forming the matter contains atoms and molecules.

Space : Space is the three dimensional region. This term is used in connection with the position of a point which is fixed by its three reference coordinates.

Mass : Mass is the quantity of matter contained in a body.

Particle : A particle is defined as a very small quantity of matter that may be considered to occupy a single point in space. It is also defined as a body whose dimensions are negligibly small and whose mass is concentrated at a point.

Rigid body : A rigid body is one in which the distance between the particles forming the body remains constant before and after the application of force on the body. Rigid bodies do not deform.

Example : Flywheel of an engine, wheel of a road roller, a block lifted by a crane, a truck, etc.

Deformable body : A body in which the dimensions change during the application of a force is called deformable body

Example : A beam that bend under the application of loads.

Vector quantity : A quantity which is completely specified by its magnitude and direction is known as a vector quantity.

Example : Velocity, acceleration, force, momentum, etc.

Scalar quantity : A quantity which is completely specified by magnitude only is known as scalar quantity.

Example : Mass, length, time, temperature, etc.

1.2 Force

Force is defined as an action which changes or tends to change the state of rest or motion of the body on which it is applied.

Effects of force:

- A force moves or tends to move a body in the direction in which it acts.
- A force may also tend to rotate the body on which it acts.

Example:

1) When a person pushes a door, he is said to apply a force on the door.
2) A bowler bowling a cricket ball is applying a force on the ball.
3) A train is pulled by the force exerted by the engine.

Characteristics of a force :

A force can be completely specified by the following :

- ***Magnitude :*** The quantity of applied force is specified by the standard unit, usually in *newton* (N).
- ***Direction :*** Direction of force is represented by the line of action and the sense of the force. The line of action of a force is the straight line of infinite length along which the force acts.
- ***Point of application on the body*** : It is the point on the body at which the force acts.

1.3 Principle of transmissibility

Principle or transmissibility states that *"if a force acting at a point on a rigid body is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged".*

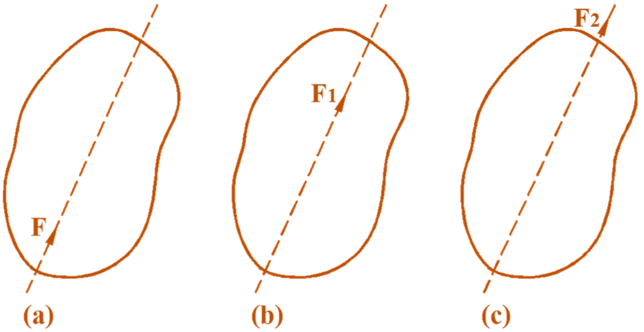


Fig. 1.1 Principle of transmissibility

If F, F_1 and F_2 have the same magnitute, the three systems shown in figure have the same effect eventhough they act at different point. Because they are shifted along the same line of action.

1.4 System of forces

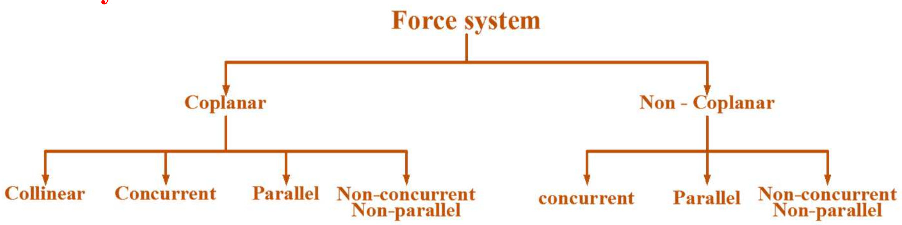


Fig.1.2 System of forces

The system of forces is generally classified as :

- **Coplanar forces :** The forces in a system acting in a same plane are called coplanar forces.

- **Non-coplanar forces :** The forces in a system acting in different planes are called non-coplanar forces.

Coplanar collinear : In this system of forces, all the forces act in the same plane and have a common line of action.

Coplanar concurrent : In this system of forces, all the forces act in the same plane and they intersect at a common point.

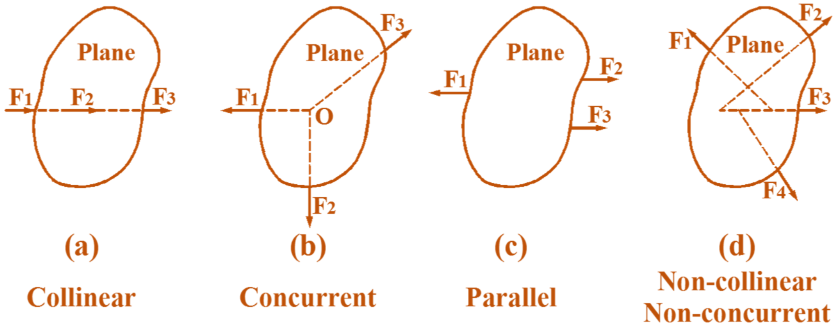


Fig.1.3 Coplanar forces

Coplanar parallel : In this system of forces, all the forces act in the same plane and are parallel to each other. If the parallel forces have same sense, they are called *like parallel forces*. If the parallel forces have different senses, they are called *unlike parallel forces*.

Coplanar non-concurrent non-parallel : In this system of forces, all the forces act in the same plane but the forces are not parallel and not meet at a common point. This force system is also known as general system of forces.

Non-coplanar concurrent : If the concurrent forces lie in different planes, they are called non-coplanar concurrent forces.

Non-coplanar parallel : If the parallel forces act in different planes, they are called non-coplanar parallel forces.

Non-coplanar non-concurrent : Non-concurrent forces lying in different planes are called non-coplanar non-concurrent forces.

1.5 Resultant of forces

Resultant is a single force which can replace a number of forces acting on a rigid body, without causing any change in the external effects on the body. Resultant is also referred to as *equivalent action.*

1.6 Parallelogram law of forces

The parallelogram law of forces sates that *"if two forces acting at a point are represented by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram gives the resultant of the two forces both in magnitude and direction".*

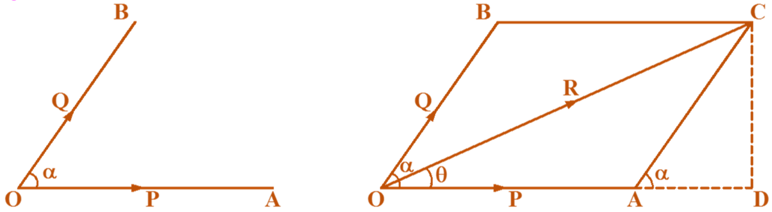


Fig.1.4 Parallelogram law of forces

Let two forces P and Q act at a point O. The force P is represented in magnitude and direction by OA. The force Q is represented in magnitude and direction by OB. Let the angle between the two forces be α. The resultant of these two forces will be obtained in magnitude and direction by the diagonal (OC) of the parallelogram with OA and OB as two adjacent sides.

To find the magnitude of resultant (R)

Construct a parallelogram $OACB$ as shown in the figure.

From C draw CD perpendicular to OA produced.

Let, α= Angle between two forces P and Q= $\angle AOB$

Now, $\angle DAC = \angle AOB = \alpha$ (∵ Corresponding angles)

In parallelogram $OACB$, AC is parallel and equal to OB.

$\therefore AC = Q$

In triangle ACD,

$$\cos\alpha = \frac{AD}{AC} = \frac{AD}{Q} \implies AD = Q\cos\alpha$$

$$\sin\alpha = \frac{CD}{AC} = \frac{CD}{Q} \implies CD = Q\sin\alpha$$

In triangle OCD,

$$OC^2 = OD^2 + CD^2$$

$$OC = R; \quad OD = OA + AD = P + Q\cos\alpha\,; \quad CD = Q\sin\alpha$$

$$\begin{aligned} R^2 &= (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2 \\ &= P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha \\ &= P^2 + Q^2\left(\cos^2\alpha + \sin^2\alpha\right) + 2PQ\cos\alpha \\ &= P^2 + Q^2 + 2PQ\cos\alpha \qquad (\because \cos^2\alpha + \sin^2\alpha = 1) \end{aligned}$$

$$\boxed{\text{Magnitude of resultant force, } \boldsymbol{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}}$$

To find the direction of resultant

Let, θ be the angle made by resultant with OA.

From triangle OCD,

$$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)}$$

1.7 Law of sines

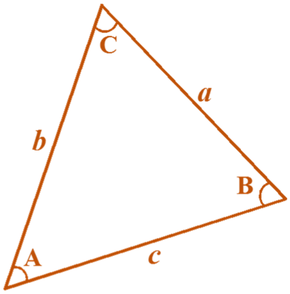


Fig.1.5 Law of sines

Law of sines is a trigonometric property of a triangle. It is an equation relating the lengths of the sides of a triangle to the sines of its angles.

According to the law, $\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$

Where, a, b, and c are the lengths of the sides of a triangle.

A, B, and C are the opposite angles.

The direction of resultant of two coplanar forces can also be obtained using the law of sines.

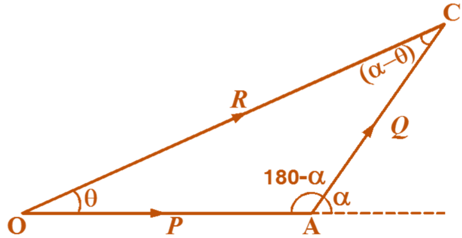


Fig.1.6 Resultant of forces

In triangle $OAC, OA = P; AC = Q; OC = R;$

$\angle OAC = (180 - \alpha); \angle ACO = 180 - (\theta + 180 - \alpha) = \alpha - \theta.$

Using the law of sines,

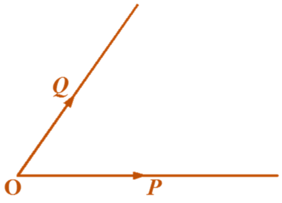
$$\frac{\sin\theta}{AC} = \frac{\sin(180-\alpha)}{OC} = \frac{\sin(\alpha-\theta)}{OA}$$

$$\boldsymbol{\frac{\sin\theta}{Q} = \frac{\sin(180-\alpha)}{R} = \frac{\sin(\alpha-\theta)}{P}}$$

Using the above relation, the *angle* $\boldsymbol{\theta}$ can be calculated.

1.8 Trianglular law of forces

The triangle law of forces states that *"if two coplanar concurrent forces acting at a point are represented in magnitude and direction by the two adjacent sides of a triangle in order, then the resultant of the two forces is given in magnitude and direction by the third side of the triangle in opposite order"*.



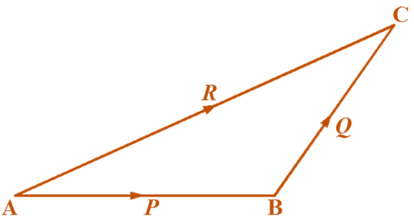


Fig.1.7 Triangular law of forces

Let two forces P and Q act at a point O. The force P is represented in magnitude and direction by AB. The force Q is represented in magnitude and direction by BC. Then, the resultant of these two forces will be represented in magnitude and direction by the third side AC of the triangle. But the direction is taken in opposite order.

1.9 Polygon law of forces

Polygon law of forces states that, *"If a number of coplanar, concurrent forces are represented in magnitude and direction by the sides of an open polygon taken in order, then the resultant of all these forces is given in magnitude and direction by the closing side of the polygon in the opposite order".*

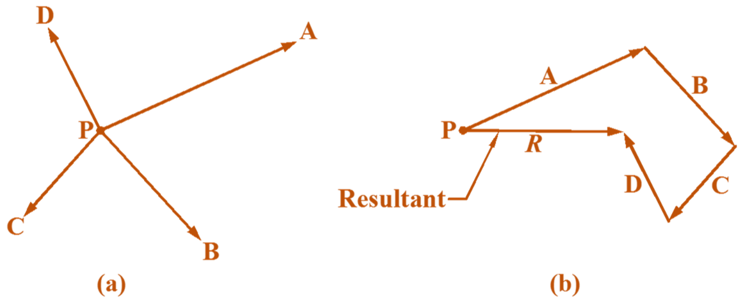


Fig.1.8 Polygon law of forces

Figure (a) shows four forces acting at point P. Using polygon law of forces, all the forces are arranged in a tip-to-tail fashion as shown in figure(b). The tail of force A is connected to the tip of last force D and the resultant R is marked as shown.

The resultant of a system of coplanar forces, $\boldsymbol{R} = \sum \boldsymbol{F}$

1.10 Resolution of a force into rectangular components

The process of finding the components of a force in two directions is called resolution of force. The components of force along two mutually perpendicular lines are called the rectangular components of a force.

Let a given force be F which makes an angle θ with X-axis. It is required to find the components of the force F along X-axis and Y-axis.

Components of F along X-axis $= F\cos\theta$

Components of F along Y-axis $= F\sin\theta$

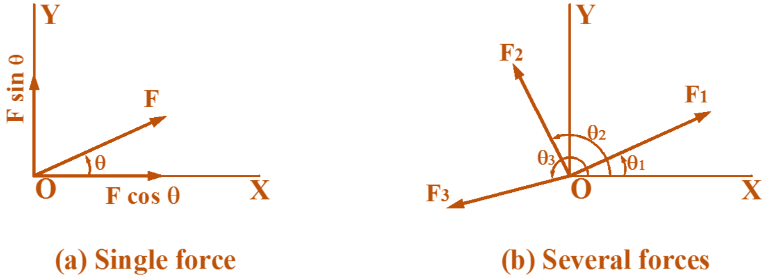


Fig.1.9 Resolution of forces

✦ **Note :** ***While resolving forces into components, the angle made by the forces with X- axis (θ) alone should be taken. The angle between two forces (α) <u>should not</u> be taken.***

Resolution of a number of coplanar forces

Let a number of coplanar forces $F_1, F_2, F_3, \ldots$ are acting at a point as shown in the figure.

Let, θ_1 = Angle made by F_1 with X – axis
θ_2 = Angle made by F_2 with X – axis
θ_3 = Angle made by F_3 with X – axis
R_x = Resultant component of all forces along X-axis.
R_y = Resultant component of all forces along Y-axis.
R = Resultant of all forces
θ = Angle made by resultant with X- axis.

Force	Component along X-axis (F_x)	Component along Y-axis (F_y)
F_1	$F_1 \cos\theta_1$	$F_1 \sin\theta_1$
F_2	$F_2 \cos\theta_2$	$F_2 \sin\theta_2$
F_3	$F_3 \cos\theta_3$	$F_3 \sin\theta_3$

Resultant components along X- axis
= Sum of components of all forces along X- axis.

$\therefore R_x = \Sigma F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + \cdots$

Resultant components along Y- axis
= Sum of components of all forces along Y- axis.

$\therefore R_y = \Sigma F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + \cdots$

Then resultant of all the forces, $\boxed{R = \sqrt{(R_x)^2 + (R_y)^2}}$

The angle made by R with X-axis is given by, $\boxed{\tan\theta = \frac{R_y}{R_x}}$

1.11 External and internal forces

External forces represent the action of other bodies on the rigid body being analysed. External forces consist of applied forces, weight of the free-body and the reactions developed at the support of contact points.

Internal force holds the particles of the body together. The internal forces cause internal stresses and strains distributed throughout the material of the body.

Example : Consider a container which is lifted by a crane by means of a cable. For the container, the lifting by the crane is an external force. For the crane, the tension in the cable is an external force. The force in the cable is an internal force for the cable.

1.12 Moment of a force

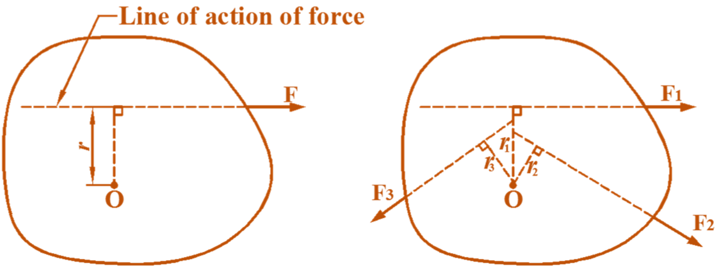
Force acting on a body tends to move the body in the direction of its application. In addition, a force may also tend to rotate a body about an axis. This rotational tendency of the force is called the *moment* of the force. Moment is also known as torque.

The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

Let, F = A force acting on a body.

r = Perpendicular distance from the point O on the line of action of froce F.

The moment (M) of the force F about O is given by, $\boldsymbol{M = F \times r}$



(a) Single force **(b) Several forces**

Fig.1.10 Moment of forces

The tendency of this moment is to rotate the body in the clockwise direction about O. Hence this moment is called clockwise moment. If the tendency of the a moment is to rotate the body in anti-clockwise direction, then that moment if known as anti-clockwise moment. If clockwise moment is taken –ve, then anti-clockwise moment will be +ve.

In S.I. system, the unit of moment is ***N-m (Newton - metre).***

Moment of several forces :

Let, F_1, F_2, F_3 = Forces acting on the body

r_1, r_2, r_3 = Perpendicular distances from the point O on the line of action of froces F_1, F_2, F_3 respectively.

Moment of F_1 about $O = F_1 \times r_1$ (clockwise) (–ve)

Moment of F_2 about $O = F_2 \times r_2$ (clockwise) (–ve)

Moment of F_3 about $O = F_3 \times r_3$ (anti-clockwise) (+ve)

The resultant moment will be algebraic sum of all the moments.

$\therefore$ The resultant moment of all the forces about O,

$$= -F_1 \times r_1 - F_2 \times r_2 + F_3 \times r_3$$

1.13 Varignon's theorem

Varignon's theorem states that *"the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point".*

Varignon's theorem is also known as *principle of moments.* It states that *" the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point".*

1.14 Moment of a couple

Two parallel, non-collinear forces of equal magnitude having opposite senses are said to form a couple. Couple has a tendency to rotate a body on which it acts.

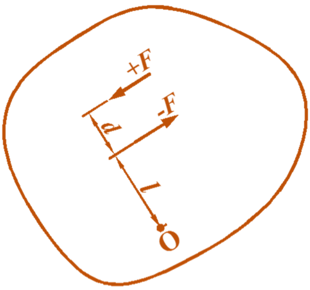


Fig.1.11 Moment of a couple

The magnitude of the moment of a couple is the algebraic sum of the moments of the two forces about any point.

From the figure, magnitude of the moment of the couple about O is $F(d + l) - Fl = Fd$.

From the above equation, it is observed that the magnitude of the moment of a couple depends only on the distance between the line of action of two forces (d). The distance d is called as *arm* of the couple.

1.15 Resolution of a force into a force and a couple

A force applied to a body at any point can be replaced by an equal force applied at another point with a couple which will be equivalent to the original force.

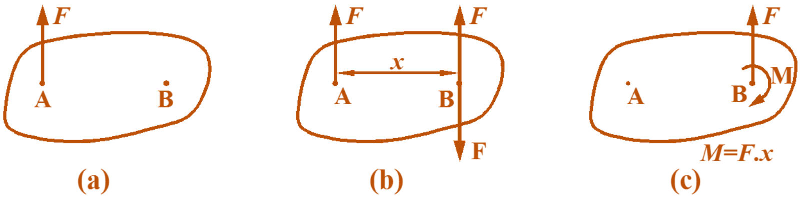


Fig.1.12 Resolution of a force into force and couple

Proof :

1) Let the given force F is acting at point A as shown in Fig.(a). This force is to be replaced at the point B.
2) Introduce two equal and opposite forces at B, each of mangitude F and acting parallel to the force at A.
3) Now, the force system of Fig.(b) is equivalent to the single force acting at A of Fig.(a).
4) The force F at A and the opposite force F at B form a couple. The moment of this couple is $F \times x$ (clockwise).
5) The third force is acting at B in the same direction in which the force at A is acting.
6) Now the given force F acting at A has been replaced by an equal and parallel force applied at point B in the same direction together with a couple of moment $F \times x$.

1.16 Necessary conditions for equilibrium of rigid bodies

A body is said to be in equilibrium when all its constituent particles are at rest or moving along a straight line uniformly. The following are the necessary and sufficient conditions for equilibrium of rigid bodies.

A body is in equilibrium when,

1) The algebraic sum of the magnitudes of the horizontal components of all the forces acting on the body is zero, i.e. $\boldsymbol{\Sigma F_x = 0}$.
2) The algebraic sum of the magnitudes of the vertical components of all the forces acting on the body is zero, i.e. $\boldsymbol{\Sigma F_y = 0}$.
3) The algebraic sum of the magnitudes of the moments of all the forces about any point is zero, i.e. $\boldsymbol{\Sigma M = 0}$.

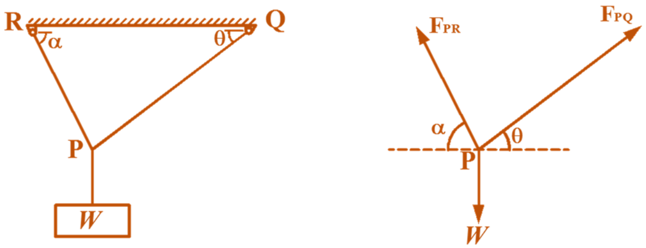
1.17 Space diagram and free body diagram

- ***Space diagram :*** The diagram showing the body and the forces acting on it may be treated as the physical representation of engineering mechanics problem. Such a diagram is said to be a *space diagram*.

- ***Free body :*** A significant part of a body which is freed or isolated from the rest of the body is called a *free-body*.

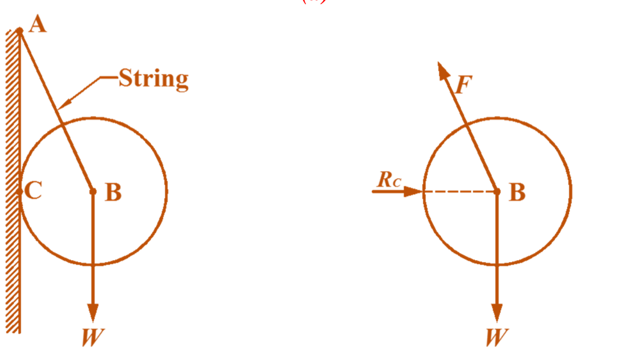
- ***Free-body diagram :*** The diagram showing the isolated significant portion of a body along with the forces acting on it is called *free-body diagram*.

Example :



Space diagram   Free - body diagram

(a)



Space diagram   Free - body diagram

(b)

Fig.1.13 Space diagram and free body diagram

- **Equilibrant :** Equilibrant is a force which is equal, collinear and opposite to the resultant in a system of forces.

1.18 Triangular law of equilibrium

Triangle law of equilibrium states that, *"if three forces acting on a particle can be represented in magnitude and direction by the three sides of a triangle taken in order, then the particle is in equilibrium."*

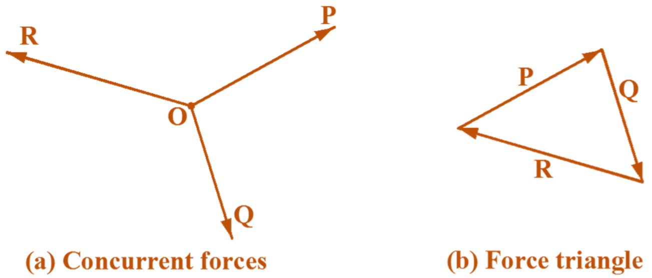


Fig.1.14 Triangular law of equilibrium

Fig(a) shows a particle O acted upon by three forces. The three forces are represented by the three sides of a triangle taken in order as shown in Fig.(b). Then the particle is in equilibrium and the triangle constructed is called as force triangle.

1.19 Lami's theorem

Lami's theorem states that, *"if three forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces."*

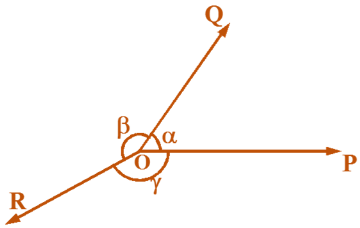


Fig.1.15 Lami's theorem

Suppose the three forces P, Q and R are acting at a point O and they are in equilibrium as shown in the figure.

Let, α = Angle between force P and Q

β = Angle between force Q and R

γ = Angle between force R and P

According to Lami's theorem, $\boxed{\dfrac{P}{\sin\beta} = \dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin\alpha}}$

1.20 Polygon law of equilibrium

Polygon law of equilibrium states that, *"if a particle is in equilibrium under the action of a system of coplanar forces, the forces can be represented in magnitude and direction by the sides of a polygon taken in order."*

1.21 Support reactions

- ***Support :*** A body that supports another body acted upon by a system of forces is called a *support*.
- ***Support reaction :*** The force exerted by the support on the supported body is called *support reaction*.

Different types of supports and reactions

1) Simple support or knife edge support

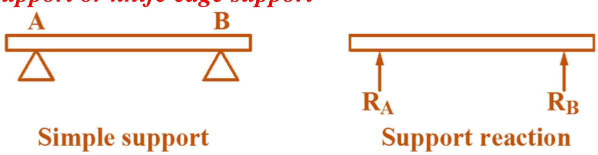


Fig.1.16(a) Simple support and support reaction

Simple support is just a support on which structural members rests. They only resist vertical forces. They cannot resist horizontal forces and moment. A beam supported on the knife edges *A* and *B* is shown in the figure. The reactions at *A* and *B* will be perpendiculat to the surface of the beam.

Example : It is used as a support for long bridges, roof spans, etc.

2) Roller support :

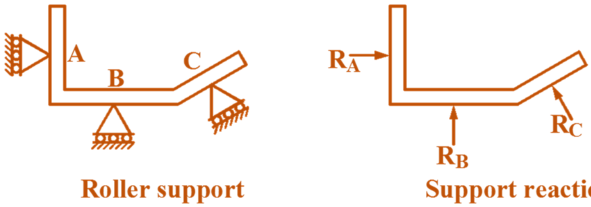


Fig.1.16(b) Roller support and support reaction

Roller supports only resist perpendicular forces. They cannot resist horizontal forces and moment. The roller support can move freely along the surface. The surface can be horizontal, vertical or inclined. The support reaction is always perpendicular to the surface.

Example : Roller supports are located at one end of long bridge. This allows the bridge structure to exapnd or contract with temperature changes.

3) Smooth surface support

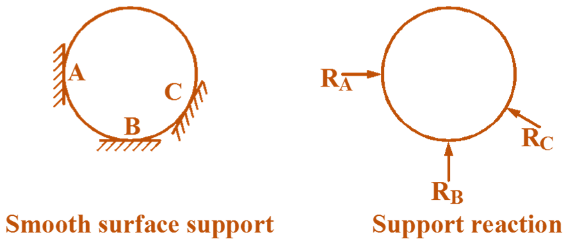


Fig.1.16(c) Smooth surface support and support reaction

A smooth or frictionless surface support will provide a support reaction always perpendicular to the support surface. The surface can be horizontal, vertical or inclined.

Example : A circular beam resting on smooth concrete blocks, etc.

4) Pin joint or hinged support

Fig.1.16(d) Hinged support and support reaction

Hinged support can resit both vertical and horizontal forces. It cannot resist a moment. They will allow the structural member to rotate but not to translate in any direction. The support reaction will be both in horizontal and vertical direction.

Example : Hinged supports can be used in trusses, door leaf, etc.

5) Fixed or built-in support

Fig.1.16(e) Fixed support and support reaction

Fixed supports cannot move or rotated in any direction. They can resist any type of force and moment. They are also called as rigid supports. The support reaction will be both in horizontal and vertical direction. They also provide a moment at the support.

Example : Beam fixed in wall, column in concrete, etc.

1.22 Calculation of support reactions in beams

- The support reactions of beams are determined by applying the three equations of equilibrium.
  - i) Sum of horizontal components of forces, $\Sigma F_x = 0$
  - ii) Sum of vertical components of forces, $\Sigma F_y = 0$
  - iii) Sum of moments about a point, $\Sigma M = 0$

- The support reactions at various supports :
  1) ***Roller support :*** Perpendicular to the plane of roller.
  2) ***Pin joint or hinged support :*** Both horizontal reaction and vertical reaction.
  3) ***Fixed support :*** A horizontal reaction, a vertical reaction and a moment.

- While determining the the support reactions, the direction of the reaction is assumed in any way. If the answer obtained is positive, the assumed direction is correct. If the answer is negative, the direction assumed should be reversed.

- ***Sign conventions :***

FORCE	+ →	− ←
	↑ +	− ↓
MOMENT	↺ +	− ↻

REVIEW QUESTIONS

1. Explain the principle of transmissibility. *(Oct.16)*
2. Classify the system of forces.
3. Explain the parallelogram law of force. *(Oct.16, Apr.17)*
4. What is law of sines?
5. State and explain triangular of law forces. *(Apr.17, Oct.17)*
6. What is polygon law of forces?
7. Explain the resolution of forces in to rectangular components.
8. What are internal and external forces? *(Oct.17)*
9. What do you mean by moment of a force?
10. How to calculate the moment of several forces?
11. Distinguish between moment and couple. *(Apr.17)*
12. State and explain Varignon's theorem. *(Oct.16, Apr.17)*
13. Explain moment of a couple.
14. How to replace a force in to a force and a couple?
15. Write down the conditions for equilibrium. *(Apr.17)*
16. What is space diagram and free body diagram? *(Apr.17)*
17. State and explain triangular law of equilibrium.
18. Explain Lam's theorem.
19. Explain the various supports and reactions. *(Apr.18)*

POINTS TO REMEMBER

1) Magnitude of resultant force, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

2) Angle made by resultant with one of the forces,

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

3) Law of sines $\Longrightarrow \dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

4) Resolution of forces :

$$R_x = \Sigma F_x = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3 + \cdots$$

$$R_y = \Sigma F_y = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3 + \cdots$$

Resultant of all the forces, $R = \sqrt{(R_x)^2 + (R_y)^2}$

The angle made by R with X-axis is given by, $\tan\theta = \dfrac{R_y}{R_x}$

5) Moment of the force F about a point is given by, $M = F \times r$

r = Perpendicular distance from the point on the line of action of force F.

6) The resultant moment of all the forces about a point,

$$= F_1 \times r_1 + F_2 \times r_2 + F_3 \times r_3 + \cdots$$

7) Necessary conditions for equilibrium :

i) Sum of horizontal components, $\Sigma F_x = 0$

ii) Sum of vertical components, $\Sigma F_y = 0$

iii) Sum of moments about a point, $\Sigma M = 0$

8) Lami's theorem $\Longrightarrow \dfrac{P}{\sin\beta} = \dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin\alpha}$

SOLVED PROBLEMS

Example : 1.1

Two forces of magnitude 10N and 8N are acting at a point. If the angle between the two forces is 60°, determine the magnitude of the resultant force and direction.

Given : Force, $P = 10$ N

Force, $Q = 8$ N

Angle between two forces, $\alpha = 60°$

To find : 1) Magnitude of resultant, R 2) Direction of resultant, θ

Solution :

The magnitude of resultant force (R) is given by,

$$R = \sqrt{P^2 + Q^2 + 2\ P\ Q \cos\alpha}$$

$$= \sqrt{10^2 + 8^2 + (2 \times 10 \times 8 \times \cos 60°)}$$

$$= \sqrt{100 + 64 + 80} = \sqrt{244} = \boxed{15.62\ \text{N}}$$

The angle made by the resultant with the direction of P is given by,

$$\theta = \tan^{-1}\left(\frac{Q \sin\alpha}{P + Q\cos\alpha}\right) = \tan^{-1}\left(\frac{8 \times \sin 60°}{10 + 8 \times \cos 60°}\right)$$

$$\theta = \tan^{-1}(0.4949) = \boxed{26.33°}$$

Result : 1) Magnitude of resultant force, R = **15.62 N**

2) Direction of resultant, θ = **26.33°**

Example : 1.2

Two equal forces are acting at a point with an angle of 60° between them. If the resultant of the forces is equal to 40N, find the mangitude of each force.

Given : Resultant of forces, $R = 40$ N

Forces are equal. i.e. $P = Q$

To find : 1) Magnitude of forces P & Q

Solution :

The magnitude of resultant force (R) is given by,

$$R = \sqrt{P^2 + Q^2 + 2\ P\ Q \cos\alpha}$$

$$= \sqrt{P^2 + P^2 + (2 \times P \times P \times \cos 60°)} \quad (\because P = Q)$$

$$40 = \sqrt{P^2 + P^2 + P^2} = \sqrt{3P^2}$$

Squaring on both sides,

$$1600 = 3P^2$$

$$P^2 = \frac{1600}{3} = 533.33$$

$$P = \sqrt{(533.33)} = \boxed{23.09\ \text{N}}$$

Result : 1) Magnitude of each force, $\boldsymbol{P = Q = 23.09}$ **N**

Example : 1.3 *(Apr.17, Oct.17)*

The magnitude of the resultant of two concurrent forces including an angle of 90° between them is $\sqrt{13}$ KN. When the included angle between the force is 60°, the magnitude of their resultant is $\sqrt{19}$ KN. Find the magnitude of the two forces.

Given :

<u>*Case : I*</u> $\Rightarrow$ Resultant, $R_1 = \sqrt{13}$ KN ; Angle, $\alpha_1 = 90°$

<u>*Case : II*</u> $\Rightarrow$ Resultant, $R_2 = \sqrt{19}$ KN ; Angle, $\alpha_2 = 60°$

To find : 1) Magnitude of forces, P & Q

Solution :

For **Case : I,**

$$R_1 = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha_1}$$

$$\sqrt{13} = \sqrt{P^2 + Q^2 + (2PQ \times \cos 90°)}$$

$$\sqrt{13} = \sqrt{P^2 + Q^2}$$

Squaring both sides , $13 = P^2 + Q^2$ ------ (1)

For **Case : II,**

$$R_2 = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha_2}$$

$$\sqrt{19} = \sqrt{P^2 + Q^2 + (2PQ \times \cos 60°)}$$

$$\sqrt{19} = \sqrt{P^2 + Q^2 + PQ}$$

Squaring both sides, $19 = P^2 + Q^2 + PQ$ ------ (2)

Subtracting equation (1) from (2),

$$19 - 13 = P^2 + Q^2 + PQ - (P^2 + Q^2)$$

$$6 = PQ \qquad ------ (3)$$

Multiplying the equation (3) $\times$ 2 $\Rightarrow$ $12 = 2PQ$ ------ (4)

Adding equation (4) and (1),

$$13 + 12 = P^2 + Q^2 + 2PQ$$

$$25 = P^2 + Q^2 + 2PQ \quad (or) \quad 5^2 = (P+Q)^2$$

$$5 = P + Q \quad (or) \quad P = 5 - Q \qquad ------(5)$$

Substituting the value of P in equation (3),

$$6 = (5 - Q) \times Q = 5Q - Q^2 \quad (or) \quad \mathbf{Q^2 - 5Q + 6 = 0}$$

This is a quadratic equation. Resolving this equation,

$$Q^2 - 3Q - 2Q + 6 = 0$$

$$Q(Q-3) - 2(Q-3) = 0$$

$$(Q-3)(Q-2) = 0$$

$$\mathbf{Q = 3} \;\; (or) \;\; \mathbf{Q = 2}$$

Substituting the value of Q in equation (5)

$$\mathbf{P} = 5 - 3 = \mathbf{2} \quad (or) \quad \mathbf{P} = 5 - 2 = \mathbf{3}$$

The two forces are **2 KN and 3 KN**.

Result : 1) The magnitude of forces, $\mathbf{P = 2}$ **KN** and $\mathbf{Q = 3}$ **KN**

Example : 1.4 *(Apr.18)*

The resultant of two concurrent forces is 1500N and the angle between the forces is 90°. The resultant makes an angle of 36° with one of the forces. Find the magnitude of each force.

Given:

Resultant, R = 1500 N

Angle between the forces, α = 90°

Angle made by the resultant with one force, θ = 36°

To find : 1) Magnitude of the forces, P & Q

Solution :

The angle made by the resultant with one force,

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$\tan 36° = \frac{Q\sin 90°}{P + Q\cos 90°} = \frac{Q}{P}$$

$$0.726 = \frac{Q}{P} \quad (or) \quad Q = 0.726\,P \qquad -----(1)$$

The magnitude of resultant,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

Squaring, $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$$1500^2 = P^2 + (0.726P)^2 + 2P(0.726\ P)\cos 90°$$
$$1500^2 = P^2 + 0.527P^2 + 0$$
$$1500^2 = 1.527P^2$$
$$P = \sqrt{\frac{1500^2}{1.527}} = \boxed{1213.86\text{ N}}$$

Substituting the value of P in (1),

$$Q = 0.726 \times 1213.86 = \boxed{881.26\text{ N}}$$

Result : 1) The magnitude of the forces, $\mathbf{P = 1213.86\ N,\ Q = 881.26\ N}$

Example : 1.5

Two forces of magnitude 240N and 200N are acting at a point O as shown in fig.P1.1. If the angle between the forces is 60°, determine the magnitude of the resultant force. Also determine the angle made by the resultant with each of the forces.

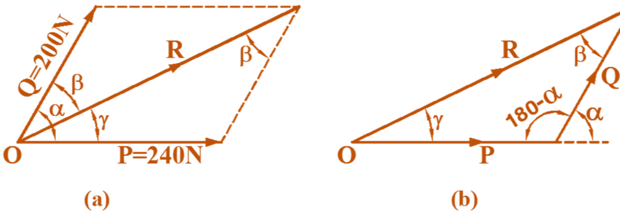


Fig.P1.1

Given :

Force, P = 240 N

Force, Q = 200 N

Angle between the forces, α = 60°

To find : 1) Magnitude of resultant, R 2) Angle, α 3) Angle, β

Solution :

The magnitude of resultant,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$
$$= \sqrt{240^2 + 200^2 + (2 \times 240 \times 200 \times \cos 60°)}$$
$$= \sqrt{57600 + 40000 + 48000} = \boxed{381.57\text{ N}}$$

Using *sine formula,*

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)} \qquad ------(1)$$

From the above equation,

$$\frac{P}{\sin\beta} = \frac{R}{\sin(180° - \alpha)}$$

$$\sin\beta = \frac{P\sin(180-\alpha)}{R} = \frac{240\sin(180-60)}{381.57}$$

$$= \frac{240 \times \sin 120°}{381.57} = 0.5447$$

$$\beta = \sin^{-1}(0.5447) = \boxed{33°}$$

From equation (1),

$$\frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)}$$

$$\sin\gamma = \frac{Q\sin(180-\alpha)}{R} = \frac{200\sin(180-60)}{381.57}$$

$$= \frac{200 \times \sin 120°}{381.57} = 0.4539$$

$$\gamma = \sin^{-1}(0.4539) = \boxed{27°}$$

Result : 1) Resultant, $R = 381.57$ N 2) Angle, $\beta = 33°$ 3) Angle, $\gamma = 27°$

Example : 1.6 *(Apr.17)*

A particle 'O' is acted upon by the following forces : i) 20N inclined at 30°to North of East, ii) 25N towards the North, iii) 30N towards North-West, iv) 35N inclined 40° to South of West. Find the magnitude and direction of resultant.

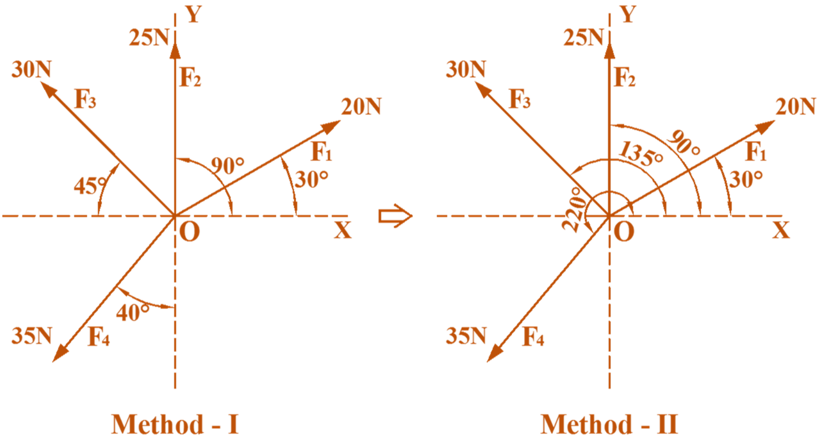


Fig.P1.2

Given : Force, $F_1 = 20$ KN ; Angle, $\theta_1 = 30°$

Force, $F_2 = 25$ KN ; Angle, $\theta_2 = 90°$

Force, $F_3 = 30$ KN ; Angle, $\theta_3 = 135°$

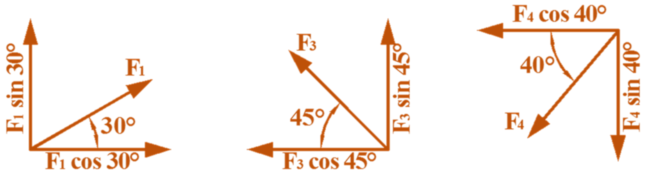
Force, $F_4 = 35$ KN ; Angle, $\theta_4 = 220°$

To find : 1) Magnitude of resultant, R 2) Direction of resultant, θ

Solution :

Method – I :

Resolving the the inclined forces



By considering the direction of resolved forces :

The resultant of forces along X-axis,

$$R_x = \Sigma F_x = F_1 \cos 30° - F_3 \cos 45° - F_4 \cos 40°$$
$$= 20 \times \cos 30° - 30 \times \cos 45° - 35 \times \cos 40°$$
$$= -30.704 \text{ (} \boldsymbol{-ve\ \ X\ direction}\text{)}$$

The resultant of forces along Y-axis,

$$R_y = \Sigma F_y = F_1 \sin 30° + F_2 + F_3 \sin 45° - F_4 \sin 40°$$
$$= (20 \times \sin 30°) + 25 + (30 \times \sin 45°) - (35 \times \sin 40°)$$
$$= 33.716 \text{ (} \boldsymbol{+ve\ Y\ direction}\text{)}$$

Method – II:

By considering the angle of direction of forces from O-X :

The resultant of forces along X-axis,

$$R_x = \Sigma F_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3 + F_4 \cos \theta_4$$
$$= (20 \times \cos 30°) + (25 \times \cos 90°) + (30 \times \cos 135°) + (35 \times \cos 220°)$$
$$= -30.704 \text{ (} \boldsymbol{-ve\ \ X\ direction}\text{)}$$

The resultant of forces along Y-axis,

$$R_y = \Sigma F_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3 + F_4 \sin \theta_4$$
$$= (20 \times \sin 30°) + (25 \times \sin 90°) + (30 \times \sin 135°) + (35 \times \sin 220°)$$
$$= 33.716 \text{ (} \boldsymbol{+ve\ Y\ direction}\text{)}$$

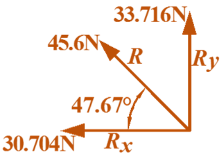
The magnitude of resultant force,

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{30.704^2 + 33.716^2} = \boxed{\textbf{45.67 KN}}$$

The direction of resultant of forces,

$$\tan\theta = \frac{R_y}{R_x} = \frac{33.716}{30.704} = 1.098$$

$$\theta = \tan^{-1}(1.098) = \boxed{47.67^\circ \text{ (from } -Ve \text{ X-axis)}}$$



Result : 1) The magnitude of resultant, $R = 45.6$ KN

2) The direction of resultant, $\theta = 47.67^\circ$ *from −ve X-axis.*

Example : 1.7

The system of four forces are acting at a point O as shown in the fig.P1.3. The resultant 72KN is acting along the Y- axis. The magnitude of forces F_1, F_2 and F_4 are 10KN, 20KN and 40KN respectively. The angles made by 10KN, 20KN and 40KN with X- axis are 30°, 90° and 120° respectively. Find the magnitude and direction of force F_2.

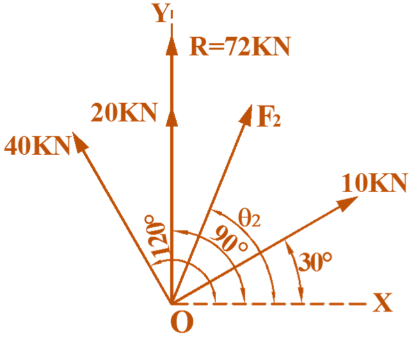


Fig.P1.3

Given : Force, $F_1 = 10$ KN ; Angle, $\theta_1 = 30^\circ$

Force, $F_3 = 20$ KN ; Angle, $\theta_3 = 90^\circ$

Force, $F_4 = 40$ KN ; Angle, $\theta_4 = 120^\circ$

Resultant, $R = 72$ KN ;

To find : 1)The magnitude of force, F_2 2) Direction of force, θ_2

Solution :

The resultant is acting along Y-axis. Hence the algebraic sum of horizontal component should be zero and algebraic sum of vertical components should be equal to the resultant.

$$\therefore R_x = \Sigma F_x = 0$$

$$R_y = \Sigma F_y = R = 72 \text{ KN}$$

$$R_x = \Sigma F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4$$
$$= (10 \times \cos 30°) + F_2 \cos\theta_2 + (20 \times \cos 90°) + (40 \times \cos 120°)$$
$$= F_2 \cos\theta_2 - 11.34$$

$$F_x = 0 \Rightarrow \; F_2 \cos\theta_2 - 11.34 = 0$$
$$\text{0r} \qquad F_2 \cos\theta_2 = 11.34 \qquad ------(1)$$

$$R_y = \Sigma F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$$
$$= (10 \times \sin 30°) + F_2 \sin\theta_2 + (20 \times \sin 90°) + (40 \times \sin 120°)$$
$$= F_2 \sin\theta_2 + 59.64$$

$$\Sigma F_y = R \Rightarrow \; F_2 \sin\theta_2 + 59.64 = 72$$
$$\text{0r} \qquad F_2 \sin\theta_2 = 72 - 59.64 = 12.36 \qquad ------(2)$$

Dividing equation (2) by (1)

$$\frac{F_2 \sin\theta_2}{F_2 \cos\theta_2} = \frac{12.36}{11.34}$$
$$\tan\theta_2 = 1.0899$$
$$\theta_2 = \tan^{-1}(1.0899) = \boxed{47.46°}$$

Substituting the value of θ_2 in equation (2)

$$F_2 \sin(47.46°) = 12.36$$
$$F_2 = \frac{12.36}{\sin(47.46°)} = \boxed{16.77\ \text{KN}}$$

Result : 1) The magnitude of force, F_2 = **16.77 KN**
2) The direction of force, θ_2 = **47. 46° from X-axis**

Example : 1.8

(Oct.16, Oct.17)

Five forces are acting on a particle. The magnitude of the forces are 300N, 600N, 900N and 'P' and their respective angles with the horizontal are 0°, 60°, 135°, 210°, 270°. If the vertical component of all the forces is –1000N, find the value of 'P'. Also calculate the magnitude and direction of the resultant force assuming that 300N force acts towards the particle while all others act away from the particle.

Given : Force, F_1 = 300 N ; Angle, θ_1 = 180°
Force, F_2 = 600 N : Angle, θ_2 = 60°
Force, F_3 = 700 N : Angle, θ_3 = 135°
Force, F_4 = 900 N : Angle, θ_4 = 210°
Force, F_5 = P ; Angle, θ_5 = 270°
ΣF_y = -1000 N

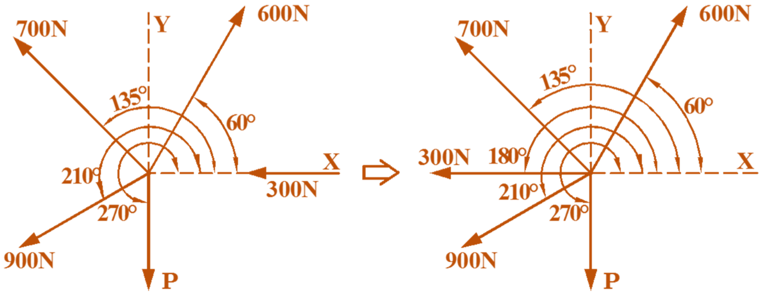


Fig.P1.4

To find : 1) Value of force P 2) Resultant, R 3) Direction of resultant, θ

Solution :

The resultant of force along Y- axis,

$$R_y = \Sigma F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4 + F_5 \sin\theta_5$$

$$= (300 \times \sin 180°) + (600 \times \sin 60°) + (700 \times \sin 135°)$$
$$+ (900 \times \sin 210°) + (P \times \sin 270°)$$
$$= 0 + 519.61 + 494.97 - 450 - P$$
$$= 564.58 - P$$

But, $\Sigma F_y = -1000 \quad (\because \textit{Given})$

$$564.58 - P = -1000$$
$$P = 564.58 + 1000 = \boxed{1564.58 \text{ N}}$$

The resultant of force along X- axis,

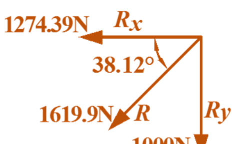
$$R_x = \Sigma F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4 + F_5 \cos\theta_5$$

$$= (300 \times \cos 180°) + (600 \times \cos 60°) + (700 \times \cos 135°)$$
$$+ (900 \times \cos 210°) + (1564.58 \times \sin 270°)$$
$$= -300 + 300 - 494.97 - 779.42 + 0 = \mathbf{-1274.39\ N}$$

The magnitude of resultant force,

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-1274.39)^2 + (-1000)^2} = \boxed{1619.9 \text{ N}}$$

The direction of resultant of force,



$$\tan\theta = \frac{R_y}{R_x} = \frac{-1000}{-1274.39} = 0.7847$$

$$\theta = \tan^{-1}(0.7847) = \boxed{38.12°}$$

Result : 1) The value of force **'P' = 1564.58 N**
2) The resultant of forces, **R = 1619.9 N**
3) The direction of resultant, **$\theta = 38.12°$ from $-$ve X-axis.**

$\mathcal{Example}$: 1.9

A force of 100N is acting at a point A as shown in the fig.P1.5. Determine the moments of this force about O.

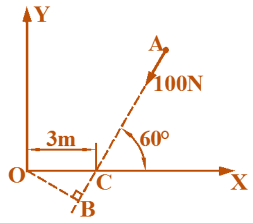


Fig.P1.5

Given : Force at A = 100 N

To find : 1) Moments of force about O, M_O

Solution :

Draw a perpendicular line (OB) from O to the line of action of force 100 N. Triangle OBC is the right angled triangle.

$$\angle OCB = 60°$$

$$\sin 60° = \frac{OB}{OC}$$

$$OB = OC \times \sin 60° = 3 \times 0.866 = 2.598 \text{ m}$$

Moment of force 100 N about O,

$$M_o = 100 \times OB = 100 \times 2.598 = \boxed{\textbf{259.8 Nm } \textit{(clockwise)}}$$

Result : 1) Moment of force about O, M_o = **259.8 Nm (clockwise)**

$\mathcal{Example}$: 1.10

Three like parallel forces 100N, 200N and 300N are acting at points A, B and C respectively on a striaght line ABC as shown in the fig.P1.6. The distances are AB = 3m and BC = 4m. Find the resultant and also the distance of the resultant from point A on the line ABC.

Given : Force at A = 100 N ; Distance AB = 3 m
Force at B = 200 N ; Distance BC = 4 m
Force at C = 300 N

To find : 1) Resultant, R 2) Distance of resultant from A

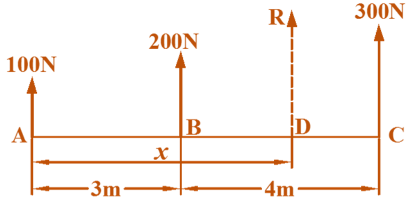


Fig.P1.6

Solution :

All the forces are parallel and acting in the same direction.

The resultant of forces,

$$R = 100 + 200 + 300 = \boxed{\mathbf{600\ N}}$$

Let x = distance of resultant from A

Taking moment of all the forces about A,

Moment of 100 N about $A = 100 \times 0 = 0$

Moment of 200 N about $A = 200 \times 3 = 600$ Nm (anti-clockwise)

Moment of 300 N about $A = 300 \times 7 = 2100$ Nm (anti-clockwise)

Algebraic sum of all the moments about A,

$$= 0 + 6000 + 21000 = 27000 \text{ Nm (anti-clockwise)}$$

Moment of resultant R about $A = R \times x = 600x$ Nm

Moment of resultant R about A

= Algebraic sum of all the moments about A

$$600x = 27000$$

$$\boldsymbol{x} = \frac{\mathbf{2700}}{\mathbf{600}} = \boxed{\mathbf{4.5\ m}}$$

Result : 1) Resultant, $\boldsymbol{R}$ **= 600 N** 2) Distance from A = **4.5 m**

Example : 1.11

A system of parallel forces are acting on a rigid bar as shown in the figP1.7(a). Reduce this system to : (i) a single force (ii) a single force and a couple at A (iii) a single force and a couple at B.

Given : Force at A = 32.5 N ; Distance AC = 1 m

Force at C = 150 N ; Distance CD = 1 m

Force at D = 67.5 N ; Distance BD = 1.5 m

Force at B = 10 N

To find : 1) A single force 2) A single force and a couple at A

3) A single force and a couple at B

Solution :

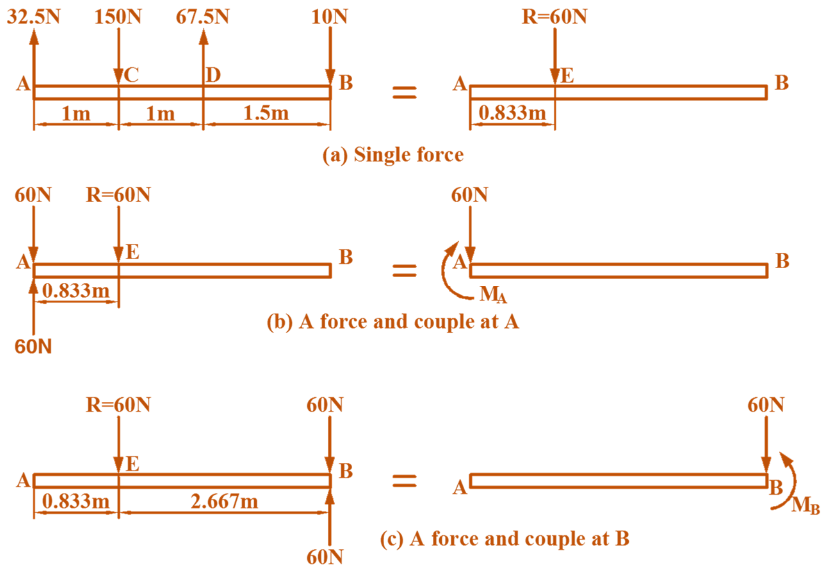


Fig.P1.7

1) Single force system

The single force system will consist of the resultant force.

Resultant of all the forces,

$$R = 32.5 - 150 + 67.5 - 10 = \boxed{-60\ \text{N}}$$

$(\because \textit{Upward force} : +Ve, \quad \textit{Downward force} : -ve)$

Let x= distance of resultan R from A

Moment of resultant R about A

$= $ Algebraic sum of all the moments about A

$$R \times x = -150 \times AC + 67.5 \times AD - 10 \times AB$$

$(\because \textit{clockwise moment} : -ve, \quad \textit{anti-clockwise moment} : +ve)$

$$-60 \times x = -150 \times 1 + 67.5 \times 2 - 10 \times 3.5$$

$$-60x = -50$$

$$x = \frac{-50}{-60} = \boxed{0.833\ \text{m}}$$

Hence, the given system of parallel forces is equivalent to a single force 60 N acting vertically downwards at a point E at a distance of 0.833 m from A as shown in the figure.

2) A single force and a couple at A

The resultant force R acting at point E can be replaced by an equal force applied at point A in the same direction together with a couple.

The moment of the couple = 60×0.833 = $\boxed{-49.98 \text{ Nm}}$

(-ve sign due to clockwise moment)

3) A single force and a couple at B

The resultant force R acting at point E can be replaced by an equal force applied at point B in the same direction together with a couple.

The moment of the couple = $60 \times (3.5 - 0.833)$

$= 60 \times 2.667 = \boxed{160 \text{ Nm}}$ *(anti-clockwise moment)*

Result :	1) Resultant, $\boldsymbol{R} = \mathbf{60\ N}$
	2) Moment of the couple at A, $\boldsymbol{M_A} = \mathbf{-49.98\ Nm}$
	3) Moment of the couple at B, $\boldsymbol{M_B} = \mathbf{160\ Nm}$

Example : 1.12

The forces F_1, F_2, F_3, F_4 and F_5 are acting at apoint on a body as shown in the fig.P1.8. The body is in equilibrium. If F_1 =18N, F_2 =22.5N, F_3=15N and F_4 =30N, find the force F_5 in magnitude and direction.

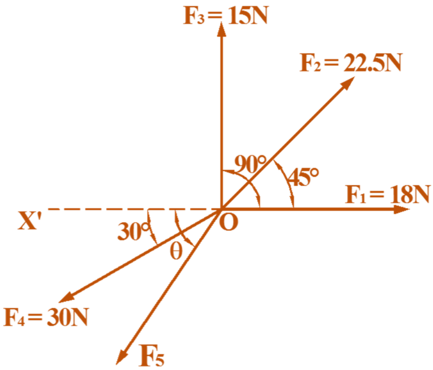


Fig.P1.8

Given : Force F_1 = 18 N ; Force F_2 = 22.5 N

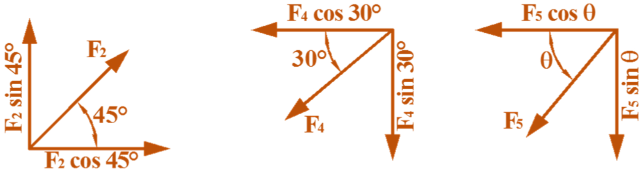
Force F_3 = 15 N ; Force F_4 = 30 N

To find : 1) Magnitude of force F_5 2) Direction of force F_5

Solution :

Let, θ= Angle mady by force F_5 with O-X' axis.

Resolving the inclined forces,



Applying the conditions for equilibrium,

(i) The sum of all the forces in X- direction = 0, i.e. $\Sigma F_x = 0$

$$F_1 + F_2 \cos 45° - F_4 \cos 30° - F_5 \cos\theta = 0$$
$$18 + 22.5 \times 0.707 - 30 \times 0.866 - F_5 \cos\theta = 0$$
$$8 + 15.91 - 25.98 - F_5 \cos\theta = 0$$
$$F_5 \cos\theta = 7.93 \qquad ------(1)$$

(ii) The sum of all the forces in Y- direction = 0, i.e. $\Sigma F_y = 0$

$$F_2 \sin 45° + F_3 - F_4 \sin 30° - F_5 \sin\theta = 0$$
$$22.5 \times 0.707 + 15 - 30 \times 0.5 - F_5 \sin\theta = 0$$
$$15.91 + 15 - 15 - F_5 \sin\theta = 0$$
$$F_5 \sin\theta = 15.91 \qquad ------(2)$$

Dividing equation (2) by (1),

$$\frac{F_5 \sin\theta}{F_5 \cos\theta} = \frac{15.91}{7.93}$$

$$\tan\theta = 2.0063$$
$$\theta = \tan^{-1}(2.0075) = \boxed{63.51°}$$

Substituting the value of θ in equation (2),

$$F_5 \sin 63.51° = 15.91$$
$$F_5 = \frac{15.91}{\sin 63.51°} = \boxed{17.78\text{ N}}$$

Result :	1) Magnitude of force F_5 **= 17.78 N**
	2) The direction of force F_5 , θ **= 63.51° from** O-X' **axis.**

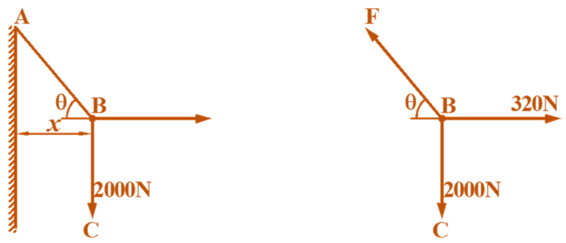
Example : 1.13

A body weighing 2000N is suspended with a chain AB of 2m long. It is pulled by a horizontal force of 320N as shown in the fig.P1.9. Find the force in the chain and the lateral displacement of the body.

To find : 1) Force in the chain, F 2) Displacement of the body, x

Solution :

Let, F = Force in chain AB, θ = Angle made by AB with horizontal.



(a) Space diagram (b) Free body diagram

Fig.P1.9

Draw the free body diagram as shown in the figure (b).

The point B is in equilibrium under the action of three forces.

Applying *Lami's theorem*,

$$\frac{F}{\sin 90°} = \frac{2000}{\sin(180-\theta)} = \frac{320}{\sin(90+\theta)}$$

$$\frac{F}{1} = \frac{2000}{\sin\theta} = \frac{320}{\cos\theta} \quad [\because \sin(180-\theta) = \sin\theta; \quad \sin(90+\theta) = \cos\theta]$$

$$F\sin\theta = 2000 \qquad ------(1)$$

$$F\cos\theta = 320 \qquad ------(2)$$

Dividing equation (1) by (2)

$$\frac{F\sin\theta}{F\cos\theta} = \frac{2000}{320}$$

$$\tan\theta = 6.25$$

$$\theta = \tan^{-1}(6.25) = \boxed{\mathbf{80.91°}}$$

Substituting the value of θ in (2),

$$F\cos 80.9° = 320$$

$$F = \frac{320}{\cos 80.9°} = \boxed{\mathbf{2025.5\ N}}$$

From figure (a),

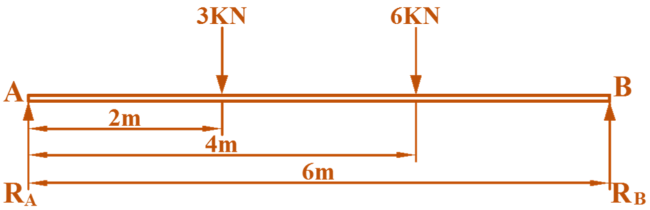
$$\cos\theta = \frac{x}{2}$$

$$x = 2\times\cos\theta = 2\times\cos 80.9° = \boxed{\mathbf{0.3161\ m}}$$

Result : 1) Force in the chain, ***F*** **= 2025.5N**
2) Displacement of the body, ***x*** **= 0.3161 m**

Example : 1.14

A simply supported beam AB of span 6m carries points loads of 3KN and 6KN at a distance of 2m and 4m from the left end A. Find the support reactions at A and B.



Solution :

Let, R_A = Support reaction at A

R_B = Support reaction at B

The beam is in equilibrium.

Applying the conditions for equilibrium,

(i) The sum of all the forces in horizontal direction = 0, i.e. $\Sigma F_x = 0$

There is no horizontal force.

(ii) The sum of all the forces in vertical direction = 0, i.e. $\Sigma F_y = 0$

$$R_A + R_B - 3 - 6 = 0$$

$$R_A + R_B = 3 + 6 = 9 \qquad ------(1)$$

(iii) The sum of all the moments about a point = 0, i.e. $\Sigma M = 0$

Now taking moment of al the forces about A, $\Sigma M_A = 0$

$$(R_B \times 6) - (6 \times 4) - (3 \times 2) = 0$$

+ | −

$(\therefore \textit{Moment} = \textit{Force} \times \textit{Distance})$

$$6\ R_B = 24 + 6 = 30$$

$$R_B = \frac{30}{6} = \boxed{5\text{ KN}}$$

Substitute the value of R_B in (1)

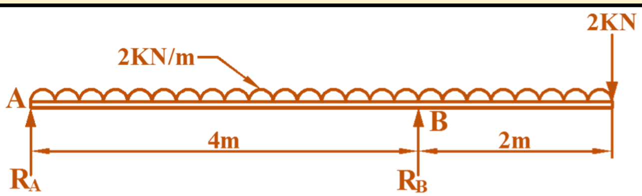
$$R_A = 9 - R_B = 9 - 5 = \boxed{4\text{ KN}}$$

Result : 1) The support reaction at A, R_A = **4 KN** (↑)

2) The support reaction at B, R_B = **5 KN** (↑)

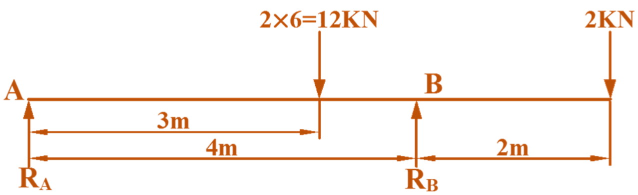
Example : 1.15

A beam AB of span 4m, overhanging on one side upto a length of 2m, carries a uniformly distributed load of 2KN/m over the entire length of 6m and a point load of 2KN/m. Calculate the reactions at A and B.



Solution :

The total udl is considered to act as a point load at the middle of the span over which it acts. Hence the given force system is reduced as follows :



Let, R_A =Support reaction at A, R_B = Support reaction at B

Applying the conditions for equilibrium,

(i) The sum of all the forces in horizontal direction = 0, i.e. $\Sigma F_x = 0$

There is no horizontal force.

(ii) The sum of all the forces in vertical direction = 0, i.e. $\Sigma F_y = 0$

$$R_A + R_B - 12 - 2 = 0$$

$$R_A + R_B = 12 + 2 = 14 \qquad ------(1)$$

(iii) The sum of all the moments about a point = 0, i.e. $\Sigma M = 0$

Now taking moment of all the forces about A, $\Sigma M_A = 0$

$$(R_B \times 4) - (12 \times 3) - (2 \times 6) = 0$$

$$4\ R_B = 36 + 12 = 48$$

$$R_B = \frac{48}{4} = \boxed{12\text{ KN}}$$

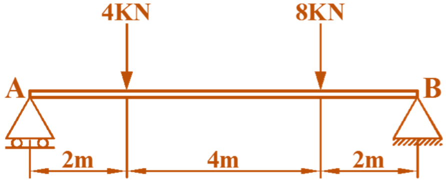
Substitute the value of R_B in (1)

$$R_A = 14 - R_B = 14 - 12 = \boxed{2\text{ KN}}$$

Result : 1)	The support reaction at A,	R_A	= **2 KN** (↑)
2)	The support reaction at B,	R_B	= **12 KN** (↑)

Example : 1.16 (Oct.16)

Determine the support reactions of the beam shown in the figure.

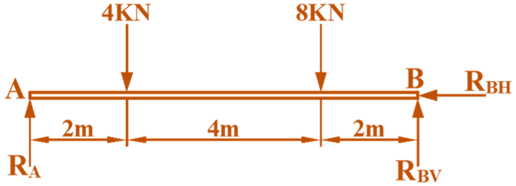


Solution :

In a roller support, only vertical reaction will act.

In a hinged support, both horizontal and vertical reactions will act.

The given force system is reduced as follows :



Let, R_A = Support reaction at A

R_{BH} = Horizontal Support reaction at B

R_{BV} = Vertical Support reaction at B

Applying the conditions for equilibrium,

(i) The sum of all the forces in horizontal direction = 0, i.e. $\Sigma F_x = 0$

There is no external horizontal force. $\therefore R_{BH} = 0$

(ii) The sum of all the forces in vertical direction = 0, i.e. $\Sigma F_y = 0$

$$R_A + R_{BV} - 4 - 8 = 0$$

$$R_A + R_{BV} = 4 + 8 = 12 \qquad ------(1)$$

(iii) The sum of all the moments about a point = 0, i.e. $\Sigma M = 0$

Now taking moment of all the forces about A, $\Sigma M_A = 0$

$$R_{BV} \times 8 - (4 \times 2) - (8 \times 6) = 0$$

$$8\ R_{BV} = 8 + 48 = 56$$

$$R_{BV} = \frac{56}{8} = \boxed{7\ KN}$$

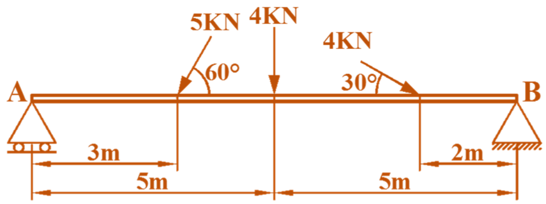
Substitute the value of R_{BV} in (1)

$$R_A = 12 - R_{BV} = 12 - 7 = \boxed{5\ KN}$$

Result :	1) The support reaction at A, R_A = **5 KN** (↑)
	2) The support reaction at B, R_B = R_{BV} = **7 KN** (↑)

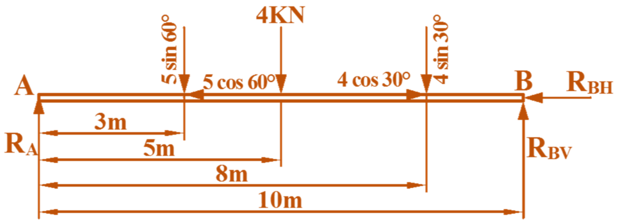
Example : 1.17

Determine the support reactions of the beam shown in the figure.



Solution :

The given force system is reduced as follows :



Let, R_A = Support reaction at A

R_{BH} = Horizontal Support reaction at B

R_{BV} = Vertical Support reaction at B

Applying the conditions for equilibrium,

(i) The sum of all the forces in horizontal direction = 0, i.e. $\Sigma F_x = 0$

$$4\cos 30° - 5\cos 60° - R_{BH} = 0$$

$$R_{BH} = 4\cos 30° - 5\cos 60° = \boxed{0.964\ \text{KN}}$$

As the result is positive, the assumed direction of R_{BH} (←) is correct.

(ii) The sum of all the forces in vertical direction = 0, i.e. $\Sigma F_y = 0$

$$R_A + R_{BV} - 5\sin 60° - 4 - 4\sin 30° = 0$$

$$R_A + R_{BV} = 5\sin 60° + 4 + 4\sin 30° = 10.33 \qquad ------(1)$$

(iii) The sum of all the moments about a point = 0, i.e. $\Sigma M = 0$

Now taking moment of al the forces about A, $\Sigma M_A = 0$

$$R_{BV}\times 10 - (4\sin 30° \times 8) - (4\times 5) - (5\sin 60° \times 3) = 0$$

The moments of the horizontal forces about A are zero.

Because the lines of action of these forces pass through A.

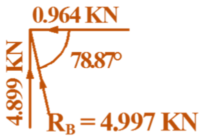
$$10\ R_{BV} = (4\sin 30° \times 8) + (4 \times 5) + (5\sin 60° \times 3) = 48.99$$

$$R_{BV} = \frac{48.99}{10} = \boxed{4.899\text{ KN}}$$

Substitute the value of R_{BV} in (1)

$$R_A = 10.33 - R_{BV} = 10.33 - 4.899 = \boxed{5.431\text{ KN}}$$

Inclination of the support reaction at B :



$$\tan\theta = \frac{R_{BV}}{R_{BH}} = \frac{4.899}{0.964} = 5.082$$

$$\theta = \tan^{-1}(5.082) = \mathbf{78.87°}$$

Magnitude of the support reaction at B :

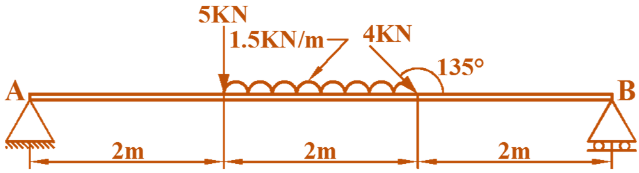
$$R_B = \sqrt{R_{BH}^2 + R_{BV}^2} = \sqrt{(0.964)^2 + (4.899)^2} = \mathbf{4.993\ KN}$$

Result : 1) The support reaction at A, R_A = **5.431 KN** (↑)

2) The support reaction at B, R_B = **4.993 KN**

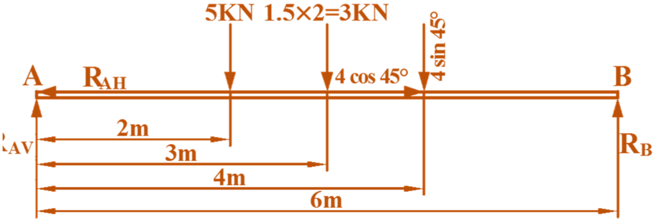
Example : 1.18

A Beam AB is loaded as shown in the figure. Determine the reactions at the supports A and B.



Solution :

The given force system is reduced as follows :



Let, R_{AH} = Horizontal Support reaction at A

R_{AV} = Vertical Support reaction at A

R_B = Support reaction at B

Applying the conditions for equilibrium,

(i) The sum of all the forces in horizontal direction = 0, i.e. $\Sigma F_x = 0$

$$4\cos 45^\circ - R_{AH} = 0$$

$$R_{AH} = 4\cos 45^\circ = \boxed{2.828\ \text{KN}}$$

As the result is positive, the assumed direction of R_{AH} ($\leftarrow$) ***is correct.***

(ii) The sum of all the forces in vertical direction = 0, i.e. $\Sigma F_y = 0$

$$R_B + R_{AV} - 5 - 3 - 4\sin 45^\circ = 0$$

$$R_B + R_{AV} = 5 + 3 + 4\sin 45^\circ = 10.828 \qquad ------(1)$$

(iii) The sum of all the moments about a point = 0, i.e. $\Sigma M = 0$

Now taking moment of all the forces about A, $\Sigma M_A = 0$

$$(R_B \times 6) - (5 \times 2) - (3 \times 3) - (4\sin 45^\circ \times 4) = 0$$

The moments of the horizontal forces about A are zero.

Because the lines of action of these forces pass through A.

$$6\ R_B = 10 + 9 + 11.314 = 30.314$$

$$R_B = \frac{30.314}{6} = \boxed{5.052\ \text{KN}}$$

Substitute the value of R_B in (1)

$$R_{AV} = 10.828 - R_B = 10.828 - 5.052 = \boxed{5.776\ \text{KN}}$$

Iinclination of the support reaction at A :

$$\tan\theta = \frac{R_{AV}}{R_{AH}} = \frac{5.776}{2.828} = 2.0424$$

$$\theta = \tan^{-1}(2.0424) = \boxed{63.9^\circ}$$

Magnitude of the support reaction at A :

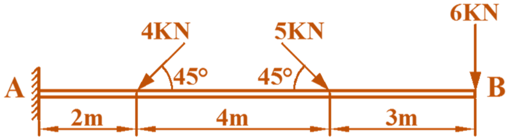
$$R_A = \sqrt{R_{AH}^2 + R_{AV}^2} = \sqrt{(2.828)^2 + (5.776)^2} = \boxed{6.43\ \text{KN}}$$

Result : 1) The support reaction at A, R_A = **6.43 KN**

2) The support reaction at B, R_B = **5.052 KN** ($\uparrow$)

Example : 1.19

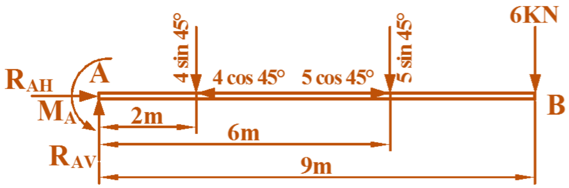
A cantilever beam of span 8 m is loaded as shown in the figure. Determine the reactions and the moment developed at the fixed end.



Solution :

In a fixed support, a horizontal, a vertical and a moment reactions will act.

The given force system is reduced as follows :



Let, R_{AH} = Horizontal support reaction at A

R_{AV} = Vertical support reaction at A

M_A = Moment developed at the fixed end A.

Applying the conditions for equilibrium,

(i) The sum of all the forces in horizontal direction = 0, i.e. $\Sigma F_x = 0$

$$R_{AH} - 4\cos 45° + 5\cos 45° = 0$$

$$R_{AH} = 4\cos 45° - 5\cos 45° = \boxed{-0.71 \text{ KN}}$$

As the result is negative, the assumed direction of R_{AH} $(\rightarrow)$***is wrong.***

Hence the direction of R_{AH} ***should be reversed*** $(\leftarrow)$

(ii) The sum of all the forces in vertical direction = 0, i.e. $\Sigma F_y = 0$

$$R_{AV} - 4\sin 45° - 5\sin 45° - 6 = 0$$

$$R_{AV} = 4\sin 45° + 5\sin 45° + 6 = \boxed{12.36 \text{ KN}}$$

(iii) The sum of all the moments about a point = 0, i.e. $\Sigma M = 0$

Now taking moment of al the forces about A.

$$M_A - (4\sin 45° \times 2) - (5\sin 45° \times 6) - (6 \times 9) = 0$$

The moments of the horizontal forces about A are zero.

Because the lines of action of these forces pass through A.

$$M_A = 5.66 + 21.21 + 54 = \boxed{80.87 \text{ KNm (anti-clockwise)}}$$

As the result is positive, the assumed direction of M_A ***(anti-clockwise) is correct.***

Iinclination of the support reaction at A :

$$\tan\theta = \frac{R_{AV}}{R_{AH}} = \frac{12.36}{0.71} = 17.408$$

$$\theta = \tan^{-1}(17.408) = \boxed{86.7°}$$

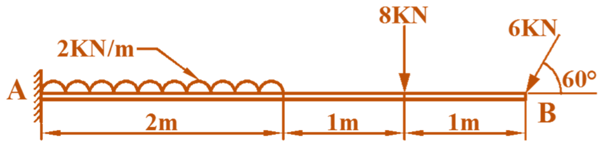
Magnitude of the support reaction at A :

$$R_A = \sqrt{R_{AH}^2 + R_{AV}^2} = \sqrt{(0.71)^2 + (12.36)^2} = \boxed{\textbf{12.38 KN}}$$

Result : 1) The support reaction at A, R_A = **12.38 KN**
2) The moment developed at the fixed support, M_A
= **80.87 KNm (anticlockwise)**

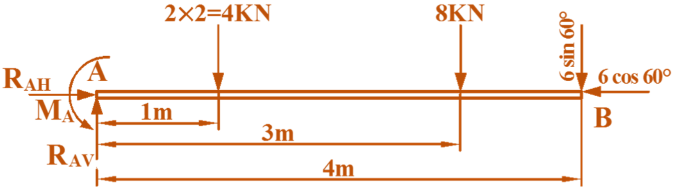
Example : 1.20

Find the support reactions at the fixed end of the cantilever beam shown in the figure.



Solution :

The given force system is reduced as follows :



Let, R_{AH} = Horizontal support reaction at A
R_{AV} = Vertical support reaction at A
M_A = Moment developed at the fixed end A.

Applying the conditions for equilibrium,

(i) The sum of all the forces in horizontal direction = 0, i.e. $\Sigma F_x = 0$

$$R_{AH} - 6\cos 60^\circ = 0$$

$$R_{AH} = 6\cos 60^\circ = \boxed{\textbf{3 KN}}$$

As the result is positive, the assumed direction of R_{AH} ***(→)is correct.***

(ii) The sum of all the forces in vertical direction = 0, i.e. $\Sigma F_y = 0$

$$R_{AV} - 4 - 8 - 6\sin 60^\circ = 0$$

$$R_{AV} = 4 + 8 + 6\sin 60^\circ = \boxed{\textbf{17.196 KN}}$$

(iii) The sum of all the moments about a point = 0, i.e. $\Sigma M = 0$

Now taking moment of al the forces about A.

$$M_A - (4 \times 1) - (8 \times 3) - (6\sin 60^\circ \times 4) = 0$$

The moments of the horizontal forces about A are zero.

Because the lines of action of these forces pass through A.

$M_A = 4 + 24 + 20.784 = \boxed{\textbf{48.784 KNm } \textit{(anti-clockwise)}}$

As the result is positive, the assumed direction of M_A (anti-clockwise) is correct.

