GOVERNMENT OF TAMILNADU
DIRECTORATE OF TECHNICAL EDUCATION
CHENNAI – 600 025
STATE PROJECT COORDINATION UNIT

Diploma in Electrical and Electronics Engineering
Course Code: 1030
M – Scheme

e-TEXTBOOK
on

ELECTRICAL CIRCUIT THEORY

for
III Semester DEEE

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DIPLOMA IN ELECTRICAL AND ELECTRONICS ENGINEERING

M - SCHEME

Course Name: Diploma in Electrical and Electronics Engineering

Subject Code: 33031 **Semester: III**

Subject Title: ELECTRICAL CIRCUIT THEORY

Rationale:

1. Electric circuit analysis is the process of determining the voltages across a component in the circuit and the currents passing through it. There are different methods to determine the voltage and current.
2. This course covers, introduction to network elements and methods for finding voltage and current across desired network Component for DC, single phase AC and 3 phase ac sources.
3. It aims at making the student conversant with different techniques of solving the problems in the field of Electric circuits and analysis.

Objectives:

The objective is to enable students to:

1. Understand the concept of electrostatics and capacitance effect and analyze different Circuit Elements, Energy Sources and analysis of Network by Kirchhoff's Laws.
2. Apply the concept of Node and Mesh methods for circuit analysis; apply superposition theorem, Thevenin's Theorem , Norton's Theorem, Maximum Power Transfer theorem and Star-delta conversion for simplification and analyzing dc circuits.
3. Analyze single phase circuits using Resistor, Inductor & Capacitor elements.
4. Understand and analyze series and parallel resonant behavior of a circuit.
5. Analyse balanced three phase ac circuit and three phase power measurement.

DETAILED SYLLABUS

33031 - ELECTRICAL CIRCUIT THEORY (M - SCHEME)

Unit –I (a) ELECTROSTATICS (b) D C CIRCUITS Page No: 5 - 51

(a) ELECTROSTATICS

Electric Flux-Electric Flux Density-electric Field Intensity-electric potential-Coulomb's laws of electrostaticsconcept of capacitance - Relationship between Voltage, Charge and capacitance – energy stored in a capacitor – capacitors in series and in parallel –Problems in above topics.

(b) D C CIRCUITS

Basic concepts of current, emf, potential difference, resistivity, temperature coefficient of resistance – Ohm's Law –application of Ohm's law – work, power energy – relationship between electrical, mechanical and thermal units – resistance – series circuits – parallel and Series parallel circuits – Kirchhoff's laws –Problems in the above topics.

Unit-II CIRCUIT THEOREMS Page No: 52 - 126

Mesh equations – Nodal equations – star/delta transformations –Superposition theorem – Thevenin's theorem – Norton's theorem – Maximum power transfer theorem. (Problems in DC circuits only)

Unit-III SINGLE PHASE CIRCUITS Page No: 127 - 193

'j' notations – rectangular and polar coordinates – Sinusoidal voltage and current – instantaneous, peak, average and effective values – form factor and peak factor(derivation for sine wave) – pure resistive, inductive and capacitive circuits – RL, RC, RLC series circuits –impedance –phase angle – phasor diagram – power and power factor – power triangle – apparent power, active and reactive power – parallel circuits (two branches only) - Conductance, susceptance and admittance –problems on all above topics.

Unit-IV RESONANT CIRCUITS Page No: 194 - 219

Series resonance – parallel resonance (R,L &C; RL&C only) – quality factor – dynamic resistance – comparison of series and parallel resonance –Problems in the above topics - Applications of resonant circuits.

Unit-V THREE PHASE CIRCUITS Page No: 220 - 263

Three phase systems-phase sequence –necessity of three phase system–concept of balanced and unbalanced load - balanced star &delta connected loads – relation between line and phase voltages and currents –phasor diagram –three phase power and power factor measurement by single wattmeter and two wattmeter methods –Problems in all above topics.

TEXT BOOK:

Sl.No	Name of the Book	Author	Publisher
1.	Electric Circuit Theory	Dr. M. Arumugam Dr. N. Premakumaran	Khanna Publishers New Delhi

REFERENCE BOOKS:

Sl.No	Name of the Book	Author	Publisher
1.	Circuits and Networks Analysis and Synthesis	A.SudhakarShyammohan S.Pali	Tata McGraw Hill Education Private Ltd.,
2.	Electric Circuits	Mohamood Nahvi Joseph A Edminister	Tata McGraw Hill Education Private Ltd.,

UNIT 1 – (A) ELECTROSTATICS (B) D.C CIRCUITS

Syllabus:

(a) ELECTROSTATICS

Electric Flux - Electric Flux Density - Electric Field Intensity - Electric potential - Coulomb's laws of electrostatics - Concept of capacitance - Relationship between Voltage, Charge and capacitance – Energy stored in a capacitor – Capacitors in series and in parallel – Problems in above topics.

(b) D.C CIRCUITS

Basic concepts of current, emf, potential difference, resistivity, temperature coefficient of resistance – Ohm's Law – application of Ohm's law – work, power energy – relationship between electrical, mechanical and thermal units – resistance – series circuits – parallel and Series parallel circuits – Kirchhoff's laws –Problems in the above topics.

1.0 Electrostatics:

Electrostatics is the study of charges at rest. Electric charges are fundamental in nature. There are two types of charges and can be produced by friction. When a glass rod is rubbed with a silk cloth, some electrons from glass rod are migrated into the silk cloth. This makes the glass rod positively charged due to loss of electrons in it and the silk cloth negatively charged due to excess of electrons in it. Similarly, when the plastic rod is rubbed with fur, electrons migrate from the fur into the plastic rod making the rod negative and the fur positive. Also there is a force of attraction between the glass rod and silk cloth. There is a force of repulsion between similarly charged bodies and a force of attraction between oppositely charged bodies.

1.1 Electrostatic force:

The forces between particles that are caused by their electric charges. It exists between moving charges as well as stationary ones. This force may be illustrated with lines as shown in Figure 1.1.

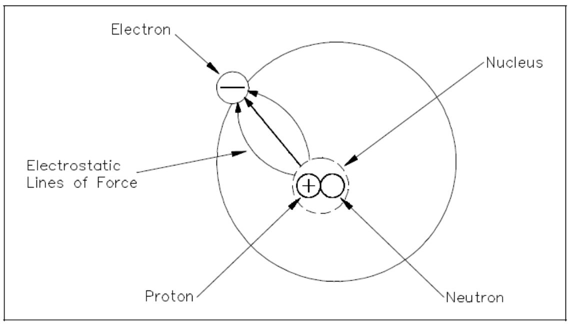


Figure 1.1 Electrostatic Force

1.2 Electric field:

An electric field is defined as the space in which an electric charge experiences a force. The electric field is represented by electric flux lines.

1.2.1 Electric Field Line:

It is defined as the direction of electric field. It leaves the positive charged conductor and enters a negatively charged conductor. The field configuration for isolated charges is shown in figure 1.2.

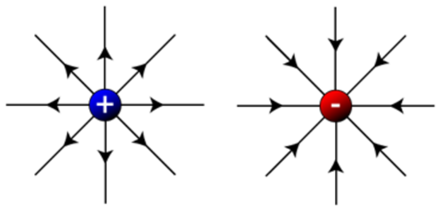


Figure 1.2 Electric Field Line

1.2.2 Properties of Electric field lines:

1. A line starts from a positive charge and ends on a negative charge.
2. The lines enter or leave a charged body at a right angle to the surface.
3. The lines go from more positive body towards the more negative body
4. The lines do not intersect.

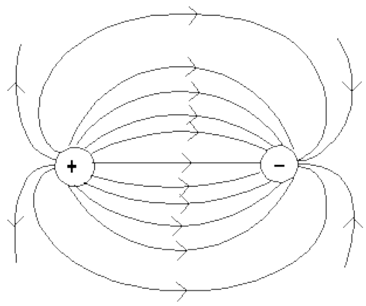


Figure 1.3 Electric field around two unlike charges

1.3 Electric Flux:

The total number of electric field lines is called electric flux. The unit of electric flux is the coulomb. The symbol for electric flux is ψ (psi)

One 'line' of flux is assumed to radiate from the surface of a positive charge of one coulomb and terminate at the surface of a negative charge of one coulomb. Hence the electric flux has the same numerical value as the charge that produces it. Therefore, the coulomb is used as the unit of electric flux.

1.4 Electric flux density:

The electric flux density is defined as the amount of flux per square metre of the electric field. This area (A) is measured at right angles to the lines of force. Electric flux density is denoted by D.

$$Electric\ Flux\ Density: \quad D = \frac{\psi}{A}\ coulomb/metre^2$$

1.5 Electric field intensity:

Electric field intensity at any point in an electric field is the force on unit positive charge placed at that point. It is expressed in Newton per coulomb (N/C). Electric flux intensity is denoted by E.

$$Electric\ Field\ Intensity: \quad E = \frac{F}{Q} \ ; Newton/coulomb$$

1.6 Electric Potential:

The potential at any point in an electric field is the amount of work done in bringing a charge of 1 coulomb of positive charge to that point against the electric field.

1.7 Permittivity of Free Space (ε_o):

When an electric field exists in a vacuum then the ratio of the electric flux density to the electric field strength is a constant, known as the permittivity of free space.

$$\varepsilon_o = 8.854x\ 10^{-12}\ farad/metre$$

$$\frac{D}{E} = \varepsilon_o \varepsilon_r$$

$$where, \varepsilon_r = relative\ permittivity\ of\ the\ insulating\ material$$

1.7.1 Absolute permittivity: (ε)

For a given system the ratio of the electric flux density to the electric field strength is a constant, known as the absolute permittivity of the dielectric being used.

$$\varepsilon = \frac{D}{E} \quad ;\ farad/metre$$

$$\varepsilon = \varepsilon_o \varepsilon_r \quad ;\ farad/metre$$

1.7.2 Relative permittivity: (ε_r)

The capacitance of two plates will be increased if, instead of a vacuum between the plates, some other dielectric is used. Thus relative permittivity is defined as the ratio of the capacitance with that dielectric to the capacitance with a vacuum dielectric.

$$\varepsilon_r = \frac{C_2}{C_1}$$

$$where, \quad C_1 = Capacitance\ with\ a\ vacuum\ dielectric$$

$$C_2 = Capacitance\ with\ other\ dielectric$$

1.8 Capacitor:

A capacitor is a passive electrical component that stores electrical charge in an electric field. The effect of a capacitor is known as capacitance.

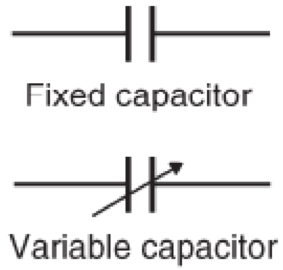


Figure 1.4 Symbol of Capacitor

1.8.1 Basic Construction:

A capacitor is an electrical device that stores electrical charge and is constructed of two parallel plates separated by an insulating material called the dielectric. Connecting leads are attached to the parallel plates as shown in figure 1.5.

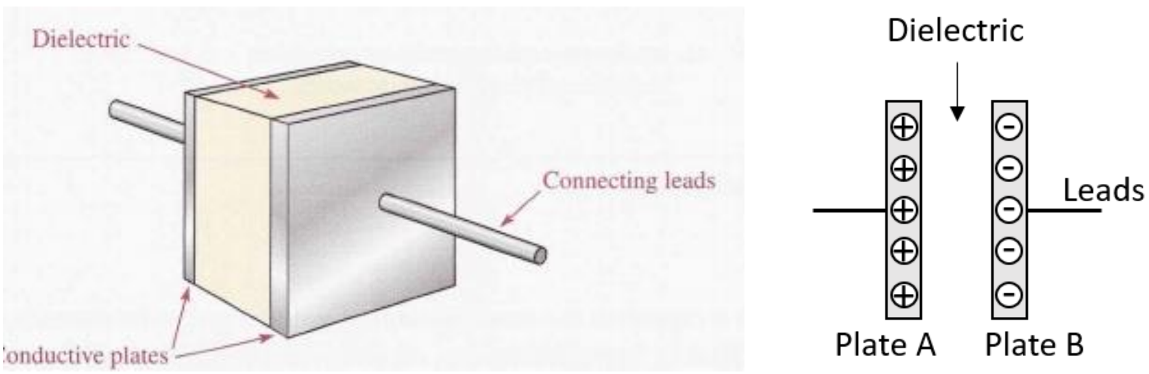


Figure 154 : Construction of Capacitor

1.8.2 Working of Capacitor:

In the natural state, both plates of a capacitor have an equal number of free electrons. When the capacitor is connected to a voltage source, electrons are removed from plate A, and an equal number are deposited on plate B. As plate A loses electrons and plate B gains electrons, plate a becomes positive with respect to plate B. During this charging process, electrons flow only through the connecting leads. No electrons flow through the dielectric of the capacitor because it is an insulator. The movement of electrons ceases when the voltage across the capacitor equal the source voltage. A charged capacitor can act as a temporary battery.

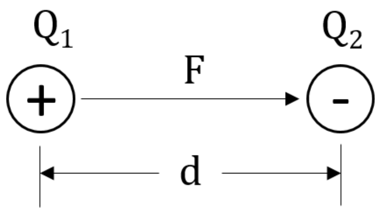
1.9 Coulomb's law of electrostatics:

First Law:

Like charges of electricity repel each other and unlike charges attract each other.

Second law:

The force (F) exists between two point-source charges (Q_1 and Q_2) is directly proportional to the product of the two charges and is inversely proportional to the square of the distance (d) between the charges.



$$F \ \alpha \ \frac{Q_1 \, Q_2}{d^2}$$

$$F = k.\frac{Q_1 \, Q_2}{d^2}$$

$$\text{Constant,} \quad k = \frac{1}{4\pi\varepsilon_o\varepsilon_r}$$

Where,

F = Force between the charges (N)

Q_1 and Q_2 = Magnitude of two charges in coulomb

F = Force between the charges (N)

d = distance between the charges (m)

ε_o = Permittivity of the free space

In S.I: ε_o = $8.854\text{x } 10^{-12}$ farad/metre

ε_r = relative permittivity of the medium

ε_r = 1 for vacuum or air

$$\text{Therefore,} \quad \text{Force:} \quad F = \frac{Q_1 \, Q_2}{4\pi\varepsilon_o\varepsilon_r d^2}$$

$$F = \frac{Q_1 \, Q_2}{4\pi \text{ x } 8.854\text{x } 10^{-12} \text{ x } \varepsilon_r d^2}$$

$$F = \frac{9 \text{ x } 10^9 \text{x } Q_1 \, Q_2}{\varepsilon_r d^2} \quad ; \text{ for a medium}$$

$$F = \frac{9 \text{ x } 10^9 \text{x } Q_1 \, Q_2}{d^2} \quad ; \text{ for air}$$

1.10 Capacitance:

The amount of charge that a capacitor can store per unit of voltage across its plates is its capacitance. It is represented by C. The capacitance is a measure of a capacitor's ability to store charge. Farad is the unit of capacitance.

One farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates.

1.11 Relationship between the voltage, capacitance and charge:

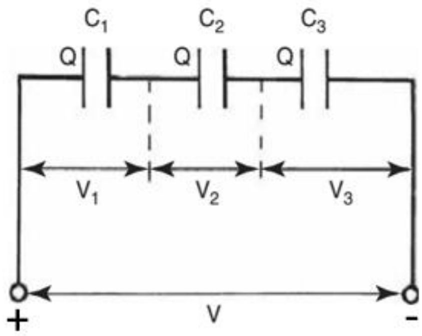
$$\text{Capacitance:} \quad C = \frac{Q}{V} \quad ; \text{ farad}$$

$$\text{Charge:} \quad Q = C.V \quad ; \text{ Coulomb}$$

$$\text{Voltage:} \quad V = \frac{Q}{C} \quad ; \text{ Volts}$$

1.12 Capacitors connected in series:

Consider three capacitors, C_1, C_2 and C_3, are connected in series to a d.c supply of 'V' volts.



Let,

C = Equivalent capacitance of series combination

V_1 = p.d. across capacitor C_1

V_2 = p.d. across capacitor C_2

V_3 = p.d. across capacitor C_3

Since the charging current at every point of the circuit is same and the charge stored on each capacitor is 'Q' coulombs.

$$V_1 = \frac{Q}{C_1}; \qquad V_2 = \frac{Q}{C_2}; \qquad V_3 = \frac{Q}{C_3} \qquad \text{-------------- (1)}$$

$$\text{In series circuit,} \quad V = V_1 + V_2 + V_3 \qquad \text{-------------- (2)}$$

Substitute the value of equation (1) in equation (2)

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad \text{-------------- (3)}$$

Where C is the total equivalent circuit capacitance, then it stores the same charge (Q) when connected to the same voltage (V).

Divide equation (3) by Q :

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If 'n' series-connected capacitors:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

In case of **two capacitors in series** :

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{C_1 \,.\, C_2}{C_1 + C_2} \qquad \text{i.e } \frac{Product}{Sum}$$

1.13 Capacitors connected in parallel:

Consider three capacitors, C_1, C_2 and C_3, are connected in parallel to a d.c supply of 'V' volts.

Let,

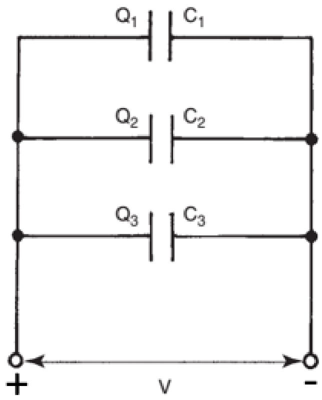
C = Equivalent capacitance of series combination

V = Voltage across each capacitor

Q_1 = Charge stored by C_1

Q_2 = Charge stored by C_2

Q_3 = Charge stored by C_3



In series circuit, $Q_T = Q_1 + Q_2 + Q_3$ -------------- (1)

Where, $Q_1 = C_1V$

$Q_2 = C_2V$ -------------- (2)

$Q_3 = C_3V$

Where C is the total equivalent circuit capacitance, then it stores the same charge (Q) when connected to the same voltage (V).

Then, $Q_T = C\,V$ -------------- (3)

Substitute the value of equation (2) & (3) in equation (1)

$$C\,V = C_1V + C_2V + C_3V \quad \text{-------------- (4)}$$

Divide equation (4) by V :

$$C = C_1 + C_2 + C_3$$

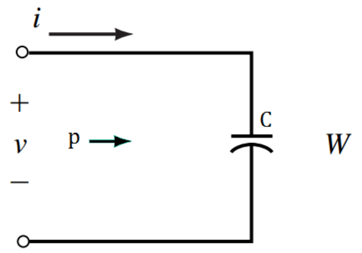
If 'n' number of capacitor in parallel:

$$C = C_1 + C_2 + .. + C_n$$

1.14 Energy stored in a capacitor:

When a capacitor is charged, Energy is stored in it. In the charging process, the potential difference across the plates in proportion to the charge stored.

During this process, the capacitor draws some current from the source called charging current and energy is stored in it. When the capacitor is fully charged, no more current is drawn from the source.



Where,
v – voltage applied to capacitor in volts
I – Current flow in amps
C- Capacitance of capacitor in farad
P – power consumed by the capacitor

Power to the capacitor, $p = vi$

Current: $i = \frac{Cdv}{dt}$

$$p = \frac{C.v.dv}{dt}$$

$$p = \frac{dW}{dt}$$

Integrating both sides,

Energy stored in Capacitor, $W = \int_0^t p.dt$

$$W = \int_0^t \frac{C.v.dv}{dt}.dt$$

$$W = C\int_0^t \frac{v.dv}{dt}.dt$$

$$W = C\int_0^v v.dv$$

$$W = \frac{1}{2}Cv^2$$

Example: 1
A capacitor supplied from 250 Volts DC mains, takes 25milli Coulombs. Find its capacitance. What will be its charge if the voltage is raised to 1000V.

Given Data:

Supply Voltage (V)	=	250V
Charge (C)	=	25 mC = 25x10^{-3} C
Raised Voltage	=	1000V

To Find:
i) Capacitance
ii) Charge at 1000V

Solution:

Capacitance: $C = \frac{Q}{V} = \frac{25x10^{-3}}{250} = 10^{-4}F$

For the supply voltage of 1000V:

Charge: $Q = C.V$

$$Q = 10^{-4} \text{ x } 1000$$

$$Q = 0.1 \text{ Coulomb}$$

Example: 2

Three capacitors have capacitance of 4mfd, 6mfd and 8mfd respectively. Find the total capacitance when they are connected (a) in series (b) Parallel.

<u>Given Data:</u>

Capacitance (C_1) = 4 mfd
Capacitance (C_2) = 6 mfd
Capacitance (C_3) = 8 mfd

<u>To Find:</u>

i) Total Capacitance in series
ii) Total Capacitance in parallel

<u>Solution:</u>

In Series Connection:

Equivalent Capacitance: $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$$

$$\frac{1}{C_{eq}} = \frac{6+4+3}{24} = \frac{13}{24}$$

$$C_{eq} = \frac{24}{13} = 1.846 \text{ mfd}$$

In parallel connection:

Equivalent Capacitance: $$C_{eq} = C_1 + C_2 + C_3$$

$$C_{eq} = 4 + 6 + 8 = 18 \text{ mfd}$$

Example: 3

Three capacitors have capacitance of 1mfd, 2mfd and 6mfd respectively. Find the total capacitance when they are connected (a) in parallel (b) series.

<u>Given Data:</u>

Capacitance (C_1) = 1 mfd
Capacitance (C_2) = 2 mfd
Capacitance (C_3) = 6 mfd

<u>To Find:</u>

i) Total Capacitance in parallel
ii) Total Capacitance in series

<u>Solution:</u>

In parallel connection:

Equivalent Capacitance: $$C_{eq} = C_1 + C_2 + C_3$$

$$C_{eq} = 1 + 2 + 6 = 9 \text{ mfd}$$

In Series Connection:

Equivalent Capacitance: $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{6}$$

$$\frac{1}{C_{eq}} = \frac{6+3+1}{6} = \frac{10}{6}$$

$$C_{eq} = \frac{6}{10} = 0.6 \text{ mfd}$$

Example: 4

If two capacitors having capacitance of 6mfd and 10mfd respectively are connected in series across a 320V supply. Find (a) the p.d across each capacitor (b) the charge on each capacitor.

Given Data:

Capacitance (C_1) = 6 mfd

Capacitance (C_2) = 10 mfd

Supply Voltage (V) = 320 V

To Find:

i) the p.d across each capacitor

ii) the charge on each capacitor

Solution:

In Series Connection:

Equivalent Capacitance:
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{10}$$
$$\frac{1}{C_{eq}} = \frac{10+6}{60} = \frac{16}{60}$$
$$C_{eq} = \frac{60}{16} = 3.75 \text{ mfd}$$

Charge : $Q = C.V$
$$Q = 3.75\text{x}10^{-6} \text{ x } 320 = 0.0012 \text{ Coulomb}$$

Charge on capacitor C_1 : $Q_1 = 0.0012$ Coulomb

Charge on capacitor C_2 : $Q_2 = 0.0012$ Coulomb

P.d across Capacitor C_1 : $V_1 = \frac{Q_1}{C_1}$
$$V_1 = \frac{0.0012}{6\text{x}10^{-6}} = 200 \text{ Volts}$$

P.d across Capacitor C_2 : $V_2 = \frac{Q_2}{C_2}$
$$V_2 = \frac{0.0012}{10\text{x}10^{-6}} = 120 \text{ Volts}$$

Example: 5

Three capacitors 5mfd, 10mfd and 15mfd are connected in parallel across 250V supply. Find the energy stored.

Given Data:

Capacitance (C_1) = 5 mfd

Capacitance (C_2) = 10 mfd

Capacitance (C_3) = 15 mfd

To Find:

i) Energy stored

Solution:

In parallel connection:

Equivalent Capacitance: $C_{eq} = C_1 + C_2 + C_3$
$$C_{eq} = 5 + 10 + 15 = 30 \text{ mfd}$$

$$\text{Energy Stored: } E = \frac{1}{2}\, C\, V^2$$
$$= \tfrac{1}{2} \text{ x } 30 \text{x } 10^{-6} \text{ x } 250^2$$
$$= 0.9375 \text{ Joules}$$

Example: 6

Two capacitors each of 3µf and 4µf are connected in series across 100V d.c supply. Calculate (i) the voltage across each capacitor (ii) the energy stored across each capacitor and (iii) the equivalent capacitance of the combination.

Given Data:

Capacitance (C_1) = 3 µf

Capacitance (C_2) = 4µf

Supply Voltage (V) = 100V

To Find:

i) Voltage across each capacitor

ii) Energy stored across each capacitor

iii) equivalent capacitance

Solution:

In parallel connection:

$$\text{Equivalent Capacitance: } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{4}$$
$$\frac{1}{C_{eq}} = \frac{4+3}{12} = \frac{7}{12}$$
$$C_{eq} = \frac{12}{7} = 1.71 \text{ mfd}$$

$$\text{Energy Stored: } E = \frac{1}{2}\, C\, V^2$$
$$= \frac{1}{2} \text{x} 1.71 \text{x } 10^{-6} \text{ x } 100^2$$
$$= 0.00855 \text{ Joules}$$

$$\text{Charge : } Q = C.\, V$$
$$Q = 1.71 \text{x} 10^{-6} \text{ x } 100 = 1.71 \text{x} 10^{-4} \text{ Coulomb}$$

$$\text{Charge on capacitor } C_1 : Q_1 = 1.71 \text{x} 10^{-4} \text{ Coulomb}$$

$$\text{Charge on capacitor } C_2 : Q_2 = 1.71 \text{x} 10^{-4} \text{Coulomb}$$

$$\text{Potential difference across Capacitor } C_1 : V_1 = \frac{Q_1}{C_1} = \frac{1.71 \text{x} 10^{-4}}{3 \text{x} 10^{-6}} = 57 \text{ Volts}$$

$$\text{Potential difference across Capacitor } C_2 : V_2 = \frac{Q_2}{C_2} = \frac{1.71 \text{x} 10^{-4}}{4 \text{x} 10^{-6}} = 42.8 \text{ Volts}$$

(B) D.C CIRCUITS

1.15 Introduction:

Matter is anything that has mass and occupies space; matter exists in three states normally solid, liquid and gaseous form. All maters in the universe are made up of fundamental building blocks called atoms. Figure shows the simplified structure of an atom. All **atoms** consist of **protons, neutrons** and **electrons**. The protons, which have positive electrical charges, and the neutrons, which have no electrical charge, are contained within the **nucleus.**

The mass of proton is approximately 1840 times that of an electron. A neutron has a mass equal to that of a proton but it has no charge. Electrons carry a negatives charge of -1.602x10^{-19} Coulomb. The electrons move round the nucleus in different orbit or shells.

The maximum number of electrons allowed in each orbit is given by $2n^2$ where n is the number of orbit. Thus first orbit can accommodate $2x1^2$ = 2 electrons; the second orbit has $2x2^2$ =8 electrons, the third electrons $2x3^2$ = 18 electrons and so on.

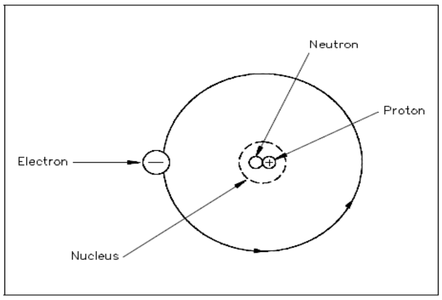


Figure 1.6 Atomic Structure

1.16 Electrical Circuit:

Electrical circuit is an interconnection of electrical devices which has atleast one closed path for current flow.

1.16.1 Conductors:

Conductors are materials that readily allow current. They have a large number of free electrons. Most metals are good conductors. Silver is the best conductor and copper is next. Copper is the most widely used conductive material because it is less expensive than silver. Copper wire is commonly used as a conductor in electric circuits.

Examples: Copper, Aluminium, Brass, Platinum, Silver, Gold and Carbon etc.,

1.16.2 Insulators:

Insulators, or nonconductors, are materials with electrons that are tightly bound to their atoms and require large amounts of energy to free them from the influence of the nucleus.

Examples: Plastic, Rubber, Glass, Porcelain, Air, Paper, Cork, Mica, Ceramics and Certain oils

1.16.3 Semiconductors:

These materials are neither good conductor nor insulators. Their electrical conductivity lies between good conductors and insulators.

Examples: Silicon and germanium

1.16.4 Super conductors:

Super conductors are a group of metals and alloys which have zero resistivity of infinite conductivity.

1.16.5 Newton:

The Newton is defined as the force required to accelerate a mass of one kg, by one metre per second squared (1m/s^2). The unit of force is the Newton (N) where one Newton is one kilogram metre per second squared.

$$Force:\ \ F = m.a\ ; N$$

1.16.6 Joule:

The unit of work or energy is the **joule (J)** where one joule is one Newton Metre. One joule is defined as the work done by the application of constant 1 Newton through a 1 metre distance.

$$Workdone \ = \ Force\ x\ Distance$$
$$W \ = \ F\ \ x\ d \quad ; N - m\ or\ Joule$$

1.17 Electricity:

It can be defined in terms of its behavior. It classified as either static or dynamic depending on whether the charge carries either static or dynamic.

1.18 Electrical Charge:

Electrical charge, denoted by Q. Charge is the characteristics property of the elementary particles of the matter. The elementary particles are electrons, protons and neutron. Charge can be either positive or negative and is usually measured in coulombs. The charge of an electron is called negative charge and charge of a proton is called positive charge.

The charge of an electron is equal to 1.602x10^{-19} coulombs.

1.18.1 Coulomb:

The coulomb is the basic unit of electrical charge. It is denoted by C. One coulomb of charge consists of approximately 6.24x10^{18} electrons or protons.

1.19 Current:

The rate of flow of electric charges is called, “electric current”. The unit of current is ampere (A). It is denoted by the letter ‘I’

$$\text{Current} = \frac{\text{Charges}}{\text{Time}}$$

$$\text{I} = \frac{\text{Q}}{\text{t}}$$

Where, Q = electric charge; coulomb (C)

T = time; second (S)

1.19.1 Ampere:

An ampere is defined as the flow of one coulomb of charge in one second.

1 ampere = 1 coulomb per second

Example : 7

A charge of 35 mC is transferred between two points in a circuit in a time of 20 ms. Calculate the value of current flowing.

Solution:

$$Q = 35 \times 10^{-3}\,\mathrm{C}; t = 20 \times 10^{-3}\,\mathrm{s}$$

$$I = \frac{Q}{t}\ \text{amp}$$

$$= \frac{35 \times 10^{-3}}{20 \times 10^{-3}}$$

$$I = 1.75\ \mathrm{A}\ \textbf{Ans}$$

Example : 8

If a current of 120µA flows for a time of 15s, determine the amount of charge transferred.

Solution:

$$I = 120 \times 10^{-6}\mathrm{A}; t = 15\,\mathrm{s}$$

$$Q = It\ \text{coulomb}$$

$$= 120 \times 10^{-6} \times 15$$

$$Q = 1.8\ \mathrm{mC}\ \textbf{Ans}$$

Example : 9

80 coulombs of charge were transferred by a current of 0.5 A. Calculate the time for which the current flowed.

Solution:

$$Q = 80\,\mathrm{C}; I = 0.5\,\mathrm{A}$$

$$t = \frac{Q}{I}\ \text{seconds}$$

$$= \frac{80}{0.5}$$

$$t = 160\ \mathrm{s}\ \textbf{Ans}$$

1.20 Electro Motive Force (E.M.F):

It is the force which causes the flow of electrons in any closed circuit. The unit of electro motive force is volt. It is represented by letter E or V.

$$\mathrm{EMF} = \frac{\mathrm{Workdone}}{\mathrm{Charge}}$$

$$\mathrm{E\ or\ V} = \frac{\mathrm{W}}{\mathrm{Q}}$$

1.21 Potential Difference (p.d):

Whenever current flows through a resistor there will be a potential difference (p.d) developed across it. Essentially, emf causes current to flow; whilst a p.d. is the result of current flowing through a resistor. The unit of potential difference is volt (V).

1.21.1 Difference between E.M.F and Potential difference (p.d):

S.No	E.M.F	Potential difference
1	It refers to source of electrical energy	It exists between any points in a circuit
2	It is measured in open circuit	It is measured in closed circuit
3	It is a cause for current flow	It is the effect of current flow
4	It is greater than the potential difference in the same circuit.	It is less than the E.M.F in the same circuit

1.22 Electrical power:

Power is defined as the rate of doing work. It is represented by 'P'.

$$P = \frac{dW}{dt} = \frac{dW}{dQ} \times \frac{dQ}{dt} = V \times I$$

Power = Voltage x current

Hence, the electrical power is given by the product of voltage and current. The unit of power is the **watt (W)**.

Note:

- *1 KW = 1000 Watts = 10^3 W*
- *1 MW = 1000000 Watts = 10^6 W*
- Wattmeter is used to measure the power

1.23 Energy:

It is defined as the capacity to do work. It is the total electrical energy consumed over a period.

Electrical Energy = Power x Time

E = P x t

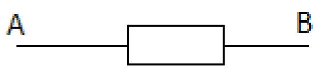
Or E = V x I x t ; watt-hour

Note:

- *1 KWH = 1 Unit*
  = 1000 Watt Hour
  = 1000 x 3600 Watt Second or Joules
  = 3 600 000 Joules
- *Energy meter is used to measure the Energy*

1.24 Electrical Circuit Elements:

Electrical circuit elements are two terminal devices which are basic building blocks of the electrical circuit. Figure shows the general form of two terminal device, where A and B forms the terminals.