Inclination of the support reaction at A :

$$\tan\theta = \frac{R_{AV}}{R_{AH}} = \frac{17.196}{3} = 5.732$$

$$\theta = \tan^{-1}(5.732) = \boxed{\mathbf{80.1^\circ}}$$

Magnitude of the support reaction at A :

$$R_A = \sqrt{R_{AH}^2 + R_{AV}^2} = \sqrt{(3)^2 + (17.196)^2} = \boxed{\mathbf{17.455\ KN}}$$

Result :	1) The support reaction at A, R_A = **17.455 KN** 2) The moment developed at the fixed support, M_A = **48.784 (anticlockwise)**

PROBLEMS FOR PRACTICE

1. Find the magnitude of the resultant of two concurrent forces of magnitude 8KN and 10KN. The angle between the two forces is 50°. ***[Ans: 16.34 KN]***

2. Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $9\sqrt{3}$ N, find the magnitude of the resultant force. ***[Ans: R =9 N]***

3. The resultant of the two forces, when they act at an angle of 60° is 14N. If the same forces are acting at right angles, their resultant is $\sqrt{136}$N. Determine the magnitude of the two forces. ***[Ans: P =10 N, Q= 6 N]***

4. Three forces of magnitude 30KN, 10KN and 15KN are acting at a point O. The angles made by 30KN force, 10 KN force and 15KN force with X-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force. ***[Ans: 21.79 KN, 83.4°]***

5. Two forces of magnitude 15N and 12N are acting at a point. If the angle between forces is 60°, determine the resultant of the forces in magnitude and direction. *[Ans : 23.43 N, 26.3°]*

6. A force 1000N is acting at a point, making an angle of 60° with the horizontal. Determine the components of this force along horizontal and vertical directions. *[Ans: 500 N, 866 N]*

7. Two forces P and Q are acting at a point O as shown in fig.P1.10. The force $P = 264.9$N and force Q=195.2N. If the resultant of the forces is equal to 400N, find the values of angles β, γ and α.

   $[Ans\ :\ \beta = 35°, \gamma = 25°, \alpha = 60°]$

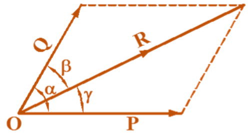


Fig.P1.10

8. The four coplanar forces are acting at a point as shown in fig.P1.11. Determine the resultant in magnitude and direction.

   [Ans : 1000 N, $\theta = 60°$ with OX]

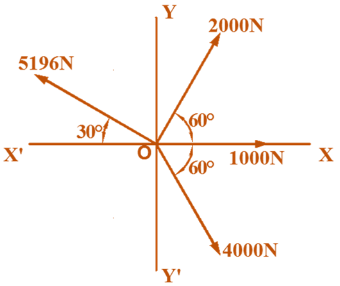


Fig.P1.11

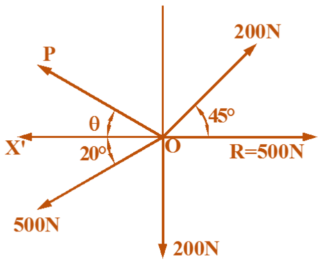


Fig.P1.12

9. The four coplanar forces are acting at a point as shown in fig. P1.12. The resultant is having a magnitude of 500N and is acting along X-axis. Determine the force P and its inclination with X-axis.

   [Ans: P =286.5 N, θ =53.25°]

10. A force of 50N is acting at a point A as shown in fig.P1.13. Determine the moment of this force about O.

    [Ans: 100 Nm clockwise]

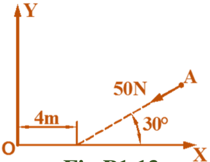


Fig.P1.13

11. Three like parallel forces 20N, 40N and 60N are acting at points A, B and C respectively on a straight line ABC. The distances are AB =3m and BC=4m. Find the resultant and also the distance of the resultant from point A on the line ABC. *[Ans: 120 N, 4.5 m]*

12. Four parallel forces of magnitudes 100N, 200N, 50N and 400N are shown in fig.P1.14. Determine the magnitude of the resultant and also the distance of the resultant from point A. *[Ans : R =350 N, 3.07 m]*

Fig.P1.14

13. A system of parallel forces are acting on a rigid bar as shown in figu.P1.15. Reduce this system to :
(i) a single force *[Ans: (i) R=120 N at 2.83 m from A*
(ii) a single force and a couple at A *(ii) R=120 N and* M_A*=-340 Nm*
(iii) a single force and a couple at B *(iii) R =120 N and* M_B*=120 nm]*

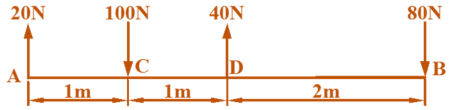


Fig.P1.15

14. Three forces F_1, F_2 and F_3 are acting on a body as shown in fig.P1.16. The body is in equilibrium. If the magnitude of force F_3 is 250N, find the magnitudes of force F_1 and F_2.
[Ans: F_1*=125 N ,* $F_2 = 215.6$ *N]*

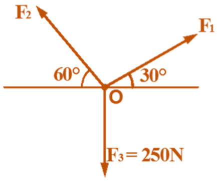
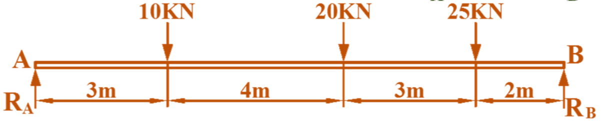


Fig.P1.16

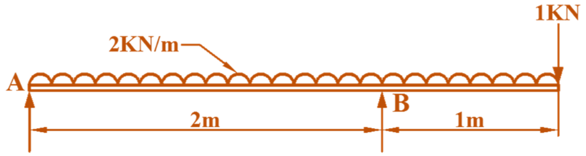
15. Three forces of magnitude 40KN, 15KN and 20KN are acting at a point O. The angles made by 40KN, 15KN and 20KN forces with X-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force. *[Ans : 30.4 KN and* $85.28°$ *with X-axis.]*

16. Determine the support reactions of the beam shown in the figure.
[Ans : R_A*=20 KN,* R_B*=35 KN]*

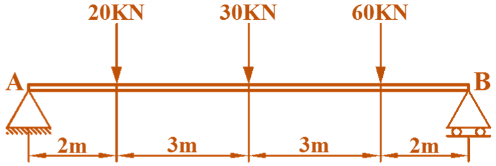


17. A simply supported beam of length 8m carries a uniformly distributed load 10KN/m for a distance of 4m, starting from a point which is at a distance of 1 m from the left end. Calculate the reactions at both ends.
[Ans : 25 KN, 15 KN]

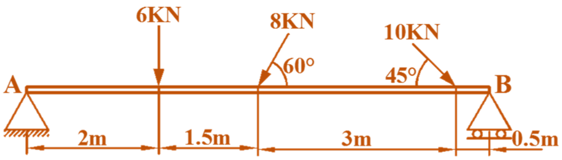
18. An overhanging beam is loaded as shown in the figure. Calculate the reactions at both ends. *[Ans : R_A=1 KN, R_B=6 KN]*



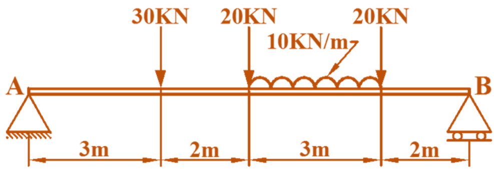
19. Determine the support reactions of the beam shown in the figure. *[Ans : R_A=43 KN, R_B=67 KN]*



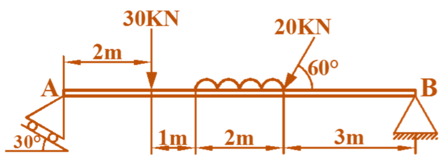
20. Determine the support reactions of the beam shown in the figure. *[Ans : R_A=8.81 KN, R_B=11.74 KN, $\theta = 69.6°$]*



21. Determine the support reactions of the beam shown in the figure. *[Ans : R_A=45.5 KN, R_B=54.5 KN]*

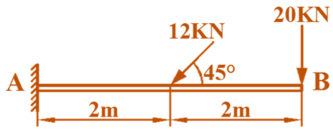


22. Find the support reactions of the beam loaded as shown in the figure. *[Ans : R_A=13.86 KN, R_B=11.82 KN; $\theta = 80.6°$]*

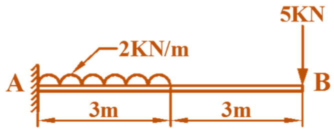


23. Determine the support reactions at the fixed end of the cantilever beam shown in the figure.

[Ans : R_A=29.72 KN; $\theta = 73.4°$] M_A=96.97 KNm(anticlockwise)]

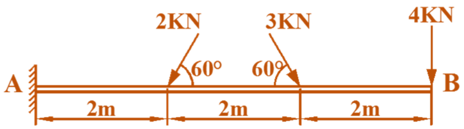


24. Find the support reactions at the fixed end of the cantilever beam shown in the figure. *[Ans : R_A=11 KN(↑), M_A=39 KNm(anticlockwise)]*



25. A cantilever beam of span 6 m is loaded as shown in the figure. Find the reactions at the fixed end of the beam.

[Ans : R_A=8.34 KN, $\theta = 86.6°$, M_A=37.86 KNm (anticlockwise)]



Unit – I

Chapter 2. FRICTION

2.1 Friction

The property of the bodies by virtue of which a force is exerted by a stationary body on the moving body to resist the motion of the moving body is called friction. Friction acts parallel to the surface of contact and depends upon the nature of the surface of contact.

2.2 Force of friction

When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called *force of friction* and is always acting in the direction of motion.

2.3 Limiting force of friction

The maximum value of frictional force acting on the body when the body just begins to slide over another body is called *limiting force of friction.*

2.4 Static friction

The frictional force acting on a body when the two surfaces of contact are at rest is called *static friction.*

2.5 Dynamic friction

The frictional force acting on a body when the body is moving, is called *dynamic friction* or *kinetic friction*.

2.6 Laws of static friction

1) The frictional force acts in the opposite direction in which surface is having tendency to move.
2) The frictional force is equal to the force applied to the surface, so long as the surface is at rest.
3) The frictional force is directly proportional to the normal reaction between the surfaces in contact.
4) The frictional force depends upon the material of the bodies in contact.
5) The frictional force is independent of the areas of contact between two surfaces.

2.7 Laws of dynamic friction

1) The frictional force acts in the opposite direction in which surface is having tendency to move.
2) The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces.
3) The limiting frictional force does not depend upon the shape and areas of the two surfaces in motion.
4) The frictional force is independent of the velocity of sliding.

2.8 Co-efficient of friction

Co-efficient of friction is defined as *the ratio of the limiting force of friction to the normal reaction between two surfaces in contact.* It is denoted by the symbol μ.

2.9 Angle of friction

Angle of friction is defined as *the angle made by the resultant of the normal reaction (R) and the limiting force of friction F_{max} with the normal reaction. It is denoted by ϕ.*

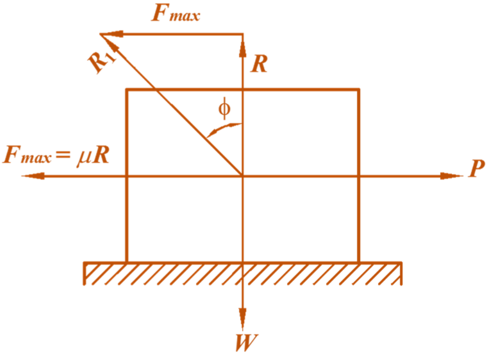


Fig.2.1 Angle of friction

Consider a body of weight W resting on a rough horizontal surface as shown in the fig.2.1. If an external force P is applied to the body and when it just begins to move, the following forces are acting on the body.

1. Weight of the body, W acting vertically downwards.
2. External force, P acting horizontally.
3. Normal reaction, R, between the plane and the body acting vertically upwards.
4. Frictional force, $F_{max} = \mu R$, acting in the opposite direction of movement of the body.

Let, R_1 be the resultant reaction of the normal reaction R and the maximum frictional force F_{max}.

From the fig.2.1

$$R = R_1 \cos\phi \qquad \text{——— (1)}$$

$$F_{max} = R_1 \sin\phi \qquad \text{——— (2)}$$

But, $F_{max} = \mu R$

$$\therefore \mu R = R_1 \sin\phi \qquad \text{——— (3)}$$

Dividing equation (3) by (1)

$$\frac{\mu R}{R} = \frac{R_1 \sin\phi}{R_1 \cos\phi}$$

$$\boxed{\mu = \tan\phi}$$

Thus, the tangent of the angle of friction is equal to the Co-efficient of friction.

2.10 Cone of friction

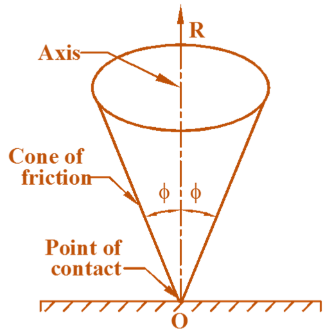


Fig.2.2 Cone of friction

Cone of friction is defined as the right circular cone with vertex at the point of contact of the two bodies (or surfaces) , axis in the direction of normal reaction (R) and semi vertical angle equal to the angle of friction (ϕ).

Fig.2.2 shows the cone of friction, in which

O = Point of contact between two bodies

R = Normal reaction and also axis of cone of friction

ϕ = Angle of friction

2.11 Angle of repose

The angle of repose is defined as *the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.*

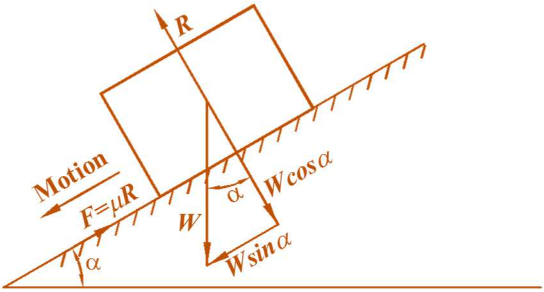


Fig.2.3 Angle of repose

Consider a body of weight W, resting on a rough inclined plane as shown in fig.2.3

Let, R = Normal reaction acting at right angle to the inclined plane

α = Inclination of the plane with the horizontal

F = Frictional force acting along the plane

Let the angle of inclination (**α**) be gradually increased, till the body just starts sliding down without any external force. This angle of inclined plane, at which a body just begins to slide down the plane, is called angle of repose.

Resolving the forces along the plane

$$F = W \sin \alpha \qquad \text{——— (1)}$$

Resolving the force normal to the plane,

$$R = W \cos \alpha \qquad \text{——— (2)}$$

Dividing equation (1) by (2)

$$\frac{F}{R} = \frac{W \sin \alpha}{W \cos \alpha} = \tan \alpha \qquad \text{- - - - - (3)}$$

Also, $\frac{F}{R} = \mu = \tan \phi$ ——— (4)

where, ϕ = Angle of friction

Hence from equation (3) and (4)

$$\boxed{\tan \phi = \tan \alpha \quad \text{(or)} \quad \alpha = \phi}$$

Thus, *angle of repose = Angle of friction.*

2.12 Friction on an inclined plane

Consider a body of weight 'W' lying on an inclined plane having an inclination of angle θ with the horizontal. If this angle of inclination of the plane with the horizontal is less than the angle of friction, the body will remain in equilibrium without any external force. If the body is to be moved upwards or downwards in this condition, an external force is required.

But, if the inclination of the plane is more than the angle of friction, the body will not remain in equilibrium. The body will move downward and upward external force will be required to keep the body in equilibrium.

REVIEW QUESTIONS

1. Distinguish between force of friction and limiting force of friction. *(Oct.03, Oct.16)*
2. Distinguish between static friction and dynamic friction. *(Apr.01, Apr.17, Apr.18)*
3. State the laws of static friction. *(Oct.04, Apr.04)*
4. State the laws of dynamic friction. *(Oct.96)*
5. Identify the factors on which the static friction depends. *(Oct.01)*
6. Define Co-efficient of friction and angle of friction. *(Oct.02, Apr.05, Oct.17)*
7. Define angle of repose.

Unit – II

Chapter 3. MECHANICAL PROPERTIES OF MATERIALS

3.1 Introduction

A vast range of materials are available today for the choice of engineer. A proper selection of material has to be made to suit the requirements. The best material is one which serves the required objective at the minimum cost. The selection of material for a particular application involves consideration of factors like mechanical properties, service requirements, manufacturing requirements and cost of the material. The mechanical properties of materials are those properties which describe the behaviour or the material under mechanical usage. Some important mechanical properties of materials are explained below.

3.2. Mechanical properties of materials

1) Elasticity

When a body is subjected to a system of external forces, deformation of body takes place. This deformation disappears at once the external forces are removed. This property of material by which a body regains its original shape and size after deformation when applied forces are removed is known as *elasticity*.

If the body regains it original shape completely, it is said to be *perfectly elastic*. However, this phenomenon holds good up to a particular value of stress known as *elastic limit*. Beyond this limit, the deformation does not entirely disappear when the force is removed. This residual deformation is known as *permanent set*. *The elasticity property is desirable in materials used for manufacturing of tools and machine elements.*

Example: Steel and rubber are some materials having good elasticity.

2) Plasticity

Plasticity is the property of a material by which a body retains the deformation due to applied load without rupture, even after the removal of applied load. Most materials become plastic under the application of heavy forces. *Plasticity plays an important role in manufacturing processes like forming, forging, swaging, coining, extrusion, etc.*

Example: Clay and lead are some materials having good plasticity.

3) Ductility

Ductility is the property of a material by which the material can be drawn out or elongated into thin wires without rupture by applying a tensile force. A ductile material should be strong and plastic in nature. Ductility of a material is usually measured by the percentage of elongation and percentage of reduction in area at fracture. *This property is very important in manufacturing processes like rolling, wire drawing, etc.*

Example: Mild steel, copper, aluminium, zinc, gold and platinum are some materials having high ductility.

4) Malleability

Malleability is the property of a material by which the material can be flattened into thin sheets without cracking by hot or cold working processes. A malleable material possesses a high degree of plasticity and can be hammered or rolled into any desired shape without rupture. *This property is very important in manufacturing processes like forging, hot rolling, drop forging, wire drawing, etc.*

Example: Mild steel, wrought iron, copper and aluminium are some materials having high malleability.

5) Machinability

Machinability is the property of a material by which the material can be easily machined by cutting tools in various machining operations. *The machinability of different materials can be compared with the help of machinability index.*

The following are the advantages, if the material having good machinability :

1) The rate of metal removal is high
2) Long life of cutting tool
3) Less power consumption
4) Good surface finish

Example: Grey cast iron has excellent machinability.

6) Castability

Castability is the property of a material by which the material can be easily cast into different size and shapes. *Castability of a material can be decided by considering the solidification rate, shrinkage during cooling, gas porosity and hot strength.*

Example: Grey cast iron has good castability.

7) Weldability

Weldability is the property of a material by which the material can be welded into a specific and suitable designed structure and to perform satisfactorily in the desired objective.

8) Strength

Strength is a property of a material by which the material can withstand or resist the action of external force or load without breaking or yielding. Strength of a material is dependent on the type of load acting on it. *This property plays an important role in the designing of various structures and machine elements.*

9) Stiffness or rigidity

Stiffness is the property of a material to resist elastic deformation or deflection due to the applied load. It is also know as *rigidity*. It is measured by the modulus of rigidity. A stiffer material undergoes smaller deformation when subjected to load. *This property is very important in the design of beams, shafts and springs.*

Example: Steel, aluminium and brass are the some materials having high stiffness.

10) Toughness

Toughness is the property of a material to resist the fracture by absorbing energy due to heavy shock loads or blow, without rupture. It is measured by the amount of energy that an unit volume of material can absorb before the point of actual failure takes place. *This property is desirable in structural members and machine elements which are subjected to absorb shock and vibrations.*

Example: Cast iron, mild steel and brass are some materials having high toughness.

11) Brittleness

Brittleness is the property of a material by which the material will fail or fracture all of sudden without any significant deformation. This property is opposite to ductility. *This property is desirable in machine parts which may be subjected to sudden compressive loads.*

Example: Cast iron, concrete, glass and stone are some material having high brittleness.

12) Hardness

Hardness is the ability of a material to resist surface penetration, abrasion and scratching. It is usually expressed as a number which is relative to the hardness of standard specimens. Hardness of a material is decreased by heating. *It is an important property involved in the design of machine members such as gears, cams, chain sprockets, etc. which are under constant rubbing action.*

Example: Hard steel, cast iron, glass and diamond are the some materials having good hardness.

13) Wear resistance

Wear resistance is the property of a material to resist wear. Wear resistance can be increased by increasing the hardness of the working surface by *case hardening process*.

14) Fatigue

Fatigue of a material may be described as the failure of the material when subjected to a number of cyclically changing loads in which the maximum stress developed in each cycle is well within the elastic range. *This property is important in machine elements such as motor shafts, bolts, springs, gear teeth, valves and turbine blades.*

Fatigue may be caused due to the application of following types of loading.

1) Repeated loading: If a member is subjected to either compressive or tensile load of same magnitude repeatedly, then the type of loading is called repeated loading.

2) Cyclic loading: If a member is subjected to compressive and tensile loads alternatively and also the magnitude of load vary from maximum value to minimum value or *vise versa* at regular intervals, then the type of loading is called cyclic loading.

Example: Load on piston and connecting rod of a double acting engine.

15) Fatigue strength

The stress at which a material fails by fatigue is known as *fatigue strength*. Fatigue strength decreases when temperature increases. *It is mainly affected by the type of loading and working nature of the material.*

16) Endurance limit or fatigue limit

Endurance limit or *fatigue limit* is a maximum stress below which a load may be repeatedly applied at infinite number of times without causing failure of material by fatigue. *This limiting stress is most important while designing machine elements which are subjected to repeated or cyclic loading.*

17) Creep

Creep is the property of a material by which the material is deformed slowly and progressively under a constant load over a long period.

Mechanical creep: If the slow and progressive deformation of material is due to constant loading which is well below the elastic limit, then the creep is called mechanical creep.

Temperature creep: If the slow and progressive deformation of material is due to rise in temperature i.e. thermal expansion of material, then the creep is called temperature creep. Creep is considered important in the following :

1) The soft metals like tin, zinc, lead and their alloys used at room temperature.
2) I.C engine components.
3) Components of gas turbines, steam turbines and boilers working at high temperature.
4) Components of rockets, missiles, etc.

3.3 Ferrous and Non-ferrous Metals

All metals can be classified as either *ferrous* or *non-ferrous.*

Ferrous metals

These are metals which contain iron. They may have small amounts of other metals or other elements added, to give the required properties. All ferrous metals are magnetic and give little resistance to corrosion.

Non-Ferrous Metals

These are metals which do not contain any iron. They are not magnetic and are usually more resistant to corrosion than ferrous metals.

Ferrous Metals

Name	Properties	Uses
Mild Steel	Tough, high tensile strength, ductile. It must be case hardened.	Girders, plates, nuts and bolts, general purpose.
High Speed Steel	Can be hardened and tempered. Can be brittle. Retains hardness at high temperatures.	Cutting tools for lathes.
Stainless Steel	Corrosion resistant	Kitchen draining boards, pipes, cutlery, aircraft.
High Tensile Steel	Very strong and very tough.	Gears, shafts, engine parts.
High Carbon Steel	The hardest of the carbon steels. Less ductile, tough and malleable.	Chisels, hammers, drills, files, lathe tools, taps and dies.
Medium Carbon Steels	Stronger and harder than mild steels. Less ductile, tough and malleable.	Metal ropes, wire, garden tools, springs.
Cast Iron	Hard, brittle, strong, cheap, self-lubricating.	Cylinder blocks, vices, machine tool parts, brake drums, gear wheels, plumbing fitments.

Non – ferrous metals

Name	Properties	Uses
Aluminium	Soft, malleable, conductive to heat and electricity, corrosion resistant.	Aircraft, boats, window frames, pistons and cranks.
Copper	Tough, ductile, high electrical conductor, corrosion resistant.	Electrical wire, cables, printed circuit boards, roofs.
Brass	Harder than copper. Good electrical conductor.	Castings, ornaments, valves, forgings.
Lead	The heaviest common metal. Soft, malleable, bright and shiny. Resistant to corrosion.	Paints, roof coverings, flashings.
Zinc	A layer of oxide protects it from corrosion, bluish-white, easily worked.	Makes brass. Coating for steel galvanized corrugated iron roofing, tanks, buckets, rust-proof paints
Tin	White and soft, corrosion resistant.	Tinplate, making bronze.

3.4 Alloying elements and their effect

An alloy is a mixture of two or more metals. When a material is needed which requires certain properties and this does not exist in a pure metal we combine metals. For example, an alloy steel is a type of steel to which one or more alloying elements have been added to give it special properties that cannot be obtained in carbon steel. The important alloying elements and their effects in steel are listed below.

Alloying element	Major effects
Aluminium (Al)	• Increases toughness, acts as deoxidizer
Boron (B)	• Increases hardenability without decreasing ductility
Calcium (Ca)	• It provides better machinability than aluminum • It improves toughness
Carbon (C)	• Increasing carbon increases hardness and strength • Increasing carbon decreases toughness and ductility
Chromium (Cr)	• Provides a moderate contribution to hardenability • Provides strength and resistance to oxidation • Provides abrasion resistance
Cobalt (Co)	• Resist softening at elevated temperatures
Copper (Cu)	• Improves resistance to atmospheric corrosion • Decreases the ability to hot work steels
Lead (Pb)	• Improves machinability
Manganese (Mn)	• Provides a moderate contribution to hardenability • Improves machinability • Increases strength and reduces ductility
Molybdenum (Mo)	• Contributes greatly to hardenability • Increases strength and creep resistance • Improves corrosion resistance
Nickel (Ni)	• Provides a moderate contribution to hardenability • Strengthens unhardened steels by solid solution • Provides toughness.
Niobium (Nb)	• Increases strength • Decreases the hardenability
Phosphorus (P)	• Increases hardenability • Improves strength • Severely reduces toughness and ductility • Improves corrosion resistance • Improves machinability

Alloying element	**Major effects**
Silicon (Si)	• Increases hardenability • Increases strength
Sulphur (S)	• Decreases strength and ductility • Decreases weldability • Increases machinability
Titanium (Ti)	• Reduces hardness
Tungsten (W)	• Decreases softening during tempering • Forms abrasion resistant carbides • Increases hardness
Vanadium (V)	• Increases hardenability • Resists softening during tempering

REVIEW QUESTIONS

1. Explain any three mechanical properties of material. *(Oct.04, Oct.01)*
2. Define the properties elasticity and plasticity of material. *(Oct.02, Oct.03, Apr.05)*
3. Distinguish between ductility and malleability of materials.
4. Define the properties machinability and weldability of materials *(Oct.04)*
5. What do you underst by strength and stiffness of material.
6. Distinguish between toughness and hardness of materials. *(Apr.04, Apr.18)*
7. Explain the properties brittleness and wear resistance of material.
8. What is meant by creep and fatigue in a metal. *(Apr.02)*
9. Differentiate between repeated loading and cyclic loading. *(Oct.96)*
10. Define fatigue strength and endurance limit. *(Apr.17)*
11. Differentiate between mechanical creep and temperature creep. *(Oct.16)*
12. Name any four ferrous metals and their uses.
13. Name any four non-ferrous metals and their uses.
14. List out the various alloying elements used in steel and explain their principal effects. *(Apr.18)*

Unit – II

Chapter 4. SIMPLE STRESSES AND STRAINS

4.1 Introduction

No engineering material is perfectly rigid. When a material is subjected to external load, it undergoes deformation. While undergoing deformation, the particles of the material exert a resisting force. When this resisting force equals applied load, the equilibrium condition exists hence deformation stops. This internal resistance is called the *stress*.

4.2 Behaviour of material when subjected to load.

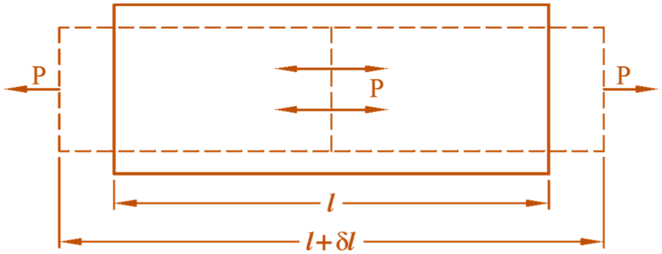


Fig.4.1 Behaviour of material when subjected to load

Consider a bar of uniform cross sectional area A and length l subjected to an axial pull of P at the ends as shown in the fig.4.1.

Consider a section X–X normal to the longitudinal axis of the bar. Due to the action of axial pull, the length of the bar is increased from l to $l + \delta l$ and lateral dimension will decrease. In order to keep this section in equilibrium, internal resistance are set up in the section. To avoid separation of the bar at this section, the internal resistance must be equal to the applied load. This internal resistance offered by the section against the deformation is called *stress*.

4.3 Definition of load, stress and strain

Load

The system of external forces acting on a body or structure is known as *load*.

Stress

The stress or intensity of stress at a section may be defined as *the ratio of the internal resistance or load acting on the section to the cross sectional area of that section.*

$$\text{Stress, } f = \frac{\text{Internal resistance}}{\text{Area of cross section}} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

The unit of stress is **N/mm^2**. The latest S.I unit for stress is **Pascal.**

1 Pascal	= 1 Pa	= 1 N/m^2	= 1×10^{-6} N/mm^2	
1 Kilo Pascal	= 1 KPa	= 1×10^3 N/m^2	= 1×10^{-3} N/mm^2	
1 Mega Pascal	= 1 MPa	= 1×10^6 N/m^2	= 1 N/mm^2	
1 Giga Pascal	= 1 GPa	= 1×10^9 N/m^2	= 1×10^3 N/mm^2	

Strain

Strain may be defined as *the ratio between the deformation produced in a body due to the applied load and the original dimension*.

$$\textbf{Strain, } e = \frac{\textbf{Change in dimension}}{\textbf{Original dimension}}$$

The strain is only the ratio between the two same quantities and hence it has no unit.

4.4 Classification of force system

According to the *applied load*, the force system is classified as follows:

1) Tensile stress 2) Compressive stress 3) Shear stress
4) Bending stress 5) Torsional stress

1) Tensile stress

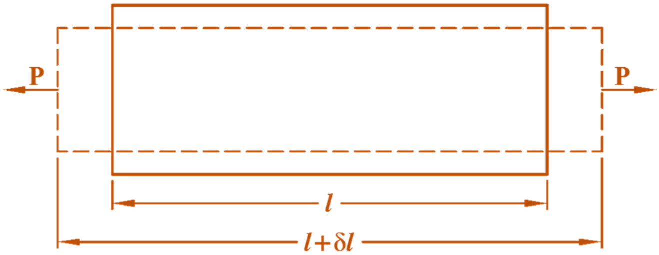


Fig.4.2 Tensile stress

When a load is such that it tends to pull apart the particles of the material causing increase in length in the direction of application of load, then the load is called *tensile load*. The resistance offered against this increase in length is called *tensile stress*. The corresponding strain is called *tensile strain*.

$$\textbf{Tensile stress, } f = \frac{\textbf{Axial pull}}{\textbf{Area of cross section}} = \frac{P}{A} \ (\text{N/mm}^2)$$

$$\textbf{Tensile strain, } e = \frac{\textbf{Increase in length}}{\textbf{Original length}} = \frac{\delta l}{l}$$

2) Compressive stress

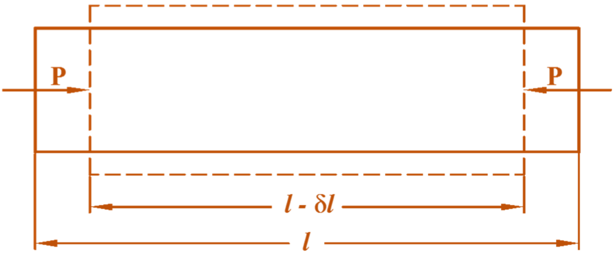


Fig.4.3 Compressive stress

When a load is such that it pushes the particles of the material nearer causing decrease in length in the direction of application of load, then the load is called *compressive load*. The resistance offered against this decrease in length is called *compressive stress* and the corresponding strain is called *compressive strain*.

$$\text{Compressive stress, } f = \frac{\text{Axial push}}{\text{Area of cross section}} = \frac{P}{A} \; (\text{N/mm}^2)$$

$$\text{Compressive strain, } e = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{\delta l}{l}$$

3) Shear stress

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, the body tends to be sheared off across the cross section. Such forces are called *shear force*. The stress induced in the section due to the shear force is called *shear stress* and the corresponding strain is called *shear strain*.

$$\text{Shear stress, } q = \frac{\text{Total shear force}}{\text{Area of resisting section}} = \frac{P}{A} \; (\text{N/mm}^2)$$

$$\text{Shear strain, } e = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

4) Bending stress

When a beam is loaded with some external forces, bending moments and shear forces are set up. The bending moment at a section tends to bend or deflect the beam. Internal stresses are developed to resist the bending. These stresses are called *bending stresses.*

5) Torsional stresses

When a machine member is subjected with two equal and opposite couples acting in parallel planes, then the member is said to be in torsion. The stress induced by this torsion is called *torsional stress*.

4.5 Behaviour of material in tension up to rupture

Stress – strain diagram

A standard specimen made of ductile material or brittle material is tested in an Universal Testing Machine (UTM) to obtain information regarding the behaviour of a material under gradually increasing stress and strain condition.

A standard specimen is subjected to gradually increasing axial load and the values of loads corresponding to deformation at regular intervals are noted. A plot of *stress Vs strain* is drawn. A typical stress – strain diagram for mild steel in tension is shown in the fig.4.4. The following salient points can be observed from the stress –strain diagram.

Proportional limit: It is the maximum stress level up to which stress is directly proportional to strain i.e. the material obeys Hooke's law. In the stress – strain diagram, the point A is called the *proportional limit* or *limit of proportionality*.

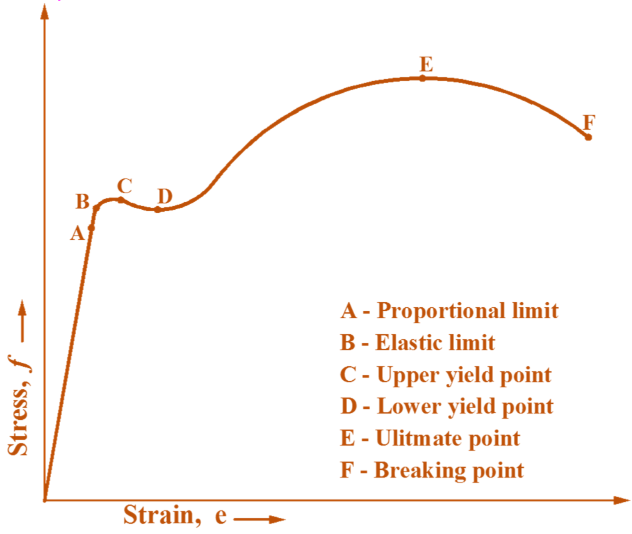


Fig.4.4 Stress – strain diagram (Mild steel)

Elastic limit: The maximum stress level up to which the material shows the characteristics of regaining its original shape and dimensions on removal of load is known as *elastic limit*. When the load is increased beyond the proportional limit, the rate of increase in elongation is more than the previous rate and the straight line actually curves down from the *point A*. In the elastic range, the material will regain its original shape, even though it does not obeys Hooke's law. The *point B* in the diagram represents the elastic limit.

Yield stress: Yield stress is the value of stress at which the material continues to deform at constant load condition. The phenomenon of increase in strain without any appreciable increase in load is called *yielding*. In the diagram, the *point C* is called the *upper yield point* and the *point D* is called the *lower yield point*. Brittle material do not show the phenomenon of yielding.

Plastic range: After the elastic limit, the specimen undergoes deformation which cannot be regained with the removal of load. This deformation is known as *plastic deformation*. In this region, deformation is not directly proportional to strain.

Ultimate stress: It is the maximum stress induced in the specimen and it occurs in the plastic region. In the diagram, the *point E* is called the *ultimate point*. Beyond this point, the load decreases while the elongation increasing at a rapid rate.

Phenomenon of necking: Weaker section in the specimen undergoes reduction in area which continues from ultimate point to breaking point. This reduction in cross sectional area is known as *necking*. The phenomenon of necking does not occur in brittle materials.

Breaking stress: As the reduction in cross sectional area continues, the load bearing capacity of specimen reduces gradually. At a certain stage, cross sectional area of specimen is so small that it cannot sustain the load and hence it breaks. The stress at which the specimen breaks is known as *breaking stress*. In the diagram, the *point F* represents the breaking point. Breaking stress is generally less than the ultimate stress.

Percentage of elongation

Let, l = Original length of the specimen

l_o = Length of specimen after fracture

$$\text{Percentage of elongation} = \frac{l_0 - l}{l} \times 100$$

Percentage reduction in area

Let, A = Original area of cross section of the specimen

A_o = Area at the neck after fracture

$$\textbf{Percentage reduction in area} = \frac{A - A_o}{A} \times 100$$

4.6 Hooke's law

Hooke's law states that *stress is directly proportional to strain within elastic limit.*

$$\textit{i.e.}\ \textbf{stress} \propto \textbf{strain} \ \text{(or)}\ \frac{\textbf{Stress}}{\textbf{Strain}} = \textbf{A constant}$$

For tensile and compressive stresses, the constant is known as *Young's modulus* or *modulus of elasticity*.

For shear stress, the constant is known as *modulus of rigidity*.

4.7 Young's modulus or modulus of elasticity

The ratio of stress to strain in tension or compression is known as *Young's modulus* or *modulus of elasticity.* It may also be defined as the slope of stress – strain curve in elastic region. It is denoted by **'*E*'** and the unit is **N/mm^2.**

Young's modulus is the measure of stiffness of the material. A member made of material with larger value of Young's modulus is said to have higher stiffness. The stiffer materials undergo smaller deformation for a given load condition.

4.8 Working stress

The maximum stress to which the material of a member or machine element is subjected in normal usage is called *working stress.* It is also known as *allowable stress* or *design stress*. To avoid permanent set, the working stress is kept less than the elastic limit.

4.9 Factor of safety and load factor

The ratio of ultimate stress to working stress is known as *factor of safety*.

$$\textbf{Factor of safety} = \frac{\textbf{Ultimate stress}}{\textbf{Working stress}}$$

The value of factor of safety varies from 3 in case of steel to as high as 20 in case of timber subjected to suddenly applied load. The value of factor safety depends on the following factors.

1) The reliability of the material
2) The accuracy with which the maximum load on the member is determined
3) The nature of loading
4) The effect of corrosion and wear
5) The effect of temperature
6) Possible manufacturing defects.

Load factor: The ratio of ultimate load to working load is called load factor.

$$\textbf{Load factor} = \frac{\textbf{Ultimate load}}{\textbf{Working load}}$$

4.10 Linear strain or longitudinal strain

Linear strain or longitudinal strain is defined as the ratio of the change in length to the original length.

$$\textbf{Linear or longitudinal strain, e} = \frac{\textbf{Change in length}}{\textbf{Original length}} = \frac{\delta l}{l}$$

4.11 Deformation due to tensile or compressive force

Consider a bar subjected to an axial pull or push at the ends. Due to this load, deformation occurs in the bar.

Let, P = Load acting on the bar

l = Length of the bar

A = Cross sectional area of the bar

f = Stress induced in the bar

e = Strain in the bar

δl = Deformation of the bar and

E = Young's modulus of the material of the bar

According to Hooke's law,

$$\frac{\text{Stress}}{\text{Strain}} = E \qquad \text{-------------} \ (1)$$

$$\text{Stress, } f = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$$\text{Strain, } e = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

Substituting the values of stress and strain in equation (1)

$$E = \frac{\left(\frac{P}{A}\right)}{\left(\frac{\delta l}{l}\right)} = \frac{P\,l}{A\,\delta l}$$

$$\boxed{\delta l = \frac{P\,l}{A\,E}} \quad \textbf{(or)} \quad \boxed{\delta l = \frac{f\,l}{E}} \quad \left(\because \frac{P}{A} = f\right)$$

4.12 Bars of varying sections

Consider a bar having different cross sections for different length as shown in the fig.4.5. Let this bar is subjected to an axial pull or push at the ends. It may be noted that each section in the bar is subjected to the same axial push or pull. Due to this variations in cross sectional area, the stresses, strain and hence change in length for each section are different. These values are calculated separately for each section as usual. The total changes in length is equal to the sum of the changes of all the individual lengths of the section.

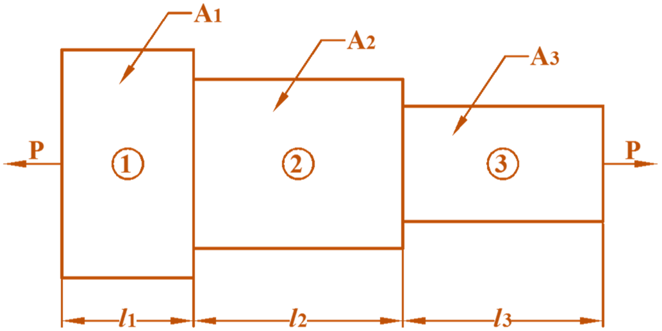


Fig.4.5 Bars of varying sections

Let l_1, l_2, l_3 and A_1, A_2, A_3 be the length and area of the sections of 1, 2, 3 respectively.

Change in length of section 1, $\delta l_1 = \dfrac{P\, l_1}{A_1\, E}$

Similarly, $\delta l_2 = \dfrac{P\, l_2}{A_2\, E}$; $\delta l_3 = \dfrac{P\, l_3}{A_3\, E}$

Total deformation of the bar, $\delta l = \delta\, l_1 + \delta\, l_2 + \delta\, l_3$

$$= \frac{P\, l_1}{A_1 E} + \frac{P\, l_2}{A_2 E} + \frac{P\, l_3}{A_3 E} = \boxed{\frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)}$$

If the modulus of elasticity is different for different sections, then

$$\boxed{\delta l = P\left(\frac{l_1}{A_1 E_1} + \frac{l_2}{A_2 E_2} + \frac{l_3}{A_3 E_3}\right)}$$

4.13 Shear stress and shear strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, the body tends to be sheared off across the cross section. Such forces are called *shear force*. The stress induced in the section due to the shear force is called *shear stress* and the corresponding strain is called *shear strain*. In shear, the strain is measured by the angle in radians through which the body is distorted by the applied force.

Consider a cube ABCD of side l fixed at the bottom face DC. Let a tangential force P be applied at the face AB. As a result of this force, the cube is distorted from $ABCD$ to $A'B'CD$ through an angle ϕ as shown in fig.4.6.

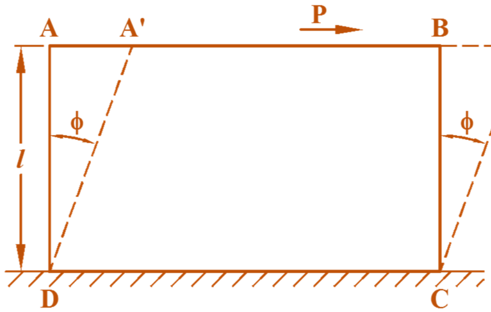


Fig.4.6 A body subjected to shear force

$$\text{Shear strain} = \frac{\textbf{Change in length}}{\textbf{Original length}} = \frac{DA' - DA}{DA} = \frac{AA'}{l} = \sin\phi = \phi$$

(∵ For small angle, $\sin\phi = \phi$)

4.14. Modulus of rigidity or shear modulus

The ratio of shear stress to shear strain within the elastic limit is known a *modulus of rigidity* or *shear modulus*. It is denoted by ***N*** or ***G*** or ***C*** and the unit is **N/mm^2**. Larger is the modulus of rigidity, lesser is the distortion when a body is subjected to shear stress.

$$\textbf{Modulus of rigidity, } C = \frac{\textbf{Shear stress}}{\textbf{Shear strain}}$$

4.15 Lateral strain

It is the ratio of *the change in lateral dimension to the original dimension.* Lateral strain is induced along the direction perpendicular to the direction of application of load.

4.16 Poisson's ratio

The ratio of the lateral strain to the corresponding longitudinal strain within elastic limit is called Poisson's ratio. It is represented by ν (nu) or $1/m$.

$$\textbf{Poisson's ratio} = \frac{1}{m} = \frac{\textbf{Lateral strain}}{\textbf{Longitudinal strain}}$$

For most of the material, Poisson's ratio lies between 0.25 to 0.33.

4.17 Volumetric strain

When a body is subjected to an axial pull or push, it undergoes change in its dimensions and hence its volume will also change.

The ratio of change in volume to the original volume is known as *volumetric strain.*

$$\textbf{Volumetric strain, } e_v = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\delta V}{V}$$

4.18 Bulk modulus

When a body is subjected to three mutually perpendicular stresses of same magnitude, the ratio of the direct stress to the corresponding volumetric strain is known as *bulk modulus* or *bulk modulus of elasticity*. It represents the resistance of a body against volumetric strain. It is usually denoted by ***K***.

$$\textbf{Bulk modulus, } K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{p}{e_v}$$

4.19 Volumetric strain of various sections

1) Rectangular bar

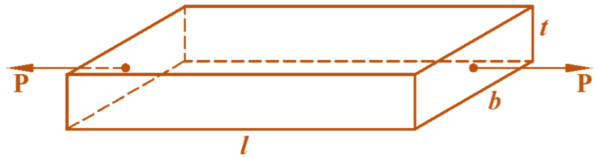


Fig.4.7 Volumetric strain in rectangular bar

Consider a rectangular bar of length l, width b and thickness t and is subjected to an axial tensile force P as shown in fig.4.7.

Let δl, δb, δt be the changes in dimensions due to the applied load.

Original volume, $V_1 = b \times t \times l$

Final volume, $V_2 = (b + \delta b)(t + \delta t)(l + \delta l)$

$$= (b + \delta b)(tl + t\delta l + l\delta t + \delta l\ \delta t)$$

$$= (b\,t\,l + b\,t\,\delta l + b\,l\,\delta t + b\,\delta l\,\delta t + t\,l\,\delta b + t\,\delta l\,\delta b + l\,\delta t\,\delta b + \delta b\,\delta l\,\delta t)$$

Neglecting the higher powers of δl, δb and δt,

Final volume, $V_2 = b\,t\,l + b\,t\,\delta l + b\,l\,\delta t + t\,l\,\delta b$

Change in volume, δV = Final volume – Original volume

$$= b\,t\,l + b\,t\,\delta l + b\,l\,\delta t + t\,l\,\delta b - b\,t\,l$$

$$= b\,t\,\delta l + b\,l\,\delta t + t\,l\,\delta b$$

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$\frac{\delta V}{V} = \frac{b\,t\,\delta l + b\,l\,\delta t + t\,l\,\delta b}{b\,t\,l} = \frac{\delta l}{l} + \frac{\delta t}{t} + \frac{\delta b}{b}$$

But, $\dfrac{\delta l}{l} = \text{Longitudinal strain} = e$

$$\frac{\delta t}{t} = \text{Lateral strain} = -\frac{1}{m}e \ (\because \textit{Thickness decreases})$$

$$\frac{\delta b}{b} = \text{Lateral strain} = -\frac{1}{m}e \ (\because \textit{Width decreases})$$

$$\text{Volumetric strain} = e - \frac{1}{m}e - \frac{1}{m}e = e - \frac{2e}{m}$$

$$\frac{\delta V}{V} = e\left(1 - \frac{2}{m}\right)$$

Change in volume, $\boxed{\delta V = e\left(1 - \frac{2}{m}\right)V}$

2) Circular bar

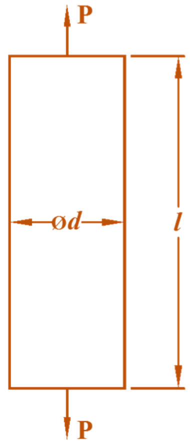


Fig.4.8 Volumetric strain in circular bar

Consider a circular bar of diameter d and length l and is subjected to a tensile force of P as shown in fig.4.8.

Let δd and δl be the change in dimension due to the applied load.

$$\text{Original volume, } V_1 = \frac{\pi}{4}d^2 l$$

$$\text{Final volume, } V_2 = \frac{\pi}{4}\left[(d + \delta d)^2 \times (l + \delta l)\right]$$

$$= \frac{\pi}{4}[(d^2 + 2d\delta d + \delta d^2) \times (l + \delta l)]$$

$$= \frac{\pi}{4}(d^2 l + d^2 \delta l + 2\ d\ l\ \delta d + 2\ d\ \delta l\ \delta d + l\ \delta d^2 + \delta d^2\ \delta l$$

Neglecting the higher powers of δd and δ*l*

$$V_2 = \frac{\pi}{4}(d^2 l + d^2 \delta l + 2\ d\ l\ \delta d)$$

Change in volume, δV = Final volume – Original volume

$$= \frac{\pi}{4}\left(d^2 l + d^2 \delta l + 2\ d\ l\ \delta d\right) - \frac{\pi}{4} d^2 l$$

$$= \frac{\pi}{4}\left(d^2 \delta l + 2\ d\ l\ \delta d\right)$$

Volumetric strain, $e_v = \dfrac{\delta V}{V} = \dfrac{\text{Change in volume}}{\text{Original volume}}$

$$= \frac{\frac{\pi}{4}\left(d^2 \delta l + 2\ d\ l\ \delta d\right)}{\frac{\pi}{4} d^2 l} = \frac{d^2 \delta l}{d^2 l} + \frac{2\ d\ l\ \delta d}{d^2 l}$$

$$= \frac{\delta l}{l} + \frac{2\ \delta d}{d}$$

But, $\dfrac{\delta l}{l}$ = Longitudinal strain $= e$

$$\frac{\delta d}{d} = \text{Lateral strain} = -\frac{1}{m} e \ (\because \textit{Diameter decreases})$$

$$\text{Volumetric strain, } \frac{\delta V}{V} = e + 2\left(-\frac{1}{m} e\right) = e\left(1 - \frac{2}{m}\right)$$

Change in volume, $\boxed{\delta V = e\left(1 - \frac{2}{m}\right) V}$

4.20 Relation between Young's modulus (E) and modulus of rigidity (N)

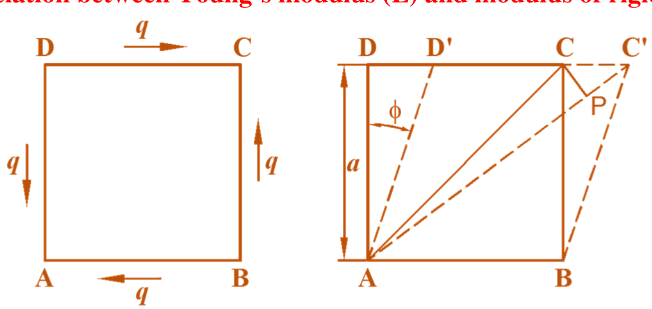


Fig.4.9 Relation between E and C

Consider a square element $ABCD$ of side 'a' and unit thickness. Let the element is distorted to $ABC'D'$ due to shear stress 'q' acting as shown in the fig.4.9. Due to the shear stress, the diagonal AC will be elongated and the diagonal BD will be shortened.

Linear strain of diagonal AC, $= \frac{q}{E} - \frac{1}{m}\left(-\frac{q}{E}\right)$

Linear strain of diagonal AC, $= \frac{q}{E}\left(1 + \frac{1}{m}\right) \qquad ------ \qquad (1)$

Let this shear stress q cause shear strain ϕ resulting in the diagonal AC to distort to AC'.

$$\text{Strain along diagonal AC} = \frac{\text{Change in length}}{\text{Original length}}$$

$$= \frac{AC' - AC}{AC} = \frac{AC' - AP}{AC} \quad (\because AC = AP)$$

$$= \frac{PC'}{AC} \qquad ------ \quad (2)$$

From triangle $CC'P$, $\quad PC' = CC' \sin 45^o = \frac{CC'}{\sqrt{2}}$

$$AC = \sqrt{AD^2 + CD^2} = \sqrt{2\,CD^2} = \sqrt{2}\;CD \quad (\because AD = CD)$$

Substitute the values of PC' and AC in equation (2)

Linear strain of diagonal AC $= \frac{CC'}{\sqrt{2}\,\sqrt{2}\,CD} = \frac{CC'}{2\,CD} = \frac{1}{2}\frac{CC'}{CD}$

From triangle CC'B, $\tan\phi = \frac{CC'}{BC} = \frac{CC'}{CD} \; (\because BC = CD)$

Since the angel is very small, $\tan\phi = \phi$

$$\therefore \phi = \frac{CC'}{CD}$$

$$\frac{q}{C} = \frac{CC'}{CD} \left(\because \text{Shear strain, } \phi = \frac{q}{C}\right)$$

$\therefore$ Linear strain of diagonal AC, $= \frac{1}{2}\frac{q}{C} \qquad ------ \; (3)$

Combining equation (1) and (3)

$$\frac{q}{E}\left(1 + \frac{1}{m}\right) = \frac{1}{2}\frac{q}{C}$$

$$\left(1 + \frac{1}{m}\right) = \frac{E}{2\,C}$$

$$\boxed{\mathbf{E = 2C\left(1 + \frac{1}{m}\right)}}$$

4.21 Relation between bulk modulus (K) and Young's modulus (E)

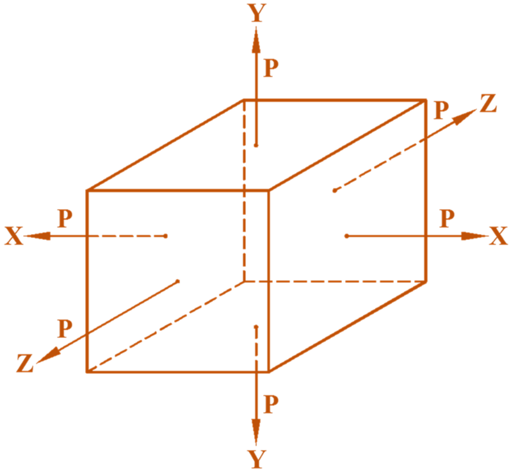


Fig.4.10 Relation between K and E

Consider a cube subjected to three mutually perpendicular tensile stresses of equal intensity as shown in fig.4.10.

Let, f be the stress acting on each face of the cube.

The strain in x direction, $e_x = \frac{f_x}{E} - \frac{1}{m}\left(\frac{f_y}{E} + \frac{f_z}{E}\right)$

$$e_x = \frac{f}{E}\left(1 - \frac{2}{m}\right) (\because f_x = f_y = f_z = f)$$

Similarly, $e_y = \frac{f}{E}\left(1 - \frac{2}{m}\right)$ and $e_z = \frac{f}{E}\left(1 - \frac{2}{m}\right)$

Volumetric strain, $\frac{\delta V}{V} = e_x + e_y + e_z = 3 \times \frac{f}{E}\left(1 - \frac{2}{m}\right)$

$$\text{Bulk modulus, K} = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$\because \text{Volumetric strain} = \frac{\text{Direct stress}}{\text{Bulk modulus}} = \frac{f}{K}$$

$$3 \times \frac{f}{E}\left(1 - \frac{2}{m}\right) = \frac{f}{K} \implies \frac{3}{E}\left[1 - \frac{2}{m}\right] = \frac{1}{K}$$

$$\boxed{E = 3K\left(1 - \frac{2}{m}\right)}$$

4.22 Relation between E, C and K

We know that, $E = 2C\left(1+\frac{1}{m}\right)$ ------------(1)

Also, $E = 3K\left(1-\frac{2}{m}\right)$ ------------(2)

Equating (1) and (2)

$$2C\left(1+\frac{1}{m}\right) = 3K\left(1-\frac{2}{m}\right)$$

$$2C+\frac{2C}{m} = 3K-\frac{6K}{m}$$

$$\frac{6K}{m}+\frac{2C}{m} = 3K-2C$$

$$\frac{1}{m}(6K+2C) = 3K-2C$$

$$\frac{1}{m} = \frac{3K-2C}{6K+2C}$$

Substituting the value of $\frac{1}{m}$ in equation (1)

$$E = 2C\left(1+\frac{3K-2C}{6K+2C}\right)$$

$$= 2C\left(\frac{6K+2C+3K-2C}{6K+2C}\right)$$

$$= \frac{2C}{2}\left(\frac{9K}{3K+C}\right)$$

$$\boxed{E = \frac{9KC}{3K+C}}$$

4.23 Composite bars

A composite bar may be defined as *a bar made of two or more different materials joined together in such a way that the system elongates or contracts as a whole equally when subjected to axial pull or push.*

Consider a composite bar made of two different materials as shown in the fig.4.11

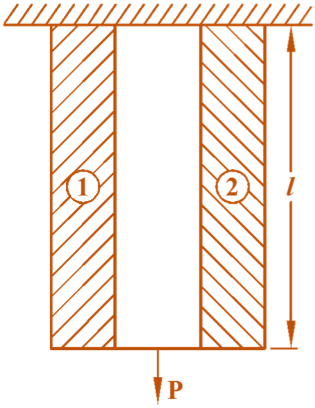


Fig.4.11 Composite bar

Let, P = Total load on the bar

l = Length of the bar

A_1 = Area of bar 1

E_1 = Young's modulus of bar 1

P_1 = Load shared by bar 1 and

A_2, E_2, P_2 are corresponding values for bar 2

According to the definition of composite bar,

THE STRAIN IN BOTH THE MATERIAL IS SAME.

i.e. $\dfrac{f_1}{E_1} = \dfrac{f_2}{E_2}$

$$f_1 = \frac{E_1}{E_2} \times f_2$$

The ratio $\dfrac{E_1}{E_2}$ is known as *modular ratio*

Total load, P = Load shared by bar 1 + Load shared by bar 2

$$P = P_1 + P_2$$

$$= f_1 A_1 + f_2 A_2$$

$$= \frac{E_1}{E_2} f_2 A_1 + f_2 A_2$$

$$= \frac{E_1\ f_2\ A_1 + E_2\ f_2 A_2}{E_2}$$

$$P = \frac{f_2 (E_1 A_1 + E_2 A_2)}{E_2}$$

$$f_2 = P \left(\frac{E_2}{E_1 A_1 + E_2 A_2} \right)$$

$$\boxed{P_2 = f_2 A_2 = P \left(\frac{E_2 \, A_2}{E_1 A_1 + E_2 A_2} \right)}$$

Similarly, $$\boxed{P_1 = f_1 A_1 = P \left(\frac{E_1 \, A_1}{E_1 A_1 + E_2 A_2} \right)}$$

Note: The following points should be remembered while solving the problems in composite bars

1) ***Extension or contraction of the bar being equal and hence the strain is also equal***
2) ***The total external load applied on the composite bar is equal to the sum of the loads shared by the different materials.***

4.24 Temperature stresses and strains.

When the temperature of a body is increased, it undergoes deformation leading to increase in dimensions. On the other hand the body contracts when its temperature is reduced.

When a body is allowed to deform freely under increased or reduced temperature condition, stresses are not induced. If the deformation is prevented completely or partially, stresses will be induced in the body.

The stresses induced in a body due to change in temperature are known as *temperature stress* or *thermal stress*. The corresponding strain in the body is known as *temperature strain* or *thermal strain*.

4.25 Expression for temperature stress and temperature strain

Fig. 4.12 Temperature stress and strain

Consider a body subjected to an increase in temperature.

Let, l = Original length of the body

T = Increase in temperature and

α = Co efficient of linear expansion

Increase in length due to increase of temperature, $\delta l = \alpha\, Tl$

If both the ends of the bar are rigidly fixed so that its expansion is prevented, then compressive stress is induced in the body.

$$\text{Strain, } e = \frac{\text{Change in length}}{\text{Original length}} = \frac{\alpha Tl}{l} = \alpha T$$

$$\text{Stress, } f = \text{Strain} \times \text{Young's modulus} = \alpha TE$$

If the supports yield by an amount equal to λ, then

the actual expansion that has taken place, $\delta l = \alpha Tl - \lambda$

$$\text{Strain, } e = \frac{\text{Change in length}}{\text{Original length}} = \frac{\alpha Tl - \lambda}{l} = \alpha T - \frac{\lambda}{l}$$

$$\text{Stress, } f = \text{Strain} \times \text{Young's modulus} = \boxed{\left(\alpha T - \frac{\lambda}{l}\right) E}$$

4.26 Strain energy or resilience due to axial load

When a body is subjected to an external load, there is deformation of the body which causes movement of the applied load. Thus work is done by the applied load. This work done is stored in the body as energy and that is why when the load is removed, the body regains its original shape and size behaving like a spring. *This energy stored in the body by virtue of strain is called strain energy or resilience.*

Analytical derivation of strain energy

Consider a body of length l and uniform cross section A and is subjected to an external load P. The deformation takes place from zero to final value of the magnitude, if the load is increased gradually.

Consider an elemental strip of thickness $d\delta$ and at a distance δ_1 from the origin. The work done by the external load P for the displacement of $d\delta$ is given by,

δw = Load × Displacement = $P.\, d\delta$ ------------- (1)

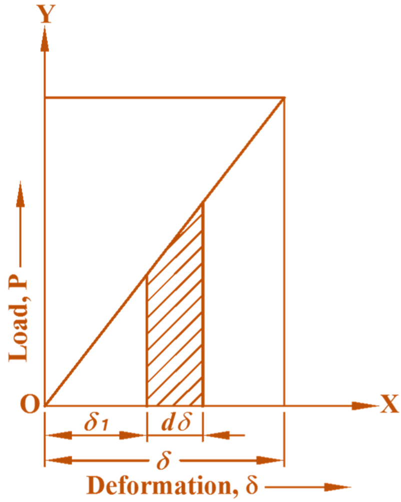


Fig.4.13 Strain energy

We know that, deformation, $\delta = \dfrac{Pl}{A\ E}$

$$P = \frac{A\ E\ \delta}{l}$$

Substitute the value of P in equation (1)

$$\delta w = \frac{A\ E}{l}\delta\ .\ d\delta$$

$$\text{Total work done} = \int_0^{\delta} \frac{AE}{l}\delta .\ d\delta = \frac{AE}{l}\left[\frac{\delta^2}{2}\right]_0^{\delta} = \frac{AE}{l}\left[\frac{\delta^2}{2}\right]$$

Substituting $\delta = \dfrac{fl}{E}$

$$\text{Total work done} = \frac{AE}{l}\left[\frac{\left(\frac{fl}{E}\right)^2}{2}\right]$$

$$= \frac{AE}{l}\left[\frac{f^2 l^2}{2E^2}\right] = \frac{f^2}{2E} \times Al = \frac{f^2}{2E} \times \text{Volume}$$

But total work done on the bar = Strain energy stored in the bar

$\therefore$ The strain energy stored, $\boxed{U = \dfrac{f^2}{2E} \times \textbf{Volume}}$

4.27 Proof resilience

The maximum strain energy which can be stored in a body without permanent deformation is called its proof resilience. If p_{max} be the maximum stress at the elastic limit, then

$$\boxed{\text{Proof resilience} \;=\frac{f_{max}^2}{2E}\times \text{Volume}}$$

4.28 Modulus of resilience

The maximum strain energy which can be stored in a body per unit volume is known as modulus of resilience.

$$\boxed{\text{Modulus of resilience} \;=\frac{f_{max}^2}{2E}}$$

4.29 Instantaneous stresses due to various types of loads

1. Gradually applied load

Consider a bar subjected to a gradually applied load.

Let, P = Gradually applied load,
A = Cross sectional area of the bar,
l = Length of the bar,
δl = Deformation of the bar
E = Young's modulus of the material of the bar and
f = Instantaneous stress induced in the bar

Since the load is applied gradually, the magnitude of he load is increasing from zero to the final value P.

$$\text{Average load} = \frac{\text{Minimum load} + \text{Maximum load}}{2} = \frac{0+P}{2} = \frac{P}{2}$$

$$\text{Work done by the load} \;=\; \text{Average load} \times \text{Deflection}$$

$$= \frac{P}{2}\times \delta l$$

$$\text{The strain energy stored in the bar , } U \;= \frac{f^2}{2\,E}\times A\;l$$

$$\text{But strain energy stored} \;=\; \text{Work done}$$

$$\frac{f^2}{2\,E}\times A\;l = \frac{P}{2}\times \delta l$$

$$\text{We know that, } \delta l = \frac{fl}{E}$$

$$\therefore \frac{f^2}{2E} \times A\,l = \frac{P}{2} \times \frac{fl}{E}$$

$$f \times A = P$$

$$f = \frac{P}{A}$$

Instantaneous stress produced due to gradually applied load, $\boxed{f = \frac{P}{A}}$

2. Suddenly applied load

Consider a bar subjected to a suddenly applied load.

Let, P = Suddenly applied load,

A = Cross sectional area of the bar,

l = Length of the bar,

δl = Deformation of the bar

E = Young's modulus of the material of the bar and

f = Instantaneous stress induced in the bar

Since the load is applied suddenly, it is constant throughout the process of deformation of the bar.

Work done by the load = Average load × Deflection = $P \times \delta l$

The strain energy stored in the bar , $U = \frac{f^2}{2E} \times A\,l$

But strain energy stored = Work done

$$\frac{f^2}{2E} \times A\,l = P \times \delta l$$

We know that, δl $\delta l = \frac{fl}{E}$

$$\therefore \frac{f^2}{2E} \times A\,l = P \times \frac{fl}{E}$$

$$\frac{f}{2} \times A = P$$

$$f = 2 \times \frac{P}{A}$$

Instantaneous stress produced due to suddenly applied load, $\boxed{f = 2 \times \frac{P}{A}}$

3. Impact by gravity

Consider a bar in which a collar is attached at the bottom. Let this bar is subjected to a load applied with impact as shown in the fig.4.14.

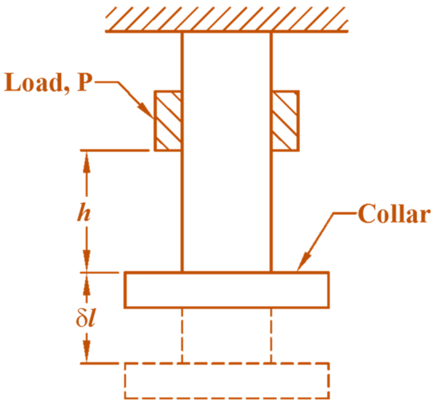


Fig.4.14 Impact by gravity

Let, P = Load applied with impact

A = Cross sectional area of the bar,

l = Length of the bar,

δl = Deformation of the bar due to the load

E = Young's modulus of the material of the bar and

f = Instantaneous stress induced in the bar

h = Height of fall of load before it strikes the collar

Work done by the load = Average load × Distance moved

$= P(h + \delta l)$

The strain energy stored in the bar , $U = \frac{f^2}{2\,E} \times A\,l$

But strain energy stored = Work done

$$\frac{f^2}{2\,E} \times A\,l = P(h + \delta l)$$

We know that, $\delta l = \frac{fl}{E}$

$$\therefore\ \frac{f^2}{2\,E} \times A\,l = P\left(h + \frac{fl}{E}\right)$$

$$\frac{f^2}{2\,E} \times A\,l = Ph + P\left(\frac{fl}{E}\right)$$

Multiply by $\frac{2E}{Al}$ on both sides,

$$\frac{f^2 \times Al}{2\,E} \times \frac{2E}{Al} = \left(Ph \times \frac{2E}{Al}\right) + \left(\frac{Pfl}{E} \times \frac{2E}{Al}\right)$$

$$f^2 = \frac{2EPh}{Al} + 2f\left(\frac{P}{A}\right)$$

$$f^2 - 2f\left(\frac{P}{A}\right) = \frac{2EPh}{Al}$$

Add $\frac{P^2}{A^2}$ on both sides

$$f^2 - 2f\left(\frac{P}{A}\right) + \frac{P^2}{A^2} = \frac{2EPh}{Al} + \frac{P^2}{A^2}$$

$$\left(f - \frac{P}{A}\right)^2 = \frac{P^2}{A^2} + \frac{2EPh}{Al}$$

Taking square root on both sides, we get,

$$f - \frac{P}{A} = \sqrt{\left(\frac{P^2}{A^2} + \frac{2EPh}{Al}\right)}$$

$$\boxed{f = \frac{P}{A} + \sqrt{\left(\frac{P^2}{A^2} + \frac{2EPh}{Al}\right)}}$$

δl is very small as compared to h, then

Work done $= P\ h$

But strain energy stored $=$ Work done

$$\frac{f^2}{2\ E} \times A\ l = Ph$$

$$f^2 = \frac{2EPh}{A\ l}$$

$$\boxed{f = \sqrt{\frac{2EPh}{A\ l}}}$$

4) Impact by shock

Consider a body subjected to a shock load

Let, A = Cross sectional area of the bar,

l = Length of the bar,

δl = Deformation of the bar due to the load

E = Young's modulus of the material of the bar and

f = Instantaneous stress induced in the bar

The strain energy is stored in the bar as kinetic energy.

$\therefore$ Shock energy $= \frac{1}{2} mv^2$

Where, m = Mass of the body, v = Velocity of the body

But strain energy stored = Shock energy

$$\boxed{\frac{f^2}{2E} \times Al = \frac{1}{2} mv^2}$$

By using the above equation, we can find out the instantaneous stress induced in the bar due to shock load.

REVIEW QUESTIONS

1. Define stress and strain. *(Oct.02, Apr.04, Apr.17)*
2. Write short notes on three simple stresses. *(Oct.92)*
3. Explain the lateral strain and linear strain. *(Oct.04, Oct.16)*
4. Sketch and explain the stress – strain diagram for a mild steel specimen with its salient points. *(Apr.01, Oct.04, Oct.01, Apr.01, Oct.17)*
5. Explain the proportional limit and elastic limit. *(Apr.17)*
6. Distinguish between yield stress and ultimate stress. *(Oct.98)*
7. Define breaking stress.
8. State and explain Hooke's law. *(Oct.04, Apr.04)*
9. Define Young's modulus and Poisson's ratio. *(Oct.03, Apr.04, Oct.16, oct.17, Aprr.18)*
10. Define factor of safety. *(Apr.17)*
11. Derive an expression for elongation of a bar due to external load.
12. Derive an expression for elongation of a stepped bar due to an external load. *(Oct.98)*
13. Explain shear stress and shear strain.
14. Define volumetric strain.
15. State and explain elastic constants. *(Oct.98)*
16. Establish the relationship $E = 3K\left[1 - \frac{2}{m}\right]$ *(Oct.96)*
17. Define composite bar. *(Oct.01, Apr.17)*
18. What do you understand by temperature stress. *(Oct.96)*
19. Define strain energy.
20. What is proof resilience and modulus of resilience. *(Oct.16)*

POINTS TO REMEMBER

1) Stress, $f = \dfrac{\text{Load}}{\text{Area}} = \dfrac{P}{A}$ (N/mm^2)

2) Strain, $e = \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{\delta l}{l}$ (No unit)

3) Young's modulus, $E = \dfrac{\text{Stress}}{\text{Strain}}$ (N/mm^2)

4) Change in length, $\delta l = \dfrac{P\,l}{AE} = \dfrac{fl}{E}$ (mm)

5) Factor of safety $= \dfrac{\text{Ultimate stress}}{\text{Working stress}}$ (No unit)

6) Percentage elongation $= \dfrac{l_0 - l}{l} \times 100$ (%)

7) Percentage reduction in area $= \dfrac{A - A_0}{A} \times 100$ (%)

8) Total elongation of bar of varying sections $\Big\}$ $\delta l = \dfrac{P}{E}\left(\dfrac{l_1}{A_1} + \dfrac{l_2}{A_2} + \dfrac{l_3}{A_3}\right)$ (mm)

9) Poisson's ratio, $\dfrac{1}{m} = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}$ (No unit)

10) Change in volume of a rectangular bar, $\delta V = e\left(1 - \dfrac{2}{m}\right) \times V$ (mm^3)

11) Change in volume of a circular bar, $\delta V = e\left(1 - \dfrac{2}{m}\right) \times V$ (mm^3)

12) Relation between E and N ⇨ $E = 2C\left(1 + \dfrac{1}{m}\right)$

13) Relation between E and K ⇨ $E = 3K\left(1 - \dfrac{2}{m}\right)$

14) Relation between E, N and K ⇨ $E = \left(\dfrac{9KC}{3K + C}\right)$

15) In a composite bar, stress induced in material 1 and material 2 are

$$f_1 = P\left(\frac{E_1}{E_1A_1 + E_2A_2}\right); \quad f_2 = P\left(\frac{E_2}{E_1A_1 + E_2A_2}\right) \quad (\text{N/mm}^2)$$

The load shared by material 1 and material 2 are

$$P_1 = f_1A_1\,;\, P_2 = f_2A_2 \quad (\text{N})$$

16) Temperature stress

(a) When the supports do not yield, $f = \alpha T E$ (N/mm^2)

(b) When the supports yield by λ, $f = \left(\alpha T - \frac{\lambda}{l}\right) E$ (N/mm^2)

17) The strain energy stored, $U = \frac{f^2}{2E} \times Al$ (N–mm)

18) Proof resilience $= \frac{f_{max}^2}{2E} \times Al$ (N–mm)

19) Modulus of resilience $= \frac{f_{max}^2}{2E}$ (N/mm^2)

20) Instantaneous stress due to various loading conditions

	Loading condition	Instantaneous stress, (N/mm^2)
1.	Gradually applied load	$f = \frac{P}{A}$
2.	Suddenly applied load	$f = 2 \times \frac{P}{A}$
3.	Impact by gravity	$f = \frac{P}{A} + \sqrt{\left(\frac{P^2}{A^2} + \frac{2EPh}{Al}\right)}$
4.	Impact by shock	'f' can be calculated by using the relation $\frac{f^2}{2E} \times Al = \frac{1}{2} mv^2$

SOLVED PROBLEMS

STRESS, STRAIN, ELONGATION AND YOUNG'S MODULUS

Example : 4.1 *(Oct.92, Oct.95, Apr.13, Apr.15)*

A circular bar of 20mm diameter and 300mm long carries a tensile load of 30KN. Find the stress, strain and elongation of the bar. Take $E = 2 \times 10^5 N/mm^2$.

Given : Diameter of the bar, $d = 20$ mm

Tensile load, $P = 30$ KN $= 30 \times 10^3$ N

Length, $l = 300$ mm

Young's modulus, $E = 2 \times 10^5$ N/mm^2

To find : 1) Stress, f 2) Strain, e 3) Elongaltion, δl

Solution :

$$\text{Area, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 20^2 = 314.159 \text{ mm}^2$$

$$\text{Stress, } f = \frac{\text{Load}}{\text{Area}} = \frac{30 \times 10^3}{314.159} = \boxed{95.493 \text{ N/mm}^2}$$

$$\text{Strain, } e = \frac{\text{Stress}}{\text{Young's Modulus}} = \frac{f}{E} = \frac{95.493}{2 \times 10^5} = \boxed{4.774 \times 10^{-4}}$$

$$\text{Elongation, } \delta l = e \times l = 4.774 \times 10^{-4} \times 300 = \boxed{0.143 \text{ mm}}$$

Result : 1) Stress, f = **95.493 N/mm^2** 2) Strain, e = **4.774×10^{-4}**
3) Elongation, δl = **0.143 mm**

Example : 4.2 *(Apr.14)*

A mild steel rod of 25mm diameter and 200mm long is subjected to an axial pull of 75KN. If $E = 2.1 \times 10^5 N/mm^2$, determine the elongation of the bar.

Given : Diameter of the rod, $d = 25$ mm

Length, $l = 200$ mm

Load , $P = 75$ KN $= 75 \times 10^3$ N

Young's modulus, $E = 2.1 \times 10^5$ N/mm^2

To find : 1) Elongation, δl

Solution :

$$\text{Area, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2 = 490.873 \text{ mm}^2$$

$$\text{Elongation, } \delta l = \frac{P\ l}{A\ E} = \frac{75 \times 10^3 \times 200}{490.873 \times 2.1 \times 10^5} = \boxed{0.1455 \text{ mm}}$$

Result : 1) Elongation, δl = **0.1455 mm**

Example : 4.3 *(Apr.02)*

A rectangular wooden column of length 3m and size 300 × 200mm carries an axial load of 300KN. The column is found to be shortened by 1.5mm under the load. Find the stress and strain.

Given : Length of the column, $l = 3\text{ m} = 3000\text{ mm}$

Width, $b = 300\text{ mm}$

Depth, $d = 200\text{ mm}$

Change in length, $\delta l = 1.5\text{ mm}$

Load , $P = 300\text{ KN} = 300 \times 10^3\text{ N}$

To find : 1) Stress, f 2) Strain, e

Solution :

$$\text{Area, } A = b \times d = 300 \times 200 = 60000\text{ mm}^2$$

$$\text{Stress , } f = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} = \frac{300 \times 10^3}{60000} = \boxed{5\text{ N/mm}^2}$$

$$\text{Strain, } e = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l} = \frac{1.5}{3000} = \boxed{0.0005}$$

Result : 1) Stress, f = **5 N/mm²** 2) Strain, e = **0.0005**

Example : 4.4 *(Oct.93, Oct.14)*

A brass tube of 50mm outside diameter and 45mm inside diameter and 300mm long is compressed between end washers with a load of 24.5KN. Reduction in length is 0.15mm. Determine the value of E.

Given : External diameter, $d_1 = 50\text{ mm}$

Internal diameter, $d_2 = 45\text{ mm}$

Length, $l = 300\text{ mm}$

Load , $P = 24.5\text{ KN} = 24.5 \times 10^3\text{ N}$

Change in length, $\delta l = 0.15\text{ mm}$

To find : 1) Young's modulus, E

Solution :

$$\text{Area, } A = \frac{\pi}{4} \times \left(d_1{}^2 - d_2{}^2\right) = \frac{\pi}{4} \times \left(50^2 - 45^2\right) = 373.064\text{ mm}^2$$

$$\text{We know that, } \delta l = \frac{P\,l}{A\,E}$$

$$\therefore E = \frac{P\,l}{A\,\delta l} = \frac{24.5 \times 10^3 \times 300}{373.064 \times 0.15} = \boxed{1.3135 \times 10^5\text{ N/mm}^2}$$

Result : 1) Young's modulus, $E = \mathbf{1.3135 \times 10^5}$ **N/mm²**

Example : 4.5 *(Apr.88)*

A rod of hydraulic lift is 1.2m long and 32mm in diameter. It is attached to a plunger of 100mm in diameter working under a pressure of 8 N/mm^2. If $E = 2 \times 10^5 N/mm^2$, find the change in length of the rod.

Given :

$$\begin{aligned} \text{Length of the rod, } l &= 1.2\text{ m} = 1200\text{ mm} \\ \text{Diameter of the rod, } d &= 32\text{ mm} \\ \text{Diameter of the plunger, } D &= 100\text{ mm} \\ \text{Pressure on the plunger, } p &= 8\text{N/mm}^2 \\ \text{Young's modulus, } E &= 2 \times 10^5 \text{N/mm}^2 \end{aligned}$$

To find : 1) Change in length, δl

Solution :

$$\text{Area of the plunger} = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 100^2 = 7853.982\text{ mm}^2$$

$$\text{Area of the rod, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 32^2 = 804.248\text{ mm}^2$$

$$\begin{aligned} \text{Load on the rod, } P &= \text{Force on the plunger} \\ &= \text{Pressure } \times \text{Area of the plunger} \\ &= 8 \times 7853.982 = 62831.856\text{ N} \end{aligned}$$

$$\text{Change in length, } \delta l = \frac{P\,l}{AE} = \frac{62831.856 \times 1200}{804.248 \times 0.2 \times 10^6} = \boxed{0.469\text{ mm}}$$

Result : 1) Change in length of the rod, δl = **0.469 mm**

WORKING STRESS, FACTOR OF SAFETY

Example : 4.6 *(Oct.92, Oct.94, Apr.01, Oct.02, Oct.03, Apr.05)*

A cement concrete cube of 150mm size crushes at a load of 337.5KN. Determine the working stress, if the factor of safety is 3.

Given :

$$\begin{aligned} \text{Side of the cube, } s &= 150\text{ mm} \\ \text{Crush load, } P &= 337.5\text{ KN} = 337.7 \times 10^3\text{ N} \\ \text{Factor of safety} &= 3 \end{aligned}$$

To find : 1) Working stress, f_w

Solution :

$$\text{Area, } A = s^2 = 150 \times 150 = 22500\text{ mm}^2$$

$$\text{Ultimate stress, } f_u = \frac{\text{Crush load}}{\text{Area}} = \frac{P}{A} = \frac{337.5 \times 10^3}{22500} = 15\text{ N/mm}^2$$

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

$$\text{Working stress, } f_w = \frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{15}{3} = \boxed{5 \text{ N/mm}^2}$$

Result : The working stress, f_w **= 5 N/mm²**

Example : 4.7 *(Aor.95)*

A hollow cast iron column 250mm diameter with a wall thickness of 25mm is subjected to an axial load. If the ultimate crushing stress for the material is 480 N/mm^2, calculate the safe load for the column using a factor of safety of 3.

Given :

External diameter, $d_1 = 250$ mm

Wall thickness, $t = 25$ mm

Ultimate stress, $f_u = 480 \text{ N/mm}^2$

Factor of safety $= 3$

To find : 1) Load, P

Solution :

$$\text{Internal diameter, } d_2 = d_1 - 2t = 250 - (2 \times 25) = 200 \text{ mm}$$

$$\text{Area, } A = \frac{\pi}{4} \times \left(d_1{}^2 - d_2{}^2\right) = \frac{\pi}{4} \times \left(250^2 - 200^2\right) = 17671.459 \text{ mm}^2$$

$$\text{Working stress, } f_w = \frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{480}{3} = 160 \text{ N/mm}^2$$

$$\text{Also, working stress, } f_w = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$$\text{Load, } P = \text{Working stress} \times \text{Area}$$

$$= 160 \times 17671.4590 = \boxed{2827433.44 \text{ N}}$$

Result : 1) Load, P = **2827433.44 N**

Example : 4.8 *(Apr.96)*

The ultimate stress for a hollow steel column which carries an axial load of 2000KN is 480 N/mm^2. If the external diameter of the column is 200mm, determine the internal diameter. Take factor of safety as 4.

Given :

Ultimate stress, $f_u = 480 \text{ N/mm}^2$

Load, $P = 2000 \text{ KN} = 2000 \times 10^3 \text{ N}$

External diameter, $d_1 = 200$ mm

Factor of safety $= 4$

To find : 1) The internal diameter, d_2

Solution :

$$\text{Working stress, } f_w = \frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{480}{4} = 120 \text{ N/mm}^2$$

$$\text{Also, working stress, } f_w = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$$\text{Area} = \frac{\text{Load}}{\text{Working Stress}} = \frac{2000 \times 10^3}{120} = 16666.666 \text{ mm}^2$$

Let d_2 be the internal diameter of the column, then

$$\text{Area, } A = \frac{\pi}{4} \times \left(d_1{}^2 - d_2{}^2\right)$$

$$16666.666 = \frac{\pi}{4} \times \left(200^2 - d_2{}^2\right)$$

$$21220.662 = 40000 - d_2{}^2$$

$$d_2{}^2 = 18779.338$$

$$d_2 = \sqrt{18779.338} = \boxed{\mathbf{137.038 \text{ mm}}}$$

Result : 1) The internal diameter, $\boldsymbol{d_2}$ = **137.038 mm**

STRESS – STRAIN DIAGRAM

Example : 4.9 *(Apr.92)*

The following observations were obtained on a mild steel specimen having an initial gauge length of 50mm and initial diameter of 16mm: Load at yield point = 60KN; Maximum load = 88KN; load at fracture = 64KN; Distance between gauge points after fracture = 68.8 mm; Diameter of the neck = 9.2mm. Determine the 1) yield stress, 2) ultimate stress, 3) nominal stress at the fracture, 4) percentage elongation and 5) percentage reduction in area.

Given :

Initial diameter, d = 16 mm

Diameter of the neck, d_0 = 9.2 mm

Initial gauge length, l = 50 mm

Distance between gauge points after fracture, l_0 = 68.8 mm

Load at yield point = 60 KN = 60×10^3 N

Maximum load = 88 KN = 88×10^3 N

Load at fracture = 64 KN = 64×10^3 N

To find :
1) Yield stress
2) Ultimate stress
3) Nominal stress at fracture
4) Percentage of elongation
5) Percentage reduction in area

Solution :

Original area of cross section, $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$

Area of neck after fracture, $A_0 = \frac{\pi}{4} \times {d_0}^2 = \frac{\pi}{4} \times 9.2^2 = 66.48 \text{ mm}^2$

$$\text{Yield stress} = \frac{\text{Load at the yield point}}{\text{Original area of cross section}}$$

$$= \frac{60 \times 10^3}{201.06} = \boxed{\mathbf{298.42 \ N/mm^2}}$$

$$\text{Ultimate stress} = \frac{\text{Maximum load}}{\text{Original area of cross section}}$$

$$= \frac{88 \times 10^3}{201.06} = \boxed{\mathbf{437.68 \ N/mm^2}}$$

$$\text{Maximum stress at fracture} = \frac{\text{Load at the fracture}}{\text{Original area of cross section}}$$

$$= \frac{64 \times 10^3}{201.06} = \boxed{\mathbf{318.31 \ N/mm^2}}$$

$$\text{Pecentage elongation} = \frac{(l_0 - l)}{l} \times 100 = \frac{(68.8 - 50)}{50} \times 100 = \boxed{\mathbf{37.6\%}}$$

$$\text{Pecentage reduction in area} = \frac{(A - A_0))}{A} \times 100$$

$$= \frac{(201.06 - 66.48)}{201.06} \times 100 = \boxed{\mathbf{66.94 \ \%}}$$

Result :
1) Yield stress = **298.42 N/mm^2**
2) Ultimate stress = **437.68 N/mm^2**
3) Nominal stress at fracture = **318.31 N/mm^2**
4) Percentage of elongation = **37.6 %**
5) Percentage reduction in area = **66.94 %**

BARS OF VARYING CROSS SECTIONS

Example : 4.10 *(Oct.92, Oct.04)*

A stepped bar of 1m length is composed of two segments of equal length. The first segment is 20×20mm square and the other is 40×40mm square in size. Calculate the elongation of the bar, when the maximum tensile stress in the material is 200 N/mm^2 due to an axial tensile force. Take $E = 2 \times 10^5 N/mm^2$.

Given :

Area of the first segment, $A_1 = 20 \times 20 = 400 \text{ mm}^2$

Area of the second segment, $A_2 = 40 \times 40 \ 1600 \text{ mm}^2$

Maximum stress in the material, $f = 200 \text{ N/mm}^2$

$$\text{Young's modulus, } E = 2\times 10^5 \text{ N/mm}^2$$
$$\text{Length of the first segment, } l_1 = 500 \text{ mm}$$
$$\text{Length of the second segment, } l_2 = 500 \text{ mm}$$

To find : 1) Total change in length, δl

Solution :

Maximum tensile stress occurs only in the segments having small area of cross section. So, the stress in the first segment, f_1=200 **N/mm²**

$$\text{Load on the material, } P = f_1 \times A_1 = 200\times 400 = 80000 \text{ N}$$

$$\text{Total change in length, } \delta l = \frac{P\, l_1}{A_1\, E} + \frac{P\, l_2}{A_2\, E}$$
$$= \frac{80000\times 500}{400\times 2\times 10^5} + \frac{80000\times 500}{1600\times 2\times 10^5} = \boxed{0.625 \text{ mm}}$$

Result : 1) Total change in length, δl = **0.625 mm**

Example : 4.11 ***(Oct.98)***

A steel bar is 500mm long. The two ends are 35mm and 25mm in diameter and each end portion is 150mm long. The middle portion is 200mm long and 20mm in diameter. Calculate the total extension in the bar if it carries an axial pull of 30KN. Take E=200KN/mm².

Given :

$$\text{Load, } P = 30\text{KN} = 30\times 10^3 \text{ N}$$
$$\text{Diameter of the first portion, } d_1 = 35 \text{ mm}$$
$$\text{Length of the first portion, } l_1 = 150 \text{ mm}$$
$$\text{Diameter of the second portion, } d_2 = 20 \text{ mm}$$
$$\text{Length of the second portion, } l_2 = 200 \text{ mm}$$
$$\text{Diameter of the third portion, } d_3 = 25 \text{ mm}$$
$$\text{Length of the third portion, } l_3 = 150 \text{ mm}$$
$$\text{Young's modulus, } E = 200 \text{ KN/mm}^2 = 2\times 10^5 \text{ N/mm}^2$$

To find : 1) Total change in length, δl

Solution :

$$\text{Area of the first portion, } A_1 = \frac{\pi}{4}\times {d_1}^2 = \frac{\pi}{4}\times 35^2 = 962.113 \text{ mm}^2$$

$$\text{Area of the second portion, } A_2 = \frac{\pi}{4}\times {d_2}^2 = \frac{\pi}{4}\times 20^2 = 314.159 \text{ mm}^2$$

$$\text{Area of the third portion, } A_3 = \frac{\pi}{4}\times {d_3}^2 = \frac{\pi}{4}\times 25^2 = 490.874 \text{ mm}^2$$

$$\text{Total change in length, } \delta l = \frac{P\, l_1}{A_1\, E} + \frac{P\, l_2}{A_2\, E} + \frac{P\, l_3}{A_3\, E}$$

$$= \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

$$= \frac{30 \times 10^3}{2 \times 10^5}\left[\frac{150}{962.113} + \frac{200}{314.159} + \frac{150}{490.874}\right] = \boxed{0.1647 \text{ mm}}$$

Result : 1) Total change in length, δl = **0.1647 mm**

Example : 4.12

(Oct.98)

A steel bar is 450mm long. The two ends are 15mm diameter and have equal lengths. It is subjected to a tensile load of 15KN. If the stress in the middle portion is limited to 160 N/mm^2, determine the diameter of that portion. Find also the length of the middle portion if the total elongation of the bar is 0.25mm. Young's modulus of the material is given as $E = 2 \times 10^5 N/mm^2$.

Given :

Total length of the bar, l = 450 mm

Diameter of two end portions, d_1 = d_2 = 15 mm

Total load, P = 15 KN = 15×10^3 N

Stress in the middle portion, f_2 = 160 N/mm^2

Total elongation, δl = 0.25 mm

Young's modulus, E = 2×10^5 N/mm^2

To find : 1) Diameter of the middle portion, d_2

2) Length of middle portion, l_2

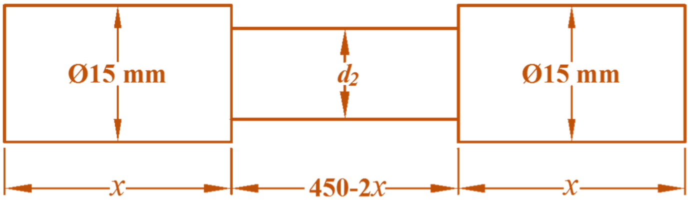


Fig.P4.1 Bar of varying sections [Exapmle 4.12]

Solution :

Let d_2 be the diameter of the middle portion

Then, $f_2 = \frac{P}{A_2}$

$$\therefore A_2 = \frac{P}{f_2} = \frac{15 \times 10^3}{160} = 93.75 \text{ mm}^2$$

Also, $A_2 = \frac{\pi}{4} \times {d_2}^2$

$93.75 = \frac{\pi}{4} \times {d_2}^2$

${d_2}^2 = 119.366$; $d_2 = \boxed{10.925 \text{ mm}}$

Area of the end portion, $A_1 = A_3 = \frac{\pi}{4} \times d_1{}^2 = \frac{\pi}{4} \times 15^2 = 176.715 \text{ mm}^2$

Let, the length of the end portion, $l_1 = l_3 = x$

Length of the middle portion, $l_2 = 450 - 2x$

Total elongation of the bar, $\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$

$$0.25 = \frac{15 \times 10^3}{2 \times 10^5}\left[\frac{x}{176.715} + \frac{450 - 2x}{93.75} + \frac{x}{176.715}\right]$$

$$0.25 = 0.075[0.0056588x + 4.8 - 0.0213333x + 0.0056588x]$$

$$3.3333333 = 4.8 - 0.0100157x$$

$$x = \frac{1.4666667}{0.0100157} = 146.437$$

Length of the middle portion, $l_2 = 450 - 2x$

$$= 450 - (2 \times 146.437) = \boxed{\mathbf{157.126 \text{ mm}}}$$

Result :	1) Diameter of middle portion, $\boldsymbol{d_2}$	=	**10.925 mm**
	2) Length of middle portion, $\boldsymbol{l_2}$	=	**157.126 mm**

SHEAR STRESS

Example : 4.13 *(Apr.93)*

A steel punch can be worked on to the compressive stress of 800 N/mm². Find the least diameter of the hole which can be punched through a steel plate 28mm thick if the ultimate shear stress for the plate is 360 N/mm².

Given : Compressive stress on punch, f = 800 N/mm²

Thickness of steel plate, t = 23 mm

Shear stress, f_s = 300 N/mm²

To find : 1) Least diameter of hole, d

Solution :

Let the least diameter of the hole = d

Diameter of the punch = Diameter of the hole = d

Compressive force from the punch = Compressive stress × Area of the punch

$$= P \times \frac{\pi}{4} \times d^2 = 800 \times \frac{\pi}{4} \times d^2$$

$$= 628.318\, d^2$$

Resisting force from the plate = Shear stress ×Resisting area of the plate

$$= f_s \times \pi d t \quad = 300 \times \pi \times d \times 23$$

$$= 21676.984\ d$$

We know that,

Compressive force from the punch = Resisting force from the plate

$$628.318\ d^2 = 21676.984\ d$$

$$d = \frac{21676.984}{628.318} = \boxed{\mathbf{34.5\ mm}}$$

Result : 1) The least diameter of the hole, d = **34.5 mm**

LATERAL STRAIN, POISSON'S RATIO, VOLUMETRIC STRAIN, ELASTIC CONSTANTS

Example : 4.14 *(Apr.01, Oct.04, Oct.13, Apr.17)*

A steel bar of 25mm diameter and length of 1m is subjected to a pull of 25KN. If $E = 2 \times 10^5\ N/mm^2$, find the elongation, decrease in diameter and increase in volume of the bar. Take 1/m = 0.25.

Given : Diameter of the steel bar, d = 25 mm

Length of the steel bar, l = 1 m = 1000 mm

Young's modulus, E = 2×10^5 N/mm^2

Poison's ratio, $1/m$ = 0.25

To find : 1) Change in length, δl 2) Change in diameter, δd

3) Change in volume, δV

Solution :

Area of the steel bar, $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2 = 490.874\ \text{mm}^2$

Volume of the steel bar, $V = A \times l = 490.874 \times 1000 = 490874\ \text{mm}^3$

Longitudinal strain, $e = \frac{P}{A\ E} = \frac{25 \times 10^3}{490.874 \times 2 \times 10^5} = 2.5465 \times 10^{-4}$

Change in length, δl = Longitudinal strain × Length

$$= 2.5465 \times 10^{-4} \times 1000 = \boxed{\mathbf{0.25465\ mm}}$$

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Lateral strain = Poisson's ratio × Longitudinal strain

$$= 0.25 \times 2.5465 \times 10^{-4} = 6.36625 \times 10^{-5}$$

Change in diameter, δd = Lateral strain × Diameter

$$= 6.36625 \times 10^{-5} \times 25 = \boxed{\mathbf{1.5916 \times 10^{-3}\ mm}}$$

$$\text{Volumetric strain} = e\left[1-\frac{2}{m}\right]$$

$$= 2.5465\times10^{-4}[1-2\times0.25] = 1.27325\times10^{-4}$$

$$\text{Change in volume, } \delta V = \text{Volumetric strain}\times\text{Volume}$$

$$= 1.27325\times10^{-4}\times490874 = \boxed{62.5\ \text{mm}^3}$$

Result : 1) Change in length, $\delta l = \mathbf{0.25465\ mm}$

2) Change in diameter, $\delta d = \mathbf{1.5916\times10^{-3}\ mm}$

3) Change in volume, $\delta V = \mathbf{62.5\ mm^3}$

Example : 4.15 *(Apr.99, Apr.02)*

A steel bar of 500mm length, 60mm width and 20mm thickness is subjected to an axial compression of 168KN. Calculate the final dimension and final volume of the bar. The modulus of elasticity of steel is $2.1\times10^5\ N/mm^2$ and the Poisson's ratio of steel is 0.3.

Given :

Length of the steel bar, l = 500 mm

Width, b = 60 mm

Thickness, t = 20 mm

Axial compressive load, P = 168 KN = 168×10^3 N

Young's modulus, E = $2.1\times10^5\text{N/mm}^2$

Poisson's ratio, $1/m$ = 0.3

To find :

1) Final length 2) Final width 3) Final thickness 4) Final volume

Solution :

$$\text{Volume of the bar, } V = b\times t\times l = 60\times20\times500 = 600000\ \text{mm}^3$$

Area of the bar along the longitudinal direction,

$$A = b\times t = 60\times20 = 1200\ \text{mm}^2$$

$$\text{Longitudinal strain, } e = \frac{P}{A\ E} = \frac{168\times10^3}{1200\times2.1\times10^5} = 6.667\times10^{-4}$$

$$\text{Change in length, } \delta l = \text{Longitudinal strain}\times\text{Length}$$

$$= 6.667\times10^{-4}\times500 = 0.3333\ \text{mm}$$

$$\text{Final length} = \text{Original length} - \text{Change in length} \quad (\because \textit{Compression})$$

$$= 500 - 0.3333 = \boxed{499.6667\ \text{mm}}$$

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Lateral strain} = \text{Poisson's ratio}\times\text{Longitudinal strain}$$

$$= 0.3\times6.667\times10^{-4} = 2\times10^{-4}$$

Change in width, δb = Lateral strain × Width

$$= 2 \times 10^{-4} \times 60 = 0.012 \text{ mm}$$

Final width = Original width + Change in width ($\because$ *Width increases*)

$$= 60 + 0.012 = \boxed{\mathbf{60.012\ mm}}$$

Change in thickness, δt = Lateral strain × Thickness

$$= 2 \times 10^{-4} \times 20 = 0.004 \text{ mm}$$

Final thickness = Original thickness + Change in thickness ($\because$ *Thickness increases*)

$$= 20 + 0.004 = \boxed{\mathbf{20.004\ mm}}$$

$$\text{Volumetric strain} = e\left[1 - \frac{2}{m}\right]$$

$$= 6.667 \times 10^{-4}[1 - 2 \times 0.3] = 2.667 \times 10^{-4}$$

Change in volume, δV = Volumetric strain × Volume

$$= 6.667 \times 10^{-4} \times 600000 = 160 \text{ mm}^3$$

Final volume = Original volume − Change in volume ($\because$ *Volume decreases*)

$$= 600000 - 160 = \boxed{\mathbf{599840\ mm^3}}$$

Result : 1) Final length = **499.6667 mm** 2) Final width = **60.012 mm** 3) Final thickness = **20.004 mm** 4) Final volume = **599840 mm^3**

Example : 4.16

(Oct.01)

A spherical ball of diameter 200mm when subjected to a hydrostatic pressure of 10 N/mm^2 is found to shrink to a ball of 199.7mm. If the Poisson's ratio of the ball is 0.3, find the Young's modulus of the material of the ball.

Given : Diameter of the spherical ball, d = 200 mm

Diameter of the ball after shrinking, d_0 = 199.7 mm

Poisson's ratio, $1/m$ = 0.3

Hydrostatic pressure = 10 N/mm^2

To find : 1) Young's modulus, E

Solution :

Stress, f = Hydrostatic pressure = 10 N/mm^2

Change in diameter, $\delta d = d - d_0 = 200 - 199.7 = 0.3$ mm

$$\text{Lateral strain} = \frac{\text{Change in diameter}}{\text{Original diameter}} = \frac{0.3}{200} = 0.0015$$

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Longitudinal strain} = \frac{\text{Lateral strain}}{\text{Poisson's ratio}} = \frac{0.0015}{0.3} = 0.005$$

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Longitudinal strain}} = \frac{10}{0.005} = \boxed{2000 \text{ N/mm}^2}$$

Result : 1) Young's modulus $E = 2000 \text{ N/mm}^2$

Example : 4.17 *(Oct.92, Oct.16, Apr.17)*

A circular bar of length 150 mm and diameter of 50mm is subjected to an axial pull of 400KN. The extension in length and contration in diameter were found to be 0.25mm and 0.02mm respectively after loading. Calculate (i) Poisson's ratio (ii) Young's modulus (iii) Modulus of rigidity and (iv) Bulk modulus.

Given :

Length of the bar, l = 150 mm

Diameter of the bar, d = 50 mm

Load, P = 400 KN = 400×10^3 N

Change in length, δl = 0.25 mm

Change in diameter, δd = 0.02 mm

To find **:** 1) Poisson's ratio, $1/m$ 2) Young's modulus, E

3) Modulus or rigidity, C 4) Bulk modulus, K

Solution :

$$\text{Area of the steel rod, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 50^2 = 1963.495 \text{ mm}^2$$

$$\text{Change in length , } \delta l = \frac{P\,l}{A\,E}$$

$$E = \frac{P\,l}{A\,\delta l} = \frac{400 \times 10^3 \times 150}{1963.495 \times 0.25} = \boxed{1.2223 \times 10^5 \text{ N/mm}^2}$$

$$\text{Lateral strain} = \frac{\delta d}{d} = \frac{0.02}{50} = 0.0004$$

$$\text{Longitudinal strain, } e = \frac{\delta l}{l} = \frac{0.25}{150} = 1.6667 \times 10^{-3}$$

$$\text{Poisson's ratio, } \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{0.0004}{1.6667 \times 10^{-3}} = \boxed{0.24}$$

$$\text{We know that, } E = 2C\left[1 + \frac{1}{m}\right]$$

$$1.2223 \times 10^5 = 2C[1 + 0.24]$$

$$C = \frac{1.2223 \times 10^5}{2 \times 1.24} = \boxed{4.9286 \times 10^4 \text{ N/mm}^2}$$

$$E = 3K\left[1 - \frac{2}{m}\right] = 3K\left[1 - 2\times\frac{1}{m}\right]$$

$$1.2223\times 10^5 = 3K[1 - 2\times 0.24]$$

$$K = \frac{1.2224\times 10^5}{3\times 0.52} = \boxed{7.8353\times 10^4\ \text{N/mm}^2}$$

Result : 1) Poisson's ratio, $\mathbf{1/m = 0.24}$
2) Young's modulus, $\boldsymbol{E = 1.2223\times 10^5}$ N/mm^2
3) Rigidity modulus, $\boldsymbol{C = 4.9286\times 10^4}$ N/mm^2
4) Bulk modulus, $\boldsymbol{K = 7.8353\times 10^4}$ N/mm^2

Example : 4.18

(Apr.01)

A steel bar of 30mm diameter is subjected to a tensile load of 70KN. Length of the bar is 400mm. Calculate (i)Extension of the bar under the load 70KN (ii)The change in diameter (iii)Bulk modulus if Young's modulus of the material is 200KN/mm² and 1/m = 0.22.

Given :

Diameter of the bar, d = 30 mm
Length of the bar, l = 400 mm
Tensile load, P = 70 KN = 70×10^3 N
Poisson's ratio, $1/m$ = 0.22
Young's modulus, E = 200×10^3N/mm^2

To find : 1) Change in length, δl 2) Change in diameter, δd
3) Bulk modulus, K

Solution :

Area of the steel bar, $A = \frac{\pi}{4}\times d^2 = \frac{\pi}{4}\times 30^2 = 706.858$ mm^2

Longitudinal strain, $e = \dfrac{P}{A\ E} = \dfrac{70\times 10^3}{706.858\times 200\times 10^3} = 4.951\times 10^{-4}$

Change in length , δl = Longitudinal strain × Length

$= 4.951\times 10^{-4}\times 400 = \boxed{0.198\text{ mm}}$

Poisson's ratio, $1/m = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}$

Lateral strain = Poisson's ration × Longitudinal strain

$= 0.22\times 4.951\times 10^{-4} = 1.0892\times 10^{-4}$

Change in diameter , δd = Lateral strain × Diameter

$= 1.0892\times 10^{-4}\times 30 = \boxed{3.2676\times 10^{-3}\text{mm}}$

We know that, $E = 3K\left[1 - \frac{2}{m}\right] = 3K\left[1 - 2\times\frac{1}{m}\right]$

$$200 \times 10^3 = 3K[1 - 2 \times 0.22]$$

$$K = \frac{200 \times 10^3}{3 \times 0.56} = \boxed{1.19048 \times 10^5 \text{ N/mm}^2}$$

Result :	1) Change in length, δl = **0.198 mm**
	2) Change in diameter, $\delta d = \mathbf{3.2676 \times 10^{-3}}$**mm**
	3) Bulk modulus, $K = \mathbf{1.19048 \times 10^5}$ **N/mm²**

Example : 4.19 *(Apr.94, Apr.03)*

For a given material, the Young's modulus is 1×10^5 N/mm² and modulus of rigidity is 0.4×10^5 N/mm². Find the bulk modulus and lateral contraction of a round bar of 50mm diameter and 2.5m long when stretched by 2.5mm.

Given :

Young's modulus, $E = 1 \times 10^5$ N/mm²

Rigidity modulus, $C = 0.4 \times 10^5$ N/mm²

Diameter of the bar, d = 50 mm

Length of the bar, l = 2.5 m = 2500 mm

Change in length, δl = 2.5 mm

To find : 1) Bulk modulus, K 2) Change in diameter, δd

Solution :

We know that, $E = 2C\left[1 + \frac{1}{m}\right]$

$$1 \times 10^5 = 2 \times 0.4 \times 10^5 \left[1 + \frac{1}{m}\right]$$

$$\left[1 + \frac{1}{m}\right] = \frac{1 \times 10^5}{2 \times 0.4 \times 10^5} = 1.25$$

$$\frac{1}{m} = 1.25 - 1 = 0.25$$

Also, $E = 3K\left[1 - \frac{2}{m}\right] = 3K\left[1 - 2 \times \frac{1}{m}\right]$

$$1 \times 10^5 = 3K[1 - 2 \times 0.25]$$

$$K = \frac{1 \times 10^5}{3 \times 0.5} = \boxed{0.667 \times 10^5 \text{ N/mm}^2}$$

Longitudinal strain, $e = \frac{\delta l}{l} = \frac{2.5}{2500} = 0.001$

Poisson's ratio, $1/m = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$$\text{Lateral strain} = \text{Poisson's ratio} \times \text{Longitudinal strain}$$
$$= 0.25 \times 0.001 = 0.25 \times 10^{-3}$$

$$\text{Change in diameter, } \delta d = \text{Lateral stain} \times \text{Diameter}$$
$$= 0.25 \times 10^{-3} \times 50 = \boxed{\mathbf{0.0125\ mm}}$$

Result : 1) Bulk modulus, $\boldsymbol{K = 0.667 \times 10^5\ \mathrm{N/mm^2}}$
2) Change in diameter, $\boldsymbol{\delta d = 0.0125\ \mathrm{mm}}$

Example : 4.20 *(Apr.90, Oct.91, Apr.04)*

In a tensile test on a hollow tube of external diameter 18mm and internal diameter 12mm, an axial load of 1700N produced an elongation of 0.0045mm in length of 75mm while diameter suffered a compression of 0.00032mm. Calculate the Poisson's ratio, Young's modulus and bulk modulus.

Given :

External diameter of the tube, $d_1 = 18$ mm
Internal diameter of the tube, $d_2 = 12$ mm
Axial load, $P = 1700$ N
Change in length, $\delta l = 0.0045$ mm
Length, $l = 75$ mm
Change in diameter, $\delta d = 0.00032$ mm

To find : 1) Poisson's ratio, $1/m$ 2) Young's modulus, E
3) Bulk modulus, K

Solution :

$$\text{Area of tube, } A = \frac{\pi}{4} \times \left(d_1{}^2 - d_2{}^2\right) = \frac{\pi}{4} \times \left(18^2 - 12^2\right) = 141.372\ \mathrm{mm^2}$$

$$\text{Lateral strain} = \frac{\delta d}{d_1} = \frac{0.00032}{18} = 1.778 \times 10^{-5}$$

$$\text{Longitudinal strain, } e = \frac{\delta l}{l} = \frac{0.0045}{75} = 6 \times 10^{-5}$$

$$\text{Poisson ratio, } 1/m = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{1.778 \times 10^{-5}}{6 \times 10^{-5}} = \boxed{\mathbf{0.2963}}$$

$$\text{Stress, } f = \frac{\text{Load}}{\text{Area}} = \frac{1700}{141.372} = 12.025\ \mathrm{N/mm^2}$$

$$\text{Young's modulus, } E = \frac{f}{e} = \frac{12.025}{6 \times 10^{-5}} = \boxed{\mathbf{2.0042 \times 10^5\ N/mm^2}}$$

$$\text{We know that, } E = 3K\left[1 - \frac{2}{m}\right] = 3K\left[1 - 2 \times \frac{1}{m}\right]$$

$$2.0042 \times 10^5 = 3K[1 - 2 \times 0.2963]$$

$$K = \frac{2.0042 \times 10^5}{3 \times 0.4074} = \boxed{\mathbf{1.6398 \times 10^5\ N/mm^2}}$$

Result :	1) Poisson's ratio, **$1/m = 0.2963$** 2) Young's modulus, **$E = 2.0042 \times 10^5$ N/mm^2** 3) Bulk modulus, **$K = 1.6398 \times 10^5$ N/mm^2**

Example : 4.21 *(Oct.94, Oct.17)*

A bar of steel 28mm diameter and 250mm long is subjected to an axial load of 80KN. It is found that the diameter has contracted by 1/240mm. If the modulus of rigidity is $0.8 \times 10^5\, N/mm^2$, calculate (1) Poisson's ratio (2) Young's modulus and (3) Bulk modulus.

Given :

Diameter, d = 28 mm

Length, l = 250 mm

Axial load, P = 80 KN = 80×10^3 N

Change in diameter, δd = 1/240 = 4.1667×10^{-3} mm

Modulus of rigidity, C = 0.8×10^5 N/mm^2

To find : : 1) Poisson's ratio, $1/m$ 2) Young's modulus, E
3) Bulk modulus, K

Solution :

$$\text{Area},\ A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 28^2 = 615.752 \text{ mm}^2$$

$$\text{Lateral strain} = \frac{\delta d}{d} = \frac{4.1667 \times 10^{-3}}{28} = 1.4881 \times 10^{-4}$$

$$\text{Longitudinal strain},\ e = \frac{P}{A\ E} = \frac{80 \times 10^3}{615.752 \times E} = \frac{129.922}{E}$$

$$\text{Poisson's ratio},\ 1/m = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{1.4881 \times 10^{-4}}{(129.922/E)} = 1.14538 \times 10^{-6} E$$

$$\text{We know that},\ E = 2C\left[1 + \frac{1}{m}\right]$$

$$E = 2 \times 0.8 \times 10^5 (1 + 1.14538 \times 10^{-6} E)$$

$$E = 1.6 \times 10^5 + 0.18326 E$$

$$(1 - 0.18326)\ E = 1.6 \times 10^5$$

$$E = \frac{1.6 \times 10^5}{0.81674} = \boxed{\mathbf{1.959 \times 10^5\ N/mm^2}}$$

$$\text{Poisson ratio},\ \frac{1}{m} = 1.14538 \times 10^{-6} \times 1.959 \times 10^5 = \boxed{\mathbf{0.2244}}$$

$$\text{Also, } E = 3K\left[1 - \frac{2}{m}\right]$$

$$1.959 \times 10^5 = 3K[1 - 2 \times 0.2244]$$

$$K = \frac{1.959 \times 10^5}{3 \times 0..5512} = \boxed{\mathbf{1.1847 \times 10^5\ N/mm^2}}$$

Result : 1) Poisson's ratio, $\mathbf{1/m = 0.2244}$

2) Young's modulus, $\mathbf{E = 1.959 \times 10^5\ N/mm^2}$

3) Bulk modulus, $\mathbf{K = 1.1847 \times 10^5\ N/mm^2}$

COMPOSITE BARS

Example : 4.22 *(Oct.92, Oct.15, Apr.17)*

Two vertical wires each 2.5mm diameter and 5m long jointly support a weight of 2.5KN. One wire is steel and the other is of different material. If the wires stretch elastically 6mm, find the load taken by each and the value of Young's modulus for the second wire if that of steel is $\mathbf{0.2 \times 10^6\ N/mm^2}$.

Given :

Diameter of the wire, d	=	2.5 mm
Length of each wire, l	=	5 m = 5000 mm
Elongation of each wire, δl	=	6 mm
Total load, P	=	2.5 KN = 2500 N
Young's modulus of steel, E_1	=	0.2×10^6 N/mm^2

To find : 1) Load taken by each wire P_1 & P_2

2) Young's modulus of the second wire, E_2

Solution :

$$\text{Area of each wire, } A_1 = A_2 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 2.5^2 = 4.909\ \text{mm}^2$$

$$\text{We know that, elongation, } \delta l = \frac{P_1\, l}{A_1\, E_1}$$

$$P_1 = \frac{A_1\, E_1\, \delta l}{l} = \frac{4.909 \times 0.2 \times 10^6 \times 6}{5000} = \boxed{\mathbf{1178.16\ N}}$$

$$\text{Total load} = P_1 + P_2$$

$$2500 = 1178.16 + P_2$$

$$P_2 = 2500 - 1178.16 = \boxed{\mathbf{1321.84\ N}}$$

$$\text{Also elongation, } \delta l = \frac{P_2\, l}{A_2\, E_2}$$

$$6 = \frac{1321.84 \times 5000}{4.909 \times E_2}$$

$$E_2 = \frac{P_2\ \ l}{A_2\ \ \delta l} = \frac{1321.84 \times 5000}{4.909 \times 6} = \boxed{\mathbf{2.244 \times 10^5\ N/mm^2}}$$

Result : 1) Load taken by first wire, P_1 = **1178.16 N**

2) Load taken by second wire, P_2 = **1321.84 N**

3) Young's modulus of second wire, $E_2 = 2.244 \times 10^5$ **N/mm²**

Example : 4.23 *(Oct.93, Oct.02)*

A solid copper rod 36mm diameter is rigidly fixed at both ends inside a tube of 45mm inside diameter and 50mm outside diameter. The composite section is then subjected to an axial pull of 98KN. Determine the stresses induced in the rod and tube and total elongation of the composite section in length of 1m. E for copper is $1.1 \times 10^5 N/mm^2$ and E for steel is $2 \times 10^5 N/mm^2$.

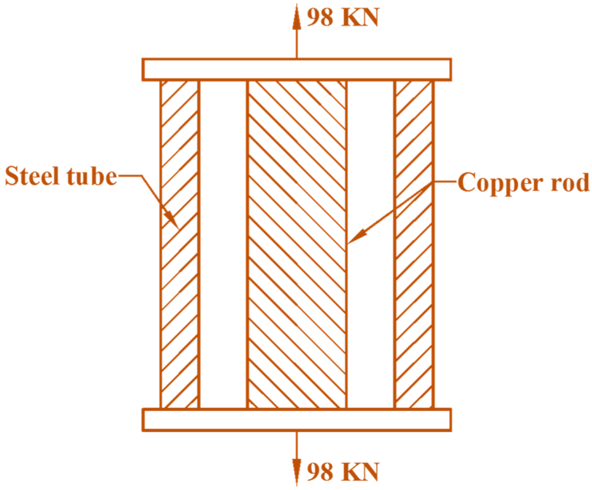


Fig.P4.2 Composite bar [Exapmle 4.23]

Given :

Diameter of solid copper rod, d_c = 36 mm

External diameter of steel tube, d_1 = 50 mm

Internal diameter of steel tube, d_2 = 45 mm

Axial pull, P = 98 KN = 98×10^3 N

Length of composite section, l = 1 m = 1000 mm

Young's modulus of copper, E_c = 1.1×10^5 N/mm²

Young's modulus of steel, E_s = 2×10^5 N/mm²

To find :

1) The stress induced in the copper, f_c

2) The stress induced in the steel, f_s

3) Total elongation, δl

Solution :

Area of copper rod, $A_c = \frac{\pi}{4} \times d_c^{\ 2} = \frac{\pi}{4} \times 36^2 = 1017.876 \text{ mm}^2$

Area of steel tube, $A_s = \frac{\pi}{4} \times \left(d_1^{\ 2} - d_2^{\ 2}\right) = \frac{\pi}{4} \times (50^2 - 45^2) = 373.064 \text{ mm}^2$

In a composite bar, the strain per unit length will be same for both the materials.

$$\text{i.e. } \frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = \frac{E_s \times f_c}{E_c} = \frac{2 \times 10^5 \times f_c}{1.1 \times 10^5} = 1.818\ f_c \qquad ------(1)$$

$$\text{Total load } = P_s + P_c = f_s\ A_s + f_c\ A_c$$

$$98000 = 373.064\ f_s + 1017.876\ f_c \qquad ------(2)$$

Substitute the value of f_s in (2), we get

$$98000 = (373.064 \times 1.818\ f_c) + 1017.876\ f_c$$

$$98000 = 1696.106\ f_c$$

$$f_c = \frac{98000}{1696.106} = \boxed{57.779 \text{ N/mm}^2}$$

Substitute the value of f_c in (1), we get

$$f_s = 1.818 \times 57.779 = \boxed{105.042 \text{ N/mm}^2}$$

$$\text{Total elongation, } \delta l = \frac{f_s\ l}{E_s}\ (or)\ \frac{f_c\ l}{E_c}$$

$$= \frac{57.779 \times 1000}{1.1 \times 10^5}\ (or)\ \frac{105.042 \times 1000}{2 \times 10^5}$$

$$= \boxed{0.5253 \text{ mm}}$$

Result : 1) The stress induced in the copper, f_c = **57.779 N/mm²**
2) The stress induced in the steel, f_s = **105.042 N/mm²**
3) Total elongation, δl = **0.5253 mm**

Example : 4.24

(Oct.13, Apr.15)

A copper rod of 30mm diameter is surrounded tightly by a cast iron tube 60mm external diameter, their ends being firmly fastened together. When they are subjected to a compressive load of 12KN axially, what load is taken by each member? Also determine the contraction of the bar if their length is 100mm originally. The Young's modulus of copper is $0.1 \times 10^6\ N/mm^2$ and that of C.I is $0.12 \times 10^6\ N/mm^2$.

Given : Diameter of the copper rod, d_c = 30 mm
External diameter of C.I tube, d_1 = 60 mm
Internal diameter of C.I tube, d_2 = 30 mm
Total load, P = 12 KN $= 12 \times 10^3$ N

$$\text{Young's modulus of copper, } E_c = 0.1 \times 10^6 \text{ N/mm}^2$$
$$\text{Young's modulus of C.I, } E_{ci} = 0.12 \times 10^6 \text{ N/mm}^2$$

To find : 1) Load taken by the copper rod, P_c

2) Load taken by the C.I tube, P_{ci}

3) Contraction of the bar, δl

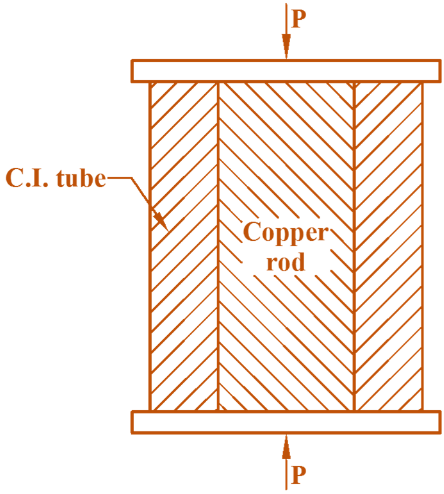


Fig.P4.3 Composite bar [Exapmle 4.24]

Solution :

$$\text{Area of copper rod, } A_c = \frac{\pi}{4} \times d_c^{\,2} = \frac{\pi}{4} \times 30^2 = 706.858 \text{ mm}^2$$

$$\text{Area of CI tube, } A_{ci} = \frac{\pi}{4} \times \left(d_1^{\,2} - d_2^{\,2}\right) = \frac{\pi}{4} \times (60^2 - 30^2) = 2120.575 \text{ mm}^2$$

In this composite bar,

$$\text{Load taken by the copper rod, } P_c = \frac{P \times A_c\, E_c}{A_c\, E_c + A_{ci}\, E_{ci}}$$

$$= \frac{12 \times 10^3 \times 706.858 \times 0.1 \times 10^6}{\left(706.858 \times 0.1 \times 10^6\right) + \left(2120.575 \times 0.12 \times 10^6\right)} = \boxed{\mathbf{2608.695\ N}}$$

$$\text{Total load , } P = P_c + P_{ci}$$

$$12 \times 10^3 = 2608.695 + P_{ci}$$

$$\text{Load taken by the CI tube, } P_{ci} = 12 \times 10^3 - 2608.695 = \boxed{\mathbf{9391.305\ N}}$$

$$\text{Contraction of the bar, } \delta l = \frac{P_c\, l}{A_c\, E_c} = \frac{2608.695 \times 100}{706.858 \times 1 \times 10^5} = \boxed{\mathbf{3.691 \times 10^{-3}\ mm}}$$

Result : 1) Load taken by the copper rod, P_c = **2608.695N**

2) Load taken by the C.I tube, P_{ci} = **9391.305N**

3) Contraction of the bar, δl = **3.691×10⁻³ mm**

Example : 4.25 *(Apr.92)*

A tube of aluminium 40mm external diameter and 20mm internal diameter is snugly fitted on to a steel rod of 20mm diameter. The composite bar is loaded in compression by an axial load P. Find the stress in aluminium when the load is such that the stress in steel rod is $70 N/mm^2$. What is the value of P, if E for steel is $2 \times 10^5\ N/mm^2$ and E for aluminium is $0.7 \times 10^5\ N/mm^2$.

Given :

Diameter of the steel rod, $d_s = 20$ mm

External diameter of aluminium tube, $d_1 = 40$ mm

Internal diameter of aluminium tube, $d_2 = 20$ mm

Stress induced in steel rod, $f_s = 70$ N/mm^2

Young's modulus of steel, $E_s = 2 \times 10^5$ N/mm^2

Young's modulus of aluminium, $E_a = 0.7 \times 10^5$ N/mm^2

To find :
1) The stress induced in aluminium tube, f_a
2) The total axial load, P

Solution :

Area of steel rod, $A_c = \frac{\pi}{4} \times d_s{}^2 = \frac{\pi}{4} \times 20^2 = 314.159$ mm^2

Area of aluminium tube, $A_a = \frac{\pi}{4} \times (d_1{}^2 - d_2{}^2) = \frac{\pi}{4} \times (40^2 - 20^2) = 942.478$ mm^2

In a composite bar, the strain per unit length will be same for both the materials.

i.e. $\frac{f_s}{E_s} = \frac{f_a}{E_a}$

$$f_a = \frac{E_a \times f_s}{E_s} = \frac{0.7 \times 10^5 \times 70}{2 \times 10^5} = \boxed{24.5\ \text{N/mm}^2}$$

Total load, $P = P_s\ A_s + P_a\ A_a$

$= (70 \times 314.159) + (24.5 \times 942.478) = \boxed{45081.841\ \text{N}}$

Result :
1) The stress induced in aluminium tube, $f_a = 24.5$ **N/mm^2**
2) The total axial load, $P = 45081.841$**N**

Example : 4.26 *(Oct.95, Apr.14)*

A steel tube 100mm internal diameter and 12.5mm thick is surrounded by a brass tube of the same thickness in such a way that the axes of the two tubes coincide. The compound tube is loaded by an axial compressive load of 5KN. Determine the load carried by each tube, the stresses and strain developed in each tube. Assume that there is no buckling of the tubes. Take Young's modulus for steel as $2 \times 10^5\ N/mm^2$ and that for brass as $1 \times 10^5\ N/mm^2$. The tubes are of the same length.

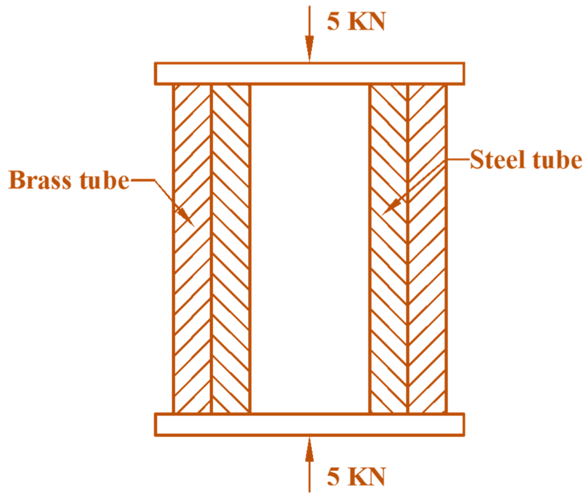


Fig.P4.4 Composite bar [Exapmle 4.26]

Given : Internal diameter of the steel tube, $d_2 = 100$ mm

Thickness, $t = 12.5$ mm

Load, $P = 5$ KN $= 5000$ N

Young's modulus of steel, $E_s = 2 \times 10^5$ N/mm^2

Young's modulus of brass, $E_b = 1 \times 10^5$ N/mm^2

To find :

1) Load carried by the steel tube, P_s
2) Load carried by the brass tube, P_b
3) Stress in steel tube, f_s
4) Stress in brass tube, f_b
5) Strain developed in each tube, e_s or e_b

Solution :

External diameter of steel tube, $d_1 = d_2 + 2t = 100 + (2 \times 12.5) = 125$ mm

Internal diameter of brass tube, $D_2 = d_1 = 125$ mm

External diameter of brass tube, $D_1 = D_2 + 2t = 125 + (2 \times 12.5) = 150$ mm

Area of steel tube, $A_s = \frac{\pi}{4} \times (d_1{}^2 - d_2{}^2) = \frac{\pi}{4} \times (125^2 - 100^2) = 4417.865$ mm^2

Area of brass tube, $A_b = \frac{\pi}{4} \times (D_1{}^2 - D_2{}^2) = \frac{\pi}{4} \times (150^2 - 125^2) = 5399.612$ mm^2

In this composite bar,

Stress induced in steel rod, $f_s = \dfrac{P \times E_s}{E_s\ A_s + E_b\ A_b}$

$$= \frac{5000 \times 2 \times 10^5}{(2 \times 10^5 \times 4417.865) + (1 \times 10^5 \times 5399.612)} = \boxed{0.7024 \text{ N/mm}^2}$$

Stress induced in brass tube, $f_b = \dfrac{P \times E_b}{E_s\ A_s + E_b\ A_b}$

$$= \frac{5000 \times 1 \times 10^5}{(2 \times 10^5 \times 4417.865) + (1 \times 10^5 \times 5399.612)} = \boxed{0.3512 \text{ N/mm}^2}$$

Load carried by steel tube, $P_s = f_s A_s = 0.7024 \times 4417.865 = \boxed{3103.108 \text{ N}}$

Load carried by brass tube, $P_b = P - P_s = 5000 - 3103.108 = \boxed{1896.892 \text{ N}}$

Stress developed in each tube, $e_b \text{ or } e_s = \dfrac{f_s}{E_s}\ (or)\ \dfrac{f_b}{E_b}$

$$= \frac{0.7024}{2 \times 10^5}\ (or)\ \frac{0.3512}{1 \times 10^5} = \boxed{3.512 \times 10^{-6}}$$

Result :	1) Load carried by the steel tube, P_s **= 3103.108 N**
	2) Load carried by the brass tube, P_b **= 1896.892 N**
	3) Stress in steel tube, f_s **= 0.7024 N/mm²**
	4) Stress in brass tube, f_b **= 0.3512 N/mm²**
	5) Strain developed in each tube, $e_s = e_b = 3.512 \times 10^{-6}$

Example : 4.27

(Oct.96)

A RCC column 300mm × 450mm has 4 number of 25mm steel rods. Calculate the safe load for the column, if the allowable stress in concrete is 5 N/mm² and E for steel is 15 times of E of concrete.

Given :

Size of the column = 300 mm × 450 mm

Diameter of one steel rod, d_s = 25 mm

Number of steel rods = 4

Stress in concrete, f_c = 5 N/mm²

Young's modulus of steel, $E_s = 15\ E_c$

To find : 1) The safe load for the column, P

Solution :

Area of the column $= 300 \times 450 = 135000 \text{ mm}^2$

Area of one steel rod $= \frac{\pi}{4} \times d_s^{\ 2} = \frac{\pi}{4} \times 25^2 = 490.874 \text{ mm}^2$

Area of one 4 steel rods $= 4 \times 490.874 = 1963.496 \text{ mm}^2$

Area of concrete, A_c = Area of column – Area of steel rods

$= 135000 - 1963.496 = 133036.51 \text{ mm}^2$

In a composite bar, the strain per unit length will be same for both the materials.

$$\text{i.e. } \frac{f_s}{E_s} = \frac{f_c}{E_c} \quad \Rightarrow \quad \frac{f_s}{15 \times E_c} = \frac{f_c}{E_c}$$

$$f_s = 15 \times f_c = 15 \times 5 = 75 \text{ N/mm}^2$$

Load taken by steel rods, $P_s = f_s\, A_s = 75 \times 1963.496 = 147262.20$ N

Load taken by concrete, $P_c = f_c\, A_c = 5 \times 133036.51 = 665182.55$ N

Total safe load for the column, $P = P_s + P_c$

$$= 147262.20 + 665182.55 = 812444.75 \text{ N} = \boxed{812.445 \text{ KN}}$$

Result : 1) The safe load for the column, $\mathbf{P = 812.445}$ **KN**

Example : 4.28

(Apr.01)

A cast iron of 200mm external diameter and 150mm internal diameter is filled with concrete. Determine the stress in cast iron and concrete when an axial compressive load of 50KN is applied. Take E for cast iron = 18 times of E for concrete.

Given : External diameter of C.I tube, $d_1 = 200$ mm
Internal diameter of C.I tube, $d_2 = 150$ mm
Total load, $P = 50 \text{ KN} = 50 \times 10^3$ N
Young's modulus of C.I, $E_{ci} = 18\, E_c$

To find : 1) Stress in cast iron tube, f_{ci} 2) Stress in concrete, f_c

Solution :

Diameter of the concrete, $d_c = d_2 = 150$ mm

Area of concrete, $A_c = \frac{\pi}{4} \times d_c^{\,2} = \frac{\pi}{4} \times 150^2 = 17671.459 \text{ mm}^2$

Area of CI tube, $A_{ci} = \frac{\pi}{4} \times (d_1^{\,2} - d_2^{\,2})$

$$= \frac{\pi}{4} \times (200^2 - 150^2) = 13744.468 \text{ mm}^2$$

In a composite bar, the strain per unit length will be same for both the materials.

$$\text{i.e. } \frac{f_c}{E_c} = \frac{f_{ci}}{E_{ci}} \quad \Rightarrow \quad \frac{f_c}{E_c} = \frac{f_{ci}}{18\, E_c}$$

$$f_{ci} = 18 \times f_c$$

Total load, $P = P_c + P_{ci}$

$$P = f_c \times A_c + f_{ci} \times A_{ci}$$

$$50 \times 10^3 = (f_c \times 17671.459) + (18\ f_c \times 13744.468)$$

$$50 \times 10^3 = 265071.883\ f_c$$

$$f_c = \frac{50 \times 10^3}{265071.883} = \boxed{\mathbf{0.18863\ N/mm^2}}$$

$$f_{ci} = 18 \times f_c = 18 \times 0.18863 = \boxed{\mathbf{3.39534\ N/mm^2}}$$

Result : 1) The stress in cast iron tube, $f_{ci} = \mathbf{3.39534\ N/mm^2}$
2) The stress in concrete, $f_c = \mathbf{0.18863\ N/mm^2}$

TEMPERATURE STRESSES

Example : 4.29 *(Apr.92)*

Two parallel walls 6 m apart are stayed together by a steel rod 20mm diameter passing through metal plates and nuts at each end. The nuts are tightened when the rod is at a temperature 100°C. Determine the stress in the rod when temperature falls down to 20°C, if (i) the ends do not yield (ii) the ends yield by 1mm. Take $E = 2 \times 10^5\ N/mm^2$ and $\alpha = 12 \times 10^{-6} / °C$. Find also the force exerted in both casees.

Given :

Length of the steel rod, l	=	6m = 6000 mm
Diameter of the steel rod, d	=	20 mm
Initial temperature, T_1	=	100°C
Final temperature, T_2	=	20°C
Amount of yield, λ	=	1 mm
Young's modulus, E	=	2×10^5 N/mm^2
Co–efficient of linear expansion, α	=	12×10^{-6} / °C

To find : 1) The stress when the ends do not yield
2) The force exerted when the ends do not yield
3) The stress when the ends yield by 1 mm
4) The force exerted when the ends yield by 1 mm

Solution :

Area of the rod, $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 20^2 = 314.159\ \text{mm}^2$

Fall in temperature, $T = T_1 - T_2 = 100 - 20 = 80°C$

The free expansion is prevented when the supports do not yield.

So, temperature stress, $f = \alpha\ T\ E$

$$= 12 \times 10^{-6} \times 80 \times 2 \times 10^5 = \boxed{\mathbf{192\ N/mm^2}}$$

Force exerted, $P = f \times A = 192 \times 314.159 = \boxed{60318.528 \text{ N}}$

When the supports yield by 1 mm,

Temperature stress, $f = \left[\alpha T - \frac{\lambda}{l}\right] E$

$$= \left[12 \times 10^{-6} \times 80 - \frac{1}{6000}\right] 2 \times 10^5 = \boxed{158.667 \text{ N/mm}^2}$$

Force exerted, $F = f \times A = 158.667 \times 314.159 = \boxed{49846.666 \text{ N}}$

Result : 1) The stress when the ends do not yield = **192 N/mm^2**
2) The force exerted when the ends do not yield = **60318.528 N**
3) The stress when the ends yield by 1mm = **158.667 N/mm^2**
4) The force exerted when the ends yield by 1mm = **49846.666 N**

Example : 4.30

(Apr.93)

A railway is laid so that there is no stress in the rail at 50°C. Calculate (i) the expansion allowance for no stress in the rail when the temperature is 150°C (ii) the maximum temperature to have no stress in the rail if the expansion allowance is 26mm per rail. Take $\alpha = 12 \times 10^{-6}/°C$ and $E = 2 \times 10^5$ N/mm^2. The length of the rails is 30m.

Given :

Initial temperature, $T_1 = 50°$ C

Final temperature, $T_2 = 150°$C

Young's modulus, $E = 2 \times 10^5$ N/mm^2

Co–efficient of linear expansion, $\alpha = 12 \times 10^{-6}/°C$

Length of the rails, $l = 30$ m $= 30 \times 10^3$ mm

Solution :

Rise in temperature, $T = T_2 - T_1 = 150 - 50 = 100°C$

(i) To find the expansion allowance for no stress in the rail

Let λ be the expansion allowance

When there is no stress in the rails, temperature stress $= 0$

$$\left[\alpha T - \frac{\lambda}{l}\right] E = 0$$

$$\left[12 \times 10^{-6} \times 100 - \frac{\lambda}{30 \times 10^3}\right] \times 2 \times 10^5 = 0$$

$$36 - \lambda = 0$$

$$\lambda = \boxed{36 \text{ mm}}$$

(ii) To find the maximum temperature to have no stress in the rails,

if λ= 26mm

When there is no stress in the rails, temperature stress = 0

$$\left[\alpha T - \frac{\lambda}{l}\right] E = 0$$

$$\left[12 \times 10^{-6} \times T - \frac{26}{30 \times 10^3}\right] \times 2 \times 10^5 = 0$$

$$0.36\, T - 26 = 0$$

$$T = \frac{26}{0.36} = 72.222°C$$

Maximum temperature $=$ Rise in temperature $+$ Initial temperature

$= 72.222 + 50 =$ $\boxed{\mathbf{122.222°C}}$

Result : 1) The expansion allowance required for no stress in the rails when the temperature is 150°C = **36 mm**

2) The maximum temperature to have no stress in the rails, if λ is 26mm = **122.222°C**

STRAIN ENERGY, RESILIENCE & TYPES OF LOADING

Example : 4.31 *(Apr.88, Apr.97, Apr.04, Apr.15, Apr.17)*

Calculate the strain energy that can be stored in a steel bar 70mm in diameter and 6m long, subjected to a pull of 200KN. Assume E=200 KN/mm^2.

Given : Diameter of the steel bar, d = 70 mm

Length of the steel bar, l = 6 m = 6000 mm

Load, P = 200 KN = 200×10^3 N

Young's modulus, E = 200 KN/mm^2 = 2×10^5 N/mm^2

To find : 1) The strain energy, U

Solution :

Area of rod, $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 70^2 = 3848.45 \text{ mm}^2$

Volume of rod, $V = A \times l = 3848.45 \times 6000 = 2.30907 \times 10^7 \text{ mm}^3$

Instantaneous stress, $f = \frac{P}{A} = \frac{200 \times 10^3}{3848.45} = 51.969 \text{ N/mm}^2$

$$\text{Strain energy, } U = \frac{f^2}{2\,E} \times Volume$$

$$= \frac{51.969^2}{2 \times 2 \times 10^5} \times 2.30907 \times 10^7$$

$$= \boxed{\mathbf{155907 \text{ N-mm}}}$$

Result : 1) The strain energy, U = **155907 N–mm**

Example : 4.32

Calculate the modulus of resilience at a point in a material subjected to a stress of 200 N/mm^2. Take $E = 0.1 \times 10^6$ N/mm^2.

Given : Maximum stress, $f_{max} = 200$ N/mm^2

Young's modulus, $E = 0.1 \times 10^6$ N/mm^2

To find : 1) Modulus of resilience

Solution :

$$\text{Modulus of resilience} = \frac{f_{max}^2}{2\,E} = \frac{200^2}{2 \times 0.1 \times 10^6} = \boxed{0.2\ \text{N/mm}^2}$$

Result : 1) Modulus of resilience = **0.2 N/mm^2**

Example : 4.33 *(Oct.89, Apr.94, Oct.97, Oct.02,Oct.03)*

A steel specimen 150mm^2 cross section stretches by 0.05mm over a 50mm gauge length under an axial load of 30KN. Calculate the strain energy stored in the specimen at this stage, if the load at the elastic limit for the specimen is 50KN. Calculate the elongation at elastic limit and the proof resilience.

Given : Area of cross section, $A = 150$ mm^2

Change in length, $\delta l = 0.05$ mm

Gauge length, $l = 50$ mm

Axial load, $P = 30$ KN $= 30 \times 10^3$ N

Load at elastic limit, $P_e = 50$ KN $= 50 \times 10^3$ N

To find : 1) Strain energy, U 2) Elongation, δl 3) Proof resilience

Solution :

Volume, $V = A \times l = 150 \times 50 = 7500$ mm^3

Assume the rod is subjected to gradually applied load.

$$\text{Instantaneous stress, } f = \frac{\text{Axila load}}{\text{Area}} = \frac{30 \times 10^3}{150} = 200 \text{ N/mm}^2$$

$$\text{Longitudinal strain, } e = \frac{\text{Change in length}}{\text{Original length}} = \frac{0.05}{50} = 1 \times 10^{-3}$$

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Longitudinal strain}} = \frac{200}{1 \times 10^{-3}} = 2 \times 10^5 \text{ N/mm}^2$$

$$\text{Strain energy stored, } U = \frac{f^2}{2\,E} \times \text{Volume} = \frac{200^2}{2 \times 2 \times 10^5} \times 7500 = \boxed{750 \text{ N-mm}}$$

Maximum instantaneous stress,

$$f_{max} = \frac{\text{Load at elastic limit}}{\text{Area}} = \frac{50 \times 10^3}{150} = 333.333 \text{ N/mm}^2$$

$$\text{Proof resilience} = \frac{f^2_{max}}{2 \times E} \times \text{Volume} = \frac{333.333^2}{2 \times 2 \times 10^5} \times 7500 = \boxed{2083.329 \text{ N-mm}}$$

$$\text{Elongation, } \delta l = \frac{f_{max} \times l}{E} = \frac{333.333 \times 50}{2 \times 10^5} = \boxed{0.0833 \text{ mm}}$$

Result :
1) Strain energy stored, U = **750 N–mm**
2) Elongation at elastic limit, δl = **0.0833 mm**
3) Proof resilience = **2083.329 N–mm**

Example : 4.34

(Oct.04)

A mild steel bar of 10mm diameter and 2m long is subjected to an axial tensile load of 25KN applied suddenly. Find the stress induced and the strain energy stored in the bar. Take $E = 2 \times 10^5 \, N/mm^2$.

Given :

Diameter of the bar, d = 10 mm

Length of the bar, l = 2 m = 2000 mm

Load, P = 25 KN = 25×10^3 N

Young's modulus, E = 2×10^5 N/mm^2

To find : 1) Stress induced, f 2) Strain energy stored, U

Solution :

$$\text{Area of the rod, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 10^2 = 78.540 \text{ mm}^2$$

$$\text{Volume, } V = A \times l = 78.540 \times 2000 = 157080 \text{ mm}^3$$

For suddenly applied load,

$$\text{Instantaneous stress, } f = 2 \times \frac{P}{A} = 2 \times \frac{25 \times 10^3}{78.540} = \boxed{636.618 \text{ N/mm}^2}$$

$$\text{Strain energy stored, } U = \frac{f^2}{2 \times E} \times Volume$$

$$= \frac{636.618^2}{2 \times 2 \times 10^5} \times 157080 = \boxed{159154.429 \text{ N-mm}}$$

Result :
1) Stress induced in the rod, f = **636.618 N/mm^2**
2) Strain energy stored, U = **159154.429 N–mm**

Example : 4.35 *(Oct.04 Apr.91, Oct.95, Oct.04, Apr.05)*

Determine the greatest weight that can be dropped from a height of 200mm on to a collar at the lower end of a vertical bar 20mm diameter and 2.5m long without exceeding the elastic limit stress 300 N/mm^2. Calculate also the instantaneous elongation. Take $E = 2 \times 10^5 N/mm^2$.

Given :

Height, h = 200 mm

Diameter of the bar, d = 20 mm

Length of the bar, l = 2.5 m = 2500 mm

Instantaneous stress, f = 300 N/mm^2

Young's modulus, E = 2×10^5 N/mm^2

To find : 1) The greatest weight that can be dropped, P

2) Elongation, δl

Solution :

$$\text{Area of the bar, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 20^2 = 314.159 \text{ mm}^2$$

$$\text{Volume, } V = A \times l = 314.159 \times 2500 = 785397.5 \text{ mm}^3$$

$$\text{Instantaneous elongation, } \delta l = \frac{f\,l}{E} = \frac{300 \times 2500}{2 \times 10^5} = \boxed{3.75 \text{ mm}}$$

$$\text{Work done by the load, } W = P(h + \delta l) = P(200 + 3.75) = 203.75\ P$$

$$\text{Strain energy stored in the bar, } U = \frac{f^2}{2 \times E} \times Volume$$

$$= \frac{300^2}{2 \times 2 \times 10^5} \times 785397.5 = 176714.438 \text{ N-mm}$$

Work done = Strain energy stored

$$203.75\ P = 176714.438$$

$$P = \frac{176714.438}{203.75} = \boxed{867.31 \text{ N}}$$

Result : 1) The greatest weight that can be dropped, P = **867.31 N**

2) Elongation, δl = **3.75 mm**

Example : 4.36 *(Oct.91)*

A load of 100N falls by gravity through a vertical distance of 3m, when it is suddenly stopped by a collar at the end of a vertical rod of length 6m and diameter 20mm. The top of the bar is rigidly fixed to a ceiling. Calculate the maximum stress and strain induced in the bar. Take $E = 1.96 \times 10^5 N/mm^2$.

Given : Falling weight, $P = 100$ N
Height of fall, $h = 3$ m $= 3000$ mm
Length of the rod, $l = 6$m $= 6000$ mm
Diameter of the rod, $d = 20$ mm
Young's modulus, $E = 1.96 \times 10^5$ N/mm^2

To find : 1) The maximum stress, f 2) Strain, e

Solution :

Area of the rod, $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 20^2 = 314.159$ mm^2

$$\text{Instantaneous stress, } p = \frac{P}{A} + \sqrt{\frac{P^2}{A^2} + \frac{2\,E\,P\,h}{A\,l}}$$

$$= \frac{100}{314.159} + \sqrt{\frac{100^2}{314.159^2} + \frac{2 \times 1.96 \times 10^5 \times 100 \times 3000}{314.159 \times 6000}}$$

$$= 0.318 + 249.778 = \boxed{\mathbf{250.096\ N/mm^2}}$$

$$\text{Instantaneous strain, } e = \frac{f}{E} = \frac{250.096}{1.96 \times 10^5} = \boxed{\mathbf{1.276 \times 10^{-3}}}$$

Result : 1) The instantaneous stress, f = **250.096 N/mm^2**
2) The Instantaneous strain, e = **1.276×10^{-3}**

Example : 4.37

(Apr.93, Apr.13, Oct.16)

A weight of 1400N is dropped on to a collar at the lower end of a vertical bar 3m long and 25mm in diameter. Calculate the height of drop, if the maximum instantaneous stress is not to exceed 120 N/mm^2. What is the corresponding instantaneous elongation. Take $E = 2 \times 10^5$ N/mm^2.

Given : Falling weight, $P = 1400$ N
Length of the bar, $l = 3$ m $= 3000$ mm
Diameter of the bar, $d = 25$ mm
Instantaneous stress, $f = 120$ N/mm^2
Young's modulus, $E = 2 \times 10^5$ N/mm^2

To find : 1) The height of drop, h 2) Eelongation, δl

Solution :

Area of the bar, $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2 = 490.874$ mm^2

Volume, $V = A \times l = 490.874 \times 3000 = 1472622$ mm^3

$$\text{Elongation, } \delta l = \frac{f\,l}{E} = \frac{120 \times 3000}{2 \times 10^5} = \boxed{\mathbf{1.8\ mm}}$$

Strain energy stored in the bar, $U = \dfrac{f^2}{2 \times E} \times Volume$

$$= \frac{120^2}{2 \times 2 \times 10^5} \times 1472622 = 53014.392 \text{ N-mm}$$

Work done by the falling weight $= P(h + \delta l) = 1400(h + 1.8)$

$$\text{Work done} = \text{Strain energy stored}$$

$$1400(h + 1.8) = 53014.392$$

$$h + 1.8 = \frac{53014.392}{1400} = 37.8674$$

$$h = 37.8674 - 1.8 = \boxed{36.0674 \text{ mm}}$$

Result : 1) The height of drop, $\boldsymbol{h = 36.0674}$ **mm**
2) The instantaneous elongation, $\boldsymbol{\delta l = 1.8}$ **mm**

Example : 4.38 *(Oct.92, Apr.01)*

It is found that a bar of 36mm in diameter stretches 2mm under a gradually applied load of 150KN. If a weight of 15KN is dropped on to a collar at the lower end of this bar through a height of 60mm. Calculate the maximum instantaneous stress and elongation produced. Assume $E = 215\ KN/mm^2$.

Given :

Diameter of the bar, d = 36 mm
Gradually applied load, P_1 = 150 KN = 150×10^3 N
Elongation under gradually applied load = 2 mm
Falling weight, P = 15 KN = 150000 N
Height of fall of weight, h = 60 mm
Young's modulus, E = $215 KN/mm^2 = 2.15 \times 10^5$ N/mm^2

To find : 1) The maximum instantaneous stress, f
2) The maximum elongation, δl

Solution :

Area of the bar, $A = \dfrac{\pi}{4} \times d^2 = \dfrac{\pi}{4} \times 36 = 1017.876$ mm^2

Elongation under gradually applied load $= \dfrac{P_1 l}{A\ E}$

$$2 = \frac{150 \times 10^3 \times l}{1017.876 \times 2.15 \times 10^5}$$

$$l = \frac{2 \times 1017.876 \times 2.15 \times 10^5}{150 \times 10^3} = 2917.911 \text{ mm}$$

Maximum instantaneous stress due to falling weight,

$$f = \frac{P}{A} + \sqrt{\frac{P^2}{A^2} + \frac{2\ E\ P\ h}{A\ l}}$$

$$= \frac{15000}{1017.876} + \sqrt{\frac{15000^2}{1017.876^2} + \frac{2 \times 2.15 \times 10^5 \times 15000 \times 60}{1017.876 \times 2917.911}}$$

$$= 14.7366 + 361.2714 = \boxed{376.008 \text{ N/mm}^2}$$

Maximum elongation, $\delta l = \frac{f\ l}{E} = \frac{376.008 \times 2917.911}{2.15 \times 10^5} = \boxed{5.103 \text{ mm}}$

Result : 1) The maximum instantaneous stress, f = **376.008 N/mm^2**
2) The maximum elongation, δl = **5.103 mm**

Example : 4.39 *(Apr.01)*

A coach weighing 20KN (is attached to a rope) is traveling down a slope at a speed of 2m/s. It is stopped suddenly by pulling the rope. What is the instantaneous stress and the maximum tension induced in the rope due to sudden stoppage. Assume the length and cross sectional area of the rope to be 100m and 1000 mm^2 respectively. Take $E = 2 \times 10^5$ N/mm^2.

Given : Weight of the coach, W = 20 KN = 20×10^3 N
Speed of the coach, v = 2 m/s = 2000 mm/s
Length of the rope, l = 100 m = 100×10^3 mm
Area of the rope, A = 1000 mm^2
Young's modulus, E = 2×10^5 N/mm^2

To find : 1) The maximum instantaneous stress in the rope, f
2) The maximum tension induced in the rope, T

Solution :

When the coach is suddenly stopped, the kinetic energy of the coach is converted into strain energy of the rope.

i.e. $$\frac{m\ v^2}{2} = \frac{f^2}{2\ E} \times \text{Volume}$$

$$\frac{W v^2}{2g} = \frac{f^2}{2\ E} \times A \times l \quad \left(\because m = \frac{W}{g}\right)$$

$$\frac{20 \times 10^3 \times 2000^2}{2 \times 9.81 \times 10^3} = \frac{f^2 \times 1000 \times 100 \times 10^3}{2 \times 2 \times 10^5} \quad (\because g = 9.81 \times 10^3 \text{mm/s}^2)$$

$$f^2 = \frac{2 \times 2 \times 10^5 \times 20 \times 10^3 \times 2000^2}{2 \times 9.81 \times 10^3 \times 1000 \times 100 \times 10^3} = 16309.89$$

$$f = \boxed{127.71 \text{ N/mm}^2}$$

Maximum tension, T = Maximum stress × Area

$$= 127.71 \times 1000 = 127710 \text{ N} = \boxed{127.71 \text{ KN}}$$

Result : 1) The maximum instantaneous stress, f = **127.71 N/mm^2**
2) The maximum tension induced in the rope, T = **127.71 KN**

PROBLEMS FOR PRACTICE

STRESS, STRAIN, ELONGATION AND YOUNG'S MODULUS

1. A steel rod of 10mm diameter and 3m long is subjected to an axial pull of 10KN. Find the stress, strain and elongation of the bar if E = 2×10^5 N/mm^2.
   [Ans: f=127.324 N/mm^2, $e = 6.366 \times 10^{-4}$, δl=1.91mm]

2. A mild steel bar 1m length and 200mm diameter is subjected to a tensile load of 20KN. If the Young's modulus of the material is 2×10^5 N/mm^2, determine the elongation produced. ***[Ans: δl=0.3183mm]***

3. A steel column of 200mm external diameter and 180mm internal diameter is 4m long. Determine the shortening of the column when it is subjected to an axial load of 200KN. Young's modulus is 2.1×10^5 N/mm^2. *(Oct.94)* ***[Ans: δl=0.6382mm]***

4. A wooden tie bar 6m long, 175mm wide and 100mm thick is subjected to an axial pull of 45KN and the stretch is 3.85mm. Find the value of E for timber. *(Apr.98)* ***[Ans: E=9348 N/mm^2]***

WORKING STRESS, FACTOR OF SAFETY

5. A wooden specimen of cross sectional dimensions 100mm × 100mm fails under an axial compression of 160KN. Determine the safe working stress in the wood in compression assuming a factor of safety 4. *(Apr.01)*
   [Ans: f_w=4N/mm^2]

6. A load of 50KN is suspended by a steel pipe of 50mm external diameter. If the ultimate tensile stress of the steel is 415 N/mm^2, and the factor of safety is taken as 6, determine the thickness of the pipe.
   [Ans: t=5.128mm]

STRESS–STRAIN DIAGRAM

7. A specimen of steel 25mm in diameter with a gauge length of 200mm was tested to destruction. The following observations were recorded:– Maximum load=140KN, load at yield point=110KN, load at fracture=120KN, the diameter of the neck = 12.5mm, distance between gauge marks after fracture = 252mm. Find 1)yield stress, 2)ultimate stress, 3)nominal stress at fracture, 4)percentage elongation and 5)percentage reduction in area.

[Ans: 224 N/mm^2, 285.21 N/mm^2, 244.46 N/mm^2, 26%, 75%]

BARS OF VARYING SECTIONS

8. A steel bar is 4m long. The two ends are 32mm and 25mm in diameter and each end portion is 1m long. The middle portion is 2m long and 28mm in diameter. Calculate the total extension in the bar if it carries an axial pull of 30KN. Take E=200KN/mm^2. *(Oct.13)*

[Ans: δl=1.6322mm]

9. A steel bar is 500mm long. The two ends are 20mm diameter and have equal lengths. It is subjected to a tensile load of 16KN. If the stress in the middle portion is limited to 150N/mm^2, determine the diameter of that portion. Find also the length of the middle portion if the total elongation of the bar is 0.2mm. Take 2×10^5 N/mm^2.

[Ans: d=11.6538mm, l=145.289mm]

SHEAR STRESS

10. A steel punch is worked on to a compressive stress of 800N/mm^2 to produce a hole through steel plate of thick 13mm. If the ultimate shear stress for the plate is 300N/mm^2, find the least diameter of the hole.

[Ans: d =19.5mm]

LATERAL STRAIN, POISSON'S RATIO, VOLUMETRIC STRAIN, ELASTIC CONSTANTS

11. A steel bar of 25mm diameter and a length of 1m is subjected to a pull of 250N. If 2×10^5 N/mm^2, find the elongation of the bar and change in diameter. If 1/m =0.25, determine the increase in volume of the bar. *(Oct.90, Apr.93, Apr.14)*

[Ans : δl=0.2546mm, $\delta d = 1.59 \times 10^{-3}$mm, δV=62.488mm^3]

12. A steel bar 400mm long, 60mm wide and 15mm thickness is subjected to an axial tension of 100KN. Calculate the final dimensions and change in volume of the bar. Take 2×10^5 N/mm^2. *(Oct.92, Oct.14, Oct.15)*

[Ans: δl=0.222mm, $\delta w = 9.667 \times 10^{-3} mm$, $\delta t = 2.499 \times 10^{-3} mm$, $\delta V = 79.992\ mm^3$]

13. A spherical ball of diameter 500mm when subjected to a hydro static pressure of 5MPa is found to shrink to a ball of 498.8mm. If the Poisson's ratio of the ball is known to be 0.24, find its Young's modulus. [Hint: 1MPa = 1N/mm^2] *(Oct.90)* **[Ans: E=500 N/mm^2]**

14. The Young's modulus and rigidity modulus of given material are 120GPa and 50GPa respectively. Find the value of 1/m and K. [Hint: 1GPa=1×10^9 N/m^2= 1×10^3 N/mm^2] *(Oct.90, Oct.12)*

[Ans: 1/m =0.2, $K = 0.667 \times 10^5 N/mm^2$]

15. A bar of steel 30mm diameter and 250mm long is subjected to an axial load of 75KN. It is found that the diameter is contracted by 1/240 mm. If the modulus of rigidity is 0.08×10^6 N/mm^2, calculate (i) Poisson's ratio, (ii) Young's modulus and (iii) Bulk modulus of steel. *(Oct.92, Apr.95, Apr.13)*

[Ans: 1/m=0.2649, $E = 2.024 \times 10^5 N/mm^2$, $K = 1.435 \times 10^5 N/mm^2$]

16. A bar of length 100mm and square in section of side 50mm is subjected to an axial pull of 150KN. The extension in length was 0.05mm and the decrease in side was 0.00625mm. Find the elastic constants and Poisson's ratio. *(Apr.01)*

[Ans: 1/m =0.25, $E = 1.2 \times 10^5 N/mm^2$, $C = 0.48 \times 10^5 N/mm^2$, $K = 0.8 \times 10^5 N/mm^2$]

COMPOSITE BARS

17. Two vertical wires each 3mm diameter and 3m long jointly support a load of 2KN. One is steel and the other is aluminium. If the wires stretch elastically 3mm, find the load taken by each wire and Young's modulus of aluminium. Take E for steel as 2×10^5 N/mm^2. *(Apr.05)*

[Ans: P_s= 1413.72N, P_a= 586.28N, E_a= 82941.65 N/mm^2]

18. A copper rod 20mm diameter is encased in a steel tube of 30mm internal diameter and 40mm external diameter. The ends are rigidly attached. The composite bar is 500mm long and is subjected to an axial pull of 30KN. Find the stress in each material and total elongation. Take $E_s=2\times10^5$ N/mm^2, $E_c=1\times10^5$ N/mm^2. *(Apr.98)*

[Ans: f_c=22.22 N/mm^2, f_s=42.44 N/mm^2, δl=0.1061mm]

19. A short steel rod of 25mm diameter is surrounded by light alloy tube. The clearance between the rod and tube are negligible. The compound member is subjected to an axial compressive load of 49KN. Calculate (a) the wall thickness of the tube if the load is equally shared between the two materials (b) the stresses in each material. E for rod is 0.21×10^6 N/mm^2 and E for the tube is 0.07×10^6 N/mm^2.

[Ans: t=12.5mm, f_s=49.911 N/mm^2, f_a=16.637 N/mm^2]

20. A 200mm long steel tube 100mm internal diameter and 10mm thick is surrounded by a brass tube of same length and thickness. The tubes carry an axial thrust of 100KN. Estimate the load carried by each tube and the amount each tube shortened. $E_s= 0.21\times10^6$ N/mm^2, $E_b= 0.1\times10^6$. *(Oct.89)* **[Ans: P_s=63.989KN, P_b=36.011KN, δl=0.0176mm]**

21. A thin copper tube of internal and external diameters 240mm and 250mm respectively is filled with mortar. The elastic modulus of copper and mortar are 125GPa and 5GPa respectively. If an axial load of 25KN is applied on the assembly, find the stress induced in mortar. *(Oct.90)*

[Ans: f_m=0.1767 N/mm^2]

TEMPERATURE STRESS

22. A copper bar 500mm long and of rectangular section 40x30mm is held in between two unyielding supports. What will be the stress in the bar if the temperature is raised by 40°C. Also find the stress if the end supports yield by 0.2mm. Assume $E= 0.11\times10^6$ N/mm^2 and $\alpha=1.5\times10^{-5}$/°C.

[Ans: 66 N/mm^2, 22 N/mm^2]

23. A steel rod 20mm diameter and 5m long is connected to two grips one at each end at a temperature of 120°C. Find the pull exerted when the temperature falls to 50°C, (i) if the end do not yield (ii) if the ends yield by 1.1mm. Take $E= 0.2\times10^6$ N/mm^2, $\alpha=12\times10^{-6}$ /°C.