1.24.1 Classifications of Electrical Circuit Elements:

1. Passive Elements (Resistor, Inductor and Capacitor)
2. Active Elements

1.24.2 Passive Elements:

- ✓ Passive elements stores or dissipates the energy.
- ✓ Example: Resistor dissipates energy and capacitor stores energy.

1.24.3 Active Elements:

- ✓ An element capable of supplying energy is called as active elements.
- ✓ An active element can be considered as source of energy.

1.25 Resistance:

It is defined as the property of a substance which opposes the flow of current through it. It is represented by R. The unit of resistance is ohm (Ω). One ohm is one volt per ampere.

1.25.1 Laws of Resistance:

The resistance of an electrical conductor depends upon the following factors:

(a) The length of the conductor,

(b) The cross-sectional area of the conductor,

(c) The type of material and

(d) The temperature of the material.

Resistance (R) is directly proportional to length (l), and inversely proportional to the cross-sectional area (a).

$$\text{i.e.,} \qquad R \, \alpha \, \frac{l}{a}$$

$$\text{i.e.,} \qquad R = \frac{\rho l}{a}$$

Where, 'ρ' is the **resistivity** of the material

1.25.2 Specific Resistance:

The specific resistance of the material is the resistance of a unit cube of the material measured between opposite faces of the cube. Resistivity varies with temperature. It is represented by the symbol 'ρ' (Greek rho) and the unit of 'ρ' is ohm metre (Ω-m)

1.26 Conductance:

Conductance may be defined as the ability of the conductor to allow the current freely through it. Conductance is reciprocal of resistance. It is represented by the letter 'G'. The unit for the conductance is siemen.

$$\text{i.e.,} \qquad G \, \alpha \, \frac{1}{R}$$

Example: 10

A coil of copper wire 200 m long and of cross sectional area of 0.8 mm^2 has a resistivity of 0.02 μΩ m at normal working temperature. Calculate the resistance of the coil.

Solution:

$$\ell = 200\,\text{m};\ \rho = 2 \times 10^{-8}\,\Omega\text{m};\ A = 8 \times 10^{-7}\,\text{m}^2$$

$$R = \frac{\rho\ell}{A}\,\text{ohm}$$

$$= \frac{2\times10^{-8}\times200}{8\times10^{-7}}$$

$$R = 5\,\Omega\ \textbf{Ans}$$

Example: 11

A wire-wound resistor is made from a 250 metre length of copper wire having a circular cross-section of diameter 0.5 mm. Given that the wire has a resistivity of 0.018 μΩ m , calculate its resistance value.

Solution:

$$\ell = 250\,\text{m};\ d = 5 \times 10^{-4}\,\text{m};\ \rho = 1.8 \times 10^{-8}\,\Omega\text{m}$$

$$R = \frac{\rho\ell}{A}\ \text{ohm, where cross-sectional area, } A = \frac{\pi d^2}{4}\ \text{metre}^2$$

$$\text{hence, } A = \frac{\pi\times(5\times10^{-4})^2}{4} = 1.9635\times10^{-7}\text{m}^2$$

$$\text{hence, } R = \frac{1.8\times10^{-8}\times250}{1.9635\times10^{-7}}$$

$$\text{so, } R = 22.92\ \Omega\ \textbf{Ans}$$

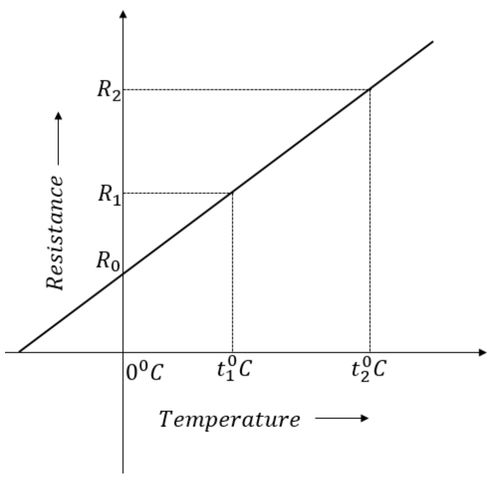
1.27 Temperature coefficient of resistance:

The resistance of almost all electricity conducting materials changes with the variation in temperature. This variation of resistance with change in temperature is governed by a property of a material called temperature coefficient of resistance (∝).

The temperature coefficient of resistance can be defined as the change in resistance per degree change in temperature and expressed as a fraction of the resistance at the reference temperature considered.

Let,

R_0 = Resistance at 0°C
R_1 = Resistance at t_1°C
R_2 = Resistance at t_2°C
α_0 = temp. coefficient of resistance at 0°C
α_1 = temp. coefficient of resistance at t_1°C



Temperature coefficient of resistance at 0^{o}C:

$$\propto_0 = \frac{\text{Change in resistance}}{\text{Original resistance x Change in temperature}}$$

$$\propto_0 = \frac{R_1 - R_0}{R_0 \text{ x } t_1}$$

$$R_1 - R_0 = \propto_0 (R_0 \text{ x } t_1) = \propto_0 R_0 \ t_1$$

$$R_1 = \propto_0 R_0 \ t_1 + R_0 = R_0 (\propto_0 \ t_1 + 1)$$

$$R_1 = R_0 (1 + \propto_0 \ t_1) \qquad \text{----------- (1)}$$

Similarly,

$$R_2 = R_0 (1 + \propto_0 t_2) \qquad \text{----------- (2)}$$

Equation $\frac{(2)}{(1)}$, we get :

$$\frac{R_2}{R_1} = \frac{R_0 (1 + \propto_0 t_2)}{R_0 (1 + \propto_0 \ t_1)}$$

$$\frac{R_2}{R_1} = \frac{(1 + \propto_0 t_2)}{(1 + \propto_0 \ t_1)} \qquad \text{----------- (3)}$$

Temperature coefficient of resistance at t_1^{o}C:

$$\propto_1 = \frac{R_2 - R_1}{(t_2 - t_1)R_1}$$

$$R_2 - R_1 = \propto_1 (t_2 - t_1)R_1$$

$$R_2 = \propto_1 (t_2 - t_1)R_1 + R_1$$

$$R_2 = R_1 [\propto_1 (t_2 - t_1) + 1]$$

$$R_2 = R_1 [1 + \propto_1 (t_2 - t_1)]$$

$$\frac{R_2}{R_1} = [1 + \propto_1 (t_2 - t_1)] \qquad \text{----------- (4)}$$

L. H. S of Equation (3) = L. H. S of Equation (4)

Hence, R. H. S of Equation (3) = R. H. S of Equation (4)

$$\frac{(1 + \propto_0 t_2)}{(1 + \propto_0 \ t_1)} = [1 + \propto_1 (t_2 - t_1)]$$

Add -1 on both sides:

$$\frac{(1 + \propto_0 t_2)}{(1 + \propto_0 \ t_1)} - 1 = [1 + \propto_1 (t_2 - t_1)] - 1$$

$$\frac{(1 + \propto_0 t_2)}{(1 + \propto_0 \ t_1)} - 1 = \propto_1 (t_2 - t_1)$$

$$\frac{(1 + \propto_0 t_2) - (1 + \propto_0 \ t_1)}{(1 + \propto_0 \ t_1)} = \propto_1 (t_2 - t_1)$$

$$\frac{\propto_0 t_2 - \propto_0 t_1}{(1 + \propto_0 \ t_1)} = \propto_1 (t_2 - t_1)$$

$$\frac{\propto_0 (t_2 - t_1)}{(1 + \propto_0 \ t_1)} = \propto_1 (t_2 - t_1)$$

$$\frac{\propto_0}{(1 + \propto_0 \ t_1)} = \propto_1$$

Temperature coefficient of resistance at t_1^{o}C:

$$\boldsymbol{\propto_1 = \frac{\propto_0}{(1 + \propto_0 \ t_1)}}$$

Example: 12

A copper coil has a resistance of 30 Ω at 0°C. Find the resistance of the coil at 40°C. Temp co-efficient of copper is 0.0043 at 0°C.

Given Data:

Resistance at 0°C (R_0) = 30Ω

Temp co-efficient at 0°C (t_0) = 0.0043

To Find:

i) Resistance at 40°C

Solution:

Resistance at 40°C : $R_t = R_0 [1+\alpha_0.t]$

$R_t = 30 [1 + 0.0043 \times 40]$

$R_t = 35.16\ \Omega$

Answer:

Resistance at 40°C = 35.16 Ω

Example: 13

An aluminium wire has a resistance of 25 Ω at 20°C. What is the resistance at 50°C. Temp co-efficient of copper is 0.00403 at 20°C.

Given Data:

Resistance at 20°C (R_1) = 25Ω

Temp co-efficient at 20°C (t_1) = 0.00403

To Find:

i) Resistance at 50°C

Solution:

Resistance at 50°C : $R_2 = R_1 [1+ \alpha_1 (t_2 - t_1)]$

$R_2 = 25 [1+ 0.00403 (50 - 20)]$

$R_2 = 25 [1+ 0.00403 (30)]$

$R_2 = 25 [1+ 0.1209] = 25 \times 1.1209 = 28.02\ \Omega$

Answer:

Resistance at 50°C = 28.02 Ω

Example: 14

The field coil of a motor has a resistance of 250 Ω at 15°C. Find the increase in resistance of the field at a temperature of 45°C. Take a = 0.00428 at 0°C.

Given Data:

Resistance at 15°C (R_1) = 250Ω

Temp co-efficient at 0°C (t_1) = 0.00428

To Find:

i) Resistance at 45°C

Solution:

Resistance at 45°C : $R_2 = R_1 [1+ \alpha_1 (t_2 - t_1)]$

$$\alpha_0 = \frac{\alpha_0}{1 + \alpha_0 t_1}$$

$$= \frac{0.00428}{1 + 0.00428 x 15}$$

$$\alpha_0 = \frac{0.00428}{1.0642} = 0.0040218$$

R_2 = 250 [1+ 0.0040218 (45 – 15)]

R_2 = 250 [1+ 0.0040218 (30)]

R_2 = 250 [1+ 0.1206] = 25 x 1.1206 = 280.15 Ω

Answer:

Resistance at 45°C = 280.15 Ω

Example: 15

If the temperature co-efficient of copper at 20°C is 0.00393 find its resistance at 80°C given that resistance at 20°C is 30Ω.

Given Data:

Resistance at 20°C (R_1)	=	30Ω
Temp co-efficient at 20°C (t_1)	=	0.00393

To Find:

i) Resistance at 80°C

Solution:

Resistance at 80°C : $R_2 = R_1 [1+ \alpha_1 (t_2 – t_1)]$

R_2 = 30 [1+ 0.00393 (80 – 20)]

R_2 = 30 [1+ 0.00393 (60)]

R_2 = 30 [1+ 0.2358] = 25 x 1.2358 = 37.07Ω

Answer:

Resistance at 80°C = 37.07 Ω

Example: 16

An aluminium wire has a resistance of 3.6 Ω at 20°C. What is its resistance at 50°C if the temperature co-efficient of resistance is 0.00403 at 20°C.

Given Data:

Resistance at 20°C (R_1)	=	3.6Ω
Temp co-efficient at 20°C (t_1)	=	0.00403

To Find:

i) Resistance at 50°C

Solution:

Resistance at 50°C : $R_2 = R_1 [1+ \alpha_1 (t_2 – t_1)]$

R_2 = 3.6 [1+ 0.00403 (50 – 20)]

R_2 = 3.6 [1+ 0.00403 (30)]

R_2 = 3.6 [1+ 0.1209] = 3.6 x 1.1209 = 4.035Ω

1.28 Ohms law:

The relationship between Voltage, Current and Resistance in any DC electrical circuit was explained by the German physicist Georg Ohm, (1787 - 1854). Georg Ohm found that, at a constant temperature, the electrical current flowing through a fixed linear resistance is directly proportional to the voltage applied across it, and also inversely proportional to the resistance.

<u>Statement:</u>

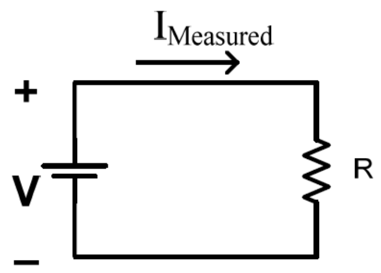
It states that the current 'I' flowing in a circuit is directly proportional to the applied voltage 'V' and inversely proportional to the resistance 'R', provided the temperature remains constant.

Thus, $I \propto V$

Current : $I = \frac{V}{R}$

Voltage : $V = I.R$

Resistance : $R = \frac{V}{I}$



According to Ohm's Law :

Power : $P = \text{Voltage x Current}$

$P = V \times I$ ------- (1)

Subsitute, $V = I.R$ in equation (1)

$P = I.R \times I$

$P = I^2R$

Substitue, $I = \frac{V}{R}$ in equation (1)

$P = V.\frac{V}{R}$

$P = \frac{V^2}{R}$

According to Ohm's Law :

Energy : $E = \text{Voltage x Current x Time}$

$E = V \times I \times t$ ------- (1)

Subsitute, $V = I.R$ in equation (1)

$E = I.R \times I \times t$

$E = I^2R\,t$

Substitue, $I = \frac{V}{R}$ in equation (1)

$E = V.\frac{V}{R}.t$

$E = \frac{V^2t}{R}$

1.29 Relation between Mechanical, Electrical & Thermal Energy:

1 N-M = 1 Joule = 1 watt-sec

1 calorie = 4.186 joules

1.29.1 Electrical to Thermal energy:

1 KWH = 1000 x 60 x 60 watt-sec	{KWH = Kilo Watt Hour}
= 36 x 10^5 watt-sec	(1 watt sec = 1 Joule)
= 36x 10^5 joules (or)	
1 KWH = 8.6 x 10^2 Kilo calories	(1 calorie = 4.186J)

1.29.2 Mechanical Power to Electrical Power:

1 horse power = 75 kg. m/sec	(1 kgm = 9.81 m)
= 75 x 9.81 Nm/sec	
= 75 x 9.81 joules/sec	
1 h.p = 735.5 watts or 736 watts	(1 joule/sec = 1 watt)

1.29.3 Electrical Power to Mechanical Power:

1 KW = 1000 watts {KW = Kilo Watts}

1 h.p = 735.5 watts

1 KW = 1.33 h.p

Example: 18

An electric heater is rated 1KW, 250V. Find the current drawn and the resistance of the heating element.

Given Data:		**To Find**:
Power (P)	= 1 KW = 1000W	Resistance (R) = ?
Voltage	= 250 V	Current (I) = ?

Solution:

$$\text{Resistance : } R = \frac{V^2}{P}$$

$$= \frac{250^2}{1000} = 62.5\ \Omega$$

$$\text{Current: } I = \frac{V}{R}$$

$$I = \frac{250}{62.5} = 4A$$

Answer:

Resistance of the heating element (R) = 62.5Ω

Current drawn by the heater (I) = 4 Amps

1.30 Resistors in series:

The circuit in which dissimilar ends are connected together is called as series circuit. In series circuit the current through all the resistors are same.

Let,

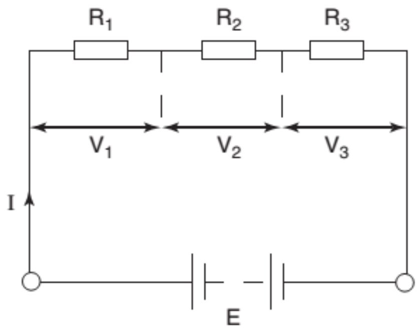
R_1, R_2 & R_3 – Three resistances connected in series.

E - Applied voltage across the circuit in volts

I – Circuit current in amps

R - The equivalent resistance in ohms

V_1, V_2 & V_3 – p.d across R_1, R_2 & R_3 respectively.



By applying KVL to the circuit,

The sum of potential drops across each resistor is equal to the applied voltage.

$E = V_1 + V_2 + V_3$ ------------- (1)

Apply ohm's law for individual resistors,

$V_1 = I.R_1$ -------------- (2)

$V_2 = I.R_2$ -------------- (3)

$V_3 = I.R_3$ -------------- (4) [current is same]

Substitute equation (2), (3) & (4) in equation (1)

⇒ $E = V_1+V_2+V_3$

$E = I.R_1 + I.R_2 + I.R_3$

$E = I\,(R_1 + R_2 + R_3)$ ----------- (5)

Where, $E = I.R_{eq}$ --------------- (6)

Equating equations (5) & (6)

$I.R_{eq} = I\,(R_1 + R_2 + R_3)$

$R_{eq} = R_1+R_2+R_3$ Ω [Three resistance in series]

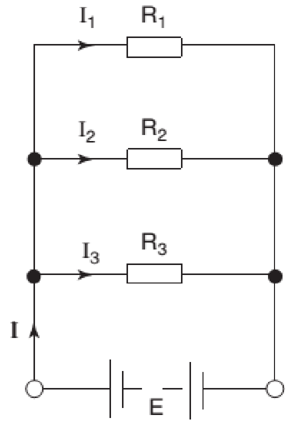
If there are 'n' resistance connected in series,

Then equivalent resistance, $R_{eq} = R_1+R_2+R_3+\ldots\ldots\ldots\ldots + R_n$

1.31 Resistors in Parallel:

The circuit in which similar ends are connected together is called as parallel circuit. In the given circuit R_1, R_2 and R_3 are connected in parallel. There are many separate paths for current flow. The voltage across all parallel branches is the same. Current in each branch is different and depends on the value of resistance in the branch.

Equivalent resistance of parallel connected resistances:



Let,

R_1, R_2 & R_3 – Three resistances connected in parallel

E - Applied voltage across the circuit in volts

I – Total current in amps

R - The equivalent resistance in ohms

I_1, I_2 & I_3 – current through R_1, R_2 & R_3 respectively

Since all resistors are connected across battery, the p.d across all are same.

$$V \text{ or } E = V_1 + V_2 + V_3$$

By applying KCL in the circuit:

$$I = I_1 + I_2 + I_3 \qquad \text{--------} (1)$$

Apply Ohm's law for individual Resistor:

$$\text{Current:} \quad I_1 = \frac{E}{R_1} \qquad \text{--------} (2)$$

$$I_2 = \frac{E}{R_2} \qquad \text{--------} (3)$$

$$I_3 = \frac{E}{R_3} \qquad \text{--------} (4)$$

Substitute equation (2), (3) & (4) in equation (1)

$$I = I_1 + I_2 + I_3$$

$$I = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

$$I = E\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\frac{I}{E} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\text{Equivalent Resistance:} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If 'n' number of Resistors are connected in parallel:

$$\text{Equivalent Resistance:} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

1.31.1 Two resistances in parallel:

This equation must be used when finding the total resistance R of a parallel circuit. For the special case of two resistors in parallel.

In case of two Resistors in parallel:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1 . R_2}{R_1 + R_2} \qquad \text{i.e } \frac{Product}{Sum}$$

1.32 Current Division in Parallel Connected Resistances:

Consider two resistances R_1 and R_2 in parallel and connected to a dc source of V Volts.

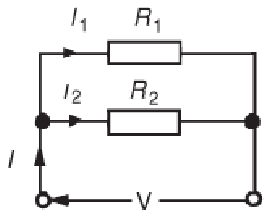
Let,

V- Supply Voltage

I - Circuit Current

I_1 – Current through R_1

I_2 – Current through R_2



$$R_{eq} = \frac{R_1 . R_2}{R_1 + R_2} \qquad \text{i.e } \frac{Product}{Sum}$$

$$V = I . R_{eq}$$

$$V = I.\left[\frac{R_1 . R_2}{R_1 + R_2}\right]$$

$$\text{Current through } R_1 ; \quad I_1 = \frac{V}{R_1}$$

$$I_1 = \frac{I.\left[\frac{R_1 . R_2}{R_1 + R_2}\right]}{R_1}$$

$$I_1 = \frac{I . R_2}{R_1 + R_2} \qquad \text{------------ (1)}$$

Similarly,

$$\text{Current through } R_2 ; \quad I_2 = \frac{I . R_1}{R_1 + R_2} \qquad \text{------------ (2)}$$

The equation (1) and (2) are called as current divide rule.

1.33 Voltage Division in Series Connected Resistances:

Consider two resistances R_1 and R_2 in series and connected to a dc source of V Volts.

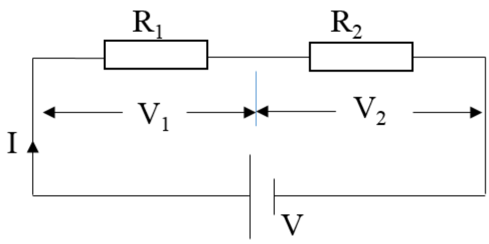
Let,

V- Supply Voltage

I - Circuit Current

V_1 – P.D across R_1

V_2 – P.D across R_2



$$R_{eq} = R_1 + R_2$$

Circuit Current ; $I = \frac{V}{R_{eq}}$

Circuit Current ; $I = \frac{V}{R_1 + R_2}$

P. D across R_1 ; $V_1 = I.R_1$ [In series circuit: $I = I_1 = I_2$]

P. D across R_1 ; $V_1 = \frac{V}{R_1 + R_2}.R_1$ ------------ (1)

Similarly,

P. D across R_2 ; $V_2 = \frac{V}{R_1 + R_2}.R_2$ ------------ (2)

The equation (1) and (2) are called as voltage divide rule.

Example: 19

A 100Watt, 250 Volts lamp is connected in series with a 100W, 200Volts lamp across a 250 Volts supply. Find the value of current flows through the lamp and voltage across each lamp.

Given Data:

Power (P_1) = 100W
Voltage (V_1) = 250 V
Power (P_2) = 100W
Voltage (V_2) = 200 V

To Find:

Current (I) = ?
Voltage drop across Lamp1 =?
Voltage drop across Lamp2=?

Solution:

Resistance: $R_1 = \frac{V^2}{P_1} = \frac{250^2}{100} = 625\Omega$

Resistance : $R_2 = \frac{V^2}{P} = \frac{200^2}{100} = 400\ \Omega$

Current: $I = \frac{V}{R_1 + R_2} = \frac{250}{625 + 400} = 0.243A$

Current flow through Lamp 1 = Current flow through Lamp 2

Voltage across lamp L_1 = $I \times R_1 = 0.243 \times 625 = 152.4V$

Voltage across lamp L_2 = $I \times R_2 = 0.243 \times 400 = 97.2V$

Answer:

Current flow through Lamp = 0.243A

Voltage across lamp (L_1) = 152.4V

Voltage across lamp (L_2) = 97.2V

1.34 Comparison between series circuit and parallel circuit:

S.NO	Series circuit	Parallel circuit
1	There are only one path for the current to flow	There are as many paths as the resistance are connected in parallel
2	Same current through all the resistor	Potential is same across all the resistor
3	The voltage drop across each resistor is different	The current in each resistor is different
4	The sum of voltage drop is equal to the applied voltage.	The sum of branch current is equal to the total current applied
5	The total resistance is equal to the sum of all resistors $R = R_1+R_2+R_3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots..R_n$	The reciprocal of the total resistance is equal to the sum of the reciprocals of all resistors $1/R = 1/R_1+1/R_2+1/R_3+\ldots\ldots\ldots.+1/R_n$
6	The total resistance is always greater than the greatest resistance in the circuit	The total resistance is always less than the smallest resistance in the circuit.
7.	Dissimilar ends of resistors are connected together to form a closed circuit	similar ends of resistors are connected together to form a closed circuit
8	**Uses:** To operate low voltage devices with high voltage source Ex. Decorative lamps(serial set)	All lamps, fans, motors etc.,are connected in parallel across the supply in house wiring

Example: 20

Three resistances of values 8Ω, 12Ω and 24Ω are connected in series. Find the equivalent resistance. Also find the equivalent resistances when they are connected in parallel.

Given Data:

Resistance (R_1) = 8Ω

Resistance (R_2) = 12Ω

Resistance (R_3) = 24Ω

To Find:

i) Equivalent resistance at series

ii) Equivalent resistance at parallel

Solution:

In Series Connection:

$$\text{Equivalent Resistance } (R_{eq}) = R_1 + R_2 + R_3$$

$$R_{eq} = 8 + 12 + 24$$

$$R_{eq} = 44\Omega$$

In Parallel Connection:

$$\text{Equivalent Resistance:} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$$

$$= \frac{3+2+1}{24} = \frac{6}{24}$$

$$R_{eq} = \frac{24}{6} = 4\Omega$$

Answer:

Equivalent Resistance in series connection = 44 Ω

Equivalent Resistance in parallel connection = 4 Ω

Example: 21

Two resistors are connected in parallel and a voltage of 200 volts is applied to the combination. The total current is 25 amps and the power dissipated in one of the resistors is 500 watts. What is the value of each resistance?

Given Data:

Two Resistors in parallel		
Supply Voltage (V)	=	200 Volts
Total Current (I)	=	25 Amps
Power Dissipated in R_1 (P_1)	=	500 Watts

To Find:

i) Value of R_1

ii) Value of R_2

Solution:

$$\text{Equivalent Resistance:} \quad R_{eq} = \frac{V}{I} = \frac{200}{25} = 8\Omega$$

$$\text{Power dissipated in } R_1\text{:} \quad P_1 = \frac{V^2}{R_1}$$

$$\text{Resistance:} \quad R_1 = \frac{V^2}{P_1} = \frac{200^2}{500} = 80\Omega$$

$$\text{Current through } R_1\text{:} \quad I_1 = \frac{V}{R_1} = \frac{200}{80} = 2.5\text{Amps}$$

$$\text{Current through } R_2\text{:} \quad I_2 = I - I_1 = 25 - 2.5 = 22.5\text{Amps}$$

$$\text{Resistance:} \quad R_2 = \frac{V}{I_2} = \frac{200}{22.5} = 8.89\Omega$$

Answer:

Resistance R_1 = 80 Ω

Resistance R_2 = 8.89 Ω

Example: 22

A circuit consists of two resistors 20 Ω and 30 Ω connected in parallel. They are connected in series with a resistor of 15 Ω. If the current through 15 Ω resistor is 3 amps find the current in the other resistors, total voltage and total power.

Given Data:

Resistance (R_1)	=	20 Ω
Resistance (R_2)	=	30 Ω
Resistance (R_3)	=	15 Ω
Current through R_3	=	3 Amps

To Find:

i) Current through R_1
ii) Current through R_2
iii) Total Voltage
iv) Total Power

Solution:

R_1 & R_2 in Parallel Connection:

$$\text{Resistance: } R_{12} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = 12\Omega$$

$$\text{Equivalent Resistance : } R_{eq} = R_{12} + R_3 = 12 + 15 = 27\Omega$$

$$\text{Current through } R_1\text{: } I_1 = \frac{I \times R_2}{R_1 + R_2} = \frac{3 \times 30}{20 + 30} = 1.8 \text{ Amps}$$

$$\text{Current through } R_2 : I_2 = \frac{I \times R_1}{R_1 + R_2} = \frac{3 \times 20}{20 + 30} = 1.2 \text{ Amps}$$

$$\text{Total Voltage : } V = I \times R_{eq} = 3 \times 27 = 81 \text{ Volts}$$

$$\text{Total Power : } P = V \times I = 81 \times 3 = 243 \text{ Watts}$$

Example: 23

A resistor of 3.6Ω is connected in series with another of 4.56 Ω. What resistance should be connected across 3.6Ω resistor so that the total resistance of the circuit shall be 6Ω?

Given Data:

Resistance (R_1)	=	3.6 Ω
Resistance (R_3)	=	4.56 Ω
Equivalent Resistance (R_{eq})	=	6 Ω

To Find:

i) Resistance R_2

Solution:

R_1 & R_2 in Parallel Connection:

$$\text{Resistance: } R_{12} + R_3 = \frac{R_1 \times R_2}{R_1 + R_2} + R_3 = \frac{3.6 \times R_2}{3.6 + R_2} + 4.56$$

$$6 = \frac{3.6 \times R_2}{3.6 + R_2} + 4.56$$

$$6 = \frac{3.6 \times R_2 + 4.56\,(3.6 + R_2)}{3.6 + R_2}$$

$$6\,(3.6 + R_2) = 3.6 \times R_2 + 4.56\,(3.6 + R_2)$$

$$21.6 + 6R_2 = 3.6R_2 + 16.41 + 4.56R_2$$
$$21.6 - 16.41 = 3.6R_2 + 4.56R_2 - 6R_2$$
$$5.19 = 2.16R_2$$
$$2.16\,R_2 = 5.19$$
$$R_2 = \frac{5.19}{2.16} = 2.4\Omega$$

Example: 24

A resistor of RΩ is connected in series with a parallel circuit consisting of 12 Ω and 8 Ω. The total power in the circuit is 80Watts when the applied voltage is 20V. Calculate the value of R.

<u>Given Data</u>:

Resistance (R_2)	=	12Ω
Resistance (R_3)	=	8 Ω
Total Power	=	80 Watts
Supply Voltage	=	20 Volts

<u>To Find</u>:

i) Resistance R_1

<u>Solution</u>:

R_2 & R_3 in Parallel Connection:

$$\text{Resistance: } R_{23} = \frac{R_2 \text{ x } R_3}{R_2 + R_3} = \frac{12 \text{ x } 8}{12 + 8} = 4.8\Omega$$
$$\text{Current: } I = \frac{P}{V} = \frac{80}{20} = 4 \text{ Amps}$$
$$\text{Equivalent Resistance: } R_{eq} = \frac{V}{I} = \frac{20}{4} = 5\Omega$$
$$R_{eq} = R_1 + R_{23}$$
$$5 = R_1 + 4.8$$
$$R_1 = 5 - 4.8 = 0.2\Omega$$

<u>Answer:</u>

Resistance R_1 = 0.2 Ω

Example: 25

A circuit consist of two resistances 10 Ω and 5 Ω connected in series across a supply of 60V.Calculate the voltage across each resistance.

<u>Given Data</u>:

Resistance (R_1)	=	10Ω
Resistance (R_2)	=	5 Ω
Supply Voltage	=	60 Volts

<u>To Find</u>:

i) Voltage across Resistance R_1

ii) Voltage across Resistance R_2

Solution:

In Series Connection:

Equivalent Resistance (R_{eq}) $= R_1 + R_2$

$$R_{eq} = 10 + 5$$

$$R_{eq} = 15\Omega$$

$$\text{Current : } I = \frac{V}{R_{eq}} = \frac{60}{15} = 4\ Amps$$

Voltage across Resistance R_1: $V_1 = I.\ R_1 = 4 \times 10 = 40$ Volts

Voltage across Resistance R_2: $V_2 = I.\ R_2 = 4 \times 5 = 20$ Volts

Answer:

Voltage across Resistance R_1 = 40 Volts

Voltage across Resistance R_2 = 20 Volts

Example: 26

A circuit consist of two resistances 6 Ω and 3 Ω connected in parallel and takes a total circuit of 12A from the supply. Calculate the current through each resistance.

Given Data:

Resistance (R_1) = 6Ω

Resistance (R_2) = 3 Ω

Total Current = 12 Amps

To Find:

i) Current through Resistance R_1

ii) Current through Resistance R_2

Solution:

$$\text{Current through } R_1 : I_1 = \frac{I \times R_2}{R_1 + R_2} = \frac{12 \times 3}{6 + 3} = \frac{36}{9} = 4 \text{ Amps}$$

$$\text{Current through } R_2 : I_2 = \frac{I \times R_1}{R_1 + R_2} = \frac{12 \times 6}{6 + 3} = \frac{72}{9} = 8 \text{ Amps}$$

Answer:

Current through *Resistance R_1* = 4 Amps

Current through Resistance R_2 = 8 Amps

Example: 27

Two resistors 4 Ω and 6 Ω are connected in parallel. The total current flowing through the resistors is 5A. Find the current flowing through each resistor.

Given Data:

Resistance (R_1) = 4Ω

Resistance (R_2) = 6 Ω

Total Current = 5 Amps

To Find:

i) Current through Resistance R_1

ii) Current through Resistance R_2

Solution:

$$\text{Current through } R_1 : I_1 = \frac{I \times R_2}{R_1 + R_2} = \frac{5 \times 6}{4 + 6} = \frac{30}{10} = 3 \text{ Amps}$$

$$\text{Current through } R_2 : I_2 = \frac{I \times R_1}{R_1 + R_2} = \frac{5 \times 4}{4 + 6} = \frac{20}{10} = 2 \text{ Amps}$$

Answer:

Current through *Resistance R_1* = 3 Amps

Current through Resistance R_2 = 2 Amps

Example: 28

The resistors of 1Ω, 2Ω and 4Ω are connected in parallel. A 5 Ω resistors is connected in series with this parallel combination and a 24V batter is connected to the circuit. Find the total current and power in each resistors.

Given Data:

Resistance (R_1) = 1Ω
Resistance (R_2) = 2Ω
Resistance (R_3) = 4Ω
Resistance (R_4) = 5Ω

To Find:

i) Total Current
ii) Power in each resistor

Solution:

In Parallel Connection:

$$\text{Equivalent Resistance: } \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4}$$

$$\frac{1}{R_{eq}} = \frac{4 + 2 + 1}{4} = \frac{7}{4}$$

$$R_{eq} = \frac{4}{7} = 0.57\Omega$$

$$\text{Equivalent Resistance: } R_{eq} = R_{13} + R_4 = 0.57 + 5 = 5.57\ \Omega$$

$$\text{Total Current : } I = \frac{V}{R_{eq}} = \frac{24}{5.57} = 4.3 \text{ Amps}$$

Voltage drop across R_4 (V_4) = $I \times R_4 = 4.3 \times 5 = 21.5$ Volts

Voltage drop across R_1 (V_1) = Supply Voltage (V) - Voltage drop across R_4

Voltage drop across R_1 (V_1) = 24 – 21.5 = 2.5Volts

Voltage drop: V_1 = V_2 = V_3 = 2.5 Volts

$$\text{Power dissipated in } R_1 = \frac{V^2}{R_1} = \frac{2.5^2}{1} = 1.25 \text{ Watts}$$

$$\text{Power dissipated in } R_2 = \frac{V^2}{R_2} = \frac{2.5^2}{2} = 3.125 \text{ Watts}$$

$$\text{Power dissipated in } R_3 = \frac{V^2}{R_3} = \frac{2.5^2}{4} = 1.5625 \text{ Watts}$$

$$\text{Power dissipated in } R_4 = \frac{V^2}{R_4} = \frac{2.5^2}{5} = 1.25 \text{ Watts}$$

Example: 29

The resistors of 2Ω, 4Ω and 12Ω are connected in parallel across a 24 Volts battery. Find the current through each resistor and the battery. Also find power dissipated in each resistors.