[Ans: 52784.256 N/mm^2, 40212.352 N/mm^2]

24. A steel rod of 25mm diameter and 10m long has no stress at 40°C. Calculate (a) the expansion allowance for no stress in the rod at 140°C (b) the maximum temperature to have no stress in the rod if the expansion allowance is 2mm. Take $\alpha = 12 \times 10^{-6}$ /°C.

[Ans: λ=12mm, T=56.667°C]

STRAIN ENERGY, RESILIENCE AND TYPES OF LOADING

25. A steel rod of diameter 50mm and length 2.5m is subjected to gradually applied load of 20KN. Calculate the strain energy stored in the rod. Take E=2×10^5 N/mm^2. *(Oct.13, Oct.14)* ***[Ans: U=1273.24 N–mm]***

26. An axial pull of 52KN is suddenly applied on to a steel rod 2.25m long and 110mm^2 in cross section. Calculate the strain energy stored in the rod. Take E= 2×10^5 N/mm^2. *(Apr.01)* ***[Ans: U=55309.091 N–mm]***

27. An axial pull of 60KN is suddenly applied to a steel rod of 50mm diameter and 4mm long. Find (i) the work done (ii) maximum instantaneous elongation. Take E=2×10^5 N/mm^2. *(Apr.04)*

[Ans: U=56415.22 N–mm, δl =0.940m]

28. A weight of 1200N is dropped on to a collar attached at the lower end of a vertical bar 4m long and 30mm in diameter. Calculate the height of drop if the maximum instantaneous stress is not to exceed 120 N/mm^2. Also calculate the instantaneous elongation.

[Ans: δl=2.4mm, h=82.423mm]

29. A trolley weighing 40KN is attached to a rope traveling down a slope at a speed of 2m/s. If it is suddenly stopped by pulling the rope, what is the maximum tension induced in the rope due to sudden braking. Area of the rope is 2000mm^2 and its length is 150m. Take E=0.8×10^5 N/mm^2. *(Apr.97)* ***[Ans: T=131.9KN]***

Unit – III

Chapter 5. GEOMETRICAL PROPERTIES OF SECTIONS

5.1 Centre of gravity

The centre of gravity of a body may be defined as *a point through which the entire weight of the body is assumed to be concentrated.* It may be noted that every body has only one centre of gravity. It is a term related with a body having volume and mass i.e. solids.

5.2 Centroid

The centroid of a section may be defined as *a point through which the entire area of the section is assumed to be concentrated.* It is the term related with plane figures like rectangle, circle, triangle, etc. having only area but no weight. The method of finding out the centroid of a plane figure is similar to that of centre of gravity of a solid body.

5.3 Centroid of a plane figure

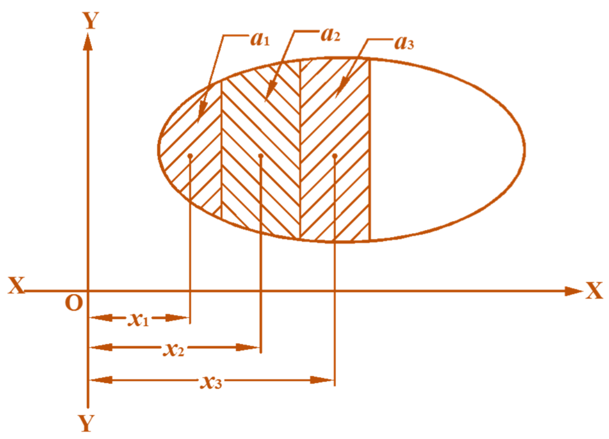


Fig. 5.1 Centroid of a plane figure

Consider a plane figure of area A whose centroid is required to be found out. Divide the plane area into number of small vertical strips as shown in fig.5.1.

Let a_1, a_2, a_3, etc. be the area of the strips and (x_1, y_1), $(x_2, y_2), (x_3, y_3)$, etc. be their co–ordinates of their centroids from a fixed point O. Let, $\bar{X}$ and $\bar{Y}$ be the co–ordinates of the centroid of the plane figure.

Taking moment about Y–Y axis,

The moment of area of first strip $= a_1 x_1$

Sum of the moment of areas of all such strips about Y–Y axis.

$$\Sigma ax = a_1 x_1 + a_2 x_2 + \cdots$$

The moment of area of the whole plane figure about Y–Y axis $= A\bar{X}$

By the principle of moment, $A\bar{X} = \Sigma ax$

$$\bar{X} = \frac{\Sigma ax}{A}$$

$$\boxed{\bar{X} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}}$$

Similarly, $$\boxed{\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}}$$

Centroidal axis

A line passing through the centroid of the plane figure is known as *centroidal axis.*

Axis of reference

A line about which the co–ordinates of centroid are calculated is known as *axis of reference* or *reference axis*.

For plane figures, the axis of reference is taken as lowermost or uppermost line of the figure for calculating $\bar{Y}$ and left extreme line or right extreme line of the figure for calculating $\bar{X}$.

Axis of symmetry

The axis which divides a section into two equal halves horizontally or vertically is known as *axis of symmetry*. The centroid of the section will lie on this axis of symmetry.

5.4 Moment of inertia

The moment of inertia of a body about an axis is defined as the internal resistance offered by the body against the rotation about that axis.

The moment of inertia of a plane figure or lamina about an axis is the product of its area and square of its distance form that axis.

Mathematically, moment of inertia, $\boldsymbol{I = \Sigma a.r^2}$

5.5 Moment of inertia a plane figure

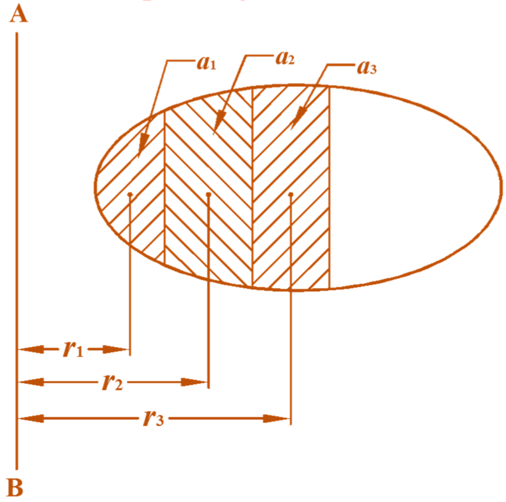


Fig.5.2 Moment of inertia of a plane figure

Consider a plane figure of area A whose moment of inertia is required to be found out. Divide the plane area into number of small elemental strips as shown in fig.5.2.

Let a_1, a_2, a_3, etc. be the areas of the elemental strips and r_1, r_2, r_3,, etc. be the distance of their centroids from a fixed line AB.

First moment of area of the first strip about AB $= a_1 r_1$

The second moment of area of the first strip about AB

$$= a_1 . r_1 . r_1 = a_1 . r_1^2$$

∴ The second moment of area of the plane figure about AB

$$= a_1 r_1^2 + a_2 r_2^2 + \cdots = \Sigma a . r^2$$

This second moment of area is known as moment of inertia.

5.6 Parallel axis theorem

It states, *if the moment of inertia of a plane area about an axis passing through its centroid is denoted by I_G then the moment of inertia of the area about any other axis AB which is parallel to the first and at a distance h from the centroidal is given by,*

$$\boxed{I_{AB} = I_G + Ah^2}$$

Where, I_{AB} = Moment of inertia of the area about an axis AB.

I_G = Moment of inertia of the area about its centroid

A = Area of the section

h = Distance between centroid of the section and axis AB.

Proof

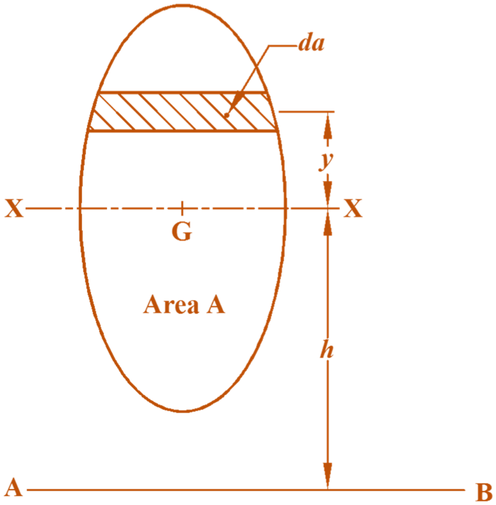


Fig.5.3 Parallel axis theorem

Consider an elemental strip in a plane whose moment of inertia is required to be found out about an axis AB as shown in the fig.5.3

Let, δa = Area of the strip

y = Distance of C.G of strip from C.G of the section

h = Distance of axis AB from the C.G of section.

We know that, the moment of inertia of the elemental strip about an axis passing through the C.G of the section,

$$I = \delta a.\, y^2$$

Moment of inertia of the whole section about an axis passing through the C.G of the section,

$$I_G = \Sigma \delta a\ y^2$$

The moment of inertia of the section about the axis AB,

$$I_{AB} = \Sigma \delta a (h + y)^2 = \Sigma \delta a \left(h^2 + y^2 + 2hy\right)$$

$$= h^2 \Sigma \delta a + y^2 \Sigma \delta a + 2hy \Sigma \delta a$$

$$= Ah^2 + I_G + 0$$

$\Sigma \delta a.\, y = Ay = 0$ ($\because$ *First moment of area about centroidal axis = 0*)

$$\boxed{\therefore I_{AB} = I_G + Ah^2}$$

5.7 Perpendicular axis theorem

It states, *if I_{xx} and I_{yy} be the moments of inertia of plane section about two perpendicular axes meeting at O, the moment of inertia I_{zz} about the axis Z–Z, perpendicular to the plane and passing through the intersection of X–X and Y–Y axes is given by,*

$$\boxed{\therefore I_{zz} = I_{xx} + I_{yy}}$$

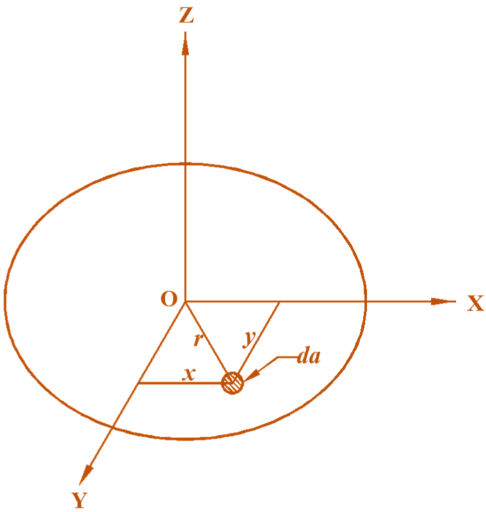


Fig.5.4 Perpendicular axis theorem

Proof

Consider three mutually perpendicular axes OX, OY and OZ. Consider a small lamina of area *da* having co–ordinates as *x* and *y* along OX and OY. Let r be the distance of the lamina form Z–Z axis.

From the geometry of the figure, $r^2 = x^2 + y^2$

The moment of inertia of the lamina about X–X axis is given by,

$$I_{xx} = da.\,y^2$$

Similarly, $I_{yy} = da.\,x^2$

$$I_{zz} = da.\,r^2 = da\left(x^2 + y^2\right)$$

$$= da.\,x^2 + da.\,y^2 = I_{xx} + I_{yy}$$

$$\boxed{\therefore I_{zz} = I_{xx} + I_{yy}}$$

5.8 Derivation of moment of inertia of some sections

1) Rectangular section

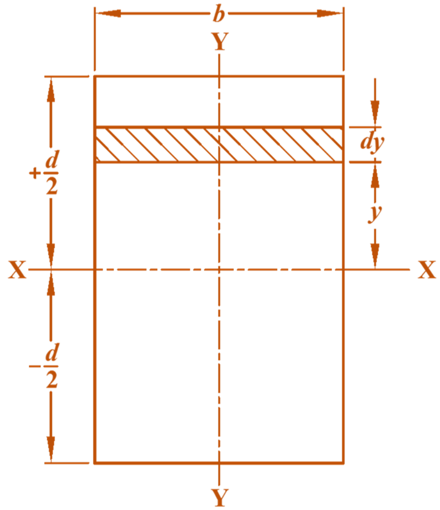


Fig.5.5 M.I of rectangular section

Consider a rectangular section of width b and depth d as shown in the fig.5.5. Now consider an elemental strip of thickness dy parallel to X–X axis and at a distance y from X–X axis.

Area of the strip $= b.\,dy$

M I of the strip about X–X axis = Area × (Distance)^2

$$= b.\,dy.\,y^2 = by^2dy$$

M. I of the whole section about X–X axis,

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} by^2dy = b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{\frac{d^3}{8}}{3} + \frac{\frac{d^3}{8}}{3}\right]$$

$$= b\left[\frac{d^3}{24} + \frac{d^3}{24}\right] = b\left[\frac{2d^3}{24}\right]$$

$\boxed{I_{xx} = \frac{bd^3}{12}}$; Similarly, $\boxed{I_{yy} = \frac{db^3}{12}}$

2) Circular section

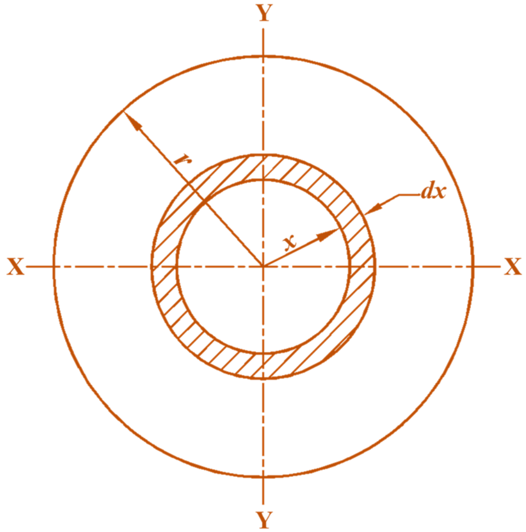


Fig.5.6 M.I of circular section

Consider a circle of radius r with centre O and X–X and Y–Y be the two axes of reference passing through O.

Now consider an elementary ring of radius x and thickness dx.

$\therefore$ The area of the ring, $da = 2\,\pi\,x.dx$

Moment of inertia of the ring about Z–Z axis

$$= \text{Area} \times (\text{Distance})^2 = 2\,\pi\,x.d\,x\,.\,x^2 = 2\pi x^3 dx$$

The moment of inertia of whole section about Z–Z axis

$$I_{zz} = \int_0^r 2\pi x^3 dx = \left[\frac{2\pi x^4}{4}\right]_0^r = \frac{2\pi r^4}{4} = \frac{\pi r^4}{2}$$

Substituting , $r = \dfrac{d}{2}$, $\quad I_{zz} = \dfrac{\pi (d/2)^4}{2} = \dfrac{\pi d^4}{32}$

From the geometry of the section, $I_{xx} = I_{yy}$.

According to perpendicular axis theorem,

$$I_{zz} = I_{xx} + I_{yy} = 2\,I_{xx} \text{ or } 2\,I_{yy}$$

$$\boxed{\boldsymbol{I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi (d^4/32)}{2} = \frac{\pi d^4}{64}}}$$

3) Triangular section

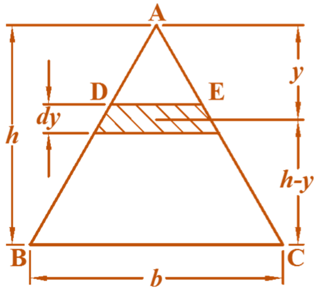


Fig.5.7 M.I of triangular section

Consider a triangular section ABC of base b and height h.

Consider an elemental strip DE of thickness dy at a distance of y from the vertex A as shown in the fig.5.7.

From the figure, the triangle ADE and ABC are similar.

$$\therefore \frac{DE}{BC} = \frac{y}{h}$$

$$DE = BC.\frac{y}{h} = \frac{by}{h}$$

Area of the strip, $da = \dfrac{by}{h} dy$

Moment of inertia of the strip about the base BC

$$= \text{Area} \times (\text{Distance})^2$$

$$= \frac{by}{h} dy (h-y)^2 = \frac{by}{h}(h-y)^2 dy$$

Moment of inertia of the whole section about the base BC,

$$I_{BC} = \int_0^h \frac{by}{h}(h-y)^2 dy$$

$$I_{BC} = \frac{b}{h}\int_0^h y(h^2 + y^2 - 2hy) dy$$

$$I_{BC} = \frac{b}{h}\int_0^h (yh^2 + y^3 - 2hy^2) dy$$

$$= \frac{b}{h}\left[\frac{y^2h^2}{2} + \frac{y^4}{4} - \frac{2hy^3}{3}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{h^4}{2} + \frac{h^4}{4} - \frac{2h^4}{3}\right]$$

$$= \frac{b}{h}\left[\frac{6h^4 + 3h^4 - 8h^4}{12}\right]$$

$$= \frac{bh^4}{12\,h} = \frac{bh^3}{12}$$

$$\boxed{\therefore I_{BC} = \frac{bh^3}{12}}$$

The moment of inertia of a triangular section about the axis passing through it centre of gravity.

In a triangular section, the distance of C.G from the base is given by,

$$h_1 = \frac{h}{3}$$

According to the parallel axis theorem,

$$I_{BC} = I_G + ah_1^2$$

$$I_G = I_{BC} - ah_1^2$$

$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{3bh^3 - 2bh^3}{36} = \frac{bh^3}{36}$$

$$\boxed{\therefore I_G = \frac{bh^3}{36}}$$

5.9 Polar moment of inertia

The moment of inertia of a plane area with respect to the centroidal axis perpendicular to the plane area is called *polar moment of inertia*.

Mathematically, I_P or $J = I_{xx} + I_{yy}$

For a circular section, $\boxed{J = \frac{\pi}{32} d^4}$

5.10 Radius of gyration

Radius of gyration may be defined as *the distance at which the whole area of the plane figure is assumed to be concentrated with respect to a reference axis.*

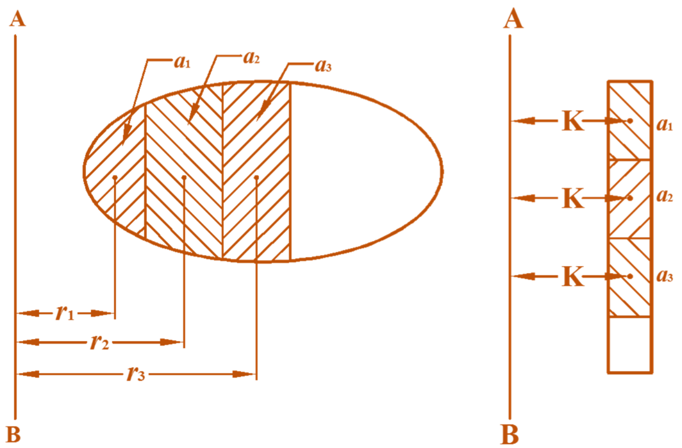


Fig.5.8 Radius of gyration

Consider a plane figure of area A. Divide the whole area into number of vertical strips as shown in the fig.5.8. Let a_1, a_2, a_3, etc. be the area of the strips and $r_1, r_2, r_3, \ldots$, etc. be the distance of these areas from a given axis AB.

The moment of inertia of the area about the reference axis AB, $I_{AB} = \Sigma ar^2$

Let us assume that the vertical strips be arranged at the same distance K from the axis AB so that the moment of inertia about the axis AB remains unchanged. Now the moment of inertia of the plane figure about the axis AB,

$$I_{AB} = a_1K^2 + a_2K^2 + a_3K^2 + \cdots = K^2\Sigma a = AK^2$$

$$\therefore \boldsymbol{I_{AB} = AK^2} \text{ (or) } \boxed{\boldsymbol{K = \sqrt{\frac{I_{AB}}{A}}}}$$

Where, K is *radius of gyration* of the plane figure about the axis AB.

5.11 Section modulus

The section modulus or modulus of section is the ratio between the moment of inertia of the figure about its centroidal axis and the distance of extreme surface from the centroidal axis. It is usually denoted by $\boldsymbol{Z}$.

$$\therefore \boldsymbol{Z} = \frac{\text{Moment of inertia about centroidal axis}}{\text{Distance of extreme surface from centroidal axis}}$$

$$\text{Section modulus of rectangle, } Z = \frac{I_G}{d/2} = \frac{bd^3}{12} \times \frac{2}{d} = \boldsymbol{\frac{bd^2}{6}}$$

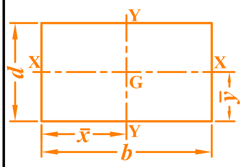
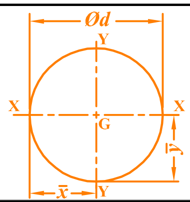
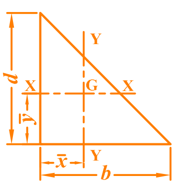
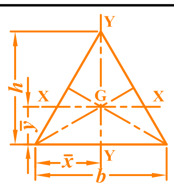
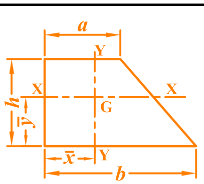
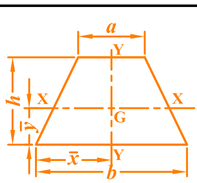
$$\text{Section modulus of circle, } Z = \frac{I_G}{d/2} = \frac{\pi d^4}{64} \times \frac{2}{d} = \boldsymbol{\frac{\pi d^3}{32}}$$

REVIEW QUESTIONS

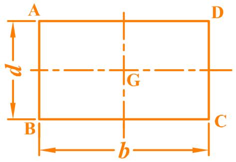
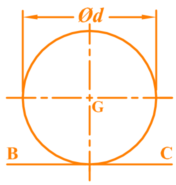
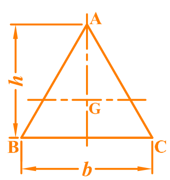
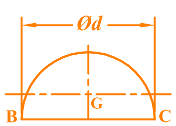
1. Distinguish between centre of gravity and centroid. *(Apr.01, Oct.16, Apr.17, Apr.18*
2. Draw any three symmetrical sections and show their centroidal positions therein. *(Apr.01)*
3. Define moment of inertia and section modulus. *(Oct.01, Oct.16)*
4. Distinguish between moment of inertia and polar moment of inertia. *(Oct.03)*
5. What is meant by polar moment of inertia. *(Apr.02, Apr.18)*
6. State parallel axis theorem and perpendicular axis theorem. *(Apr.04, Oct.04, Apr.02, Apr.01)*
7. State and prove parallel axis theorem . *(Oct.23, Oct.04, Apr.05)*
8. State and prove perpendicular axis theorem. *(Oct.98, Oct.02, Oct.01, Apr.17)*
9. Derive from first principle the moment of inertia of a rectangle about its base. *(Oct.96, Oct.01)*
10. Derive the moment of inertia of a circular section about its centroidal axis.
11. Derive the moment of inertia of triangle about its centroidal horizontal axis and about its base.
12. Explain the term radius of gyration. *(Apr.01, Oct.02, Apr.17)*

POINTS TO REMEMBER

1) Position of centroid of plane geometrical figures

Shape	Figure	Area	$\bar{X}$	$\bar{Y}$
Rectangle		bd	$\frac{b}{2}$	$\frac{d}{2}$
Circle		$\frac{\pi d^2}{4}$	$\frac{d}{2}$	$\frac{d}{2}$
Triangle		$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{h}{3}$
Triangle		$\frac{bh}{2}$	Intersection of medians	$\frac{h}{3}$
Trapezium		$\frac{(a+b)h}{2}$	$\frac{(a^2+b^2+ab)}{3(a+b)}$	$\frac{(2a+b)h}{3(a+b)}$
Trapezium		$\frac{(a+b)h}{2}$	$\frac{b}{2}$	$\frac{(2a+b)h}{3(a+b)}$

2) Moment of inertia of plane geometrical figures

Shape	Figure	M.I about centroidal axis (I_G)	M.I about base (I_{BC})
Rectangle		$I_G = \frac{bd^3}{12}$	$I_{BC} = \frac{bd^3}{3}$
Circle		$I_G = \frac{\pi d^4}{64}$	$J = \frac{\pi d^4}{32}$
Triangle		$I_G = \frac{bh^3}{36}$	$I_{BC} = \frac{bh^3}{12}$
Semi circle		$I_G = \frac{\pi d^4}{24} - \frac{d^4}{18\pi}$	$I_{BC} = \frac{\pi d^4}{128}$

3) $\bar{X} = \dfrac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$ (mm)

4) $\bar{Y} = \dfrac{a_1y_1 + a_2y_2 + a_3y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$ (mm)

5) Parallel axis theorem, $I_{AB} = I_G + ah^2$ (mm^4)

6) Perpendicular axis theorem, $I_{zz} = I_{xx} + I_{yy}$ (mm^4)

7) Radius of gyration, $K = \sqrt{\frac{I}{A}}$ (mm)

SOLVED PROBLEMS

DETERMINATION OF CENTROID

Example : 5.1

Determine the centroid of an angle section 100mm × 80mm × 20mm thick with its longer arm being placed vertical.

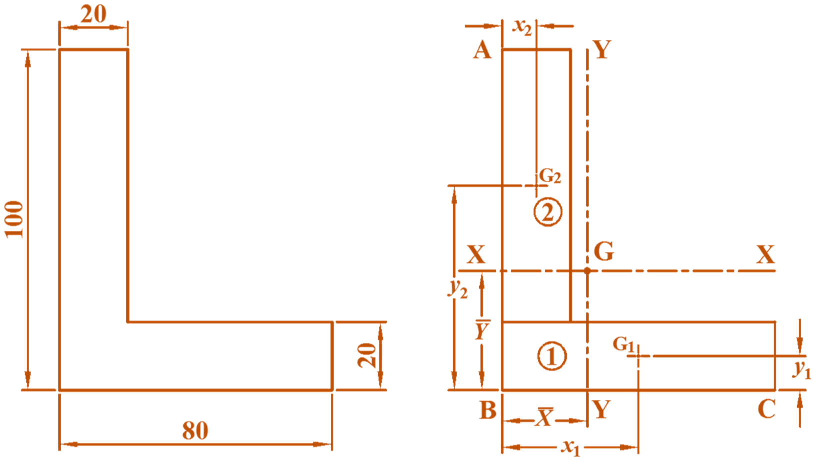


Fig.P5.1 Centroid of 'L' section [Example 5.1]

Solution :

Split the section into two rectangles as shown.

Let, AB and BC be the reference axes

Let $\bar{X}$ and $\bar{Y}$ be the distance of C.G from AB and BC respectively.

$$a_1 = 80 \times 20 = 1600 \text{ mm}^2; \quad x_1 = \frac{80}{2} = 40 \text{ mm}; \quad y_1 = \frac{20}{2} = 10 \text{ mm}$$

$$a_2 = 20 \times 80 = 1600 \text{ mm}^2; \quad x_2 = \frac{20}{2} = 10 \text{ mm}; \quad y_2 = 20 + \frac{80}{2} = 60 \text{ mm}$$

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(1600 \times 40) + (1600 \times 10)}{1600 + 1600} = \frac{80000}{3200} = \boxed{25 \text{ mm}}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(1600 \times 10) + (1600 \times 60)}{1600 + 1600} = \frac{112000}{3200} = \boxed{35 \text{ mm}}$$

Result : The coordinate of centroid from reference axes

$\bar{X}$ = **25 mm** and $\bar{Y}$ = **35 mm**

Example : 5.2

Find the centroid of the section shown in the fig.P5.2

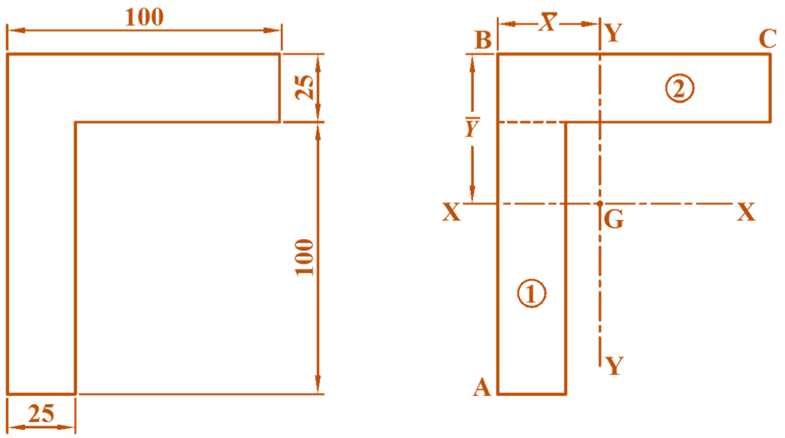


Fig.P5.2 Centroid of 'L' section [Example 5.2]

Solution :

$$a_1 = 25 \times 100 = 2500 \text{ mm}^2; \; a_2 = 100 \times 25 = 2500 \text{ mm}^2$$

$$x_1 = \frac{25}{2} = 12.5 \text{ mm}; \; y_1 = 25 + \frac{100}{2} = 75 \text{ mm}$$

$$x_2 = \frac{100}{2} = 50 \text{ mm}; \; y_2 = \frac{25}{2} = 12.5 \text{ mm}$$

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2500 \times 12.5) + (2500 \times 50)}{2500 + 2500} = \frac{156250}{5000} = \boxed{\mathbf{31.25 \text{ mm}}}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2500 \times 75) + (2500 \times 12.5)}{2500 + 2500} = \frac{218750}{5000} = \boxed{\mathbf{43.75 \text{ mm}}}$$

Result : $\bar{X}$ = **31.25 mm** and $\bar{Y}$ = **43.75 mm** from reference axes

Example : 5.3

(Apr.14)

Find the centroid of a T–section with flange 100mm × 30mm and web 120mm × 30mm.

Solution :

This section is symmetrical about Y–Y axis. So the C.G will lie on this axis

$$\therefore \bar{X} = \frac{100}{2} = \boxed{\mathbf{50 \text{ mm}}}$$

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2; \; a_2 = 30 \times 120 = 3600 \text{ mm}^2$$

$$y_1 = \frac{30}{2} = 15 \text{ mm}; \; y_2 = 30 + \frac{120}{2} = 90 \text{ mm}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 15) + (3600 \times 90)}{3000 + 3600} = \frac{369000}{6600} = \boxed{\mathbf{55.91 \text{ mm}}}$$

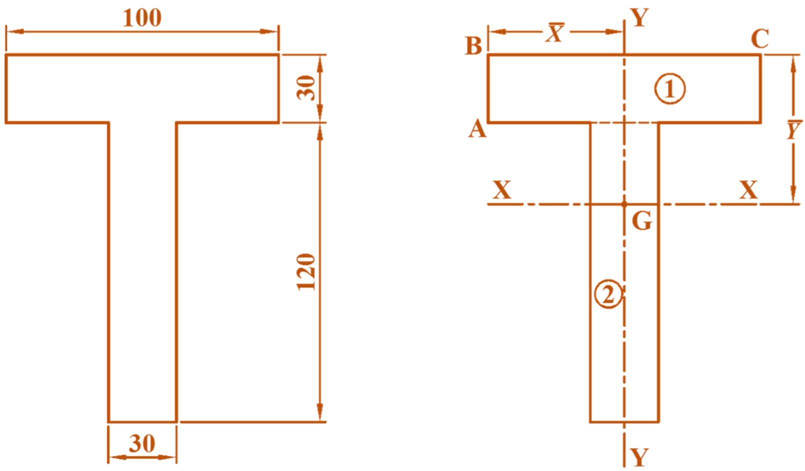


Fig.P5.3 Centroid of 'T' section [Example 5.3]

Result : $\bar{X}$ **= 50 mm** and $\bar{Y}$ **= 55.91 mm** from reference axes

Example : 5.4 *(Apr.04, Oct.12)*

Find the centroid of an inverted T–section with flange 150mmx20mm and web 100mm × 25mm.

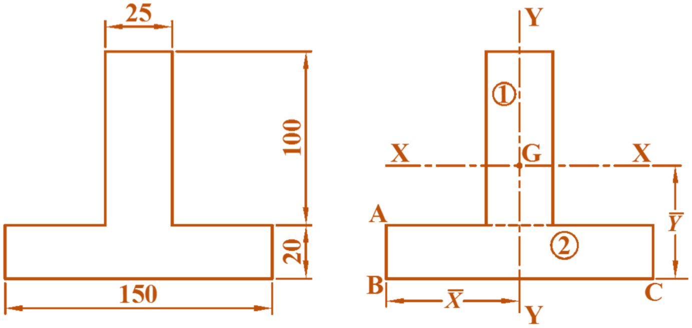


Fig.P5.4 Centroid of inverted 'T' section [Example 5.4]

Solution :

This section is symmetrical about Y–Y axis. So the C.G will lie on this axis

$$\therefore \bar{X} = \frac{150}{2} = \boxed{75\text{ mm}}$$

$$a_1 = 25 \times 100 = 2500\text{ mm}^2;\ a_2 = 150 \times 20 = 3000\text{ mm}^2$$

$$y_1 = 20 + \frac{100}{2} = 70\text{ mm};\ y_2 = \frac{20}{2} = 10\text{ mm}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2500 \times 70) + (3000 \times 10)}{2500 + 3000} = \frac{205000}{5500} = \boxed{37.273 \text{ mm}}$$

Result : $\bar{X}$ **= 75 mm** and $\bar{Y}$ **= 37.273 mm** from reference axes

Example : 5.5

A channel section of size 100mm × 50mm overall. The base as well as the flanges of the channel are 15mm thick. Determine the centroid for the section.

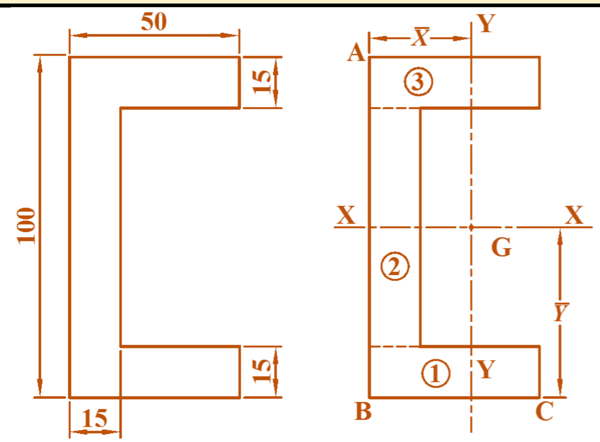


Fig.P5.5 Centroid of channel section [Example 5.5]

Solution :

This section is symmetrical about X–X axis. So the C.G will lie on this axis

$$\therefore \bar{Y} = \frac{100}{2} = \boxed{50 \text{ mm}}$$

a_1=50×15=750 mm^2; a_2=70×15=1050 mm^2; a_3=50×15=750 mm^2

$$x_1 = \frac{50}{2} = 25 \text{ mm}; \ x_2 = \frac{15}{2} = 7.5 \text{ mm}; \ x_3 = \frac{50}{2} = 25 \text{ mm}$$

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750}$$

$$= \frac{45375}{2550} = \boxed{17.794 \text{ mm}}$$

Result : $\bar{X}$ **= 17.794 mm** and $\bar{Y}$ **= 50 mm** from reference axes

Example : 5.6

(Oct.14)

Find the centroid of an I–section having top flange 150mm × 25mm, web 160mm × 25mm and bottom flange 200mm × 25mm.

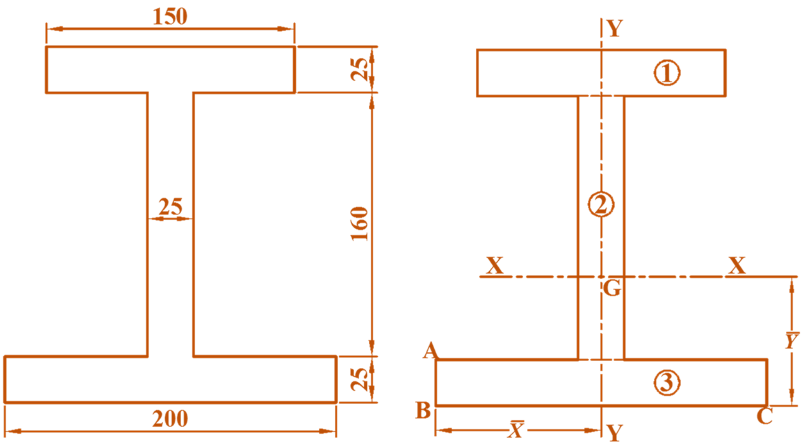


Fig.P5.6 Centroid of 'I' section [Example 5.6]

Solution :

This section is symmetrical about Y–Y axis. So the C.G will lie on this axis

$$\therefore \bar{X} = \frac{200}{2} = \boxed{100 \text{ mm}}$$

$$a_1 = 150 \times 25 = 3750 \text{ mm}^2; \quad y_1 = 25 + 160 + \frac{25}{2} = 197.5 \text{ mm}$$

$$a_2 = 25 \times 160 = 4000 \text{ mm}^2; \quad y_2 = 25 + \frac{160}{2} = 105 \text{ mm}$$

$$a_3 = 200 \times 25 = 5000 \text{ mm}^2; \quad y_3 = \frac{25}{2} = 12.5 \text{ mm}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(3750\times197.5)+(4000\times105)+(5000\times12.5)}{3750+4000+5000}$$

$$= \frac{1223125}{12750} = \boxed{95.931 \text{ mm}}$$

Result : $\bar{X}$ = **100 mm** and $\bar{Y}$ = **95.931** mm from reference axes

DETERMINATION MOMENT OF INERTIA

Example : 5.7 *(Oct.01)*

Determine the polar moment of inertia of rectangle 100mm ×150mm.

Solution :

Moment of inertia of rectangular section about X–X axis,

$$I_{xx} = \frac{bd^3}{12} = \frac{100 \times 150^3}{12} = 28125000 \text{ mm}^4$$

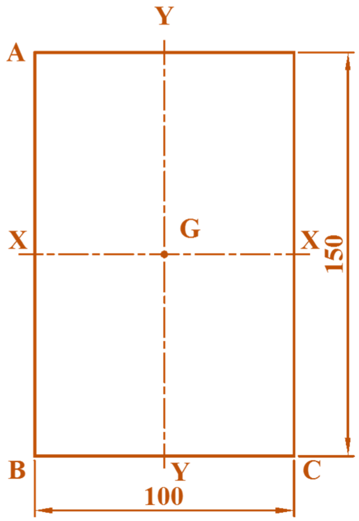


Fig.P5.7 M.I of rectangular section [Example 5.7]

Moment of inertia of rectangular section about Y–Y axis,

$$I_{yy} = \frac{db^3}{12} = \frac{150 \times 100^3}{12} = 12500000 \text{ mm}^4$$

Polar moment of inertia,

$$I_{zz} = I_{xx} + I_{yy} = 28125000 + 12500000 = \boxed{40625000 \text{ mm}^4}$$

Result : The polar moment of inertia, I_{zz} **= 40625000 mm^4**

Example : 5.8 *(Apr.01)*

Determine the polar moment of inertia of a circle of diameter 100mm.

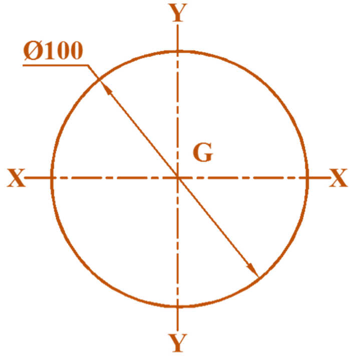


Fig.P5.8 M.I of circular section [Example 5.8]

Solution :

Diameter of the circle, $d = 100$ mm

Moment of inertia of circular section about X–X or Y–Y axis,

$$I_{xx} = I_{yy} = \frac{\pi d^4}{64} = \frac{\pi \times 100^4}{64} = 4908738.521 \text{ mm}^4$$

Polar moment of inertia,

$$I_{zz} = I_{xx} + I_{yy} = 4908738.521 + 4908738.521 = \boxed{9817477.042 \text{ mm}^4}$$

Result : The polar moment of inertia, $I_{zz} = \mathbf{9817477.042 \text{ mm}^4}$

Example : 5.9 *(Apr.03, Oct.16)*

An angle section is of 100 mm wide and 120 mm deep overall. Both the flanges of the angle are 10 mm thick. Determine the moment of inertia about the centroidal axes X-X and Y-Y. Also find its radius of gyration about its centroidal axes.

Solution :

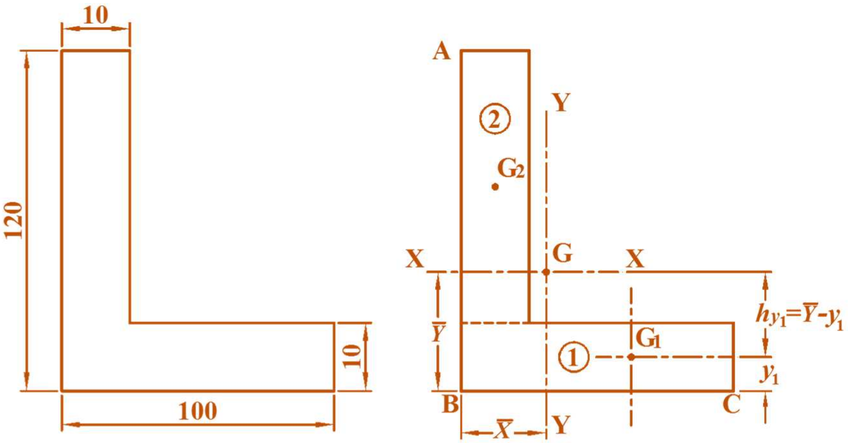


Fig.P5.9 M.I of 'L' section [Example 5.9]

Split the section into two rectangles as shown.

$$a_1 = 100 \times 10 = 1000 \text{ mm}^2; x_1 = \frac{100}{2} = 50 \text{ mm}; y_1 = \frac{10}{2} = 5 \text{ mm}$$

$$a_2 = 10 \times 110 = 1100 \text{ mm}^2; x_2 = \frac{10}{2} = 5 \text{ mm}; y_2 = 10 + \frac{110}{2} = 65 \text{ mm}$$

$$\overline{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(1000 \times 50) + (1100 \times 5)}{1000 + 1100} = \frac{55500}{2100} = 26.43 \text{ mm}$$

$$\overline{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(1000 \times 5) + (1100 \times 65)}{1000 + 1100} = \frac{76500}{2100} = 36.43 \text{ mm}$$

Calculation for I_{xx}

Distance of C.G of section (1) from X–X axis,

$$h_{y1} = \overline{Y} - y_1 = 36.43 - 5 = 31.43 \text{ mm}$$

Distance of C.G of section (2) from X–X axis,

$$h_{y2} = \overline{Y} - y_2 = 36.43 - 65 = -28.57 \text{ mm}$$

Moment of inertia of section (1) about an axis parallel to X–X and passing through its C.G (G_1),

$$I_{Gx1} = \frac{b_1 {d_1}^3}{12} = \frac{100 \times 10^3}{12} = 8333.333 \text{ mm}^4$$

Moment of inertia of section (2) about an axis parallel to X–X and passing through its C.G (G_2),

$$I_{Gx2} = \frac{b_2 {d_2}^3}{12} = \frac{10 \times 110^3}{12} = 1109166.667 \text{ mm}^4$$

According to parallel axis theorem,

the moment of inertia of section (1) about X–X axis,

$$I_{xx1} = I_{Gx1} + a_1 h_{y1}^2 = 8333.333 + [1000 \times 31.43^2] = 996178.233 \text{ mm}^4$$

Similarly,

$$I_{xx2} = I_{Gx2} + a_2 h_{y2}^2 = 1109166.667 + [1100 \times (-28.57)^2] = 2007036.057 \text{ mm}^4$$

Moment of inertia of the whole section about X–X axis,

$$I_{xx} = I_{xx1} + I_{xx2} = 996178.233 + 2007036.057 = 3003214.29$$

$$= \boxed{\mathbf{3.0032 \times 10^6 \text{ mm}^4}}$$

Calculation for I_{yy}

Distance of C.G of section (1) from Y–Y axis,

$$h_{x1} = \overline{X} - x_1 = 26.43 - 50 = -23.57 \text{ mm}$$

Distance of C.G of section (2) from Y–Y axis,

$$h_{x2} = \overline{X} - x_2 = 26.43 - 5 = 21.43 \text{ mm}$$

Moment of inertia of section (1) about an axis parallel to Y–Y and passing through its C.G(G_1),

$$I_{Gy1} = \frac{d_1 {b_1}^3}{12} = \frac{10 \times 100^3}{12} = 833333.333 \text{ mm}^4$$

Moment of inertia of section (2) about an axis parallel to Y–Y and passing through its C.G(G_2),

$$I_{Gy2} = \frac{d_2 {b_2}^3}{12} = \frac{110 \times 10^3}{12} = 9166.667 \text{ mm}^4$$

According to parallel axis theorem,

the moment of inertia of section (1) about Y–Y axis,

$$I_{yy1} = I_{Gy1} + a_1 h_{x1}^2 = 833333.333 + \left[1000 \times (-23.57)^2\right] = 1388878.233 \text{ mm}^4$$

$$I_{yy2} = I_{Gy2} + a_2 h_{x2}^2 = 9166.667 + \left[1100 \times 21.43^2\right] = 514336.057 \text{ mm}^4$$

Moment of inertia of the whole section about Y–Y axis,

$$I_{yy} = I_{yy1} + I_{yy2} = 1388878.233 + 514336.057 = 1903214.29$$

$$= \boxed{\mathbf{1.9032 \times 10^6 \text{ mm}^4}}$$

Calculation for K_{xx}

Radius of gyration about centroidal axis X-X,

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{3003214.29}{2100}} = \boxed{\mathbf{37.817 \text{ mm}}}$$

Calculation for K_{yy}

Radius of gyration about centroidal axis Y-Y,

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{1903214.29}{2100}} = \boxed{\mathbf{30.105 \text{ mm}}}$$

Result : 1) The moment of inertia about centroidal axes,

$\mathbf{I_{xx} = 2.088 \times 10^6 \text{ mm}^4}$; $\mathbf{I_{yy} = 1.2974 \times 10^6 \text{ mm}^4}$

2) The radius of gyration about centroidal axes,

$\mathbf{K_{xx} = 37.817 \text{ mm}}$; $\mathbf{K_{yy} = 30.105 \text{ mm}}$

Example : 5.10 *(Oct.03, Oct.04, Apr.13, Apr.18)*

Find the values of I_{xx} and I_{yy} of a T–section 120mm wide and 120mm deep overall. Both the web and flange are 10mm thick. Also calculate K_{xx} and K_{yy}.

Solution :

This section is symmetrical about Y–Y axis. So the C.G will lie on this axis.

$$\therefore \bar{X} = \frac{120}{2} = 60 \text{ mm}$$

$$a_1 = 120 \times 10 = 100 \text{ mm}^2;\ a_2 = 10 \times 110 = 1100 \text{ mm}^2$$

$$y_1 = \frac{10}{2} = 5 \text{ mm};\quad y_2 = 10 + \frac{110}{2} = 65 \text{ mm}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(1200 \times 5) + (1100 \times 65)}{1200 + 1100} = \frac{77500}{2300} = 33.696 \text{ mm}$$

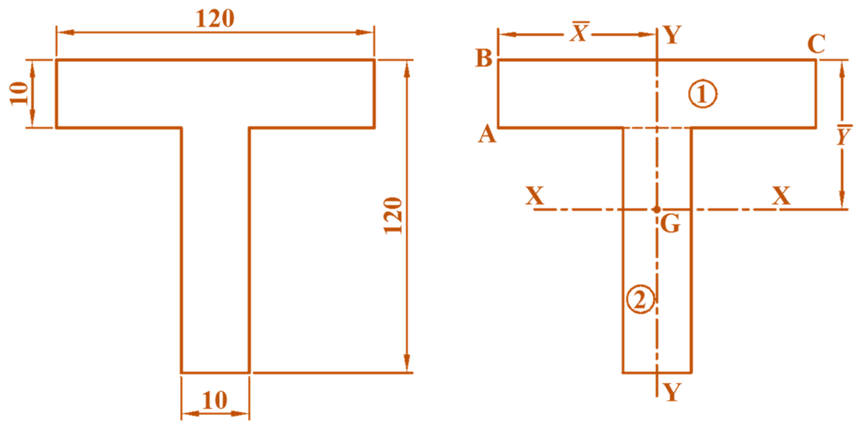


Fig.P5.10 M.I of 'T' section [Example 5.10]

Calculation for I_{xx}

$$h_{y1} = \bar{Y} - y_1 = 33.696 - 5 = 28.696 \text{ mm}$$
$$h_{y2} = \bar{Y} - y_2 = 33.696 - 65 = -31.304 \text{ mm}$$

$$I_{Gx1} = \frac{b_1 d_1^{\,3}}{12} = \frac{120 \times 10^3}{12} = 10000 \text{ mm}^4$$

$$I_{Gx2} = \frac{b_2 d_2^{\,3}}{12} = \frac{10 \times 110^3}{12} = 1109166.667 \text{ mm}^4$$

$$I_{xx1} = I_{Gx1} + a_1 h_{y1}^2 = 10000 + \left[1200 \times (28.696)^2\right] = 998152.5 \text{ mm}^4$$

$$I_{xx2} = I_{Gx2} + a_2 h_{y2}^2$$
$$= 1109166.667 + \left[1100 \times (-31.304)^2\right] = 2187101.125 \text{ mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2} = 998152.5 + 2187101.125 = \boxed{3.185 \times 10^6 \text{mm}^4}$$

Calculation for I_{yy}

$$h_{x1} = \bar{X} - x_1 = 60 - 60 = 0$$
$$h_{x2} = \bar{X} - x_2 = 60 - 60 = 0$$

$$I_{Gy1} = \frac{d_1 b_1^{\,3}}{12} = \frac{10 \times 120^3}{12} = 144000 \text{ mm}^4$$

$$I_{Gy2} = \frac{d_2 b_2^{\,3}}{12} = \frac{110 \times 10^3}{12} = 9166.667 \text{ mm}^4$$

$$I_{yy1} = I_{Gy1} + a_1 h_{x1}^2 = 144000 + 0 = 1440000 \text{ mm}^4$$
$$I_{yy2} = I_{Gy2} + a_2 h_{x2}^2 = 9166.667 + 0 = 9166.667 \text{ mm}^4$$

$$I_{yy} = I_{yy1} + I_{yy2} = 1440000 + 9166.667 = \boxed{1.449 \times 10^6 \text{ mm}^4}$$

Calculation for radius of gyration

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{3.185 \times 10^6}{2300}} = \boxed{37.213 \text{ mm}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{1.449 \times 10^6}{2300}} = \boxed{25.1 \text{ mm}}$$

Result :	1) $I_{xx} = 3.185 \times 10^6 \text{mm}^4$	2) $I_{yy} = 1.449 \times 10^6 \text{ mm}^4$
	3) $K_{xx} = 37.213 \text{ mm}$	4) $K_{yy} = 25.1 \text{ mm}$

Example : 5.11 *(Apr.90)*

Calculate I_{xx} and I_{yy} for the section shown in the fig.P5.11. Also find K_{xx} and K_{yy}.

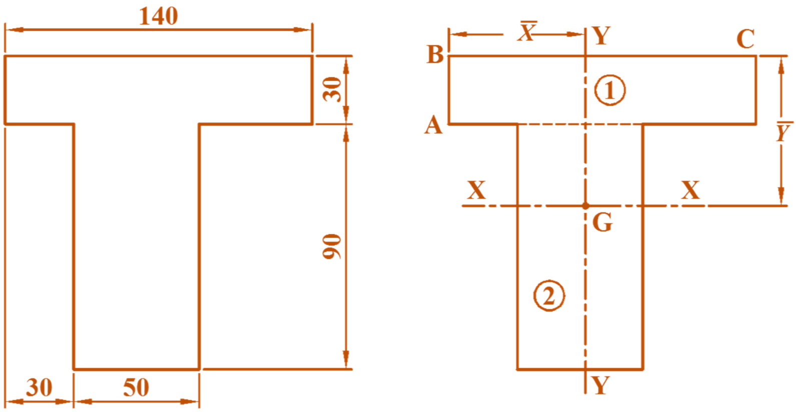


Fig.P5.11 M.I of 'T' section [Example 5.11]

Solution :

$$a_1 = 140 \times 30 = 4200 \text{ mm}^2; x_1 = \frac{140}{2} = 70 \text{ mm}; y_1 = \frac{30}{2} = 15 \text{ mm}$$

$$a_2 = 50 \times 90 = 4500 \text{ mm}^2; \quad x_2 = 30 + \frac{50}{2} = 55 \text{ mm}; \quad y_2 = 30 + \frac{90}{2} = 75 \text{ mm}$$

$$\overline{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(4200 \times 70) + (4500 \times 55)}{4200 + 4500} = \frac{5415100}{8700} = 64.241 \text{ mm}$$

$$\overline{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(4200 \times 15) + (4500 \times 75)}{4200 + 4500} = \frac{400500}{8700} = 46.034 \text{ mm}$$

Calculation for I_{xx}

$$h_{y1} = \bar{Y} - y_1 = 46.034 - 15 = 31.034 \text{ mm}$$
$$h_{y2} = \bar{Y} - y_2 = 46.034 - 75 = -28.966 \text{ mm}$$

$$I_{Gx1} = \frac{b_1 d_1{}^3}{12} = \frac{140 \times 30^3}{12} = 315000 \text{ mm}^4$$

$$I_{Gx2} = \frac{b_2 d_2{}^3}{12} = \frac{50 \times 90^3}{12} = 3.0375 \times 10^6 \text{ mm}^4$$

$$I_{xx1} = I_{Gx1} + a_1 h_{y1}^2$$
$$= 315000 + \left[4200 \times (31.034)^2\right] = 4360058.455 \text{ mm}^4$$

$$I_{xx2} = I_{Gx2} + a_2 h_{y2}^2$$
$$= 3.0375 \times 10^6 + \left[4500 \times (-28.966)^2\right] = 6813131.202 \text{ mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2} = 4360058.455 + 6813131.202 = \boxed{\mathbf{11.173 \times 10^6 mm^4}}$$

Calculation for I_{yy}

$$h_{x1} = \bar{X} - x_1 = 62.241 - 70 = -7.759 \text{ mm}$$
$$h_{x2} = \bar{X} - x_2 = 62.241 - 55 = 7.241 \text{ mm}$$

$$I_{Gy1} = \frac{d_1 b_1{}^3}{12} = \frac{30 \times 140^3}{12} = 6.86 \times 10^6 \text{mm}^4$$

$$I_{Gy2} = \frac{d_2 b_2{}^3}{12} = \frac{90 \times 50^3}{12} = 937500 \text{ mm}^4$$

$$I_{yy1} = I_{Gy1} + a_1 h_{x1}^2 = 6.86 \times 10^6 + \left[4200 \times (-7.759)^2\right] = 7112848.74 \text{ mm}^4$$

$$I_{yy2} = I_{Gy2} + a_2 h_{x2}^2 = 937500 + \left[4500 \times (7.241)^2\right] = 1173444.365 \text{ mm}^4$$

$$I_{yy} = I_{yy1} + I_{yy2} = 7112848.74 + 1173444.365 = \boxed{\mathbf{8.2863 \times 10^6 \ mm^4}}$$

Calculation for radius of gyration

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{11.173 \times 10^6}{8700}} = \boxed{\mathbf{35.836 \ mm}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{8.2863 \times 10^6}{8700}} = \boxed{\mathbf{30.862 \ mm}}$$

Result : 1) $I_{xx} = \mathbf{11.173 \times 10^6 mm^4}$ 2) $I_{yy} = \mathbf{8.2863 \times 10^6 \ mm^4}$
3) $K_{xx} = \mathbf{35.836 \ mm}$ 4) $K_{yy} = \mathbf{30.862 \ mm}$

Example : 5.12 *(Apr.05, Oct.12)*

A channel section is of size 300mm×100mm overall. The base as well as the flanges of the channel are 10mm thick. Determine the values of I_{xx} and I_{yy}. A.lso find K_{xx} and K_{yy}.

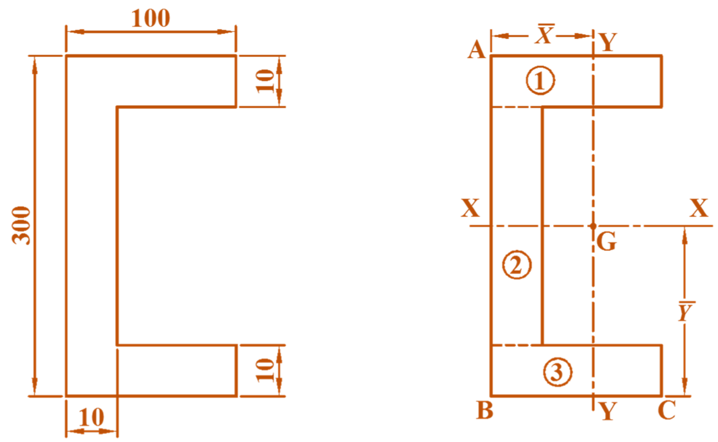


Fig.P5.12 M.I of channel section [Example 5.12]

Solution :

This section is symmetrical about X–X axis. So the C.G will lie on this axis

$$\therefore \overline{Y} = \frac{300}{2} = 150 \text{ mm}$$

$$a_1 = a_3 = 100 \times 10 = 1000 \text{ mm}^2; a_2 = 10 \times 280 = 2800 \text{ mm}^2$$

$$x_1 = x_3 = \frac{100}{2} = 50 \text{ mm}; x_2 = \frac{10}{2} = 5 \text{ mm}$$

$$\overline{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1000 \times 50) + (2800 \times 5) + (1000 \times 50)}{1000 + 2800 + 1000} = \frac{114000}{4800} = 23.75 \text{ mm}$$

Calculation for I_{xx}

$$h_{y1} = \overline{Y} - y_1 = 150 - 5 = 145 \text{ mm}$$

$$h_{y2} = \overline{Y} - y_2 = 150 - \left(10 + \frac{280}{2}\right) = 0 \text{ mm}$$

$$h_{y3} = \overline{Y} - y_3 = 150 - \left(10 + 280 + \frac{10}{2}\right) = -145 \text{ mm}$$

$$I_{Gx1} = I_{Gx3} = \frac{b_1 d_1{}^3}{12} = \frac{100 \times 10^3}{12} = 8333.333 \text{ mm}^4$$

$$I_{Gx2} = \frac{b_2 {d_2}^3}{12} = \frac{10 \times 280^3}{12} = 18.2933 \times 10^6 \text{ mm}^4$$

$$I_{xx1} = I_{Gx1} + a_1 h_{y1}^2 = 83333.333 \ + \left[1000 \times (145)^2\right] = 21.0333 \times 10^6 \text{ mm}^4$$

$$I_{xx2} = I_{Gx2} + a_2 h_{y2}^2 = 18.2933 \times 10^6 + \left[2800 \times (0)^2\right] = 18.2933 \times 10^6 \text{ mm}^4$$

$$I_{xx3} = I_{Gx3} + a_3 h_{y3}^2 = 83333.333 + \left[1000 \times (145)^2\right] = 21.0333 \times 10^6 \text{ mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$$
$$= 21.033 \times 10^6 + 18.2933 \times 10^6 + 21.0333 \times 10^6 = \boxed{\mathbf{60.36 \times 10^6 mm^4}}$$

Calculation for I_{yy}

$$h_{x1} = \bar{X} - x_1 = 23.75 - 50 = -26.25 \text{ mm}$$
$$h_{x2} = \bar{X} - x_2 = 23.75 - 5 = 18.75 \text{ mm}$$
$$h_{x3} = \bar{X} - x_3 = 23.75 - 50 = -26.25 \text{ mm}$$

$$I_{Gy1} = I_{Gy3} = \frac{d_1 {b_1}^3}{12} = \frac{10 \times 100^3}{12} = 0.8333 \times 10^6 \text{mm}^4$$

$$I_{Gy2} = \frac{d_2 {b_2}^3}{12} = \frac{280 \times 10^3}{12} = 23.333 \times 10^3 \text{ mm}^4$$

$$I_{yy1} = I_{Gy1} + a_1 h_{x1}^2 = 0.8333 \times 10^6 + \left[1000 \times (-26.25)^2\right] = 1.5224 \times 10^6 \text{ mm}^4$$

$$I_{yy2} = I_{Gy2} + a_2 h_{x2}^2 = 23.333 \times 10^3 + \left[2800 \times (18.75)^2\right] = 1.0077 \times 10^6 \text{ mm}^4$$

$$I_{yy3} = I_{yy1} = 1.5224 \times 10^6 \text{mm}^4$$

$$I_{yy} = I_{yy1} + I_{yy2} + I_{yy3}$$
$$= 1.5224 \times 10^6 + 1.0077 \times 10^6 + 1.5224 \times 10^6 = \boxed{\mathbf{4.0525 \times 10^6 \ mm^4}}$$

Calculation for radius of gyration

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{60.36 \times 10^6}{4800}} = \boxed{\mathbf{112.138 \ mm}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{4.0525 \times 10^6}{4800}} = \boxed{\mathbf{29.056 \ mm}}$$

Result :	1) I_{xx} = **60.36 × 10^6 mm^4**	2) I_{yy} = **4.0525 × 10^6 mm^4**
	3) K_{xx} = **112.138 mm**	4) K_{yy} = **29.056 mm**

Example : 5.13

Find the moment of inertia of the section shown in the fig.P5.13 about the horizontal centroidal axis. Also find the radius of gyration about that axis.

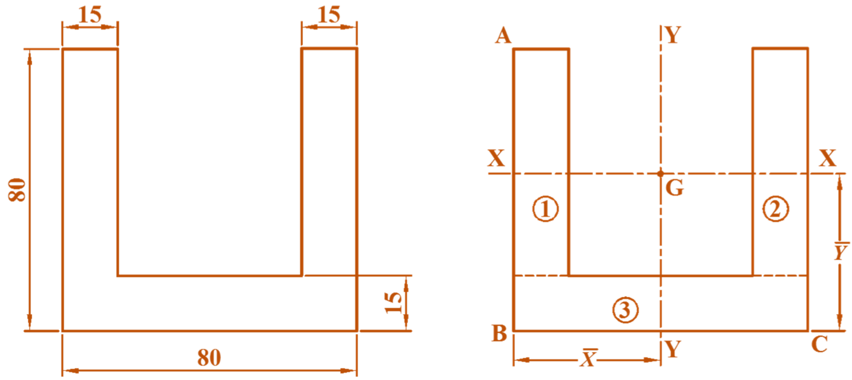


Fig.P5.13 M.I of channel section [Example 5.13]

Solution :

$a_1 = a_2 = 15 \times (80 - 15) = 975 \text{ mm}^2; a_3 = 80 \times 15 = 1200 \text{ mm}^2$

$y_1 = y_2 = 15 + \dfrac{65}{2} = 47.5 \text{ mm}; y_3 = \dfrac{15}{2} = 7.5 \text{ mm}$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(975 \times 47.5) + (1200 \times 7.5) + (975 \times 47.5)}{975 + 1200 + 975}$$

$$= \frac{101625}{3150} = 32.262 \text{ mm}$$

Calculation for I_{xx}

$$h_{y1} = h_{y2} = \bar{Y} - y_1 = 32.262 - 47.5 = -15.238 \text{ mm}$$

$$h_{y3} = \bar{Y} - y_3 = 32.262 - 7.5 = 24.762 \text{ mm}$$

$$I_{Gx1} = I_{Gx2} = \frac{b_1 {d_1}^3}{12} = \frac{15 \times 65^3}{12} = 343281.25 \text{ mm}^4$$

$$I_{Gx3} = \frac{b_3 {d_3}^3}{12} = \frac{80 \times 15^3}{12} = 22500 \text{ mm}^4$$

$$I_{xx1} = I_{Gx1} + a_1 h_{y1}^2$$
$$= 343281.25 + \left[975 \times (-15.238)^2\right] = 569672.978 \text{ mm}^4$$

$$I_{xx2} = I_{xx1} = 569672.978 \text{ mm4}$$

$$I_{xx3} = I_{Gx3} + a_3 h_{y3}^2 = 22500 + \left[1200 \times (24.762)^2\right] = 1758287.973 \text{ mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$$
$$= 569672.978 + 1758287.973 + 569672.978 = \boxed{1.8976 \times 10^6 \text{mm}^4}$$

$$\text{Radius of gyration, } K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{18.976 \times 10^6}{3150}} = \boxed{25.544 \text{ mm}}$$

Result : 1) $I_{xx} = 1.8976 \times 10^6 \text{mm}^4$ 2) $K_{xx} = 24.544 \text{ mm}$

Example : 5.14 *(Apr.02, Oct.13)*

Determine the moment of inertia about centroidal co–ordinate axes of an I–section having equal flanges 120mm × 20mm size and web 120mm × 20mm thick. Also find K_{xx} and K_{yy}.

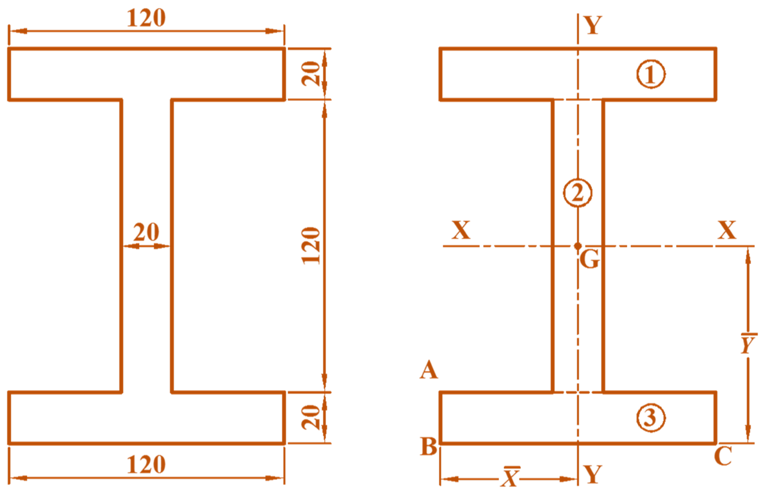


Fig.P5.14 M.I of 'I' section [Example 5.14]

Solution :

This section is symmetrical about X–X and Y-Y axis.

$$\therefore \bar{X} = \frac{120}{2} = 60 \text{ mm}; \qquad \bar{Y} = \frac{160}{2} = 80 \text{ mm}$$

$$a_1 = a_3 = 120 \times 20 = 2400 \text{ mm}^2; \quad a_2 = 20 \times 120 = 2400 \text{ mm}^2$$

$$x_1 = x_2 = x_3 = 60 \text{ mm}; \quad y_1 = \frac{20}{2} = 10 \text{ mm}; \quad y_2 = 20 + \frac{120}{2} = 80 \text{ mm};$$

$$y_3 = 20 + 120 + \frac{20}{2} = 150 \text{ mm}; \quad \Sigma a = 2400 + 2400 + 2400 = 7200 \text{ mm}^2$$

Calculation for I_{xx}

$$h_{y1} = \bar{Y} - y_1 = 80 - 10 = 70 \text{ mm}$$

$$h_{y2} = \bar{Y} - y_2 = 80 - 80 = 0 \text{ mm}$$

$$h_{y3} = \bar{Y} - y_3 = 80 - 150 = -70 \text{ mm}$$

$$I_{Gx1} = I_{Gx3} = \frac{b_1 {d_1}^3}{12} = \frac{120 \times 20^3}{12} = 80000 \text{ mm}^4$$

$$I_{Gx2} = \frac{b_2 {d_2}^3}{12} = \frac{20 \times 120^3}{12} = 2.88 \times 10^6 \text{ mm}^4$$

$$I_{xx1} = I_{Gx1} + a_1 h_{y1}^2 = 80000 + \left[2400 \times (70)^2\right] = 11.84 \times 10^6 \text{mm}^4$$

$$I_{xx2} = I_{Gx2} + a_2 h_{y2}^2 = 2.88 \times 10^6 + \left[2400 \times 0^2\right] = 2.88 \times 10^6 \text{ mm}^4$$

$$I_{xx3} = I_{Gx3} + a_3 h_{y3}^2 = 80000 + \left[2400 \times (-70)^2\right] = 11.84 \times 10^6 \text{mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$$

$$= 11.84 \times 10^6 + 2.88 \times 10^6 + 11.84 \times 10^6 = \boxed{\mathbf{26.56 \times 10^6 mm^4}}$$

Calculation for I_{yy}

$$h_{x1} = h_{x2} = h_{x3} = \bar{X} - x_1 = 60 - 60 = 0 \text{ mm}$$

$$I_{Gy1} = I_{Gy3} = \frac{d_1 b_1^{\,3}}{12} = \frac{20 \times 120^3}{12} = 2.88 \times 10^6 \text{mm}^4$$

$$I_{Gy2} = \frac{d_2 b_2^{\,3}}{12} = \frac{120 \times 20^3}{12} = 80000 \text{ mm}^4$$

$$I_{yy1} = I_{yy3} = I_{Gy1} + a_1 h_{x1}^2 = 2.88 \times 10^6 + 0 = 2.88 \times 10^6 \text{ mm}^4$$

$$I_{yy2} = I_{Gy2} + a_2 h_{x2}^2 = 80000 + 0 = 80000 \text{ mm}^4$$

$$I_{yy} = I_{yy1} + I_{yy2} + I_{yy3}$$

$$= 2.88 \times 10^6 + 80000 + 2.88 \times 10^6 = \boxed{\mathbf{5.84 \times 10^6 \ mm^4}}$$

Calculation for radius of gyration

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{26.56 \times 10^6}{7200}} = \boxed{\mathbf{60.736 \ mm}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{5.84 \times 10^6}{7200}} = \boxed{\mathbf{28.480 \ mm}}$$

Result : 1) $I_{xx} = \mathbf{26.56 \times 10^6 mm^4}$ 2) $I_{yy} = \mathbf{5.84 \times 10^6 \ mm^4}$

3) $K_{xx} = \mathbf{60.736 \ mm}$ 3) $K_{yy} = \mathbf{28.480 \ mm}$

Example : 5.15

(Apr.04, Apr.15, Oct.17)

An I–section has the top flange 100mm × 15mm, web 150mm × 20mm and the bottom flange 180mm × 30mm. Calculate I_{xx} and I_{yy} of the section. Also find K_{xx} and K_{yy} of the section.

Solution :

This section is symmetrical about Y-Y axis.

$$\therefore \bar{X} = \frac{180}{2} = 90 \text{ mm}$$

$$a_1 = 180 \times 30 = 5400 \text{ mm}^2; \quad y_1 = \frac{30}{2} = 15 \text{ mm}$$

$$a_2 = 20 \times 150 = 3000 \text{ mm}^2; \quad y_2 = 30 + \frac{150}{2} = 105 \text{ mm}$$

$$a_3 = 100 \times 15 = 1500 \text{ mm}^2; \quad y_3 = 30 + 150 + \frac{15}{2} = 187.5 \text{ mm}$$

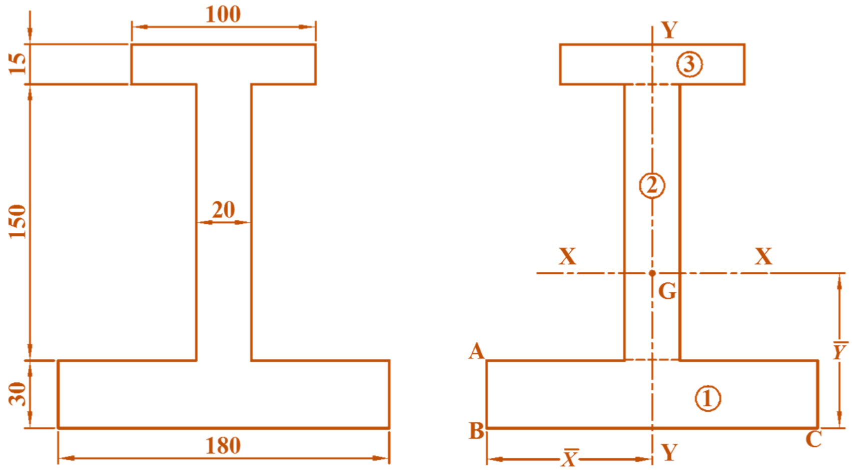


Fig.P5.15 M.I. of 'I' section [Example 5.15]

$$\overline{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(5400 \times 15) + (3000 \times 105) + (1500 \times 187.5)}{5400 + 3000 + 1500}$$

$$= \frac{677250}{9900} = 68.41 \text{ mm}$$

Calculation for I_{xx}

$$h_{y1} = \overline{Y} - y_1 = 68.41 - 15 = 53.41 \text{ mm}$$

$$h_{y2} = \overline{Y} - y_2 = 68.41 - 105 = -36.59 \text{ mm}$$

$$h_{y3} = \overline{Y} - y_3 = 68.41 - 187.5 = -119.09 \text{ mm}$$

$$I_{Gx1} = \frac{b_1 d_1^{\ 3}}{12} = \frac{180 \times 30^3}{12} = 0.405 \times 10^6 \text{ mm}^4$$

$$I_{Gx2} = \frac{b_2 d_2^{\ 3}}{12} = \frac{20 \times 150^3}{12} = 5.625 \times 10^6 \text{ mm}^4$$

$$I_{Gx3} = \frac{b_3 d_3^{\ 3}}{12} = \frac{100 \times 15^3}{12} = 28125 \text{ mm}^4$$

$$I_{xx1} = I_{Gx1} + a_1 h_{y1}^2 = 0.405 \times 10^6 + \left[5400 \times (53.41)^2\right] = 15.809 \times 10^6 \text{ mm}^4$$

$$I_{xx2} = I_{Gx2} + a_2 h_{y2}^2 = 5.625 \times 10^6 + \left[3000 \times (-36.59)^2\right] = 9.6415 \times 10^6 \text{ mm}^4$$

$$I_{xx3} = I_{Gx3} + a_3 h_{y3}^2 = 28125 + \left[1500 \times (-119.09)^2\right] = 21.302 \times 10^6 \text{ mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$$
$$= 15.809\times10^6 + 9.6415\times10^6 + 21.302\times10^6 = \boxed{46.7525\times10^6 \text{mm}^4}$$

Calculation for I_{yy}

$$h_{x1} = h_{x2} = h_{x3} = \bar{X} - x_1 = 90 - 90 = 0 \text{ mm}$$

$$I_{Gy1} = I_{Gy3} = \frac{d_1 {b_1}^3}{12} = \frac{30\times180^3}{12} = 14.58\times10^6 \text{mm}^4$$

$$I_{Gy2} = \frac{d_2 {b_2}^3}{12} = \frac{150\times20^3}{12} = 0.1\times10^6 \text{ mm}^4$$

$$I_{Gy3} = \frac{d_3 {b_3}^3}{12} = \frac{15\times100^3}{12} = 1.25\times10^6 \text{ mm}^4$$

$$I_{yy1} = I_{Gy1} + a_1 h_{x1}^2 = 14.58\times10^6 + 0 = 14.58\times10^6 \text{ mm}^4$$

$$I_{yy2} = I_{Gy2} + a_2 h_{x2}^2 = 0.1\times10^6 + 0 = 0.1\times10^6 \text{ mm}^4$$

$$I_{yy3} = I_{Gy3} + a_3 h_{x3}^2 = 1.25\times10^6 + 0 = 1.25\times10^6 \text{ mm}^4$$

$$I_{yy} = I_{yy1} + I_{yy2} + I_{yy2}$$
$$= 14.58\times10^6 + 0.1\times10^6 + 1.25\times10^6 = \boxed{15.93\times10^6 \text{ mm}^4}$$

Calculation for radius of gyration

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{46.7525\times10^6}{9900}} = \boxed{68.72 \text{ mm}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{15.93\times10^6}{9900}} = \boxed{40.119 \text{ mm}}$$

Result : 1) $I_{xx} = 46.7525\times10^6 \text{mm}^4$ 2) $I_{yy} = 15.85\times10^6 \text{ mm}^4$
3) $K_{xx} = 68.72$ mm 4) $K_{yy} = 40.119$ mm

Example : 5.16 *(Oct.01)*

A rectangular hole of breadth 60mm and depth 100mm is made at the centre of rectangular plate of breadth 120mm and depth 200mm. Determine the moment of inertia of the hollow plate about its centroidal axis. Also find K_{xx} and K_{yy}.

Solution :

$$a_1 = 120\times200 = 24000 \text{ mm}^2; a_2 = 60\times100 = 6000 \text{ mm}^2;$$
$$\Sigma a = a_1 - a_2 = 24000 - 6000 = 18000 \text{ mm}^2$$

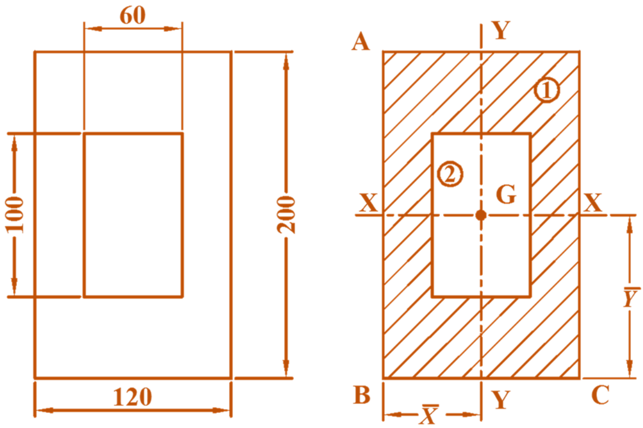


Fig.P5.16 M.I. of hollow rectangular section [Example 5.16]

Calculation for I_{xx}

Moment of inertia of outer rectangle about X-X axis,

$$I_{xx1} = \frac{b_1 d_1^{\,3}}{12} = \frac{120 \times 200^3}{12} = 80 \times 10^6 \text{ mm}^4$$

Moment of inertia of inner rectangle about X-X axis,

$$I_{xx2} = \frac{b_2 d_2^{\,3}}{12} = \frac{60 \times 100^3}{12} = 5 \times 10^6 \text{ mm}^4$$

Moment of inertia of the whole section about X–X axis,

$$I_{xx} = I_{xx1} - I_{xx2} = 80 \times 10^6 - 5 \times 10^6 = \boxed{75 \times 10^6 \text{ mm}^4}$$

Calculation for I_{yy}

Moment of inertia of outer rectangle about Y–Y axis,

$$I_{yy1} = \frac{d_1 b_1^{\,3}}{12} = \frac{200 \times 120^3}{12} = 28.8 \times 10^6 \text{mm}^4$$

Moment of inertia of inner rectangle about Y–Y axis,

$$I_{yy2} = \frac{d_2 b_2^{\,3}}{12} = \frac{100 \times 60^3}{12} = 1.8 \times 10^6 \text{ mm}^4$$

Moment of inertia of the whole section about Y–Y axis,

$$I_{yy} = I_{yy1} - I_{yy2} = 28.8 \times 10^6 - 1.8 \times 10^6 = \boxed{27 \times 10^6 \text{ mm}^4}$$

Calculation for radius of gyration

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma a}} = \sqrt{\frac{75 \times 10^6}{18000}} = \boxed{64.55 \text{ mm}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma a}} = \sqrt{\frac{27 \times 10^6}{18000}} = \boxed{38.73 \text{ mm}}$$

Result : 1) $I_{xx} = 75 \times 10^6$ mm^4 2) $I_{yy} = 27 \times 10^6$ mm^4
3) $K_{xx} = 64.55$ mm ; 4) $K_{yy} = 38.73$ mm

PROBLEMS FOR PRACTICE

DETERMINATION OF CENTROID

1. Determine the centroid of an angle section 90mm × 70mm × 10mm thick with its longer arm being placed vertical.

   [Ans: $\bar{X}$ = 19mm, $\bar{Y}$ = 29mm from bottom flange]

2. Find the centroid of the section shown in the fig.P5.17

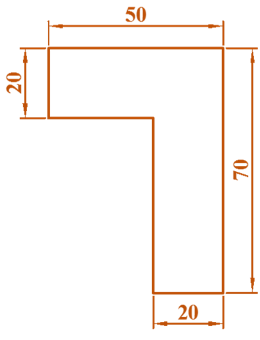


Fig.P5.17

[Ans: $\bar{X}$ = 17.5mm from right end, $\bar{Y}$ = 27.5mm from top face]

3. Find the centroid of a T–section with flange 120mm × 15mm and web 100mm × 15mm.

   [Ans: $\bar{X}$ = 50mm from left end, $\bar{Y}$ = 33.636mm from top flange]

4. Find the centroid of a T–section with flange 200mm × 20mm and web 150mm × 20mm.

   [Ans: $\bar{X}$ = 100mm from left end, $\bar{Y}$ = 46.429mm from bottom flange]

5. A channel section is of size 300mm × 150mm overall. the base as well as the flange of the channel are 15mm thick. Determine the centroid for the section.

   [Ans: $\bar{X}$ = 43.026mm from base, $\bar{Y}$ = 150mm from bottom flange]

6. Find the centroid of an I–section having top flange 150mm × 50mm, web 300mm × 50mm and bottom flange 300mm x100mm.

[Ans: $\bar{X}$ = 150mm from left end, $\bar{Y}$ = 160.714mm from bottom flange]

DETERMINATION OF MOMENT OF INERTIA

7. Calculate the moment of inertial about Y–axis of a circular section of radius 2m. *(Oct.04)* *[Ans: $I_{yy} = 1.2566 \times 10^{13} mm^4$]*

8. An unequal angle section is 100mm wide and 120mm deep overall and both flanges of the angle are 10mm thick. Find I_{xx}, I_{yy}, K_{xx} and K_{yy}. *(Apr.96, Oct.15)* *[Ans: $I_{xx} = 3.0032 \times 10^6\ mm^4$, $I_{yy} = 1.903 \times 10^6\ mm^4$, $K_{xx} = 37.817mm$, $K_{yy} = 30.103mm$]*

9. Find the moment of inertia of a T–section whose flange is 140mm × 12mm and the web is 100mm × 12mm about X-X and Y–Y centroidal axis. Also find K_{xx} and K_{yy}. *(Apr.04)*
[*Ans*: $I_{xx} = 3.255 \times 10^6\ mm^4$, $I_{yy} = 2.888 \times 10^6\ mm^4$, $K_{xx} = 33.619mm$, $K_{yy} = 31.667mm$]

10. The thickness of flanges and web of a 150mm × 75mm channel section are 9mm and 6mm respectively. Find the position of centroid of the section and its I_{xx}. Also find its radius of gyration.
[Ans: $\bar{X}$ = 24.744mm, $\bar{Y}$ = 75mm, $I_{xx} = 7.87 \times 10^6\ mm^4$, K_{xx} = 60.615mm]

11. Determine the value of I_{xx} and I_{yy} for the section shown in the fig.P5.18 about horizontal and vertical centroidal axes. Also find K_{xx} and K_{yy}.

[Ans: $I_{xx} = 17.799 \times 10^7\ mm^4$,
$I_{yy} = 67.973 \times 10^6\ mm^4$,
$K_{xx} = 92.728mm$,
$K_{yy} = 57.304mm$]

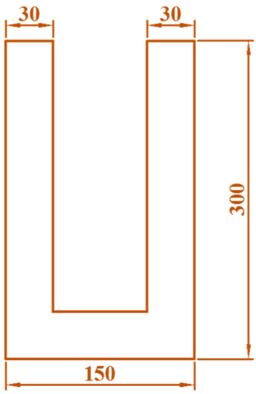


Fig.P5.18

12. Calculate the moment of inertia of an I–section whose overall dimensions are 150 × 250 × 15mm about the centroidal X-X and Y–Y axes. Also find K_{xx} and K_{yy}. *(Oct.01)*

[*Ans*: $I_{xx} = 7.552 \times 10^7 mm^4$, $I_{yy} = 8.499 \times 10^6\ mm^4$, $K_{xx} = 98.397mm$, $K_{yy} = 33.009mm$]

13. Determine the moment of inertia about centroidal coordinate axes of an I–section having equal flanges 150mm × 10mm size and web 130mm × 10mm thick. Also find K_{xx} and K_{yy}. *(Oct.88)*

[*Ans:* $I_{xx} = 16.55 \times 10^6\ mm^4$, $I_{yy} = 5.636 \times 10^6\ mm^4$, K_{xx} = *62.039mm*, K_{yy} = *36.204mm*]

14. An I–section is made up of three rectangular sections top flange 120mm × 20mm, web 180mm × 20mm and the bottom flange 200mm × 40mm. Find I_{xx}, I_{yy}, K_{xx} and K_{yy} of the section. *(Oct.95, Apr.14)*

[*Ans:* $I_{xx} = 1.0241 \times 10^8\ mm^4$, $I_{yy} = 29.67 \times 10^6\ mm^4$, K_{xx} = *85.528mm*, K_{yy} = *46.036mm*]

Unit - III

Chapter 6. THIN CYLINDERS AND THIN SPHERICAL SHELLS

6.1 Introduction

Some engineering components like pipes, steam boilers, liquid storage tanks and compressed air reservoirs have greater strength by virtue of their curved shape more than the material by which they are made. These are called *shells*. Generally the walls of such shells are very thin and compared to their diameter. Shells having cylindrical and spherical shapes are widely used. Whenever a shell is subjected to an internal pressure, its walls are subjected to tensile stresses. The shell wall will behave as a membrane in which the stresses are tangential to the middle surface of the wall uniformly distributed across its thickness.

6.2 Comparison of thin and thick cylindrical shells.

	Thin cylindrical shell	Thick cylindrical shell
1)	The thickness of this cylindrical shell is less than 1/10 to 1/15 times of its diameter.	The thickness of this cylindrical shell is greater than 1/15 times of its diameter.
2)	The normal stresses are assumed to be uniformly distributed throughout the wall thickness	The normal stresses are not uniformly distributed.
3)	Longitudinal stress is uniformly distributed	Longitudinal stress is not uniformly distributed.
4)	The radial stress induced is very small and is neglected.	A finite value of radial stress is induced.

6.3 Assumptions made in design of thin cylindrical shells

The following assumptions are made while designing thin cylindrical shells.

1) The normal stress distribution over a cross section is uniform.
2) Radial stress is small and hence neglected.
3) Loading is assumed to be uniform by neglecting the self weight of the shell.

4) Cylindrical shell is assumed to be subjected to an internal pressure above the atmospheric pressure.
5) Degradation of wall due to corrosion and chemical reaction of contents is neglected.

6.4 Failure of thin cylindrical shell due to internal pressure

Whenever a thin cylindrical shell is subjected to an internal pressure, its walls are subjected to tensile stresses. If the tensile stresses exceed the permissible limit, the cylinder may fail in any one of the following two ways.

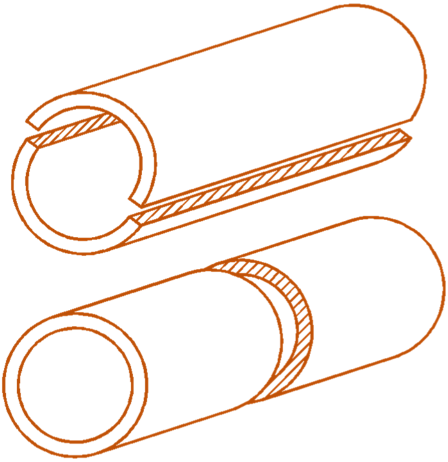


Fig.6.1 Failure of thin cylindrical shell

1) It may split up into two troughs
2) It may split up into two cylinders.

6.5 Stress in cylindrical shell due to internal pressure

Whenever a thin cylindrical shell is subjected to an internal pressure, its walls will be subjected to the following two types of tensile stresses.

1) Circumferential stress or hoop stress
2) Longitudinal stress

1) Circumferential stress or hoop stress

Consider a thin cylindrical shell subjected to an internal pressure as shown in the fig.6.2. As a result of this pressure, the cylinder may split up in to two troughs.

Let, l = Length of the shell

d = Diameter of the shell

t	$=$	Thickness of the shell
p	$=$	Intensity of internal pressure and
f_1	$=$	Circumferential stress induced in the shell

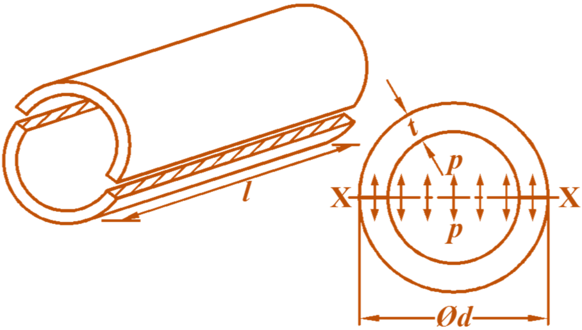


Fig.6.2 Circumferential stress or hoop stress

Let us consider a longitudinal section through the diameter of the shell.

Total force normal to this section

$= $ Intensity of pressure $\times$ Projected area

$= p \times (d \times l) = pdl$

Resisting force offered by this section

$=$ Circumferential stress $\times$ Area of the resisting section

$= f_1(2tl) = 2f_1tl$

Resisting force offered by the section $=$ Total force normal to the section

$$pd\,l = 2f_1tl$$

$$f_1 = \frac{pdl}{2tl} = \boxed{\frac{pd}{2t}}$$

2) Longitudinal stress

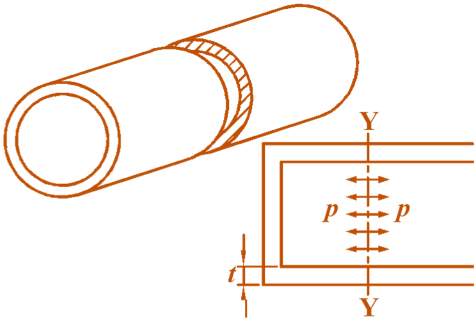


Fig.6.3 Longitudinal stress

Consider a thin cylindrical shell subjected to an internal pressure as shown in the fig.6.3. As a result of this pressure, the cylinder may split up in to two pieces.

Let, l = Length of the shell

d = Diameter of the shell

t = Thickness of the shell

p = Intensity of internal pressure and

f_2 = Longitudinal stress induced in the shell

Let us consider a normal section at equilibrium.

The bursting force acts on one end of the shell

= Intensity of pressure× Area

$= p \times \frac{\pi}{4} d^2$

Resisting force offered by this section

= Longitudinal stress x Area of the resisting section

$= f_2(\pi d t)$

Resisting force offered by the section = Bursting force acts on one end

$$f_2(\pi d t) = p \times \frac{\pi}{4} d^2$$

$$f_2 = \frac{p \times \frac{\pi}{4} d^2}{\pi\ d\ t} = \boxed{\frac{pd}{4t} = \frac{f_1}{2}}$$

6.6. Maximum shear stress

Let f_1 and f_2 be the circumferential stress and longitudinal stress acting at any point on its circumference of a thin cylindrical shell.

The maximum shear stress, $\boxed{f_s = \frac{f_1 - f_2}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t}}$

6.7 Changes in dimensions of a thin cylindrical shell due to an internal pressure

Consider a thin shell subjected to an internal pressure.

Let, f_1 = Circumferential or hoop stress which acts in the direction perpendicular to the axis of the cylinder.

f_2 = Longitudinal stress which acts in the direction of length.

e_1 = Circumferential strain

e_2 = Volumetric strain

V = Volume of cylindrical shell

$1/m$ = Poisson's ratio

$$\delta d = \text{Change in diameter of the shell and}$$
$$\delta l = \text{Change in length of the shell}$$

We know that, circumferential strain, $e_1 = \frac{1}{E}\left(f_1 - \frac{1}{m} f_2\right)$

$$= \frac{1}{E}\left(f_1 - \frac{1}{m}\frac{f_1}{2}\right) \quad \left(\because f_2 = \frac{f_1}{2}\right)$$

$$e_1 = \frac{f_1}{E}\left(1 - \frac{1}{2m}\right)$$

Also circumferential strain, $e_1 = \frac{\delta d}{d}$

$\therefore$ Change in diameter, $\delta d = e_1 \times d = \frac{f_1}{E}\left(1 - \frac{1}{2m}\right) \times d$

Longitudinal strain, $e_2 = \frac{1}{E}\left(f_2 - \frac{1}{m} f_1\right)$

$$= \frac{1}{E}\left(\frac{f_1}{2} - \frac{1}{m} f_1\right)$$

$$e_2 = \frac{f_1}{E}\left(\frac{1}{2} - \frac{1}{m}\right)$$

Also, longitudinal strain, $e_2 = \frac{\delta l}{l}$

$\therefore$ Change in length, $\boldsymbol{\delta l = e_2 \times l =} \boxed{\boldsymbol{\frac{f_1}{E}\left(\frac{1}{2} - \frac{1}{m}\right) \times l}}$

Volume of the cylindrical shell, $V = \frac{\pi}{4} d^2 l$

Taking log on both sides,

$$\log V = \log\frac{\pi}{4} + \log d^2 + \log l$$

$$\log V = \log\frac{\pi}{4} + 2\log d + \log l$$

Taking differential on both sides,

$$\frac{\delta V}{V} = 0 + 2\frac{\delta d}{d} + \frac{\delta l}{l} = 2e_1 + e_2$$

$$= \frac{2 f_1}{E}\left(1 - \frac{1}{2m}\right) + \frac{f_1}{E}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{f_1}{E}\left(2 - \frac{1}{m} + \frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{f_1}{E}\left(\frac{5}{2} - \frac{2}{m}\right)$$

$$\delta V = \frac{f_1}{E}\left(\frac{5}{2} - \frac{2}{m}\right) V$$

Change in volume, $\boxed{\delta V = \frac{f_1}{E}\left(2.5 - \frac{2}{m}\right) V}$

6.8 Thin spherical shells

Consider a thin spherical shell subjected to an internal pressure as shown in the fig.6.4

Let, p = Intensity of internal pressure

d = Internal diameter of the spherical shell

t = Thickness of the spherical shell

As a result of this internal pressure, the shell is likely to be torn away along the centre of the sphere.

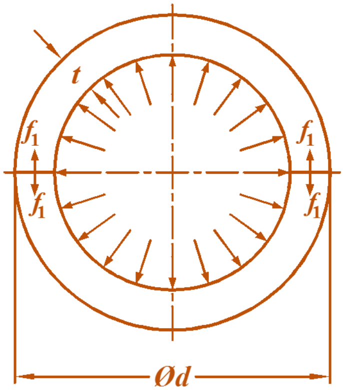


Fig.6.4 Thin spherical shell

Let us consider a section X-X through the centre of the shell.

The bursting force acting along X-X,

F = Intensity of internal pressure × Projected area $= p \times \frac{\pi}{4} d^2$

Let f_1 be the tensile stress induced in the shell at the section X-X.

Resisting force = Tensile stress x Resisting area $= f_1 \times \pi\ d\ t$

But, resisting force = Bursting force

$$f_1 \times \pi d t = p \times \frac{\pi}{4} d^2$$

$$f_1 = \frac{pd}{4t}$$

The tensile stress induced in Y-Y axis, $\boxed{f_2 = f_1 = \frac{pd}{4t}}$

If η is the efficiency of the riveted joint of the spherical shell, then

$$\text{Stress, } f = \frac{pd}{4t\eta}$$

6.9 Change in diameter and volume of thin spherical shell subjected to an internal pressure

Consider a thin spherical shell subjected to an internal pressure as shown in the fig.4.4

Let, p = Intensity of internal pressure
d = Internal diameter of the spherical shell
t = Thickness of the spherical shell
$1/m$ = Poisson's ratio

The tensile stress induced in any direction due to the internal pressure,

$$f_1 = f_2 = f = \frac{pd}{4t}$$

The strain in any direction, $e_1 = e_2 = e = \frac{f_1}{E}\left(1 - \frac{1}{m}\right) = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$

Change in diameter, $\delta d = e \times d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$

Original volume of the shell, $V = \frac{\pi}{6} d^3$

Taking log on both sides,

$$\log V = \log \frac{\pi}{6} + \log d^3 = \log \frac{\pi}{6} + 3 \log d$$

Taking differential on both sides,

$$\frac{\delta V}{V} = 0 + 3\frac{\delta d}{d} = 3e_1 = 3 \times \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$$

Change in volume, $\delta V = 3 \times \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) V$

$$= \frac{3}{4} \times \frac{pd}{tE}\left(1 - \frac{1}{m}\right) \times \frac{\pi}{6} d^3$$

Change in volume, $\boxed{\delta V = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)}$

REVIEW QUESTIONS

1. Distinguish between thin cylinder and thick cylinder. *(Apr.01, Oct.16, Apr.17, Apr.18)*
2. What are the assumptions made while designing a thin cylinder. (*Oct.96)*
3 Derive an expression to determine the hoop stress developed in the wall of a thin cylinder subjected to internal pressure. (*Oct.01)*
4. Derive an expression to determine the longitudinal stress developed in the wall of a thin cylinder subjected to internal pressure.
5. Derive an expression to determine the change in diameter, change in length and change in volume of a thin cylindrical shell subjected to internal pressure.
6. Derive an expression to find out the change in diameter and change in volume of a thin spherical shell subjected to internal pressure.

POINTS TO REMEMBER

Thin cylindrical shell

1) Circumferential or hoop stress, $f_1 = \frac{pd}{2t}$

2) Longitudinal stress, $f_2 = \frac{pd}{4t} = \frac{f_1}{2}$

3) Maximum shear stress, $f_s = \frac{pd}{8t}$

4) Circumferential strain, $e_1 = \frac{f_1}{E}\left(1 - \frac{1}{2m}\right)$

5) Longitudinal strain, $e_2 = \frac{f_1}{E}\left(\frac{1}{2} - \frac{1}{m}\right)$

6) Change in diameter, $\delta d = e_1 \times d = \frac{f_1}{E}\left(1 - \frac{1}{2m}\right) \times d$

7) Change in length, $\delta l = e_2 \times l = \frac{f_1}{E}\left(\frac{1}{2} - \frac{1}{m}\right) \times l$

8) Change in volume, $\delta V = (2e_1 + e_2)V = \frac{f_1}{E}\left(2.5 - \frac{2}{m}\right) V$

(Note : If permissible tensile stress is given, assume the stress as circumferential stress)

Thin spherical shell

9) Tensile stress in any direction, $f_1 = f_2 = \frac{pd}{4t}$

10) If η is the efficiency of joint, then stress, $f = \frac{pd}{4t\eta}$

11) Strain in any direction, $e = \frac{f}{E}\left(1 - \frac{1}{m}\right) = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$

12) Change in diameter, $\delta d = e_1 \times d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$

13) Change in volume, $\delta V = 3e_1 \times V = \frac{\pi pd^4}{8tE}\left(1 - \frac{1}{m}\right)$

14) Volume of spherical shell, $V = \frac{\pi}{6} d^3$

Where,
- d = Diameter of shell (mm)
- t = Thickness of shell (mm)
- l = Length of the cylindrical shell (mm)
- p = Pressure (N/mm^2)
- $1/m$ = Poisson's ratio
- E = Young's modulus (N/mm^2)

SOLVED PROBLEMS

DETERMINATION OF HOOP STRESS AND LONGITUDINAL STRESS

Example : 6.1 *(Apr.01, Apr.15, Apr.17)*

A boiler 2.8m diameter is subjected to a steam pressure of 0.68 N/mm². Find the hoop stress and longitudinal stresses, if the thickness of the boiler plate is 10mm.

Given : Diameter of boiler, $d = 2.8 \text{ m} = 2800 \text{ mm}$

Internal pressure, $p = 0.68 \text{ N/mm}^2$

Thickness of the cylinder, $t = 10 \text{ mm}$

To find : Hoop stress, f_1 2) Longitudinal stress, f_2

Solution :

$$\text{Hoop stress, } f_1 = \frac{p\,d}{2\,t} = \frac{0.68 \times 2800}{2 \times 10} = \boxed{95.2 \text{ N/mm}^2}$$

$$\text{Longitudinal stress, } f_2 = \frac{f_1}{2} = \frac{95.2}{2} = \boxed{47.6 \text{ N/mm}^2}$$

Result : 1) Hoop stress, $f_1 = 95.2 \text{ N/mm}^2$
2) Longitudinal stress, $f_2 = 47.6 \text{ N/mm}^2$

Example : 6.2

A water pipe 1.5m diameter and 15mm wall thickness is subjected to an internal pressure of 1.5 N/mm². Calculate the circumferential and longitudinal stress induced in the pipe.

Given : Diameter of pipe, $d = 1.5 \text{ m} = 1500\text{mm}$

Wall thickness, $t = 15 \text{ mm}$

Internal pressure, $p = 1.5 \text{ N/mm}^2$

To find : 1) Circumferential stress, f_1 2) Longitudinal stress, f_2

Solution :

$$\text{Circumferential stress, } f_1 = \frac{p\,d}{2\,t} = \frac{1.5 \times 1500}{2 \times 15} = \boxed{75 \text{ N/mm}^2}$$

$$\text{Longitudinal stress, } f_2 = \frac{f_1}{2} = \frac{75}{2} = \boxed{37.5 \text{ N/mm}^2}$$

Result : 1) Circumferential stress, $f_1 = 75 \text{ N/mm}^2$
2) Longitudinal stress, $f_2 = 37.5 \text{ N/mm}^2$

Example : 6.3 *(Apr.04)*

A boiler 3m internal diameter is subjected to a boiler pressure of 5bar. Find the hoop and longitudinal stresses, if the thickness of the boiler plate is 14mm.

Given : Diameter of boiler, d = 3 m = 3000 mm

Thickness of plate, t = 10 mm

Steam pressure, p = 5 bar= 5×10^5 N/m^2 = 0.5 N/mm^2

To find : 1) Hoop stress, f_1 2) Longitudinal stress, f_2

Solution :

$$\text{Hoop stress, } f_1 = \frac{p\,d}{2\,t} = \frac{0.5\times 3000}{2\times 10} = \boxed{75 \text{ N/mm}^2}$$

$$\text{Longitudinal stress, } f_2 = \frac{f_1}{2} = \frac{75}{2} = \boxed{37.5 \text{ N/mm}^2}$$

Result : 1) Hoop stress, f_1 = **75 N/mm^2**

2) Longitudinal stress, f_2 = **37.5 N/mm^2**

Example : 6.4 *(Oct.97, Apr.93, Oct.04)*

A gas cylinder of internal diameter 1.5m is 30mm thick. Find the allowable pressure of the gas inside the cylinder if the permissible tensile stress is not to exceed 150N/mm^2.

Given : Internal diameter of gas cylinder, d = 1.5m = 1500 mm

Thickness of the gas cylinder, t = 30 mm

Permissible tensile stress = 150 N/mm^2

To find : 1) Allowable pressure of gas inside the cylinder, p

Solution :

Assume the given tensile stress as hoop stress.

$$\text{We know that, hoop stress, } f_1 = \frac{p\,d}{2\,t}$$

$$150 = \frac{p\times 1500}{2\times 30}$$

$$p = \frac{150\times 2\times 30}{1500} = \boxed{6 \text{ N/mm}^2}$$

Result : Allowable pressure of gas inside the cylinder, p = **6 N/mm^2**

Example : 6.5 *(Oct.03)*

A thin cylindrical shell of 1m diameter is subjected to an internal pressure of 1N/mm^2. Find the suitable thickness of the shell, if the tensile stress in the material is not to exceed 100N/mm^2.

Given : Diameter of the cylindrical shell, d = 1m = 1000 mm

Internal pressure, p = 1 N/mm^2

Allowable stress = 100 N/mm^2

To find : The thickness of the shell, t

Solution :

Assume the given tensile stress as hoop stress.

We know that, hoop stress, $f_1 = \dfrac{p\,d}{2\,t}$

$$100 = \frac{1 \times 1000}{2 \times t}$$

$$t = \frac{1 \times 1000}{2 \times 100} = \boxed{\textbf{5 mm}}$$

Result : The thickness of the shell, t = **5mm**

Example : 6.6

(Oct.03)

A thin cylindrical shell of 2m diameter is subjected to an internal pressure of 1.5 N/mm^2. Find out the suitable thickness of the shell, if the ultimate tensile strength of the plate is 500 N/mm^2. Use a factor of safety of 4.

Given : Diameter of cylinder, d = 2m = 2000 mm

Internal pressure, p = 1.5 N/mm^2

Ultimate stress = 100 N/mm^2

Factor of safety = 4

To find : 1) The thickness of the shell, t

Solution :

$$\text{Working stress} = \frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{500}{4} = 125\ N/mm^2$$

Assume the given tensile stress as hoop stress.

Hoop stress, $f_1 = \dfrac{p\,d}{2\,t}$

$$125 = \frac{1.5 \times 2000}{2 \times t}$$

$$t = \frac{1.5 \times 2000}{2 \times 125} = \boxed{\textbf{12 mm}}$$

Result : 1) The thickness of the shell, t = **12 mm**

Example : 6.7 *(Apr.92)*

A water main 500mm diameter contains water at a pressure head of 100mm. The weight of the water is $10 KN/mm^3$. Find the thickness of the metal required if the permissible stress is 25 N/mm^2.

Given : Diameter of water main, d = 500 mm

Pressure head, h = 100 m = 100×10^3 mm

Permissible stress, f_1 = 25 N/mm^2

Weight of water, w = 10 KN/mm^3 = $\frac{10 \times 10^3}{10^9}$ N/mm^3

To find : 1) The thickness of the metal, t

Solution :

Internal pressure of water, $p = w \times h$

$$= \frac{10 \times 10^3}{10^9} \times 100 \times 10^3 = 1 \text{ N/mm}^2$$

Let the permissible stree be the hoop stress

$$\text{Hoop stress, } f_1 = \frac{p\,d}{2\,t}$$

$$25 = \frac{1 \times 500}{2 \times t}$$

$$t = \frac{1 \times 500}{2 \times 25} = \boxed{\textbf{10 mm}}$$

Result : 1) The thickness of the metal required, t = **10 mm**

Example : 6.8 *(Oct.97, Oct.01, Apr.05, Apr.18)*

A long steel tube 70mm internal diameter and wall thickness 2.5mm has closed ends and subjected to an internal pressure of $10 N/mm^2$. Calculate the magnitude of hoop stress and longitudinal stresses set up in the tube. If the efficiency of the longitudinal joint is 80%, state the stress which is affected and what is its revised value.

Given : Diameter of the steel tube, d = 70 mm

Wall thickness, t = 2.5 mm

Internal pressure, p = $10 N/mm^2$

Efficiency of the joint, η = 80 % = 0.8

To find : 1) Hoop stress, f_1 2) Longitudinal stress, f_2

Solution :

$$\text{Hoop stress, } f_1 = \frac{p\,d}{2\,t} = \frac{10 \times 70}{2 \times 2.5} = 140 \text{ N/mm}^2$$

Longitudinal stress, $f_2 = \frac{f_1}{2} = \frac{140}{2} = 70 \text{ N/mm}^2$

The hoop is affected by the longitudinal joint.

When the efficiency is 0.8,

Revised value of hoop stress, $f_1 = \frac{p\,d}{2\,t\,\eta} = \frac{10 \times 70}{2 \times 2.5 \times 0.8} = \boxed{175 \text{ N/mm}^2}$

Result : 1)Hoop stress, f_1=140 N/mm^2 2)Longitudinal stress, f_2=70 N/mm^2

3) Revised value of hoop stress when the efficiency of longitudinal joint is 80%, $f_1 = 175 \text{ N/mm}^2$

DETERMINATION OF CHANGE IN DIMENSIONS OF THIN CYLINDRICAL SHELLS

Example : 6.9 *(Oct.03, Oct.13, Oct.17)*

A cylindrical shell 3m long and 500 mm in diameter is made up of 20 mm thick plate. If the cylindrical shell is subjected to an internal pressure of $5\,N/mm^2$, find the Result :ing hoop stress, longitudinal stress, changes in diameter, length and volume. Take $E = 2 \times 10^5\,N/mm^2$ and Poisson's ratio = 0.3.

Given :

Length of cylinder, l = 3m = 3000 mm

Internal diameter, d = 500 mm

Metal thickness, t = 20 mm

Internal pressure, p = 5 N/mm^2

Young's modulus, E = 2×10^5 N/mm^2

Poisson's ratio, $1/m$ = 0.3

To find :

1) Hoop stress, f_1

2) Longitudinal stress, f_2

3) Change in diameter, δd

4) Change in length, δl

5) Change in volume, δV

Solution :

Volume of the shell, $V = \frac{\pi}{4} d^2 l = \frac{\pi}{4} \times 500^2 \times 3000 = 589.0486 \times 10^6 \text{ mm}^3$

Circumferential stress, $f_1 = \frac{p\,d}{2\,t} = \frac{5 \times 500}{2 \times 20} = \boxed{62.5 \text{ N/mm}^2}$

The longitudinal stress, $f_2 = \frac{f_1}{2} = \frac{62.5}{2} = \boxed{31.25 \text{ N/mm}^2}$

Circumferential strain, $e_1 = \frac{1}{E}\left[f_1 - \frac{1}{m} f_2\right]$

$$= \frac{1}{2 \times 10^5}[62.5 - 0.3 \times 31.25] = \boxed{\mathbf{2.65625 \times 10^{-4}}}$$

$$\text{Longitudinal strain, } e_2 = \frac{1}{E}\left[f_2 - \frac{1}{m} f_1\right]$$

$$= \frac{1}{2 \times 10^5}[31.25 - 0.3 \times 62.5] = \boxed{\mathbf{6.25 \times 10^{-5}}}$$

$$\text{Change in diameter, } \delta d = e_1 \times d = 2.65625 \times 10^{-4} \times 500 = \boxed{\mathbf{0.1328\ mm}}$$

$$\text{Change in length, } \delta l = e_2 \times l = 6.25 \times 10^{-5} \times 3000 = \boxed{\mathbf{0.1875\ mm}}$$

$$\text{Change in volume, } \delta V = (2e_1 + e_2) \times V$$

$$= \left(2 \times 2.65625 \times 10^{-4} + 6.25 \times 10^{-5}\right) \times 589.0486 \times 10^6$$

$$= \boxed{\mathbf{349.748 \times 10^3\ mm^3}}$$

Result : 1) Hoop stress, $f_1 = \mathbf{62.5\ N/mm^2}$
2) Longitudinal stress, $f_2 = \mathbf{31.25\ N/mm^2}$
3) Change in diameter, $\delta d = \mathbf{0.1328\ mm}$
4) Change in length, $\delta l = \mathbf{0.1875\ mm}$
5) Change in volume, $\delta V = \mathbf{349.748 \times 10^3\ mm^3}$

Example : 6.10

(Apr. 04, Oct.12, Apr.17)

Calculate the increase in volume of a boiler 3m long and 1.5m diameter, when subjected to an internal pressure of $2 N/mm^2$. The thickness is such that the maximum tensile stress is not to exceed $30 N/mm^2$. Take $E = 2.1 \times 10^5 N/mm^2$ and 1/m = 0.28. Also calculate the changes in diameter and length.

Given :

Length of the boiler shell, l	=	3m = 3000 mm
Diameter of the boiler shell, d	=	1.5 m = 1500 mm
Internal pressure, p	=	2 N/mm^2
Maximum tensile stress, f_1	=	30 N/mm^2
Young's modulus, E	=	2.1×10^5 N/mm^2
Poisson's ratio, $1/m$	=	0.28

To find : 1) Increase in volume, δV 2) Change in diameter, δd
3) Change in length, δl

Solution :

$$\text{Longitudinal stress, } f_2 = \frac{f_1}{2} = \frac{30}{2} = 15\ N/mm^2$$

$$\text{Volume of the shell, } V = \frac{\pi}{4} \times d^2 l$$

$$= \frac{\pi}{4} \times 1500^2 \times 3000 = 5.3014 \times 10^9 \text{ mm}^3$$

$$\text{Increase in volume, } \delta V = \frac{f_1}{E}\left[2.5 - 2 \times \frac{1}{m}\right] \times V$$

$$= \frac{30}{2.1 \times 10^5}[2.5 - 2 \times 0.28] \times 5.3014 \times 10^9 = \boxed{\mathbf{1.469 \times 10^6 \ mm^3}}$$

$$\text{Change in diameter, } \delta d = \frac{1}{E}\left[f_1 - \frac{1}{m} \times f_2\right] \times d$$

$$= \frac{1}{2.1 \times 10^5}[30 - 0.28 \times 15] \times 1500 = \boxed{\mathbf{0.1843 \ mm}}$$

$$\text{Change in length, } \delta l = \frac{1}{E}\left[f_2 - \frac{1}{m} \times f_1\right] \times l$$

$$= \frac{1}{2.1 \times 10^5}[15 - 0.28 \times 30] \times 3000 = \boxed{\mathbf{0.0943 \ mm}}$$

Result : 1) Increase in volume, $\boldsymbol{\delta V} = \mathbf{1.469 \times 10^6 \ mm^3}$
2) Change in diameter, $\boldsymbol{\delta d} = \mathbf{0.1843}$ mm
3) Change in length, $\boldsymbol{\delta l} = \mathbf{0.0943 \ mm}$

THIN SPHERICAL SHELLS

Example : 6.11

A vessel in the shape of a thin spherical shell 2m in diameter and 5mm thickness is completely filled with a fluid at a pressure of 0.1 N/mm². Determine the stress induced in the shell material.

Given : Diameter of the shell, d = 2 m = 2000 mm
Thickness of the shell, t = 5 mm
Intensity of pressure, p = 0.1 N/mm^2

To find : 1) Tensile stress, f

Solution :

$$\text{Tensile stress, } f = \frac{pd}{4t} = \frac{0.1 \times 2000}{4 \times 5} = \boxed{\mathbf{10 \ N/mm^2}}$$

Result : Tensile stress, $\boldsymbol{f}$ = **10 N/mm^2**

Example : 6.12

A spherical vessel of 3m diameter is subjected to an internal pressure of 1.5 N/mm². Find the thickness of the plate, if the maximum stress is not to exceed 90 N/mm². The efficiency of the joint is 75%.

Given : Diameter of spherical shell, d = 3 m = 3000 mm
Internal pressure, p = 1.5 N/mm^2
Tensile stress, f = 90 N/mm^2
Efficiency of the joint, η = 75 % = 0.75

To find : The thickness of the plate, t

Solution :

We know that, tensile stress, $f = \dfrac{pd}{4t\eta}$

$$90 = \frac{1.5 \times 3000}{4 \times t \times 0.75}$$

$$t = \frac{1.5 \times 3000}{90 \times 4 \times 0.75} = \boxed{16.667 \text{ mm}}$$

Result : 1) The thickness of the plate, t = **16.667 mm**

Example : 6.13

(Oct.01, Oct.18)

Determine the change in diameter and change in volume of spherical shell 2m in diameter and 12mm thick subjected to an internal pressure of 2N/mm^2. Assume $E = 2 \times 10^5$ N/mm^2 and Poisson's ratio = 0.25.

Given : Diameter of spherical shell, d = 2 m = 2000 mm
Thickness of the shell, t = 12 mm
Internal pressure, p = 2 N/mm^2
Young's modulus, E = 2×10^5 N/mm^2
Poisson's ratio, $1/m$ = 0.25

To find : 1) Change in diameter, δd 2) Change in volume, δV

Solution :

Volume of shell, $V = \dfrac{\pi}{6} \times d^3 = \dfrac{\pi}{6} \times 2000^3 = 4.18879 \times 10^9 \text{ mm}^3$

Strain in the spherical shell, $e = \dfrac{f_1}{E}\left[1 - \dfrac{1}{m}\right] = \dfrac{pd}{4tE}\left[1 - \dfrac{1}{m}\right]$

$$= \frac{2 \times 2000}{4 \times 12 \times 2 \times 10^5}[1 - 0.25] = 3.125 \times 10^{-4}$$

Change in diameter, $\delta d = e \times d = 3.125 \times 10^{-4} \times 2000 = \boxed{0.625 \text{ mm}}$

Change in volume, $\delta V = 3e \times V$

$$= 3 \times 3.125 \times 10^{-4} \times 4.18879 \times 10^9 = \boxed{3.927 \times 10^6 \text{ mm}^3}$$

Result : 1) Change in diameter, δd = **0.625 mm**
2) Change in volume, δV = $\mathbf{3.927 \times 10^6}$ **mm^3**

Example : 6.14 *(Apr.01, Apr.13)*

Determine the depth to which a spherical float 200mm diameter and 6mm thickness have to be immersed in water in order that its diameter is decreased by 0.05mm. Assume $E = 2 \times 10^5 N/mm^2$, 1/m = 0.25 and weight of water = 9810 N/m^3.

Given : Diameter of float, d = 200 mm

Thickness of float, t = 6 mm

Change in diameter, δd = 0.05 mm

Young's modulus, E = 2×10^5 N/mm^2

Poisson's ratio, $1/m$ = 0.25

Weight of water, w = 9810 N/m^3 = 9810×10^{-9} N/mm^3

To find : 1) Depth to which float to be immersed, h

Solution :

Change in diameter of spherical float,

$$\delta d = \frac{pd^2}{4tE}\left[1 - \frac{1}{m}\right]$$

$$0.05 = \frac{p \times 200^2}{4 \times 6 \times 2 \times 10^5}[1 - 0.25]$$

$$p = \frac{0.05 \times 4 \times 6 \times 2 \times 10^5}{200^2} = 8 \text{ N/mm}^2$$

We know that, pressure, $p = w \times h$

$$h = \frac{p}{w} = \frac{8}{9810 \times 10^{-9}} = \boxed{815494.394 \text{ mm}}$$

Result : 1) Depth to which float to be immersed, **h =815494.394 mm**

Example : 6.15 *(Apr.01, Oct.16)*

A spherical shell of 1m internal diameter and 5mm thick is filled with a fluid until its volume increases by $0.2 \times 10^6 mm^3$. Calculate the pressure exerted by the fluid on the shell. Take $E = 2 \times 10^5 N/mm^2$, 1/m = 0.3 for the material.

Given : Internal diameter of spherical shell = 1000 mm

Thickness of spherical shell, t = 5 mm

Increase in volume δV = 0.2×10^6mm^3

Young's modulus, E = E = 2×10^5N/mm^2

Poisson's ratio, $1/m$ = 0.3

To find : 1) Pressure exerted by the fluid, p

Solution :

Volume of shell, $V = \frac{\pi}{6} \times d^3 = \frac{\pi}{6} \times 1000^3 = 5.236 \times 10^8 \text{ mm}^3$

Change in volume of spherical shell, $\delta V = 3 \times \frac{pd}{4tE}\left[1 - \frac{1}{m}\right] \times V$

$$0.2 \times 10^6 = \frac{3 \times p \times 1000}{4 \times 5 \times 2 \times 10^5}[1 - 0.3] \times 5.236 \times 10^8$$

$$p = \frac{0.2 \times 10^6 \times 4 \times 5 \times 2 \times 10^5}{3 \times 1000 \times 0.7 \times 5.236 \times 10^8} = \boxed{\mathbf{0.7276\ N/mm^2}}$$

Result : 1) Pressure exerted by the fluid, p = **0.7276 N/mm²**

PROBLEMS FOR PRACTICE

DETERMINATION HOOP STRESS AND LONGITUDINAL STRESS

1. A pipe 2m diameter and 20mm thick contains a fluid under a pressure of 2N/mm^2. Calculate the hoop stress and longitudinal stress induced in the pipe. ***[Ans: f_1=100 N/mm^2, f_2 =50 N/mm^2]***

2. A thin cylinder of internal diameter 2m is subjected to an internal pressure of 3N/mm^2. Determine the hoop stress and longitudinal stress in the wall of thin cylinder if the thickness of the cylinder is 20mm. ***[Ans: f_1=150 N/mm^2, f_2=75 N/mm^2]***

3. A gas cylinder of internal diameter 1.5m and 30mm thick. Find the allowable pressure inside the cylinder if the tensile stress in the material is not to exceed 100N/mm^2. ***[Ans: p = 4 N/mm^2]***

4. A boiler shell is 1.8m in diameter and 15mm in thickness. The permissible tensile stress in the boiler is not to exceed 70N/mm^2. Determine the allowable working pressure of the boiler. *(Oct.02)* ***[Ans: p=1.1667 N/mm^2]***

5. A thin cylindrical shell of 1.2m and length 2.4m with flat covers is subjected to internal pressure of 3N/mm^2. Find the thickness of shell wall required if the allowable stress is 140N/mm^2. *(Apr.90)* ***[Ans: t=12.857 mm]***

6. A pipe of 500mm diameter is carrying water under a head of 150m. The weight of water is 10KN/m^3. Determine the thickness of the pipe if the allowable stress in the pipe material is 100N/mm^2. ***[Ans: 3.75 mm]***

DETERMINATION OF CHANGE IN DIMENSIONS

8. A cylindrical shell 3m long has 1m internal diameter and 1.5mm metal thickness. Calculate the circumferential and longitudinal stresses induced. Also calculate the changes in diameter, length and volume of the steel if it is subjected to an internal pressure of 1.5N/mm^2. Take E = 2×10^5N/mm^2 and Poisson's ratio =0.3. *(Oct.03, Oct.04, Oct.13)*
   [Ans: δd =2.125 mm, δl=1.5 mm, $\delta V = 1.119195 \times 10^7 mm^3$]

9. A cylindrical thin drum 800mm in diameter and 3m long has a shell thickness of 12mm. If the shell is subjected to an internal pressure of 2.5N/mm^2, determine the change in dimensions. Take E = 2×10^5N/mm^2 and Poisson's ratio=0.25. *(Apr.01)*
   [Ans: δl =0.3125 mm, δd = 0.125 mm, $\delta V = 1.2566 \times 10^5 mm^3$]

9. Calculate the increase in volume enclosed by a boiler shell 2.5m long, 1m diameter, when it is subjected to an internal pressure of 150N/mm^2. The wall thickness is such that the maximum tensile stress in the shell is 2150N/mm^2 under this pressure. E=200KN/mm^2, $1/m$ =0.3
   [Ans: $\delta V = 40.1045 \times 10^6 mm^3$]

10. Calculate the increase in volume enclosed by a boiler shell 2.4m long and 1m diameter when it is subjected to an internal pressure of 1.6N/mm^2. The wall thickness is such that the permissible stress is 250N/mm^2 under this pressure. E = 2×10^5N/mm^2, $1/m$ =0.28.
    [Ans: $4.571 \times 10^6 mm^3$]

THIN SPHERICAL SHELLS

11. A thin spherical shell 2m in diameter is subjected to an internal pressure of 1N/mm^2. Find the thickness of the plate required if the permissible stress in the plate material is not to exceed 80N/mm^2. Take the joint efficiency is 75%. ***[Ans: t=8.333 mm]***

12. Calculate the change in diameter and change in volume of spherical shell 1m in diameter and 10mm thick, when it is subjected to an internal pressure of 1.2N/mm^2. Assume E = 2×10^5N/mm^2 and 1/m = 0.25
    [Ans: δd=0.1125 mm, $\delta V = 1.767 \times\times 10^5 mm^3$]

13. To what depth would a steel float 300mm in diameter and 5mm thick have to be immersed in water in order that its diameter is decreased by 0.04mm? E for steel = 2×10^5N/mm^2, $1/m$ = 0.25 and weight of water = 9810N/m^3. ***[Ans: h=241590.214 mm]***

14. Calculate the depth to which a copper float 250mm diameter and 3mm thick have to be sunk in water so that the diameter is reduced by 0.03mm. Take $E = 1 \times 10^5 N/mm^2$ and $1/m = 0.27$. The specific weight of water is $9.81 KN/m^3$. ***[Ans: h=80.432 m]***

15. A spherical shell of 1.5m diameter has 10mm thick wall. Determine the pressure that can increase its volume by $1.8 \times 10^6 mm^3$. Take $E = 2 \times 10^5 N/mm^2$, $1/m = 0.3$. ***[Ans: $p = 2.587\ N/mm^2$]***

Unit – IV

Chapter 7. SHEAR FORCE AND BENDING MOMENT DIAGRAMS

7.1 Beam

Beam is a structural member which is subjected to a system of external forces acting perpendicular to its axis.

Whenever a beam is subjected to vertical loads it bends due to the action of the load. The amount with which a beam bends, depends upon the type of loads, length of the beam, elasticity of the beam and the type of beam.

7.2 Classification of beams

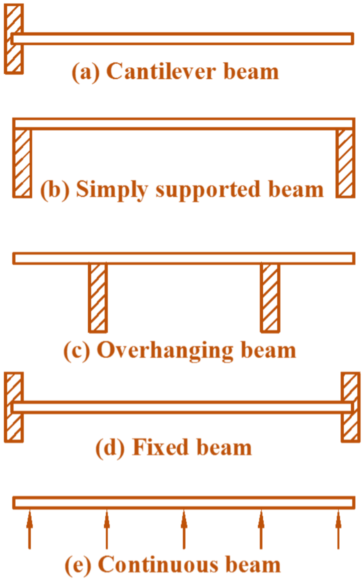


Fig.7.1 Types of beam

The beams are generally classified according to the supporting conditions as follows.

1) Cantilever beam 2) Simply supported beam 3) Overhanging beam
4) Fixed beam 5) Continuous beam

1) Cantilever beam

If one end of the beam is fixed and the other end is free, then such type of beam is called cantilever beam.

2) Simply supported beam

If both the ends of the beam are made to rest freely on supports, then such type of beam is called simply supported beam.

3) Overhanging beam

If the ends of the beam are extended beyond the supports in a simply supported beam, then it is called as overhanging beam.

4) Fixed beam

If both the ends of a beam are rigidly fixed or built into the walls, then it is called fixed beam.

5) Continuous beam

If a beam is provided with more than two supports, then it is called as continuous beam.

7.3 Types of loading

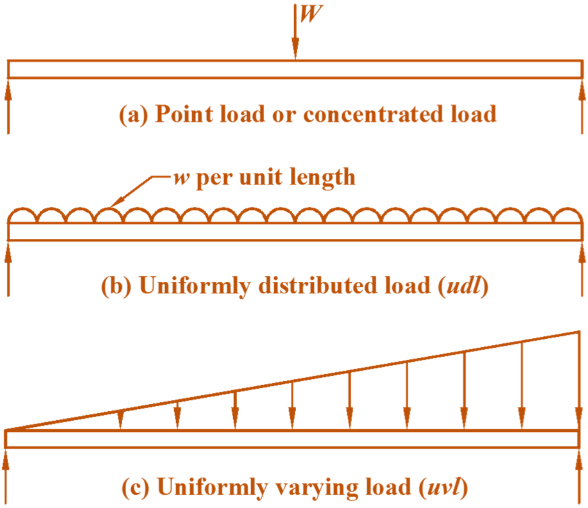


Fig.7.2 Types of loading

A beam may be subjected to the following types of loads.

1) Point load or concentrated load.
2) Uniformly distributed load (udl).
3) Uniformly varying load.

1) Point load or concentrated load

If a load is acting exactly at a point in the beam then it is called point load or concentrated load.

2) Uniformly distributed load (udl)

If a load is spread over the beam in such a way that its magnitude is same for each and every unit length of the beam, then it is called uniformly distributed load (udl).

3) Uniformly varying load

If a load is spread over the beam in such a way that its magnitude is gradually varying within an unit length of the beam, then it is called uniformly varying load.

7.4 Shear force

The shear force at a cross section of beam may be defined as the unbalanced vertical forces to the left or right of the section. It is denoted as **SF.**

7.5 Bending moment

The bending moment at a cross section of a beam may be defined as the algebraic sum of the moments of the forces to the left or right of the section. It is denoted as **BM.**

7.6 Sign conventions.

Shear force

Consider a section X–X perpendicular to the axis of the beam. All the upward forces to the left of the section and all the downward forces to the right of the section cause positive shear force.

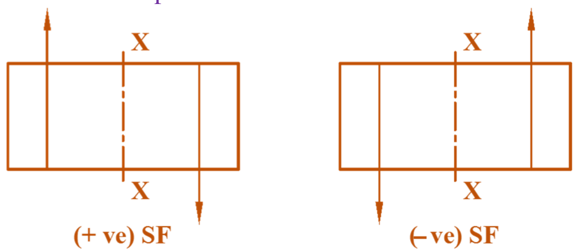


Fig.7.3 Sign convention of shear force

All the upward forces to the right of the section and all the downward forces to the left of the section cause negative shear force.

Bending moment

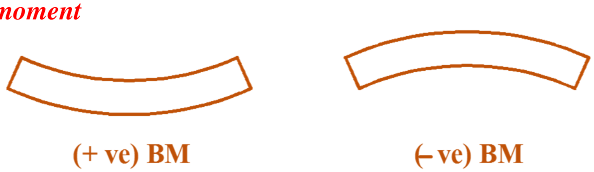


Fig.7.4 Sign convention of bending moment

If the bending moment at a section is such a way that it tends to bend the beam at that point to a curvature having concavity at the top is taken as positive bending moment. The positive bending moment is often called as ***sagging*** moment. The right anti–clockwise moment and left clockwise moment are taken as positive moment.

If the bending moment at a section is such a way that it tends to bend the beam at that point to a curvature having convexity at the top is taken as negative bending moment. The negative bending moment is often called as ***hogging*** moment. The right clockwise moment and left anti–clockwise moment are taken as negative moment.

7.7 Relationship between load, shear force and bending moment

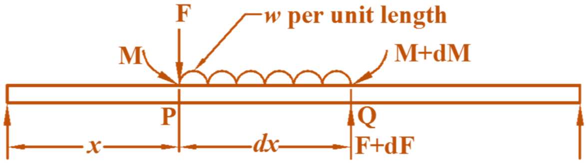


Fig.7.5 Relationship between load, SF and BM.

Consider a beam carrying a udl of w per unit length. Let us consider a portion PQ of length dx and at a distance x from the left hand support of the beam as shown in fig.7.5. Total load acting on the beam length PQ is equal to $w.dx$

Let, shear force at P = F, and shear force at Q = $F + dF$

Bending moment at P = M and Bending moment at Q = $M + dM$

For equilibrium condition, $\Sigma SF = 0$

$$F + w.dx - (F + dF) = 0$$

$$dF = w.dx$$

$$\boxed{\frac{dF}{dx} = w} \text{ ----------------(1)}$$

The above relation shows that ***the rate of change of shear force is the rate of loading per unit length of the beam.***

The force system in fig.7.5 may be simplified as shown in fig.7.5(a). ***The total udl is considered to act as a point load at the middle of the span over which it acts.***

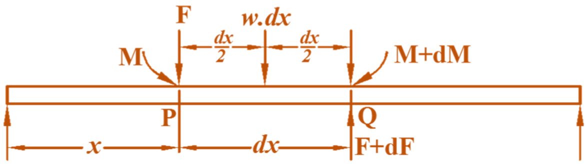


Fig.7.5(a) Relationship between load, SF and BM.

Taking moment of forces and couples about P,

$$-(M+dM)+M-w.dx\frac{dx}{2}+(F+dF)dx=0$$

$$-M-dM+M-w\frac{(dx)^2}{2}+F.dx+dF.dx=0$$

Neglecting the small quantities

$$-dM+F.dx=0$$

$$dM=F.dx$$

$$\boxed{\frac{dM}{dx}=F}$$

The above relation shows that ***the rate of change of bending moment about a section is equal to the SF at that section.***

For maximum bending moment, $\frac{dM}{dx}=0$ ***i.e.*** $F=0$.

Therefore, the bending moment is maximum at a section where shear force is zero.

7.8 Standard cases of loading

1) Cantilever beam with a point load at its free end

Consider a cantilever AB of length l and carrying a point load W at its free end B as shown in the fig.7.6. Consider a section X–X at a distance x from the free end.

Shear force :

SF at B $=+W$ *(Plus sign due to right downward)*

SF at X–X $=+W$ *($\because$ There is no load between B and X–X)*

SF at A $=+W$ *($\because$ There is no load between X–X and A)*

Bending moment :

Bending moment at X–X $=-W\ x$ (Minus sign due to hogging)

The bending moment at any section is proportional to the distance of that section from the free end.

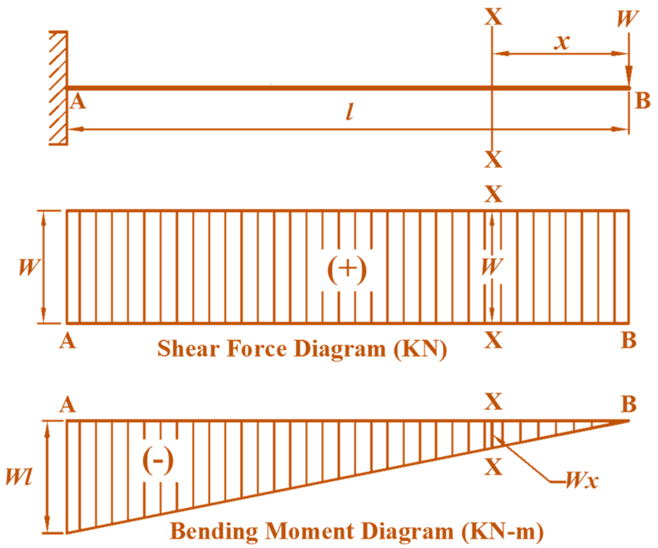


Fig.7.6 Cantilever with a point load at its free end

At B, $x = 0$; $\therefore$ BM $= -W \times 0 = 0$

At A, $x = l$; $\therefore$ BM $= -W \times l = -Wl$

2) Cantilever beam with uniformly distributed load

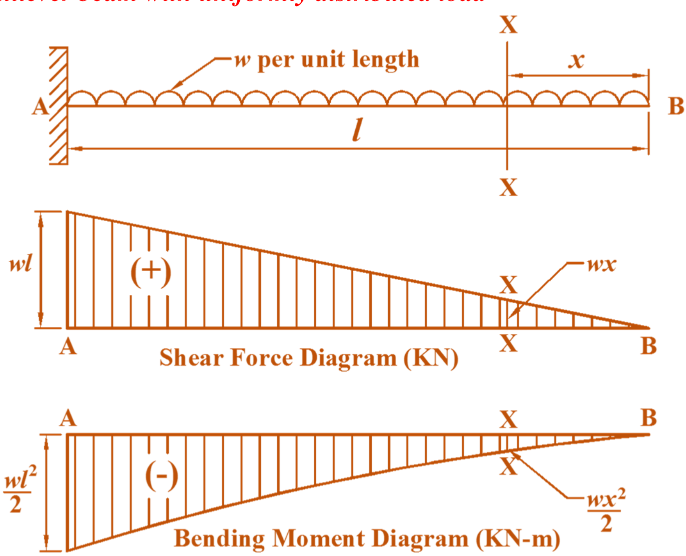


Fig.7.7 Cantilever with uniformly distributed load

Consider a cantilever AB of length l and carrying a uniformly distributed load w per unit length over the entire length of the beam as shown in the fig.7.7. Consider a section X–X at a distance x from the free end.

SF at X–X = $+wx$ *(∵ Plus sign due to right downward)*

Bending moment at X–X $= -wx \times \frac{x}{2} = -\frac{wx^2}{2}$ *(Hogging moment)*

From the above two equations, the shear force varies according to a ***straight line law***, while the bending moment varies according to ***parabolic law***.

Shear force :

At B, $x = 0;\quad SF = 0$

At X–X , $x = x;\quad SF = wx$

At A, $x = l;\quad SF = wl$

Bending moment :

At B, $x = 0;\quad \mathrm{BM} = 0$

At X-X, $x = x;\quad \mathrm{BM} = -\frac{wx^2}{2}$

At A, $x = l;\quad \mathrm{BM} = -\frac{wl^2}{2}$

3) Simply supported beam with point load at the mid span

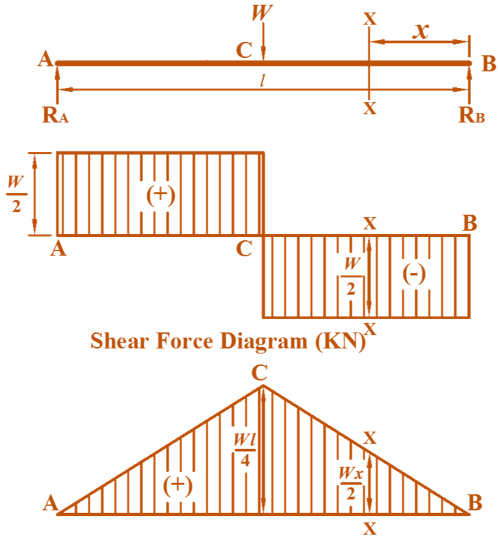


Fig.7.8 Simply supported beam with point load at mid span.

Consider a simply supported beam AB of length l and carrying a point load W at its mid point C as shown in the fig.7.8.

Let R_A and R_B be the reactions at the supports A and B. Taking moment about the support A,

$$R_B \times l = W \times \frac{l}{2}$$

$$R_B = \frac{Wl}{2l} = \frac{W}{2}$$

But, $R_A + R_B = W$

$$R_A = W - \frac{W}{2} = \frac{W}{2}$$

Consider a section X–X at a distance x from B.

Shear force :

Shear force at B $= -\frac{W}{2}$ *($\because$ Minus sign due to right upward)*

Shear force at X–X $= -\frac{W}{2}$

Shear force remains constant between B and C and is equal to $-\frac{W}{2}$

Shear force at C $= -\frac{W}{2} + W = \frac{W}{2}$

Shear force remains constant between C and A and is equal to $\frac{W}{2}$

Shear force at A $= +\frac{W}{2}$

Bending moment :

Bending moment at X–X $= +\frac{W}{2}x$ *($\because$ Plus due to sagging)*

At B, $x = 0$; BM $= 0$

At C, $x = \frac{l}{2}$; BM $= +\frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$

At A, BM $= 0$

4) Simply supported beam with uniformly distributed load over entire span

Consider a simply supported beam AB of length l and carrying a udl of w per unit length, over the entire length as shown in the fig.7.9.

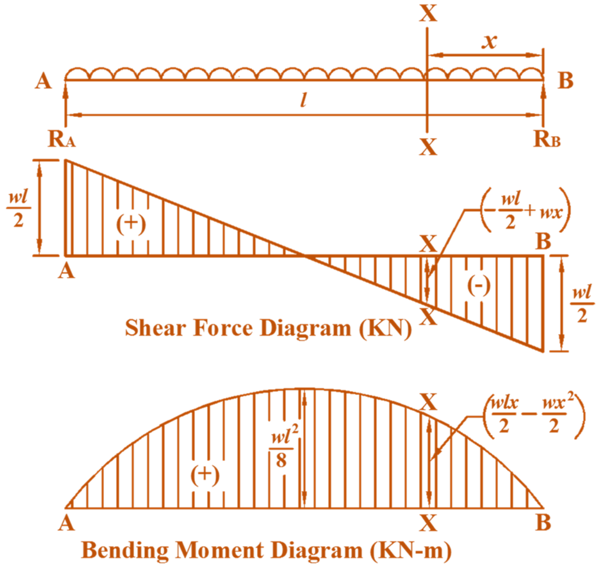


Fig.7.9 Simply supported beam with udl over the entire length

Let R_A and R_B be the reactions at the supports A and B. Taking moment about the support A,

$$R_B \times l = wl \times \frac{l}{2}$$

$$R_B = \frac{wl^2}{2l} = \frac{wl}{2}$$

But, $\quad R_A + R_B = wl$

$$R_A = wl - \frac{wl}{2} = \frac{wl}{2}$$

Consider a section X–X at a distance x from B.

Shear force :

Shear force at B $= -\dfrac{wl}{2}$ *($\because$ Minus sign due to right upward)*

Shear force at X–X $= -\dfrac{wl}{2} + wx$

Shear force at C $\left(x = \dfrac{l}{2}\right) = -\dfrac{wl}{2} + \dfrac{wl}{2} = 0$

Shear force at A($x = l$) $= -\dfrac{\boldsymbol{wl}}{\boldsymbol{2}} + \boldsymbol{wl} = \dfrac{\boldsymbol{wl}}{\boldsymbol{2}}$

Bending moment :

$$\text{Bending moment at X–X} = R_B x - wx\frac{x}{2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

At B, $x = 0$; BM $= 0$

At C, $\left(x = \frac{l}{2}\right)$; $\text{BM} = \frac{wl}{2} \times \frac{l}{2} - \frac{w}{2}\left(\frac{l}{2}\right)^2 = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$

At B $(x = l$ $\text{BM} = \frac{wl^2}{2} - \frac{wl^2}{2} = 0$

7.9 Hints for calculating SF and BM at a section

1) Calculation of shear force

(a) Consider a section at which shear force is to be calculated

(b) Consider all the loads which act either to the right or to the left of the section.

(c) Find the algebraic sum of the loads by using sign conventions for shear force. This sum gives the value of shear force at that section.

2) Calculation of bending moment

(a) Consider a section at which bending moment is to be calculated

(b) Consider all the loads which act either to the right or to the left of the section.

(c) Take moment of these loads about that section.

(d) Find the algebraic sum of the moments by using sign convention of bending moment. This sum gives the value of bending moment at that section.

(e) A concentrated load which passes through the considered section have zero moment about that section.

(f) The bending moment at the free end of a cantilever beam and the two supports of SSB will be zero.

(g) The udl is considered to act as a point load at the middle of the span over which it acts.

7.10 Hints for drawing SF and BM diagrams

1) Shear force diagram

(a) If there is a point load at a section, the shear force line will suddenly increase or decrease by a vertical line.

(b) If there is no load between any two sections, the shear force will remain constant and shear force line will be a horizontal straight line parallel to the base line.

(c) If there is a uniformly distributed load between two sections, the shear force line will be an inclined straight line.

(d) When a point load acts along with a uniformly distributed load, the SF diagram will have two inclined lines separated by a vertical straight line at a point where point load acts.

(e) In a cantilever beam, the maximum shear force will occur at the fixed end. In a simply supported beam, the maximum shear force will occur at the supports.

2) Bending moment diagram

(a) The bending moment line in a region between two point loads will be an inclined straight line.

(b) The bending moment line in a region of udl will be a parabolic line.

7.11 Point of contraflexure

Overhanging beam can be considered as combination of simply supported beam and a cantilever beam. We know that the bending moment in the simply supported beam is positive, whereas the bending moment in the cantilever beam is negative. It is thus known that in an overhanging beam, there will be a point, where the bending moment will change sign from positive to negative and *vice versa*. Such a point, where the bending moment changes sign, is known as a ***point of contraflexure.***

REVIEW QUESTIONS

1. Classify the beams on the basis of the types of supports used. *(Oct.02, Apr.17)*
2. Show the different types of beams with the help of line sketches. *(Oct.01, Oct.02, Apr.04, Apr.17)*
3. Define point load and uniformly distributed load. *(Oct.96, Oct.04, Apr.05, Apr.18)*
4. Define shear force and bending moment at a section along the length of a loaded beam. *(Apr.01, Oct.04, Oct.01, Apr.17)*
5. Write down the relationship between load, shear force and bending moment. *(Oct.16, Oct.17)*
6. Write sign conventions for shear force and bending moment. *(Oct.17)*
7. Distinguish between sagging and hogging moments. *(Oct.04)*
8. Draw the SFD and BMD for a cantilever beam of length l and subjected to a point load W at the free end. *(Oct.01)*
9. Draw the SFD and BMD for a cantilever beam subjected to a udl of w per unit length over the entire length l.
10. Draw the SFD and BMD for a simply supported beam subjected to a point load W at its mid point.
11. Draw the SFD and BMD for a simply supported beam subjected to a udl of w per unit length, over the entire length.
12. State and explain the significance of point of contraflexure. *(Apr.05)*

SOLVED PROBLEMS

CANTILEVER BEAMS

Example : 7.1 *(Apr.01)*

A cantilever 2m long carries a point load of 3KN at its free end and another point load of 2KN at a distance of 0.5m from the free end. Draw the shear force and bending moment diagram.

Solution :

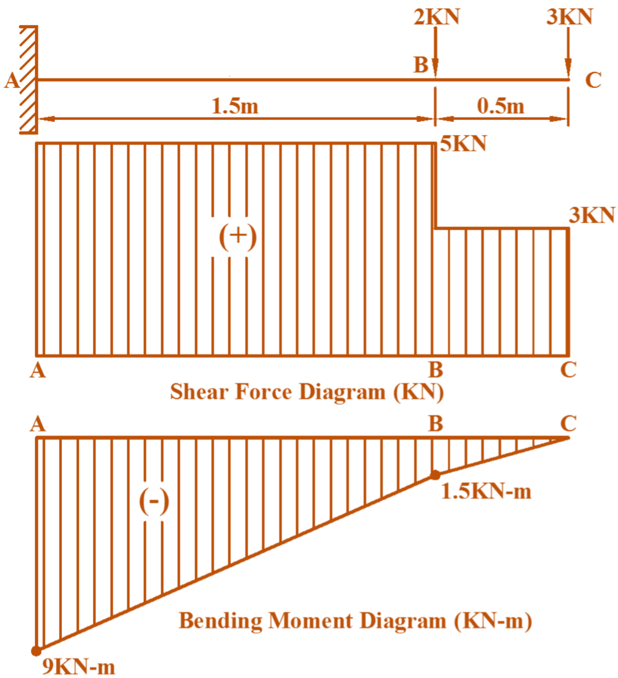


Fig.P7.1 SF and BM diagram [Example 7.1]

Calculation for shear force :

Shear force at C = +3 KN

Shear force at B = +3 + 2 = 5 KN

Shear force at A = +5 KN *(There is no load between B & A)*

Calculation for bending moment :

Bending moment at C = 0

Bending moment at B = $-3 \times 0.5 = -1.5$ KN–m

Bending moment at A = $-3 \times 2 - 2 \times 1.5 = -9$ KN–m

Example : 7.2

A cantilever of span 10 m carries point loads of 6KN and 8KN at 4m and 7m from the fixed end. Draw SF and BM diagram.

Solution :

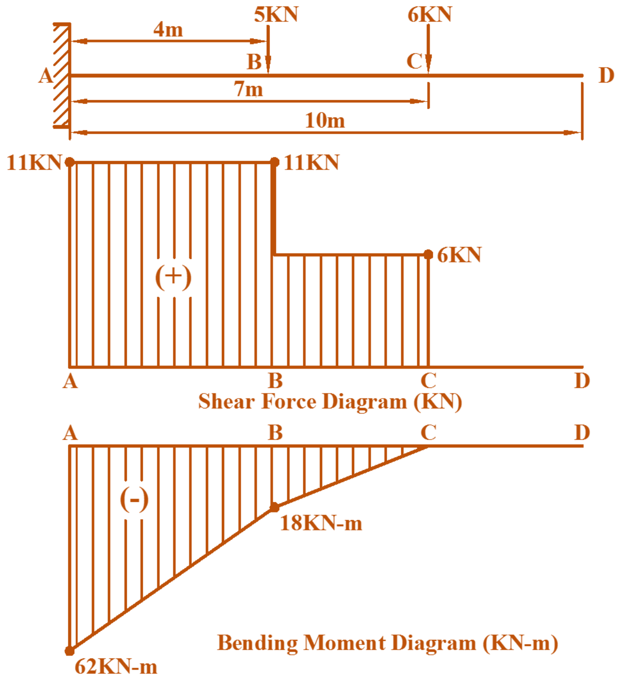


Fig.P7.2 SF and BM diagram [Example 7.2]

Calculation for shear force :

SF at D = 0 (∵*There is no load)*

SF at C = + 6 KN

SF at B = + 6 + 5 = +11 KN

SF at A = + 11 KN (∵ *There is no load between B and A)*

Calculation for bending moment :

BM at D = 0

BM at C = 0

BM at B = – 6 × 3 = –18 KN–m

BM at A = – 6 × 7 – 5 × 4 = – 62 KN–m

Example : 7.3 *(Apr.89, Oct.96, Oct.03, Oct.12, Apr.17)*

A cantilever 4m long carries a udl of 30KN/m over half of its length adjoining the free end. Draw SF and BM diagrams.

Solution :

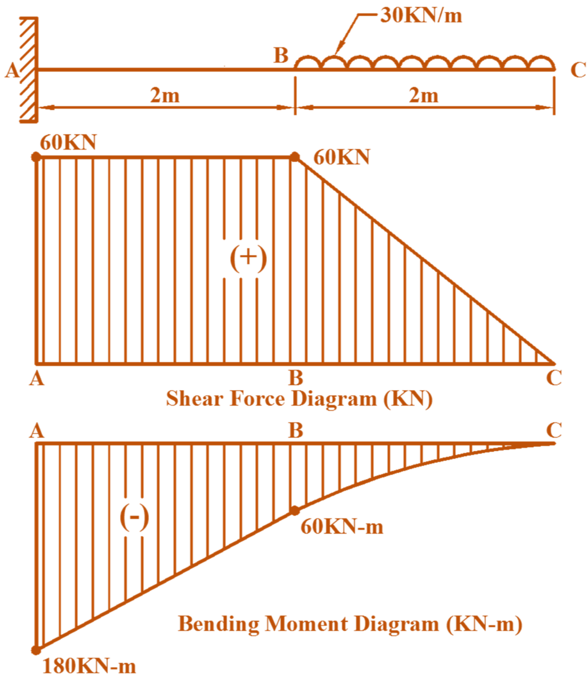


Fig.P7.3 SF and BM diagram [Example 7.3]

Calculation for shear force :

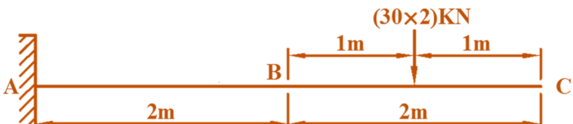
SF at C $=$ 0 ($\because$ *There is no load)*

SF at B $= +30 \times 2 = +60$ KN

SF at A $= +60$ KN ($\because$ *There is no load between B and A)*

Calculation for bending moment :

Note : udl is assumed as a point load acting at the middle of udl span.



$$\text{BM at C} = 0$$

$$\text{BM at B} = -30 \times 2 \times \left(\frac{2}{2}\right) = -60 \text{ KN-m}$$

$$\text{BM at A} = -30 \times 2 \times \left(2 + \frac{2}{2}\right) = -180 \text{ KN-m}$$

Example : 7.4 *(Oct.88, Apr.92, Oct.03)*

A cantilever of 2m long carries a point load of 20KN at 0.8mm from the fixed end and another point load of 5KN at the free end. In addition a udl of 15KN/m is spread over the entire length of the cantilever. Draw the SF and BM diagrams.

Solution :

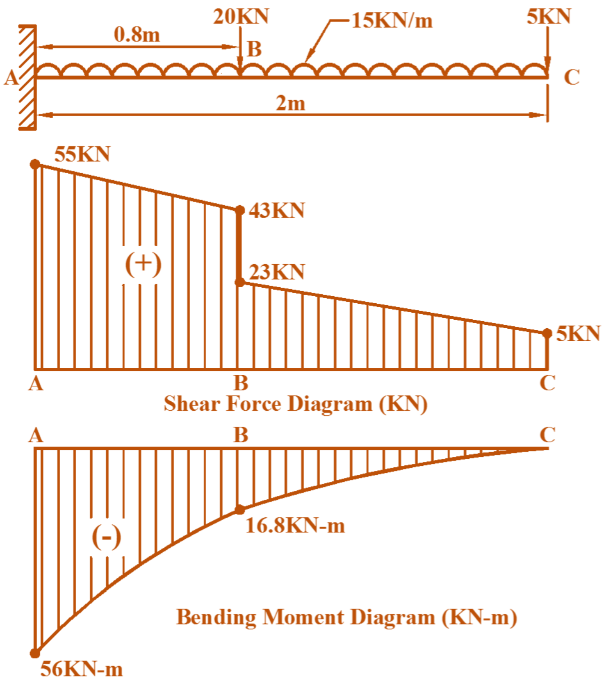


Fig.P7.4 SF and BM diagram [Example 7.4]

Calculation for shear force :

$$\text{SF at C} = +5 \text{ KN}$$

$$\text{SF at B (Due to udl)} = +5 + (15 \times 1.2) = +23 \text{ KN}$$

$$\text{SF at B (Due to point load)} = +23 + 20 = +43 \text{ KN}$$

$$\text{SF at A} = +43 + (15 \times 0.8) = +55 \text{ KN}$$

Calculation for bending moment :

$$\text{BM at C} = 0$$

$$\text{BM at B} = -(5 \times 1.2) - \left(15 \times 1.2 \times \frac{1.2}{2}\right) = -16.8 \text{ KN–m}$$

$$\text{BM at A} = -(5 \times 2) - \left(15 \times 2 \times \frac{2}{2}\right) - (20 \times 0.8) = -56 \text{ KN–m}$$

Example : 7.5 *(Oct.92, Apr.13)*

Draw the shear force and bending moment diagrams for the loaded beam shown in the fig.P7.5

Solution :

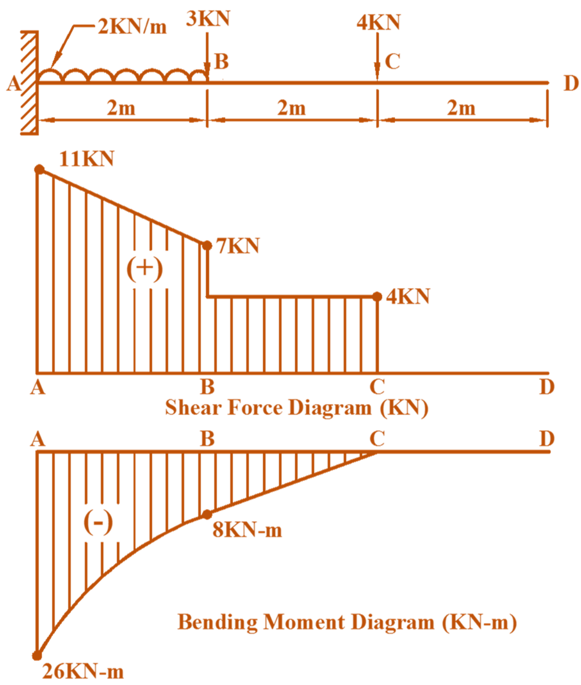


Fig.P7.5 SF and BM diagram [Example 7.5]

Calculation for shear force :

$$\text{SF at D} = 0$$

$$\text{SF at C} = +4 \text{ KN}$$

$$\text{SF at B} = +4 + 3 = +7 \text{ KN}$$

$$\text{SF at A} = +7 + (2 \times 2) = +11 \text{ KN}$$

Calculation for bending moment :

BM at D $=$ 0

BM at C $=$ 0

BM at B $= -4 \times 2 = -8$ KN–m

BM at A $= -(4 \times 4) - (3 \times 2) - \left(2 \times 2 \times \frac{2}{2}\right) = -26$ KN–m

Example : 7.6 *(Apr.93)*

Draw the shear force and bending moment diagrams for the loaded beam shown in the fig.P7.6

Solution :

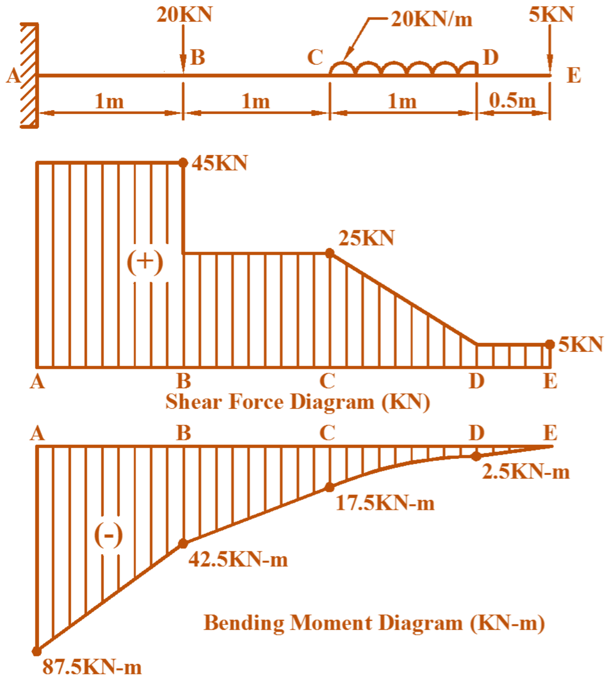


Fig.P7.6 SF and BM diagram [Example 7.6]

Calculation for shear force :

SF at E $= +5$ KN

SF at D $= +5$ KN

SF at C $= +5 + (20 \times 1) = +25$ KN

SF at B $= +5 + (20 \times 1) + 20 = +45$ KN

SF at A $= +45$ KN ($\because$ *There is no load between B & A)*

Calculation for bending moment :

$$\text{BM at E} = 0$$

$$\text{BM at D} = -5 \times 0.5 = -2.5 \text{ KN–m}$$

$$\text{BM at C} = -(5 \times 1.5) - \left(20 \times 1 \times \frac{1}{2}\right) = -17.5 \text{ KN–m}$$

$$\text{BM at B} = -(5 \times 2.5) - \left[20 \times 1 \times \left(1 + \frac{1}{2}\right)\right] = -42.5 \text{ KN–m}$$

$$\text{BM at A} = -(5 \times 3.5) - \left[20 \times 1 \times \left(2 + \frac{1}{2}\right)\right] - 20 \times 1) = -87.5 \text{ KN–m}$$

SIMPLY SUPPORTED BEAMS

Example : 7.7 *(Apr.97)*

A simply supported beam 5m span carries a point load of 20KN at 2m from left support. Draw the shear force and bending moment diagrams.

Solution :

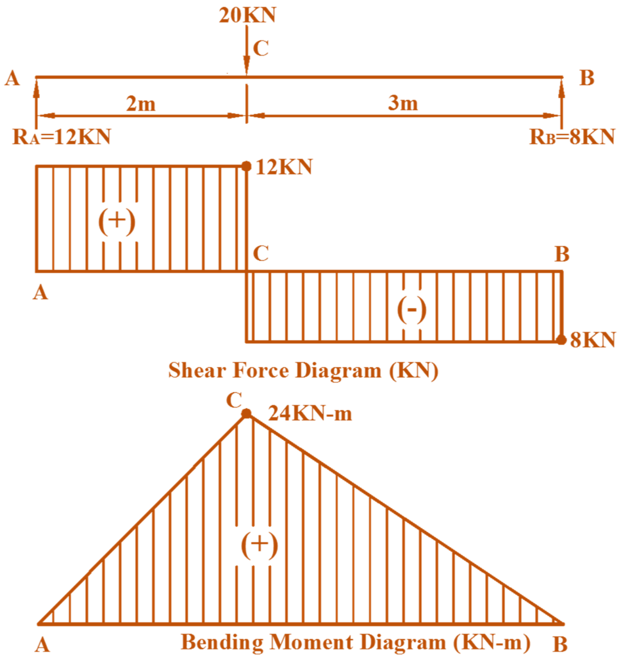


Fig.P7.7 SF and BM diagram [Example 7.7]

Taking moment about A,

$$R_B \times 5 = 20 \times 2$$

$$R_B = \frac{40}{5} = 8 \text{ KN}$$

But, $R_A + R_B \ = \ 20$ KN

$R_A \ = \ 20 - R_B \ = \ 20 - 8 \ = \ 12$ KN

Calculation for shear force :

Shear force at B $=$ -8 KN ($\because$ Minus sign due to right upward)

Shear force at C $=$ $-8 + 20 = +12$ KN

Shear force at A $=$ $+12$ KN ($\because$ There is no load between C and A)

Calculation for bending moment :

Bending moment at B $=$ 0

Bending moment at C $=$ $+8 \times 3 = +24$ KN–m

Bending moment at A $=$ $+(8 \times 5) - (20 \times 2) = 0$

Example : 7.8 *(Oct.04)*

A simply supported beam of 10m span is loaded with point loads of 20KN, 40KN at 2m and 8m from left support respectively. Draw the shear force and bending moment diagrams.