<u>Given Data</u>:

Resistance (R_1) = 2Ω
Resistance (R_2) = 4Ω
Resistance (R_3) = 12Ω
Supply Voltage (V) = 24V

<u>To Find</u>:

i) Current through each resistor
ii) Power in each resistor

<u>Solution</u>:

$$\text{Current through Resistor } R_1 : I_1 = \frac{V}{R_1}$$

$$= \frac{24}{2} = 12 \text{ Amps}$$

$$\text{Current through Resistor } R_2 : I_2 = \frac{V}{R_2}$$

$$= \frac{24}{4} = 6 \text{ Am}$$

$$\text{Current through Resistor } R_3 : I_3 = \frac{V}{R_3}$$

$$= \frac{24}{12} = 2 \text{ Amps}$$

$$\text{Power dissipated in } R_1 = \frac{V^2}{R_1} = \frac{24^2}{2} = 288 \text{ Watts}$$

$$\text{Power dissipated in } R_2 = \frac{V^2}{R_2} = \frac{24^2}{4} = 144 \text{ Watts}$$

$$\text{Power dissipated in } R_3 = \frac{V^2}{R_3} = \frac{24^2}{12} = 48 \text{ Watts}$$

Kirchhoff's laws:

Kirchhoff's laws are more comprehensive than ohm's law and are used for solving electrical networks.

The two Kirchhoff's laws are:

1. Kirchhoff's Current Law (KCL)
2. Kirchhoff's Voltage Law (KVL)

Kirchhoff's Current Law (KCL)

It states that "the algebraic sum of the currents in a junction of a circuit is zero.

(or)

The sum of current entering the junction is equal to the sum of the current leaving the junction.

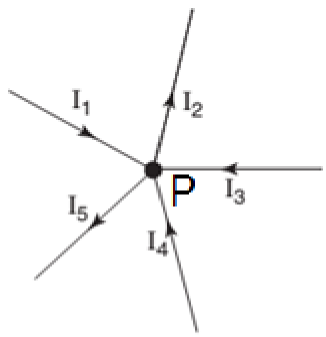
Explanation:

Consider 5 conductors carrying currents. Assume positive sign for the current flowing towards the junction and negative sign for the current flowing away from the junction.

Let **P** is a junction

I_1, I_3, & I_4 = incoming current towards the junction P

I_2, & I_5 = outgoing current away from the junction P



By applying Kirchhoff ' s current law at junction P,

$$I_1 - I_2 + I_3 + I_4 - I_5 = 0$$

$$I_1 + I_3 + I_4 = I_2 + I_5$$

Sum of incoming current = sum of outgoing current

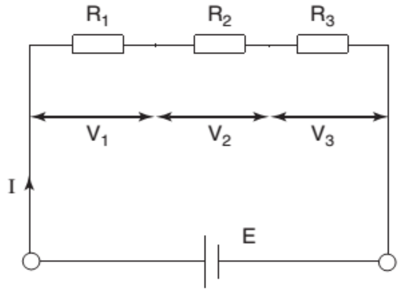
Kirchhoff's Voltage Law (KVL):

It states that in any closed electrical circuit, "the algebraic sum of voltage is zero". (OR)

That is the sum of voltage rises (EMF) in a closed network is equal to the sum of voltages drops (P.D)

Explanation:

Consider a simple closed electrical circuit,
Let,
E - Supply voltage
R_1, R_2 & R_3 - Resistors in the circuit.
I - Current flow through the circuit
V_1 – p.d across R_1
V_2 – p.d across R_2
V_3 – p.d across R_3



Applying Kirchhoff's voltage law to the closed circuit,

$$E - V_1 - V_2 - V_3 = 0$$

$$E = V_1+V_2+V_3 \quad \text{-----------} \ (1)$$

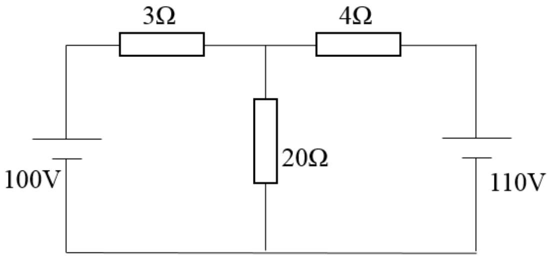
Substitute, $V_1 = I.R_1$, $V_2 = I.R_2$ & $V_3 = I.R_3$ in equation (1)

$$E = IR_1+IR_2+IR_3$$

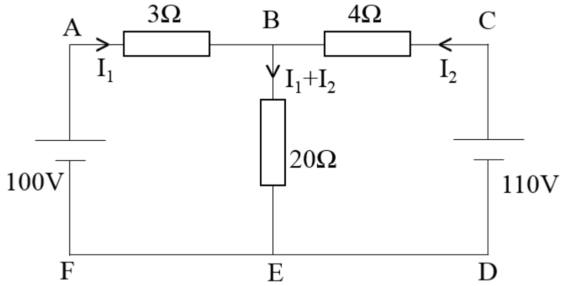
$$E - IR_1 - IR_2 - IR_3 = 0 \quad \text{[Algebraic sum of voltages is zero]}$$

Example: 30

Determine the currents in different branches of the circuit shown in figure by applying Kirchhoff's law.



<u>Solution</u>:



Apply KVL in Loop ABEFA

Branch	Potential Drop	Potential Rise
AB	$3I_1$	-
BE	$20(I_1+I_2)$	-
EF	-	-
FA	-	100
	$3I_1 + 20(I_1+I_2)$	100

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$

$$3I_1 + 20(I_1+I_2) = 100$$

$$3I_1 + 20I_1 + 20I_2 = 100$$

$$23I_1 + 20I_2 = 100 \quad \text{------------ (1)}$$

Apply KVL in Loop CBEDC:

Branch	Potential Drop	Potential Rise
CB	$4I_2$	-
BE	$20(I_1+I_2)$	-
ED	-	-
DC	-	110
	$4I_2 + 20(I_1+I_2)$	110

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$

$$4I_2 + 20(I_1+I_2) = 110$$

$$4I_2 + 20I_1 + 20I_2 = 110$$

$$20I_1 + 24I_2 = 110 \quad \text{------------ (2)}$$

$$23I_1 + 20I_2 = 100 \quad \text{------------ (1)}$$

$$20I_1 + 24I_2 = 110 \quad \text{------------ (2)}$$

$$\begin{vmatrix} 23 & 20 \\ 20 & 24 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 100 \\ 110 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 23 & 20 \\ 20 & 24 \end{vmatrix}$$

$$\Delta = (23x24) - (20 \text{ x } 20) = 152$$

$$\Delta I_1 = \begin{vmatrix} 100 & 20 \\ 110 & 24 \end{vmatrix}$$

$$= (100x24) - (20 \text{ x } 110) = 200$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{200}{152} = 1.315 Amps$$

$$\Delta I_2 = \begin{vmatrix} 23 & 100 \\ 20 & 110 \end{vmatrix}$$

$$= (23x110) - (100 \text{ x } 20) = 530$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{530}{152} = 3.486\ Amps$$

Answer:

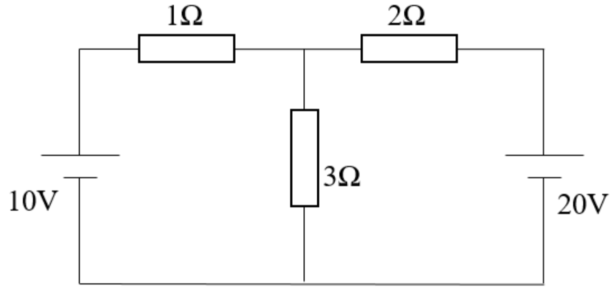
Current through 3Ω resistor : I_1 = 1.315 Amps

Current through 4Ω resistor : I_2 = 3.486 Amps

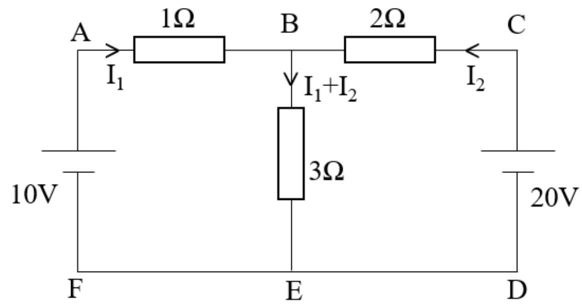
Current through 20Ω resistor : $I_1 + I_2$ = 1.315 + 3.486 = 4.801 Amps

Example: 31

Find the current in the 3 ohm resistor in the circuit shown in figure.



Solution:



Apply KVL in Loop ABEFA:

Branch	Potential Drop	Potential Rise
AB	$1I_1$	-
BE	$3(I_1+I_2)$	-
EF	-	-
FA	-	10
	$1I_1 + 3(I_1+I_2)$	10

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$

$$1I_1 + 3(I_1+I_2) = 10$$

$$1I_1 + 3I_1 + 3I_2 = 10$$

$$4I_1 + 3I_2 = 10 \qquad \text{------------ (1)}$$

Apply KVL in Loop CBEDC:

Branch	Potential Drop	Potential Rise
CB	$2I_2$	-
BE	$3(I_1+I_2)$	-
ED	-	-
DC	-	20
	$2I_2 + 3(I_1+I_2)$	20

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$

$$2I_2 + 3(I_1+I_2) = 20$$

$$2I_2 + 3I_1 + 3I_2 = 20$$

$$3I_1 + 5I_2 = 20 \qquad \text{------------ (2)}$$

$$4I_1 + 3I_2 = 10 \qquad \text{------------ (1)}$$

$$3I_1 + 5I_2 = 20 \qquad \text{------------ (2)}$$

$$\begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 10 \\ 20 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix}$$

$$\Delta = (4x5) - (3 \text{ x } 3) = 11$$

$$\Delta I_1 = \begin{vmatrix} 10 & 3 \\ 20 & 5 \end{vmatrix}$$

$$= (10x5) - (20 \text{ x } 3) = -10$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{-10}{11} = -0.909 Amps$$

$$\Delta I_2 = \begin{vmatrix} 4 & 10 \\ 3 & 20 \end{vmatrix}$$

$$= (4x20) - (3 \text{ x } 10) = 50$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{50}{11} = 4.54\ Amps$$

Answer:

Current through 1Ω resistor : $I_1 = -0.909$ Amps

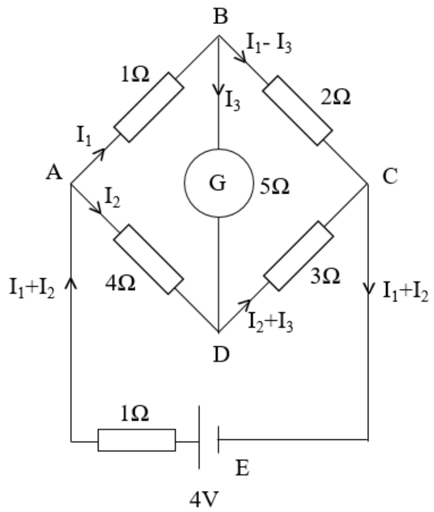
Current through 2Ω resistor : $I_2 = 4.54$ Amps

Current through 30Ω resistor : $I_1 + I_2 = -0.909 + 4.54 = 3.631$ Amps

Example: 32

A wheat stone bridge ABCD is arranged as follows AB=1Ω, BC=2Ω, CD=3Ω, DA=4Ω. A galvanometer of resistance 5Ω is connected between B and D. A battery of 4 volt with internal resistance 1Ω is connected between A and C. Calculate current flow through 5Ω.

Solution:



Apply KVL in Loop ABDA:

Branch	Potential Drop	Potential Rise
AB	$1I_1$	-
BD	$5I_3$	-
DA	-	$4I_2$
	$1I_1 + 5I_3$	$4I_2$

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$
$$1I_1 + 5I_3 = 4I_2$$
$$1I_1 + 5I_3 - 4I_2 = 0$$
$$1I_1 - 4I_2 + 5I_3 = 0 \quad \text{------------ (1)}$$

Apply KVL in Loop BCDB:

Branch	Potential Drop	Potential Rise
BC	$2(I_1 - I_3)$	-
CD	-	$3(I_2+I_3)$
DB	-	$5I_3$
	$2(I_1 - I_3)$	$3(I_2+I_3)+ 5I_3$

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$
$$2(I_1 - I_3) = 3(I_2+I_3)+ 5I_3$$
$$2(I_1 - I_3) - 3(I_2+I_3) - 5I_3 = 0$$
$$2(I_1 - I_3) - 3(I_2+I_3) - 5I_3 = 0$$
$$2I_1 - 2I_3 - 3I_2 - 3I_3 - 5I_3 = 0$$
$$2I_1 - 3I_2 - 10I_3 = 0 \quad \text{------------ (2)}$$

Apply KVL in Loop ADCA:

Branch	Potential Drop	Potential Rise
AD	$4I_2$	-
DC	$3(I_2+I_3)$	-
CA	$1(I_1+I_2)$	4
	$4I_2 + 3(I_2+I_3)+ 1(I_1+I_2)$	4

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$
$$4I_2 + 3(I_2+I_3)+ 1(I_1+I_2) = 0$$
$$4I_2 + 3I_2+3I_3+ 1I_1+1I_2 = 0$$
$$1I_1 + 8I_2 + 3I_3 = 4 \quad \text{------------ (3)}$$

$$1I_1 - 4I_2 + 5I_3 = 0 \quad \text{------------ (1)}$$
$$2I_1 - 3I_2 - 10I_3 = 0 \quad \text{------------ (2)}$$
$$1I_1 + 8I_2 + 3I_3 = 4 \quad \text{------------ (3)}$$

$$\begin{vmatrix} 1 & -4 & 5 \\ 2 & -3 & -10 \\ 1 & 8 & 3 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 4 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & -4 & 5 \\ 2 & -3 & -10 \\ 1 & 8 & 3 \end{vmatrix}$$

$$\Delta = 1[(-3x3)-(-10x8)] - (-4)[(2x3)-(-10x1)] + 5[(2x8) - (-3)]$$

$$\Delta = 1[(-9)-(-80)] - (-4)[(6)-(-10)] + 5[(16) - (-3)]$$

$$\Delta = 1[-9 + 80] + 4[6+10] + 5[16+3]$$

$$\Delta = 1[71] + 4[16] + 5[19]$$

$$\Delta = 71 + 64 + 95$$

$$\Delta = 230$$

$$\Delta I_3 = \begin{vmatrix} 1 & -4 & 0 \\ 2 & -3 & 0 \\ 1 & 8 & 4 \end{vmatrix}$$

$$\Delta I_3 = 1[(-3x4)-(0x8)] - (-4)[(2x4)-(0x1)] + 0$$

$$\Delta I_3 = 1[-12 - 0] + 4[8-0] + 0$$

$$\Delta I_3 = 1[-12] + 4[8]$$

$$\Delta I_3 = -12 + 32$$

$$\Delta I_3 = 20$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{20}{230}$$

$$I_3 = 0.087\ Amps$$

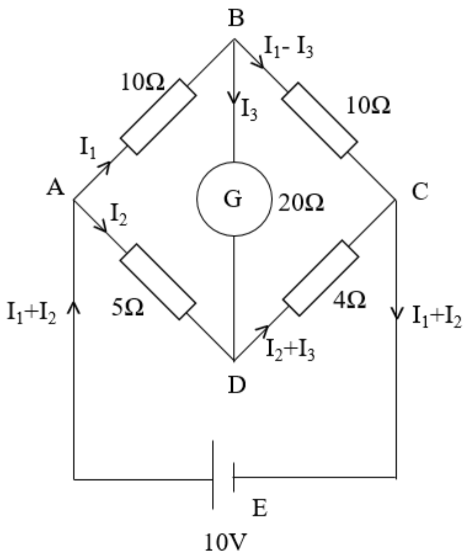
<u>**Answer:**</u>

Current through 5Ω resistor : $I_3 = 0.087$ Amps

Example: 33

A wheat stone bridge consists of AB=10Ω, BC=10Ω, CD=4Ω, DA=5Ω. A galvanometer of resistance 20Ω is connected across BD. Calculate the current through the galvanometer when a p.d of 10V is maintained across AC.

Solution:



Apply KVL in Loop ABDA

Branch	Potential Drop	Potential Rise
AB	$10I_1$	-
BD	$20I_3$	-
DA	-	$5I_2$
	$10I_1 + 20I_3$	$5I_2$

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$

$$10I_1 + 20I_3 = 5I_2$$

$$10I_1 + 20I_3 - 5I_2 = 0$$

$$10I_1 - 5I_2 + 20I_3 = 0 \qquad \text{------------ (1)}$$

Apply KVL in Loop BCDB

Branch	Potential Drop	Potential Rise
BC	$10(I_1 - I_3)$	-
CD	-	$4(I_2+I_3)$
DB	-	$20I_3$
	$10(I_1 - I_3)$	$4(I_2+I_3)+ 20I_3$

According to KVL:

$$\text{Sum of Potential Drop} = \text{Sum of Potential Rise}$$

$$10(I_1 - I_3) = 4(I_2+I_3)+ 20I_3$$

$$10(I_1 - I_3) - 4(I_2+I_3) - 20I_3 = 0$$

$$10I_1 - 10I_3 - 4I_2 - 4I_3 - 20\,I_3 = 0$$

$$10I_1 - 4I_2 - 34I_3 = 0$$

$$10I_1 - 4I_2 - 34I_3 = 0 \qquad \text{------------ (2)}$$

Apply KVL in Loop ADCA:

Branch	Potential Drop	Potential Rise
AD	$5I_2$	-
DC	$4(I_2+I_3)$	-
CA	-	10
	$5I_2 + 4(I_2+I_3)$	10

According to KVL:

Sum of Potential Drop = Sum of Potential Rise

$$5I_2 + 4(I_2+I_3) = 10$$

$$5I_2 + 4I_2+4I_3 = 10$$

$$9I_2 + 4I_3 = 10 \quad \text{------------ (3)}$$

$$10I_1 - 5I_2 +20I_3 = 0 \quad \text{------------ (1)}$$

$$10I_1 - 4I_2 - 34I_3 = 0 \quad \text{------------ (2)}$$

$$9I_2 + 4I_3 = 10 \quad \text{------------ (3)}$$

$$\begin{vmatrix} 10 & -5 & 20 \\ 10 & -4 & -34 \\ 0 & 9 & 4 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 10 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 10 & -5 & 20 \\ 10 & -4 & -34 \\ 0 & 9 & 4 \end{vmatrix}$$

$$\Delta = 10[(-4x4)-(-34x9)] – (-5)[(10x4)-(-34x0)] +20[(10x9) – (0)]$$

$$\Delta = 10[(-16)-(-306)] +5[40-0] +20[90 – 0]$$

$$\Delta = 10[-16+306] +5[40] +20[90]$$

$$\Delta = 10[290] +5[40] +20[90]$$

$$\Delta = 2900 +200 +1800$$

$$\Delta = 4900$$

$$\Delta I_3 = \begin{vmatrix} 10 & -5 & 0 \\ 10 & -4 & 0 \\ 0 & 9 & 10 \end{vmatrix}$$

$$\Delta I_3 = 10[(-4x10)-(0x9)] – (-5)[(10x10)-(0x0)] +0$$

$$\Delta I_3 = 10[-40 -0] +5[100 - 0] +0$$

$$\Delta I_3 = 10[-40] +5[100] = -400 +500$$

$$\Delta I_3 = 100$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{100}{4900}$$

$$I_3 = 0.0204\ Amps$$

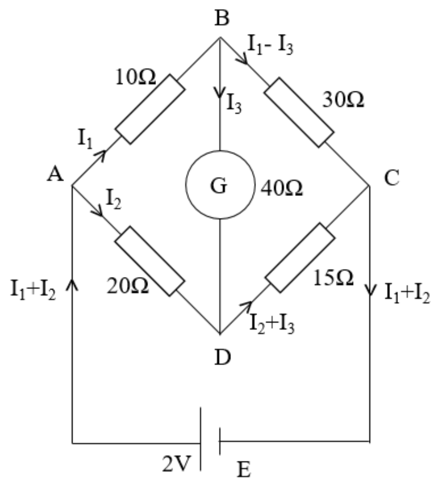
Answer:

Current through 20Ω resistor : $I_3 = 0.0204$ Amps

Example: 34

A wheat stone bridge consist of AB=10Ω, BC=30Ω, CD=15Ω, DA=20Ω. A galvanometer of resistance 40Ω is connected across BD. Calculate the current through the galvanometer when a p.d of 2V is maintained across A.C.

Solution:



Apply KVL in Loop ABDA

Branch	Potential Drop	Potential Rise
AB	$10I_1$	-
BD	$40I_3$	-
DA	-	$20I_2$
	$10I_1 + 40I_3$	$20I_2$

According to KVL:

Sum of Potential Drop = Sum of Potential Rise

$$10I_1 + 40I_3 = 20I_2$$

$$10I_1 + 40I_3 - 20I_2 = 0$$

$$10I_1 - 20I_2 + 40I_3 = 0 \quad \text{------------ (1)}$$

Apply KVL in Loop BCDB:

Branch	Potential Drop	Potential Rise
BC	$30(I_1 - I_3)$	-
CD	-	$15(I_2+I_3)$
DB	-	$40I_3$
	$30(I_1 - I_3)$	$15(I_2+I_3) + 40I_3$

According to KVL:

Sum of Potential Drop = Sum of Potential Rise

$$30(I_1 - I_3) = 15(I_2+I_3) + 40I_3$$

$$30(I_1 - I_3) - 15(I_2+I_3) - 40I_3 = 0$$

$$30I_1 - 30I_3 - 15I_2 - 15I_3 - 40\,I_3 = 0$$

$$30I_1 - 15I_2 - 85I_3 = 0 \quad \text{------------ (2)}$$

Apply KVL in Loop ADCA:

Branch	Potential Drop	Potential Rise
AD	$20I_2$	-
DC	$15(I_2+I_3)$	-
CA	-	2
	$20I_2 + 15(I_2+I_3)$	2

According to KVL:

Sum of Potential Drop = Sum of Potential Rise

$$20I_2 + 15(I_2+I_3) = 2$$

$$20I_2 + 15I_2+15I_3 = 2$$

$$35I_2 + 15I_3 = 2 \quad \text{------------ (3)}$$

$$10I_1 - 20I_2 +40I_3 = 0 \quad \text{------------ (1)}$$

$$30I_1 - 15I_2 - 85I_3 = 0 \quad \text{------------ (2)}$$

$$35I_2 + 15I_3 = 2 \quad \text{------------ (3)}$$

$$\begin{vmatrix} 10 & -20 & 40 \\ 30 & -15 & -85 \\ 0 & 35 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 10 & -20 & 40 \\ 30 & -15 & -85 \\ 0 & 35 & 15 \end{vmatrix}$$

$$\Delta = 10[(-15x15)-(-85x35)] – (-20)[(30x15)-(0)] +40[(30x35) – (0)]$$

$$\Delta = 10[-225 + 2975] – (-20)[450 - 0] +40[1050 – 0]$$

$$\Delta = 10[2750] +20[450] +40[1050]$$

$$\Delta = 27500 +9000 +42000$$

$$\Delta = 78500$$

$$\Delta I_3 = \begin{vmatrix} 10 & -20 & 0 \\ 30 & -15 & 0 \\ 0 & 35 & 2 \end{vmatrix}$$

$$\Delta I_3 = 10[(-15x2)-(0x35)] – (-20)[(30x2)-(0x0)] +0$$

$$\Delta I_3 = 10[-30- 0] +20[60-0] +0$$

$$\Delta I_3 = 10[-30] +20[60]$$

$$\Delta I_3 = 900$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{900}{78500}$$

$$I_3 = 0.0114\ Amps$$

Answer:

Current through 40Ω resistor : $I_3 = 0.0114$ Amps

REVIEW QUESTIONS
UNIT : I ELECTROSTATICS & D.C CIRCUITS

PART - A : 2 Mark Questions.

1. Define Electric flux.
2. Define flux density.
3. Define electric field intensity.
4. Define electric potential.
5. Define a capacitance.
6. Write the expression for energy stored in a capacitor.
7. State expression for the equivalent capacitance of two capacitors in series.
8. State expression for the equivalent capacitance of two capacitors in parallel.
9. If a 2µF capacitor and 4 µF capacitor are connected in parallel. What is its equivalent capacitance?
10. Define permittivity?
11. Define relative permittivity?
12. State the relation between electric flux density and field intensity?
13. Define electric current.
14. Define electromotive force.
15. Define potential difference.
16. Define Resistance.
17. Define electric power.
18. Define electrical energy.
19. State the unit of electric current and Resistance.
20. State the unit of electrical power and Electrical Energy.
21. Define Specific resistance or resistivity.
22. State ohm's law.
23. Define temperature coefficient of resistance.
24. State Kirchhoff's Current Law.
25. State Kirchhoff's Voltage Law.

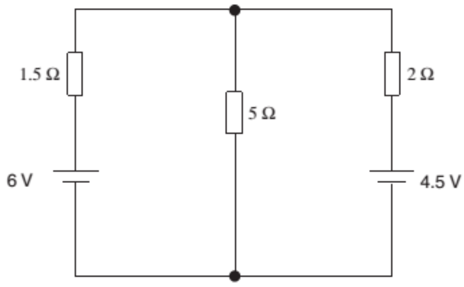
PART – B : 3 Mark Questions

1. Define: (i) Electric Flux (ii) Electric Flux Density
2. Define: (i) Electric field Intensity (ii) Electric Potential
3. State Coulomb's law of electrostatics.
4. Define Capacitance and state its fundamental practical unit.
5. Derive an expression for energy stored in a capacitor.
6. Derive an equation for the equivalent capacitance of two capacitors connected in series.
7. Derive an equation for the equivalent capacitance of two capacitors connected in parallel.

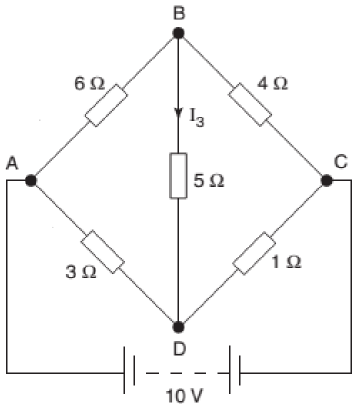
8. Define: (i) Current (ii) E.M.F (iii) Resistance
9. Define power. What is the relationship between power, voltage and current?
10. Define resistance and conductance.
11. Define resistivity or specific resistance and state its unit.
12. What are the factors on which resistance of a material depend?
13. Define temperature co-efficient of resistance. Give its unit of measurements.
14. State ohm's law and write its expression in 3 different forms
15. Prove that Power P = V.I Watts.
16. Define Electric power and energy. Mention the unit of measurement
17. State Kirchhoff's laws.
18. A 100µF capacitance is charged to a steady voltage of 500V. What is the energy stored in the capacitance?
19. Two resistors are connected in series and connected across a battery. The p.d across first resistor is 10V and p.d across second resistor is 25V. The current through the resistors is 2 Amps. Find the value of each resistor.
20. A battery of 20V is connected across a resistor so that the current is limited to 0.2Amps. Calculate he value of resistance and its power rating.
21. A resistor has a resistance of 150Ω at 20˚C and 166Ω at 55˚C. Find the temperature coefficient of the resistance at 0˚C.
22. Find the resistances in ohm of incandescent lamps of (a) 100W at 120V and (b) 60W at 12V.

PART – C : 10 Mark Questions

1. Define the following terms and state their units :
   (i) Electric flux (ii) Electric flux density (iii) Field intensity (iv) Electric Potential
2. Show that $R_t = R_0+(1+\alpha_0 t)$
3. State and explain coulomb's laws of electrostatics.
4. Derive equivalent capacitance of 3 capacitors in series.
5. Derive equivalent capacitance of 3 capacitors in parallel.
6. Define the following terms and state their units :
   (i) Electric Current (ii) E.M.F (iii) Resistance (iv) Electrical Power
7. Derive equivalent resistance of 3 resistors in series.
8. Derive equivalent resistance of 3 resistors in parallel.
9. State and explain kirchhoff's Laws.
10. For the given circuit, use Kirchhoff ' s Laws to calculate (a) the current flowing in each branch of the circuit, and (b) the p.d. across the 5 Ω resistor.



11. For the given bridge, calculate the current through 5 Ω resistor, and the current drawn from the supply.



12. Capacitors of 3 μF, 6μF and 12μF are connected in series across a 400 V supply. Determine the p.d. across each capacitor.

13. Three resistors of resistance 10Ω, 15Ω and 20Ω are connected once in series and then parallel. Find the equivalent resistance for each case. Also calculate the power dissipated in each case, if the supply voltage is 15V.

14. Two incandescent lamps of 100W and 60W and rated voltage of 110V are supplied by a source of 110V d.c. Determine the current drawn from the source (a) when the lamps are connected in parallel, (b) when the lamps are connected in series.

UNIT 2 - CIRCUIT THEOREMS

Syllabus:

Mesh equations – Nodal equations – star/delta transformations – Superposition theorem – Thevenin's theorem – Norton's theorem – Maximum power transfer theorem. (Problems in DC circuits only).

2.0 Introductions:

In unit 1, we analysed the various types of circuits using ohm's law and Kirchhoff's laws. Some circuits are difficult to solve using only basics laws and require additional methods in order to simplify the analysis. In this unit, we shall understand a variety of techniques such as Mesh Current Analysis, Nodal analysis, the Superposition theorem, Thévenin's theorem and Norton's theorem, which will speed up the analysis of the more complicated networks.

The theorems and conversions make analysis easier for certain types of circuit. These methods do not replace Ohm's law and Kirchhoff's laws, but they are normally used in conjunction with laws in certain situation.

Electric circuit theorems are always beneficial to find voltage and currents in multi loop circuits. These theorems use fundamental rules or formulae and basic equations of mathematics to analyze basic components of electrical or electronics parameters such as voltages, currents, resistance, and so on. These fundamental theorems include the basic theorems like Superposition theorem, Thevenin's theorems, Norton's theorem and Maximum power transfer theorem.

Not all loads are connected in series or in parallel. There are two other arrangements known as star and delta. They are not so common but, because they are interchangeable, we can readily find a solution to any network in which they appear – so long as we can transform the one into the other.

Because all electric circuits are driven by either voltage sources or current sources, it is important to understand how to work with these elements. Source conversion technique also useful to get the solution easily. The superposition theorem will help you to deal with circuits that have multiple sources. Super position theorem is used only in linear networks. This theorem is used in both AC and DC circuits wherein it helps to construct Thevenin and Norton equivalent circuit

Thevenin's and Norton's theorems provide methods for reducing a circuit to a simple equivalent form for ease of analysis. This theorem can be applied to both linear and bilateral networks. The maximum power transfer theorem is used in applications where it is important for a given circuit to provide maximum power to a load.

2.1 Terms and Definition:

Sl.No	Term	Definition
1	Network Element	Any individual element which can be connected to some other element is called network element.
2	Active Element	These are elements which can deliver electrical energy. *Example: Voltage source and Current source*
3	Passive Element	These are elements which consumes electrical energy either by absorbing or storing in it. *Example: R, L and C*
4	Electric Network	Interconnection of elements like resistor or inductor or capacitor or electrical energy sources are known as networks.
5	Electric circuit	A closed energized network is known as circuit.
6	Node or Junction in a network	A point at which two or more elements are joined together is called node.
7	Simple Node	A node where only two elements are joined is called simple node.
8	Principal Node	A node where three or more elements are joined is called Principal Node.
9	Branch of a network	A branch is a path that connects two nodes.
10	Loop in a network	A closed path obtained by starting at a node and returning back to the same node through a set of connected circuit elements.
11	Mesh in a circuit	A loop that does not contain any other loops within it.

2.2 Active Network Vs Passive Network:

Sl.No	Active network	Passive network
1	A network which contains a source of energy is called active network.	A network which contains no energy source of energy is called passive network.
2	Example: Voltage source and current source	Example: It contains R, L and C

2.3 Unilateral Circuit Vs Bilateral Circuit:

Unilateral circuit	Bilateral circuit
It is that circuit whose properties or characteristics change with the direction of its operation. A diode rectifier is a unilateral circuit, because it cannot perform rectification in both directions.	A bilateral circuit is one whose properties or characteristics are the same in either direction. The usual transmission line is bilateral, because it can be made to perform its function equally well in either direction.

2.4 Linear Circuit Vs Non-Linear Circuit:

Sl.No	Linear circuit	Non-Linear circuit
1	A circuit or network whose parameters i.e., elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as linear network.	A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as nonlinear network.
2	The ohm's law can be applied to such network. Such network follow the law of superposition.	The ohm's law may not be applied to such network. Such network does not follow the law of superposition.

2.5 Ideal voltage source:

A voltage source which maintains a constant potential difference at a given magnitude and a prescribed waveform across its terminals independent of the current supplied by it.

2.5.1 Characteristics of Ideal voltage source:

i) It is the energy source whose output voltage remains constant whatever be the value of the output current.
ii) It has zero internal resistance so that voltage drop in the source is zero.

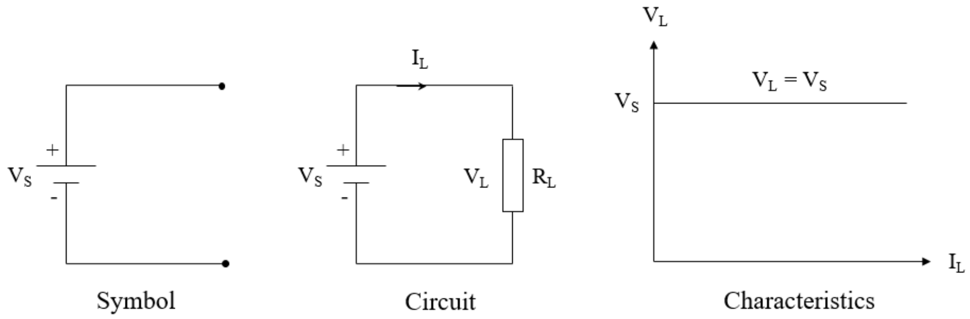


Figure 2.1: Ideal Voltage Source

2.5.2 Characteristics of Practical voltage source:

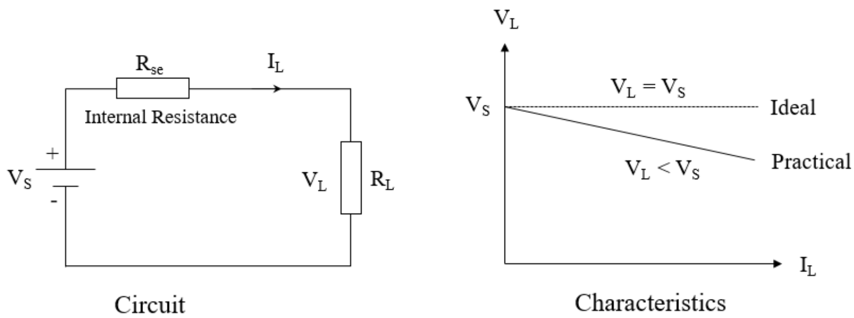


Figure 2.2: Practical Voltage Source

Practically, every voltage source has small internal resistance in series with voltage source and is represented by R_{se} as shown in figure 2.2. In this source, the voltage does not remain constant, but falls slightly with addition of load.

2.6 Ideal Current source:

A current source is one which maintains a current of a given magnitude and a prescribed waveform across its terminals independent of the potential difference appearing across its terminals. Symbolically, we represent a general current source as a two terminal device as shown in figure.

2.6.1 Characteristics of Ideal current source:

i) It produces a constant current irrespective of the value of the voltage across it.
ii) It has infinity resistance.
iii) It is capable of supplying infinity power.

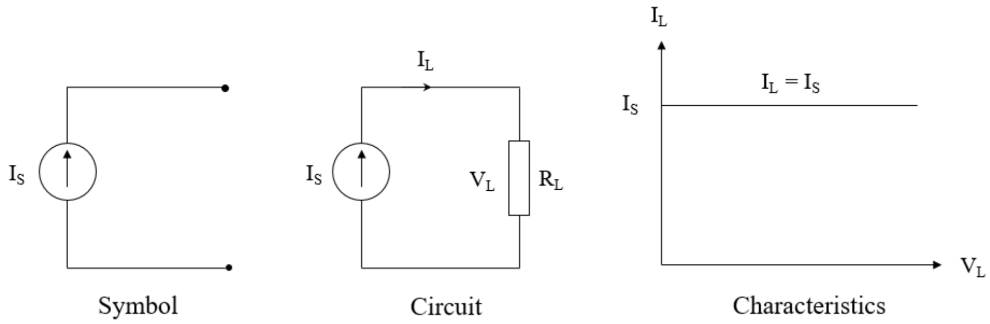


Figure 2.3: Ideal Current Source

2.6.2 Characteristics of Practical current source:

Practically, every current source has high internal resistance in parallel with current source and it is represented by R_{sh}. This is shown in figure 2.4. In practical current source, the current does not remain constant, but falls slightly.

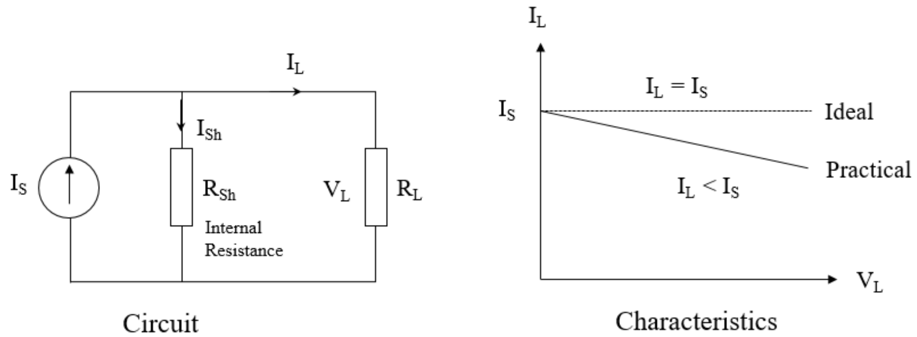


Figure 2.4: Practical Current Source

2.6.3 Loading of sources:

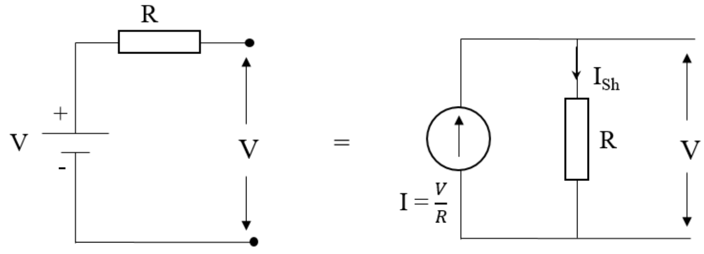
The output voltage of a voltage source decreases as the load current increases. If the source is loaded in such a way that the output voltage falls below a specified full load value, then the source is said to be loaded and the situation is known as loading of source.

2.7 Source Transformation:

Transformation of several voltage (or current) sources into a single voltage (or current) source and a voltage source into a current source or vice-versa is known as source transformation. This makes circuit analysis easier.

2.7.1 Ideal voltage sources be converted into ideal current sources:

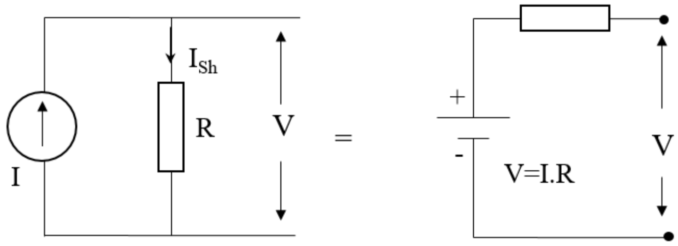
A voltage source V with an internal resistance R can be converted into a current source I in parallel with the same resistance R where, I = V/R.



Voltage Source Current Source

2.7.2 Ideal current sources be converted into ideal voltage sources:

A current source I with an internal resistance R can be converted into a voltage source V in series with the same resistance R where, V = I.R.

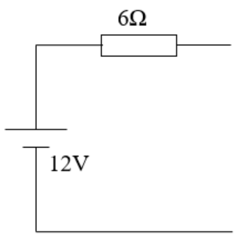


Current Source Voltage Source

Example: 1

Convert the given voltage source into an equivalent current source.

Given:

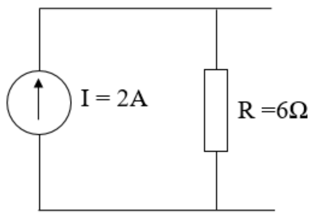


Solution:

V = 12V

R = 6Ω

$$I = \frac{V}{R} = \frac{12}{6} = 2 \text{ Amps}$$



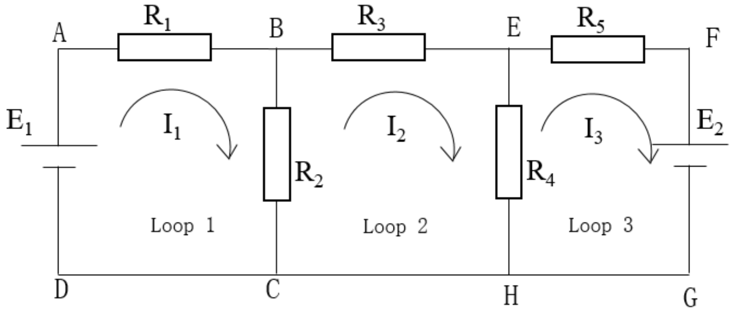
Current source

2.8 Mech Equation:

This method is also called as mesh current analysis. In this method, we consider loop or mesh current instead of branch current. For each loop, an equation is written based on Kirchhoff's Voltage Law KVL and there are a many equations as the number of loops. The current directions are chosen arbitrarily.

Procedure:

i) Circulating currents are allocated to closed loops or meshes in the circuit.
ii) An equation for each loop of the circuit is then obtained by Kirchhoff 's voltage law.
iii) Branch currents are found thereafter by taking the algebraic sum of the loop currents common to individual branches.



By applying KVL in Loop 1 (ABCD):

$$\begin{aligned}
\text{Potential Rise} &= \text{Potential Drop} \\
E_1 &= I_1 R_1 + (I_1 - I_2) R_2 \\
E_1 &= I_1 R_1 + I_1 R_2 - I_2 R_2 \\
E_1 &= I_1 (R_1 + R_2) - I_2 R_2 \\
I_1 (R_1 + R_2) - I_2 R_2 &= 0 \qquad \text{-------------} (1)
\end{aligned}$$

By applying KVL in Loop 2 (BEHC):

$$\begin{aligned}
\text{Potential Rise} &= \text{Potential Drop} \\
0 &= I_2 R_3 + (I_2 - I_3) R_4 + (I_2 - I_1) R_2 \\
0 &= I_2 R_3 + I_2 R_4 - I_3 R_4 + I_2 R_2 - I_1 R_2 \\
0 &= - I_1 R_2 + I_2(R_2 + R_3 + R_4) - I_3 R_4 \\
- I_1 R_2 + I_2(R_2 + R_3 + R_4) - I_3 R_4 &= 0 \qquad \text{-------------} (2)
\end{aligned}$$

By applying KVL in Loop 3 (EFGH):

$$\begin{aligned}
\text{Potential Rise} &= \text{Potential Drop} \\
-E_2 &= I_3 R_5 + (I_3 - I_2) R_4 \\
-E_2 &= I_3 R_5 + I_3 R_4 - I_2 R_4 \\
-E_2 &= - I_2 R_4 + I_3 (R_4 + R_5) \\
- I_2 R_4 + I_3 (R_4 + R_5) &= -E_2 \qquad \text{-------------} (3)
\end{aligned}$$

The equation (1), (2) and (3) can be arranged in matrix form:

$$[R]\,[I] = [V]$$

$$\begin{vmatrix} R_1+R_2 & -R_2 & 0 \\ -R_2 & R_2+R_3+R_4 & -R_4 \\ 0 & -R_4 & R_4+R_5 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} E_1 \\ 0 \\ -E_2 \end{vmatrix}$$

The resistance matrix and its formation is shown below:

$$[R] = \begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix}$$

Where, R_{11} = Self-Resistance of Loop 1

R_{22} = Self-Resistance of Loop 2

R_{33} = Self-Resistance of Loop 3

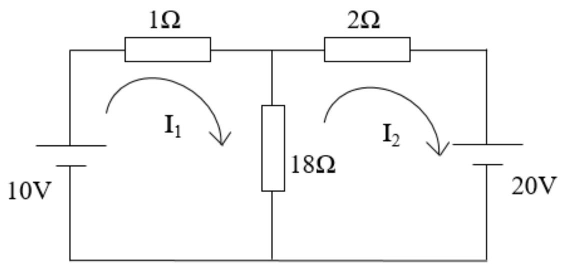
$R_{12} = R_{21}$ = Common Resistance between Loop 1 & 2

$R_{13} = R_{31}$ = Common Resistance between Loop 1 & 3

$R_{23} = R_{32}$ = Common Resistance between Loop 2 & 3

Example: 2

Find the current through 18Ω resistor in the given circuit using mesh current analysis.



<u>Given Data:</u>

Number of loops = 2

<u>To Find:</u>

Current through 18Ω Resistor = ?

<u>Solution:</u>

By inspection Method:

$$\begin{vmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{vmatrix} \begin{vmatrix} I_1 \\ -I_2 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}$$

$$\begin{vmatrix} 1+18 & -18 \\ -18 & 2+18 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 10 \\ -20 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 19 & -18 \\ -18 & 20 \end{vmatrix} = 56$$

$$\Delta I_1 = \begin{vmatrix} 10 & -18 \\ -20 & 20 \end{vmatrix} = -160$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{-160}{56} = -2.86\text{Amps}$$

$$\Delta I_2 = \begin{vmatrix} 19 & 10 \\ -18 & -20 \end{vmatrix} = -200$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{-200}{56} = -3.57\text{Amps}$$

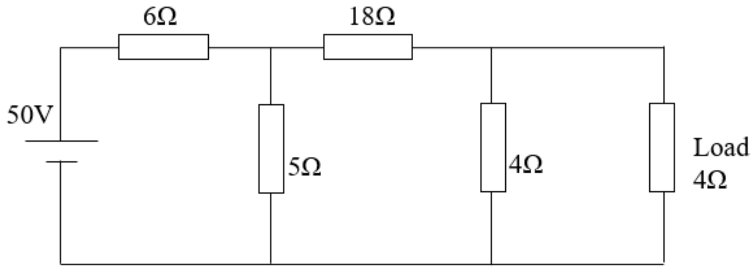
Current through 18Ω Resistor $= I_1 - I_2 = -2.86 - (-3.57) = 0.71\text{Amps}$

<u>Answer:</u>

Current through 18Ω Resistor = 0.71 Amps

Example: 3

Find the current through 4Ω load resistor using mesh current analysis.



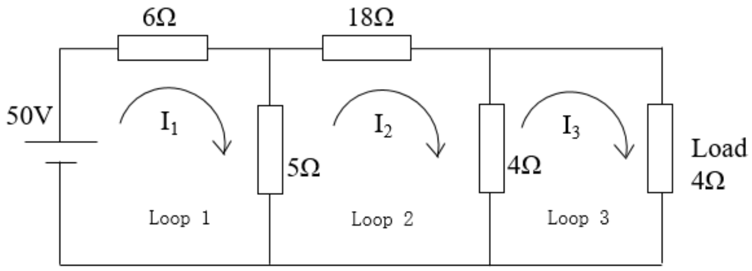
Given Data:

Number of loops = 3

To Find:

i) Current through 4Ω Resistance =?

Solution:



By inspection Method:

$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

Where, $R_{11} = 6 + 5 = 11\Omega$

$R_{22} = 5 + 18 + 4 = 27\Omega$

$R_{33} = 4 + 4 = 8\ \Omega$

$R_{12} = R_{21} = 5\Omega$

$R_{23} = R_{32} = 4\Omega$

$R_{13} = R_{31} = 0$

$$\begin{vmatrix} 11 & -5 & 0 \\ -5 & 27 & -4 \\ 0 & -4 & 8 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 50 \\ 0 \\ 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & 0 \\ -5 & 27 & -4 \\ 0 & -4 & 8 \end{vmatrix}$$

$$\Delta = 11[27 \text{ x } 8 - (-4 \text{ x} -4)] - (-5)\,[(-5 \text{ x } 8) - (0 \text{ x} -4)] + 0$$

$$\Delta = 11[216 - 16] + 5\,[-40] + 0 = 11\,[200] -200$$

$$\Delta = 2000$$

$$\Delta I_3 = \begin{vmatrix} 11 & -5 & 50 \\ -5 & 27 & 0 \\ 0 & -4 & 0 \end{vmatrix}$$

$$\Delta I_3 = 11[27 \text{ x } 0 - (0)] - (-5)\,[0 - 0] + 50[(-5 \text{ x} -4) - (0 \text{ x } 27)]$$

$$\Delta I_3 = 50[(-5 \text{ x} -4) - (0 \text{ x } 27)] = 50[20]$$

$$\boldsymbol{\Delta I_3} = 1000$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{1000}{2000} = 0.5\text{Amps}$$

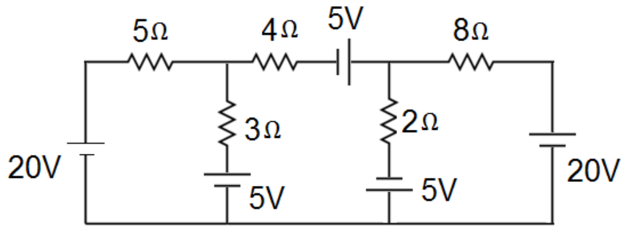
$$I_3 = 0.5\text{Amps}$$

Answer:

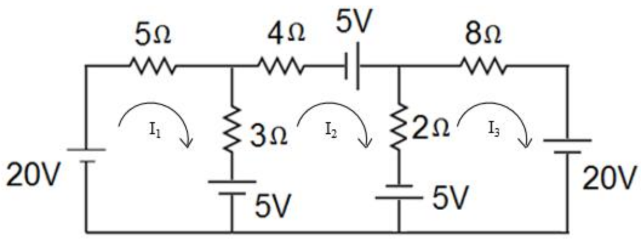
Current through 4Ω Resistor = 0.5 Amps

Example: 4

By mesh current method determine the current through 5Ω, 4Ω and 8Ω Resistor.



Solution:



By inspection Method:

$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

Where, $R_{11} = 5 + 3 = 8\Omega$

$R_{22} = 3 + 4 + 2 = 9\Omega$

$R_{33} = 2 + 8 = 10\ \Omega$

$R_{12} = R_{21} = 3\Omega$

$R_{23} = R_{32} = 2\Omega$

$R_{13} = R_{31} = 0$

$$\begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 15 \\ 15 \\ -25 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix}$$

$$\Delta = 8[9 \text{ x } 10 - (-2 \text{ x } -2)] - (-3)\ [(-3 \text{ x } 10) - (0 \text{ x } -2)] + 0$$

$$\Delta = 8[90 - 4] + 3\ [-30] + 0$$

$$\Delta = 688 - 90$$

$$\Delta = 598$$

$$\Delta I_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -25 & -2 & 10 \end{vmatrix}$$

$$\Delta I_1 = 15[(9 \text{ x } 10) - (-2 \text{ x } -2)] + 3\ [(15 \text{ x } 10) - (-2 \text{ x } -25)] +$$

$$\Delta I_1 = 15[90 - 4] + 3\ [150 - 50] + 0$$

$$\Delta I_1 = 15[86] + 300 + 0$$

$$\Delta I_1 = 1290 + 300$$

$$\Delta I_1 = 1590$$

$$I_1 = \frac{\Delta I_1}{\Delta}$$

$$I_1 = \frac{1590}{598}$$

$$I_1 = 2.65\text{Amps}$$

$$\Delta I_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -25 & 10 \end{vmatrix}$$

$$\Delta I_2 = 8[(15 \text{ x } 10) - (-2 \text{ x } -25)] - 15[(-3 \text{ x } 10) - 0] + 0]$$

$$\Delta I_2 = 8[150 - 50] - 15[-30] + 0$$

$$\Delta I_2 = 8[100] + 450 = 800 + 450$$

$$\Delta I_2 = 1250$$

$$I_2 = \frac{\Delta I_2}{\Delta}$$

$$I_2 = \frac{1250}{598} = 2.09\text{Amps}$$

$$\Delta I_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -25 \end{vmatrix}$$

$$\Delta I_3 = 8[(9 \text{ x } -25) - (-2 \text{ x } 15)] + 3[(-3 \text{ x } -25) - 0] + 15[-3 \text{ x } -$$

$$\Delta I_3 = 8[-225 + 30] + 3[75] + 15[6]$$

$$\Delta I_3 = 8[-195] + 225 + 90 = -1560 + 225 + 90$$

$$\Delta I_3 = -1245$$

$$I_3 = \frac{\Delta I_3}{\Delta}$$

$$I_3 = \frac{-1245}{598} = -2.08\text{Amps}$$

Answer:

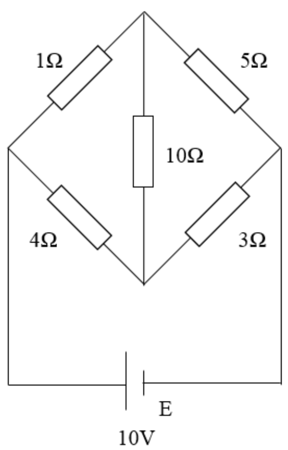
Current through 5Ω Resistor = 2.65 Amps

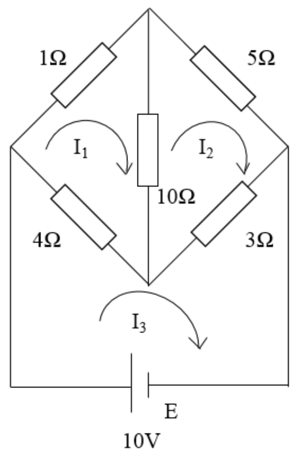
Current through 4Ω Resistor = 2.09 Amps

Current through 8Ω Resistor = -2.08 Amps

Example: 5

Find the current through 10Ω load resistor using mesh current analysis.





Given Data:

Number of loops = 3

To Find:

i) Current through 10Ω =?

Solution:

By inspection Method:

$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

$$\begin{vmatrix} 15 & -10 & -4 \\ -10 & 18 & -3 \\ -4 & -3 & 7 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 10 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 15 & -10 & -4 \\ -10 & 18 & -3 \\ -4 & -3 & 7 \end{vmatrix}$$

Δ = 15(126 – 9) + 10 (-70 - 12) - 4 (30 + 72)

Δ = 15(117) + 10(-82] -4(102)

Δ = 1755 - 820 – 408

Δ = 527

$$\Delta I_1 = \begin{vmatrix} 0 & -10 & -4 \\ 0 & 18 & -3 \\ 10 & -3 & 7 \end{vmatrix}$$

ΔI_1 = 0 +10(0 + 30) -4(0 -180)

ΔI_1 = 10(30) - 4(-180) = 300 + 720

ΔI_1 = 1020

$$I_1 = \frac{\Delta I_1}{\Delta}$$

$$I_1 = \frac{1020}{527}$$

I_1 = 1.94Amps

$$\Delta I_2 = \begin{vmatrix} 15 & 0 & -4 \\ -10 & 0 & -3 \\ -4 & 10 & 7 \end{vmatrix}$$

ΔI_2 = 15 (0 + 30) -0 - 4(-100 +0)

ΔI_2 = 15(30) - 4(-100)

ΔI_2 = 450 + 400 = 850

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{850}{527}$$

I_2 = 1.61Amps

Current through 10Ω Resistor = $I_1 - I_2$

= 1.94 – 1.61

= 0.33 Amps

Answer:

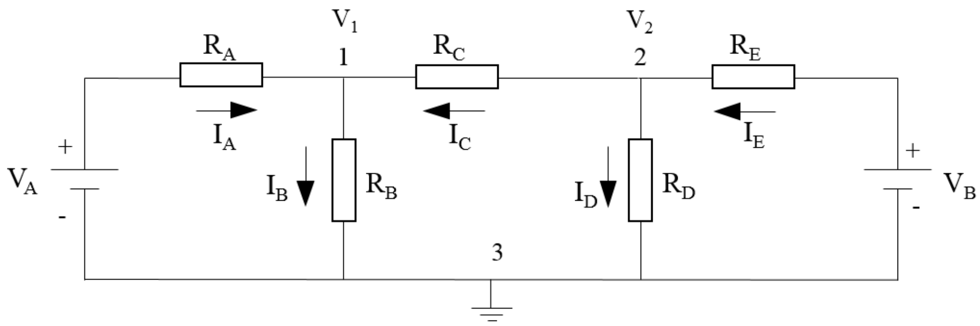
Current through 10Ω Resistor = 0.33 Amps

2.9 Nodal Equations :

This method of circuit solution, also known as the Node Voltage method, is based on the application of Kirchhoff 's Current Law at each junction (node) of the circuit, to find the node voltages.

Procedure:

i) Select a node as the reference node.
ii) Assign voltages V_1, V_2 .....V_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
iii) Apply KCL to each of the n-1 non-reference nodes. Use ohm's law to express the branch currents in terms of node voltages.
iv) Solve the resulting simulatnetous equations to obtain the unknown node voltage.



Let, V_1 and V_2 – Voltage of Node 1 and 2 with respect to Reference Node 3

Number of Node = 3

Independent Node = Node 1 and 2

By applying KCL at Node 1 (Junction of R_A,R_B & R_C):

Sum of Incoming Current = Sum of Outgoing Current

$$I_A + I_C = I_B$$

$$\frac{V_A - V_1}{R_A} + \frac{V_2 - V_1}{R_C} = \frac{V_1}{R_B}$$

$$\frac{V_A}{R_A} - \frac{V_1}{R_A} + \frac{V_2}{R_C} - \frac{V_1}{R_C} = \frac{V_1}{R_B}$$

$$-\frac{V_1}{R_A} + \frac{V_2}{R_C} - \frac{V_1}{R_C} - \frac{V_1}{R_B} = -\frac{V_A}{R_A}$$

$$\frac{V_1}{R_A} - \frac{V_2}{R_C} + \frac{V_1}{R_C} + \frac{V_1}{R_B} = \frac{V_A}{R_A}$$

$$V_1\left[\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right] - V_2\left[\frac{1}{R_C}\right] = \frac{V_A}{R_A} \qquad \text{----------- (1)}$$

By applying KCL at Node 2:(Junction of R_C,R_D & R_E):

Sum of Incoming Current = Sum of Outgoing Current

$$I_E = I_C + I_D$$

$$\frac{V_B - V_2}{R_E} = \frac{V_2 - V_1}{R_C} + \frac{V_2}{R_D}$$

$$\frac{V_B}{R_E} - \frac{V_2}{R_E} = \frac{V_2}{R_C} - \frac{V_1}{R_C} + \frac{V_2}{R_D}$$

$$-\frac{V_2}{R_E} - \frac{V_2}{R_C} + \frac{V_1}{R_C} - \frac{V_2}{R_D} = -\frac{V_B}{R_E}$$

$$\frac{V_2}{R_E} + \frac{V_2}{R_C} - \frac{V_1}{R_C} + \frac{V_2}{R_D} = \frac{V_B}{R_E}$$

$$-V_1\left[\frac{1}{R_C}\right] + V_2\left[\frac{1}{R_C} + \frac{1}{R_D} + \frac{1}{R_E}\right] = \frac{V_B}{R_E} \qquad \text{----------- (2)}$$

Equation (1) & (2) can be put in matric form as follows:

$$\begin{vmatrix} \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C} + \frac{1}{R_D} + \frac{1}{R_E} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} \frac{V_A}{R_A} \\ \frac{V_B}{R_E} \end{vmatrix}$$

The resistance matrix and its formation is shown below:

$$[G] = \begin{vmatrix} \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C} + \frac{1}{R_D} + \frac{1}{R_E} \end{vmatrix}$$

Where,

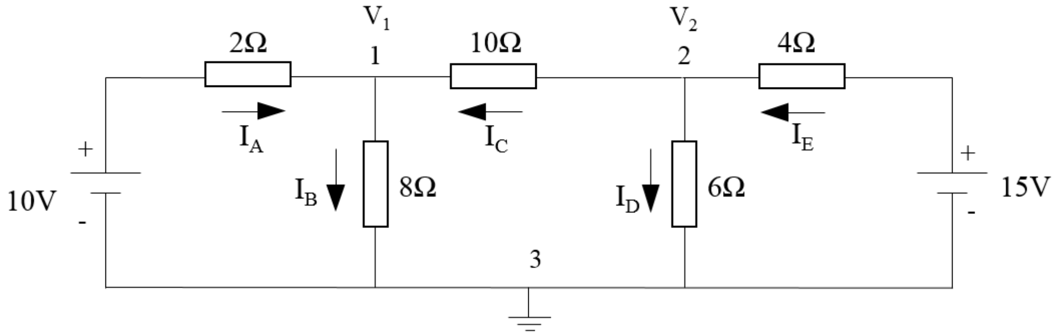
Diagonal elements (Top to bottom) = Conductance of resistors of Node 1 & 2

Other diagonal (Bottom left to top right) = Conductance of resistor common to Nodes 1 & 2

Note: *For nodal analysis, the number of equations required to solve a network is less than what we require in other methods.*

Example: 6

Find by nodal analysis, the current I_A and I_C in the circuit shown.



Given Data:

Number of Node = 3

Number of equation = 3-1 =2

To Find:

i) Current I_A =?

i) Current I_C =?

Solution:

$$\begin{vmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} \frac{V_A}{R_A} \\ \frac{V_B}{R_E} \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{2}+\frac{1}{8}+\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{6}+\frac{1}{4} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} \frac{10}{2} \\ \frac{15}{4} \end{vmatrix}$$

$$\begin{vmatrix} 0.5+0.125+0.1 & -0.1 \\ -0.1 & 0.1+0.167+0.25 \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} 5 \\ 3.75 \end{vmatrix}$$

$$\begin{vmatrix} 0.725 & -0.1 \\ -0.1 & 0.517 \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} 5 \\ 3.75 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 0.725 & -0.1 \\ -0.1 & 0.517 \end{vmatrix}$$

$$\Delta = (0.725 \text{ x } 0.517) - (0.1 \text{ x } 0.1)$$

$$\Delta = 0.375 - 0.01$$

$$\Delta = 0.365$$

$$\Delta V_1 = \begin{vmatrix} 5 & -0.1 \\ 3.75 & 0.517 \end{vmatrix}$$

$$\Delta V_1 = (5 \text{ x } 0.517) - (-0.1 \text{ x } 3.75)$$

$$\Delta V_1 = 2.585 + 0.375$$

$$\Delta V_1 = 2.96$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{2.96}{0.365} = 8.1\text{V}$$

$$\Delta V_2 = \begin{vmatrix} 0.725 & 5 \\ -0.1 & 3.75 \end{vmatrix}$$

$$\Delta V_2 = (0.725 \text{ x } 3.75) - (-0.1 \text{ x } 5)$$

$$\Delta V_2 = 2.719 + 0.5$$

$$\Delta V_2 = 3.219$$

$$V_2 = \frac{\Delta V_2}{\Delta} = \frac{3.219}{0.365} = 8.8\text{V}$$

$$I_A = \frac{V_A - V_1}{R_A} = \frac{10-8.1}{2} = 0.95\text{Amps}$$

$$I_B = \frac{V_1}{R_B} = \frac{8.1}{8} = 1.01 \text{ Amps}$$

$$I_C = \frac{V_2 - V_1}{R_C} = \frac{8.8-8.1}{10} = 0.07 \text{ Amps}$$

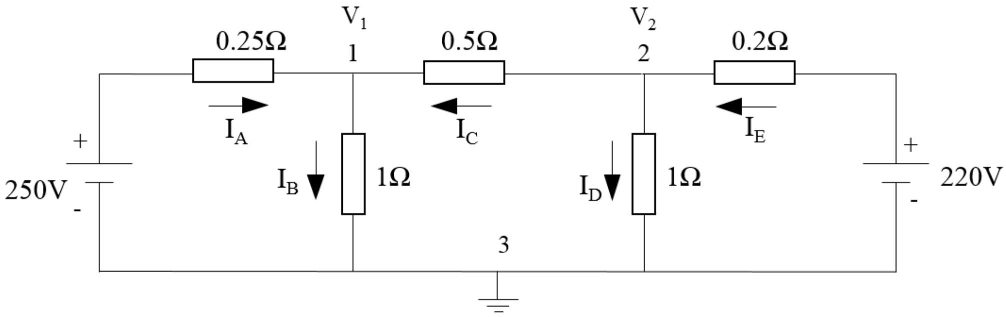
Answer:

Current through 8Ω Resistor = I_A = 0.95 Amps

Current through 6Ω Resistor = I_C = 0.07 Amps

Example: 7

Find by nodal analysis, the current I_A and I_C in the circuit shown.



Given Data:

Number of Node = 3

Number of equation = 3-1 =2

To Find:

i) Current I_A =?

i) Current I_C =?