Solution :

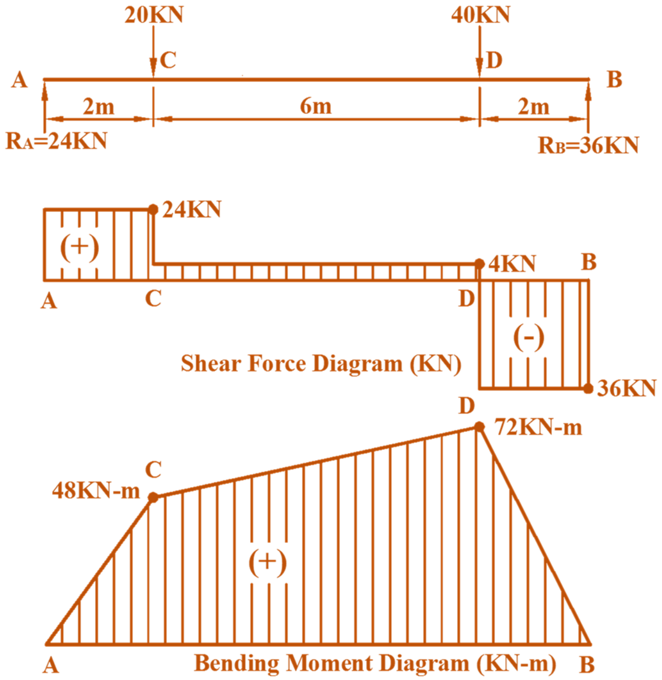


Fig.P7.8 SF and BM diagram [Example 7.8]

Taking moment about A,

$$R_B \times 10 = (40 \times 8) + (20 \times 2) = 360$$

$$R_B = \frac{360}{10} = 36 \text{ KN}$$

$$\text{But, } R_A + R_B = 60 \text{ KN}$$

$$R_A = 60 - R_B = 60 - 36 = 24 \text{ KN}$$

Calculation for shear force :

SF at B = – 36 KN

SF at D = – 36 + 40 = +4 KN

SF at C = + 4 + 20 = 24 KN

SF at A = + 24 KN ($\because$ There is no load between C and A)

Calculation for bending moment :

BM at B = 0

BM at D = $+36 \times 2 = +72$ KN–m

BM at C = $+(36 \times 8) - (40 \times 6) = +48$ KN–m

BM at A = 0

Example : 7.9 *(Apr.88, Oct.03, Oct.16)*

A simply supported beam of effective span 6m carries three point loads of 30KN, 25KN and 40KN at 1m, 3m and 4.5m respectively from the left support. Draw the SF and BM diagrams. Also indicate the maximum value of bending moment.

Solution :

Taking moment about A,

$$R_B \times 6 = (30 \times 1) + (25 \times 3) + (40 \times 4.5) = 285$$

$$R_B = \frac{285}{6} = 47.5 \text{ KN}$$

$$\text{But, } R_A + R_B = 30 + 25 + 40 = 95 \text{ KN}$$

$$R_A = 95 - R_B = 95 - 47.5 = 47.5 \text{ KN}$$

Calculation for shear force :

SF at B = – 47.5 KN

SF at E = – 47.5 + 40 = – 7.5 KN

SF at D = – 7.5 + 25 = + 17.5 KN

SF at C = + 17.5 + 30 = + 47.5 KN

SF at A = + 47.5 KN ($\because$ *There is no load between C and A)*

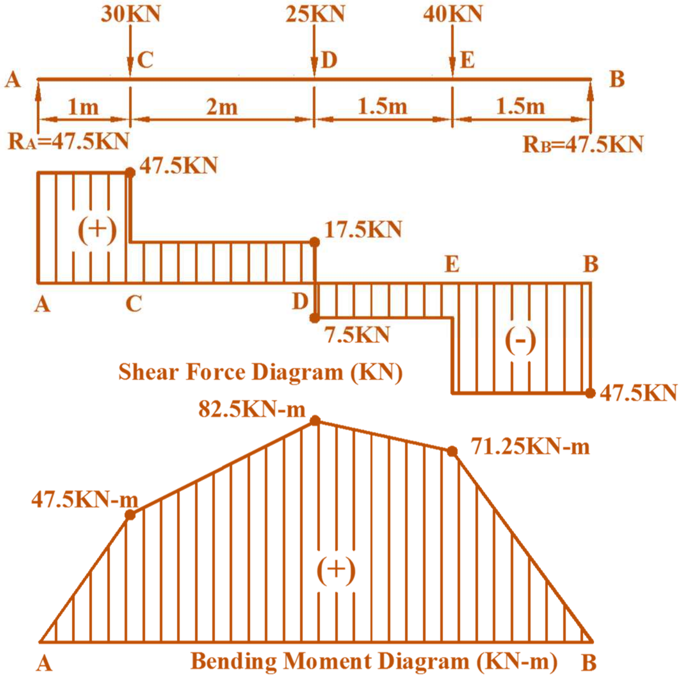


Fig.P7.9 SF and BM diagram [Example 7.9]

Calculation for bending moment :

BM at B = 0

BM at E = $+47.5 \times 1.5 = +71.25$ KN–m

BM at D = $+(47.5 \times 3) - (40 \times 1.5) = +82.5$ KN–m

BM at C = $+(47.5 \times 5) - (40 \times 3.5) - (25 \times 2) = +47.5$ KN–m

BM at A = 0

Example : 7.10 *(Oct.96, Oct.17)*

A beam is freely supported over a span of 8m. It carries a point load of 8KN at 2m from the left hand support and a udl of 2KN/m run from the centre up to the right hand support. Construct the SF and BM diagram.

Solution :

Taking moment about A,

$$R_B \times 8 = \left[(2 \times 4) \times \left(4 + \frac{4}{2}\right)\right] + (8 \times 2) = 64$$

$$R_B = \frac{64}{8} = 8 \text{ KN}$$

But, $R_A + R_B = (2 \times 4) + 8 = 16$ KN

$R_A = 16 - R_B = 16 - 8 = 8$ KN

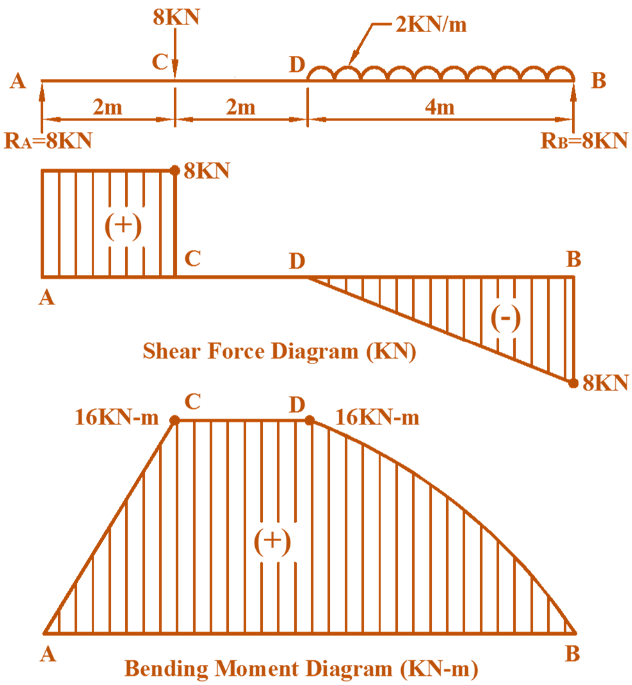


Fig.P7.10 SF and BM diagram [Example 7.10]

Calculation for shear force :

SF at B $= -8$ KN

SF at D $= -8 + (2 \times 4) = 0$ KN

SF at C $= 0 + 8 = +8$ KN

SF at A $= 8$ KN ($\because$ There is no load between C and A)

Calculation for bending moment :

BM at B $= 0$

BM at D $= +(8 \times 4) - \left(2 \times 4 \times \frac{4}{2}\right) = +16$ KN–m

BM at C $= +(8 \times 6) - \left[2 \times 4 \times \left(2 + \frac{4}{2}\right)\right] = +16$ KN–m

BM at A $= 0$

Example : 7.11 *(Oct.88, Apr.93, Oct.01, Apr.14, Oct.14, Apr.17)*

A simply supported beam of length 6m carries a udl of 20KN/m throughout its length and a point load of 30KN at 2m from the right support. Draw the shear force and bending moment diagram. Also find the position and magnitude of maximum bending moment.

Solution :

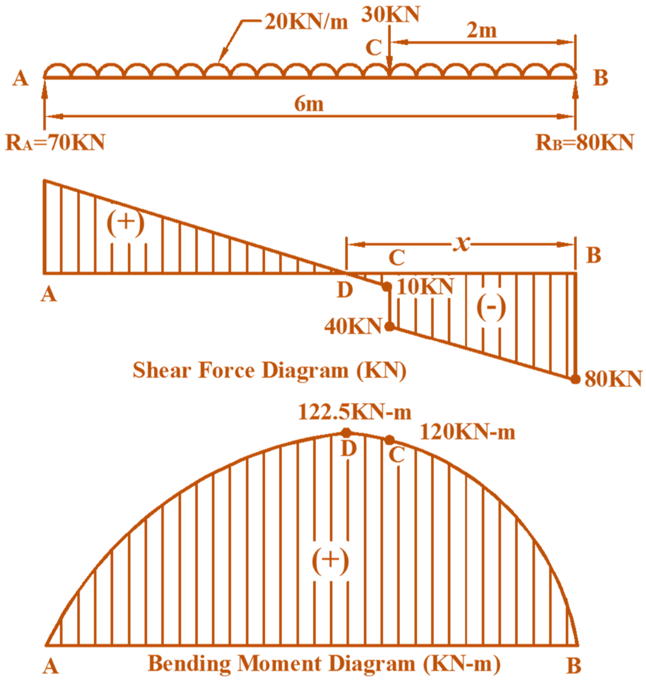


Fig.P7.11 SF and BM diagram [Example 7.11]

Taking moment about A,

$$R_B \times 6 = \left(20 \times 6 \times \frac{6}{2}\right) + (30 \times 4) = 480$$

$$R_B = \frac{480}{6} = 80 \text{ KN}$$

But, $R_A + R_B = (20 \times 6) + 30 = 150$ KN

$$R_A = 150 - R_B = 150 - 80 = 70 \text{ KN}$$

Calculation for shear force :

SF at B = – 80 KN

SF at C (Due to udl) = – 80 + (20 × 2) = –40 KN

SF at C (Due to point load) = – 40 + 30 = –10 KN

SF at A = –10+(20 × 4) = + 70 KN

Calculation for bending moment :

$$\text{BM at B} = 0$$

$$\text{BM at C} = +(80 \times 2) - \left(20 \times 2 \times \frac{2}{2}\right) = +120 \text{ KN–m}$$

$$\text{BM at A} = 0$$

To find the maximum bending moment :

The bending moment will be maximum at a point where the shear force is equal to zero. Let D be the point at a distance '***x***' from B at which the shear force is zero.

$$\text{Shear force at D} = -80 + 20x + 30 = 0$$

$$20x = 50$$

$$x = \frac{50}{20} = \mathbf{2.5}$$

The bending moment will be maximum at a distance **2.5 m** from the right support (B).

Maximum bending moment at D

$$= +(80 \times 2.5) - (30 \times 0.5) - \left(20 \times 2.5 \times \frac{2.5}{2}\right) = \mathbf{122.\ \ KN}$$

Example : 7.12 ***(Oct.04, Apr.18)***

A simply supported beam of span 10m carries a udl of 20kN/m over the left half of the span and a point load of 30KN at the mid span. Draw the SFD and BMD. Find also the position and magnitude of maximum bending moment.

Solution :

Taking moment about A,

$$R_B \times 10 = (30 \times 5) + \left(20 \times 5 \times \frac{5}{2}\right) = 400$$

$$R_B = \frac{400}{10} = 40 \text{ KN}$$

$$\text{But, } R_A + R_B = 30 + (20 \times 5) = 130 \text{ KN}$$

$$R_A = 130 - R_B = 130 - 40 = 90 \text{ KN}$$

Calculation for shear force :

$$\text{SF at B} = -40 \text{ KN}$$

$$\text{SF at C} = -40 + 30 = -10 \text{ KN}$$

$$\text{SF at A} = -10 + (20 \times 5) = +90 \text{ KN}$$

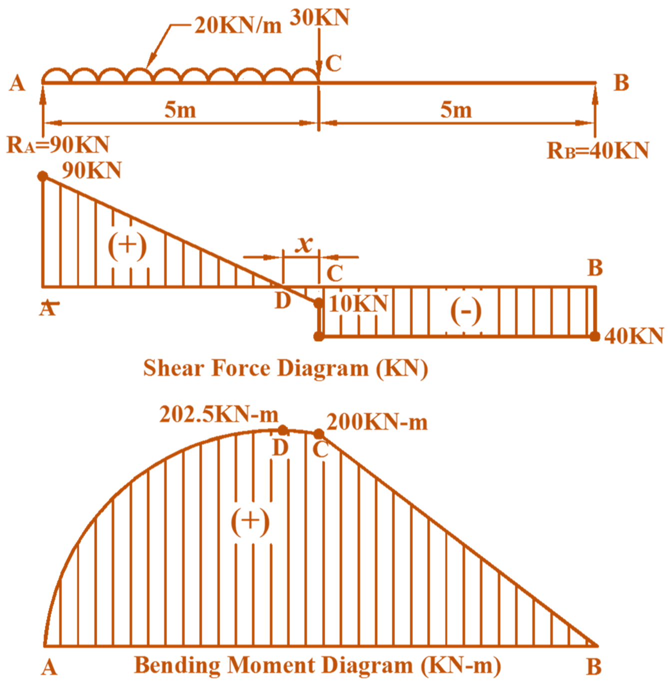


Fig.P7.12 SF and BM diagram [Example 7.12]

Calculation for bending moment :

BM at B $= 0$

BM at C $= +(40 \times 5) = +200$ KN–m

BM at A $= 0$

To find the maximum bending moment :

The bending moment will be maximum at a point where the shear force is equal to zero. Let D be the point at a distance 'x' from C at which the shear force is zero.

$$\text{Shear force at D} = -40 + 30 + 20x = 0$$

$$20x = 10$$

$$x = \frac{10}{20} = \mathbf{0.5}$$

The bending moment will be maximum at a distance **5.5m** from the point B.

Maximum bending moment at D

$$= +(40 \times 5.5) - (30 \times 0.5) - \left(20 \times 0.5 \times \frac{0.5}{2}\right) = \mathbf{202.5\ KN\text{–}m}$$

Example : 7.13 *(Apr.01)*

A simply supported beam AB of 8m length carries an udl of 5KN/m for a distance of 4m from the left end support A. The rest of the beam of 4m carries an udl of 10KN/m. Draw SF and BM diagrams.

Solution :

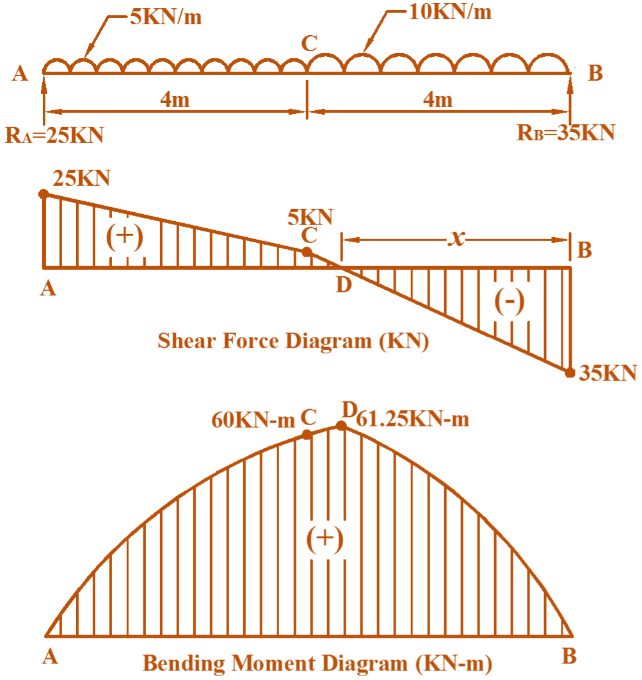


Fig.P7.13 SF and BM diagram [Example 7.13]

Taking moment about A,

$$R_B \times 8 = \left[10 \times 4 \times \left(4 + \frac{4}{2}\right)\right] + \left(5 \times 4 \times \frac{4}{2}\right) = 280$$

$$R_B = \frac{280}{8} = 35 \text{ KN}$$

But, $R_A + R_B = (10 \times 4) + (5 \times 4) = 60$ KN

$$R_A = 60 - R_B = 60 - 35 = 25 \text{ KN}$$

Calculation for shear force :

SF at B $= -35$ KN

SF at C $= -35 + (10 \times 4) = +5$ KN

SF at A $= +5 + (5 \times 4) = +25$ KN

Calculation for bending moment :

$$\text{BM at B} = 0$$

$$\text{BM at C} = +(35 \times 4) - \left(10 \times 4 \times \frac{4}{2}\right) = +60 \text{ KN–m}$$

$$\text{BM at A} = 0$$

To find the maximum bending moment :

The bending moment will be maximum at a point where the shear force is equal to zero. Let D be the point at a distance '$\boldsymbol{x}$' from B at which the shear force is zero.

$$\text{Shear force at D} = -35 + 10\boldsymbol{x} = 0$$

$$\boldsymbol{x} = \frac{35}{10} = \mathbf{3.5}$$

The bending moment will be maximum at a distance **3.5m** from the point B.

Maximum bending moment at D

$$= +(35 \times 3.5) - \left(10 \times 3.5 \times \frac{3.5}{2}\right) = \mathbf{61.25} \text{ KN–m}$$

Example : 7.14 *(Oct.94)*

Draw the SF and BM diagrams for the beam shown in the fig.P.7.14 and also calculate the maximum bending moment.

Solution :

Taking moment about A,

$$R_B \times 5 = (4 \times 4) + (8 \times 3 \times 2.5) + (2 \times 1) = 78$$

$$R_B = \frac{78}{5} = 15.6 \text{ KN}$$

$$\text{But, } R_A + R_B = 4 + (8 \times 3) + 2 = 30 \text{ KN}$$

$$R_A = 30 - R_B = 30 - 15.6 = 14.4 \text{ KN}$$

Calculation for shear force :

$$\text{SF at B} = -15.6 \text{ KN}$$

$$\text{SF at D} = -15.6 + 4 = -11.6 \text{ KN}$$

$$\text{SF at C}(\textit{due to udl}) = -11.6 + (8 \times 3) = +12.4 \text{ KN}$$

$$\text{SF at C}(\textit{due to point load}) = +12.4 + 2 = 14.4 \text{ KN}$$

$$\text{SF at A} = +14.4 \text{ KN}$$

(∵There is no load between C and A)

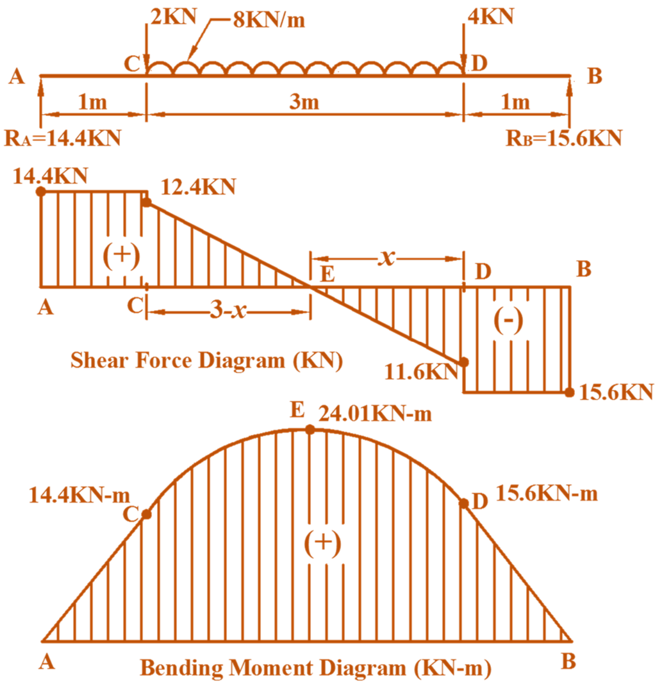


Fig.P7.14 SF and BM diagram [Example 7.14]

Calculation for bending moment :

BM at B $=$ 0

BM at D $= +(15.6 \times 1) = +15.6$ KN–m

BM at C $= +(15.6 \times 4) - \left(8 \times 3 \times \dfrac{3}{2}\right) = 14.4$ KN–m

BM at A $=$ 0

To find the maximum bending moment :

The bending moment will be maximum at a point where the shear force is equal to zero. Let E be the point at a distance 'x' from D at which the shear force is zero.

Shear force at E $= -15.6 + 4 + 8x = 0$

$$x = \frac{11.6}{8} = \mathbf{1.45}$$

The bending moment will be maximum at a distance **1.45m** from the point D.

Maximum bending moment at E

$$= +(15.6 \times 2.45) - (4 \times 1.45) - \left(8 \times 1.45 \times \frac{1.45}{2}\right) = \mathbf{24.01} \text{ KN–m}$$

Example : 7.15 *(Oct.91)*

Draw the SF and BM diagrams for the beam shown in the fig.P.7.15 and also calculate the maximum bending moment.

Solution :

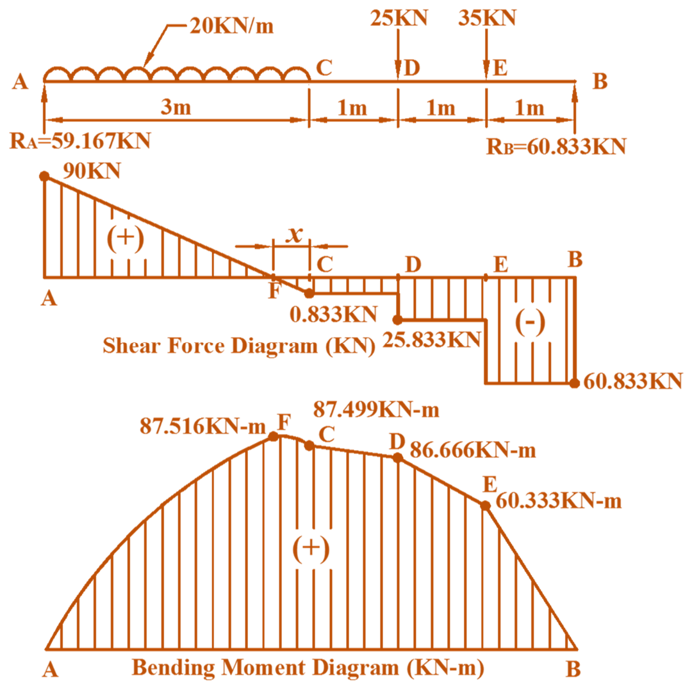


Fig.P7.15 SF and BM diagram [Example 7.15]

Taking moment about A,

$$R_B \times 6 = (35 \times 5) + (25 \times 4) + \left(20 \times 3 \times \frac{3}{2}\right) = 365$$

$$R_B = \frac{365}{6} = 60.833 \text{ KN}$$

But, $R_A + R_B = 35 + 25 + (20 \times 3) = 120$ KN

$$R_A = 120 - R_B = 120 - 60.833 = 59.167 \text{ KN}$$

Calculation for shear force :

SF at B $= -60.833$ KN

SF at E $= -60.833 + 35 = -25.83$ KN

SF at D $= -25.833 + 25 = -0.833$ KN

SF at C $= -0.833$ KN ($\because$*There is no load between D and C)*

SF at A $= 0.833 + (20 \times 3) = +59.167$ KN

Calculation for bending moment :

BM at B $= 0$

BM at E $= +(60.833 \times 1) = +60.833$ KN–m

BM at D $= +(60.833 \times 2) - (35 \times 1) = +86.666$ KN–m

BM at C $= +(60.833 \times 3) - (35 \times 2) - (25 \times 1) = +87.499$ KN–m

BM at A $= 0$

To find the maximum bending moment :

The bending moment will be maximum at a point where the shear force is equal to zero. Let F be the point at a distance 'x' from C at which the shear force is zero.

Shear force at F $= -60.833 + 35 + 25 + 20x = 0$

$$x = \frac{0.833}{20} = \mathbf{0.04165}$$

The bending moment will be maximum at a distance **0.04165m** from the point C.

Maximum bending moment at F

$$= +(60.833 \times 3.04165) - (35 \times 2.04165) - (25 \times 1.04165) - (20 \times 0.04165 \times 0.04165/2) = +\mathbf{87.516}\ \text{KN–m}$$

Example : 7.16

A simply supported beam of span 7m is subjected to a udl of 10KN/m for 3m from left support and a udl of 5KN/m for 2m from the right support. Draw the SF and BM diagrams. Also calculate the maximum bending moment.

Solution :

Taking moment about A,

$$R_B \times 7 = \left[5 \times 2 \times \left(5 + \frac{2}{2}\right)\right] + \left(10 \times 3 \times \frac{3}{2}\right) = 105$$

$$R_B = \frac{105}{8} = 15\ \text{KN}$$

But, $R_A + R_B = (5 \times 2) + (10 \times 3) = 40$ KN

$R_A = 40 - R_B = 40 - 15 = 25$ KN

Calculation for shear force :

SF at B $= -15$ KN

SF at D $= -15 + (5 \times 2) = -5$ KN

SF at C $= -5$ KN

SF at A $= +25$ KN

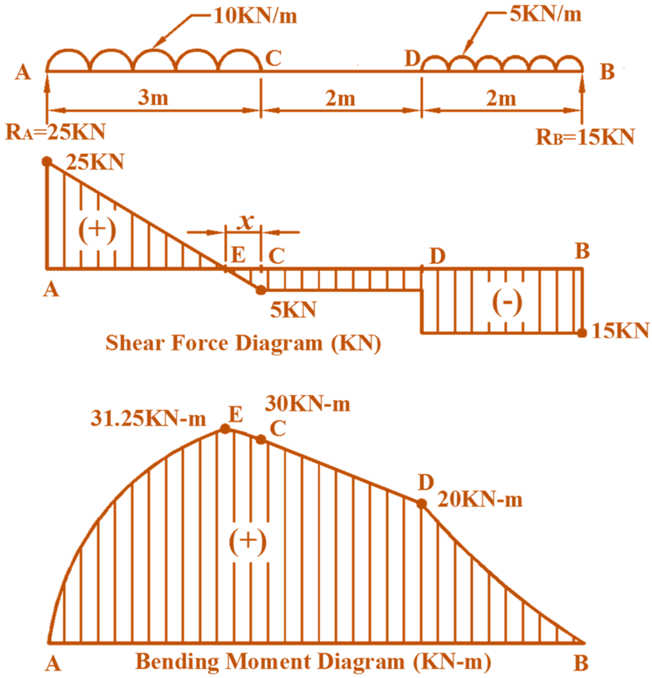


Fig.P7.16 SF and BM diagram [Example 7.16]

Calculation for bending moment :

BM at B $= 0$

BM at D $= +(\mathbf{15 \times 2}) - \left(\mathbf{5 \times 2 \times \frac{2}{2}}\right) = +\mathbf{20}$ KN–m

BM at C $= +(15 \times 4) - (5 \times 2 \times 3) = +30$ KN–m

BM at A $= 0$

To find the maximum bending moment :

The bending moment will be maximum at a point where the shear force is equal to zero. Let E be the point at a distance 'x' from C at which the shear force is zero.

Shear force at E $= -15 + (5 \times 2) + 10\,x = 0$

$$x = \frac{5}{10} = \mathbf{0.5}$$

The bending moment will be maximum at a distance **0.5m** from the point C.

Maximum bending moment at E

$$= +(15 \times 4.5) - \left[5 \times 2 \times \left(2.5 + \frac{2}{2}\right)\right] - \left(10 \times 0.5 \times \frac{0.5}{2}\right) = +\mathbf{31.25\ KN–m}$$

PROBLEMS FOR PRACTICE

CANTILEVER BEAMS

1. A cantilever beam 2m long carries a point load of 1.8KN at its free end. Draw shear force and bending moment diagrams for the cantilever.
   [Ans: Max.SF=+1.8KN, Max.BM=–3.6KN–m]

2. A cantilever beam of 1.5m long carries point loads of 1KN, 2KN and 3KN at 0.5m, 1.0m, 1.5m from the fixed end. Draw the SF and BM diagrams for the beam. ***[Ans: Max.SF=+6KN, Max.BM=–7KN–m]***

3. A cantilever beam of 1.4m length carries a uniformly distributed load of 1.5KN/m over its entire length. Draw SF and BM diagrams for the beam.
   [Ans: Max.SF=+2.1KN, Max.BM=–1.47KN–m]

4. A cantilever of 4m length is fixed at the left end A. It carries point load of 40KN, 30KN and 20KN at 1m, 2m and 3m respectively from fixed end A. In addition a udl of 10KN/m is acted throughout its length. Sketch the SFD and BMD for the beam. *(Oct.04)*
   [Ans: Max.SF=+130KN, Max.BM=–240KN–m]

5. A cantilever beam of span 4m is loaded with two point loads of 6KN and 5KN at 1m and 2m from the fixed end in addition to its self weight 2KN/m. Draw SF and BM diagrams. *(Apr.01)*
   [Ans: Max.SF=+19KN, Max.BM=–32KN–m]

6. A cantilever 4m long carries a udl of 20KN/m over half of its length adjoining the free end. Draw the SFD and BMD. *(Oct.91)*
   [Ans: Max.SF=+40KN, Max.BM=–120KN–m]

7. A cantilever 3m long is loaded with a udl of 2KN/m run over a length of 2.5m from the free end. It also carries a load of 3KN at a distance of 1m from the free end. Draw the SFD and BMD of the beam.
   [Ans: Max.SF=+8KN, Max.BM=–14.75KN–m]

8. A cantilever 4m span carries a udl of 10KN/m for a length of 2.5m from fixed end and two point loads of 20KN and 30KN at free end and at 1.5m from free end respectively. Draw the SF and BM diagrams.*(Oct.12)*
   [Ans: Max.SF=+75KN, Max.BM=–186.25KN–m]

SIMPLY SUPPORTED BEAMS

9. A simply supported beam of length 8m is loaded as shown in fig.P7.17. Construct the SFD and BMD.

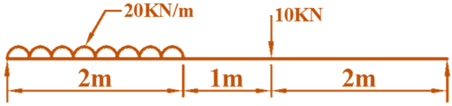


Fig.P7.17

[Ans: R_A=36KN, R_B=14KN, Max.BM=+32.4KN–m at a distance 3.2m from the right support]

10. A simply supported beam 5m long caries point loads of 70KN, 90KN, 50KN and 80KN at distance 1m, 3m, 4m and 4.5m respectively from the left hand support. Find the reactions and draw the SFD and BMD.

[Ans: R_A=128KN, R_B=162KN, Max.BM=+154KN–m at a distance 2m from the right support]

11. A simply supported beam of 4m span carries a udl of 6KN/m for a distance of 2m from the left support. Draw the SFD.

[Ans: R_A=9KN, R_B=3KN]

12. A simply supported beam 6m long is carrying a udl of 2KN/m over a length of 3m from the right end. Construct SFD and BMD. Also find the position and magnitude of maximum BM. *(Oct.94)*

[Ans: Max.BM=+5.0625KN–m at a distance 2.25m from the right support]

13. Draw the SF and BM diagrams for a simply supported beam loaded as shown in fig.P7.18. Determine the maximum BM in the beam. *(Apr.93, Oct.95)*

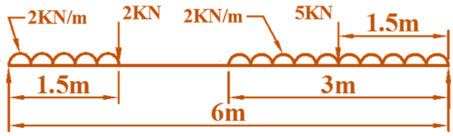


Fig.P7.18

[Ans: Max.BM=+11.754KN–m at a distance 2.0625m from the right support]

Unit – IV

Chapter 8. THEORY OF SIMPLE BENDING OF BEAMS

8.1 Introduction

When a beam is loaded with some external forces, bending moment and shear forces are set up. The bending moment at a section tends to bend or deflect the beam and internal stresses are developed to resist this bending. These stresses are called *bending stresses* and the relevant theory is called *theory of simple bending.*

8.2 Simple bending or pure bending

If a beam tends to bend or deflect only due to the application of constant bending moment and not due to shear force, then it is said to be in a state of *simple bending* or *pure bending*.

8.3 Theory of simple bending

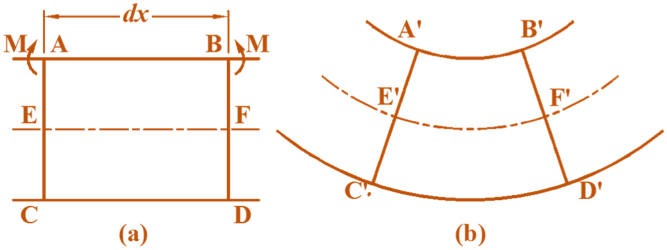


Fig.8.1 Theory of simple bending

Consider a small length dx of simply supported beam subjected to a bending moment M as shown in the fig.8.1(a). Due to the action of the bending moment, the beam as a whole will bend as shown in fig.8.1(b). Due to bending, the length of the beam is changed. Let us consider a top most layer AB and bottom most layer CD. The layer AB is subjected to compression and shortened to A'B' while the layer CD is subjected to tension and stretched to C'D'.

Let us consider the beam length dx consists of large number of such layers. The length of all the layers are changed due to bending. Some of them may be shortened while some others may be stretched. However, there exists a layer EF in between the top and bottom layers which will retain its original length even after bending. This layer EF which is neither shortened nor stretched is known as the *neutral layer* or *neutral plane.*

8.4 Assumptions made in the theory of simple bending

The following are the assumptions made in the theory of simple bending.

1) The material of the beam is uniform throughout.
2) The material of the beam has equal elastic properties in all directions.
3) The beam material is stressed within elastic limit and thus obeys Hooke's law.
4) The beam material has same value of Young's modulus both in tension and compression.
5) The radius of curvature of the beam is very large when compared with the cross sectional dimensions of the beam.
6) The resultant pull or push on a transverse section of the beam is zero.
7) Each layer of the beam is free to expand or contract independently of the layer, above or below it.
8) The cross section of the beam which is plane and normal before bending will remain plane and normal even after bending.

8.5 Neutral axis

The line of intersection of the neutral layer with any normal cross–section of the beam is known as *neutral axis* of that section. It is denoted as N.A. A beam is subjected to compressive stresses on one side of the neutral axis and tensile stresses on the other side of the neutral axis. There is no stress of any kind at the neutral axis.

8.6 Bending stress distribution

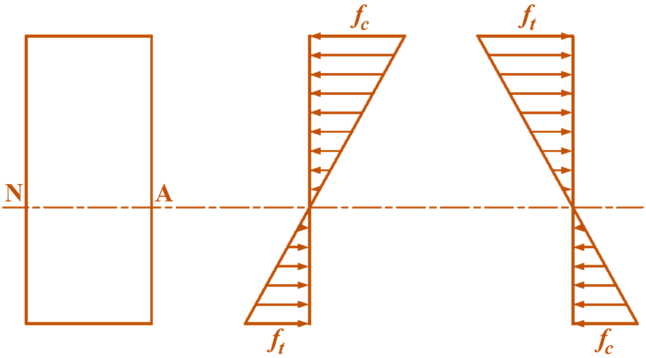


Fig.8.2 Bending stress distribution

There is no stress at the neutral axis. The magnitude of stress at a point is directly proportional to its distance from the neutral axis. The maximum stress taken place at the outer most layer.

In a simply supported beam, compressive stresses are developed above the neutral axis and tensile stresses are developed below the neutral axis. But in cantilever beam, tensile stresses are developed above the neutral axis and compressive stresses are developed below the neutral axis.

8.7 Moment of resistance

The maximum bending moment that a beam can withstand without failure is called moment of resistance.

From the theory of simple bending, we know that one side of the neutral axis is subjected to compressive stresses and other side of the neutral axis is subjected to tensile stresses. These compressive and tensile stresses form a couple, whose moment must equal to the external moment (M). The moment of this couple which resist the external bending moment is known as moment of resistance.

8.8 Derivation of flexural formula

a) To prove $\frac{f}{y} = \frac{E}{R}$

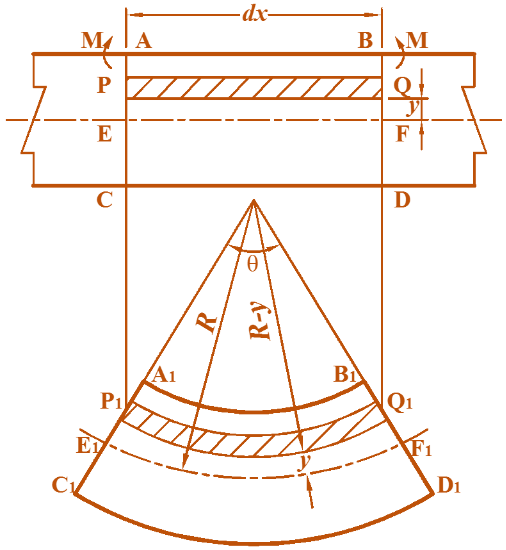


Fig.8.3 Bending stress

Consider a small length dx of a beam subjected to a bending moment as shown in the fig.8.3. As a result of this bending moment, this small length of beam bend into an arc of circle with O as centre.

Let, M = Moment acting at the beam

θ = Angle subtended at the centre by the arc and

R = Radius of curvature of the beam

Now consider a length PQ at a distance $'y'$ from the neutral axis EF. Let this layer be compressed to P_1Q_1 after bending.

We know that, decrease in length of this layer,

$$\delta l = PQ - P_1Q_1 = R\theta - (R - y)\theta$$

$$\text{Strain in the layer, e} = \frac{\text{change in length}}{\text{Original length}} = \frac{y\theta}{R\theta} = \frac{y}{R}$$

If 'f' be the bending stress in the layer, then

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{f}{e}$$

$$f = E \times e = E \times \frac{y}{R}$$

$$\boxed{\frac{f}{y} = \frac{E}{R}}$$

Since E and R for a beam are constant, the bending stress is directly proportional to the distance of the layer from the neutral axis.

$$\therefore \frac{f_1}{y_1} = \frac{f_2}{y_2} = \cdots = \frac{f_{max}}{y_{max}}$$

b) To prove $\frac{M}{I} = \frac{E}{R}$

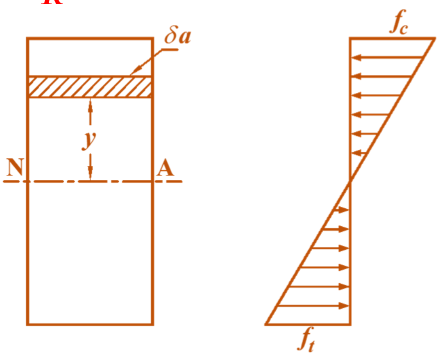


Fig.8.4 Neutral axis

Consider a small elemental area δa of a beam at a distance 'y' from neutral axis as shown in fig.8.4

Let 'f' be the bending stress in the elemental area.

The force on the elemental area $= f \times \delta a$

Moment of this force about neutral axis,

$$\delta M = f \times \delta a \times y \qquad \text{------------ (1)}$$

Substitute, $f = y \times \frac{E}{R}$ in equation (1)

$$\delta M = \frac{yE}{R} \times \delta a \times y = \frac{E}{R} \delta a \; y^2$$

By definition, moment of resistance

$$M = \Sigma \delta M = \sum \frac{E}{R} \delta a \; y^2 = \frac{E}{R} \Sigma \delta a \; y^2$$

We know that $\Sigma \delta a \; y^2 = \textit{Moment of inertia}$ of the area of the section about neutral axis i.e. I

$$\therefore M = \frac{E}{R} \times \mathrm{I} \quad \text{(or)}$$

$$\frac{M}{I} = \frac{E}{R} \qquad \text{------------(2)}$$

Also, $\frac{f}{y} = \frac{E}{R}$ ----------- (3)

Combining the equations (2) and (3)

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

The above equation is called *flexural equation.*

8.9 Section modulus

The ratio of moment of inertia about the neutral axis to the distance of the extreme layer from the neutral axis is known as *section modulus* or *modulus of section.*

$$\text{Section modulus} = \frac{\text{Moment of inertial about N.A}}{\text{Distance of extreme layer from N.A}}$$

We know that the maximum bending stress occurs at the outermost layer. Let y_{max} be the distance of the outermost layer and f_{max} be the maximum stress.

From the flexural formula, $f_{max} = \frac{M}{I} \times y_{max}$ (or)

$$M = f_{max} \times \frac{I}{y_{max}} = f_{max} \times Z$$

Where Z= Section modulus or modulus of section.

Section modulus of various sections

1) Rectangular section

Consider a rectangular section of width 'b' and depth 'd'.

Moment of inertia about the neutral axis, $I = \frac{bd^3}{12}$

Distance of extreme layer from N.A, $y_{max} = \frac{d}{2}$

$\therefore$ Section Modulus, $Z = \frac{I}{y_{max}} = \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^2}{6}$

2. Circular section

Consider a circular section of diameter 'd'.

Moment of inertia about the neutral axis, $I = \frac{\pi d^4}{64}$

Distance of extreme layer from N.A, $y_{max} = \frac{d}{2}$

$\therefore$ Section Modulus, $Z = \frac{I}{y_{max}} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$

1.10 Strength and stiffness of beam

Strength : The moment of resistance offered by the beam is known as *strength* of a beam.

We know that, moment of resistance, $M = f \times Z$

From the above relation, it is known that, for a given value of bending stress, the moment of resistance depends upon the section modulus. Therefore, if the value of Z is greater, the beam will be strong. This ideal is put into practice, by providing beam of I –section, where the flanges alone withstand almost all the bending stress.

Stiffness : The resistance offered by a beam against deflection from its original straight condition is known as *stiffness* of the beam.

REVIEW QUESTIONS

1. What do you mean by simple bending?
2. Explain the theory of simple bending. *(Oct.92)*
3. State the assumptions made in theory of simple bending. *(Oct.96, Apr.04, Apr.01, Oct.16, Oct.17, Apr.18)*
4. Define neutral axis. *(Apr.97, Oct.17, Apr.18)*
5. Draw the sketch to show the bending stress distribution in beam.
6. What is moment of resistance?
7. Derive the flexural formula $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ *(Oct.17)*
8. Define section modulus of a beam. Write down the expression for rectangular and circular sections. *(Apr.04, Oct.04, Apr.18)*
9. Define strength and stiffness of beam. *(Oct.01)*

POINTS TO REMEMBER

1) Flexural formula $\Longrightarrow \frac{M}{I} = \frac{f}{y} = \frac{E}{R}$

Where, f = Maximum bending stress (N/mm^2)

y = Distance of extreme layer from N.A. (mm)

M = Bending moment (N–mm)

I = Moment of inertia (mm^4)

E = Young's modulus (N/mm^2)

R = Radius of curvature (mm)

2) Maximum bending moment for

(a) Simply supported beam with central point load , $M = \frac{Wl}{4}$

(b) Simply supported beam with udl over entire span, $M = \frac{wl^2}{8}$

(c) Cantilever beam with point load at the free end, $M = Wl$

(d) Cantilever beam with udl over entire span, $M = \frac{wl^2}{2}$

Where, W = Point load (N)

w = uniformly distributed load (N/mm)

l = Length of beam (mm)

SOLVED PROBLEMS

Example : 8.1

A steel wire of 5mm diameter is bent into a circular shape of 5m radius. Determine the maximum stress induced in the wire. Take $E = 2 \times 10^5 N/mm^2$.

Given : Diameter of the steel wire, d = 5 mm

Radius of circular shape, R = 5 m = 5000 mm

Young's modulus, E = 2×10^5N/mm^2

To find : 1) The maximum stress induced, f_{max}

Solution :

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{5}{2} = 2.5 \text{ mm}$$

We know that, $\dfrac{f_{max}}{y_{max}} = \dfrac{E}{R}$

$$f_{max} = \frac{E}{R} \times y_{max} = \frac{2 \times 10^5 \times 2.5}{5000} = \boxed{100 \text{ N/mm}^2}$$

Result : 1) The maximum stress induced in the wire, f_{max} **= 100 N/mm^2**

Example : 8.2 *(Apr.93, Oct.02)*

A steel rod 100mm diameter is to be bent into circular shape. Find the maximum radius of curvature which it should be bent so that stress in the steel should not exceed $120 N/mm^2$. Take $E = 2 \times 10^5 N/mm^2$.

Given : Diameter of the steel rod, d = 100 mm

Maximum bending stress, f_{max} = 120 N/mm^2

Young's modulus, E = 2×10^5N/mm^2

To find : 1) The radius of curvature, R

Solution :

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{100}{2} = 50 \text{ mm}$$

We know that, $\dfrac{f_{max}}{y_{max}} = \dfrac{E}{R}$

$$R = \frac{E}{f_{max}} \times y_{max} = \frac{2 \times 10^5 \times 50}{120} = \boxed{83333 \text{ mm}}$$

Result : 1) The radius of curvature, R **= 83333 mm**

Example : 8.3

A metallic rod of 10mm diameter is bent into a circular form of radius 6m. If the maximum bending stress developed in the rod is 125 N/mm², find the value of Young's modulus for the rod material.

Given : Diameter of the rod, d = 10 mm
Maximum bending stress, f_{max} = 125 N/mm^2
Radius of curvature, R = 6 m = 6000 mm

To find : 1) Young's modulus, E

Solution :

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{100}{2} = 50 \text{ mm}$$

We know that, $\frac{f_{max}}{y_{max}} = \frac{E}{R}$

$$E = \frac{R}{y_{max}} \times f_{max} = \frac{6000 \times 125}{5} = \boxed{\mathbf{1.5 \times 10^5 \ N/mm^2}}$$

Result : 1) Young's modulus of the material, $E = \mathbf{1.5 \times 10^5}$ **N/mm^2**

Example : 8.4

(Oct.01)

Determine the resisting moment of a timber beam rectangular in section 125mm × 250mm, if the permissible bending stress is 8 N/mm².

Given : Maximum bending stress, f_{max} = 8 N/mm^2
Width of the beam, b = 125 mm
Depth of the beam, d = 250 mm

To find : 1) Resisting moment, M

Solution :

Moment of inertia, $I = \frac{bd^3}{12} = \frac{125 \times 250^3}{12} = 1.6276 \times 10^8 \text{ mm}^4$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{250}{2} = 125 \text{ mm}$$

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$M = \frac{f_{max}}{y_{max}} \times I = \frac{8 \times 1.6276 \times 10^8}{125} = \boxed{\mathbf{10.417 \times 10^6 \ N\text{-}mm}}$$

Result : 1) Resisting moment, M = **10.417×10^6 N-mm**

Example : 8.5 *(Oct.92, Oct.14, Oct.15)*

A simply supported beam is 300mm wide and 400mm deep. Determine the bending stress at 40mm above N.A, if the maximum bending stress is 15 N/mm².

Given :

Width of the beam, b = 300 mm

Depth of the beam, d = 400 mm

Distance of layer from the N.A, y_1 = 40 mm

Maximum bending stress, f_{max} = 15 N/mm^2

To find : 1) Bending stress at a distance 40mm above the N.A, f_1

Solution :

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{400}{2} = 200 \text{ mm}$$

We know that, $\dfrac{f_1}{y_1} = \dfrac{f_{max}}{y_{max}}$

$$f_1 = \frac{f_{max}}{y_{max}} \times y_1 = \frac{15}{200} \times 40 = \boxed{3 \text{ N/mm}^2}$$

Result : 1) Bending stress at a distance 40mm above N.A, f_1 = **3 N/mm^2**

Example : 8.6 *(Oct.88, Oct.91, Oct.12, Oct.13)*

A rectangular beam 200mm deep and 100mm wide is simply supported over a span of 8m and carries a central point load of 25KN. Determine the maximum stress in the beam. Also calculated the value of longitudinal fibre stress at a distance of 25mm from the surface of the beam.

Given :

Width of the beam, b = 100 mm

Depth of the beam, d = 200 mm

Length of the beam, l = 8m = 8000 mm

Central point load, W = 12 KN = 12×10^3 N

To find : 1) Maximum bending stress, f_{max}

2) Bending stress at 25mm from the surface of the beam, f_1

Solution :

$$\text{Moment of inertia, } I = \frac{bd^3}{12} = \frac{100 \times 200^3}{12} = 66.667 \times 10^6 \text{ mm}^4$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{200}{2} = 100 \text{ mm}$$

In case of simply supported beam subjected to a central point load,

$$\text{Maximum bending moment, } M = \frac{W\,l}{4}$$

$$= \frac{25 \times 10^3 \times 8000}{4} = 50 \times 10^6 \text{ N-mm}$$

$$\text{We know that, } \frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{50 \times 10^6 \times 100}{66.667 \times 10^6} = \boxed{75 \text{ N/mm}^2}$$

To find the bending stress at 25mm from the surface of the beam :

The distance of layer from N.A, $= y_1 = 100 - 25 = 75$ mm

$$\frac{f_1}{y_1} = \frac{f_{max}}{y_{max}}$$

$$f_1 = \frac{f_{max}}{y_{max}} \times y_1 = \frac{75}{100} \times 75 = \boxed{56.25 \text{ N/mm}^2}$$

Result : 1) The maximum bending stress, f_{max} **= 75 N/mm^2**

2) Bending stress at 25mm from surface of beam, f_1 **= 56.25 N/mm^2**

Example : 8.7

(Apr.14, Apr.15, Oct.15)

A simply supported beam of rectangular cross section carries a central load of 25 KN over a span of 6m. The bending stress should not exceed 7.5N/mm^2. The depth of the section is 400mm. Calculate the necessary width of the section.

Given :

Central point load, W = 25 KN = 25×10^3N

Length of the beam, l = 6m = 6000 mm

Bending stres, f_{max} = 7.5 N/mm^2

Depth of the beam, d = 150 mm

To find : 1) Width of the beam, b

Solution :

$$\text{Moment of inertia, } I = \frac{bd^3}{12} = \frac{b \times 400^3}{12} = 5.333 \times 10^6\ b \text{ mm}^4$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{400}{2} = 200 \text{ mm}$$

In case of simply supported beam subjected to a central point load,

$$\text{Maximum bending moment, } M = \frac{W\,l}{4}$$

$$= \frac{25\times10^3\times6000}{4} = 37.5\times10^6 \text{ N-mm}$$

$$\text{We know that, } \frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$\frac{37.5\times10^6}{5.333\times10^6\,b} = \frac{7.5}{200}$$

$$b = \frac{37.5\times10^6\times200}{7.5\times5.333\times10^6} = \boxed{187.5 \text{ mm}}$$

Result : 1) Width of the beam, $\boldsymbol{b} = \mathbf{187.5\ mm}$

Example : 8.8

(Apr.87, Oct.89, Oct.04, Apr.17)

A rectangular beam 300mm deep is simply supported over a span of 4m. What udl per metre, the beam may carry if the bending stress is not to exceed 120 N/mm^2. Given $I = 8\times10^6\ mm^4$.

Given :

Depth of the beam, d = 300 mm

Length of the beam, l = 4m = 4000 mm

Maximum bending stress, f_{max} = 120 N/mm^2

Moment of inertia, I = $8\times10^6\ mm^4$

To find : 1) The of udl per metre, w

Solution :

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{300}{2} = 150 \text{ mm}$$

In case of simply supported beam subjected to a udl,

$$\text{Maximum bending moment, } M = \frac{wl^2}{8} = \frac{w\times4000^2}{8} = 2\times10^6 w \text{ N-mm}$$

$$\text{We know that, } \frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$\frac{2\times10^6 w}{8\times10^6} = \frac{120}{150}$$

$$w = \frac{120\times8\times10^6}{150\times2\times10^6} = 3.2 \text{ N/mm} = \boxed{3.2 \text{ KN/m}}$$

Result : 1) The udl per metre, $\boldsymbol{w}$ = **3.2 KN/m**

Example : 8.9 *(Apr.13)*

A rectangular beam 60mm wide and 150mm deep is simply supported over a span of 4m. If the beam is subjected to a uniformly distributed load of 4.5KN/m, find the maximum bending stress induced in the beam.

Given :

Width of the beam, $b = 60$ mm

Depth of the beam, $d = 150$ mm

Length of the beam, $l = 4\text{m} = 4000$ mm

Uniformly distributed load, $w = 4.5$ KN/m $= 4.5$ N/mm

To find : 1) Maximum bending stress, f_{max}

Solution :

$$\text{Moment of inertia, } I = \frac{bd^3}{12} = \frac{60 \times 150^3}{12} = 16.875 \times 10^6 \text{ mm}^4$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{150}{2} = 75 \text{ mm}$$

In case of simply supported beam subjected to a udl,

$$\text{Maximum bending moment, } M = \frac{wl^2}{8} = \frac{4.5 \times 4000^2}{8} = 9 \times 10^6 \text{ N-mm}$$

We know that, $\dfrac{M}{I} = \dfrac{f_{max}}{y_{max}}$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{9 \times 10^6 \times 75}{16.875 \times 10^6} = \boxed{40 \text{ N/mm}^2}$$

Result : 1) Maximum bending stress induced, f_{max} **= 40 N/mm²**

Example : 8.10

A timber beam of rectangular section supports a load of 20KN uniformly distributed over a span of 3.6m. If depth of the beam section is twice the width and maximum stress is not to exceed 7N/mm², find the dimension of the beam section.

Given :

Total load, $W = 20 \text{ KN} = 20 \times 10^3$ N

Length of the beam, $l = 3.6 \text{ m} = 3600$ mm

Depth of the beam, $d = 2 \times$ width of the beam (b)

Maximum bending stress, $f_{max} = 7 \text{ N/mm}^2$

To find : 1) Depth of the beam, d 2) Width of the beam, b

Solution :

$$\text{Moment of inertia, } I = \frac{bd^3}{12} = \frac{b \times (2b)^3}{12} = 0.667\, b^4$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{2b}{2} = b$$

In case of simply supported beam subjected to a udl,

$$\text{Maximum bending moment, } M = \frac{wl^2}{8} = \frac{Wl}{8}$$

$$= \frac{20 \times 10^3 \times 3600}{8} = 9 \times 10^6 \text{ N-mm}$$

We know that, $\dfrac{M}{I} = \dfrac{f_{max}}{y_{max}}$

$$\frac{9 \times 10^6}{0.667\ b^4} = \frac{7}{b}$$

$$7 \times 0.667\ b^4 = 9 \times 10^6 \times b$$

$$b^3 = \frac{9 \times 10^6}{7 \times 0.667} = 1.9276 \times 10^6$$

$$b = \boxed{124.453 \text{ mm}}$$

Result : 1) Depth of the beam, d = **248.906 mm**
2) Width of the beam, b = **124.453 mm**

Example : 8.11 *(Oct.02)*

A beam of T–section flange 150mm × 50mm, web thickness 50mm, overall depth 200mm and 10m long is simply supported (with flange uppermost) and carries a central point load of 10KN. Determine the maximum fibre stress in the beam.

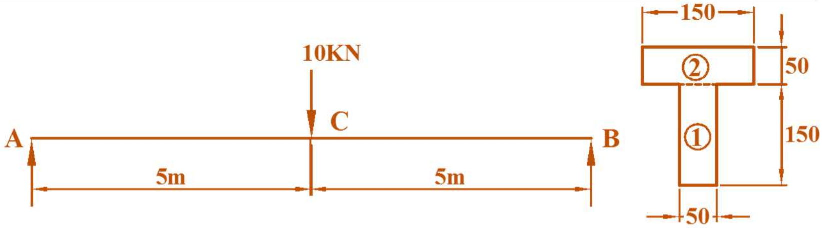


Fig.P8.1 Maximum BM in T–sectional beam [Example. 8.11]

Given : Central point load, $W = 10 \text{ KN} = 10 \times 10^3$ N
Length of the beam, $l = 10\text{m} = 10 \times 10^3$ mm

To find : 1) Maximum fibre stress, f_{max}

Solution :

In case of simply supported beam subjected to a point load,

Maximum bending moment, $M = \frac{Wl}{4}$

$$= \frac{10 \times 10^3 \times 10 \times 10^3}{4} = 25 \times 10^6 \text{ N-mm}$$

Distance of extreme layer from N.A, $y_{max} = \bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$

$$= \frac{(50 \times 150 \times 75) + (150 \times 50 \times 175)}{(50 \times 150) + (150 \times 50)} = 125 \text{ mm}$$

Moment of inertia of the section about an axis passing through the centroid and parallel to the bottom face,

$$I = \left[I_{G1} + a_1 h_1^2\right] + \left[I_{G2} + a_2 h_2^2\right]$$

$$= \left[\frac{50 \times 150^3}{12} + (50 \times 150)(125 - 75)^2\right]$$

$$+ \left[\frac{150 \times 50^3}{12} + (150 \times 50)(125 - 175)^2\right]$$

$$= 32.8125 \times 10^6 + 20.3125 \times 10^6 = 53.125 \times 10^6 \text{ mm}^4$$

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{25 \times 10^6 \times 125}{53.125 \times 10^6} = \boxed{58.824 \text{ N/mm}^2}$$

Result : 1) Maximum fibre stress, f_{max} **= 58.824 N/mm²**

Example : 8.12 *(Oct.90)*

A simply supported beam of span 6m carries uniformly distributed load of intensity 40KN/m over half of the span. The cross section of the beam is symmetrical I–section with following dimensions: Overall depth=300mm, flange width=120mm, flange thickness=25mm, web thickness=12mm. Evaluate the maximum bending stress induced in the beam.

To find : 1) Maximum bending stress induced in the beam, f_{max}

Solution :

Let R_A and R_B be the reactions at the supports of the beam.

Taking moment about A,

$$R_B \times 6 = (40 \times 3 \times 3/2) = 180$$

$$R_B = \frac{180}{6} = 30 \text{ KN}$$

But, $R_A + R_B = (40 \times 3) = 120$ KN

$$R_A = 120 - RB = 120 - 30 = 90 \text{ KN}$$

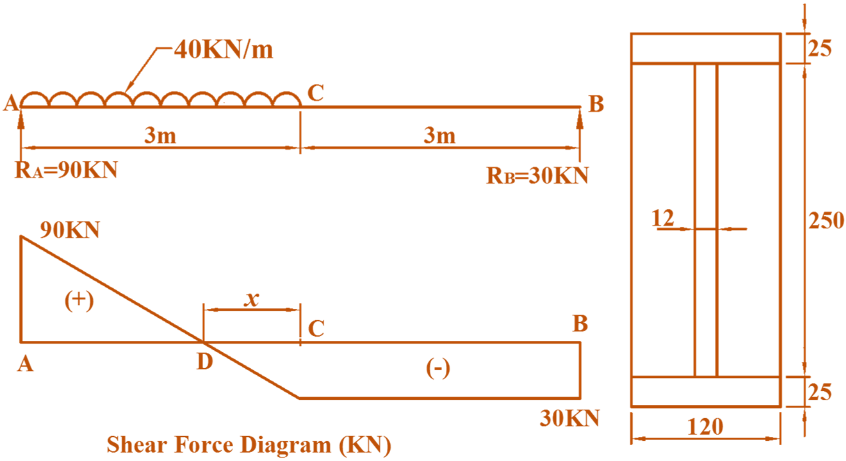


Fig.P8.2 Maximum BM in I–sectional beam [Example. 8.12]

The shear force diagram for the beam is shown in the fig.P8.2. The bending moment will be maximum at a point where the shear force is equal to zero. Let D be the point at a distance 'x' from the point C at which the shear force is zero.

Shear force at D $= -30 + 40\,x = 0$

$$x = \frac{30}{40} = \mathbf{0.75\ m}$$

Maximum bending moment at D

$$= +(30 \times 3.75) - (40 \times 0.75 \times 0.75/2)$$

$$= 101.25 \text{ KN–m} = 101.25 \times 10^6 \text{ N–mm}$$

Moment of inertia of the section about an axis passing through the centroid and parallel to the bottom face,

$$I = \left[\frac{120 \times 300^3}{12}\right] - \left[\frac{108 \times 250^3}{12}\right] = 1.294 \times 10^8 \text{mm}^4$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \bar{Y} = \frac{300}{2} = 150 \text{ mm}$$

We know that, $\dfrac{M}{I} = \dfrac{f_{max}}{y_{max}}$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{101.25 \times 10^6 \times 150}{1.294 \times 10^8} = \boxed{117.369 \text{ N/mm}^2}$$

Result : 1) Maximum bending stress, f_{max} = **117.369 N/mm^2**

Example : 8.13 *(Apr.01, Oct.03, Oct.18)*

A wooden beam of rectangular section 100mm × 200mm is simply supported over a span of 6m. Determine the udl it may carry if the bending stress is not to exceed 7.5N/mm^2. Estimate the concentrated load it may carry at the centre of the beam with the same permissible stress.

Given :

Width of the beam, b = 100 mm

Depth of the beam, d = 200 mm

Length of the beam, l = 6m = 6000 mm

Maximum bending stress, f_{max} = 7.5 N/mm^2

To find : 1) The udl over the entire span, w

2) The point load at the centre for the same stress, W

Solution :

Moment of inertia, $I = \frac{bd^3}{12} = \frac{100 \times 200^3}{12} = 66.667 \times 10^6 \text{ mm}^4$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{200}{2} = 100 \text{ mm}$$

(a) In case of simply supported beam subjected to a udl

Maximum bending moment, $M = \frac{wl^2}{8} = \frac{w \times 6000^2}{8} = 4.5 \times 10^6 w$ N-mm

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$\frac{4.5 \times 10^6 w}{66.667 \times 10^6} = \frac{7.5}{100}$$

$$w = \frac{7.5 \times 66.667 \times 10^6}{100 \times 4.5 \times 10^6} = 1.1111 \text{ N/mm} = \boxed{1.1111 \text{ KN/m}}$$

(b) In case of simply supported beam subjected to a point load,

Maximum bending moment, $M = \frac{Wl}{4} = \frac{W \times 6000}{4} = 1500\ W$ N-mm

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$\frac{1500\ W}{66.667 \times 10^6} = \frac{7.5}{100}$$

$$W = \frac{7.5 \times 66.667 \times 10^6}{100 \times 1500} = 3333.35\text{ N} = \boxed{3.3333\text{ KN}}$$

Result : 1) The udl over the entire span, ***w* = 1.1111 KN/m**
2) The point load at the centre of the beam, ***W* = 3.3333 KN**

Example : 8.14 *(Oct.93, Apr.13)*

The moment of inertia of a rolled steel joist girder of symmetrical section about N.A is $2460 \times 10^4 mm^4$. The total depth of the girder is 240mm. Determine the longest span when simply supported such that the beam would carry a udl of 5KN/m run and the bending stress should not to exceed $120 N/mm^2$.

Given : Moment of inertia, $I = 2460 \times 10^4 \text{mm}^4$
Depth of the girder, $d = 240$ mm
Load, $w = 6$ KN/m $= 6$ N/mm
Maximum bending stress, $f_{max} = 120\text{ N/mm}^2$

To find : 1) The longest span, l

Solution :

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{240}{2} = 120\text{ mm}$$

In case of simply supported beam subjected to a udl,

$$\text{Maximum bending moment, } M = \frac{wl^2}{8} = \frac{6 \times l^2}{8} = 0.75\, l^2$$

$$\text{We know that, } \frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$\frac{0.75\, l^2}{2460 \times 10^4} = \frac{120}{120}$$

$$l^2 = \frac{2460 \times 10^4}{0.75} = 32.8 \times 10^6$$

$$l = \sqrt{32.8 \times 10^6} = \mathbf{5727.128\text{ mm}} = \boxed{5.727\text{ m}}$$

Result : 1) The longest span, ***l* = 5.727 m**

Example : 8.15 *(Oct.92, Oct.94, Oct.12)*

Find the dimensions of a timber joist span 10m to carry a brick wall 0.2m thick and 4m height if the weight of the brick wall is $19 KN/mm^3$ and the maximum permissible stress is limited to $8 N/mm^2$. The depth of the joist is to be twice its width.

Given : Thickness of the wall, t = 0.2 m = 200 mm
Height of the wall, h = 4m = 4000 mm
Length of the wall, l = 10 m = 10000 mm
Weight of the brick wall = 19 KN/mm^3
Depth of the joist, d = $2 \times$ Width of the joist (b)
Maximum bending stress, f_{max} = 8 N/mm^2

To find : 1) Width of joist, b 2) Depth of joist, d

Solution :

Volume of the brick wall over full length, V=Length×thickness×height

$$= 10 \times 0.2 \times 4 = 8 \text{ m}^3$$

Total weight of the wall over full length, $W = 19 \times 8 = 152$ KN

Load on the brick wall per unit length,

$$w = \frac{152}{10} = 15.2 \text{ KN/m} = 15.2 \text{ N/mm}$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{2b}{2} = b$$

Moment of inertia, $I = \dfrac{bd^3}{12} = \dfrac{b \times (2b)^3}{12} = 0.667\ b^4$

In case of simply supported beam subjected to a udl,

Maximum bending moment, $M = \dfrac{wl^2}{8} = \dfrac{15.2 \times 10000^2}{8} = 1.9 \times 10^8$ N-mm

We know that, $\dfrac{M}{I} = \dfrac{f_{max}}{y_{max}}$

$$\frac{1.9 \times 10^8}{0.667\ b^4} = \frac{8}{b}$$

$$8 \times 0.667\ b^4 = 1.9 \times 150^8 \times b$$

$$b^3 = \frac{1.9 \times 10^8}{8 \times 0.667} = 35.607 \times 10^6$$

$$b = 328.98 \text{ mm} \approx \boxed{\textbf{330 mm}}$$

$$d = 2 \times b = 2 \times 330 = \boxed{\textbf{660 mm}}$$

Result : 1) Width, $\boldsymbol{b}$ **= 330 mm**	2) Depth, $\boldsymbol{d}$ **= 660 mm**

Example : 8.16 *(Oct.96, Apr.04, Apr.05, Oct.17)*

A cast iron water pipe 450 mm bore and 20 mm thick is supported at two points 6 m apart. Assuming each span as simply supported, find the maximum stress in the metal when (a) the pipe is running full (b) the pipe is empty. Specific weight of cast iron is 70 KN/mm^3 and that of water is 9.81KN/mm^3.

Given : Inside diameter of pipe, d_2 = 450 mm

Thickness of the pipe, t = 20 mm

Length of the pipe, l = 6 m = 6000 mm

Specific weight of cast iron = 70 KN/mm^3 = 70×10^{-6} N/mm^3

Specific weight of water = 9.81KN/mm^3 = 9.81×10^{-6}N/mm^3

To find : 1) Maximum stress in the pipe when it is running full, f_{max}

2) Mmaximum stress in the pipe when it is empty, f_{max}

Solution :

Outside diameter of pipe, $d_1 = d_2 + 2t = 450 + (2 \times 20) = 490$ mm

Cross sectional area of pipe, $A_1 = \frac{\pi}{4}\left(d_1^{\,2} - d_2^{\,2}\right)$

$$= \frac{\pi}{4}\left(490^2 - 450^2\right) = 29531 \text{ mm}^2$$

Weight of the pipe per unit length, $w_1 = A_1 \times Sp.wt.of\ pipe$

$$= 29531 \times 70 \times 10^{-6} = 2.067 \text{ N/mm}$$

Cross sectional area of the water section,

$$A_2 = \frac{\pi}{4} \times d_2^2 = \frac{\pi}{4} \times 450^2 = 1.5904 \times 10^5 \text{ mm}^2$$

Weight of water per unit length, $w_2 = A_2 \times Sp.wt.of\ water$

$$= 1.5904 \times 10^5 \times 9.81 \times 10^{-6} = 1.56 \text{ N/mm}$$

(a) When the pipe is running full

Total weight per unit length, $w = w_1 + w_2 = 2.067 + 1.56 = 3.627$ N/mm

In case of simply supported beam subjected to a udl,

Maximum bending moment, $M = \frac{wl^2}{8}$

$$= \frac{3.627 \times 6000^2}{8} = 16.3215 \times 10^6 \text{ N-mm}$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d_1}{2} = \frac{490}{2} = 245 \text{ mm}$$

Moment of inertia, $I = \frac{\pi}{64}\left(d_1^{\,4} - d_2^{\,4}\right)$

$$= \frac{\pi}{64}\left(490^4 - 450^4\right) = 8.169 \times 10^8 \text{ mm}^4$$

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{16.3215 \times 10^6 \times 245}{8.169 \times 10^8} = \boxed{4.895 \text{ N/mm}^2}$$

(b) When the pipe is empty, only pipe weight is considered.

Weight per unit length, $w = w_1 = 2.067$ N/mm

In case of simply supported beam subjected to a udl,

Maximum bending moment, $M = \frac{wl^2}{8}$

$$= \frac{2.067 \times 6000^2}{8} = 9.3015 \times 10^6 \text{ N-mm}$$

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{9.3015 \times 10^6 \times 245}{8.169 \times 10^8} = \boxed{2.79 \text{ N/mm}^2}$$

Result : 1) Stress in the pipe when it is running full, f_{max} **= 4.895 N/mm²**
2) Stress in the pipe when it is empty, f_{max} **= 2.79 N/mm²**

CANTILEVER BEAMS

Example : 8.17 *(Oct.92, Apr.13, Apr.14)*

A cantilever of span 1.5m carries a point load of 5KN at the free end. Find the modulus of section required, if the bending stress is not to exceed 150 N/mm².

Given : Load at the free end, W = 5 KN = 5000 N
Length of the beam, l = 1.5 m = 1500 mm
Maximum bending stress, f_{max} = 150 N/mm²

To find : 1) Section modulus, Z

Solution :

In case of cantilever subjected to a point load at the free end,

Maximum bending moment, $M = Wl = 5000 \times 1500 = 7.5 \times 10^6$ N-mm

$$\text{Section modulus, } Z = \frac{M}{f_{max}} = \frac{7.5 \times 10^6}{150} = \boxed{50000 \text{ mm}^3}$$

Result : 1) Section modulus, Z = **50000 mm³**

Example : 8.18 *(Apr.90, Oct.16)*

A cantilever beam of span 2m carries a point load of 600N at the free end. If the cross–section of the beam is rectangular 100mm wide and 150mm deep, find the maximum bending stress induced.

Given :

Length of the beam, $l = 2 \text{ m} = 2000 \text{ mm}$

Load at the free end, $W = 600 \text{ N}$

Width of the beam, $b = 100 \text{ mm}$

Depth of the beam, $d = 150 \text{ mm}$

To find : 1) Maximum bending stress, f_{max}

Solution :

Moment of inertia, $I = \dfrac{bd^3}{12} = \dfrac{100 \times 150^3}{12} = 28.125 \times 10^6 \text{ mm}^4$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{150}{2} = 75 \text{ mm}$$

In case of cantilever subjected to a point load at the free end,

Maximum bending moment, $M = Wl = 600 \times 2000 = 1.2 \times 10^6 \text{N-mm}$

We know that, $\dfrac{M}{I} = \dfrac{f_{max}}{y_{max}}$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{1.2 \times 10^6 \times 75}{28.125 \times 10^6} = \boxed{3.2 \text{ N/mm}^2}$$

Result : 1) Maximum bending stress, $f_{max} = 3.2 \text{ N/mm}^2$

Example : 8.19

A cantilever beam is rectangular in section having 80mm width and 120mm depth. If the cantilever is subjected to a point load of 6KN at the free end and the bending stress is not to exceed 40N/mm², find the span of the cantilever beam.

Given :

Width of the beam, $b = 80 \text{ mm}$

Depth of the beam, $d = 120 \text{ mm}$

Point load, $W = 6 \text{ KN} = 6000 \text{ N}$

Maximum bending stress, $f_{max} = 40 \text{ N/mm}^2$

To find : 1) Span of the beam, l

Solution :

Moment of inertia, $I = \frac{bd^3}{12} = \frac{80 \times 120^3}{12} = 11.52 \times 10^6 \text{ mm}^4$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{120}{2} = 60 \text{ mm}$$

In case of cantilever subjected to a point load at the free end,

Maximum bending moment, $M = Wl = 6000\ l$

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$\frac{6000\ l}{11.52 \times 10^6} = \frac{40}{60}$$

$$l = \frac{40 \times 11.52 \times 10^6}{6000 \times 60} = 1280 \text{ mm} = \boxed{\mathbf{1.28\ m}}$$

Result : 1) Span of the beam, $\boldsymbol{l} = \mathbf{1.28\ m}$

Example : 8.20

A square beam 20mm × 20mm in section and 2m in long is supported at the ends. The beam fails when a point load of 400N is applied at the centre of the beam. What udl per metre will break a cantilever of the same material 40mm width and 60mm deep and 3m long.

(i) Simply supported beam

Given :

Width of the beam, b = 20 mm

Depth of the beam, d = 20 mm

Length of the beam, l = 2m = 2000 mm

Central point load, W = 400 N

To find : 1) Maximum bending stress, f_{max}

Solution :

Moment of inertia, $I = \frac{bd^3}{12} = \frac{20 \times 20^3}{12} = 1.333 \times 10^4 \text{ mm}^4$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{20}{2} = 10 \text{ mm}$$

In case of simply supported beam subjected to a point load,

Maximum bending moment, $M = \frac{Wl}{4} = \frac{400 \times 2000}{4} = 2 \times 10^5$ N-mm

We know that, $\frac{M}{I} = \frac{f_{max}}{y_{max}}$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{2 \times 10^5 \times 10}{1.222 \times 10^4} = \boxed{150 \text{ N/mm}^2}$$

Result : 1) Maximum bending stess, f_{max} = **150 N/mm²**

(ii) Cantilever beam

Given : Width of the beam, b = 40 mm
Depth of the beam, d = 60 mm
Length of the beam, l = 3m = 3000 mm

To find : 1) Safe udl spread over the entire span, w

Solution :

$$\text{Moment of inertia, } I = \frac{bd^3}{12} = \frac{40 \times 60^3}{12} = 7.2 \times 10^5 \text{ mm}^4$$

Distance of extreme layer from neutral axis (N.A.)

$$y_{max} = \frac{d}{2} = \frac{60}{2} = 30 \text{ mm}$$

For the same material, the bending stress should be equal

∴ Maximum bending stress in the beam, $f_{max} = 150 \text{ N/mm}^2$

In case of cantilever beam subjected to a udl over entire span,

$$\text{Maximum bending moment, } M = \frac{wl^2}{2} = \frac{\text{w} \times 3000^2}{2} = 4.5 \times 10^6 w \text{ N-mm}$$

$$\text{We know that, } \frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$\frac{4.5 \times 10^6 \, w}{7.2 \times 10^5} = \frac{150}{30}$$

$$w = \frac{150 \times 7.2 \times 10^5}{30 \times 4.5 \times 10^6} = 0.8 \text{ N/mm} = \boxed{0.8 \text{ KN/m}}$$

Result : 1) Safe udl spread over the entire span, w = **0.8 KN/m**

Example : 8.21 *(Oct.95)*

A beam of I–section 300mm × 150mm has flanges 20mm thick and web 13mm thick. Compare its flexural strength with that of a rectangular section of the same weight and same material, when the depth being twice the width.

Solution :

$$\text{Area of I–section} = (300 \times 20) + (13 \times 110) + (300 \times 20)$$
$$= 13430 \text{ mm}^2$$

Moment of inertia of the I–section,

$$I = \left[\frac{300 \times 150^3}{12}\right] - \left[\frac{(300-13) \times 110^3}{12}\right] = 52.542 \times 10^6 \text{ mm}^4$$

The section is symmetrical about X-X and Y–Y axis.

$$\therefore y_{max} = \bar{Y} = \frac{150}{2} = 75 \text{ mm}$$

Section modulus of I section, $Z_1 = \dfrac{I}{y_{max}}$

$$= \frac{52.542 \times 10^6}{75} = 7.0056 \times 10^5 \text{ mm}^3$$

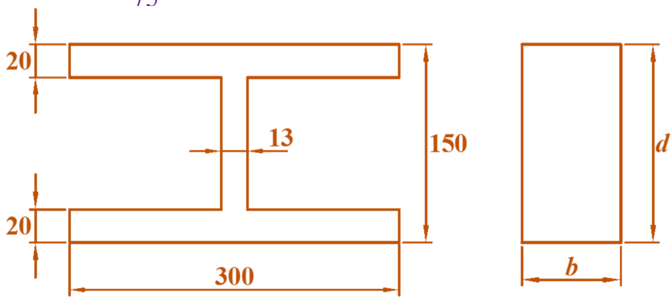


Fig.P8.3 Comparison of flexural strength [Example. 8.21]

Let, b = Width of the required rectangular section

d = Depth of the required rectangular section

Then, $d = 2b$

For same weight of two beams made of same material, the area of two beams must be equal.

$\therefore$ Area of I section = Area of rectangular section

$$13430 = bd = b(2b) = 2b^2$$

$$b^2 = \frac{13430}{2} = 6715$$

$$b = \boxed{\mathbf{81.945\ mm}}$$

$$d = 2b = 2 \times 81.945 = \boxed{\mathbf{163.89\ mm}}$$

Section modulus of rectangular section, $Z_2 = \dfrac{bd^2}{6}$

$$= \frac{81.945 \times 163.89^2}{6} = 3.668 \times 10^5 \text{ mm}^3$$

The strength of the beam is proportional to its section modulus.

$$\therefore \frac{\text{Flexural strength of I beam}}{\text{Flexural strength of rectangular beam}} = \frac{Z_1 \times E_1}{Z_2 \times E_2} = \frac{Z_1}{Z_2}$$

($\because$ For same material, $E_1 = E_2$)

$$= \frac{7.0056 \times 10^5}{3.668 \times 10^5} = \boxed{1.9099}$$

Result : 1) The ratio of flexural strength of two beams = **1.9099**

Example : 8.22

Compare the weights of two beams of same material and of equal flexural strengths, one being circular solid section and other being hollow circular section. The internal diameter being 7/8 of the external diameter.

Solution :

Let, D = Diameter of the solid beam

d_1 = External diameter of the hollow beam

d_2 = Internal diameter of the hollow beam

Then, $d_2 = \frac{7}{8} d_1 = 0.875\ d_1$

$$\text{Area of solid beam} = \frac{\pi}{4} D^2$$

$$\text{Area of hollow beam} = \frac{\pi}{4}\left(d_1^2 - d_2^2\right)$$

$$= \frac{\pi}{4}\left[d_1^2 - (0.875\ d_1)^2\right]$$

$$= \frac{\pi}{4}\left[d_1^2 - 0.765625\ d_1^2\right] = \frac{\pi}{4} \times 0.234375\ d_1^2$$

$$\text{Section modulus of solid beam, } Z_1 = \frac{\pi}{32} D^3$$

$$\text{Section modulus of hollow beam, } Z_2 = \frac{\pi}{32}\left[\frac{d_1^4 - d_2^4}{d_1}\right]$$

$$= \frac{\pi}{32 \times d_1}\left[d_1^4 - (0.875 d_1)^4\right]$$

$$= \frac{\pi}{32 \times d_1}\left[d_1^4 - 0.5862\ d_1^4\right]$$

$$= \frac{\pi}{32 \times d_1} \times 0.4138\ d_1^4 = \frac{\pi}{32} \times 0.4138\ d_1^3$$

Since both the beams have the same flexural strength, the section modulus of both the beams must be equal.

$$\therefore Z_1 = Z_2$$

$$\frac{\pi}{32} \times D^3 = \frac{\pi}{32} \times 0.4138\ d_1^3$$

$$D^3 = 0.4138\ d_1^3$$

Taking cube root on both sides,

$$D = 0.7452\, d_1$$

Weight of two beams are proportional to their cross sectional areas.

$$\frac{\text{Weight of solid beam}}{\text{Weight of hollow beam}} = \frac{\text{Area of solid beam}}{\text{Area of hollow beam}}$$

$$= \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} \times 0.234375\, d_1^{\,2}}$$

$$= \frac{(0.7452\, d_1)^2}{0.234375\, d_1^{\,2}}$$

$$= \frac{0.5553\, d_1^{\,2}}{0.234375\, d_1^{\,2}} = \boxed{2.369}$$

Result : 1) The ratio of weight of solid and hollow beams = **2.369**

PROBLEMS FOR PRACTICE

1. Determine the resisting moment of a timber beam, rectangular in section of size 100mm × 200mm, if the permissible bending stress is equal to 10N/mm^2 . *(Oct.93)* ***[Ans:*** $\boldsymbol{M = 6.667 \times 10^6}$ ***N-mm]***

2. A copper wire of 2mm diameter is required to be wound around a drum. Find the minimum radius of the drum, if the stress in the wire is not to exceed 80N/mm^2. Take modulus of elasticity of copper as 1 × 10^{5}N/mm^2. ***[Ans: R = 1.25mm]***

3. An alloy of wire of 5mm diameter is wound around a circular drum of 3m diameter. If the maximum bending stress in the wire is not to exceed 200N/mm^2, find the value of Young's modulus for the alloy. ***[Ans:*** $\boldsymbol{E = 1.2 \times 10^5\, N/mm^2}$***]***

SIMPLY SUPPORTED BEAMS

4. A rectangular beam of size 60mm × 100mm is subjected to a central concentrated load of 4KN on a simply supported span of 4m. Find the maximum bending stress. *(Oct.95)* ***[Ans:*** $f_{max} = 40 N/mm^2$***]***

5. A rectangular beam 300mm deep is simply supported over a span of 4m. What uniformly distributed load the beam may carry, if the bending stress is not to exceed 120N/mm^2. Take I = 2.25×10^6mm^4.

[Ans: w = 90KN/m]

6. A wooden beam 10m long, 360mm deep and 300mm wide is simply supported and loaded with a udl of 50KN/m over the entire span. Find the intensity of stress. ***[Ans: f_{max}= 96.45 N/mm^2]***

7. A rectangular beam simply supported over a span of 4m, is carrying an uniformly distributed load of 50KN/m. Find the dimension of the beam, if depth of the beam section is 2.5 times its width. Take maximum bending stress in the beam section as 60N/mm^2.

[Ans: b = 125mm, d = 300mm]

8. A rectangular timber beam 200mm×300mm in section is used as a simply supported beam over a span of 3m. Determine the udl including self weight of the beam can carry, if the bending stress is not to exceed 10N/mm^2. *(Oct.93)* ***[Ans: w = 26.667KN/m]***

9. A beam of rectangular section simply supported over a span of 4m carries an udl of 20KN/m run over the entire span and also a point load of 15KN at 1.5m from the right support. Calculate the width and depth of the beam, if the stress is not to exceed 120N/mm^2. Take the ratio of depth to width is 1.6

[Ans: b = 101mm, d = 162mm]

10. A wooden beam rectangular section 150mm × 300mm is simply supported over a span of 6m. What udl it may carry, if the maximum bending stress is not to exceed 7.5N/mm^2. What concentrated load may be carried by the beam at the centre with the same permissible stress. *(Oct.87)*

[Ans: w = 3.75KN/m, W = 11.25KN]

11. Find the dimensions of a timber joist of span 6m to carry a brick wall 230mm thick and 3m height, if the weight of the brick is 20KN/m^3 and the maximum permissible stress is limited to 80N/mm^2. The depth of the joist is twice its breadth. ***[Ans: b = 106mm, d = 212mm]***

12. A cast iron water pipe of 500mm inside diameter and 20mm thick is supported over a span of 10m. Find the maximum stress in the pipe metal, when the pipe is running full. Take density of cast iron as 70.6 KN/m^3 and that of water as 9.8 KN/m^3. ***[Ans: f_{max} = 12.9 N/mm^2]***

CANTILEVER BEAMS

13. A cantilever of span 1.2m carries a point load of 6KN at free end. Find the modulus of section required if the bending stress is not to exceed 120N/mm^2. *(Apr.01)* ***[Ans: Z=60000 mm^3]***

14. A cantilever of span 10m carries a point load of 25KN at the free end. If the cross section of the beam is rectangular of size 150mm $\times$ 300mm, find the maximum bending stress induced.
[Ans: f_{max}= 111.111N/mm^2]

15. A cast iron test beam 20mm $\times$ 20mm in section and 1m long and supported at the ends fails when a central load of 600N is applied. What udl will run over a cantilever of the same material 50mm wide, 100mm deep and 2m long. ***[Ans: w = 4.688KN/m]***

16. A beam of I –section 300mm $\times$ 150mm has flanges 15mm thick and web 8mm thick. Compare its flexural strength with that of a beam of (a) solid circular section (b) hollow circular section with inside diameter is equal to 0.6 times the outside diameter for the same weight and same material. ***[Ans: 0.2485, 0.4243]***

Unit – V

Chapter 9. TORSION OF CIRCULAR SHAFTS

9.1 Introduction

Power is generally transmitted through shafts. While transmitting power, a turning force is applied in a vertical plane perpendicular to the axis of the shaft. The product of this turning force and distance of its application from the centre of the shaft is known as *torque, turning moment* or *twisting moment*. A shaft of a circular section is said to be in torsion when it is subjected to torque.

9.2 Pure torsion

A circular shaft is said to be in a state of pure torsion when it is subjected to pure torque and not accompanied by any other force such as bending or axial force. Due to this torsion, the state of stress at any point in the cross–section is one of pure shear. The shearing stress thus induced in the shaft produces a moment of resistance, equal and opposite to the applied torque.

9.3 Assumption made in theory of pure torsion

The following assumptions are made in the theory of pure torsion which relates shear stress and the angle of twist to the applied torque.

1) The material of the shaft is uniform throughout.
2) The material of the shaft obeys Hooke’s law.
3) The shaft is of uniform circular section throughout.
4) The shaft is subjected to twisting couples whose planes are exactly perpendicular to the longitudinal axis.
5) The twist along the shaft is uniform.
6) Stresses do not exceed the limit of proportionality.
7) All diameters which are straight before twist remain straight after twist.
8) Normal cross–sections at the shaft, which were plane and circular before the twist, remain plane and circular after the twist.

9.4 Derivation of torsion equation

a) To prove $\dfrac{f_s}{r} = \dfrac{C\theta}{l}$

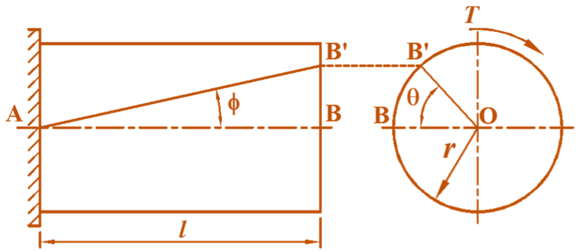


Fig.9.1 Shaft under pure torsion

Consider a shaft fixed at one end and subjected to a torque at the other end as shown in the fig.9.1.

Let, T = Torque

l = Length of the shaft

r = Radius of circular shaft

As a result of the torque, every cross section of the shaft is subjected to shear stresses. Let the line AB on the surface of the shaft be deformed to AB' and OB to OB' as shown in the fig.

Let, $\angle BAB' = \phi$ in degrees

$\angle BOB' = \theta$ in radians

f_s = Shear stress induced in the surface

C = Modulus of rigidity of the shaft material.

We know that,

$$\text{Shear strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{BB'}{l} = \tan\phi = \phi \quad \text{------- (1)}$$

Since ϕ is very small, $\tan\phi = \phi$

We also know that, $arc\ BB' = r\theta$

$$\phi = \frac{BB'}{l} = \frac{r\theta}{l} \quad \text{------- (2)}$$

If f_s is the intensity of shear stress on the outermost layer, then

$$\text{Modulus of rigidity, } C = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{f_s}{\phi}$$

$$\phi = \frac{f_s}{C} \quad \text{------- (3)}$$

Equating (2) and (3) $\Rightarrow \dfrac{f_s}{C} = \dfrac{r\theta}{l}$ (or) $\boxed{\dfrac{f_s}{r} = \dfrac{C\theta}{l}}$

Since θ, C and l are constants, the intensity of stress at any section of the shaft is proportional to the distance of the point from the axis of the shaft.

$$i.e.\quad \frac{f_1}{r_1} = \frac{f_2}{r_2} = \cdots = \frac{f_s}{r}$$

b) To prove $\dfrac{T}{J} = \dfrac{C\theta}{l}$

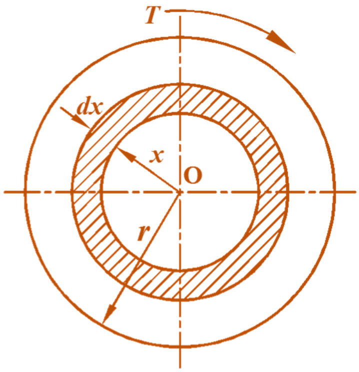


Fig.9.2 Shaft under pure torsion

Consider a shaft subjected to torque T as shown in the fig.9.2

Consider an elemental area 'da' of thickness 'dx' at a distance 'x' from the centre of the shaft.

Let, r = Radius of the shaft and

f_s = Shear stress developed in the outermost layer of the shaft.

Shear stress at this section $= f_s \times \dfrac{x}{r}$

Area of the elemental strip, $da = 2\pi x \times dx$

Turning force on the elemental area = Shear stress × Area

$$= f_s \frac{x}{r} \times 2\pi x\, dx$$

$$= \frac{2\pi}{r} \times f_s (x^2 dx)$$

Turning moment (torque) of this element,

dT = Shear force × Distance of element from axis

$$= \frac{2\pi}{r} f_s \left(x^2 dx\right) x = \frac{2\pi}{r} f_s\, x^3 dx$$

Total torque can be found out by integrating the above equation between '0' and 'r'.

$$T = \int_0^r \frac{2\pi f_s}{r} x^3 dx = \frac{2\pi f_s}{r}\left[\frac{x^4}{4}\right]_0^r = \frac{2\pi}{r} f_s \times \frac{r^4}{4}$$

$$T = \frac{\pi}{2} f_s r^3 = \frac{\pi}{16} f_s d^3 \quad \left(\because r = \frac{d}{2}\right)$$

$$f_s = \frac{16T}{\pi d^3} \qquad \text{--------- (1)}$$

We know that, $\frac{f_s}{r} = \frac{C\theta}{l}$ -------- (2)

Substituting the value of f_s in equation (2)

$$\frac{16T}{\pi d^3 \times \frac{d}{2}} = \frac{C\theta}{l} \Rightarrow \frac{T}{\frac{\pi}{32} d^4} = \frac{C\theta}{l}$$

$$\boxed{\frac{T}{J} = \frac{C\theta}{l}} \qquad \text{---------(3)}$$

Where, $J = \frac{\pi}{32} d^4$ which is known as ***polar moment of inertia***

Combining equation (2) and (3) $\Rightarrow$ $\boxed{\frac{T}{J} = \frac{f_s}{r} = \frac{C\theta}{l}}$

The above relation can be rewritten as $\Rightarrow$ $\boxed{\frac{T}{J} = \frac{f_s}{r}; \quad \frac{T}{J} = \frac{C\theta}{l}}$

9.5 Strength of hollow shaft

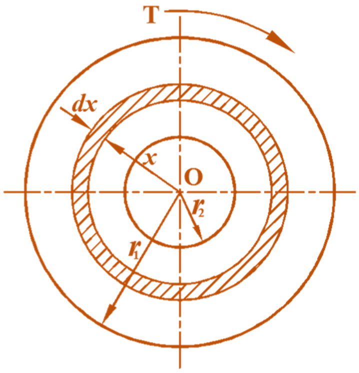


Fig.9.3 Hollow circular shaft subjected to pure torsion

Consider a hollow shaft subjected to toque 'T' as shown in the fig.9.3. Let r_1 and r_2 be the outside and inside radius of the hollow shaft respectively. Let us consider an elemental area 'da' at distance 'x' from the centre of the shaft and of thickness 'dx' as shown in the fig.

Area of the elemental strip, $da = 2\pi x . dx$

Shear stress at this section, $f_x = f_s \dfrac{x}{r_1}$

Turning force = Stress × Area $= f_s \dfrac{x}{r_1} \; 2\pi x \, dx = \dfrac{f_s}{r_1} \, 2\pi x^2 dx$

Turning moment (torque) of this element,

dT = Shear force × Distance of element from axis

$$= \frac{2\pi}{r_1} f_s \, x^2 dx \, . x = \frac{2\pi}{r_1} \, f_s \, x^3 dx$$

Total torque can be found out by integrating the above equation between r_2 and r_1 .

$$T = \int_{r_2}^{r_1} \frac{2\pi f_s}{r_1} \, x^3 dx = \frac{2\pi f_s}{r_1} \left[\frac{x^4}{4}\right]_{r_2}^{r_1}$$

$$= \frac{2\pi f_s}{r_1} \left[\frac{r_1^4}{4} - \frac{r_2^4}{4}\right]$$

$$= \frac{2\pi f_s}{(d_1/2)} \left[\frac{(d_1/2)^4 - (d_1/2)^4}{4}\right]$$

$$= \frac{4\pi f_s}{d_1} \left[\frac{d_1^4 - d_2^4}{4 \times 16}\right]$$

$$\boldsymbol{T = \frac{\pi}{16} f_s \left[\frac{d_1^4 - d_2^4}{d_1}\right]}$$

9.6 Stress distribution in the shaft under pure torsion

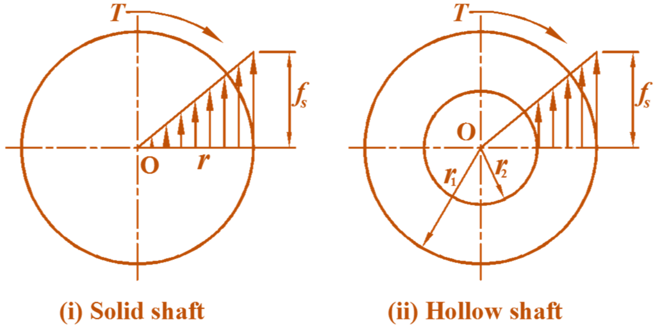


Fig.9.4 Shear stress distribution

The intensity of shear stress at any point in the cross–section of a shaft subjected to pure torsion is proportional to its distance from the centre. In other words, the shear intensity is zero at the axis of the shaft and increases linearly to maximum of f_s at the surface. The shear stress at any point on the circumference is same. The intensity of shear stress in hollow shaft is more or less uniform throughout the section.

9.7 Power transmitted by the shaft

Consider a rotating shaft which transmits power from one of its ends to another.

Let, N = Speed of the shaft in rpm and

T = Average torque in KN–m

Work done per minute = Force × Distance

$$= T \times 2\pi\ N = 2\pi\ N\ T$$

$$\therefore \text{ Work done per second } = \frac{2\pi\ N\ T}{60} \ (\text{KN–m})$$

Power transmitted = Work done per second

$$\boxed{P = \frac{2\pi N\ T}{60} \quad (\text{KW})}$$

9.8 Polar modulus

The ratio between the polar moment of inertia of the cross– section of the shaft and the maximum radius of the section is known as polar modulus or polar section modulus. It is an important parameter, generally used in the design of shaft. It is denoted by Z.

$$Z = \frac{\text{Polar moment of inertia}}{\text{Maximum radius}} = \frac{J}{r}$$

For a solid circular shaft, $J = \frac{\pi}{32} d^4$; $r = \frac{d}{2}$

$$\boxed{Z = \frac{J}{r} = \frac{\frac{\pi}{32} d^4}{(d/2)} = \frac{\pi d^3}{16}}$$

For a hollow circular shaft, $J = \frac{\pi}{32}(d_1^4 - d_2^4)$; $r_1 = \frac{d_1}{2}$

$$\boxed{Z = \frac{J}{r_1} = \frac{\frac{\pi}{32}(d_1^4 - d_2^4)}{(d_1/2)} = \frac{\pi}{16\ d_1}(d_1^4 - d_2^4)}$$

9.9 Torsional strength

It is defined as the torque developed per unit maximum shear stress. Torsional strength is also known as the *efficiency* of a shaft.

$$\text{Torsional strength} = \frac{T}{f_s}$$

$$\text{From the equation } \frac{T}{J} = \frac{f_s}{r}$$

$$\boldsymbol{\frac{T}{f_s} = \frac{J}{r} = Z}$$

Therefore, torsional strength may also be represented by the section modulus. For a given material and weight, a hollow shaft withstands larger value of torque when compared to that of solid shaft. Because for a given cross–sectional area, hollow circular section has larger section modulus when compared to that of solid circular section.

9.10 Torsional rigidity or stiffness

Torsional rigidity or stiffness is defined as the torque required to produce an unit angle of twist in a specified length of the shaft.

$$\text{Torsional rigidity} = \frac{T}{\theta}$$

$$\text{From the equation } \frac{T}{J} = \frac{C\theta}{l}$$

$$\boldsymbol{\frac{T}{\theta} = \frac{CJ}{l}}$$

From the above equation it is evident that torsional rigidity or stiffness is the product of modulus of rigidity and polar moment of inertia over a unit length of the shaft. For a given cross–sectional area, torsional rigidity of a hollow circular shaft is larger when compared to that of solid circular shaft.

9.11 Comparison of hollow shaft and solid shaft

Let, d = Diameter of the solid shaft

d_1 = Outside diameter of the hollow shaft

d_2 = Inside diameter of the hollow shaft

a) Comparison by strength consideration

$$\frac{\text{Strength of the hollow shaft}}{\text{Strength of the solid shaft}} = \frac{\text{Section modulus of hollow shaft}}{\text{Section modulus of solid shaft}}$$

$$= \frac{\frac{\pi}{16\, d_1}\left(d_1^{\,4} - d_2^{\,4}\right)}{\frac{\pi d^3}{16}} = \frac{\left(d_1^{\,4} - d_2^{\,4}\right)}{d_1 \times d^3}$$

For a given cross–sectional area a hollow circular shaft has larger value of section modulus when compared with that of a solid circular shaft. So the hollow shaft has more strength than that of a solid shaft.

b) Comparison by weight consideration

Let, l = Length of both the solid and hollow shaft

ρ = Density of both the material of solid and hollow shaft

A_s = Cross–sectional area of the solid shaft

A_h = Cross–sectional area of the hollow shaft

Weight of the solid shaft, W_s = Density × Volume

$$= \rho \times l \times A_s = \rho\, l\, \frac{\pi}{4} d^2$$

Weight of the hollow shaft, W_h = Density × Volume

$$= \rho \times l \times A_h = \rho\, l\, \frac{\pi}{4}(d_1^{\,2} - d_2^{\,2})$$

$$\frac{\text{Weight of the solid shaft}}{\text{Weight of the hollow shaft}} = \frac{\rho\, l\, \frac{\pi}{4} d^2}{\rho\, l\, \frac{\pi}{4}(d_1^{\,2} - d_2^{\,2})} = \frac{d^2}{(d_1^{\,2} - d_2^{\,2})}$$

For a given material, length and torsional strength, the weight of a hollow shaft is less than that of a solid shaft. When using hollow shaft, the material requirement is considerably reduced.

$$\% \text{ Saving in material} = \frac{W_s - W_h}{W_s} \times 100 = \frac{A_s - A_h}{A_s} \times 100$$

9.12 Advantages of hollow shaft over solid shaft

The following are the advantages of hollow shaft over solid shaft.

1) A hollow shaft has greater torsional strength than a solid shaft of same material.
2) A hollow shat has more stiffness than a solid shaft of same cross–sectional area.
3) The material required for hollow shaft is comparatively lesser than the solid shaft for same strength.
4) Hollow shaft is lighter in weight than a solid shaft of equal strength.
5) The removal of core from large shafts increase their reliability.
6) The material in the hollow shaft is effectively utilized.
7) The shear stress induced in the hollow shaft is almost uniform throughout the section.

REVIEW QUESTIONS

1. What is pure torsion?
2. State the assumptions made in theory of pure torsion.
   (Apr.01, Oct.01, Oct.04, Apr.17)
3. Indicate the shear stress distribution on the cross section of a shaft subjected to pure torsion with a sketch. *(Apr.04, Oct.01)*
4. Derive an equation to determine the torque in a circular shaft.
   (Apr.01, Apr.05)
5. Derive the relation $\frac{T}{J} = \frac{f_s}{r} = \frac{C\theta}{l}$ *(Oct.02, Apr.05)*
6. Define polar modulus and torsional rigidity. *(Oct.03, Oct.16)*
7. Derive polar modulus for (i) solid shaft (ii) hollow shaft. *(Oct.17)*
8. Distinguish between flexural rigidity and torsional rigidity. *(Oct.02)*
9. Explain the strength and stiffness of the shaft. *(Apr.17, Apr.18)*
10. Derive the equation for power transmitted by a solid shaft. *(Apr.04)*
11. Compare hollow and solid shafts. *(Oct.04)*
12. What are the advantages of hollow shaft over a solid shaft.
    (Oct.96. Apr.02, Apr.03, Oct.16, Apr.18)

POINTS TO REMEMBER

1) Torsion equation $\dfrac{T}{J} = \dfrac{f_s}{r} = \dfrac{C\theta}{l}$

2) Torque in solid shaft, $T = \dfrac{\pi}{16} f_s d^3$

3) Torque in hollow shaft, $T = \dfrac{\pi}{16} f_s \left[\dfrac{d_1^4 - d_2^4}{d_1}\right]$

4) Power transmitted, $P = \dfrac{2\pi\ N\ T}{16}$

5) Polar modulus of solid shaft, $Z = \dfrac{\pi}{16} d^3$

6) Polar modulus of hollow shaft, $Z = \dfrac{\pi}{16\ d_1}(d_1^4 - d_2^4$

7) % Saving in material $= \dfrac{A_s - A_h}{A_s} \times 100$

Where,
T = Torque (N–mm)
J = Polar moment of inertia (mm^4)
f_s = Shear stress (N/mm^2)
r = Radius of the shaft (mm)
C = Modulus of rigidity (N/mm^2)
θ = Angle of twist (radians)
l = Length of the shaft (mm)
d = Diameter of solid shaft (mm)
d_1 = Outside diameter of hollow shaft (mm)
d_2 = Inside diameter of hollow shaft (mm)
N = Speed of the shaft (rpm)
A_s = Cross–sectional area of the solid shaft (mm^2)
A_h = Cross–sectional area of the hollow shaft (mm^2)

SOLVED PROBLEMS

Example : 9.1 *(Apr.01)*

Calculate the torque in a solid circular shaft 120mm diameter, if the shear stress is not to exceed $80 N/mm^2$.

Given : Diameter of shaft, $d = 120$ mm

Maximum shear stress, $f_s = 80$ N/mm^2

To find : 1) Torque, T

Solution :

Torque in a solid circular shaft,

$$T = \frac{\pi}{16} f_s \, d^3 = \frac{\pi}{16} \times 80 \times 120^3 = \boxed{27.143 \times 10^6 \text{ N-mm}}$$

Result : 1) Torque in the shaft, $\mathbf{T = 27.143 \times 10^6}$ **N-mm**

Example : 9.2

A solid steel shaft is to transmit a torque of 10KN–m. If the shearing stress is not to exceed $45 N/mm^2$, find the minimum diameter of the shaft.

Given : Torque, $T = 10$ KN–m $= 10 \times 10^6$ N–mm

Maximum shearing stress, $f_s = 45$ N/mm^2

To find : 1) Minimum diameter of shaft, d

Solution :

Torque in a solid circular shaft,

$$T = \frac{\pi}{16} f_s \, d^3$$

$$d^3 = \frac{16 \times T}{\pi \times f_s} = \frac{16 \times 10 \times 10^6}{\pi \times 45} = 1.13177 \times 10^6$$

$$d = \boxed{104 \text{ mm}}$$

Result : 1) Minimum diameter of the shaft, $\mathbf{d = 104}$ **mm**

Example : 9.3

A hollow shaft of external and internal diameter of 80mm and 50mm is required to transmit torque from one end to the other. What is the safe torque it can transmit, if the allowable shear stress is $45 N/mm^2$?

Given : External diameter of the shaft, $d_1 = 80$ mm

Inter diameter of the shaft, $d_2 = 50$ mm

Allowable shear stress, $f_s = 45$ N/mm^2

To find : 1) Torque transmitted by the shaft, T

Solution :

Torque transmitted by the hollow circular shaft,

$$T = \frac{\pi}{16} \times f_s \times \frac{(d_1^4 - d_2^4)}{d_1} = \frac{\pi \times 45}{16} \times \frac{80^4 - 50^4}{80}$$

$$= \boxed{3.834 \times 10^6 \text{ N-mm}}$$

Result : 1) Torque transmitted by the shaft, $T = 3.834 \times 10^6$ **N-mm**

Example : 9.4 *(Oct.12, Apr.15, Apr.17)*

Calculate the power transmitted by a shaft 100 mm diameter running at 250 rpm, if the shear stress in the shaft material is not to exceed 75 N/mm²^2***.***

Given : Diameter of the shaft, d = 100 mm

Speed of the shaft, N = 250 rpm

Maximum shear stress, f_s = 75 N/mm^2

To find : 1) Power transmitted by the shaft, P

Solution :

Torque transmitted by the shaft,

$$T = \frac{\pi}{16} f_s\, d^3 = \frac{\pi}{16} \times 75 \times 100^3 = 14.726 \times 10^6 \text{ N-mm} = 14.726 \text{ KN-m}$$

Power transmitted by the shaft,

$$P = \frac{2\,\pi\,N\,T}{60} = \frac{2 \times \pi \times 250 \times 14.726}{60} = \boxed{385.53 \text{ KW}}$$

Result : 1) The power transmitted by the shaft, P **= 385.53 KW**

Example : 9.5 *(Oct.13)*

A hollow shaft of external and internal diameters as 100mm and 40mm is transmitting power at 120 rpm. Find the power the shaft can transmit, if the shearing stress is not to exceed 50 N/mm^2***.***

Given : External diameter of the shaft, d_1 = 100 mm

Inter diameter of the shaft, d_2 = 40 mm

Speed of the shaft, N = 120 rpm

Allowable shear stress, f_s = 50 N/mm^2

To find : 1) Power transmitted by the shaft, P

Solution :

Torque transmitted by the hollow circular shaft,

$$T = \frac{\pi}{16} \times f_s \times \frac{(d_1^4 - d_2^4)}{d_1} = \frac{\pi \times 50}{16} \times \frac{100^4 - 40^4}{100}$$

$$= 9.566 \times 10^6 \text{ N-mm} = 9.566 \text{ KN-m}$$

Power which can be transmitted by the shaft,

$$P = \frac{2\,\pi\,N\,T}{60} = \frac{2 \times \pi \times 120 \times 9.566}{60} = \boxed{\mathbf{120.21\ KW}}$$

Result : 1) Power transmitted by the shaft, $\boldsymbol{P}$ **= 120.21 KW**

Example : 9.6

A solid circular shaft of 100mm diameter is transmitting 120KW at 150 rpm. Find the intensity of shear stress in the shaft.

Given : Diameter of the shaft, d = 100 mm
Power transmitted, P = 120 KW
Speed of the shaft, N = 150 rpm

To find : 1) Intensity of shear stress, f_s

Solution :

Power transmitted by the shaft,

$$P = \frac{2\,\pi\,N\,T}{60}$$

$$T = \frac{P \times 60}{2 \times \pi \times N} = \frac{120 \times 60}{2 \times \pi \times 150} = 7.639 \text{ KN-m} = 7.639 \times 10^6 \text{ N-mm}$$

Also, torque transmitted by the shaft,

$$T = \frac{\pi}{16} f_s\, d^3$$

$$f_s = \frac{16 \times T}{\pi \times d^3} = \frac{16 \times 7.639 \times 10^6}{\pi \times 100^3} = \boxed{\mathbf{38.905\ N/mm^2}}$$

Result : 1) Intensity of shear stress, $\boldsymbol{f_s}$ **= 38.905 N/mm²**

Example : 9.7

(Oct.17)

A hollow circular shaft of 25 mm outside diameter and 20 mm inside diameter is subjected to a torque of 50 N-m. Find the shear stress induced at the outside and inside layer of shaft.

Given : Outside diameter, $d_1 = 25$ mm
Inside diameter, $d_2 = 20$ mm
Torque transmitted, $T = 50$ N-m $= 50 \times 10^3$ N-mm

To find : 1) Shear stress at outside layer, f_{s1}
2) Shear stress at inside layer, f_{s2}

Solution :

Polar moment of inertia,

$$J = \frac{\pi}{32}\left[d_1^4 - d_2^4\right] = \frac{\pi}{32}\left[25^4 - 20^4\right] = 22641.556 \text{ mm}^4$$

We know that, $\dfrac{T}{J} = \dfrac{f_s}{r} \implies f_s = \dfrac{T}{J} \times r$

At the outside layer, $r = r_1 = \dfrac{d_1}{2} = \dfrac{25}{2} = 12.5$ mm

$$f_{s1} = \frac{T}{J} \times r_1 = \frac{50 \times 10^3}{22641.556} \times 12.5 = \boxed{27.6 \text{ N/mm}^2}$$

At the inside layer, $r = r_2 = \dfrac{d_2}{2} = \dfrac{20}{2} = 10$ mm

$$f_{s2} = \frac{T}{J} \times r_2 = \frac{50 \times 10^3}{22641.556} \times 10 = \boxed{22.08 \text{ N/mm}^2}$$

Result : 1) Shear stress at outside layer, $f_{s1} = 27.6$ N/mm^2
2) Shear stress at inside layer, $f_{s2} = 22.08$ N/mm^2

Example : 9.8

A hollow shaft is to transmit 200KW at 80 rpm. If the stress is not to exceed 60N/mm^2 and internal diameter is 0.6 times of the external diameter, find the diameter of the shaft.

Given : Power transmitted, $P = 200$ KW $= 200 \times 10^6$ N-mm/s
Speed of the shaft, $n = 80$ rpm
Allowable shear stress, $f_s = 60$ N/mm^2
Internal diameter, $d_2 = 0.6 \times$ External diameter (d_1)

To find : 1) External diameter, d_1 2) Internal diameter, d_2

Solution :

Torque transmitted by the hollow circular shaft,

$$T = \frac{\pi}{16} \times f_s \times \frac{(d_1^4 - d_2^4)}{d_1}$$

$$= \frac{\pi \times 60}{16} \times \frac{d_1^4 - (0.6 d_1)^4}{d_1} = 10.254\, d_1^3 \text{ N-mm}$$

Power transmitted by the shaft,

$$P = \frac{2\,\pi\,N\,T}{60} = \frac{2\,\pi \times 80 \times 10.254\,d_1^3}{60} = 85.904\,d_1^{\,3}$$

$$200 \times 10^6 = 85.904\,d_1^{\,3}$$

$$d_1^{\,3} = \frac{200 \times 10^6}{85.904} = 2.328 \times 10^6$$

$$d_1 = \boxed{132.5\text{ mm}}\ ; \quad d_2 = 0.6 \times d_1 = 0.6 \times 132.5 = \boxed{79.5\text{ mm}}$$

Result : 1) External diameter, d_1 **= 132.5 mm**
2) The internal diameter, d_2 **= 79.5 mm**

Example : 9.9

(Apr.93)

A solid circular shaft has to transmit a power of 40KW at 120rpm. The permissible shear stress is 100 N/mm². Determine the diameter of the shaft, if the maximum torque exceeds the mean torque by 25%.

Given : Power transmitted, P = 40 KW
Shear stress, f_s = 100 N/mm^2
Maximum torque, T_{max} = 1.25 × Mean torque = 1.25 T_{mean}

To find : 1) Diameter of shaft, d

Solution :

Power transmitted by the shaft,

$$P = \frac{2\,\pi\,N\,T_{mean}}{60}$$

$$T_{mean} = \frac{P \times 60}{2 \times \pi \times N} = \frac{40 \times 60}{2 \times \pi \times 120} = 3.183\text{ KN-m} = 3.183 \times 10^6\text{ N-mm}$$

$$T_{max} = 1.25 \times T_{mean} = 1.25 \times 3.183 \times 10^6 = 3.979 \times 10^6\text{ N-mm}$$

Torque transmitted by the shaft,

$$T_{max} = \frac{\pi}{16} f_s\,d^3$$

$$d^3 = \frac{16 \times T_{max}}{\pi \times f_s} = \frac{16 \times 3.979 \times 10^6}{\pi \times 100} = 202648.806$$

$$d = \boxed{58.737\text{ mm}}$$

Result : 1) Diameter of shaft, d **= 58.737 mm**

Example : 9.10 *(Oct.91, Oct.96)*

Find the torque transmitted by (i) solid shaft of diameter 0.4m (ii) hollow shaft of external diameter 0.4m and internal diameter 0.2m, if the angle of twist is not to exceed 1° in a length of 10m. Take $C = 0.8 \times 10^5 \ N/mm^2$.

Given : Angle of twist, $\theta = 1° = 1 \times (\pi/180) = 0.01745$ rad.

Modulus of rigidity, $C = 0.8 \times 10^5$ N/mm^2

Length of the shaft, $l = 10$ m $= 10000$ mm

To find : 1) Torque transmitted, T

Solution :

(i) Solid shaft

Diameter of the shaft, $d = 0.4$ m $= 400$mm

Polar moment of inertia, $J = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 400^4 = 25.133 \times 10^8$ mm^4

Relation for troque transmitted by the shaft,

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$T = \frac{C\theta}{l} \times J = \frac{0.8 \times 10^5 \times 0.01745 \times 25.133 \times 10^8}{10000}$$

$$= 3.509 \times 10^8 \text{ N-mm} = 3.509 \times 10^2 \text{ KN-m} = \boxed{350.9 \text{ KN-m}}$$

(ii) Hollow shaft

External diameter of the shaft, $d_1 = 0.4$ m $= 400$mm

Internal diameter of the shaft, $d_2 = 0.2$ m $= 200$ mm

Polar moment of inertia, $J = \frac{\pi}{32}(d_1^4 - d_2^4) = \frac{\pi}{32}(400^4 - 200^4)$

$$= 23.562 \times 10^8 \text{ mm}^4$$

Relation for troque transmitted by the shaft,

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$T = \frac{C\theta}{l} \times J = \frac{0.8 \times 10^5 \times 0.01745 \times 23.562 \times 10^8}{10000}$$

$$= 3.289 \times 10^8 \text{ N-mm} = 3.289 \times 10^2 \text{ KN-m} = \boxed{328.9 \text{ KN-m}}$$

Result : 1) Torque transmitted by solid shaft, T = **350.9 KN–m**

2) Torque transmitted by hollow shaft, T = **328.9 KN–m**

Example : 9.11

Find the angle of twist per metre length of a hollow shaft of 100mm external diameter and 60mm internal diameter, if the shear stress is not to exceed 35N/mm². Take $C = 85 \times 10^3 N/mm^2$.

Given :

Length of the shaft, l = 1m = 1000 mm

External diameter, d_1 = 100 mm

Internal diameter, d_2 = 60 mm

Maximum shear stress, f_s = 35 N/mm^2

Modulus of rigidity, C = 85×10^3N/mm^2

To find : 1) Angle of twist, θ

Solution :

Torque transmitted by the hollow circular shaft,

$$T = \frac{\pi}{16} \times f_s \times \frac{(d_1^4 - d_2^4)}{d_1} = \frac{\pi \times 35}{16} \times \frac{100^4 - 60^4}{100} = 5.9816 \times 10^6 \text{ N-mm}$$

Polar moment of inertia, $J = \frac{\pi}{32}(d_1^4 - d_2^4) = \frac{\pi}{32}(100^4 - 60^4)$

$$= 8.545 \times 10^6 \text{mm}^4$$

Relation for angle of twist,

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\theta = \frac{T\ l}{C\ J} = \frac{5.9816 \times 10^6 \times 1000}{85 \times 10^3 \times 8.545 \times 10^6}$$

$$= 8.235 \times 10^{-3} \text{rad.} = 8.235 \times 10^{-3} \times \frac{180}{\pi} = \boxed{0.472^\circ}$$

Result : 1) Angle of twist in the shaft, $\theta = \mathbf{0.472^\circ}$

Example : 9.12

A solid shaft of 120mm diameter is required to transmit 200KW at 100 rpm. If the angle of twist is not to exceed 2°, find the length of the shaft. Take $C = 90 \times 10^3 N/mm^2$.

Given :

Diameter of the shaft, d = 120 mm

Power transmitted, P = 200 KW

Speed of the shaft, N = 100 rpm

Angle of twist, θ = $2^\circ = 2 \times (\pi/180) = 0.0349$ rad.

Modulus of rigidity, C = 90×10^3N/mm^2

To find : 1) Length of shaft, l

Solution :

Power transmitted by the shaft, $P = \dfrac{2\,\pi\,N\,T}{60}$

$$T = \frac{P \times 60}{2 \times \pi \times N} = \frac{200 \times 60}{2 \times \pi \times 100} = 19.1 \text{ KN-m} = 19.1 \times 10^6 \text{ N-mm}$$

Polar moment of inertia, $J = \dfrac{\pi}{32} d^4 = \dfrac{\pi}{32} \times 120^4 = 20.358 \times 10^6 \text{mm}^4$

Relation for length of the shaft,

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$l = \frac{C\theta \times J}{T} = \frac{90 \times 10^3 \times 0.0349 \times 20.358 \times 10^6}{19.1 \times 10^6} = \boxed{3347.878 \text{ mm}}$$

Result : 1) Length of shaft, $\boldsymbol{l}$ **= 3347.878 mm = 3.348 m**

Example : 9.13 ***(Oct.04, Oct.13, Oct.18)***

A solid shaft 20mm diameter transmits 10KW at 1200 rpm. Calculate the maximum intensity of shear stress induced and the angle of twist in degrees in a length of 1m, if modulus of rigidity for the material of the shaft is $\mathbf{8 \times 10^4}$ ***N/mm***$^{\mathbf{2}}$.

Given : Diameter of the shaft, d = 20 mm

Power transmitted, P = 10 KW

Speed of the shaft, N = 1200 rpm

Length of the shaft, l = 1 m = 1000 mm

Modulus of rigidity, C = $8 \times 10^4 \text{N/mm}^2$

To find : 1) Shear stress, f_s 2) Angle of twist, θ

Solution :

Power transmitted by the shaft,

$$P = \frac{2\,\pi\,N\,T}{60}$$

$$T = \frac{P \times 60}{2 \times \pi \times N} = \frac{10 \times 60}{2 \times \pi \times 1200}$$

$$= 79.577 \times 10^{-3} \text{ KN-m} = 79.577 \times 10^3 \text{ N-mm}$$

Torque transmitted by the shaft, $T = \dfrac{\pi}{16} f_s\, d^3$

$$f_s = \frac{16 \times T}{\pi \times d^3} = \frac{16 \times 79.577 \times 10^3}{\pi \times 20^3} = \boxed{50.66 \text{ N/mm}^2}$$

Polar moment of inertia, $J = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 20^4 = 15.708 \times 10^3 \text{mm}^4$

Relation for angle of twist $\Longrightarrow \quad \frac{T}{J} = \frac{C\theta}{l}$

$$\theta = \frac{T\ l}{C\ J} = \frac{79.577 \times 10^3 \times 1000}{8 \times 10^4 \times 15.708 \times 10^3}$$

$$= 0.0633 \text{ rad.} = 0.0633 \times \frac{180}{\pi} = \boxed{3.628^\circ}$$

Result : 1) Shear stress induced, f_s = **50.66 N/mm²**
2) Angle of twist, θ = **3.628°**

Example : 9.14 *(Apr.04)*

Calculate the power transmitted by a shaft of diameter 150mm at 120 rpm, if the maximum shear stress is not to exceed 80 N/mm². What will be the angle of twist in a length of 10m? Take $C = 0.84 \times 10^5\ N/mm^2$.

Given :

Diameter of the shaft, d	=	150 mm
Speed of the shaft, N	=	120 rpm
Maximum shear stress, f_s	=	80 N/mm²
Length of the shaft, l	=	10 m = 10000 mm
Modulus of rigidity, C	=	0.84×10^5 N/mm²

To find : 1) Power transmitted, P 2) Angle of twist, θ

Solution :

Torque transmitted by the shaft,

$$T = \frac{\pi}{16} f_s\, d^3 = \frac{\pi}{16} \times 80 \times 150^3 = 53.014 \times 10^6 \text{ N-mm} = 53.014 \text{ KN-m}$$

Power transmitted by the shaft,

$$P = \frac{2\ \pi\ N\ T}{60} = \frac{2 \times \pi \times 120 \times 53.014}{60} = \boxed{666.194 \text{ KW}}$$

Polar moment of inertia, $J = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 150^4 = 49.7 \times 10^6 \text{mm}^4$

Relation for angle of twist $\Longrightarrow \quad \frac{T}{J} = \frac{C\theta}{l}$

$$\theta = \frac{T\ l}{C\ J} = \frac{53.014 \times 10^6 \times 10000}{0.84 \times 10^5 \times 49.7 \times 10^6}$$

$$= 0.127 \text{ rad.} = 0.127 \times \frac{180}{\pi} = \boxed{7.276^\circ}$$

Result : 1) Power transmitted, P = **666.194 KW**
2) Angle of twist, $\theta = 7.276^\circ$

Example : 9.15 *(Apr.04)*

Find the maximum torque that can be applied to a shaft of 80mm diameter. The permissible angle of twist is 1.5° in a length of 5m and shear stress not to exceed $42\,N/mm^2$. Take $C = 84\times10^3\,N/mm^2$.

Given : Diameter of shaft, d = 80 mm

Angle of twist, θ = 1.5° = 1.5 × (π /180) = 0.02618 rad.

Length of the shaft, l = 5 m = 5000 mm

Maximum shear stress, f_s = 42 N/mm^2

Modulus of rigidity, C = 84×10^3 N/mm^2

To find : 1) Torque that can be applied, T

Solution :

(a) Torque based on shear stress.

$$T_1 = \frac{\pi}{16} f_s\, d^3 = \frac{\pi}{16}\times 42\times 80^3 = \boxed{4.222\times10^6 \text{ N-mm}}$$

(b) Torque based on angle of twist

Polar moment of inertia, $J = \frac{\pi}{32} d^4 = \frac{\pi}{32}\times 80^4 = 4.021\times10^6\text{mm}^4$

Relation for torque $\Longrightarrow \frac{T_2}{J} = \frac{C\theta}{l}$

$$T_2 = \frac{C\,\theta\times J}{l} = \frac{84\times10^3\times0.02618\times4.021\times10^6}{5000}$$

$$= \boxed{1.769\times10^6 \text{ N-mm}}$$

We shall apply the torque which is lesser.

i. e $T = T_2 = \mathbf{1.769\times10^6}$ **N-mm**

Result : 1) Torque that can be applied, $T = \mathbf{1.769\times10^6}$ **N-mm**

Example : 9.16 *(Oct.89)*

The external and internal diameters of a hollow shaft are 400mm and 200mm respectively. Find the maximum torque that can be transmitted, if the angle of twist is not to exceed 0.5° in a length of 10m and the shear stress is not to exceed $70\,N/mm^2$. Take $C = 80\ KN/mm^2$.

Given : External diameter, d_1 = 400 mm

Internal diameter, d_2 = 200 mm

Angle of twist, θ = 0.5° = 0.5 × (π /180) = 8.727×10^{-3} rad.

Length of the shaft, l = 10 m = 10000 mm

Maximum shear stress, f_s = 70 N/mm^2

Modulus of rigidity, C = 80 KN/mm^2 = 80×10^3N/mm^2

To find : 1) Maximum torque that can be transmitted, T

Solution :

(a) Torque based on shear stress

$$T_1 = \frac{\pi}{16} \times f_s \times \frac{(d_1^{\,4} - d_2^{\,4})}{d_1} = \frac{\pi \times 70}{16} \times \frac{400^4 - 200^4}{100}$$

$$= \boxed{\mathbf{8.247 \times 10^8 \text{ N-mm}}}$$

(b) Torque based on angle of twist

Polar moment of inertia, $J = \frac{\pi}{32}(d_1^{\,4} - d_2^{\,4}) = \frac{\pi}{32}(400^4 - 200^4)$

$$= 2.3562 \times 10^9 \text{ mm}^4$$

Relation for torque $\Longrightarrow \frac{T_2}{J} = \frac{C\theta}{l}$

$$T_2 = \frac{C\,\theta \times J}{l} = \frac{80 \times 10^3 \times 8.727 \times 10^{-3} \times 2.3562 \times 10^6}{10 \times 10^3}$$

$$= \boxed{\mathbf{1.645 \times 10^8 \text{ N-mm}}}$$

We shall apply the torque which is lesser.

i. e $T = T_2 = \mathbf{1.645 \times 10^8 \text{ N-mm}}$

Result : 1) Torque that can be transmitted, $T = \mathbf{1.645 \times 10^8}$ **N-mm**

Example : 9.17 ***(Oct.03)***

A solid shaft is subjected to a torque of 15KN–m. Find the suitable diameter of the shaft, if the allowable shear stress is 60 N/mm². The allowable twist is 1° for every 20 diameters length of the shaft. Take C = 80 KN/mm².

Given :

Torque, T = 15 KN–m = 15×10^6 N–mm

Angle of twist, θ = $1^\circ = 1 \times (\pi/180) = 0.1745$ rad.

Length of the shaft, l = $20 \times$ diameter (d)

Maximum shear stress, f_s = 60 N/mm^2

Modulus of rigidity, C = 80 KN/mm^2 = 80×10^3 N/mm^2

To find : 1) Diameter of shaft, d

Solution :

(a) Diameter for strength

Torque transmitted, $T = \frac{\pi}{16} f_s d^3$

$$d^3 = \frac{16 \times T}{\pi \times f_s} = \frac{16 \times 15 \times 10^6}{\pi \times 60} = 1.27324 \times 10^6$$

$$d = \boxed{\mathbf{108.385 \text{ mm}}}$$

(b) Diameter for stiffness

Polar moment of inertia, $J = \frac{\pi}{32} d^4 = 0.098175\, d^4$

Relation for diameter $\Longrightarrow \frac{T}{J} = \frac{C\theta}{l}$

$$\frac{15 \times 10^6}{0.098175\, d^4} = \frac{80 \times 10^3 \times 0.01745}{20 \times d}$$

$$\frac{152.788 \times 10^6}{d^4} = \frac{69.8}{d}$$

$$d^3 = \frac{152.796 \times 10^6}{69.8} = 2.1889 \times 10^6$$

$$d = \boxed{\mathbf{129.84\ mm}}$$

We shall provide a shaft of greater diameter.

i.e. $d =$ **129.84 mm**

Result : 1) Diameter of shaft, d = **129.84 mm**

Example : 9.18 *(Apr.01, Apr.15, Apr.17)*

A solid shaft is transmitting 100 KW at 180 rpm. If the allowable stress is 60 N/mm², find the necessary diameter for the shaft. The shaft is not to twist more than 1° in a length of 3 m. Take C = 80 KN/mm².

Given :

Speed of the shaft, N	=	180 rpm
Power transmitted, P	=	100 KW
Maximum shear stress, f_s	=	60 N/mm^2
Angle of twist, θ	=	$1° = 1 \times (\pi/180) = 0.01745$ rad.
Length of the shaft, l	=	3 m = 3000 mm
Modulus of rigidity, C	=	80 KN/mm^2 = 80×10^3 N/mm^2

To find : 1) Diameter of shaft, d

Solution :

Power transmitted by the shaft, $P = \frac{2\,\pi\,N\,T}{60}$

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{100 \times 60}{2\,\pi \times 180} = 5.3052 \text{ KN-m} = 5.3052 \times 10^6 \text{ N-mm}$$

(a) Diameter for strength

Torque transmitted, $T = \frac{\pi}{16} f_s d^3$

$$d^3 = \frac{16 \times T}{\pi \times f_s} = \frac{16 \times 5.3052 \times 10^6}{\pi \times 60} = 450319.36$$

$$d = \mathbf{76.65\ mm} \approx \boxed{\mathbf{77\ mm}}$$

(b) Diameter for stiffness

Polar moment of inertia, $J = \frac{\pi}{32} d^4$

Relation for diameter $\Longrightarrow \quad \frac{T}{J} = \frac{C\theta}{l}$

$$\frac{T \times 32}{\pi d^4} = \frac{C\ \theta}{l}$$

$$d^4 = \frac{T \times 32 \times l}{\pi \times C \times \theta} = \frac{5.3052 \times 10^6 \times 32 \times 3000}{\pi \times 80 \times 10^3 \times 0.01745} = 116.128 \times 10^6$$

$$d = \mathbf{103.809\ mm} \approx \boxed{\mathbf{104\ mm}}$$

We shall provide a shaft of greater diameter.

i.e. $d = \mathbf{104\ mm}$

Result : 1) Diameter of shaft, d = **109.76 mm**

Example : 9.19

A solid steel shaft of 60mm diameter is to be replaced by a hollow steel shaft of the same material with internal diameter equal to half of the external diameter. Find the diameters of the hollow shaft and saving in material, if the maximum allowable shear stress is same for both the shafts.

Given : Diameter of solid shaft, d = 60 mm

External diameter of hollow shaft, d_1 = $0.5 \times$ Internal diameter (d_2)

To find : 1) Diameters of the hollow shaft, d_1 and d_2

2) Percent saving in material

Solution :

Torque transmitted by the solid shaft,

$$T_1 = \frac{\pi}{16} f_s d^3 = \frac{\pi}{16} \times f_s \times 60^3 \qquad \text{---------} \quad (1)$$

Torque transmitted by the hollow shaft,

$$T_2 = \frac{\pi}{16} \times f_s \times \frac{(d_1^4 - d_2^4)}{d_1} = \frac{\pi \times f_s}{16} \times \frac{d_1^4 - (0.5 d_1)^4}{d_1}$$

$$T_2 = \frac{\pi}{16} \times f_s \times 0.9375\ d_1^3 \qquad \text{---------} \quad (2)$$

Power transmitted and allowable shear stress in both the cases are same

$$\therefore T_1 = T_2$$

$$\frac{\pi}{16} \times f_s \times 60^3 = \frac{\pi}{16} \times f_s \times 0.9375\ d_1^3$$

$$d_1^{\,3} = \frac{60^3}{0.9375} = 230400$$

$$d_1 = \boxed{61.305 \text{ mm}} \; ; \qquad d_2 = \frac{d_1}{2} = \frac{61.305}{2} = \boxed{30.653 \text{ mm}}$$

Area of the solid shaft, $A_s = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 60^2 = 2827.433 \text{ mm}^2$

Area of the hollow shaft,

$$A_h = \frac{\pi}{4} \times (d_1^{\,2} - d_2^{\,2}) = \frac{\pi}{4} \times (61.305^2 - 30.653^2) = 2213.799 \text{ mm}^2$$

Saving in material,

$$= \frac{A_s - A_h}{A_s} \times 100 = \frac{2827.433 - 2213.799}{2827.433} \times 100 = \boxed{21.7 \text{ \%}}$$

Result :	1)	External diameter of hollow shaft, d_1 **= 61.305 mm**
	2)	Internal diameter of hollow shaft, d_2**= 30.635 mm**
	3)	Saving in material **= 21.7 %**

Example : 9.20 *(Apr.13, Apr.14, Oct.16)*

A hollow shaft having inner diameter 0.6 times the outer diameter is to be replaced by a solid shaft of the same material to transmit 550KW at 220 rpm. The permissible shear stress is 80 N/mm². Calculate the diameters of the hollow and solid shafts. Also calculate the percentage of saving in material.

Given : Power transmitted, P = 550 KW

Speed of the shaft, N = 220 rpm

Shear stress, f_s = 80 N/mm²

To find :
1) Diameter of solid shaft, d
2) Diameters of hollow shaft, d_1 and d_2
3) Percentage saving in material

Solution :

Power transmitted by the shaft, $P = \frac{2\,\pi\,N\,T}{60}$

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{550 \times 60}{2\,\pi \times 220} = 23.873 \text{ KN-m} = 23.873 \times 10^6 \text{ N-mm}$$

(a) Solid shaft

Torque transmitted, $T = \frac{\pi}{16} f_s d^3$

$$d^3 = \frac{16 \times T}{\pi \times f_s} = \frac{16 \times 23.873 \times 10^6}{\pi \times 80} = 1519802.383$$

$$d = \boxed{114.973 \text{ mm}}$$

(b) Hollow shaft

Torque transmitted by the hollow shaft,

$$T = \frac{\pi}{16} \times f_s \times \frac{(d_1^4 - d_2^4)}{d_1} = \frac{\pi \times 80}{16} \times \frac{d_1^4 - (0.6d_1)^4}{d_1}$$

$$23.873 \times 10^6 = 13.672\, d_1^3$$

$$23.873 \times 10^6 = 13.672\, d_1^3$$

$$d_1^3 = \frac{23.873 \times 10^6}{13.672} = 1746123.464$$

$$d_1 = \boxed{120.418 \text{ mm}}$$

$$d_2 = 0.6 \times d_1 = 0.6 \times 120.418 = \boxed{72.251 \text{ mm}}$$

Area of the solid shaft, $A_s = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 114.973^2 = 10382 \text{ mm}^2$

Area of the hollow shaft,

$$A_h = \frac{\pi}{4} \times (d_1^2 - d_2^2) = \frac{\pi}{4} \times (120.418^2 - 72.251^2) = 7288.72 \text{ mm}^2$$

Saving in material,

$$= \frac{A_s - A_h}{A_s} \times 100 = \frac{10382 - 7288.72}{10382} \times 100 = \boxed{29.79\ \%}$$

Result : 1) Diameter of solid shaft, $\boldsymbol{d}$ **= 114.973 mm**
2) External diameter of hollow shaft, $\boldsymbol{d_1}$ **= 120.418 mm**
3) Internal diameter of hollow shaft, $\boldsymbol{d_2}$ **= 72.251 mm**
4) Saving in material **= 29.79 %**

Example : 9.21 *(Oct.92)*

Compare the weight of a solid shaft with that of a hollow shaft for the same material, length and designed to reach the same maximum shear stress when subjected to same torque. Assume the inside diameter of the hollow shaft equal to two third of the external diameter.

Solution :

Let, T = Torque transmitted by the shaft, f_s = Maximum shear stress
l = Length of the shaft

(a) Solid shaft

Let, d = Diameter of solid shaft

Torque transmitted by the shaft, $T = \frac{\pi}{16} f_s d^3$

$$d^3 = \frac{16 \times T}{\pi \times f_s} = 5.093 \left(\frac{T}{f_s}\right)$$

$$d = 1.7205\left(\frac{T}{f_s}\right)^{\frac{1}{3}}$$

Weight of the solid shaft,

$$W_1 = \rho l A_1 = \rho l \times \frac{\pi}{4}d^2 = \rho l \times \frac{\pi}{4}\left[1.7205\left(\frac{T}{f_s}\right)^{\frac{1}{3}}\right]^2$$

$$= 2.3249\ \rho l\ \left(\frac{T}{f_s}\right)^{\frac{2}{3}}$$

(b) Hollow shaft

Let, d_1 = External diameter, d_2 = Internal diameter

Then, $d_2 = \frac{2}{3}d_1 = 0.667\ d_1$

Torque transmitted by the hollow shaft,

$$T = \frac{\pi}{16}\times f_s \times \frac{(d_1^4 - d_2^4)}{d_1} = \frac{\pi \times f_s}{16}\times\frac{d_1^4 - (0.667d_1)^4}{d_1}$$

$$T = 0.157488\ f_s\ d_1^3$$

$$d_1^3 = \frac{T}{0.157488\ f_s} = 6.3497\frac{T}{f_s}$$

$$d_1 = 1.8518\left(\frac{T}{f_s}\right)^{\frac{1}{3}}$$

$$d_2 = 0.667 \times d_1 = 0.667 \times 1.8518\left(\frac{T}{f_s}\right)^{\frac{1}{3}} = 1.235\left(\frac{T}{f_s}\right)^{\frac{1}{3}}$$

Weight of the hollow shaft,

$$W_2 = \rho l A_2 = \rho l \times \frac{\pi}{4}(d_1^2 - d_2^2)$$

$$= \rho l \times \frac{\pi}{4}\left\{\left[1.8518\left(\frac{T}{f_s}\right)^{\frac{1}{3}}\right]^2 - \left[1.235\left(\frac{T}{f_s}\right)^{\frac{1}{3}}\right]^2\right\}$$

$$= 1.4954\ \rho l\ \left(\frac{T}{f_s}\right)^{\frac{2}{3}}$$

The ratio of weight of solid shaft to hollow shaft,

$$\frac{W_1}{W_2} = \frac{2.3249\ \rho l\ \left(\frac{T}{f_s}\right)^{\frac{2}{3}}}{1.4954\ \rho l\ \left(\frac{T}{f_s}\right)^{\frac{2}{3}}} = \mathbf{1.5547}$$

Result : 1) The ratio of weight of solid shaft to hollow shaft = **1.5547**

PROBLEMS FOR PRACTICE

1. A shaft of 50mm diameter is required to transmit torque from one shaft to another. Find the safe torque which the shaft can transmit, if the shear stress is not to exceed $40N/mm^2$. ***[Ans: T = 0.982 KN–m]***

2. A solid steel shaft is required to transmit a torque of 6.5KN–m. What should be the minimum diameter of the shaft, if the maximum shear stress is $40N/mm^2$. ***[Ans: d = 94mm]***

3. A hollow shaft of external and internal diameter of 60mm and 40mm is transmitting torque. Find the torque it can transmit, if the shear stress is not to exceed $40N/mm^2$. ***[Ans: T=1.66KN–m]***

4. A circular shaft of 80mm diameter is required to transmit power at 120rpm. If the shear stress is not to exceed $40N/mm^2$, find the power transmitted by the shaft. ***[Ans: P = 50.5 KW]***

5. A hollow circular shaft of external diameter 75mm and internal diameter 40mm is to transmit power at a speed of 40 rpm. If the maximum shear stress is not to exceed $80N/mm^2$, calculate the power transmitted by the shaft. *(Apr.95)* ***[Ans: P = 25.51 KW]***

6. A hollow circular shaft of outer and inner diameters 150mm and 90mm respectively transmits a power of 200KW running at a speed of 600 rpm. Calculate the maximum shear stress induced in the shaft. *(Oct.90)* ***[Ans: $f_s = 5.52 N/mm^2$]***

7. A hollow shaft has to transmit 53KW at 160 rpm. If the maximum stress is $50N/mm^2$ and internal diameter is half of its external diameter, find the diameters of the shaft. ***[Ans: $d_1 = 70mm$, $d_2 = 35mm$]***

8. A solid steel shaft has to transmit 100KW at 160 rpm. Taking allowable shear stress as $70N/mm^2$, find the suitable diameter of the shaft. The maximum torque transmitted in each revolution exceeds the mean by 20%. ***[Ans: d = 80mm]***

9. Find the torque a solid shaft of 100mm diameter can transmit, if the maximum angle of twist is 1.5° in a length of 2m. Take $C = 70 \times 10^3 N/mm^2$. ***[Ans: T = 9.0 KN–m]***

10. A hollow shaft of external and internal diameters as 80mm and 40mm is required to transmit torque from one pulley to another. What is the value of torque transmitted, if the angle of twist is not to exceed 1° in a length of 2m. Take $C = 80 \times 10^3$ N/mm^2. ***[Ans: T = 5.26 KN–m]***

11. A shaft is transmitting 160KW at 180 rpm. If the allowable shear stress in the shaft material is 60N/mm^2, determine the suitable diameter for the shaft. The shaft is not to twist more than 1° in a length of 3m. Take $C = 80 \times 10^3$ N/mm^2. ***[Ans: d = 103.8 mm]***

12. A solid shaft of diameter 100mm has working in a shear of 60N/mm^2. The allowable angle of twist is 2° for a length of 8m of the shaft. Calculate the safe torque that the shaft can transmit. Take $C = 0.8 \times 10^5$ N/mm^2. ***[Ans: T = 3.426 KN–m]***

13. A solid shaft is subjected to a torque of 1.6KN–m. Find the necessary diameters of the shaft, if the allowable shear stress is 60N/mm^2. The allowable twist is 1° for every 20 diameters length of the shaft. Take $C = 80 \times 10^3$ N/mm^2. ***[Ans: d = 61.6 mm]***

14. A shaft running at 200 rpm has to transmit 125KW. The shaft should not be stressed beyond 65N/mm^2 and should not twist more than 1.5° in a length of 5m. Select a suitable diameter. Take $C = 0.8 \times 10^5$ N/mm^2. ***[Ans: d = 109.76 mm]***

15. A solid shaft 150mm diameter is to be replaced by a hollow shaft of the same material with internal diameter equal to 60% of the external diameter. Find the saving in material, if the maximum allowable shear stress is same for both the shafts. ***[Ans: 30.9%]***

16. A hollow steel shaft of 300mm external diameter and 200mm internal diameter has to be replaced by a solid shaft. Assuming the same values of polar modulus for both, calculate the diameter of the later and work out the ratio of their tensional rigidities. Take C for steel as 2.4 times as that of alloy. ***[Ans: d = 278.8 mm, ratio of tensional rigidities = 2.58]***

17. A solid shaft and a hollow circular shaft, whose inside diameter is ¾ of the outside diameter, are of equal lengths and are required to transmit a given torque. Compare the weights of these two shafts, if maximum shear stress developed in both the shaft is also equal. ***[Ans: Ratio of weights = 1.76]***

Unit – V

Chapter 10. SPRINGS

10.1 Introduction

A spring is a device which can undergo considerable amount of deformation without permanent distortion. The general purpose of all kinds of springs is to absorb energy and to release it as and when required. Springs are also used to provide a means of restoring various mechanisms to their original configurations against the action of some external force.

10.2 Types of springs

The springs are classified as follows based on their forms :

1) Laminated or leaf springs
2) Coiled helical springs
3) Spiral springs
4) Disc springs

1) Laminated or leaf springs

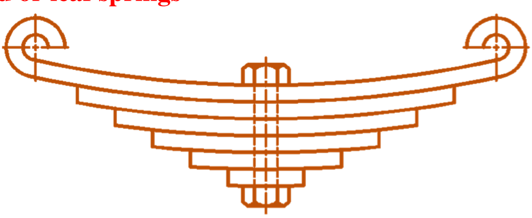


Fig.10.1 Laminated or Leaf spring

A laminated spring consists of a number of arc shaped strips of metal having different lengths but same width and thickness. They are placed one over the other in laminations. The strips are bolted together. The two types of laminated springs are :

(i) Semi - elliptical laminated springs

(ii) Quarter - elliptical laminated springs.

Uses : Thsese springs are used in railway wagons, coaches and road vehicles to absorb shocks.

2) Coiled helical springs

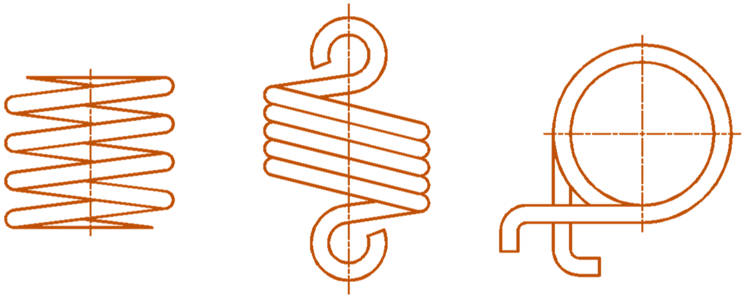
A helical spring is made up of a wire wound in helix form. The following two types of helical springs are used.

i) Closely coiled helical spring   *ii) Open coiled helical spring*

Comparison of closely coiled helical spring and open coiled helical spring

	Closely coiled helical spring	Open coiled helical spring
1)	The pitch of the coil is very small	The pitch of the coil is large
2)	The gap between the successive turn is small	The gap between the successive turn is large
3)	The helix angle is less (7° to 10°)	The helix angle is more ($>10^{\circ}C$)
4)	Under axial load, it is subjected to torsion only	It is subjected to both torsion and bending
5)	It can withstand more load	It can withstand less load

The helical springs are further classified as follows :



(a) Compression Spring *(b) Tension Spring* *(c) Torsion Spring*

Fig.10.2 Coiled helical springs

(a) Compression springs

A helical spring is said to be a compression spring, if the coils close when subjected to axial load and open out when the load is removed.

Uses : These springs are used in automobiles and railway coaches as shock absorbers.

(b) Tension springs

A helical spring is said to be a tension spring, if the coils open out when subjected to axial load and closes when the load is removed.

Uses : These springs are used in spring balances and cycle stands.

(c) Torsion springs or extension springs

The coils of torsion springs are fully compressed. Both the ends of the coil are straightened out. When one end is fixed and other end rotated, the coil deforms and creates a force opposing the rotation.

Uses : These springs are used in mouse trap, automobile starters, door hinges, etc.

3) Spiral springs or constant force springs

It consists of a uniform thin strip wound into a spiral shape. The outer end is pinned. The inner end is wound on a spindle by applying a torque. The wound spring is released slowly over a period of time. It gives a constant force.

Uses : These springs are widely used in clocks.

Fig.10.3 Spiral spring

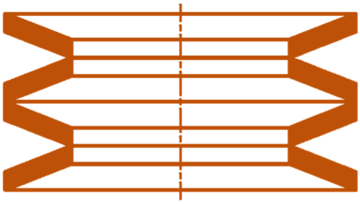


Fig.10.4 Disc spring

4) Disc springs or Belleville washer

It is a convex disc shaped spring with a hole at the centre. It can be used singly or in stacks to achieve a desired load. This spring requires less space for installation. It can withstand a very large load.

Uses : These springs are used in clutches, high pressure valves, drill bit shock absorbers, etc.

10.3 Closely coiled helical spring subjected to an axial load

Consider a closely coiled helical spring subjected to an axial load as shown in the fig.10.5.

Let, d = Diameter of the spring wire

R = Mean radius of the spring coil

n = Number of turn

C = Modulus of rigidity of spring material

W = Axial load the spring

f_s = Maximum shear stress induced in the wire due to twisting

θ = Angle of twist in the spring wire and

δ = Deflection of the spring due to axial load

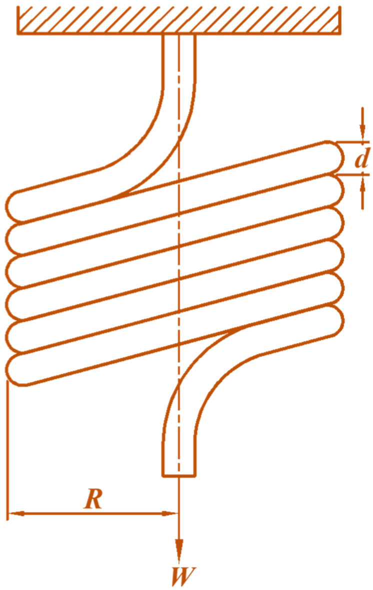


Fig.10.5 Closely coiled helical spring

Twisting moment on the coil due to the axial load, $T = W \cdot R$ ----(1)

We know that, $T = \frac{\pi}{16} f_s \, d^3$ ----(2)

$$\therefore WR = \frac{\pi}{16} f_s \, d^3$$

$$f_s = \frac{16 \, W \, R}{\pi d^3}$$

Length of the wire, $l = 2 \pi R \, . \, n$

From the equation, $\frac{T}{J} = \frac{C\theta}{l}$

$$\theta = \frac{T \, l}{C \, J} = \frac{WR \times 2\pi \, R \, n}{C \times \frac{\pi}{32} d^4}$$

$$\theta = \frac{64 \, W \, R^2 n}{C \, d^4}$$

Deflection of the spring, $\delta = R\theta = R \times \frac{64 \, W \, R^2 n}{C \, d^4}$

$$\boxed{\delta = \frac{64 \, W \, R^3 n}{C \, d^4}}$$

10.4 Stiffness of the spring

The stiffness of the spring is defined as the load required to produce unit deflection. It is denoted by '*s*'.

$$s = \frac{W}{\delta} = \frac{W}{\dfrac{64\,W\,R^3 n}{C\,d^4}} = \frac{C\,d^4}{64\,R^3 n}$$

It is also known as ***spring constant.***

10.5 Resilience or strain energy stored in a closely coiled helical spring.

Energy stored = Average load × Deflection

$$= \frac{W}{2} \times \frac{64\,W\,R^3 n}{C\,d^4} = \frac{32\,W^2\,R^3 n}{C\,d^4}$$

10.6 Applications of springs

1) To apply forces and controlling motion, as in brakes and clutches.
2) Measuring forces, as in spring balances.
3) Storing energy, as springs used in watches and toys.
4) Reducing the effect of shock and vibrations in vehicles and machine foundations.

REVIEW QUESTIONS

1. Briefly explain the types of springs. *(Apr.02, Oct.04, Apr.17)*
2. Distinguish between closely coiled helical spring and open coiled helical spring. *(Oct.96, Oct.03, Oct.17)*
3. Define: stiffness of a spring. *(Apr.18)*
4. Write down the formula to determine the stiffness of a spring closely coiled when subjected to an axial loading. *(Apr.01)*
5. Derive an expression for the deflection and stiffness of a closely coiled helical spring when it is subjected to an axial load.

POINTS TO REMEMBER

1) Deflection of the spring, $\delta = \frac{64\,W\,R^3 n}{C\,d^4}$ (mm)

2) Stiffness, $s = \frac{W}{\delta}$ (N/mm)

3) Energy stored $= \frac{W}{2}\delta$ (N–mm)

Where, W = Axial load (N)
R Mean radius of coil (mm)
n = Number of turns
d = Diameter of wire (mm)
C = Modulus of rigidity (N/mm^2)

SOLVED PROBLEMS

Example : 10.1 *(Apr.89, Oct.90)*

A closely coiled helical spring of alloy steel wire of 10mm diameter having 15 complete turns with the mean coil diameter as 10mm. Calculate the stiffness of the spring. Take $C = 90 \times 10^3$ N/mm^2.

Given :

Diameter of wire, d = 10 mm

Mean diameter of coil, D = 100 mm

Number of turns, n = 15

Modulus of rigidity, C = 90×10^3 N/mm^2

To find : 1) Stiffness of spring, s

Solution :

Mean radius, $R = \dfrac{D}{2} = \dfrac{100}{2} = 50$ mm

The stiffness of spring, $s = \dfrac{Cd^4}{64\ R^3 n} = \dfrac{90 \times 10^3 \times 10^4}{64 \times 50^3 \times 15} = \boxed{7.5 \text{ N/mm}}$

Result : 1) Stiffness of spring, s = **7.5 N/mm**

Example : 10.2 *(Oct.03)*

Calculate the modulus of rigidity of a spring of 10 turns 65mm mean diameter and wire of 6.5mm diameter. The spring compresses 10mm under a load of 70N.

Given :

Number of turns, n = 10

Mean diameter of coil, D = 65 mm

Diameter of wire, d = 6.5 mm

Load, W = 70 N

Deflection, δ = 10 mm

To find : 1) Modulus of rigidity, C

Solution :

Mean radius, $R = \dfrac{D}{2} = \dfrac{65}{2} = 32.5$ mm

Relation for modulus of rigidity $\Rightarrow \delta = \dfrac{64\ WR^3 n}{Cd^4}$

$$C = \frac{64\ WR^3 n}{\delta\ d^4} = \frac{64 \times 70 \times 32.5^3 \times 10}{10 \times 6.5^4} = \boxed{86.154 \times 10^3 \text{ N/mm}^2}$$

Result : 1) Modulus of rigidity, C = **86.154×10^3** N/mm^2

Example : 10.3 *(Oct.92)*

A closely coiled helical spring has the stiffness of 40N/mm. Determine its number of turns when the diameter of the wire of the spring is 10mm and mean diameter of the coil is 80mm. Take $C = 0.8 \times 10^5\ N/mm^2$.

Given :

$$\begin{aligned} \text{Stiffness, } s &= 40 \text{ N/mm} \\ \text{Mean diameter of coil, } D &= 80 \text{ mm} \\ \text{Diameter of wire, } d &= 10 \text{ mm} \\ \text{Modulus of rigidity, } C &= 0.8 \times 10^5 \text{ N/mm}^2 \end{aligned}$$

To find : 1) Number of turns in the spring, n

Solution :

$$\text{Mean radius, } R = \frac{D}{2} = \frac{80}{2} = 40 \text{ mm}$$

$$\text{Stiffness, } s = \frac{Cd^4}{64\ R^3 n}$$

$$n = \frac{Cd^4}{64\ R^3 s} = \frac{0.8 \times 10^5 \times 10^4}{64 \times 40^3 \times 40} = \boxed{5.2 \approx 6}$$

Result : 1) Number of turns in the spring, $\boldsymbol{n = 6}$

Example : 10.4 *(Oct.15)*

A closely coiled helical spring made of 12mm steel wire having 12 turns of mean radius 60mm elongates by 15mm under a load. Find the magnitude of the load if the modulus of rigidity is given as $7.5 \times 10^4\ N/mm^2$.

Given :

$$\begin{aligned} \text{Diameter of wire, } d &= 12 \text{ mm} \\ \text{Number fo turns, } n &= 12 \\ \text{Mean radius of coil, } R &= 60 \text{ mm} \\ \text{Deflection of spring, } \delta &= 15 \text{ mm} \\ \text{Modulus of rigidity, } C &= 7.5 \times 10^4 \text{ N/mm}^2 \end{aligned}$$

To find : 1) Magnitude of load, W

Solution :

$$\text{Deflection of spring, } \delta = \frac{64\ WR^3 n}{Cd^4}$$

$$W = \frac{\delta \times C\ d^4}{64\ R^3\ n} = \frac{15 \times 7.5 \times 10^4 \times 12^4}{64 \times 60^3 \times 12} = \boxed{140.63 \text{ N}}$$

Result : 1) Magnitude of load, $\boldsymbol{W = 140.63}$ **N**

Example : 10.5 *(Apr.01, Oct.13)*

A closely coiled helical spring is to carry a load of 100KN. The mean coil diameter is 15 times that of the wire diameter. Calculate these diameters if the shear stress is limited to $120\,N/mm^2$.

Given : Load, $W = 100\text{ KN} = 100 \times 10^3$ N

Shear stress, $f_s = 120$ N/mm^2

To find : 1) Diameter of wire, d 2) Diameter of coil, D

Solution :

Let, d = Diameter of wire ; D = Diameter of coil

Then, $D = 15 \times d$; $R = \dfrac{D}{2} = \dfrac{15\,d}{2} = 7.5\,d$

Torque, $T = W \times R = 100 \times 10^3 \times 7.5\,d = 7.5 \times 10^5\,d$

Also, torque, $T = \dfrac{\pi}{16} f_s\, d^3 = \dfrac{\pi}{16} \times 120 \times d^3 = 23.562\,d^3$

$$\therefore 23.562\,d^3 = 7.5 \times 10^5\,d$$

$$d^2 = \frac{7.5 \times 10^5}{23.562} = 31830.91$$

$$d = \boxed{178.4\text{ mm}}\ ;\ D = 15\,d = 15 \times 178.4 = \boxed{2676\text{ mm}}$$

Result : 1) Diameter of wire, $\boldsymbol{d}$ **= 178.4 mm**
2) Diameter of coil, $\boldsymbol{D}$ **= 2676 mm**

Example : 10.6 *(Apr.04, Oct.14, Apr.18)*

The mean diameter of a closely coiled helical spring is 5 times the diameter of wire. It elongates 8mm under an axial pull of 120N. If the permissible shear stress is $40\,N/mm^2$, find the size of wire and number of coils in the spring. Take $C = 0.8 \times 10^5\ N/mm^2$.

Given : Deflection, δ = 8 mm

Axial load, W = 120 N

Shear stress, f_s = 40 N/mm^2

Modulus of rigidity, $C = 0.8 \times 10^5$ N/mm^2

To find : 1) Diameter of wire, d 2) Number of turns, n

Solution :

Let, d = Diameter of wire ; D = Diameter of coil

Then, $D = 5 \times d$; $R = \dfrac{D}{2} = \dfrac{5\,d}{2} = 2.5\,d$

Torque, $T = W \times R = 120 \times 2.5\,d = 300\,d$

Also, torque, $T = \frac{\pi}{16} f_s d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.854\ d^3$

$\therefore 7.854\ d^3 = 300\ d$

$$d^2 = \frac{300}{7.854} = 38.197$$

$$d = \boxed{6.18\ \text{mm}}\ ; \quad R = 2.5\ d = 2.5 \times 6.18 = \boxed{15.45\ \text{mm}}$$

Relation for number of turns $\Rightarrow \delta = \frac{64\ WR^3 n}{C d^4}$

$$n = \frac{C d^4 \times \delta}{64\ W R^3} = \frac{0.8 \times 10^5 \times 6.18^4 \times 8}{64 \times 120 \times 40^3} = 32.96 \approx \boxed{33}$$

Result : 1) Diameter of wire, $\boldsymbol{d} = \mathbf{6.18\ mm}$
2) Number of turns, $\boldsymbol{n} = \mathbf{33}$

Example : 10.7 *(Oct.02, Apr.14, Oct.16, Apr.17)*

A closely coiled helical spring made of steel wire of 10mm diameter has 10 coils of 120mm mean diameter. Calculate the deflection of the spring under an axial load of 100N and the stiffness of the spring. Take $C = 1.2 \times 10^5\ N/mm^2$.

Given : Diameter of wire, d = 10 mm
Number of turns, n = 10
Mean diameter of coil, D = 120 mm
Axial load, W = 100 N
Modulus of rigidity, C = 1.2×10^5 N/mm^2

To find : 1) Deflection, δ 2) Stiffness, s

Solution :

Mean radius, $R = \frac{D}{2} = \frac{120}{2} = 60$ mm

$$\text{Deflection, } \delta = \frac{64\ WR^3 n}{C d^4} = \frac{64 \times 100 \times 60^3 \times 10}{1.2 \times 10^5 \times 10^4} = \boxed{11.52\ \text{mm}}$$

$$\text{Stiffness, } s = \frac{W}{\delta} = \frac{100}{11.52} = \boxed{8.68\ \text{N/mm}}$$

Result : 1) Deflection, $\boldsymbol{\delta} = \mathbf{11.52\ mm}$ 2) Stiffness, $\boldsymbol{s} = \mathbf{8.68\ N/mm}$

Example : 10.8 *Oct.88, Apr.92, Apr.01, Oct.12, Apr.13*

Design a closely coiled helical spring of stiffness 20N/mm deflection. The maximum shear stress in the spring material is not to exceed $80 N/mm^2$ under a load of 600N. The diameter of the coil is to be 10 times the diameter of the wire. Take $C = 85 \times 10^3\ N/mm^2$.