Solution:

$$\begin{vmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} \frac{V_A}{R_A} \\ \frac{V_B}{R_E} \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{0.25}+\frac{1}{1}+\frac{1}{0.5} & -\frac{1}{0.5} \\ -\frac{1}{0.5} & \frac{1}{0.5}+\frac{1}{1}+\frac{1}{0.2} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} \frac{250}{0.25} \\ \frac{220}{0.2} \end{vmatrix}$$

$$\begin{vmatrix} 4+1+2 & -2 \\ -2 & 2+1+5 \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} 1000 \\ 1100 \end{vmatrix}$$

$$\begin{vmatrix} 7 & -2 \\ -2 & 8 \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} 1000 \\ 1100 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -2 \\ -2 & 8 \end{vmatrix}$$

$$\Delta = 56 - 4$$

$$\Delta = 52$$

$$\Delta V_1 = \begin{vmatrix} 1000 & -2 \\ 1100 & 8 \end{vmatrix}$$

$$\Delta V_1 = 8000 + 2200$$

$$\Delta V_1 = 10200$$

$$V_1 = \frac{\Delta V_1}{\Delta}$$

$$V_1 = \frac{10200}{52}$$

$$V_1 = = 196.15\text{V}$$

$$\Delta V_2 = \begin{vmatrix} 7 & 1000 \\ -2 & 1100 \end{vmatrix}$$

$$\Delta V_2 = 7700 + 2000$$
$$\Delta V_2 = 9700$$
$$V_2 = \frac{\Delta V_2}{\Delta}$$
$$V_2 = \frac{9700}{52}$$
$$V_2 = = 186.5\text{V}$$

$$I_A = \frac{V_A - V_1}{R_A}$$
$$I_A = \frac{250 - 196.15}{0.25} = 215.4 \text{ Amps}$$
$$I_B = \frac{V_1}{R_B}$$
$$I_B = \frac{196.15}{1} = 196.15 \text{ Amps}$$
$$I_C = \frac{V_2 - V_1}{R_C}$$
$$I_C = \frac{186.5 - 196.15}{0.5} = -19.3 \text{ Amps}$$

Answer:

Current through 1Ω Resistor = I_A = 215.4 Amps

Current through 0.5Ω Resistor = I_C = -19.3 Amps

2.10 Superposition Theorem:

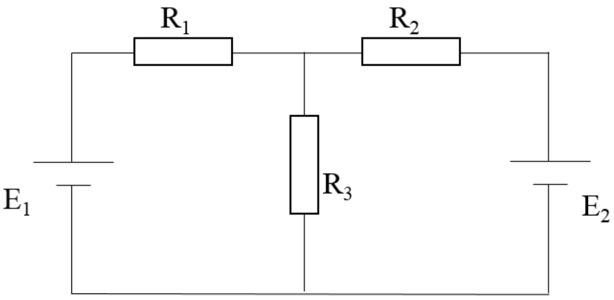
Some circuits require more than one voltage or current source. For example, most amplifiers operate with two voltage sources. When multiple sources are used in a circuit, the superposition theorem provides a methods for analysis. Hence the superposition theorem is a way to determine currents in a circuit with multiple sources, by considering one source at a time and replacing the other sources by their internal resistances.

Statement:

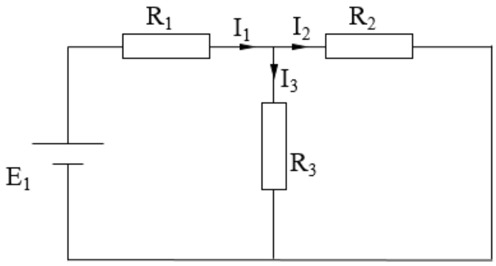
In a linear, bilateral network having more than one source, the current and voltage in any part of the network can be found by adding algebraically the effect of each source separately.

Procedure:

i) Consider only one source E_1 and replace the other source E_2 by its internal resistance. (If it is voltage source open circuit it and Current source short circuit it.)
ii) Determine the current (I_L) through load resistance.
iii) Then consider the other source E_2 and replace the source E_1 by its internal resistance.
iv) Determine the current (I_L') through load resistance.
v) Final load current is the algebraic sum of current supplied by E_1 and E_2 i.e I_L and I_L'.



Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\text{Equivalent Resistance} \quad R_{eq} \quad = \quad R_1 + \left(\frac{R_2 \text{ x } R_3}{R_2 + R_3}\right)$$

$$\text{Current} \quad I_1 \quad = \quad \frac{E_1}{R_{eq}}$$

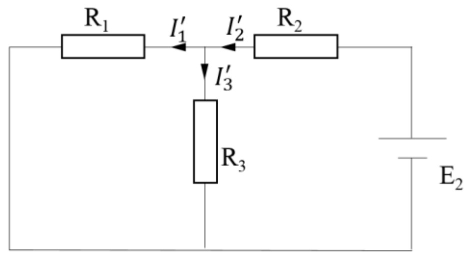
$$I_1 \quad \frac{E_1}{R_1 + \left(\frac{R_2 \text{ x } R_3}{R_2 + R_3}\right)}$$

When E_1 is Working:

Current through Load(R_3) $= I_3$

$$\text{So,} \quad I_3 \quad = \quad \frac{I_1 \text{ x } R_2}{R_2 + R_3}$$

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\text{Equivalent Resistance} \quad R'_{eq} \quad = \quad R_2 + \left(\frac{R_1 \text{ x } R_3}{R_1 + R_3}\right)$$

$$\text{Current} \quad I'_2 \quad = \quad \frac{E_2}{R'_{eq}}$$

$$I'_2 \quad \frac{E_1}{R_2 + \left(\frac{R_1 \text{ x } R_3}{R_1 + R_3}\right)}$$

When E_2 is Working:

Current through Load(R_3) $= I'_3$

$$\text{So, } I_3' = \frac{I_2' \times R_1}{R_1 + R_3}$$

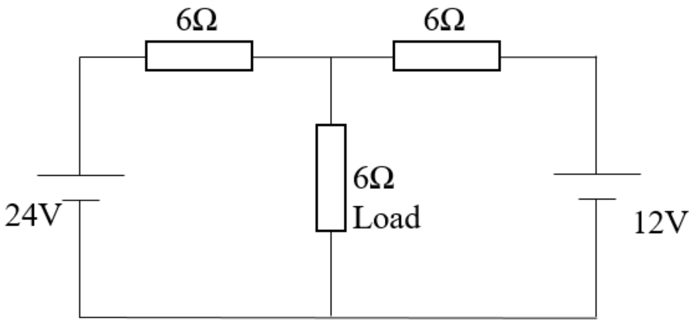
When E_1 and E_2 are working: Current through Load R_3 $=$ Current supplied by E_1 + Current supplied by E_2

$$= I_3 + I_3'$$

When E_1 and E_2 are working: Current through Load R_3 $= \frac{I_1 \times R_2}{R_2 + R_3} + \frac{I_2' \times R_1}{R_1 + R_3}$

Example: 8

Find the current through Load Resistor (6Ω) resistor in the following circuit by the principle of superposition theorem



Given Data:

Voltage source E_1 = 24V

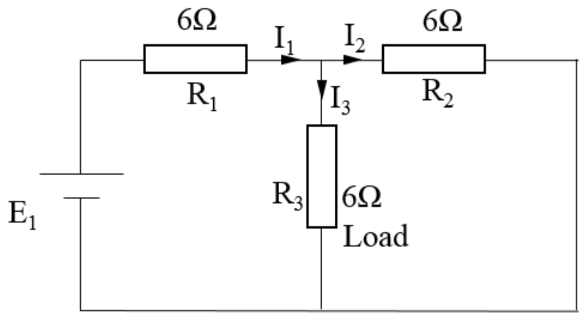
Voltage source E_2 = 12V

To Find:

i) Current through 6Ω Resistor=?

Solution:

Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\text{Equivalent Resistance } R_{eq} = R_1 + \left(\frac{R_2 \times R_3}{R_2 + R_3}\right)$$

$$\text{Equivalent Resistance } R_{eq} = 6 + \left(\frac{6 \times 6}{6 + 6}\right) = 6 + 3 = 9\Omega$$

$$\text{Current } I_1 = \frac{E_1}{R_{eq}}$$

$$\text{Current } I_1 = \frac{24}{9} = 2.67 \text{ Amps}$$

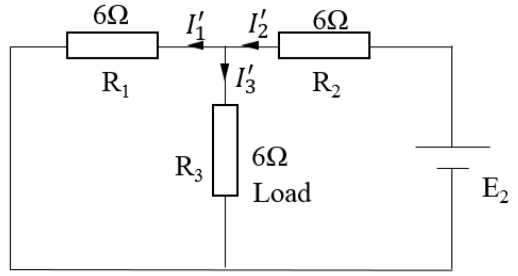
When E_1 is Working:

Current through Load(R_3) $= I_3$

$$\text{So,} \quad I_3 = \frac{I_1 \times R_2}{R_2 + R_3}$$

$$\text{So,} \quad I_3 = \frac{2.67 \times 6}{6 + 6} = 1.335 \text{ Amps}$$

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\text{Equivalent Resistance} \quad R'_{eq} = R_2 + \left(\frac{R_1 \times R_3}{R_1 + R_3}\right)$$

$$\text{Equivalent Resistance} \quad R'_{eq} = 6 + \left(\frac{6 \times 6}{6 + 6}\right) = 9\Omega$$

$$\text{Current} \quad I'_2 = \frac{E_2}{R'_{eq}}$$

$$\text{Current} \quad I'_2 = \frac{12}{9} = 1.33\text{Amps}$$

When E_2 is Working:

Current through Load(R_3) $= I'_3$

$$\text{So,} \quad I'_3 = \frac{I'_2 \times R_1}{R_1 + R_3}$$

$$\text{So,} \quad I'_3 = \frac{1.33 \times 6}{6 + 6} = 0.665\text{Amps}$$

When E_1 and E_2 are working:

Current through Load R_3 = Current supplied by E_1 + Current supplied by E_2

$= I_3 + I'_3$

$= 1.335 + 0.665$

When E_1 and E_2 are working:

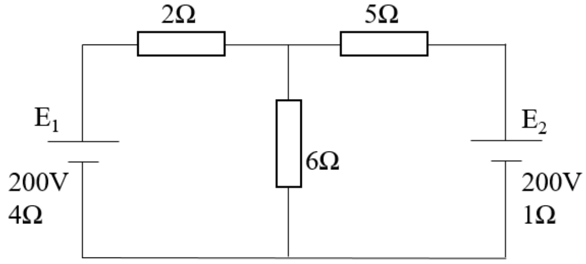
Current through Load R_3 = 2 Amps

<u>Answer:</u>

Current through 6Ω Load Resistor = 2 Amps

Example: 9

Find the current through 6Ω resistor in the following circuit by the principle of superposition theorem.



Given Data:

Voltage source E_1 = 200V

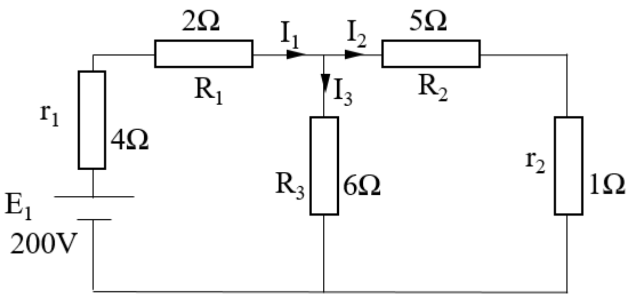
Voltage source E_2 = 200V

To Find:

i) Current through 6Ω Resistor=?

Solution: **[Method -1]**

Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\text{Equivalent Resistance } R_{eq} = (r_1 + R_1) + \left(\frac{(R_2 + r_2) \text{ x } R_3}{(R_2 + r_2) + R_3}\right)$$

$$\text{Equivalent Resistance } R_{eq} = (4 + 2) + \left(\frac{(5 + 1) \text{ x } 6}{(5 + 1) + 6}\right) = 6 + 3 = 9\Omega$$

$$\text{Equivalent Resistance } R_{eq} = 6 + \left(\frac{6 \text{ x } 6}{6 + 6}\right) = 6 + 3 = 9\Omega$$

$$\text{Current } I_1 = \frac{E_1}{R_{eq}}$$

$$\text{Current } I_1 = \frac{200}{9} = 22.22 \text{ Amps}$$

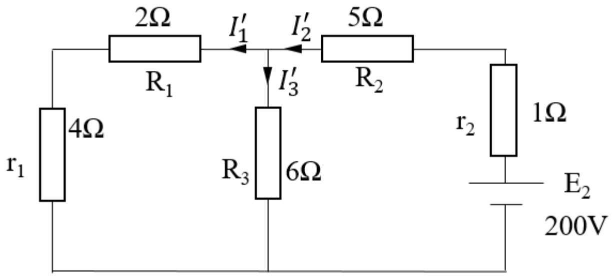
When E_1 is Working:

Current through Load(R_3) $= I_3$

$$\text{So, } I_3 = \frac{I_1 \text{ x } (R_2 + r_2)}{(R_2 + r_2) + R_3}$$

$$\text{So, } I_3 = \frac{22.22 \text{ x } 6}{6 + 6} = 11.11 \text{ Amps}$$

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\text{Equivalent Resistance } R'_{eq} = (r_2+R_2) + \left(\frac{(R_1+r_1) \times R_3}{(R_1+r_1) + R_3}\right)$$

$$\text{Equivalent Resistance } R'_{eq} = (1+5) + \left(\frac{(2+4) \times 6}{(2+4) + 6}\right)$$

$$6 + \left(\frac{6 \times 6}{6+6}\right) = 6 + 3 = 9\Omega$$

$$\text{Current } I_2' = \frac{E_2}{R'_{eq}}$$

$$\text{Current } I_2' = \frac{200}{9} = 22.22\text{Amps}$$

When E_2 is Working:
Current through Load(R_3) $= I_3'$

$$\text{So, } I_3' = \frac{I_2' \times (R_1+r_1)}{(R_1+r_1) + R_3}$$

$$\text{So, } I_3' = \frac{22.22 \times 6}{6+6} = 11.11 \text{ Amps}$$

When E_1 and E_2 are working:
Current through Load R_3 = Current supplied by E_1 + Current supplied by E_2

$= I_3 + I_3'$

$= 11.11 + 11.11$

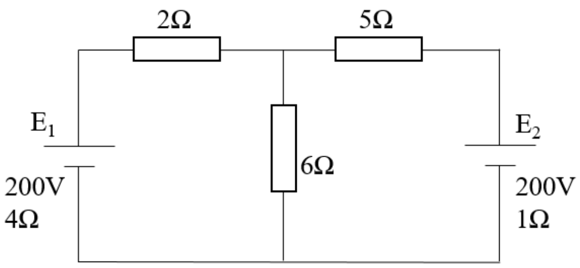
$= 22.22$ Amps

<u>Answer:</u>

Current through 6Ω Resistor = 22.22 Amps

Example: 10

Find the current through 6Ω resistor in the following circuit by the principle of superposition theorem.



<u>Given Data:</u>

Voltage source E_1 = 200V

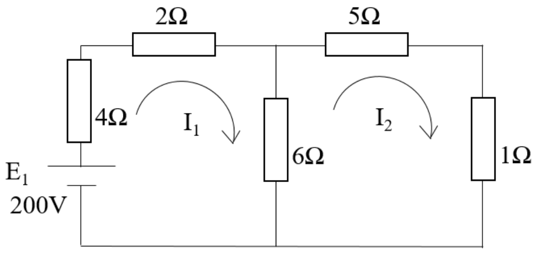
Voltage source E_2 = 200V

<u>To Find:</u>

i) Current through 6Ω Resistor=?

Solution: **[Method -2]**

Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\begin{vmatrix} 4+2+6 & -6 \\ -6 & 6+5+1 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \end{vmatrix}$$

$$\begin{vmatrix} 12 & -6 \\ -6 & 12 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 200 \\ 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 12 & -6 \\ -6 & 12 \end{vmatrix}$$

$$\Delta = 144 - 36$$

$$\Delta = 108$$

$$\Delta I_1 = \begin{vmatrix} 200 & -6 \\ 0 & 12 \end{vmatrix}$$

$$\Delta I_1 = 200 \text{ x } 12 = 2400$$

$$\Delta I_1 = 2400$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{2400}{108} = 22.22 \text{ Amps}$$

$$\Delta I_2 = \begin{vmatrix} 12 & 200 \\ -6 & 0 \end{vmatrix}$$

$$\Delta I_2 = 200 \text{ x } 6 = 1200$$

$$\Delta I_2 = 1200$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{1200}{108} = 11.11 \text{ Amps}$$

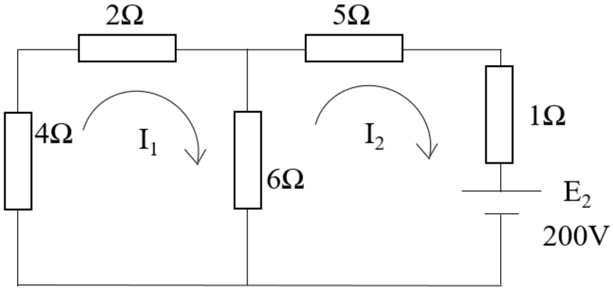
When E_1 is working:

Current through 6Ω Resistor $= I_1 - I_2$

$= 22.22 - 11.11$

$= 11.11$ Amps (downwards)

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\begin{vmatrix} 4+2+6 & -6 \\ -6 & 6+5+1 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \end{vmatrix}$$

$$\begin{vmatrix} 12 & -6 \\ -6 & 12 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 0 \\ -200 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 12 & -6 \\ -6 & 12 \end{vmatrix}$$

$$\Delta = 144 - 36$$

$$\Delta = 108$$

$$\Delta I_1 = \begin{vmatrix} 0 & -6 \\ -200 & 12 \end{vmatrix}$$

$$\Delta I_1 = -200 \times 6 = -1200$$

$$\Delta I_1 = -1200$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{-1200}{108} = -11.11 \text{ Amps}$$

$$\Delta I_2 = \begin{vmatrix} 12 & 0 \\ -6 & -200 \end{vmatrix}$$

$$\Delta I_2 = 12 \times -200 = -2400$$

$$\Delta I_2 = -2400$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{-2400}{108} = -22.22 \text{ Amps}$$

When E_2 is working:

Current through 6Ω Resistor $= I_1 - I_2$

$= -11.11 - (-22.22) = -11.11 + 22.22$

$= 11.11$ Amps (downwards)

When E_1 and E_2 are working:

Current through 6Ω Resistor $= I_1 + I_2$

$= 11.11 + 11.11$

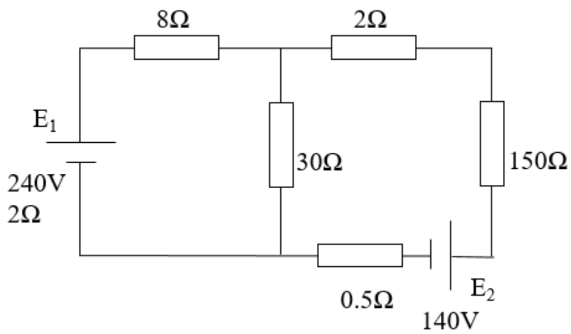
$= 22.22$ Amps

Answer:

Current through 6Ω Resistor $= 22.22$ Amps

Example: 11

Find the current in the 150Ω resistor and the power consumed in it by the by the principle of superposition theorem.



Given Data:

Voltage source E_1 = 240V

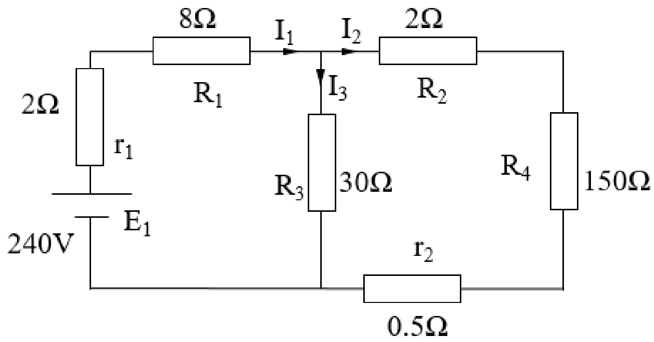
Voltage source E_2 = 140V

To Find:

i) Current through 150Ω Resistor=?

Solution: **[Method -1]**

Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\text{Equivalent Resistance } R_{eq} = (r_1+R_1) + \left(\frac{(R_2 + R_4+r_2) \times R_3}{(R_2 + R_4+r_2) + R_3}\right)$$

$$\text{Equivalent Resistance } R_{eq} = (2 + 8) + \left(\frac{(2 + 150 + 0.5) \times 30}{(2 + 150 + 0.5) + 30}\right)$$

$$\text{Equivalent Resistance } R_{eq} = 10 + \left(\frac{152.5 \times 30}{152.5 + 30}\right)$$

$$\text{Equivalent Resistance } R_{eq} = 35.07\Omega$$

$$\text{Current } I_1 = \frac{E_1}{R_{eq}}$$

$$\text{Current } I_1 = \frac{240}{35.07} = 6.84 \text{ Amps}$$

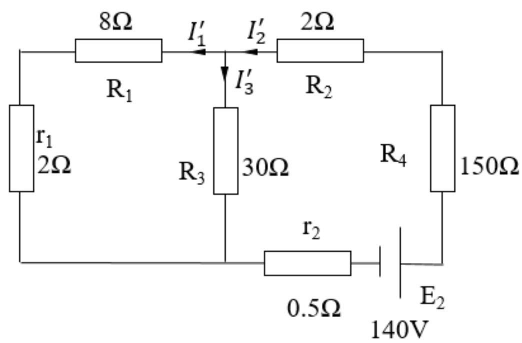
When E_1 is Working:

Current through 150Ω Resistor $= I_2$

$$\text{So, } I_2 = \frac{I_1 \times R_3}{R_3 + (R_2 + R_4+r_2)}$$

$$\text{So, } I_2 = \frac{6.84 \times 30}{30 + 152.5} = 1.125 \text{ Amps}$$

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\text{Equivalent Resistance } R'_{eq} = (r_2+R_4+R_2) + \left(\frac{(R_1+r_1) \times R_3}{(R_1+r_1) + R_3}\right)$$

$$\text{Equivalent Resistance } R'_{eq} = (0.5 + 150 + 2) + \left(\frac{(8 + 2) \times 30}{(8 + 2) + 30}\right)$$

$$\text{Equivalent Resistance } R'_{eq} = 152.5 + \left(\frac{10 \times 30}{10 + 30}\right)$$

$$\text{Equivalent Resistance } R'_{eq} = 152.5 + 7.5 = 160\Omega$$

$$\text{Current } I'_2 = \frac{E_2}{R'_{eq}}$$

$$\text{Current } I'_2 = \frac{140}{160} = 0.875\text{Amps}$$

When E_2 is Working:

Current through 150Ω Resistor $= I'_2$

So, $I'_2 = 0.875$ Amps

When E_1 and E_2 are working:

Current through 150Ω = Current supplied by E_1 + Current supplied by E_2

$= I_2 - I'_2$

$= 1.125 - 0.875 = 0.25$ Amps

Power consumed by 150Ω Resistor $= I^2 R$

$P = 0.25^2 \text{ x } 150 = 9.375$ Watts

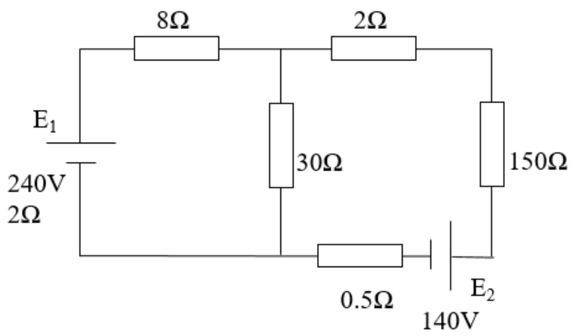
Answer:

Current through 150Ω Resistor = 0.25 Amps

Power consumed by 150Ω Resistor = 9.375 Watts

Example: 12

Find the current in the 150Ω resistor and the power consumed in it by the by the principle of superposition theorem.



Given Data:

Voltage source E_1 = 240V

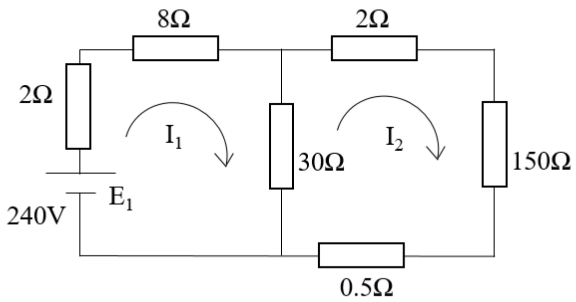
Voltage source E_2 = 140V

To Find:

i) Current through 150Ω Resistor =?

Solution: **[Method -2]**

Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\begin{vmatrix} 2+8+30 & -30 \\ -30 & 30+2+150+0.5 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \end{vmatrix}$$

$$\begin{vmatrix} 40 & -30 \\ -30 & 182.5 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 240 \\ 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 40 & -30 \\ -30 & 182.5 \end{vmatrix}$$

$$\Delta = (40 \text{ x } 182.5) - (-30 \text{ x } -30)$$

$$\Delta = 7300 - 900$$

$$\Delta = 6400$$

$$\Delta I_2 = \begin{vmatrix} 40 & 240 \\ -30 & 0 \end{vmatrix}$$

$$\Delta I_2 = 240 \text{ x } 30 = 7200$$

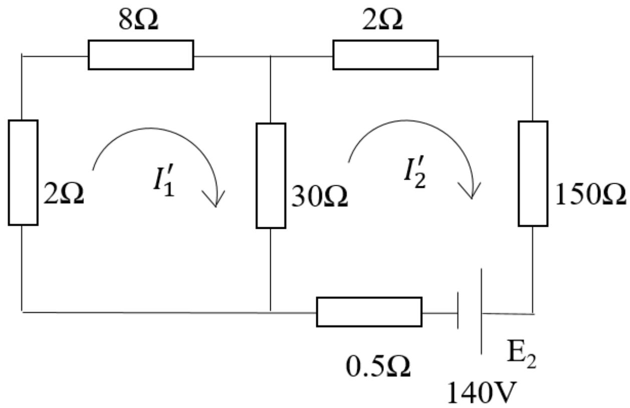
$$\Delta I_2 = 7200$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{7200}{6400} = 1.125 \text{ Amps}$$

When E_1 is working:

Current through 150Ω Resistor = I_2 = 1.125 Amps (downwards)

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\begin{vmatrix} 40 & -30 \\ -30 & 182.5 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \end{vmatrix}$$

$$\begin{vmatrix} 40 & -30 \\ -30 & 182.5 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 0 \\ -140 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 40 & -30 \\ -30 & 182.5 \end{vmatrix}$$

$$\Delta = (40 \text{ x } 182.5) - (-30 \text{ x } -30)$$

$$\Delta = 7300 - 900$$

$$\Delta = 6400$$

$$\Delta I_2' = \begin{vmatrix} 40 & 0 \\ -30 & -140 \end{vmatrix}$$

$$\Delta I_2' = 40 \text{ x } -140 = -5600$$

$$\Delta I_2' = -5600$$

$$I_2' = \frac{\Delta I_2'}{\Delta}$$

$$I_2' = \frac{-5600}{6400} = -0.875 \text{ Amps}$$

When E_2 is working:

Current through 150Ω Resistor $= I_2' = -0.875$ Amps

$= I_2 + I_2'$

$= 1.125 - 0.875$

$= 0.25$ Amps

Power consumed by 150Ω Resistor $= I^2 R$

$P = 0.25^2 \times 150 = 9.375$ Watts

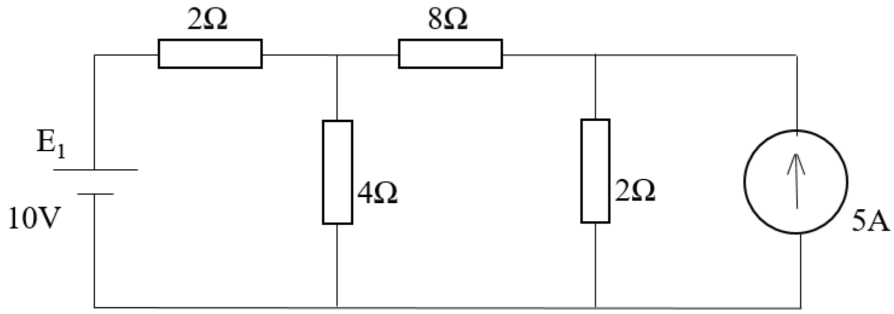
Answer:

Current through 150Ω Resistor = 0.25 Amps

Power consumed by 150Ω Resistor = 9.375 Watts

Example: 13

Find the current through 8Ω resistor and its direction for the network shown below.



Given Data:

Voltage source E_1 = 10V

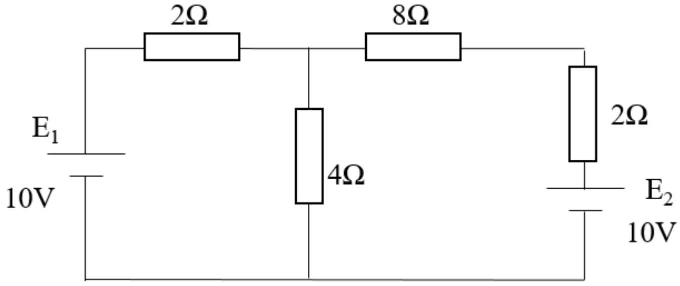
Current source I = 5A

To Find:

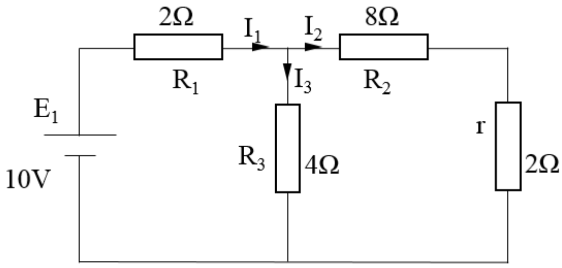
i) Current through 8Ω Resistor =?

Solution: **[Method -1]**

Current source is converted into voltage source as follows.



Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\text{Equivalent Resistance} \quad R_{eq} = R_1 + \left(\frac{(R_2 + r) \text{ x } R_3}{(R_2 + r) + R_3}\right)$$

$$\text{Equivalent Resistance} \quad R_{eq} = 2 + \left(\frac{(8 + 2) \text{ x } 4}{(8 + 2) + 4}\right)$$

$$\text{Equivalent Resistance} \quad R_{eq} = 2 + \left(\frac{10 \text{ x } 4}{10 + 4}\right) = 2 + 2.86 = 4.86\Omega$$

$$\text{Current} \quad I_1 = \frac{E_1}{R_{eq}}$$

$$\text{Current} \quad I_1 = \frac{10}{4.86} = 2.06 \text{ Amps}$$

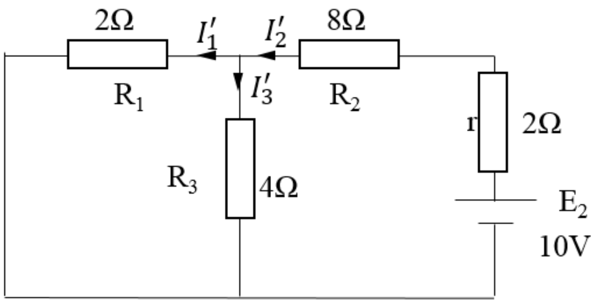
When E_1 is Working:

$$\text{Current through } 8\Omega \text{ Resistor} = I_2$$

$$\text{So,} \quad I_2 = \frac{I_1 \text{ x } R_3}{R_3 + (R_2 + r_2)}$$

$$So, \quad I_2 = \frac{2.06 \, x \, 4}{4 + 10} = 0.59 \, Amps$$

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\text{Equivalent Resistance} \quad R'_{eq} = (r + R_2) + \left(\frac{R_1 \text{ x } R_3}{R_1 + R_3}\right)$$

$$\text{Equivalent Resistance} \quad R'_{eq} = (2 + 8) + \left(\frac{2 \text{ x } 4}{2 + 4}\right)$$

$$10 + \left(\frac{8}{6}\right) = 10 + 1.33 = 11.33\Omega$$

$$\text{Current} \quad I'_2 = \frac{E_2}{R'_{eq}}$$

$$\text{Current} \quad I'_2 = \frac{10}{11.33} = 0.88\text{Amps}$$

When E_2 is Working:

$$\text{Current through Load}(R_2) = I'_2$$

$$\text{So,} \quad I'_2 = 0.88 \text{ Amps}$$

When E_1 and E_2 are working:

$$
\begin{aligned}
\text{Current through } 8\Omega \text{ Resistor} &= \text{Current supplied by } E_1 + E_2 \\
&= I_2 - I_2' \\
&= 0.59 - 0.88 \\
&= -0.29 \, Amps
\end{aligned}
$$

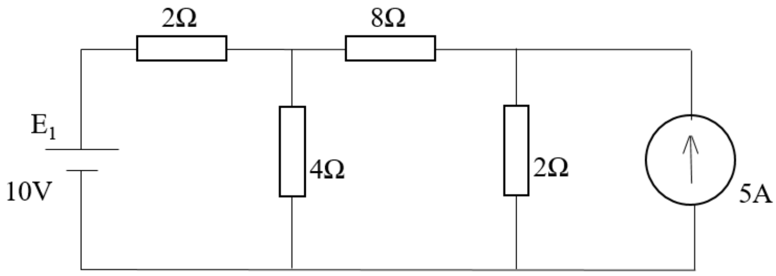
Answer:

Current through 8Ω Resistor = -0.29 Amps

Direction of current through 8Ω = Right to Left

Example: 14

Find the current through 8Ω resistor and its direction for the network shown below.



Given Data:

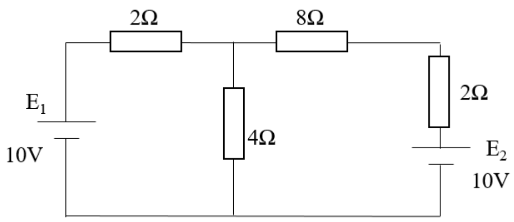
Voltage source E_1 = 10V

Voltage source E_2 = 10V

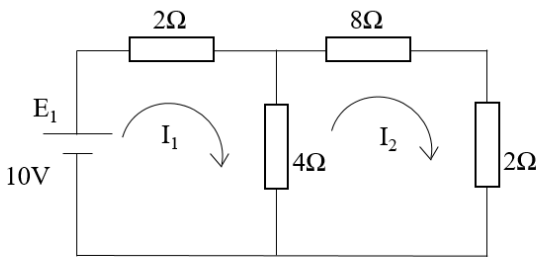
To Find:

i) Current through 8Ω Resistor =?

Solution: **[Method -2]**



Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\begin{vmatrix} 2+4 & -4 \\ -4 & 4+8+2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \end{vmatrix}$$

$$\begin{vmatrix} 6 & -4 \\ -4 & 14 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 10 \\ 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -4 \\ -4 & 14 \end{vmatrix}$$

$$\Delta = 84 - 16 = 68$$

$$\Delta I_2 = \begin{vmatrix} 6 & 10 \\ -4 & 0 \end{vmatrix}$$

$$\Delta I_2 = 40$$

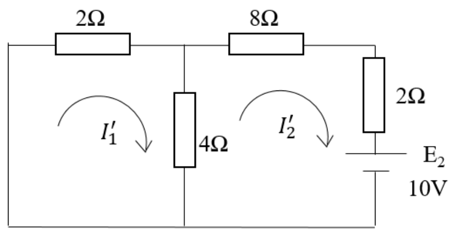
$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{40}{68} = 0.59\ Amps$$

When E_1 is working:

Current through 8Ω Resistor = I_2

= 0.59 Amps

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\begin{vmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \end{vmatrix}$$

$$\begin{vmatrix} 6 & -4 \\ -4 & 14 \end{vmatrix} \begin{vmatrix} I_1' \\ I_2' \end{vmatrix} = \begin{vmatrix} 0 \\ -10 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -4 \\ -4 & 14 \end{vmatrix}$$

$$\Delta = 84 - 16$$

$$\Delta = 68$$

$$\Delta I_2' = \begin{vmatrix} 6 & 0 \\ -4 & -10 \end{vmatrix}$$

$$\Delta I_2' = -60$$

$$I_2' = \frac{\Delta I_2'}{\Delta} = \frac{-60}{68} = -0.88\text{Amps}$$

When E_2 is working:

Current through 8Ω Resistor = $I_2' = -0.88$ Amps

= $I_1 + I_2$

= 0.59 + (-0.88)

= 0.59 - 0.88

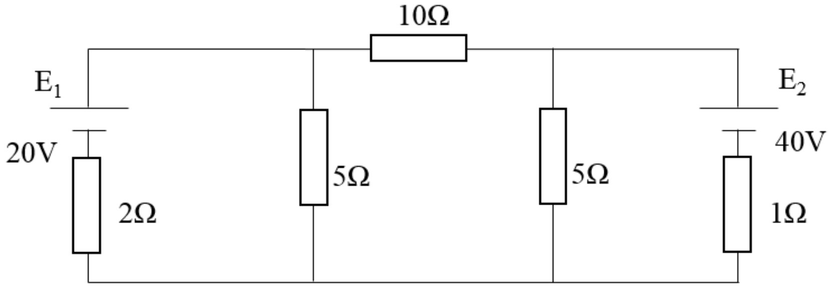
= -0.29 Amps

Answer:

Current through 8Ω Resistor = -0.29 Amps (Direction: Right to Left)

Example: 15

Find the current through 10Ω resistor using superposition theorem.



<u>Given Data:</u>

Voltage source E_1 = 20V

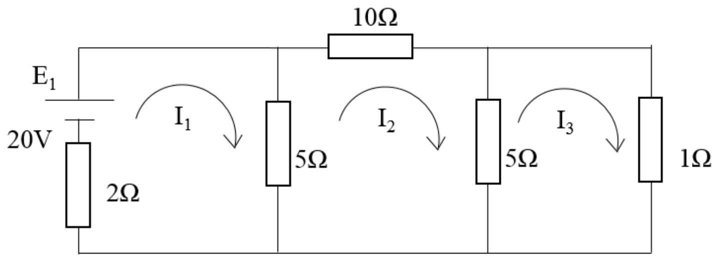
Voltage source E_2 = 40V

<u>To Find:</u>

i) Current through 10Ω Resistor =?

<u>Solution:</u>

Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \\ E_3 \end{vmatrix}$$

$$\begin{vmatrix} 7 & -5 & 0 \\ -5 & 20 & -5 \\ 0 & -5 & 6 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 20 \\ 0 \\ 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -5 & 0 \\ -5 & 20 & -5 \\ 0 & -5 & 6 \end{vmatrix}$$

$$\Delta = 7\,[20 \times 6 - (-5 \times -5)] - (-5)\,[(-5 \times 6) - 0] + 0$$

$$\Delta = 7[120 - 25] + 5\,[-30]$$

$$\Delta = 7\,[95] - 150 = 665 - 150$$

$$\Delta = 515$$

$$\Delta I_2 = \begin{vmatrix} 7 & 20 & 0 \\ -5 & 0 & -5 \\ 0 & 0 & 6 \end{vmatrix}$$

$$\Delta I_2 = -20(-30) = 600$$

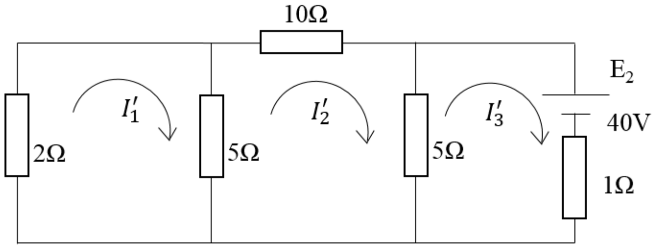
$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{600}{515} = 1.16\ Amps$$

When E_1 is working:

Current through 10Ω Resistor $= I_2$

$= 1.16$ Amps

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\begin{vmatrix} 7 & -5 & 0 \\ -5 & 20 & -5 \\ 0 & -5 & 6 \end{vmatrix} \begin{vmatrix} I'_1 \\ I'_2 \\ I'_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -40 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -5 & 0 \\ -5 & 20 & -5 \\ 0 & -5 & 6 \end{vmatrix}$$

$$\Delta = 895$$

$$\Delta I'_2 = \begin{vmatrix} 7 & 0 & 0 \\ -5 & 0 & -5 \\ 0 & -40 & 6 \end{vmatrix}$$

$$\Delta I'_2 = 7\,(0 - 200) = 7(-200)$$

$$\Delta I'_2 = -1400$$

$$I'_2 = \frac{\Delta I'_2}{\Delta} = \frac{-1400}{515} = -2.72\text{Amps}$$

When E_2 is working:
Current through 10Ω Resistor $= I'_2$

$= -2.72$ Amps

When E_1 and E_2 are working:
Current through 10Ω Resistor $= I_1 + I_2$

$= 1.16 + (-2.72)$

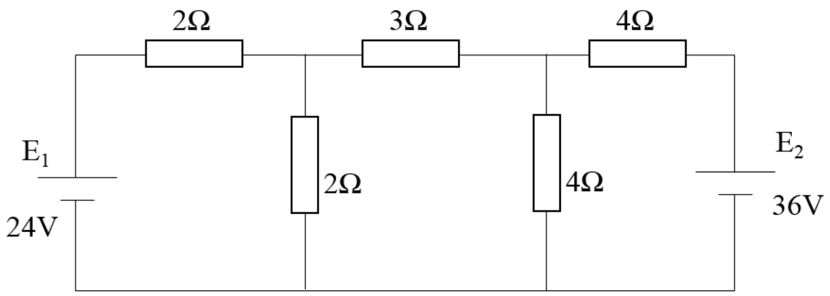
$= 0.67 - 2.72$

$= -1.55$ Amps

<u>Answer:</u>
Current through 10Ω Resistor $= -1.55$ Amps

Example: 16

Find the voltage drop in 3Ω resistor by applying superposition theorem.



<u>Given Data:</u>

Voltage source E_1 = 24V

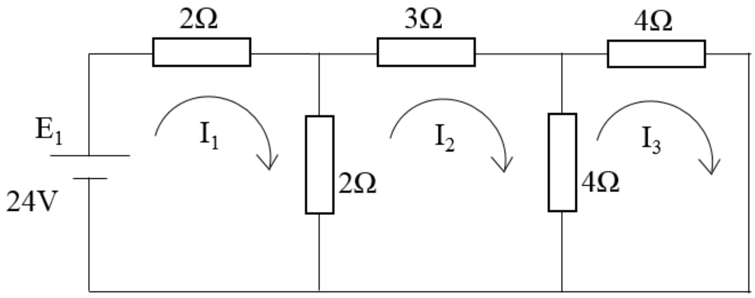
Voltage source E_2 = 36V

<u>To Find:</u>

i) Current through 3Ω Resistor =?

Solution:

Step 1: Voltage Source E_1 is Working and E_2 is replaced by its internal resistance.



$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} E_1 \\ E_2 \\ E_3 \end{vmatrix}$$

$$\begin{vmatrix} 4 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 24 \\ 0 \\ 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 4 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix}$$

$$\Delta = 4\,[9 \times 8 - (-4 \times -4)] - (-2)\,[(-2 \times 8) - 0] + 0$$

$$\Delta = 4[72 - 16] + 2\,[-16]$$

$$\Delta = 4\,[56] - 32 \quad = 224 - 32$$

$$\Delta = 192$$

$$\Delta I_2 = \begin{vmatrix} 4 & 24 & 0 \\ -2 & 0 & -4 \\ 0 & 0 & 8 \end{vmatrix}$$

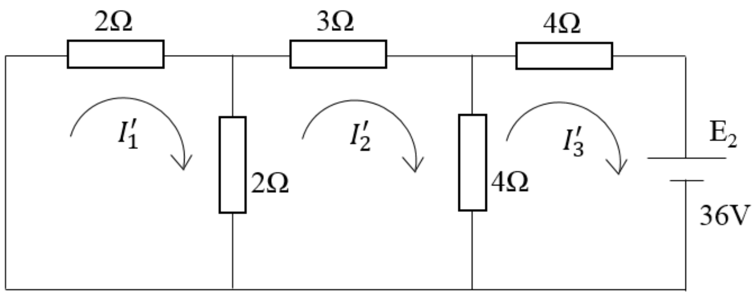
$$\Delta I_2 = -24(-16) = 384$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{384}{192} = 2 \text{ Amps}$$

When E_1 is working:

Current through 3Ω Resistor $= I_2 = 2$ Amps

Step 2: Voltage Source E_2 is Working and E_1 is replaced by its internal resistance.



$$\begin{vmatrix} 4 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix} \begin{vmatrix} I'_1 \\ I'_2 \\ I'_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -36 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 4 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix}$$

$$\Delta = 192$$

$$\Delta I_2' = \begin{vmatrix} 4 & 0 & 0 \\ -2 & 0 & -4 \\ 0 & -36 & 8 \end{vmatrix}$$

$$\Delta I_2' = 4\,(0 - 144) = 4(-144)$$

$$\Delta I_2' = -576$$

$$I_2' = \frac{\Delta I_2'}{\Delta} = \frac{-576}{192} = -3\text{Amps}$$

When E_2 is working:

Current through 10Ω Resistor $= I_2'$

$= -3$ Amps

When E_1 and E_2 are working:

Current through 10Ω Resistor $= I_1 + I_2$

$= 2 + (-3)$

$= 2 - 3$

$= -1$ Amps

Voltage drop across 3Ω Resistor = Current through 3Ω resistor x 3

$= -1 \times 3 = -3$Volts

<u>Answer:</u>

Current through 3Ω Resistor $= -1$ Amps

Voltage drop across 3Ω Resistor $= -3$ Volts

2.11 Thevenin's theorem:

Thevenin's theorem provides a method for simplifying a circuit to a standard equivalent form. This theorem can be used to simplify the analysis of complex circuits.

Statement:

Any linear bilateral network may be reduced to a simplified two-terminal circuit consisting of a single voltage source (V_{th}) in series with a single resistor (R_{th})

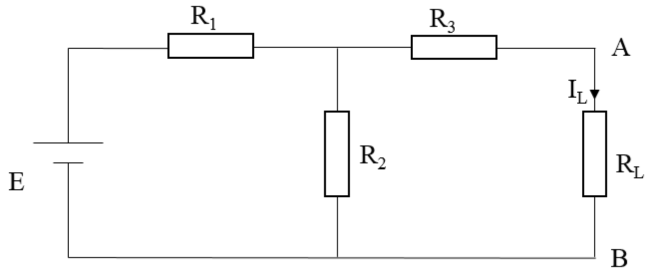
Procedure:

i) Remove the load resistance RL and put an open circuit across terminals.
ii) Find the voltage across open circuited terminals (V_{th} or V_{oc})
iii) Find the equivalent resistance R_{th} as seen from open circuited terminals by replacing the voltage source by its internal resistance.
iv) Draw the thevenin's equivalent circuit by a voltage V_{th} in series with the equivalent resistance R_{th}

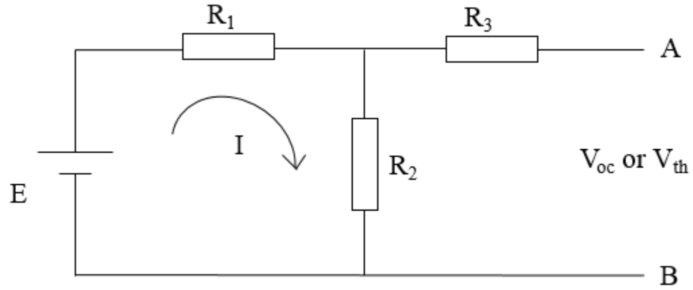
v) Find the current through R_L by applying ohms law.

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Explanation:



Step 1: Open Circuit the Load Resistor (R_L)



Step 2: Find Open Circuit Voltage (V_{th})

V_{th} = P.D across terminals A

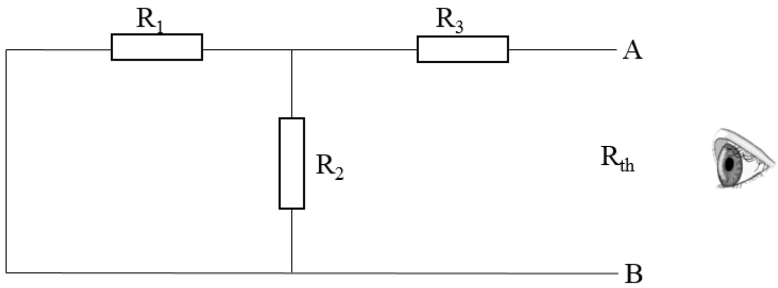
P.D across terminals A and B = P.D across R_2 Resistor

V_{th} = I x R_2

$$I = \frac{E}{(R_1 + R_2)}$$

$$V_{th} = \frac{E.R_2}{(R_1 + R_2)}$$

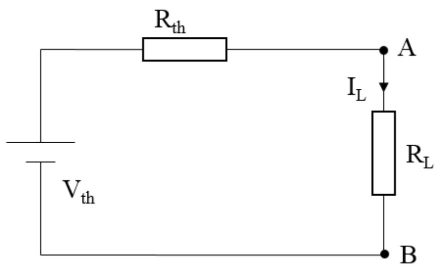
Step 3: Find Thevenin's equivalent Resistance: R_{th}



$$R_{12} = \frac{R_1 \text{ x } R_2}{R_1 + R_2}$$

$$R_{th} = R_{12} + R_3$$

Step 4: Draw Thevenin's equivalent circuit

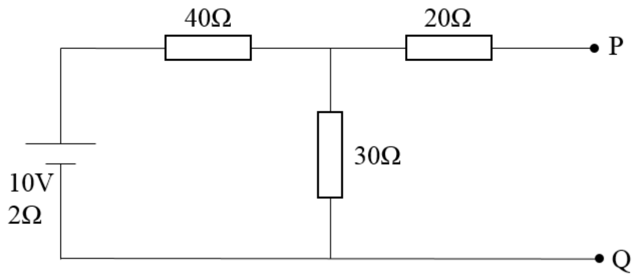


Step 5: Find the Load Current (I_L):

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Example: 17

Obtain the Thevenin's equivalent circuit at terminals PQ of the following active network.



<u>Given Data:</u>

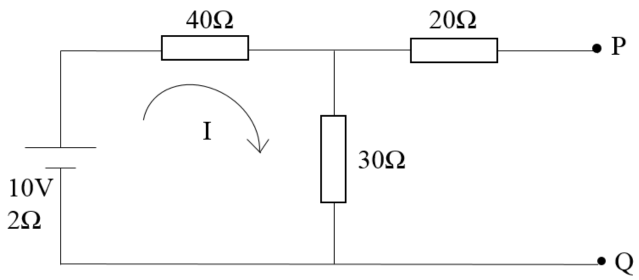
Supply Voltage = 10V

<u>To Find:</u>

i) Draw thevenin's equivalent circuit

<u>Solution:</u>

Step 1: Find Open Circuit Voltage (V_{th})



V_{th} = P.D across terminals P and Q (or)

P.D across terminals P and Q = P.D across 30 Ω Resistor

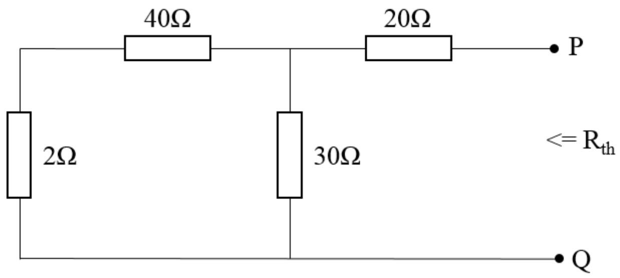
$V_{th} = I \times 30$

$$I = \frac{10}{(40+30+2)} = \frac{10}{72} = 0.14 Amps$$

$V_{th} = 0.14 \times 30$

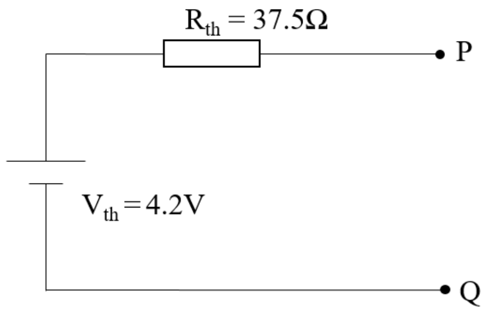
$V_{th} = 4.2$ Volts

Step 2: Find R_{th}



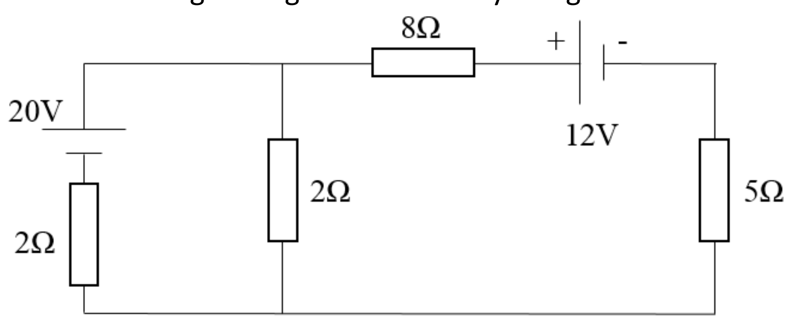
$$R_{12} = R_1 + R_2$$
$$R_{12} = 2 + 40 = 42\ \Omega$$
$$R_{123} = \frac{R_{12} \text{ x } R_3}{R_{12} + R_3} = \frac{42 \text{ x } 30}{42 + 30} = \frac{1260}{72}$$
$$R_{123} = 17.5\ \Omega$$
$$R_{th} = R_{123} + R_4$$
$$R_{th} = 17.5 + 20$$
$$R_{th} = 37.5\ \Omega$$

Step 3: Draw thevenin's equivalent circuit



Example: 18

Determine the current flowing through 5Ω resistor by using Thevenin's theorem.



<u>Given Data:</u>

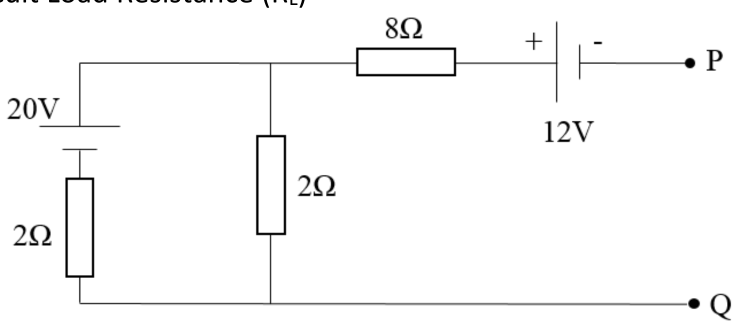
Load Resistance = 5 Ω

<u>To Find:</u>

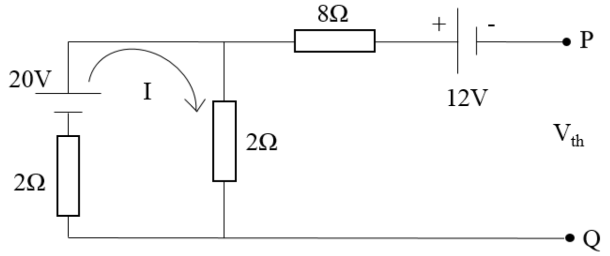
i) Current through 5Ω Resistor = ?

<u>Solution:</u>

Step 1: Open Circuit Load Resistance (R_L)



Step 1: Find Open Circuit Voltage (V_{th})



$$V_{th} = \text{P.D across terminals P and Q} \quad \text{(or)}$$

$$\text{P.D across terminals P and Q} = \text{P.D across } 2\ \Omega \text{ Resistor } -12V$$

$$V_{th} = I \times 30$$

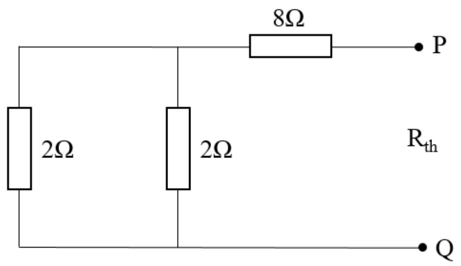
$$I = \frac{20}{(2+2)} = \frac{20}{4} = 5\text{Amps}$$

$$V_{th} = (5 \times 2) - 12$$

$$V_{th} = 10 - 12$$

$$V_{th} = -2V$$

Step 2: Find R_{th}



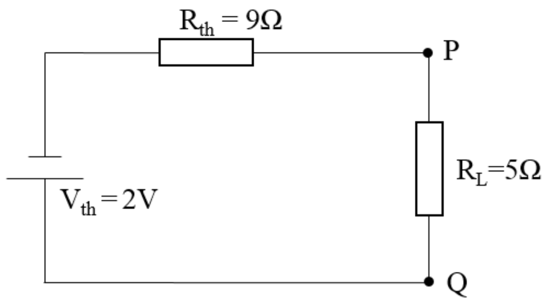
$$R_{th} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{2 \times 2}{2+2} = \frac{4}{4}$$

$$R_{12} = 1\ \Omega$$

$$R_{th} = R_{12} + R_3 = 1 + 8 = 9\ \Omega$$

$$R_{th} = 9\ \Omega$$

Step 3: Draw thevenin's equivalent circuit



Step 3: Find the Load Current (I_L):

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$I_L = \frac{-2}{9+5} = \frac{-2}{14}$$

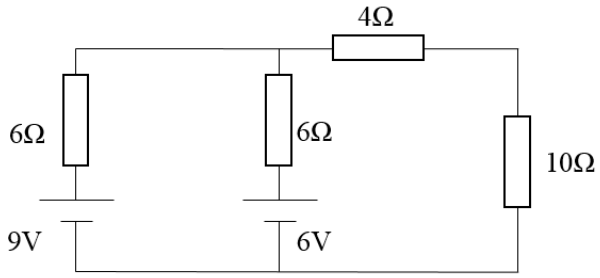
$$I_L = -0.143 \text{ Amps}$$

<u>Answer:</u>

$$\text{Current through } 5\Omega \text{ Resistor} = -0.143 \text{ Amps}$$

Example: 19

Determine the current flowing through 10Ω resistor by using Thevenin's theorem.



Given Data:

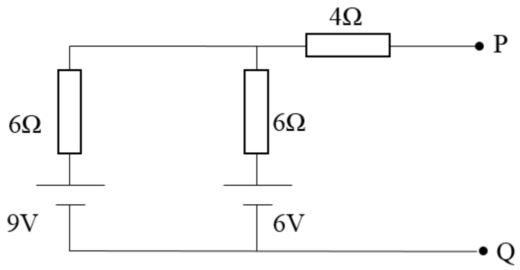
Load Resistance = 10Ω

To Find:

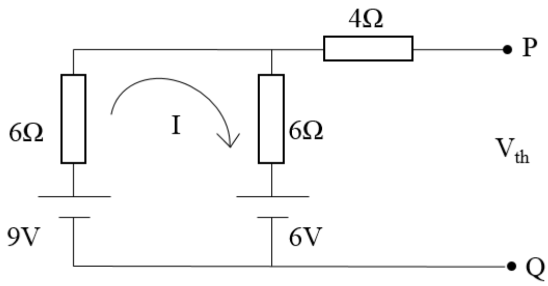
i) Current through 10Ω Resistor = ?

Solution:

Step 1: Open Circuit Load



Step 1: Find Open Circuit



V_{th} = 9V - P.D across 6Ω (or)6V + P.D across 6Ω

P.D across 6Ω Resistor = I x 6

$$I = \frac{9-6}{(6+6)} = \frac{3}{12} = 0.25\,Amps$$

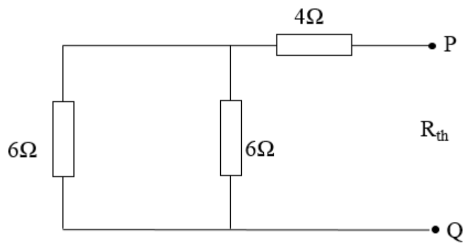
P.D across 6Ω Resistor = 0.25 x 6 = 1.5V

V_{th} = 9 – 1.5 = 7.5V (or)

V_{th} = 6 + 1.5 = 7.5V

V_{th} = 7.5V

Step 2: Find R_{th}



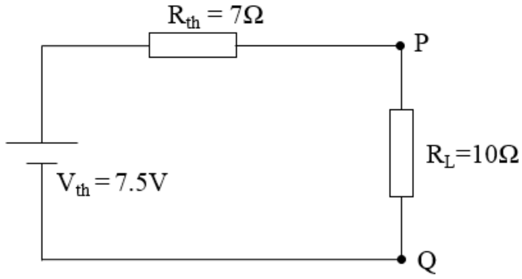
$$R_{th} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{6 \times 6}{6+6} = \frac{36}{12}$$

R_{12} = 3 Ω

$R_{th} = R_{12} + R_3 = 3 + 4 = 7\ \Omega$

R_{th} = 7 Ω

Step 3: Draw thevenin's equivalent circuit



Step 3: Find the Load Current (I_L):

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$I_L = \frac{7.5}{7+10} = \frac{7.5}{17}$$

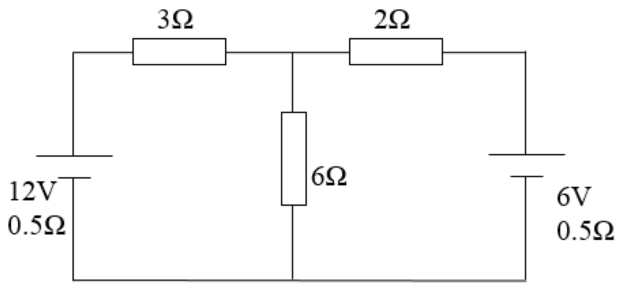
$$I_L = 0.44 \text{ Amps}$$

<u>Answer:</u>

Current through 10Ω Resistor = 0.44 Amps

Example: 20

Determine the current flowing through 6Ω resistor by using Thevenin's theorem.



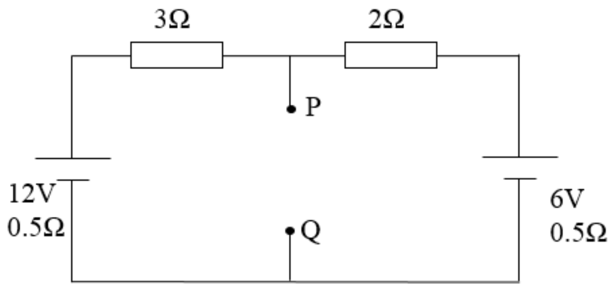
<u>Given Data:</u>

<u>To Find:</u>

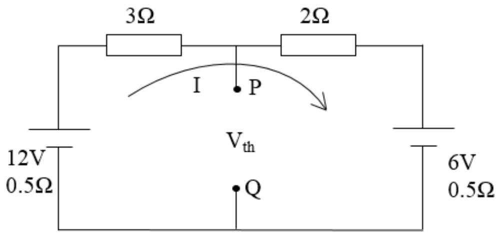
i) Current through 6Ω Resistor = ?

<u>Solution:</u>

Step 1: Open Circuit Load Resistance (R_L)

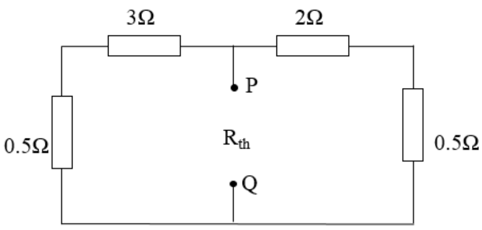


Step 1: Find Open Circuit Voltage (V_{th})



$$V_{th} = 12V - \text{P.D across } 3.5\Omega \text{ Resistor} \quad \text{(or)}$$
$$= 6V + \text{P.D across } 2.5\Omega \text{ Resistor}$$
$$\text{P.D across } 3\Omega \text{ Resistor} = I \times 3.5$$
$$I = \frac{12-6}{(3+2+0.5+0.5)} = \frac{6}{6} = 1\text{Amps}$$
$$\text{P.D across } 3\Omega \text{ Resistor} = 1 \times 3.5 = 3.5V$$
$$V_{th} = 12 - 3.5 = 8.5V \quad \text{(or)}$$
$$V_{th} = 6 + 2.5 = 8.5V$$
$$V_{th} = 8.5V$$

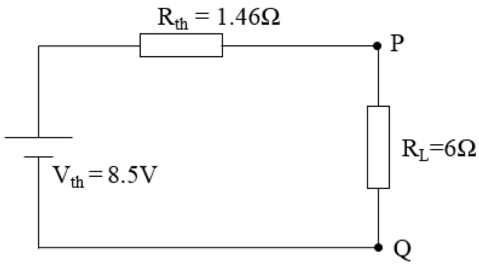
Step 2: Find R_{th}



$$R_{th} = \frac{3.5 \times 2.5}{3.5 + 2.5} = \frac{8.75}{6}$$

$$R_{th} = 1.46\ \Omega$$

Step 3: Draw thevenin's equivalent circuit



Step 3: Find the Load Current (I_L):

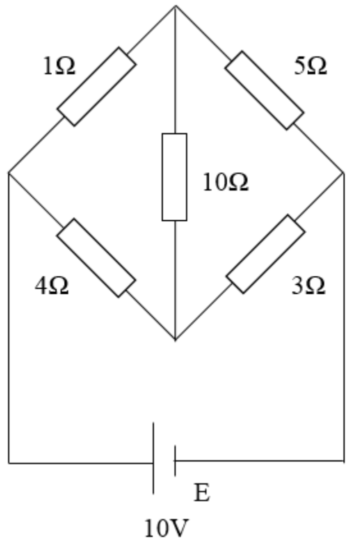
$$I_L = \frac{V_{th}}{R_{th} + R_L}$$
$$I_L = \frac{8.5}{1.46 + 6} = \frac{8.5}{7.46}$$
$$I_L = 1.14 \text{ Amps}$$

<u>Answer:</u>

Current through 6Ω Resistor = 1.14 Amps

Example: 21

Find the current through 10Ω load resistor using Thevenin's Theorem.



Given Data:

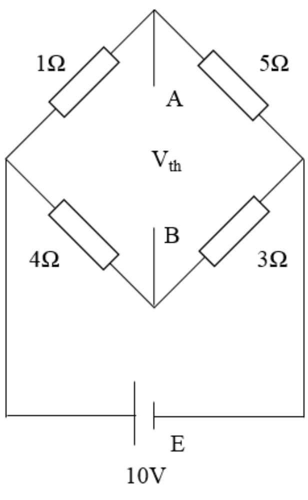
Supply Voltage = 10V

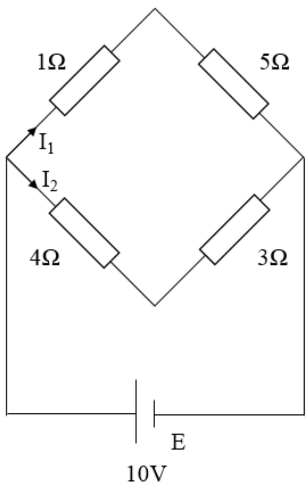
To Find:

i) Current through 10Ω Resistor = ?