Given : Stiffness of the spring, $s = 20$ N/mm
Shear stress, $f_s = 80$ N/mm^2
Axial load, $W = 600$ N
Modulus of rigidity, $C = 85 \times 10^3$ N/mm^2

Solution :

Let, d = Diameter of wire ; D = Diameter of coil

Then, $D = 10\,d$; $R = \dfrac{D}{2} = \dfrac{10\,d}{2} = 5\,d$

Torque, $T = W \times R = 600 \times 5\,d = 3000\,d$

Also, torque, $T = \dfrac{\pi}{16}\,f_s\,d^3 = \dfrac{\pi}{16} \times 80 \times d^3 = 15.708\,d^3$

$$\therefore 15.708\,d^3 = 3000\,d$$

$$d^2 = \frac{3000}{15.708} = 190.986$$

$$d = 13.82 \text{ mm} \approx \boxed{\mathbf{14\ mm}}$$

$$D = 10\,d = 10 \times 14 = \boxed{\mathbf{140\ mm}}\ ; \quad R = 5d = 5 \times 14 = \boxed{\mathbf{70\ mm}}$$

Relation for number of turns $\Rightarrow s = \dfrac{C d^4}{64\,R^3 n}$

$$n = \frac{C d^4}{64\,R^3 s} = \frac{85 \times 10^3 \times 14^4}{64 \times 70^3 \times 20} = \boxed{\mathbf{7.44 \approx 8}}$$

Result : 1) Diameter of coil, D = **140 mm**
2) Diameter of wire, d = **14 mm**
3) Number of turns, n = **8**

Example : 10.9

A closely coiled helical spring is to be designed to carry an axial load 2500N under a deflection of 70mm. The number of coil is to be limited to 10 and the coil diameter is 10 times the wire diameter. Calculate the diameter of the coil and shear stress produced in the spring. Take $C = 85\,KN/mm^2$.

Given : Axial load, $W = 2500$ N
Deflection, $\delta = 70$ mm
Number of coil, $n = 10$
Modulus of rigidity, $C = 85$ KN/mm^2 $= 85 \times 10^3$ N/mm^2

To find : 1) Diameter of coil, D 2) Shear stress, f_s

Solution :

Let, d = Diameter of wire ; D = Diameter of coil

Then, $D = 10\,d$; $R = \dfrac{D}{2} = \dfrac{10\,d}{2} = 5\,d$

$$\text{Deflection, } \delta = \frac{64\,WR^3 n}{C d^4} = \frac{64 \times 2500 \times (5d)^3 \times 10}{85 \times 10^5 \times d^4}$$

$$70 = \frac{2352.94}{d}$$

$$d = \frac{2352.94}{70} = \mathbf{33.61\ mm} \approx \boxed{\mathbf{34\ mm}}$$

$$D = 10\,d = 10 \times 34 = \boxed{\mathbf{340\ mm}}$$

$$\text{Torque, } T = W \times R = 2500 \times (5 \times 34) = 425000 \text{ N-mm}$$

$$\text{Also, torque, } T = \frac{\pi}{16}\, f_s\, d^3$$

$$f_s = \frac{16\,T}{\pi\, d^3} = \frac{16 \times 425000}{\pi \times 34^3} = \boxed{\mathbf{55.07\ N/mm^2}}$$

Result : 1) Diameter of coil, D = **340 mm** 2) Shear stress, f_s = **55.07 N/mm²**

Example : 10.10 *(Oct.92)*

A closely coiled helical spring has to absorb 50N–m of energy when compressed by 50mm. The coil diameter is 12 times the wire diameter. The number of coil is 10. Determine the diameters of the wire and coil, if $C = 0.08 \times 10^6\ N/mm^2$.

Given : Energy absorbed = 50 N–m = 50×10^3 N–mm

Deflection, δ = 50 mm

Number of coil, n = 10

Modulus of rigidity, C = 0.08×10^6 N/mm²

To find : 1) Diameter of coil, D 2) Diameter of wire, d

Solution :

Let, d = Diameter of wire ; D = Diameter of coil

Then, $D = 12\,d$; $R = \dfrac{D}{2} = \dfrac{12\,d}{2} = 6\,d$

Energy absorbed by the coil = Average load × deflection

$$50 \times 10^3 = \frac{W}{2} \times 50$$

$$W = \frac{2 \times 50 \times 10^3}{50} = 2000 \text{ N}$$

Deflection, $\delta = \frac{64\ WR^3 n}{C d^4} = \frac{64 \times 2000 \times (6d)^3 \times 10}{0.08 \times 10^6 \times d^4}$

$$50 = \frac{3456}{d}$$

$$d = \frac{3456}{50} = \mathbf{69.12} \approx \boxed{\mathbf{70\ mm}}$$

$$D = 12\ d = 12 \times 70 = \boxed{\mathbf{840\ mm}}$$

Result : 1) Diameter of coil, $\boldsymbol{D = 840}$ **mm** 2) Diameter of wire, $\boldsymbol{d = 70}$ **mm**

Example : 10.11 *(Oct.03, Oct.17)*

A truck weighing 30KN and moving at 5Km/hr has to be brought to rest by a buffer. Find how many springs, each of 18 coils will be required to store the energy of motion during compression of 200mm. The spring is made out of 25mm diameter steel rod coiled to a mean diameter of 240mm. Take $\boldsymbol{C = 0.84 \times 10^5\ N/mm^2}$.

Given : Weight of the truck, $W_1 = 30$ KN $= 30 \times 10^3$ N

Velocity of the truck, $v = 5$Km/hr $= \frac{5\times10^3\times10^3}{60\times60} = 1388.889$ mm/s

Number of coil, $n = 18$

Deflection, $\delta = 200$ mm

Diameter of wire, $d = 25$ mm

Diameter of coil, $D = 240$ mm

Modulus of rigidity, $C = 0.84 \times 10^5$ N/mm^2

To find : 1) Number of springs

Solution :

Mean radius, $R = \frac{D}{2} = \frac{240}{2} = 120$ mm

Kinetic energy stored in the truck,

$$K.E = \frac{W_1 v^2}{2\ g} = \frac{30 \times 10^3 \times 1388.889^2}{2 \times 9.81 \times 10^3} = 2.95 \times 10^6 \text{ N-mm}$$

Let. W = Axial load act on each spring

Then deflection, $\delta = \frac{64\ WR^3 n}{C d^4}$

$$W = \frac{C d^4 \times \delta}{64\ R^3 n} = \frac{0.84 \times 10^5 \times 25^4 \times 200}{64 \times 120^3 \times 18} = 3296.65 \text{ N}$$

Energy stored in each spring = Average load × deflection

$$= \frac{W}{2} \times \delta = \frac{3296.65}{2} \times 200 = 329665 \text{ N-mm}$$

$$\text{No. of springs} = \frac{\text{Kinetic energy stored in the truck}}{\text{Energy stored in each spring}}$$

$$= \frac{2.95 \times 10^6}{3296.65} = \mathbf{8.95} \approx \boxed{9}$$

Result : 1) Number of springs required = **9**

Example : 10.12 *(Oct.04, Oct.16)*

A weight of 150 N is dropped on to a compression spring with 10 coils of 12 mm diameter closely coiled to a mean diameter of 150 mm. If the instantaneous contraction is 140 mm, calculate the height of drop. Take $C = 0.8 \times 10^5$ N/mm^2.

Given : Weight dropped on the spring, P = 150 N
Number of turns, n = 10
Deflection, δ = 140 mm
Diameter of wire, d = 12 mm
Diameter of coil, D = 150 mm
Modulus of rigidity, C = 0.8×10^5 N/mm^2

To find : 1) Height of drop of weight, h

Solution :

$$\text{Mean radius, } R = \frac{D}{2} = \frac{150}{2} = 75 \text{ mm}$$

Let, h = Height of drop of weight before strike

Potential energy stored in the weight,

$$= P(h + \delta l) = 150\,(h + 140)$$

Let. W = Axial load act on each spring

$$\text{Then, deflection, } \delta = \frac{64\, WR^3 n}{Cd^4}$$

$$W = \frac{Cd^4 \times \delta}{64\, R^3 n} = \frac{0.8 \times 10^5 \times 12^4 \times 140}{64 \times 75^3 \times 10} = 860.16 \text{ N}$$

Energy stored in spring = Average load × deflection

$$= \frac{W}{2} \times \delta = \frac{860.16}{2} \times 140 = 60211.2 \text{ N-mm}$$

After striking,

the potential energy stored in the weight is lost to compress the spring.

$\therefore$ Potential energy stored in weight = Energy stored in spring

$$150(h + 140) = 60211.2$$

$$h + 140 = \frac{60211.2}{150} = 401.408 \text{ mm}$$

$$h = 401.408 - 140 = \boxed{261.408 \text{ mm}}$$

Result : 1) Height of drop of weight, **h = 261.408 mm**

PROBLEMS FOR PRACTICE

1. Calculate the stiffness of a closely coiled helical spring made of steel wire 10mm diameter having 15 complete turns. The mean coil diameter is 50mm. Take $C = 90$ KN/mm^2. *(Oct.95)* ***[Ans: $s = 2.4$ N/mm]***

2. A closely coiled helical spring has the stiffness of 40N/mm. Determine the number of turns, when the diameter of wire is 10mm and mean diameter of coil is 90mm. Take $C = 80$ KN/mm^2. ***[Ans: $n = 4$]***

3. A closely coiled helical spring is to carry a load of 120KN. The mean coil diameter is 10 times that of the wire diameter. Calculate these diameters, if the shear stress is limited to 100N/mm^2. *(Apr.85)*
[Ans: $d = 175$mm, $D = 1750$ mm]

4. A closely coiled helical spring is made of 6mm wire. The maximum shear stress and the deflection under the load of 200N is not to exceed 80N/mm^2 and 11mm respectively. Determine the number of coils and the mean coil radius. Take $C = 0.8 \times 10^5$ N/mm^2. *(Apr.92)*
[Ans: $R = 17$ mm, $n = 19$]

5. A closely coiled helical spring of round steel rod of 10mm diameter having 20 complete turns of 100mm in mean diameter is subjected to an axial load of 100N. Find the shear stress induced and the deflection of the spring. Take $C = 0.85 \times 10^5$N/mm^2.
[Ans: $f_s = 25.46$ N/mm^2, $\delta = 19.05$ mm]

6. A truck weighing 20KN and moving at 6Km/hr has to be brought to rest by a buffer. Find how many springs each of 15 coils will be required to store the energy of motion during compression of 200mm. The spring is made out of 25mm diameter steel rod coiled to a mean diameter of 200mm. Take $C = 0.945 \times 10^5$ N/mm^2. *(Apr.86, Oct.91, Apr.05)*

   [Ans: 4]

7. A weight of 2.5KN is dropped on a closely coiled compression spring with 15 coils. Calculate the height of drop before strike, so that the spring is compressed by 200mm. Diameter of the rod of which the spring is made is 25mm and the mean diameter of he coil is 200mm. Assume $C = 0.94 \times 10^5$ N/mm^2. ***[Ans: h = 106 mm]***

TWO & THREE MARKS Questions & Answers

Chapter 1. STATICS OF PARTICLE

1. Define force. State the effects of force.

Force is defined as an action which changes or tends to change the state of rest or motion of the body on which it is applied.

Effects of force:

✦ A force moves or tends to move a body in the direction in which it acts.

✦ A force may also tend to rotate the body on which it acts.

2. What are the characteristics of force?

1) Magnitude 2) Direction 3) Point of application on the body

3. State the principle of transmissibility of forces.

Principle or transmissibility states that "if a force acting at a point on a rigid body is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged".

4. Classify the system of forces.

1) Coplanar forces

a) Collinear
b) Concurrent
c) Parallel
d) Non-concurrent, Non-parallel

2) Non-coplanar

a) Concurrent
b) Parallel
c) Non-concurrent, Non-parallel

5. What are coplanar and non-coplanar forces?

✦ The forces in a system acting in a same plane are called *coplanar forces*.

✦ The forces in a system acting in different planes are called *non-coplanar forces.*

6. Differentiate between collinear and concurrent forces.

✦ In *collinear* system of forces, all the forces act in the same plane and have a common line of action.

✦ In *concurrent* system of forces, all the forces act in the same plane and they intersect at a common point.

7. **Define resultant of forces.**

Resultant is a single force which can replace a number of forces acting on a rigid body, without causing any change in the external effects on the body. Resultant is also referred to as *equivalent action.*

8. **State Parallelogram law of forces.**

The parallelogram law of forces sates that *"if two forces acting at a point are represented by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram gives the resultant of the two forces both in magnitude and direction".*

9. **Write the equation to find out the magnitude and direction of resultant of two collinear forces.**

Magnitude of resultant, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

Direction of resultant, $\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$

Where, P *and* Q = Two collinear forces

α=Angle between the two forces

10. **Write down the relationship in law of sines.**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Where, a, b, and c are the lengths of the sides of a triangle.

A, B, and C are the opposite angles.

11. **State the triangular law of forces.**

The triangular law of forces states that *"if two coplanar concurrent forces acting at a point are represented in magnitude and direction by the two adjacent sides of a triangle in order, then the resultant of the two forces is given in magnitude and direction by the third side of the triangle in opposite order".*

12. **State the polygon law of forces.**

Polygon law of forces states that, *"if a number of coplanar, concurrent forces are represented in magnitude and direction by the sides of an open polygon taken in order, then the resultant of all these forces is denoted in magnitude and direction by the closing side of the polygon in the opposite order".*

13. **Write down the formula to find out the magnitude and direction of resultant of several forces.**

Resultant of all the forces, $R = \sqrt{(R_x)^2 + (R_y)^2}$

The angle made by R with X-axis is given by, $\tan\theta = \frac{R_y}{R_x}$

Where,

$R_x = \Sigma F_x$= Sum of components of all forces along X- axis.

$R_y = \Sigma F_y$= Sum of components of all forces along Y- axis.

14. What are external and internal forces?

- *External forces* represent the action of other bodies on the rigid body being analysed. External forces consist of applied forces, weight of the free-body and the reactions developed at the support of contact points.
- *Internal force* holds the particles of the body together. The internal forces cause internal stresses and strains distributed throughout the material of the body.

15. Define moment of a force.

The product of a force ($\boldsymbol{F}$) and the perpendicular distance ($\boldsymbol{r}$) of the line of action of the force from a point is known as moment of the force about that point. Moment of the force $\boldsymbol{F}$ about a point, $\boldsymbol{M} = \boldsymbol{F} \times \boldsymbol{r}$

16. State Varignon's theorem.

Varignon's theorem states that *"the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point"*.

17. Define couple. What is arm of the couple?

- Tow parallel, non-collinear forces of equal magnitude having opposite senses are said to form a *couple*.
- *Arm* of a couple is a perpendicular distance between the line of action of two forces.

18. State the necessary conditions for the equilibrium of rigid bodies?

1) The algebraic sum of the magnitudes of the horizontal components of all the forces acting on the body is zero, i.e. $\boldsymbol{\Sigma F_x = 0}$.
2) The algebraic sum of the magnitudes of the vertical components of all the forces acting on the body is zero, i.e. $\boldsymbol{\Sigma F_y = 0}$.
3) The algebraic sum of the magnitudes of the moments of all the forces about any point is zero, i.e. $\boldsymbol{\Sigma M = 0}$.

19. What are space diagram and free body diagram?

- *Space diagram* is the physical representation showing the body and the forces acting on it.
- The diagram showing the isolated significant portion of a body along with the forces acting on it is called *free-body diagram.*

20. What is equilibrant?

Equilibrant is a force which is equal, collinear and opposite to the resultant in a system of forces.

21. State triangular law of equilibrium.

Triangular law of equilibrium states that, *"if three forces acting on a particle can be represented in magnitude and direction by the three sides of a triangle taken in order, then the particle is in equilibrium."*

22. State Lami's theorem.

Lami's theorem states that, *"if three forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces."*

23. State polygon law of equilibrium.

Polygon law of equilibrium states that, *"if a particle is in equilibrium under the action of a system of coplanar forces, the forces can be represented in magnitude and direction by the sides of a polygon taken in order."*

24. What is support and support reaction?

- ✦ A body that supports another body acted upon by a system of forces is called a support.
- ✦ The force exerted by the support on the supported body is called support reaction.

25. List out the different types of supports.

1) Simple support or knife edge support
2) Roller support
3) Pin joint or hinged support
4) Smooth surface support
5) Fixed or built-in support

1. **Define friction.**

The property of the bodies by virtue of which a force is exerted by a stationary body on the moving body to resist the motion of the moving body is called *friction*.

2. **What is force of friction and limiting force of friction?**

- When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called *force of friction*.
- The maximum value of frictional force acting on the body when the body just begins to slide over another body is called *limiting force of friction*.

3. **Differentiate between static friction and dynamic friction.**

- The frictional force acting on a body when the two surfaces of contact are at rest is called *static friction*.
- The frictional force acting on a body when the body is moving, is called *dynamic friction* or *kinetic friction*.

4. **State the laws of static friction.**

1) The frictional force acts in the opposite direction in which surface is having tendency to move.
2) The frictional force is equal to the force applied to the surface, so long as the surface is at rest.
3) The frictional force is directly proportional to the normal reaction between the surfaces in contact.
4) The frictional force depends upon the material of the bodies in contact.

5. **State the laws of dynamic friction.**

1) The frictional force acts in the opposite direction in which surface is having tendency to move.
2) The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces.
3) The limiting frictional force does not depend upon the shape and areas of the two surfaces in motion.
4) The frictional force is independent of the velocity of sliding.

6. **Define co-efficient of friction.**

   Co-efficient of friction is defined as the ratio of the limiting force of friction to the normal reaction between two surfaces in contact. It is denoted by the symbol μ.

7. **Define angle of friction.**

   Angle of friction is defined as the angle made by the resultant of the normal reaction ($\boldsymbol{R}$) and the limiting force of friction ($\boldsymbol{F_{max}}$) with the normal reaction. It is denoted by $\boldsymbol{\phi}$.

8. **Define cone of friction.**

   Cone of friction is defined as the right circular cone with vertex at the point of contact of the two bodies, axis in the direction of normal reaction ($\boldsymbol{R}$) and semi vertical angle equal to the angle of friction ($\boldsymbol{\phi}$).

9. **What is angle of repose?**

   Angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.

Unit – II
Chapter 3. MECHANICAL PROPERTIES OF MATERIALS

1. **Define elasticity and plasticity.**

   ✦ The property of material by which a body regains its original shape and size after deformation when applied forces are removed is known as *elasticity*.

   ✦ *Plasticity* is the property of a material by which a body retains the deformation due to applied load without rupture, even after the removal of applied load.

2. **Differentiate between ductility and malleability.**

   ✦ *Ductility* is the property of a material by which the material can be drawn out or elongated into thin wires without rupture by applying a tensile force.

   ✦ *Malleability* is the property of a material by which the material can be flattened into thin sheets without cracking by hot or cold working processes.

3. **Give examples of materials having ductility and malleability.**

- Mild steel, copper, aluminium, zinc, gold and platinum are some materials having high ductility.
- Mild steel, wrought iron, copper and aluminium are some materials having high malleability.

4. **What is machinability? Give its advantages.**

Machinability is the property of a material by which the material can be easily machined by cutting tools in various machining operations.

Advantages :

1) The rate of metal removal is high
2) Long life of cutting tool
3) Less power consumption
4) Good surface finish

5. **Define castability and weldability of a material.**

- Castability is the property of a material by which the material can be easily cast into different size and shapes.
- Weldability is the property of a material by which the material can be welded into a specific and suitable designed structure and to perform satisfactorily in the desired objective.

6. **Differentiate between strength and toughness.**

- Strength is a property of a material by which the material can withstand or resist the action of external force or load without breaking or yielding.
- Toughness is the property of a material to resist the fracture by absorbing energy due to heavy shock loads or blow, without rupture.

7. **What is stiffness or rigidity? Give its importance.**

- Stiffness or rigidity is the property of a material to resist elastic deformation or deflection due to the applied load.
- This property is very important in the design of beams, shafts and springs.

8. **Define brittleness. List out the high brittle materials.**

- Brittleness is the property of a material by which the material will fail or fracture all of sudden without any significant deformation. This property is opposite to ductility.
- Cast iron, concrete, glass and stone are some material having high brittleness.

9. Define hardness. What is its importance?

- Hardness is the ability of a material to resist surface penetration, abrasion and scratching.
- It is an important property involved in the design of machine members such as gears, cams, chain sprockets, etc. which are under constant rubbing action.

10. What is meant by fatigue and creep in materials?

- Fatigue is described as the failure of the material when subjected to a number of cyclically changing loads.
- Creep is the property of a material by which the material is deformed slowly and progressively under a constant load over a long period.

11. Differentiate between repeated loading and cyclic loading.

Repeated loading : A member is subjected to either compressive or tensile load of same magnitude repeatedly.

Cyclic loading : A member is subjected to compressive and tensile loads alternatively and also the magnitude of load vary from maximum value to minimum value at regular intervals.

12. Define fatigue strength and endurance limit.

- The stress at which a material fails by fatigue is known as *fatigue strength*.
- *Endurance limit* or *fatigue limit* is a maximum stress below which a load may be repeatedly applied at infinite number of times without causing failure of material by fatigue.

13. Differentiate between mechanical creep and temperature creep.

Mechanical creep : If the slow and progressive deformation of material is due to constant loading, then the creep is called mechanical creep.

Temperature creep : If the slow and progressive deformation of material is due to rise in temperature, then the creep is called temperature creep.

14. Give any four ferrous materials and its uses.

- *Mild Steel :* Girders, plates, nuts and bolts, general purpose.
- *High Speed Steel :* Cutting tools for lathes.
- *Stainless Steel :*Kitchen draining boards, pipes, cutlery, aircraft.
- *Cast Iron :*Cylinder blocks, vices, machine tool parts, brake drums, gear wheels, plumbing fitments.

15. **List any four non-ferrous metals and their uses.**

- *Aluminium :*Aircraft, boats, window frames, pistons and cranks.
- *Copper :* Electrical wire, cables, printed circuit boards, roofs.
- *Brass :* Castings, ornaments, valves, forgings.
- *Lead :* Paints, roof coverings, flashings.

16. **List any four alloying elements and their major effects.**

1) *Aluminium (Al)*
   - Increases toughness, acts as deoxidizer
   - Provides abrasion resistance

2) *Chromium (Cr)*
   - Provides a moderate contribution to hardenability
   - Provides strength and resistance to oxidation
   - Provides abrasion resistance

3) *Copper (Cu)*
   - Improves resistance to atmospheric corrosion
   - Decreases the ability to hot work steels

4) *Manganese (Mn)*
   - Provides a moderate contribution to hardenability
   - Improves machinability
   - Increases strength and reduces ductility

Unit – II
Chapter 4. SIMPLE STRESSES AND STRAINS

1. **Define stress and strain.**

- The stress or intensity of stress at a section may be defined as the ratio of the internal resistance or load acting on the section to the cross sectional area of that section.

$$\text{stress, } f = \frac{\text{Internal resistance}}{\text{Area of cross section}} = \frac{\text{Load}}{\text{Area}}$$

- Strain may be defined as the ratio between the deformation produced in a body due to the applied load and the original dimension.

$$\text{Strain, } e = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

2. **Name the types of stresses.**

1) Tensile stress 2) Compressive stress 3) Shear stress
4) Bending stress 5) Torsional stress

3. **Differentiate between tensile stress and compressive stress.**

- ✦ The resistance induced against the increase in length due to the tensile load is called *tensile stress.*
- ✦ The resistance induced against the decrease in length due to the compressive load is called *compressive stress.*

4. **What is shear stress and bending stress?**

- ✦ The stress induced in a section due to the shear force is called *shear stress.*
- ✦ The stress developed in a beam to resist the bending due to external forces is called *bending stress.*

5. **Define torsional stress.**

When a machine member is subjected with two equal and opposite couples acting in parallel planes, then the member is said to be in *torsion*. The stress induced by this torsion is called *torsional stress.*

6. **What is proportional limit and elastic limit?**

- ✦ *Proportional limit* is the maximum stress level up to which stress is directly proportional to strain.
- ✦ The maximum stress level up to which the material shows the characteristics of regaining its original shape and dimensions on removal of load is known as *elastic limit.*

7. **Define : Yield stress, Ultimate stress and Breaking stress.**

- ✦ *Yield stress:* Yield stress is the value of stress at which the material continues to deform at constant load condition.
- ✦ *Ultimate stress:* It is the maximum stress induced in the specimen.
- ✦ *Breaking stress:* The stress at which the specimen breaks.

8. **Sate Hooke's law.**

Hooke's law state that *stress is directly proportional to strain within elastic limit.*

$$\frac{\text{Stress}}{\text{Strain}} = \text{A constant}$$

9. **Define Young's modulus. Give its importance.**

- ✦ The ratio of stress to strain in tension or compression is known as *Young's modulus* or *modulus of elasticity.*
- ✦ Young's modulus is the measure of stiffness of the material. A member made of material with larger value of Young's modulus is said to have higher stiffness.

10. Define working stress.

The maximum stress to which the material of a member or machine element is subjected in normal usage is called *working stress*.

11. Distinguish between factor of safety and load factor.

- ✦ The ratio of ultimate stress to working stress is known as *factor of safety*.

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

- ✦ The ratio of ultimate load to working load is known as *load factor.*

$$\text{Load factor} = \frac{\text{Ultimate load}}{\text{Working load}}$$

12. Write down the formula for change in length due to tensile load.

$$\text{Change in length, } \delta l = \frac{P\ l}{A\ E}$$

Where, P= Load, l= Length , A=Area, E= Young's modulus

13. Define modulus of rigidity.

The ratio of shear stress to shear strain within the elastic limit is known a *modulus of rigidity* or *shear modulus*.

$$\text{Modulus of rigidity, } \boldsymbol{C} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

14. Distinguish between linear strain and lateral strain

- ✦ The ratio of the change in length to the original length is called *linear strain* or *longitudinal strain*.
- ✦ The ratio of the change in lateral dimension to the original dimension is called *lateral strain*.

15. Define Poisson's ratio.

- ✦ The ratio of the lateral strain to the corresponding longitudinal strain within elastic limit is called *Poisson's ratio.*

$$\text{Poisson's ratio, } \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

16. Define volumetric strain and Bulk modulus.

- ✦ The ratio of change in volume to the original volume is known as *volumetric strain*.

$$\text{Volumetric strain, } e_v = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\delta V}{V}$$

- ✦ The ratio of the direct stress to the corresponding volumetric strain is known as *bulk modulus.*

$$\text{Bulk modulus, } K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{f}{e_v}$$

17. **Write down the formula for change in volume of rectangular bar.**

$$\text{Change in volume, } \delta V = e\left(1 - \frac{2}{m}\right) V$$

Where, e = strain, $1/m$ = Poission's ration, V = Original volume

18. **Write down the relationship between the elastic constants.**

$$E = \frac{9KC}{3K + C}$$

Where, E = Young's modulus, K = Bulk modulus,
C = Rigidity modulus

19. **Define composite bar.**

A *composite bar* may be defined as a bar made of two or more different materials joined together in such a way that the system elongates or contracts as a whole equally when subjected to axial pull or push.

20. **What are the characteristics of composite bar?**

- ✦ Extension or contraction of the bar being equal and hence the strain is also equal.
- ✦ The total external load applied on the composite bar is equal to the sum of the loads shared by the different materials.

21. **Define temperature stress and strain.**

The stresses induced in a body due to change in temperature are known as *temperature stress* or *thermal stress*. The corresponding strain in the body is known as *temperature strain* or *thermal strain*.

22. **Write down the formula for temperature stress.**

$$\text{Temperature stress, } f = \left(\alpha T - \frac{\lambda}{l}\right) E$$

Where, α = coefficient of linear expansion,
T = Change in temperature, λ = Yielding in the support
l = Length of the bar, E = Young's modulus

23. **Define strain energy or resilience.**

- ✦ The energy stored in the body by virtue of strain is called *strain energy* or *resilience*.

24. **Define proof resilience and modulus of resilience.**

- ✦ The maximum strain energy which can be stored in a body without permanent deformation is called its *proof resilience.*

$$\text{Proof resilience} = \frac{f_{max}^2}{2E} \times Volume$$

- ✦ The maximum strain energy which can be stored in a body per unit volume is known as *modulus of resilience.*

$$\text{Modulus of resilience} = \frac{f_{max}^2}{2E}$$

25. **What is the instantaneous stress produced in gradually applied load and suddenly applied load?**

For gradually applied load, $f = \frac{P}{A}$

For suddenly applied load, $f = 2 \times \frac{P}{A}$

26. **Write down the expression for the stress induced due to impact load.**

$$\text{Stress, } f = \frac{P}{A} + \sqrt{\left(\frac{P^2}{A^2} + \frac{2EPh}{Al}\right)}$$

Unit – III

Chapter 5. GEOMETRICAL PROPERTIES OF SECTIONS

1. **Define centre of gravity and centroid.**

- The *centre of gravity* of a body may be defined as a point through which the entire weight of the body is assumed to be concentrated.
- The *centroid* of a section may be defined as a point through which the entire area of the section is assumed to be concentrated.

2. **Write down the formula for centroid of a section.**

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}, \quad \bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

3. **What is centroidal axis and axis of reference?**

- A line passing through the centroid of the plane figure is known as *centroidal axis*.
- A line about which the co–ordinates of centroid are calculated is known as axis of *reference* or *reference axis*.

4. **Define axis of symmetry.**

The axis which divides a section into two equal halves horizontally or vertically is known as axis of symmetry. The centroid of the section will lie on this *axis of symmetry*.

5. **Define moment of inertia.**

The *moment of inertia* of a body about an axis is defined as the internal resistance offered by the body against the rotation about that axis. Moment of inertia, $I = \Sigma a . r^2$

6. **State parallel axis theorem.**

It states that, *if the moment of inertia of a plane area about an axis passing through its centroid is denoted by* I_G *then the moment of inertia of the area about any other axis AB which is parallel to the centroidal axis and at a distance h from the centroid is given by,* $I_{AB} = I_G + Ah^2$

7. **State perpendicular axis theorem.**

It states that, *if* I_{xx} *and* I_{yy} *be the moments of inertia of plane section about two perpendicular axes meeting at O, the moment of inertia* I_{zz} *about the axis Z–Z, perpendicular to the plane and passing through the intersection of X–X and Y–Y axes is given by,* $I_{zz} = I_{xx} + I_{yy}$

8. **What is the moment of inertia of rectangular section about X-X and Y-Y axis.**

$$I_{xx} = \frac{bd^3}{12}; \qquad I_{yy} = \frac{db^3}{12}$$

Where, b = width, d = depth of rectangular section.

9. **What is moment of inertia of circular section about X-X axis?**

$$I_{xx} = I_{yy} = \frac{\pi d^4}{64}; \qquad \text{Where, } d = \text{ diameter of the circular section}$$

10. **State the moment of inertia of a triangle about its base.**

$$\therefore I_{BC} = \frac{bh^3}{12}; \qquad \text{Where, } b = \text{base side, } h = \text{height of triangle}$$

11. **Define polar moment of inertia.**

The moment of inertia of a plane area with respect to the centroidal axis perpendicular to the plane area is called *polar moment of inertia*. I_P or $J = I_{xx} + I_{yy}$

12. **Define radius of gyration.**

Radius of gyration may be defined as the distance at which the whole area of the plane figure is assumed to be concentrated with respect to a reference axis.

$$\text{Radius of gyration, } K = \sqrt{\frac{I}{A}}$$

I = Moment of Inertia ; A = Total Area

13. **What is section modulus?**

The *section modulus* or *modulus of section* is the ratio between the moment of inertia of the figure about its centroidal axis and the distance of extreme surface from the centroidal axis. It is usually denoted by Z.

$$\therefore Z = \frac{\text{Moment of inertia about centroidal axis}}{\text{Distance of extreme surface from centroidal axis}}$$

14. Write down the section modulus for rectangle and circular section.

Section modulus of rectangle, $Z = \dfrac{bd^2}{6}$

Section modulus of circle, $Z = \dfrac{\pi d^3}{32}$

Unit – III

Chapter 6. THIN CYLINDERS AND THIN SPHERICAL SHEELS

1. Distinguish between thin and thick cylinders.

	Thin cylindrical shell	Thick cylindrical shell
1.	The thickness of this cylindrical shell is less than 1/10 to 1/15 times of its diameter.	The thickness of this cylindrical shell is greater than 1/15 times of its diameter.
2.	The normal stresses are assumed to be uniformly distributed throughout the wall thickness	The normal stresses are not uniformly distributed.
3.	Longitudinal stress is uniformly distributed	Longitudinal stress is not uniformly distributed.
4.	The radial stress induced is very small and is neglected.	A finite value of radial stress is induced.

2. State the nature of stresses induced in thin cylindrical shells.

1) Circumferential stress or hoop stress 2) Longitudinal stress

3. Write down the formula for hoop stress and longitudinal stress in thin cylindrical shell.

Hoop stress, $f_1 = \dfrac{pd}{2t}$; Longitudinal stress, $f_2 = \dfrac{pd}{4t}$

Where, p = internal pressure, d= diameter of the shell,
t = thickness of the shell

4. What is the maximum shear stress in thin cylindrical shells?

Maximum shear stress, $f_s = \dfrac{pd}{8t}$

5. **Write down the formula for change in diameter and change in length in thin cylindrical shells.**

Change in diameter, $\delta d = e_1 \times d = \dfrac{f_1}{E}\left(1 - \dfrac{1}{2m}\right) \times d$

Change in volume, $\delta V = \dfrac{f_1}{E}\left(2.5 - \dfrac{2}{m}\right) V$

6. **Write down the expression for the stress induces in thin spherical shells.**

$$\text{Stress, } f = \frac{pd}{4t\eta}$$

Where, p= internal pressure, d=diameter,

t= thickness, η= efficiency of riveted joint

7. **Write down the expression for change in volume of thin spherical shell.**

$$\text{Change in volume, } \delta V = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)$$

Unit – IV

Chapter 7. SHEAR FORCE AND BENDING MOMENT DIAGRAMS

1. **Define beam.**

Beam is a structural member which is subjected to a system of external forces acting perpendicular to the axis.

2. **State the types of beams.**

1) Cantilever beam 2) Simply supported beam
3) Overhanging beam 4) Fixed beam
5) Continuous beam

3. **What is cantilever beam and simply supported beam?**

✦ If one end of the beam is fixed and the other end is free, then such type of beam is called *cantilever beam.*

✦ If both the ends of the beam are made to rest freely on supports, then such type of beam is called *simply supported beam.*

4. **What are the types of loading?**

1) Point load or concentrated load.
2) Uniformly distributed load (*udl*).
3) Uniformly varying load (*uvl*)

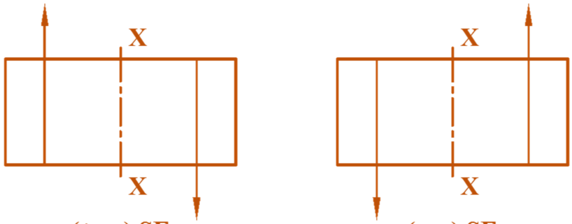
5. **What is *udl* and *uvl*?**

- If a load is spread over the beam in such a way that its magnitude is same for each and every unit length of the beam, then it is called *uniformly distributed load (udl).*
- If a load is spread over the beam in such a way that its magnitude is gradually varying within an unit length of the beam, then it is called *uniformly varying load (uvl).*

6. **Define shear force and bending moment.**

- The *shear force* at a cross section of beam may be defined as the unbalanced vertical forces to the left or right of the section.
- The *bending moment* at a cross section of a beam may be defined as the algebraic sum of the moments of the forces to the left or right of the section.

7. **Draw the sign convention of shear force.**



(+ ve) SF **(–ve) SF**

- All the upward forces to the left of the section and all the downward forces to the right of the section cause positive shear force.
- All the upward forces to the right of the section and all the downward forces to the left of the section cause negative shear force.

8. **Draw the sign convention of bending moment.**

(+ ve) BM **(–ve) BM**

- Bending moment that produce concavity at the top is +ve.
- Bending moment that produce convexity at the top is –ve.

9. **Distinguish between sagging and hogging moment.**

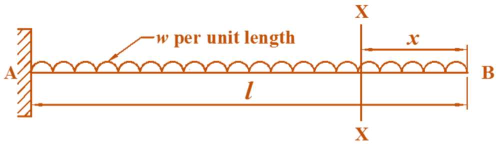
- The positive bending moment is often called as *sagging* moment.
- The negative bending moment is often called as *hogging* moment.

10. **Write the relationship between load, shear force and bending moment.**

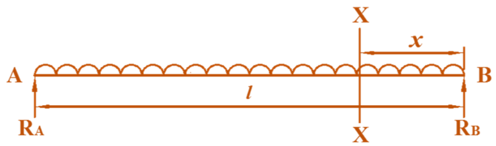
$$1)\quad \frac{dM}{dx} = F \qquad\qquad 2)\ w = \frac{dF}{dx}$$

1) The rate of change of bending moment about a section is equal to the SF at that section.
2) The rate of change of shear force is equal to the rate of loading per unit length of the beam.

11. **Draw a cantilever beam with udl.**



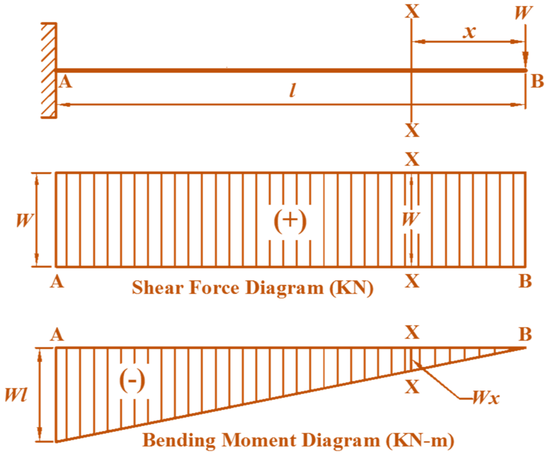
12. **Draw a simply supported beam with udl.**



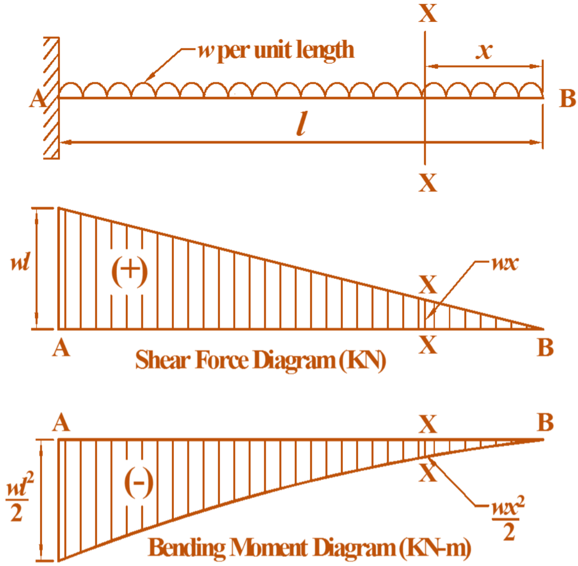
13. **Write down the maximum bending moment in a cantilever beam with udl and simply supported beam with udl.**

Cantilever beam $\Longrightarrow -\dfrac{wl^2}{2}$; Simply supported beam $\Longrightarrow \dfrac{wl^2}{8}$

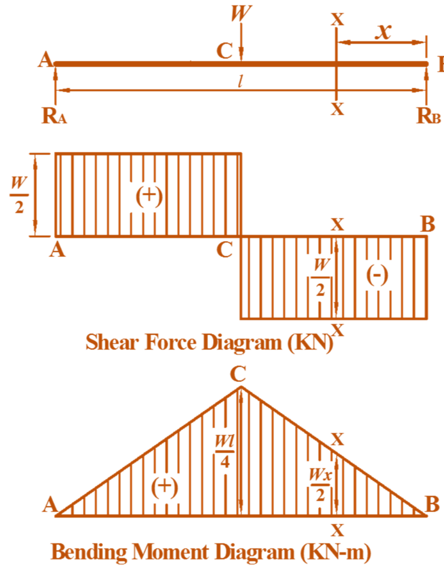
14. **Draw the shear force and bending moment diagram for a cantilever beam with a point load at its free end.**



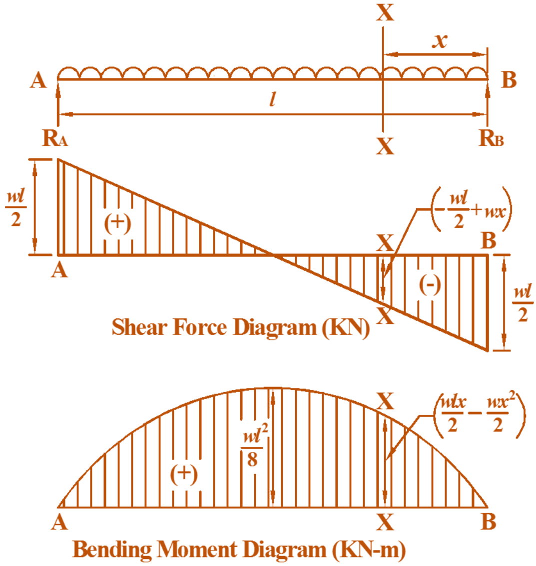
15. **Draw the shear force and bending diagram for a cantilever beam with a udl .**



16. **Draw the shear force and bending diagram for a simply supported beam with a point load at the mid point.**



17. **Draw the shear force and bending diagram for a simply supported beam with udl.**



18. **What is point of contraflexure?**

The point, where the bending moment changes sign, is known as a ***point of contraflexure.***

Unit – IV
Chapter 8. THEORY OF SIMPLE BENDING OF BEAMS

1. **Define simple bending or pure bending.**

If a beam tends to bend or deflect only due to the application of constant bending moment and not due to shear force, then it is said to be in a state of simple bending or pure bending.

2. **Write down the assumptions made in theory of simple bending.**

   1) The material of the beam is uniform throughout.
   2) The material of the beam has equal elastic properties in all directions.
   3) The radius of curvature of the beam is very large when compared with the cross sectional dimensions of the beam.

4) Each layer of the beam is free to expand or contract independently of the layer, above or below it.

5) The cross section of the beam which is plane and normal before bending will remain plane and normal even after bending.

3. **Define neutral axis.**

The line of intersection of the neutral layer with any normal cross–section of the beam is known as *neutral axis* of that section.

4. **What is moment of resistance.**

The maximum bending moment that a beam can withstand without failure is called *moment of resistance*.

5. **Write down the flexural equation.**

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

Where, M = bending moment, I = Moment of inertia

f = bending stress, y = distance from neutral layer

E = Young's modulus, R = radius of curvature

6. **Define section modulus.**

The ratio of moment of inertia about the neutral axis to the distance of the extreme layer from the neutral axis is known as section modulus of modulus of section.

$$\text{Section modulus, } Z = \frac{\text{Moment of inertial about N.A}}{\text{Distance of extreme layer from N.A}}$$

7. **Define strength and stiffness of a beam.**

- ✦ The moment of resistance offered by the beam is known as strength of a beam.
- ✦ The resistance offered by a beam against deflection from its original straight condition is known as stiffness of the beam.

Unit – V
Chapter 9. TORSION OF CIRCULAR SHAFTS

1. **What is pure torsion?**

A circular shaft is said to be in a state of *pure torsion* when it is subjected to pure torque and not accompanied by any other force such as bending or axial force.

2. **Write down the assumptions made in theory of pure torsion.**

1) The material of the shaft is uniform throughout.
2) The shaft is subjected to twisting couples whose planes are exactly perpendicular to the longitudinal axis.
3) The twist along the shaft is uniform.
4) All diameters which are straight before and after twist.
5) Normal cross–sections at the shaft, which were plane and circular before the twist, remain plane and circular after the twist.

3. **Write down the torsion equation.**

$$\frac{T}{J} = \frac{f_s}{r} = \frac{C\theta}{l}$$

Where, f_s=shear stress, r=radius of shaft, T=Torque
J=Polar moment of inertia, N= Rigidity modulus
θ= Angle of twist, l= length of shaft.

4. **Write down the formula for the torque produced in hollow shaft.**

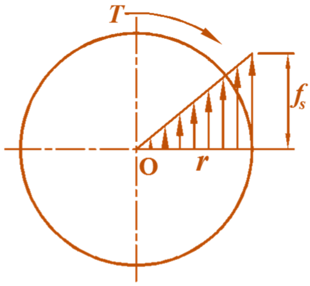
$$\text{Torque, } T = \frac{\pi}{16} f_s \left[\frac{d_1^4 - d_2^4}{d_1}\right]$$

5. **How do you find the power transmitted by the shat?**

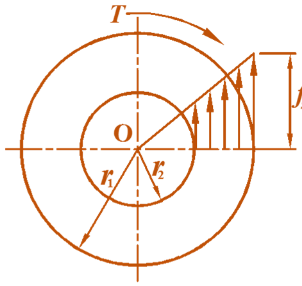
$$\text{Power transmitted, } P = \frac{2\,\pi\,N\,T}{60}$$

Where, T = Torque, N = Speed of shaft (rpm)

6. **Draw the stress distribution in solid and hollow shaft.**



(i) Solid shaft



(ii) Hollow shaft

7. **Define polar modulus.**

The ratio between the polar moment of inertia of the cross– section of the shaft and the maximum radius of the section is known as *polar modulus.*

$$Z = \frac{\text{Polar moment of inertia}}{\text{Maximum radius}} = \frac{J}{r}$$

8. **State the polar modulus for solid and hollow circular shafts.**

Solid shaft $\Rightarrow Z = \dfrac{\pi d^3}{16}$; Hollow shaft $\Rightarrow Z = \dfrac{\pi}{16\, d_1}\left(d_1^4 - d_2^4\right)$

9. **Define torsional strength and torsional rigidity.**

- *Torsional strength* is defined as the torque developed per unit maximum shear stress. Torsional strength is also known as the *efficiency* of a shaft.

$$\text{Torsional strength} = \frac{T}{f_s}$$

- *Torsional rigidity* or *stiffness* is defined as the torque required to produce an unit angle of twist in a specified length of the shaft.

$$\text{Torsional rigidity} = \frac{T}{\theta}$$

10. **Compare the strength of hollow shaft and solid shaft of same weight and length.**

$$\frac{\text{Strength of hollow shaft}}{\text{Strength of solid shaft}} = \frac{\left(d_1^4 - d_2^4\right)}{d_1 \times d^3}$$

For a given cross–sectional area, a hollow circular shaft has larger value of section modulus when compared with that of a solid circular shaft. So the hollow shaft has more strength than that of a solid shaft.

11. **Compare the weight of hollow shaft and solid shaft of same material and length.**

$$\frac{\text{Weight of solid shaft}}{\text{Weight of hollow shaft}} = \frac{d^2}{(d_1^2 - d_2^2)}$$

For a given material, length and torsional strength, the weight of a hollow shaft is less than that of a solid shaft. When using hollow shaft, the material requirement is considerably reduced.

12. **List out the advantages hollow shaft over solid shaft.**

1) A hollow shaft has greater torsional strength than a solid shaft of same material.
2) A hollow shat has more stiffness than a solid shaft of same cross–sectional area.
3) The material required for hollow shaft is comparatively lesser than the solid shaft for same strength.
4) The shear stress induced in the hollow shaft is almost uniform throughout the section.

1. **What are the types of springs?**

1) Laminated or leaf springs 2) Coiled helical springs
3) Spiral springs 4) Disc springs

2. **What are laminated springs? Give its uses.**

✦ A laminated spring consists of a number of parallel strips of metal having different lengths but same width and placed one over the other in laminations.

✦ This type of springs are widely used in railway wagons, coaches and road vehicles to absorb shocks.

3. **Compare closely coiled and open coiled helical springs.**

	Closely coiled helical spring	Open coiled helical spring
1.	Pitch of coil is very small	Pitch of coil is large
2.	Gap between the successive turn is small	Gap between the successive turn is large
3.	Helix angle is less	Helix angle is more
4.	Under axial load, it is subjected to torsion only	It is subjected to both torsion and bending
5.	It can withstand more load	It can withstand less load

4. **Write down the deflection formula for closely coiled helical spring.**

$$\text{Deflection of the spring, } \delta = \frac{64\ W\ R^3 n}{C\ d^4}$$

Where, W = Load, R=Mean radius of coil, n = Number of turns,
C = Modulus of rigidity, d= Diameter of spring wire.

5. **Define stiffness or spring constant.**

The *stiffness* of the spring is defined as the load required to produce unit deflection. It is denoted by 's'.

$$s = \frac{W}{\delta} = \frac{C\ d^4}{64\ R^3 n}$$

6. **Give the formula for resilience of the spring.**

$$\text{Energy stored or resilience of spring} = \frac{32\ W^2\ R^3 n}{C\ d^4}$$

7. **State the applications of spring.**

1) To apply forces and controlling motion, as in brakes and clutches.
2) Measuring forces, as in spring balances.
3) Storing energy, as springs used in watches and toys.
4) Reducing the effect of shock and vibrations in vehicles and machine foundations.

32031 – STRENGTH OF MATERIALS

MODEL QUESTION PAPER - I

Time: 3 Hrs   Max Marks : 75

*[N.B: (1) Q.No.8 in PART – A and Q.No.16 in PART – B are compulsory. Answer any **FOUR** questions from the remaining in each PART – A and PART – B.*

(2) Answer division (a) or division (b) of each question in PART – C.

(3) Each question carries 2 marks in PART – A, 3 marks in PART – B and 10 marks in PART – C.]

PART - A

1. State the principle of transmissibility of forces.
2. Define co-efficient of friction.
3. Define Poisson's ratio.
4. Define centroid.
5. Write perpendicular axis theorem.
6. Write down the relationship between load, shear force and bending moment.
7. Define polar modulus.
8. State the applications of spring.

PART - B

9. Differentiate between static friction and dynamic friction.
10. Define hardness. What is its importance?
11. Define proof resilience and modulus of resilience.
12. Distinguish between thin and thick cylinders.
13. Write the assumptions made in theory of simple bending.
14. Define section modulus. Write the section modulus for rectangular section.
15. Compare closely coiled and open coiled helical springs.
16. Two equal forces are acting at a point with an angle of 60° between them. If the resultant of the forces is equal to 40 N, find the magnitude of each force.

PART – C

17.(a) Five forces are acting on a particle. The magnitude of the forces are 300 N, 600 N, 900 N and 'P' and their respective angles with the horizontal are 0°, 60°, 135°, 210°, 270°. If the vertical component of all the forces is –1000 N, find the value of 'P'. Also calculate the magnitude and direction of the resultant force assuming that 300 N force acts towards the particle while all others act away from the particle.

(Or)

(b) i) State the laws of dynamic friction.

ii) The resultant of two concurrent forces is 1500 N and the angle between the forces is 90°. The resultant makes an angle of 36° with one of the forces. Find the magnitude of each force.

18.(a) Sketch and explain the stress strain diagram for a mild steel specimen with its salient point.

(Or)

(b) A weight of 1400 N is dropped on to a collar at the lower end of a vertical bar 3 m long and 25 mm in diameter. Calculate the height of drop, if the maximum instantaneous stress is not to exceed 120 N/mm^2. What is the corresponding instantaneous elongation? Take E=2×10^5N/ mm^2.

19.(a) Find the values of I_{xx} and I_{yy} of a T–section 120 mm wide and 120 mm deep overall. Both the web and flange are 10 mm thick. Also calculate K_{xx} and K_{yy}.

(Or)

(b) A spherical shell of 1 m internal diameter and 5 mm thick is filled with a fluid until its volume increases by 0.2×10^6 mm^3. Calculate the pressure exerted by the fluid on the shell. Take $E = 2 \times 10^5$ N/mm^2, $1/m = 0.3$ for the material.

20.(a) i) A cantilever 4 m long carries a udl of 30 KN/m over half of its length adjoining the free end. Draw SF and BM diagrams.

ii) A cantilever beam of span 2 m carries a point load of 600 N at the free end. If the cross–section of the beam is rectangular 100 mm wide and 150 mm deep, find the maximum bending stress induced.

(Or)

(b) (i) A cast iron water pipe 450 mm bore and 20 mm thick is supported at two points 6 m apart. Assuming each span as simply supported, find the maximum stress in the metal when (a) the pipe is running full (b) the pipe is empty. Specific weight of cast iron is 70 KN/m^3 and that of water is 9.81 KN/m^3.

21.(a) A solid shaft is transmitting 100 KW at 180 rpm. If the allowable stress is 60 N/mm^2, find the necessary diameter for the shaft. The shaft is not to twist more than $1°$ in a length of 3 m. Take $C = 80$ KN/mm^2.

(Or)

(b) A weight of 150 N is dropped on to a compression spring with 10 coils of 12 mm diameter closely coiled to a mean diameter of 150 mm. If the instantaneous contraction is 140 mm, calculate the height of drop. Take $C = 0.8 \times 10^5 N/mm^2$.

32031 – STRENGTH OF MATERIALS

MODEL QUESTION PAPER - II

Time: 3 Hrs | Max Marks : 75

*[N.B: (1) Q.No.8 in PART – A and Q.No.16 in PART – B are compulsory. Answer any **FOUR** questions from the remaining in each PART – A and PART – B.*

(2) Answer division (a) or division (b) of each question in PART – C.

(3) Each question carries 2 marks in PART – A, 3 marks in PART – B and 10 marks in PART – C.]

PART - A

1. Define resultant of forces.
2. What is limiting force of friction?
3. Define creep.
4. Define moment of inertia.
5. Define radius of gyration.
6. Define neutral axis.
7. Define stiffness of shaft.
8. What is laminated spring? List its application.

PART - B

9. State parallelogram law of forces.
10. Define bulk modulus.
11. Define composite bar.
12. State parallel axis theorem.
13. Write sign conventions for shear force and bending moment.
14. State the assumptions made in theory of pure torsion.
15. List out the advantages of hollow shaft over solid shaft.
16. A closely coiled helical spring made of steel wire of 10 mm diameter has 10 coils of 120 mm mean diameter. Calculate the deflection of the spring under an axial load of 100 N. Take $C = 1.2 \times 10^5 \text{N/mm}^2$.

PART – C

17.(a) The magnitude of the resultant of two concurrent forces including an angle of 90° between them is $\sqrt{13}$ KN. When the included angle between the force is 60°, the magnitude of their resultant is $\sqrt{19}$ KN. Find the magnitude of the two forces.

(Or)

(b) Explain the various types of supports and reaction with neat sketches.

18.(a) A circular bar of length 150 mm and diameter of 50 mm is subjected to an axial pull of 400 KN. The extension in length and contraction in diameter were found to be 0.25 mm and 0.02 mm respectively after loading. Calculate (i) Poisson's ratio (ii) Young's modulus (iii) Modulus of rigidity and (iv) Bulk modulus

(Or)

(b) Determine the greatest weight that can be dropped from a height of 200 mm on to a collar at the lower end of a vertical bar 20 mm diameter and 2.5 m long without exceeding the elastic limit stress 300 N/mm^2. Calculate also the instantaneous elongation. Take $E = 2 \times 10^5$ N/mm^2.

19.(a) An I–section has the top flange 100 mm × 15 mm, web 150 mm × 20 mm and the bottom flange 180 mm × 30 mm. Calculate I_{xx} and I_{yy} of the section. Also find K_{xx} and K_{yy} of the section.

(Or)

(b) A cylindrical shell 3 m long and 500 mm in diameter is made up of 20 mm thick plate. If the cylindrical shell is subjected to an internal pressure of 5 N/mm^2, find the resulting hoop stress, longitudinal stress, changes in diameter, length and volume. Take $E = 2 \times 10^5$ N/mm^2 and Poisson's ratio = 0.3.

20.(a) A simply supported beam of length 6m carries a udl of 20 KN/m throughout its length and a point load of 30 KN at 2 m from the right support. Draw the shear force and bending moment diagram. Also find the position and magnitude of maximum bending moment.

(Or)

(b) i) Write down the expression for section modulus of rectangular and circular sections.

ii) A simply supported beam of rectangular cross section carries a central load of 25 KN over a span of 6m. The bending stress should not exceed 7.5 N/mm^2. The depth of the section is 400 mm. Calculate the necessary width of the section.

21.(a) A hollow shaft having inner diameter 0.6 times the outer diameter is to be replaced by a solid shaft of the same material to transmit 550 KW at 220 rpm. The permissible shear stress is 80 N/mm^2. Calculate the diameters of the hollow and solid shafts. Also calculate the percentage of saving in material.

(Or)

(b) Design a closely coiled helical spring of stiffness 20 N/mm deflection. The maximum shear stress in the spring material is not to exceed 80 N/mm^2 under a load of 600 N. The diameter of the coil is to be 10 times the diameter of the wire. Take $C = 85 \times 10^3$ N/mm^2.

BOARD EXAMINATION QUESTION PAPER

OCTOBER – 2016

PART - A

1. State the principle of transmissibility of forces.
[Ans : Q&A – Chapter.1 – Qn.3]
2. Define limiting friction. *[Ans : Q&A – Chapter.2 – Qn.2]*
3. Define temperature creep. *[Ans : Q&A – Chapter.3 – Qn.13]*
4. Define Poisson's ratio. *[Ans : Q&A – Chapter.4 – Qn.15]*
5. Define centre of gravity. *[Ans : Q&A – Chapter.5 – Qn.1]*
6. Write down the relationship between load, shear force and bending moment.
[Ans : Q&A – Chapter.7 – Qn.10]
7. Define polar modulus. *[Ans : Q&A – Chapter.9 – Qn.7]*
8. Define moment of inertia. *[Ans : Q&A – Chapter.5 – Qn.5]*

PART - B

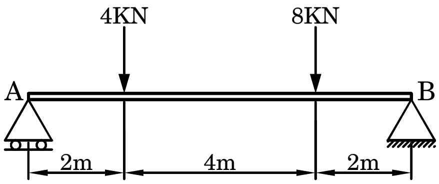
9. State parallelogram law of forces. *[Ans : Q&A – Chapter.1 – Qn.6]*
10. State Varignon's theorem. *[Ans : Q&A – Chapter.1 – Qn.16]*
11. Distinguish between linear strain and lateral strain.
[Ans : Q&A – Chapter.4 – Qn.14]
12. Define proof resilience and modulus of resilience.
[Ans : Q&A – Chapter.4 – Qn.24]
13. Distinguish between thin and thick cylinders.
[Ans : Q&A – Chapter.6 – Qn.1]
14. Write down the assumptions made in theory of simple bending.
[Ans : Q&A – Chapter.8 – Qn.2]
15. Write down the advantages of hollow shafts over solid shafts.
[Ans : Q&A – Chapter.9 – Qn.12]
16. A closely coiled helical spring made of steel wire of 10 mm diameter has 10 coils of 120 mm mean diameter. Calculate the deflection of the spring under an axial load of 100 N. Take $C = 1.2 \times 10^5 \text{N/mm}^2$.
[Ans : Example – 10.7]

PART – C

17.(a) Five forces are acting on a particle. The magnitude of the forces are 300 N, 600 N, 900 N and ‘P’ and their respective angles with the horizontal are 0°, 60°, 135°, 210°, 270°. If the vertical component of all the forces is –1000 N, find the value of ‘P’. Also calculate the magnitude and direction of the resultant force assuming that 300 N force acts towards the particle while all others act away from the particle. *[Ans : Example – 1.8]*

(Or)

(b) Determine the support reactions of the beam shown in the figure.



[Ans : Example – 1.16]

18.(a) A circular bar of length 150 mm and diameter of 50 mm is subjected to an axial pull of 400 KN. The extension in length and contraction in diameter were found to be 0.25 mm and 0.02 mm respectively after loading. Calculate (i) Poisson’s ratio (ii) Young’s modulus (iii) Modulus of rigidity and (iv) Bulk modulus.

[Ans : Example – 4.17]

(Or)

(b) A weight of 1400 N is dropped onto a collar at the lower end of a vertical bar 3 m long and 25 mm in diameter. Calculate the height of drop, if the maximum instantaneous stress produced is not to exceed 120 N/mm^2. Take $E = 0.2 \times 10^6$ N/mm^2. *[Ans : Example – 4.37]*

19.(a) An angle section is of 100 mm wide and 120 mm deep overall. Both the flanges of the angle are 10 mm thick. Determine the moment of inertia about the centroidal axes X-X and Y-Y.

[Ans : Example – 5.9]

(Or)

(b) A spherical shell of 1m internal diameter and 5mm thick is filled with a liquid under pressure until its volume increases by 0.2×10^6 mm^3. Determine the pressure exerted by the liquid on the shell. Take $E = 2 \times 10^5$N/mm^2 and $1/m = 0.3$.

[Ans : Example – 6.15]

20.(a) A simply supported beam of span 6 m caries three point loads of 30 KN, 25 KN and 40 KN at 1 m, 3 m and 4.5 m respectively from the left support. Draw the SFD and BMD and indicates the maximum value of bending moment. *[Ans : Example – 7.9]*

(Or)

(b) i) A cantilever beam of span 2 m carries a point load of 600 N at its free end. If the beam is rectangular section of 100 mm wide and 150 mm deep, find the maximum bending stress induced.

[Ans : Example – 8.18]

ii) Derive an expression for section modulus of a rectangular section.

[Ans : Theory – Chapter.8 – 8.9]

21.(a) A hollow shaft having inner diameter 0.6 times the outer diameter is to replace a solid shaft of the same material to transmit 550 KW at 220 rpm. Calculate the diameters of the hollow and solid shafts. Also calculate the percentage of saving in material. The allowable shear stress is 80 N/mm^2 . *[Ans : Example – 9.20]*

(Or)

(b) A weight of 150N is dropped onto a compression spring with 10 coils of 12mm diameter closely coiled to a mean diameter of 150mm. If the instantaneous contraction is 140mm, calculate the height of drop. Take $C = 0.8 \times 10^5$N/mm^2. *[Ans : Example – 10.12]*

BOARD EXAMINATION QUESTION PAPER

APRIL – 2017

PART – A

1. Write down the conditions of equilibrium.
[Ans : Q&A – Chapter.1 – Qn.18]
2. What is dynamic friction? *[Ans : Q&A – Chapter.2 – Qn.3]*
3. Define fatigue strength. *[Ans : Q&A – Chapter.3 – Qn.12]*
4. What is meant by elastic limit? *[Ans : Q&A – Chapter.4 – Qn.6]*
5. Define centroid. *[Ans : Q&A – Chapter.5 – Qn.1]*
6. Mention different types of loading. *[Ans : Q&A – Chapter.7 – Qn.4]*
7. State the applications of springs. *[Ans : Q&A – Chapter.10 – Qn.7]*
8. State any two assumptions made in theory of torsion.
[Ans : Q&A – Chapter.9 – Qn.2]

PART - B

9. State the triangular law of forces. *[Ans : Q&A – Chapter.1 – Qn.11]*
10. Distinguish between moment and couple.
[Ans : Q&A – Chapter.1 – Qn.15 & 17]
11. Distinguish between factor of safety and load factor.
[Ans : Q&A – Chapter.4 – Qn.11]
12. Define bulk modulus. *[Ans : Q&A – Chapter.4 – Qn.16]*
13. State parallel axis theorem. *[Ans : Q&A – Chapter.5 – Qn.6]*
14. Define shear force and bending moment. *[Ans : Q&A – Chapter.7 – Qn.6]*
15. Write down the various types of springs. *[Ans : Q&A – Chapter.10 – Qn.1]*
16. A boiler 2.8 m diameter is subjected to a steam pressure of 0.68 N/mm^2. Find the hoop stress and longitudinal stress, if the thickness of the boiler plate is 10 mm. *[Ans : Example – 6.1]*

PART – C

17.(a) A particle 'O' is acted upon by the following forces : (i) 20N inclined 30° to north of east, (ii) 25N towards the north., (iii) 30N towards north-west, (iv) 35N inclined 40° to south of west. Find the magnitude and direction of the resultant force.

[Ans : Example – 1.6]

(Or)

(b) Discuss the various types of supports with neat sketches and show the reaction components of each. *[Ans : Theory – Chapter.1 – 1.21]*

18.(a) A steel bar of 25 mm diameter and a length of 1 m is subjected to a pull of 25 KN. If $E = 2 \times 10^5$ N/mm^2, find the elongation, decrease in diameter and increase in the volume of the bar. Take $m = 4$.

[Ans : Example – 4.14]

(Or)

(b) Two vertical wires, each of 2.5 mm diameter and 5 m long jointly support a weight of 2.5 KN. One wire is of steel and the other is of different material. If the wires stretch 6 mm elastically, find the load taken by each and the value of Young's modulus for second wire. The Young's modulus of steel wire is 2×10^5 N/mm^2.

[Ans : Example – 4.22]

19.(a) Find I_{xx}, I_{yy}, K_{xx} and K_{yy} of a 'T' section with flange 150 mm × 20 mm and web 100 mm × 20 mm.

[Ans : Example – 5.10 (Same Model)]

(Or)

(b) Calculate the increase in volume of a boiler shell 3 m long and 1.5 m in diameter, when subjected to an internal pressure of 2 N/mm^2. The thickness is such that the maximum tensile stress is not to exceed 30 N/mm^2. Take $E = 2.1 \times 10^5$ N/mm^2 and $1/m = 0.28$. Also calculate the changes in diameter and length.

[Ans : Example – 6.10]

20.(a) A simply supported beam of length 6 m carries an udl of 20 KN/m throughout its length and a point load of 30 KN at 2 m from the right support. Draw the SFD and BMD. Also find the position and magnitude of maximum bending moment. *[Ans : Example – 7.11]*

(Or)

(b) i) A cantilever beam of 4 m long carries an udl of 20 KN/m over half of its length from the free end. Draw the SF and BM diagrams. *[Ans : Example – 7.3]*

ii) A rectangular beam of 300 mm deep is simply supported over a span of 4 m. What udl the beam may carry, if the bending stress is not to exceed 120 N/mm^2? Take $I = 8 \times 10^6 \text{mm}^4$. *[Ans : Example – 8..8]*

21.(a) A solid shaft is transmitting 100 KW power at 180 rpm. If the allowable stress is 60 N/mm^2, find the necessary diameter for the shaft. The shaft is not to twist more than 1° in a length of 3 m. Take $C = 80$ KN/mm^2. *[Ans : Example – 9.18]*

(Or)

(b) (i) Calculate the power transmitted by a shaft of 100 mm diameter running at 250 rpm, if the shear stress in the shaft material is not to exceed 75 N/mm^2. *[Ans : Example – 9.4]*

(ii) A closely coiled helical spring made of steel wire of 100 mm diameter has 10 coils of 120 mm mean diameter. Calculate the deflection under an axial load of 100 N and the stiffness of the spring. Take $C = 1.2$ N/mm^2. *[Ans : Example – 10.7]*

BOARD EXAMINATION QUESTION PAPER

OCTOBER – 2017

PART – A

1. State triangular law of forces. *[Ans : Q&A – Chapter.1 – Qn.11]*
2. Define angle of friction. *[Ans : Q&A – Chapter.2 – Qn.7]*
3. Define creep. *[Ans : Q&A – Chapter.3 – Qn.10]*
4. Define Poisson's ratio. *[Ans : Q&A – Chapter.4 – Qn.15]*
5. State perpendicular axis theorem. *[Ans : Q&A – Chapter.5 – Qn.7]*
6. Write torsion equation. *[Ans : Q&A – Chapter.9 – Qn.3]*
7. Define neutral axis. *[Ans : Q&A – Chapter.8 – Qn.3]*
8. Write sign conventions for shear force and bending moment.
   [Ans : Q&A – Chapter.7 – Qn.7 & 8]

PART – B

9. Explain external and internal forces. *[Ans : Q&A – Chapter.1 – Qn.14]*
10. Explain cone of friction. *[Ans : Q&A – Chapter.2 – Qn.8]*
11. Explain bulk modulus. *[Ans : Q&A – Chapter.4 – Qn.16]*
12. Derive the moment of inertia for rectangular area.
    [Ans : Theory – Chapter.5 – 5.8]
13. State the relationship between load, force and bending moment at a section.
    [Ans : Q&A – Chapter.7– Qn.10]
14. Write the assumptions made on theory of simple bending.
    [Ans : Q&A – Chapter.8 – Qn.2]
15. Derive polar modulus for i) solid shaft (ii) Hollow shaft.
    [Ans : Theory – Chapter.9 – 9.8]
16. State the difference between open and closely coiled helical spring.
    [Ans : Q&A – Chapter.10 – Qn.3]

PART – C

17.(a) Five forces are acting on a particle. The magnitude of the forces are 300 N, 600 N, 700 N, 900 N and 'P'. Their respective angles with the horizontal are 0°, 60°, 135°, 210° and 270°. If the vertical component of all the forces is -1000 N, find the value of 'P'. Also calculate the magnitude and direction of the resultant, assuming that the first force acts towards the point, while all the remaining forces act away from the point.

[Ans : Example – 1.8]

(Or)

(b) The magnitude of the resultant of two concurrent forces including and angle of 90° between them is $\sqrt{13}$ KN. When the included angle between the forces is 60°, the magnitude of their resultant is $\sqrt{19}$ KN. Find the magnitudes of the two forces. *[Ans : Example – 1.3]*

18.(a) Explain the stress-strain diagram for a mild steel specimen with its salient point parameters. *[Ans : Theory – Chapter.3 – 4.5]*

(Or)

(b) A bar of steel 28 mm diameter and 250 mm long is subjected to an axial load of 80 KN. It is found that the diameter has contracted by 1/240 mm. If the modulus of rigidity is $0.8 \times 10^5 \text{N/mm}^2$, calculate (1) Poission's ratio (2) Bulk modulus and (3) Young's modulus.

[Ans : Example – 4.21]

19.(a) An I-section has the top flange 100 mm × 15 mm, web 150 mm × 20 mm and the bottom flange 180 mm × 30 mm. Calculate I_{xx}, I_{yy} and also radius of gyration about the centroidal axes.

[Ans : Example – 5.15]

(Or)

(b) A cylinder shell 3 m long 500 mm in diameter is made up of 20 mm thick plate. If the cylinder shell is subjected to an internal pressure of 5 N/mm^2, find the resulting hoop stress, longitudinal stress, change in length and change in volume. Take $E = 2 \times 10^5 \text{N/mm}^2$ and $1/m = 0.3$.

[Ans : Example – 6.9]

20.(a) A beam is freely supported over a span of 8 m. It carries point load of 3 KN at 2 m from the left hand support and an udl of 2 KN/m run from the centre to the right hand support. Construct SFD and BMD.

[Ans : Example – 7.10]

(Or)

(b) A cast iron water main 450 mm bore and 20 mm thick is supported at intervals of 6 m. Assuming each span as simply supported, find the maximum stress in the metal when (i) the pipe is running full and (ii) the pipe is empty. Specific weight of cast iron is 70 KN/m^3 and specific weight of water is 9.81 KN/m^3. ***[Ans : Example – 8.16]***

21.(a) Hollow circular shaft of 25 mm outside diameter and 20 mm inside diameter is subjected to a torque of 50 N-m. Find the shear stress induced at the outside and inside layer of shaft. ***[Ans : Example – 9.7]***

(Or)

(b) A truck weighing 30 KN and moving at 5 Km/hr. has to be brought to rest by buffer. Find how many springs each of 18 coils will be required to the energy of motion during a compression of 200 mm. The spring is made of 25 mm diameter steel rod coiled to a mean diameter of 240 mm. Take $C = 0.84 \times 10^5$ N/mm^2. ***[Ans : Example – 10.11]***

BOARD EXAMINATION QUESTION PAPER

APRIL – 2018

PART – A

1. Define resultant of forces. *[Ans : Q&A – Chapter.1 – Qn.7]*
2. What is meant by coplanar force? *[Ans : Q&A – Chapter.1 – Qn.5]*
3. Define Poisson's ratio. *[Ans : Q&A – Chapter.4 – Qn.15]*
4. Define polar moment of inertia. *[Ans : Q&A – Chapter.5 – Qn.11]*
5. Define neutral axis. *[Ans : Q&A – Chapter.8 – Qn.3]*
6. Define section modulus. *[Ans : Q&A – Chapter.8 – Qn.6]*
7. Define stiffness of a spring. *[Ans : Q&A – Chapter.10 – Qn.5]*
8. Distinguish between centre of gravity and centroid.
   [Ans : Q&A – Chapter.5 – Qn.1]

PART - B

9. Differentiate between static friction and dynamic friction.
   [Ans : Q&A – Chapter.2 – Qn.3]
10. Define hardness and lists its characteristics.
    [Ans : Q&A – Chapter.2 – Qn.9]
11. Differentiate between thin cylinder and thick cylinder.
    [Ans : Q&A – Chapter.6 – Qn.1]
12. What is point load and uniformly distributed load?
    [Ans : Q&A – Chapter.7 – Qn.5]
13. Write down the assumptions made in theory of simple bending.
    [Ans : Q&A – Chapter.8 – Qn.2]
14. What is laminated spring? List its applications.
    [Ans : Q&A – Chapter.10 – Qn.2]
15. List out the advantages of hollow shaft over solid shaft.
    [Ans : Q&A – Chapter.9 – Qn.12]
16. Define support. List the various types of loads.
    [Ans : Q&A – Chapter.1 – Qn.24 & Chapter.7 – Qn.4]

PART – C

17.(a) (i) Explain the various methods of supports and reactions.

[Ans : Theory – Chapter.1 – 1.21]

(ii) State the laws of dynamic friction.

[Ans : Theory – Chapter.2 – 2.7]

(Or)

(b) The resultant of two concurrent forces is 1500 N and angle between the forces is 90°. The resultant makes an angle of 36° with one of the forces. Find the magnitude of each force.

[Ans : Example – 1.4]

18.(a) State and explain the three types of elastic constants.

[Ans : Theory – Chapter.4 – 4.7, 4.14 & 4.18]

(Or)

(b) List out the various alloying elements used in steel and explain their major effects. *[Ans : Theory – Chapter.3 – 3.4]*

19.(a) Find the values of I_{xx} and I_{yy} of a T-section 120 mm wide, 120 mm deep overall. Both the web and flange are 10 mm thick.

[Ans : Example – 5.10]

(Or)

(b) A long steel tube 70 mm internal diameter and wall thickness 2.5 mm has closed ends and subjected to an internal pressure of 10 N/mm^2. Calculate the magnitude of hoop stress and longitudinal stresses setup in the tube. If the efficiency of the longitudinal joint is 80%, state the stress which is affected and what is its revised value? *[Ans : Example – 6.8]*

20.(a) Write down the expressions for sectional modulus of rectangular and circular beam. ***[Ans : Theory – Chapter.8 – 8.9]***

(Or)

(b) A simply supported beam of span 10 m carries an udl of 20 KN/m over the left half of the span and a point load of 30 KN at the mid span. Draw SFD and BMD. Find also, the position and magnitude of maximum bending moment. ***[Ans : Example – 7.12]***

21.(a) Prove the torsion equation. ***[Ans : Theory – Chapter.9 – 9.4]***

(Or)

(b) The mean diameter of a closely coiled helical spring is 5 times the diameter of wire. It elongates 8 mm under an axial pull of 120 N. If the permissible shear stress is 40 N/mm^2, find the size of wire and number of coils in the spring. Take $C = 0.8 \times 10^5$ N/mm^2.

[Ans : Example – 10.6]

BOARD EXAMINATION QUESTION PAPER

OCTOBER – 2018

PART – A

1. What are the characteristics of force? *[Ans : Q&A – Chapter.1 – Qn.2]*
2. State Hooke's law. *[Ans : Q&A – Chapter.4 – Qn.8]*
3. What is meant by temperature stress? *[Ans : Q&A – Chapter.4 – Qn.21]*
4. List out the stresses, induced in thin cylindrical shells.
   [Ans : Q&A – Chapter.6 – Qn.2]
5. What is beam? *[Ans : Q&A – Chapter.7 – Qn.1]*
6. Define polar modulus. *[Ans : Q&A – Chapter.9 – Qn.7]*
7. List the various types of springs. *[Ans : Q&A – Chapter.10 – Qn.1]*
8. Define moment of inertia. *[Ans : Q&A – Chapter.5 – Qn.5]*

PART - B

9. Distinguish between force of friction and limiting force of friction.
   [Ans : Q&A – Chapter.2 – Qn.2]
10. Define co-efficient of friction. *[Ans : Q&A – Chapter.2 – Qn.6]*
11. Differentiate between repeated loading and cyclic loading.
    [Ans : Q&A – Chapter.3 – Qn.11]
12. What is centroidal axis and axis of reference.
    [Ans : Q&A – Chapter.5 – Qn.3]
13. What is cantilever beam and simply supported beam?
    [Ans : Q&A – Chapter.7 – Qn.3]
14. Write down the assumptions made in theory of pure torsion.
    [Ans : Q&A – Chapter.9 – Qn.2]
15. Compare closely coiled and open coiled helical spring.
    [Ans : Q&A – Chapter.10 – Qn.3]
16. What is UDL and UVL? *[Ans : Q&A – Chapter.7 – Qn.5]*

PART – C

17.(a) (i) What are the essential conditions for equilibrium of rigid body?

[Ans : Theory – Chapter.1 – 1.16]

(ii) State the laws of static friction. *[Ans : Theory – Chapter.2 – 2.6]*

(Or)

(b) The magnitude of the resultant of two concurrent forces including and angle of 90° between them is $\sqrt{13}$ KN. When the included angle between the forces is 60°, the magnitude of their resultant is $\sqrt{19}$ KN. Find the magnitudes of the two forces.

[Ans : Example – 1.3]

18.(a) List and explain the various mechanical properties of materials.

[Ans : Theory – Chapter.3 – 3.2]

(Or)

(b) A mild steel specimen 25 mm rod diameter was subjected to an axial pull of 100 KN. An extension of 0.25 mm was noted on a gauge length of 300 mm and a decrease in diameter of 0.00595 mm was observed. Find the values of Poisson's ratio and Young's modulus of the material. *[Ans : Example – 4.20 (Same Model)]*

19.(a) Find the centroid of an I-section having top flange 150 mm × 25 mm, web 160 mm × 25 mm and bottom flange 200 mm × 25 mm.

[Ans : Example – 5.14 (Same Model)]

(Or)

(b) Determine the change in diameter and change in volume of spherical shell 2 m in diameter and 12 mm thick subjected to an internal pressure of 2 N/mm^2. Assume $E = 2 \times 10^5$ N/mm^2 and Poisson's ratio = 0.25. *[Ans : Example – 6.13]*

20.(a) Draw the SFD and BMD for a simply supported beam subjected to a point load W at its mid point. *[Ans : Theory – Chapter.7 – 7.8.3)]*

(Or)

(b) A wooden beam of rectangular section 100 mm $\times$ 200 mm is simply supported over a span of 6 m. Determine the UDL it may carry, if the bending stress is not to exceed 7.5 N/mm^2. Estimate the concentrated load it may carry at the centre of the beam with the same permissible stress. ***[Ans : Example – 8.13]***

21.(a) With neat sketches, explain he various types of springs.

[Ans : Theory – Chapter.10 – 10.2]

(Or)

(b) A solid shaft 20 mm diameter transmits 10 KW at 1200 rpm. Calculate the maximum intensity of shear stress induced and the angle of twist in degrees in a length of 1 m. If modulus of rigidity for the material of the shaft is 8×10^4 N/mm^2.

[Ans : Example – 9.13]

Note :

The original book is

printed in BLACK colour only