Solution:

Step 1: Open Circuit Load Resistance (R_L)





Step 1: Find Open Circuit Voltage (V_{th})

V_{th} = Voltage at point A – Voltage at point B

$V_{th} = V_A - V_B$

V_A = Supply Voltage – P.D across 1Ω Resistor

P.D across 1Ω Resistor $= I_1 \times 1$

$$I_1 = \frac{10}{1+5} = \frac{10}{6} = 1.67 Amps$$

P.D across 1Ω Resistor $= 1.67 \times 1 = 1.67V$

V_B = Supply Voltage – P.D across 1Ω Resistor

P.D across 4Ω Resistor $=$ $I_2 \times 4$

$$I_2 = \frac{10}{4+3} = \frac{10}{7} = 1.43\ Amps$$

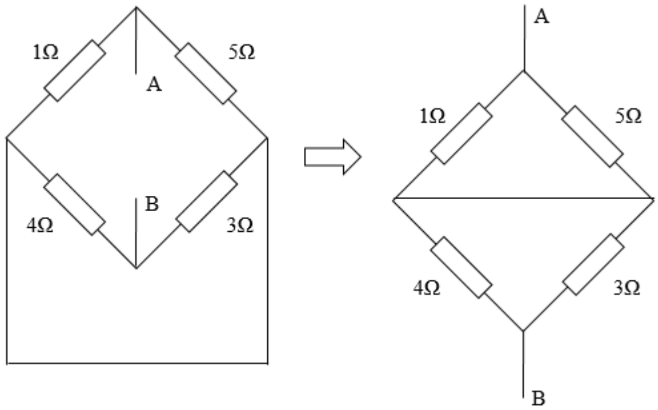
P.D across 4Ω Resistor $=$ $1.43 \times 4 = 5.72V$

$$V_{th} = V_A - V_B$$

$$V_{th} = 1.67 - 5.72 \quad = -4.05V$$

$$V_{th} = -4.05V$$

Step 3: Find Thevenin's equivalent Resistance R_{th}



Step 2: Find R_{th}

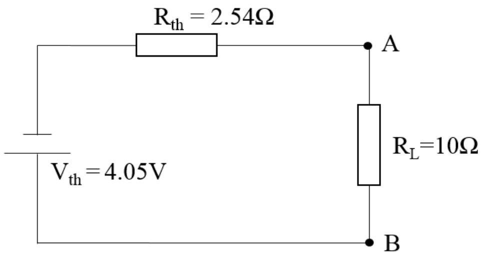
$$R_{th} = \left(\frac{1 \times 5}{1+\ 5}\right) + \left(\frac{4 \times 3}{4+\ 3}\right)$$

$$R_{th} = \left(\frac{5}{6}\right) + \left(\frac{12}{7}\right)$$

$$R_{th} = 0.833 + 1.71$$

$$R_{th} = 2.54\ \Omega$$

Step 4: Draw Thevenin's equivalent circuit



Step 3: Find the Load Current (I_L):

$$I_L = \frac{V_{th}}{R_{th} +\ R_L}$$

$$I_L = \frac{4.05}{2.54+10} = \frac{4.05}{12.54}$$

$$I_L = 0.32\ \text{Amps}$$

<u>Answer:</u>

Current through 10Ω Resistor = 0.32 Amps

2.12 Norton's Theorem:

Norton's theorem is a method for simplifying a two terminal linear circuit to an equivalent circuit with only a current source in parallel with a resistor. This basic difference between Thevenin's Theorem and Norton's theorem is that, Norton's theorem results in an equivalent source in parallel with an equivalent resistance. The equivalent current source is designated I_N, and the equivalent resistance is designated R_N. To apply Norton's theorem, you must know how to find the two quantities I_N and R_N.

Statement:

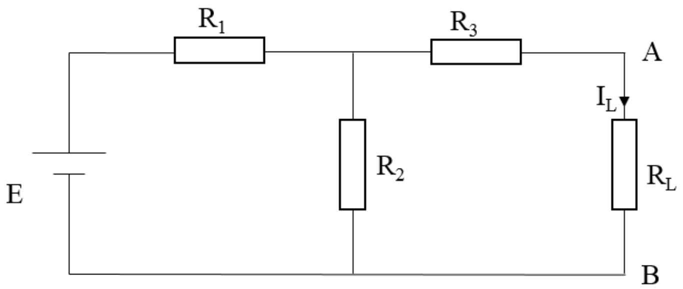
Norton's theorem states that any two terminal network can be replaced by a single current source I_N in parallel with a single resistance R_N.

Procedure:

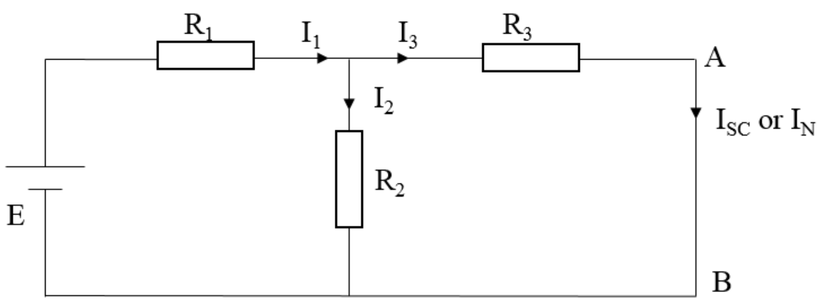
i) Remove the load resistance R_L and put a short circuit across terminals.
ii) Find the short circuit current (I_N or I_{SC})
iii) Find the equivalent resistance R_N as seen from open circuited terminals by replacing the voltage source by its internal resistance.
iv) Draw the Norton's equivalent circuit by a current I_{SC} in series with the equivalent resistance R_{th} or R_N
v) Find the current through R_L by applying current divider rule.

$$I_L = \frac{I_N \times R_N}{R_N + R_L}$$

Explanation: Unit: 2



Step 1: Remove the Load Resistor (R_L) and Short circuit its terminals.



Step 2: Find Short Circuit Current or Norton's Current (I_{SC} or I_N)

$$I_{SC} \text{ or } I_N = \text{Current through branch A and B}$$

$$I_{SC} \text{ or } I_N = I_3$$

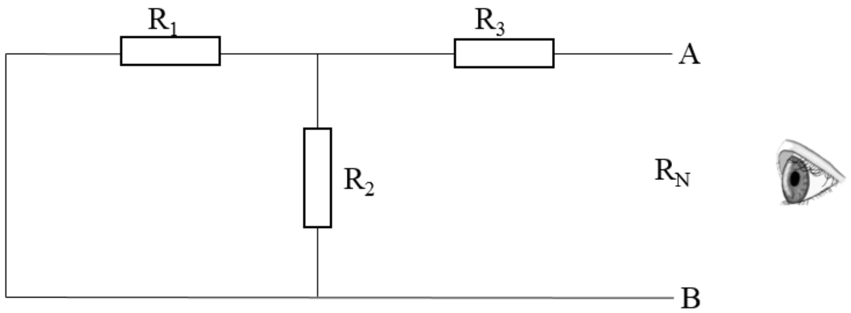
$$\text{Equivalent Resistance } R_{eq} = R_1 + \left(\frac{R_2 \text{ x } R_3}{R_2 + R_3}\right)$$

$$\text{Current } I_1 = \frac{E}{R_{eq}}$$

$$\text{Current through Shorted path}(I_{SC} \text{or } I_N) = I_3$$

$$\text{So, } I_{SC} \text{or } I_N = \frac{I_1 \text{ x } R_2}{R_2 + R_3}$$

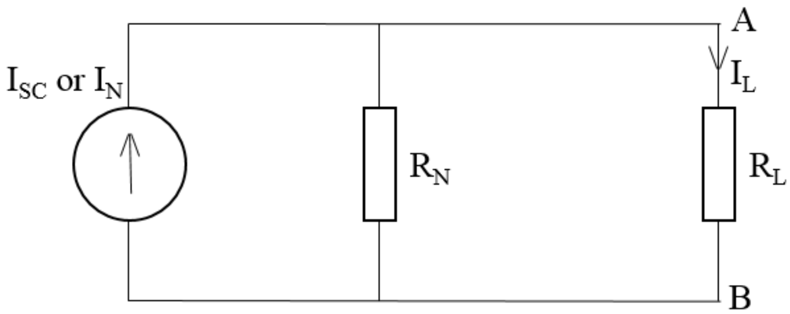
Step 3: Find R_N (Equivalent Resistance between Terminal A and B)



$$R_{12} = \frac{R_1 \text{ x } R_2}{R_1 + R_2}$$

$$R_N = R_{12} + R_3$$

Step 4: Draw Norton's equivalent circuit

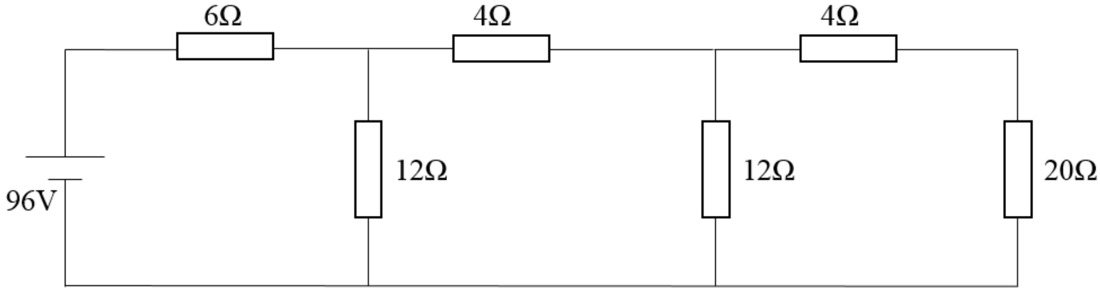


Step 5: Find the Load Current (I_L):

$$I_L = \frac{I_N \text{ x } R_N}{R_N + R_L}$$

Example: 22

Find the current through 20Ω load resistor using Norton's Theorem.



Given Data:

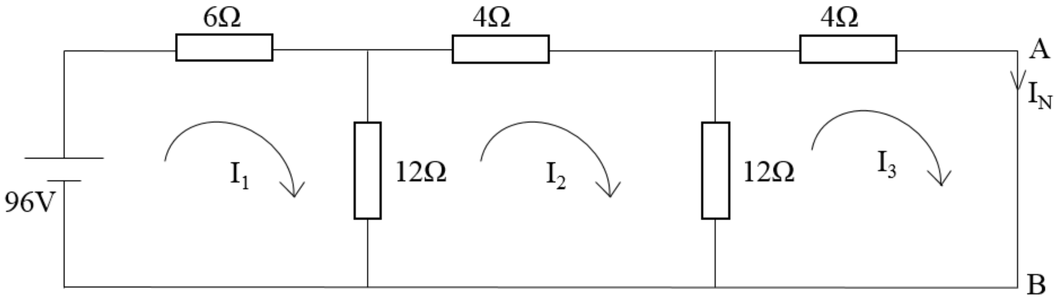
Supply Voltage = 96V

To Find:

i) Current through 20Ω Resistor = ?

Solution:

Step 1: Remove the Load Resistance and Short Circuit its terminal.



Step 2: Find Current through short circuited Terminals I_N

$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

$$\begin{vmatrix} 18 & -12 & 0 \\ -12 & 28 & -12 \\ 0 & -12 & 16 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 96 \\ 0 \\ 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 18 & -12 & 0 \\ -12 & 28 & -12 \\ 0 & -12 & 16 \end{vmatrix}$$

$$\Delta = 18[28 \text{ x } 16 - (-12 \text{ x } -12)] + 12 [(-12 \text{ x } 16) - 0] + 0$$

$$\Delta = 18[448 - 144] + 12 [-192]$$

$$\Delta = 5472 - 2304$$

$$\Delta = 3168$$

$$\Delta I_3 = \begin{vmatrix} 18 & -12 & 96 \\ -12 & 28 & 0 \\ 0 & -12 & 0 \end{vmatrix}$$

$$\Delta I_3 = 18 (0) – 12(0) + 96 [(-12 \text{ x } -12) – (0 \text{ x } 28)]$$

$$\Delta I_3 = 96 \text{ x } 144$$

$$\Delta I_3 = 13824$$

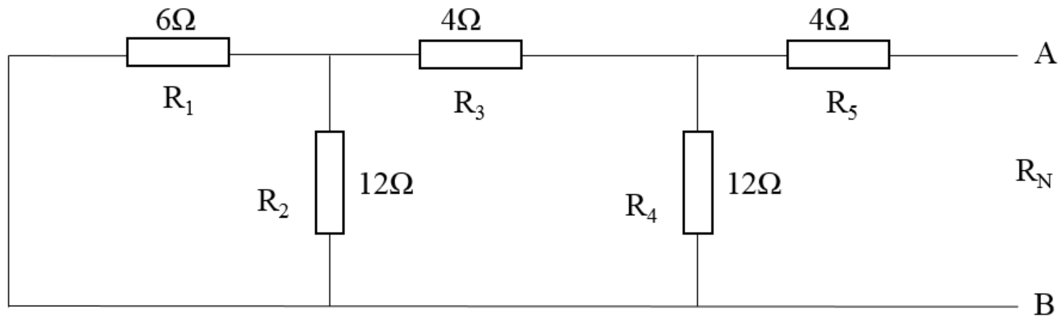
$$I_3 = \frac{\Delta I_3}{\Delta}$$

$$I_3 = \frac{13824}{3168}$$

$$I_3 = 4.36\text{Amps}$$

$$I_3 = I_N = 4.36\text{Amps}$$

Step 3: Find Norton equivalent Resistance R_N



$$R_{12} = \frac{R_1 \text{ x } R_2}{R_1 + R_2} = \frac{6 \text{ x } 12}{6 + 12} = \frac{72}{18} = 4\Omega$$

$$R_{123} = R_{12} + R_3$$

$$4 + 4 = 8\Omega$$

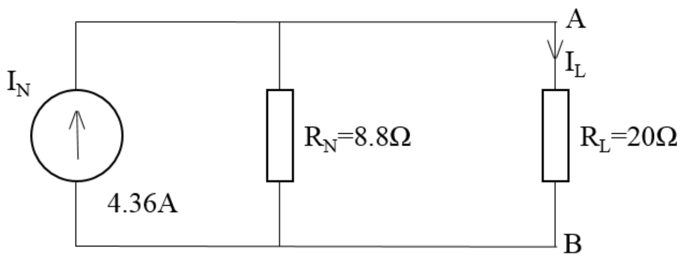
$$R_{1234} = \frac{R_{123} \text{ x } R_4}{R_{123} + R_4} = \frac{8 \text{ x } 12}{8 + 12} = \frac{96}{20} = 4.8\Omega$$

$$R_N = R_{1234} + R_5$$

$$R_{th} = 4.8 + 4$$

$$R_N = 8.8\ \Omega$$

Step 4: Draw Norton's equivalent circuit



Step 5: Find Load Current (I_L)

$$I_L = \frac{I_N \text{ x } R_N}{R_N + R_L}$$

$$I_L = \frac{4.36 \text{ x } 8.8}{8.8 + 20} = \frac{38.37}{28.8} = 1.33 \text{ Amps}$$

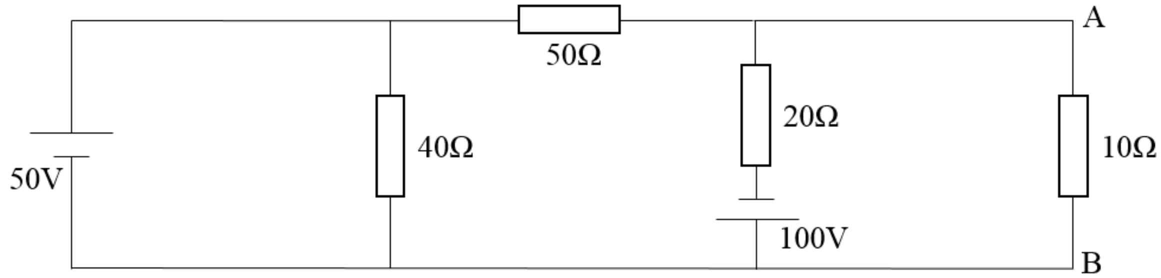
$$I_L = 1.33\text{Amps}$$

<u>Answer:</u>

Current through 10Ω Resistor = 1.33 Amps

Example: 23

Calculate the current flow through 10Ω resistor by using Norton's theorem.



Given Data:

Voltage source E_1 = 50V

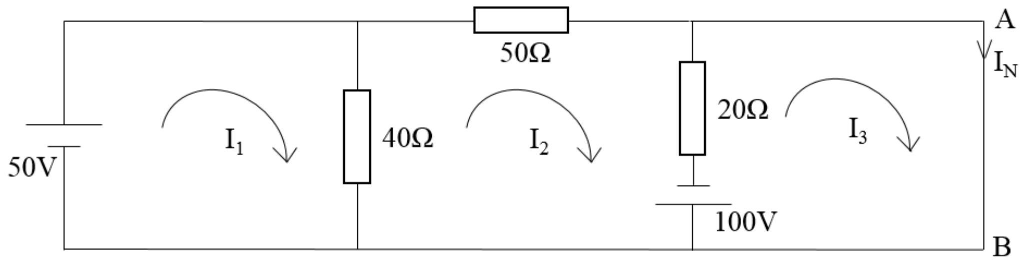
Voltage source E_2 = 100V

To Find:

i) Current through 10Ω Resistor = ?

Solution:

Step 1: Remove the Load Resistance and Short Circuit its terminal.



Step 2: Find Current through short circuited Terminals I_N

$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

$$\begin{vmatrix} 40 & -40 & 0 \\ -40 & 110 & -20 \\ 0 & -20 & 20 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 50 \\ 100 \\ -100 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 40 & -40 & 0 \\ -40 & 110 & -20 \\ 0 & -20 & 20 \end{vmatrix}$$

$$\Delta = 40[110 \text{ x } 20 - (-20 \text{ x } -20)] + 40\,[(-40 \text{ x } 20) - 0] + 0$$

$$\Delta = 40[2200 - 400] + 40\,(-800)$$

$$\Delta = 72000 - 32000$$

$$\Delta = 40000$$

$$\Delta I_3 = \begin{vmatrix} 40 & -40 & 50 \\ -40 & 110 & 100 \\ 0 & -20 & -100 \end{vmatrix}$$

$$\Delta I_3 = 40\,(-11000 + 2000) + 40(4000) + 50\,(800)$$

$$\Delta I_3 = -360000 + 160000 + 40000$$

$$\Delta I_3 = -160000$$

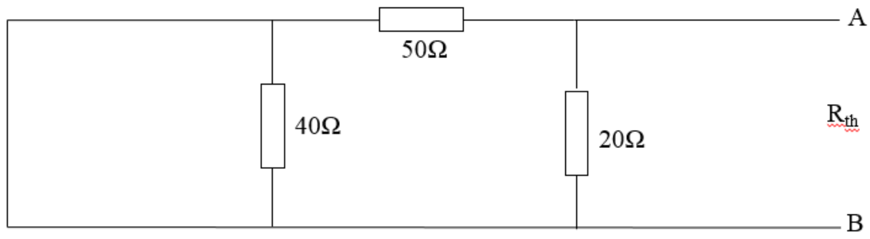
$$I_3 = \frac{\Delta I_3}{\Delta}$$

$$I_3 = \frac{-160000}{40000}$$

$$I_3 = -4 \text{ Amps}$$

$$I_3 = I_N = -4 \text{ Amps}$$

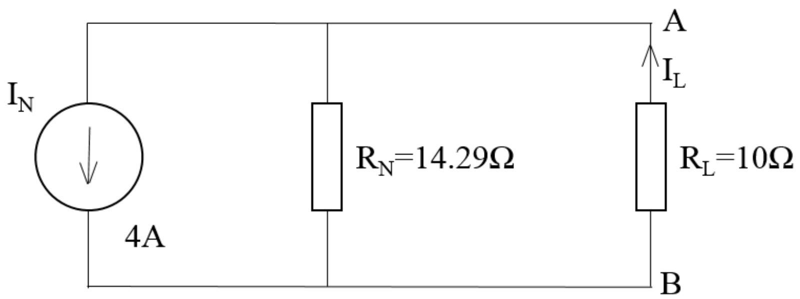
Step 3: Find Norton equivalent Resistance R_N



$$R_N = \frac{50 \text{ x } 20}{50 + 20} = \frac{1000}{70} = 14.29\Omega$$

$$R_N = 14.29\ \Omega$$

Step 4: Draw Norton's equivalent circuit



Step 5: Find Load Current (I_L)

$$I_L = \frac{I_N \text{ x } R_N}{R_N + R_L}$$

$$I_L = \frac{-4 \text{ x } 14.29}{14.29 + 10} = \frac{-57.16}{24.29} = -2.35 \text{ Amps}$$

$$I_L = -2.35 \text{ Amps}$$

Voltage across 10Ω Resistor $V_L = I_L \text{ x } R_L$

$$V_L = -2.35 \text{ x } 10$$

$$V_L = -23.5 \text{ Volts}$$

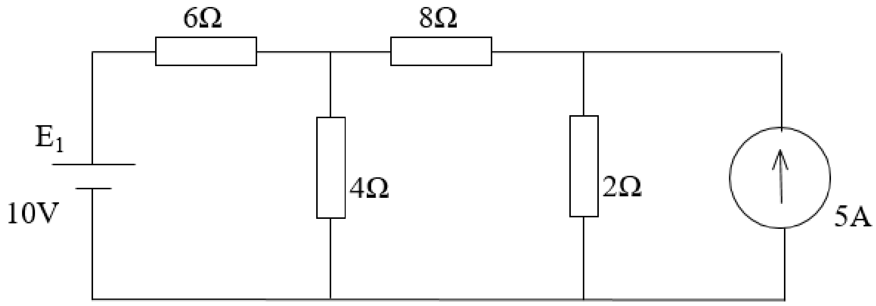
Answer:

Current through 10Ω Resistor = -2.35 Amps

Voltage drop across10Ω Resistor = -23.5 Volts

Example: 24

Find the current through 4Ω resistance by using Norton's theorem.



<u>Given Data:</u>

Voltage source E_1 = 50V

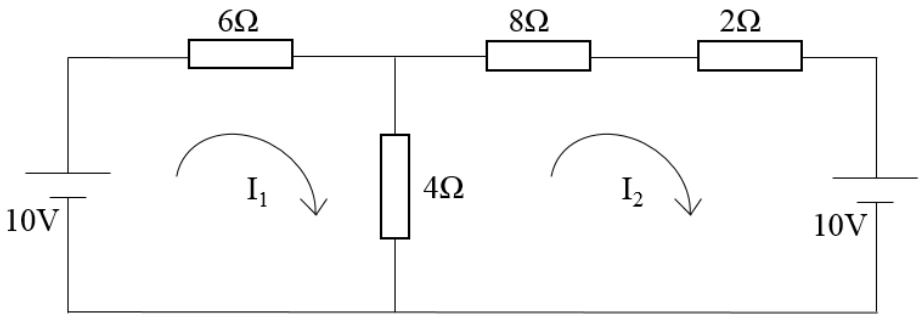
Voltage source E_2 = 100V

<u>To Find:</u>

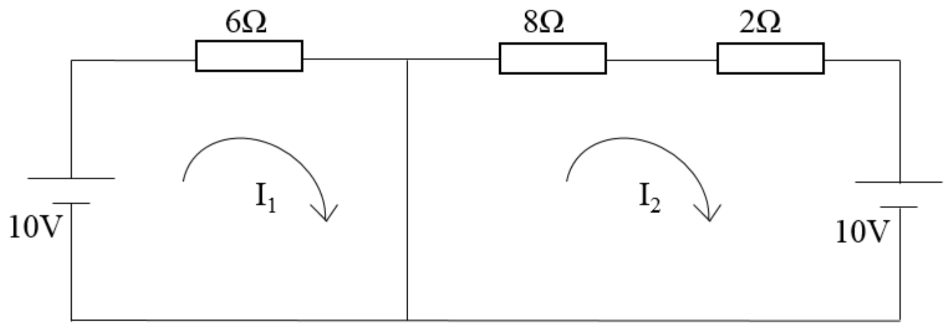
i) Current through 10Ω Resistor = ?

<u>Solution:</u>

Convert the given current source into equivalent voltage source.



Step 1: Remove the Load Resistance and Short Circuit its terminal.



Step 2: Find Current through short circuited Terminals I_N

$$\begin{vmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}$$

$$\begin{vmatrix} 6 & 0 \\ 0 & 10 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 10 \\ -10 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 6 & 0 \\ 0 & 10 \end{vmatrix}$$

$$\Delta = 60$$

$$\Delta I_1 = \begin{vmatrix} 10 & 0 \\ -10 & 10 \end{vmatrix}$$

$$\Delta I_1 = 100$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{100}{60} = 1.67 \text{ Amps}$$

$$\Delta I_2 = \begin{vmatrix} 6 & 10 \\ 0 & -10 \end{vmatrix}$$

$$\Delta I_2 = -60$$

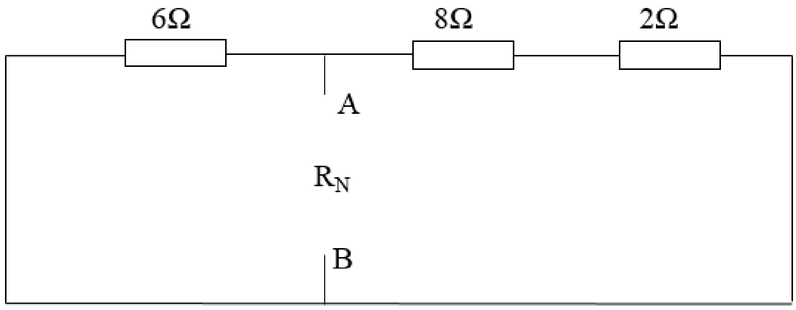
$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{-60}{60} = -1 \text{ Amps}$$

$$I_2 = 0.67\text{Amps}$$

$$I_N = I_1 - I_2$$

$$I_N = 1.67 - (-1) = 2.67 \text{ Amps}$$

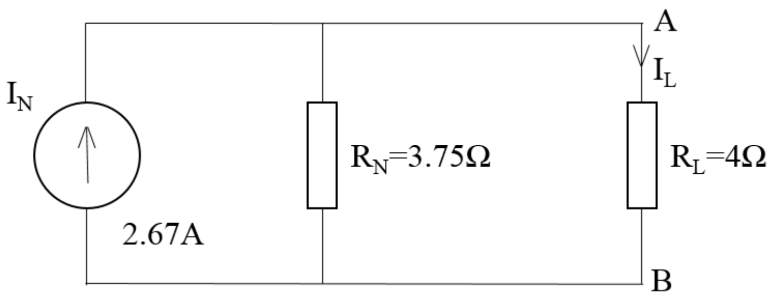
Step 3: Find Norton equivalent Resistance R_N



$$R_N = \frac{6 \, x \, 10}{6 + 10} = \frac{60}{16} = 3.75\Omega$$

$$R_N = 3.75 \, \Omega$$

Step 4: Draw Norton's equivalent circuit



Step 5: Find Load Current (I_L)

$$I_L = \frac{I_N \text{ x } R_N}{R_N + R_L}$$

$$I_L = \frac{2.67 \text{ x } 3.75}{3.75 + 4} = \frac{2.51}{7.75} = 1.29 \text{ Amps}$$

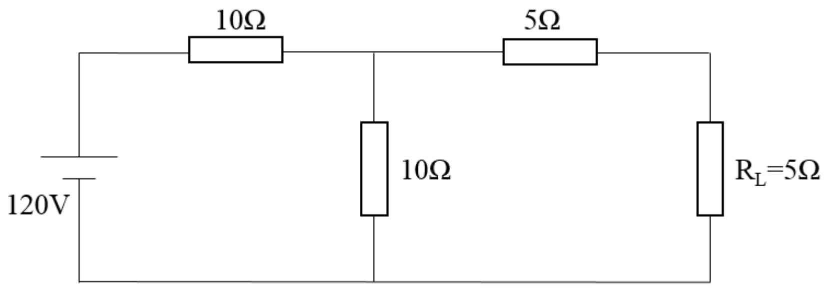
Current through 4Ω Resistance I_L = 1.29 Amps

<u>Answer:</u>

Current through 4Ω Resistor = 1.29 Amps

Example: 25

Using Norton's theorem find the current in the load resistance R_L of the circuit given below.



<u>Given Data:</u>

Voltage source E_1 = 120V

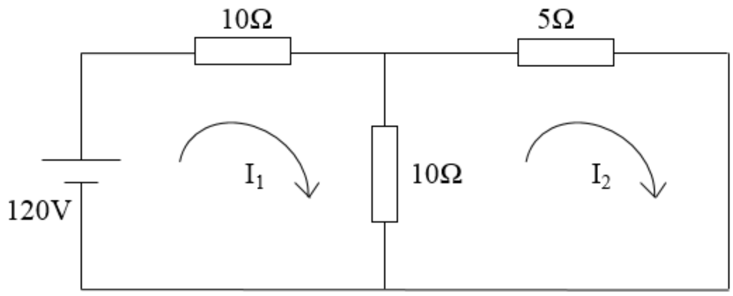
Load Resistance R_L = 5Ω

<u>To Find:</u>

i) Current through 10Ω Resistor = ?

<u>Solution:</u>

Step 1: Remove the Load Resistance and Short Circuit its terminal.



Step 2: Find Current through short circuited Terminals I_N

$$\begin{vmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}$$

$$\begin{vmatrix} 20 & -10 \\ -10 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 120 \\ 0 \end{vmatrix}$$

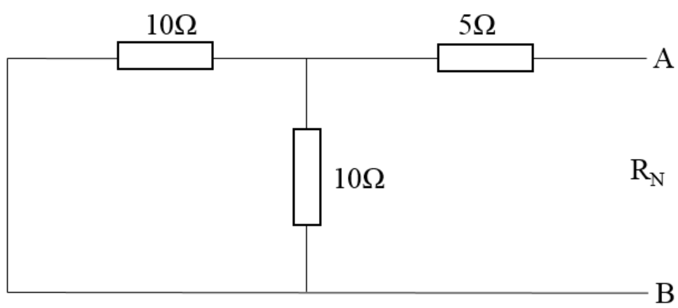
$$\Delta = \begin{vmatrix} 20 & -10 \\ -10 & 15 \end{vmatrix} = 300 - 100 = 200$$

$$\Delta I_2 = \begin{vmatrix} 20 & 120 \\ -10 & 0 \end{vmatrix} = 1200$$

$$I_2 = \frac{\Delta I_2}{\Delta} = \frac{1200}{200} = 6 \text{ Amps}$$

$$I_2 = I_N = 6 \text{ Amps}$$

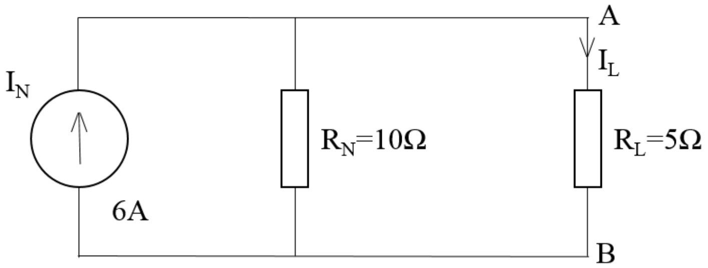
Step 3: Find Norton equivalent Resistance R_N



$$R_N = \left(\frac{10 \text{ x } 10}{10 + 10}\right) + 5 = 5$$

$$R_N = 10\ \Omega$$

Step 4: Draw Norton's equivalent circuit



Step 5: Find Load Current (I_L)

$$I_L = \frac{I_N \text{ x } R_N}{R_N + R_L}$$

$$I_L = \frac{6 \text{ x } 10}{10 + 5} = \frac{60}{15} = 4 \text{ Amps}$$

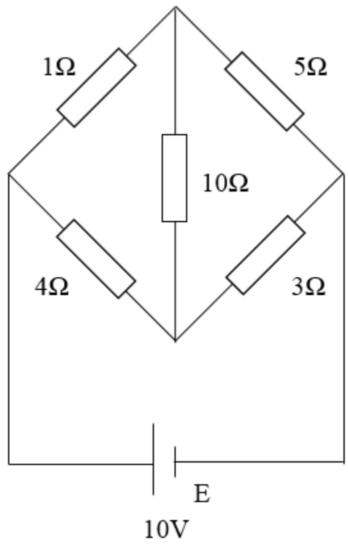
Current through 5Ω Resistor I_L = 4 Amps

Answer:

Current through 5Ω Resistor = 4 Amps

Example: 26

Find the current through 10Ω load resistor using Norton's Theorem.



Given Data:

Voltage source E = 10V

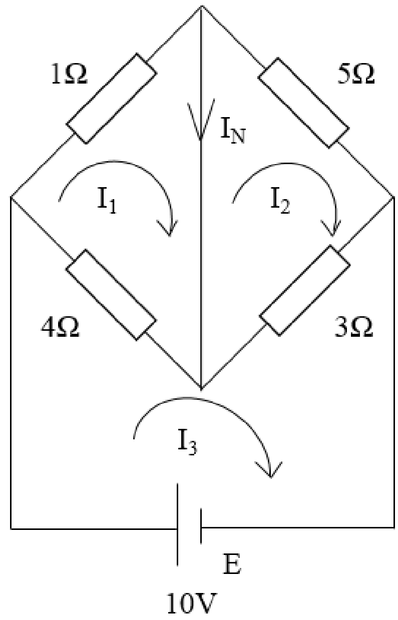
Load Resistance R_L = 10Ω

To Find:

i) Current through 10Ω Resistor = ?

Solution:

Step 1: Remove the Load Resistance and Short Circuit its terminal.



Step 2: Find Current through short circuited Terminals I_N

$$\begin{vmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

$$\begin{vmatrix} 5 & -0 & -4 \\ -0 & 8 & -3 \\ -4 & -3 & 7 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 10 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -0 & -4 \\ -0 & 8 & -3 \\ -4 & -3 & 7 \end{vmatrix}$$

$$\Delta = 5(56 - 9) + 0 - 4\,(0 + 32)$$

$$\Delta = 5(47) - 4(32)$$

$$\Delta = 235 - 128$$

$$\Delta = 107$$

$$\Delta I_1 = \begin{vmatrix} 0 & -0 & -4 \\ 0 & 8 & -3 \\ 10 & -3 & 7 \end{vmatrix}$$

$$\Delta I_1 = -4(\,0 - 80)$$

$$\Delta I_1 = -\,4(-80)$$

$$\Delta I_1 = 320$$

$$I_1 = \frac{\Delta I_1}{\Delta}$$

$$I_1 = \frac{320}{107}$$

$$I_1 = 2.99\text{Amps}$$

$$\Delta I_2 = \begin{vmatrix} 5 & 0 & -4 \\ -0 & 0 & -3 \\ -4 & 10 & 7 \end{vmatrix}$$

$$\Delta I_2 = 5(0 + 30) - 0 - 4(0)$$

$$\Delta I_2 = 150$$

$$I_2 = \frac{\Delta I_1}{\Delta}$$

$$I_2 = \frac{150}{107}$$

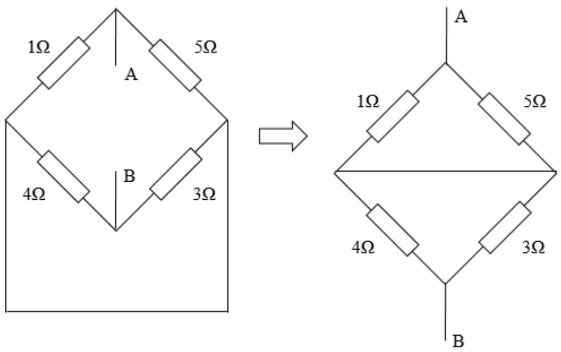
$$I_2 = 1.4\text{Amps}$$

Short circuit current $I_N = I_1 - I_2$

$$I_N = 2.99 - 1.4$$

$$I_N = 1.59 \text{ Amps}$$

Step 3: Find Norton equivalent Resistance R_N



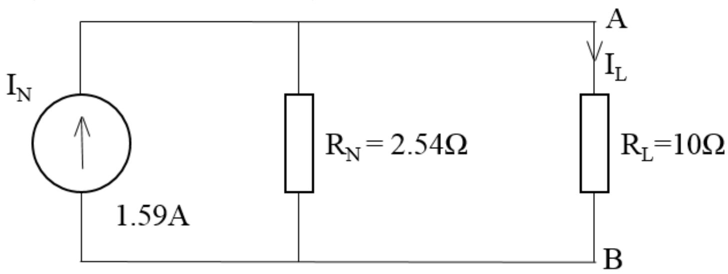
$$R_N = \left(\frac{1 \times 5}{1 + 5}\right) + \left(\frac{4 \times 3}{4 + 3}\right)$$

$$R_N = \left(\frac{5}{6}\right) + \left(\frac{12}{7}\right)$$

$$R_N = 0.833 + 1.71$$

$$R_N = 2.54\ \Omega$$

Step 4: Draw Norton's equivalent circuit



Step 5: Find Load Current (I_L)

$$I_L = \frac{I_N \times R_N}{R_N + R_L}$$

$$I_L = \frac{1.59 \times 2.54}{2.54 + 10} = \frac{4.04}{12.54} = 0.32 \text{ Amps}$$

$$I_L = 0.32 \text{ Amps}$$

<u>Answer:</u>

Current through 10Ω Resistor = 0.32 Amps

2.13 Maximum Power Transfer Theorem:

Statement:

It states that maximum power will be transferred from source to load when load resistance is equal to source resistance.

Procedure:

i) Remove the load resistance and find the Thevenin's resistance R_{th} of the source network looking through the open circuited load terminals.
ii) Calculate thevenin's voltage V_{th} or Norton's current I_N
iii) Draw thevenin's equivalent circuit.
iv) As per maximum power transfer theorem this R_{th} is the load resistance of the network, i.e., $R_L = R_{th}$
v) Maximum power is given by,

$$P_{\max} = \frac{V_{th}^2}{4R_{th}}$$

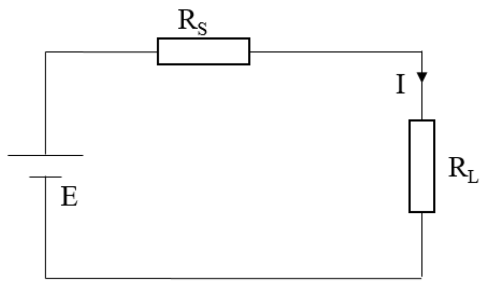
Proof:

Consider the given circuit, which supplies power directly to the load.

Let,

E – Supply Voltage
R_S – Internal Resistance of the Source
R_L – Load Resistance
P_L – Power delivered to the load

I – Circuit Current



$$\text{Current through Load (I)} = \frac{E}{R_S + R_L} \quad \text{----------- (1)}$$

$$\text{Load Power (P)} = I^2 R_L$$

$$\text{Load Power (P)} = \left(\frac{E}{R_S + R_L}\right)^2 R_L$$

$$P = \frac{E^2 R_L}{(R_s + R_L)^2} \quad \text{----------- (2)}$$

The power delivery will be maximum when:

$$\frac{dP}{dR_L} = 0 \quad \text{----------- (3)}$$

Differentiating the equation (2) w.r.t R_L and equating to zero

$$\frac{d}{dR_L}\left[\frac{E^2 R_L}{(R_s + R_L)^2}\right] = 0$$

$$\frac{E^2\,[(R_s + R_L)^2.1 - R_L.2(R_S + R_L)]}{(R_s + R_L)^4} = 0$$

$(R_S + R_L)(R_S + R_L) - 2R_L(R_S + R_L) = 0$

$(R_S + R_L)(R_S + R_L) - 2R_L R_S - 2R^2{}_L = 0$

${R_S}^2 + R_S R_L + R_L R_S + {R_L}^2 - 2R_L R_S - 2{R_L}^2 = 0$

${R_S}^2 - {R_L}^2 = 0$

$R_S = R_L$ ----------- (4)

Source Resistance = Load Resistance

Hence from the equation (4), the maximum power is transferred from the source to load when the load resistance equals the source resistance.

$Substitue, R_S = R_L\ in\ equation\ (1)$

$$I_{max} = \frac{E}{2R_L}$$

$Substitue, R_S = R_L\ in\ equation\ (2)$

$$\text{Maximum Power } (P_{max}) = \frac{E^2 R_L}{(R_L + R_L)^2}$$

$$P_{max} = \frac{E^2 R_L}{(2R_L)^2}$$

$$P_{max} = \frac{E^2}{4R_L}$$

The total power supplied is, thus

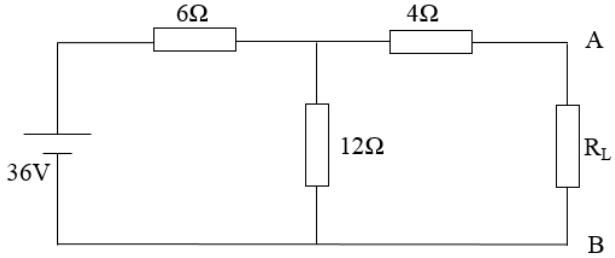
$$P = 2\frac{V_{th}^2}{4R_{th}} = \frac{V_{th}^2}{2R_{th}}$$

During maximum power transfer, the efficiency becomes

$$\eta = \frac{P_{max}}{P} \times 100 = 50\%$$

Example: 27

For the circuit shown below, find the value of R_L for which the maximum power is transferred from the source. Find the value of that power.



Given Data:

Supply Voltage = 36 V

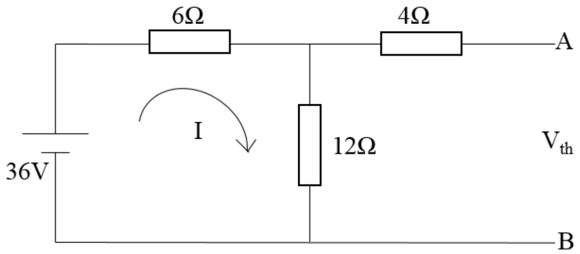
To Find:

i) Load Resistance (R_L) =?

ii) Load Power (P) =?

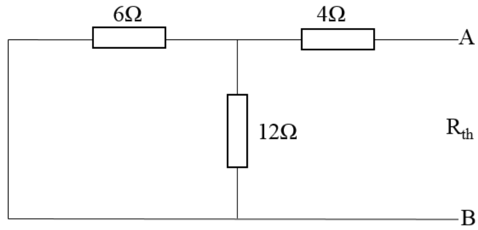
Solution:

Step 1: Find Open Circuit Voltage (V_{th})



$$V_{th} = \text{P.D across } 12\Omega \text{ Resistor}$$
$$V_{th} = I \times 12$$
$$I = \frac{36}{(6+12)} = \frac{36}{18} = 2\text{Amps}$$
$$V_{th} = 2 \times 12$$
$$V_{th} = 24 \text{ Volts}$$

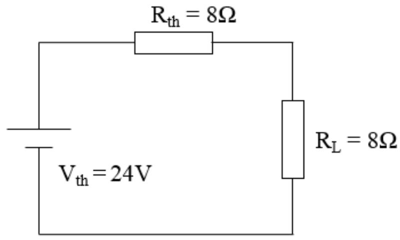
Step 2: Find Thevenin Equivalent Resistance (R_{th}):



$$R_{123} = = \frac{6 \times 12}{6+12} = \frac{72}{18}$$
$$R_{123} = 4\ \Omega$$
$$R_{th} = 4 + 4 = 8$$
$$R_{th} = 8\ \Omega$$

According to Max. Power Transfer Theorem: $R_L = R_{th} = 8\ \Omega$

Step 3: Draw thevenin's equivalent circuit

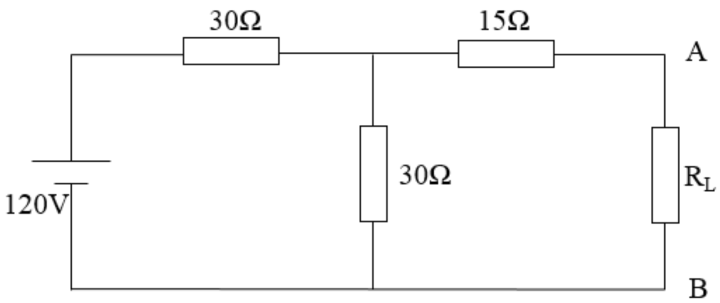


Step 4: Calculate Maximum Power:

$$\text{Maximum Power} \quad (P_{max}) = \frac{V_{th}^2}{4R_L} = \frac{24^2}{4 \times 8} = \frac{576}{32}$$
$$P_{max} = 18 \text{ Watts}$$

Example: 28

Calculate the value of load resistance for maximum power transferred from the circuit shown below. Also find the value of that maximum power.



Given Data:

Supply Voltage = 120 V

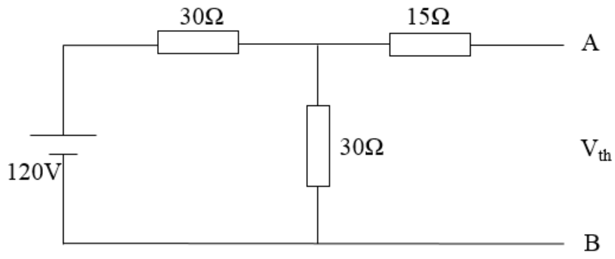
To Find:

i) Load Resistance (R_L) =?

ii) Load Power (P) =?

Solution:

Step 1: Find Open Circuit Voltage (V_{th})



$$V_{th} = \text{P.D across } 30\Omega \text{ Resistor}$$

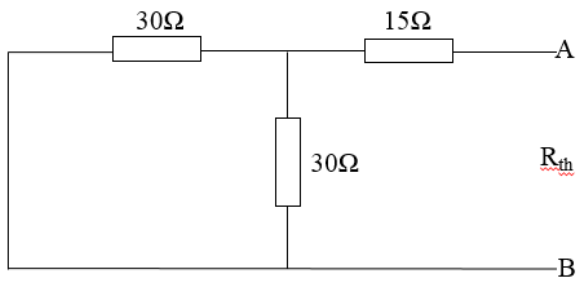
$$V_{th} = I \times 30$$

$$I = \frac{120}{(30+30)} = \frac{120}{60} = 2 \text{ Amps}$$

$$V_{th} = 2 \times 30$$

$$V_{th} = 60 \text{ Volts}$$

Step 2: Find R_{th}



$$R_{123} = \frac{30 \times 30}{30+30} = \frac{900}{60}$$

$$R_{123} = 15\ \Omega$$

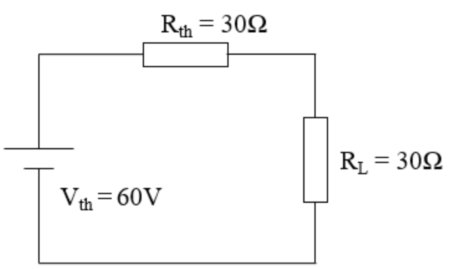
$$R_{th} = 15 + 15$$

$$R_{th} = 30\ \Omega$$

According to Maximum Power Transfer theorem:

$$R_L = R_{th} = 30\ \Omega$$

Step 3: Draw thevenin's equivalent circuit



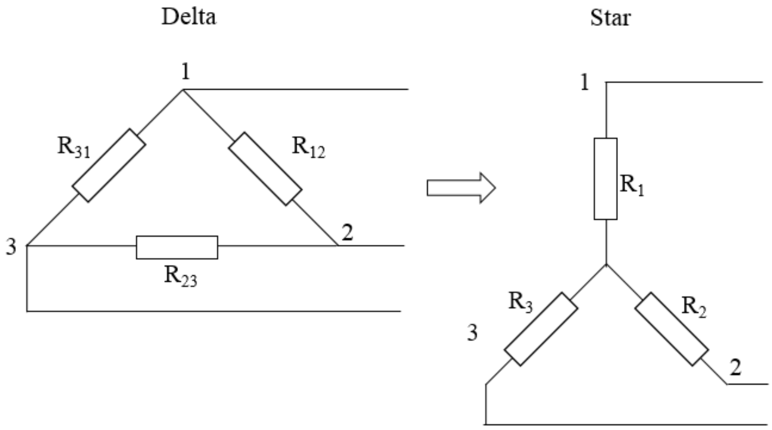
Step 4: Calculate Maximum Power:

$$\text{Maximum Power} \quad (P_{max}) = \frac{V_{th}^2}{4R_L} = \frac{60^2}{4 \times 30} = \frac{3600}{120}$$

$$P_{max} = 30 \text{ Watts}$$

2.14 Delta to Star Conversion:

Consider a three terminal network in which the resistors are connected in the form Δ. Such a network is known as delta network. Let the resistor values are R_{12}, R_{23} and R_{31}. Now we find its equivalent Y-network such that both the circuits are identical as far as the terminals 1, 2 and 3 are concerned. Let the resistor values are R_1, R_2 and R_3.



In Delta (Δ) Connection:

$$\text{Equivalent Resistance between 1 \&2} = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

In Star (Y) Connection:

$$\text{Equivalent Resistance between 1 \&2} = R_1 + R_2$$

$$\text{Resistance between terminal 1 \& 2 in } \mathbf{Y} = \text{Resistance between terminal 1 \& 2 in } \boldsymbol{\Delta}$$

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{---------- (1)}$$

Similarly,

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad \text{---------- (2)}$$

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \quad \text{---------- (3)}$$

Subtract Equation (2) from Equation (3):

$$R_1 - R_2 = \frac{(R_{12}R_{31} + R_{23}R_{31} - R_{23}R_{31} - R_{12}R_{23})}{R_{12} + R_{23} + R_{31}}$$

$$R_1 - R_2 = \frac{R_{12}(R_{31} - R_{23})}{R_{12} + R_{23} + R_{31}} \quad \text{---------- (4)}$$

Add Equation (1) and Equation (4):

$$2R_1 = \frac{2\, R_{12}\, R_{31}}{R_{12} + R_{23} + R_{31}}$$

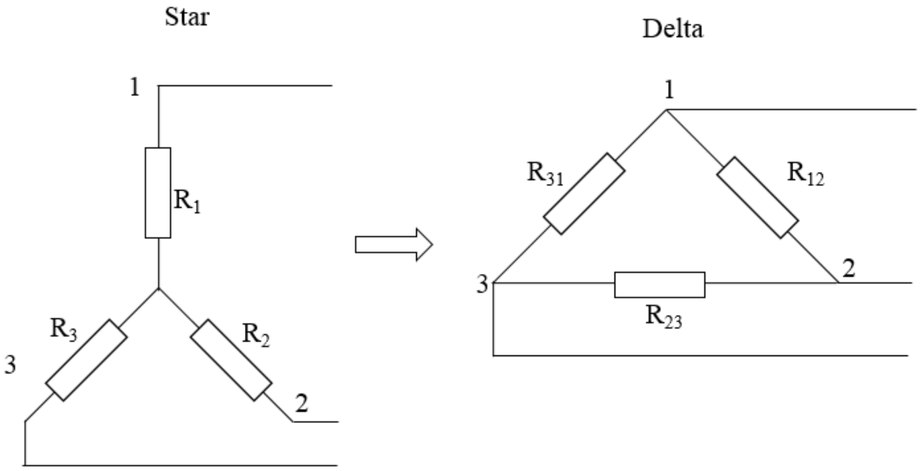
$$R_1 = \frac{R_{12}\, R_{31}}{R_{12} + R_{23} + R_{31}}$$

Similarly,

$$R_2 = \frac{R_{23}\, R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{31}\, R_{23}}{R_{12} + R_{23} + R_{31}}$$

2.15 Star to Delta Conversion:



We know, $$R_1 = \frac{R_{12}\,R_{31}}{R_{12} + R_{23} + R_{31}} \qquad \text{---------- (1)}$$

$$R_2 = \frac{R_{23}\,R_{12}}{R_{12} + R_{23} + R_{31}} \qquad \text{---------- (2)}$$

$$R_3 = \frac{R_{31}\,R_{23}}{R_{12} + R_{23} + R_{31}} \qquad \text{---------- (3)}$$

From the above relations:

(1) X (2) => $$R_1\,R_2 = \frac{R_{12}^{\,2}\,R_{23}\,R_{31}}{R_{12} + R_{23} + R_{31}} \qquad \text{---------- (4)}$$

(2) X (3) => $$R_2\,R_3 = \frac{R_{12}\,R_{23}^{\,2}\,R_{31}}{R_{12} + R_{23} + R_{31}} \qquad \text{---------- (5)}$$

(3) X (1) => $$R_3\,R_1 = \frac{R_{12}\,R_{23}\,R_{31}^{\,2}}{R_{12} + R_{23} + R_{31}} \qquad \text{---------- (6)}$$

By adding Equation: (4) + (5) + (6)

$$R_1\,R_2 + R_2\,R_3 + R_3\,R_1 = \frac{R_{12}\,R_{23}\,R_{31}\,(R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$= \frac{R_{12}\,R_{23}\,R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_{12} = \frac{R_{23}\,R_{31}}{R_{12} + R_{23} + R_{31}} \text{ x } (R_1\,R_2 + R_2\,R_3 + R_3\,R_1)$$

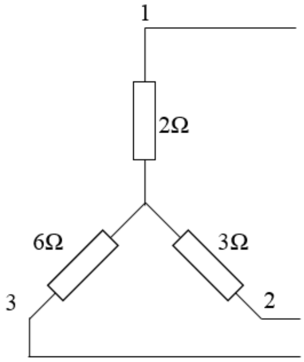
$$R_{12} = \frac{R_1\,R_2 + R_2\,R_3 + R_3\,R_1}{R_3}$$

$$R_{23} = \frac{R_1\,R_2 + R_2\,R_3 + R_3\,R_1}{R_1}$$

$$R_{31} = \frac{R_1\,R_2 + R_2\,R_3 + R_3\,R_1}{R_2}$$

Example: 29

Find the DELTA Resistance for given STAR.



Given Data:

Resistance R_1	=	2Ω	i) R_{12} =?
Resistance R_2	=	3Ω	ii) R_{23} =?
Resistance R_3	=	6Ω	iii) R_{31} =?

To Find:

Solution:

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{(2 \times 3) + (3 \times 6) + (6 \times 2)}{6}$$

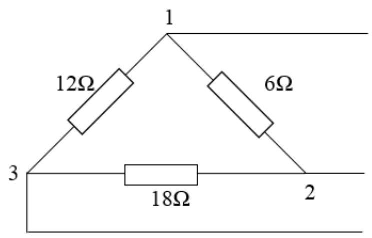
$$R_{12} = \frac{6 + 18 + 12}{6} = \frac{36}{6} = 6\Omega$$

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{(2 \times 3) + (3 \times 6) + (6 \times 2)}{2} = \frac{36}{2} = 18\Omega$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{(2 \times 3) + (3 \times 6) + (6 \times 2)}{3} = \frac{36}{3} = 12\Omega$$

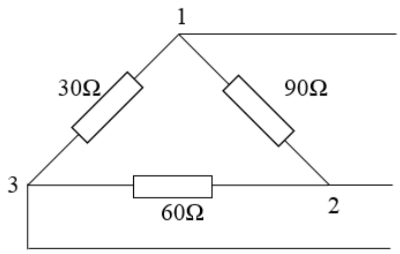
Answer:

R_{12} =6Ω, R_{23} = 18Ω and R_{31} = 12Ω



Example: 30

Find the STAR Resistance for given DELTA.



Given Data:

To Find:

Resistance R_{12}	=	90Ω	i) R_1 =?
Resistance R_{23}	=	60Ω	ii) R_2 =?
Resistance R_{31}	=	30Ω	iii) R_3 =?

Solution:

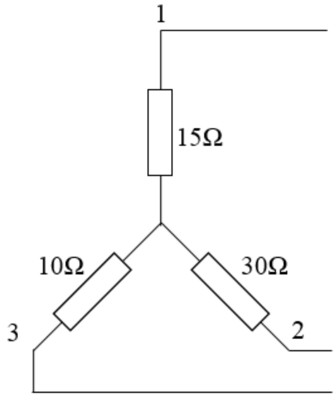
$$R_1 = \frac{R_{12}\, R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{90 \times 30}{90 + 60 + 30} = \frac{2700}{180} = 15\Omega$$

$$R_2 = \frac{R_{23}\, R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{60 \times 90}{90 + 60 + 30} = \frac{5400}{180} = 30\Omega$$

$$R_3 = \frac{R_{31}\, R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{30 \times 60}{90 + 60 + 30} = \frac{1800}{180} = 10\Omega$$

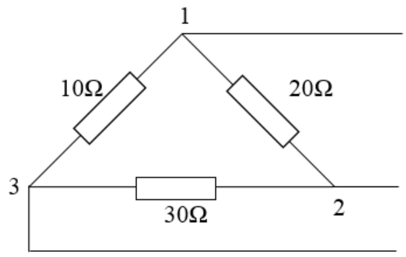
Answer:

R_1 = 15Ω, R_2=30Ω and R_3 =



Example: 31

Find the STAR Resistance for given DELTA.



Given Data:

Resistance R_{12}	=	90Ω	
Resistance R_{23}	=	60Ω	
Resistance R_{31}	=	30Ω	

To Find:

i) R_1 =?
ii) R_2 =?
iii) R_3 =?

Solution:

Step 1: Convert the **Delta** 123 into equivalent **Star** :

$$R_1 = \frac{R_{12}\, R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{20 \times 10}{20 + 30 + 10} = \frac{200}{60} = 3.33\Omega$$

$$R_2 = \frac{R_{23}\, R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{20 \times 30}{20 + 30 + 10} = \frac{600}{60} = 10\Omega$$

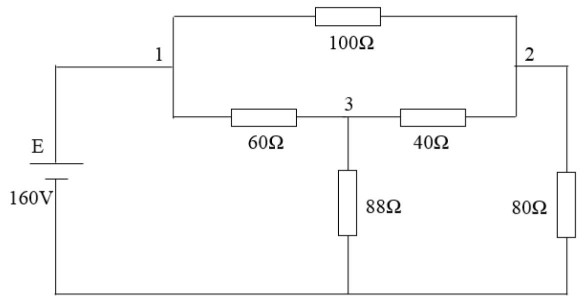
$$R_3 = \frac{R_{31}\, R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{10 \times 30}{20 + 30 + 10} = \frac{300}{60} = 5\Omega$$

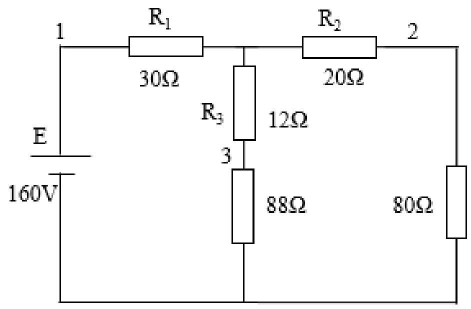
Answer:

R_1 = 3.33Ω, R_2 = 10Ω and R_3 = 5Ω

Example: 32

Determine the total resistance and circuit current of the given circuit.





<u>Given Data:</u>

Resistance R_{12} = 100Ω

Resistance R_{23} = 40Ω

Resistance R_{31} = 60Ω

<u>To Find:</u>

i) R_1 =?

ii) R_2 =?

iii) R_3 =?

<u>Solution:</u>

Step 1: Convert the **Delta** 123 into equivalent **Star** :

$$R_1 = \frac{R_{12}\ R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{100 \text{ x } 60}{100 + 40 + 60} = \frac{6000}{200} = 30\Omega$$

$$R_2 = \frac{R_{23}\ R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{40 \text{ x } 100}{100 + 40 + 60} = \frac{4000}{20} = 20\Omega$$

$$R_3 = \frac{R_{31}\ R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{60 \text{ x } 40}{100 + 40 + 60} = \frac{2400}{200} = 12\Omega$$

$$\text{Total Resistance } R = 30\Omega + [(12\Omega + 88\Omega) \text{ II}^{el} (20\Omega + 80\Omega)]$$

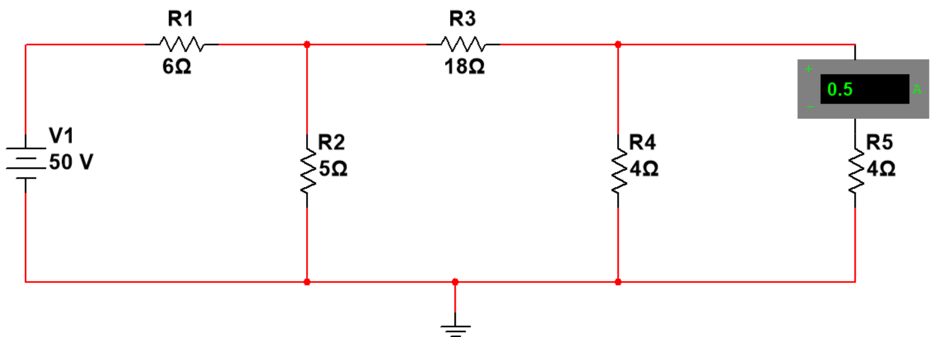
$$\text{Total Resistance } R = 30\Omega + [100\Omega \text{ II}^{el} 100\Omega] = 30\Omega + 50\Omega$$

$$\text{Total Resistance } R = 80\Omega$$

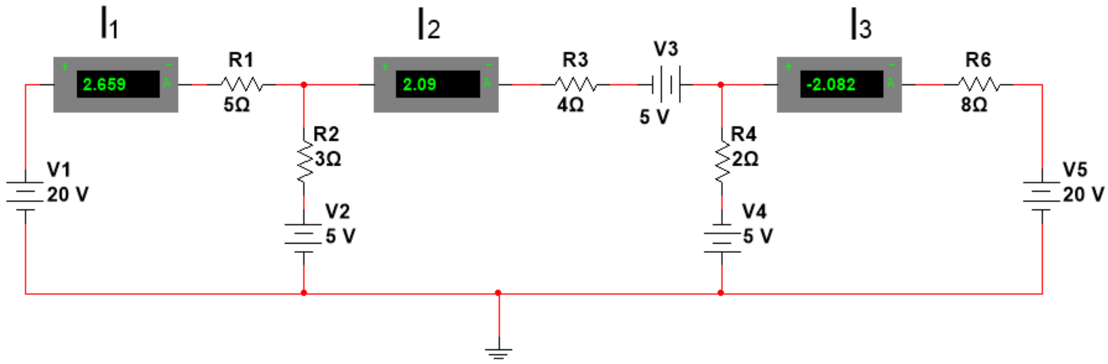
$$\text{Circuit Current } I = \frac{V}{R} = \frac{240}{80} = 3\text{Amps}$$

SIMULATED RESULT OF EXAMPLE PROBLEMS IN UNIT 2

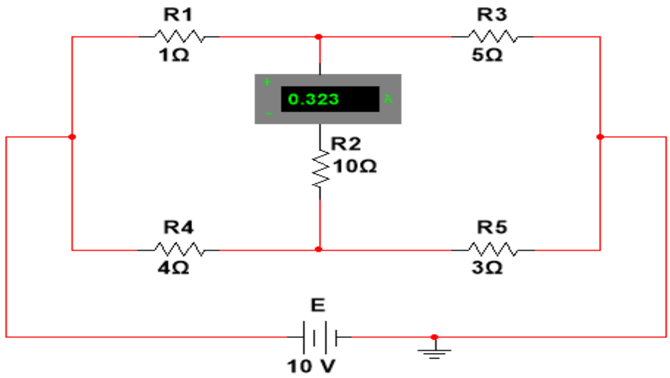
Example: 3 <u>Mesh Current Analysis</u>



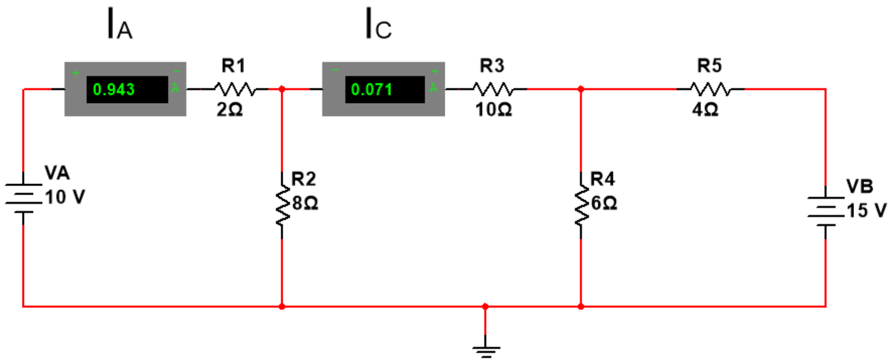
Example: 4 <u>Mesh Current Analysis</u>



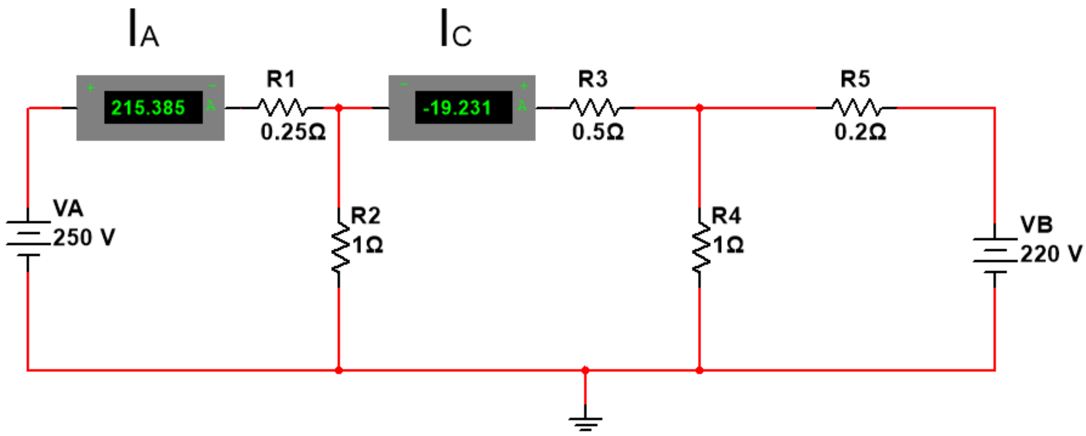
Example: 5 <u>Mesh Current Analysis</u>



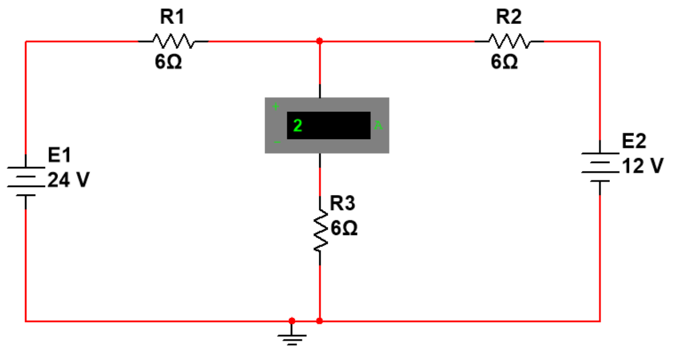
Example: 6 <u>Nodal Voltage Analysis</u>



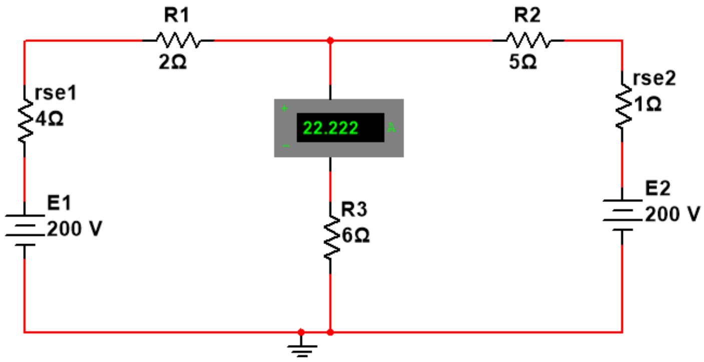
Example: 7 <u>Nodal Voltage Analysis</u>



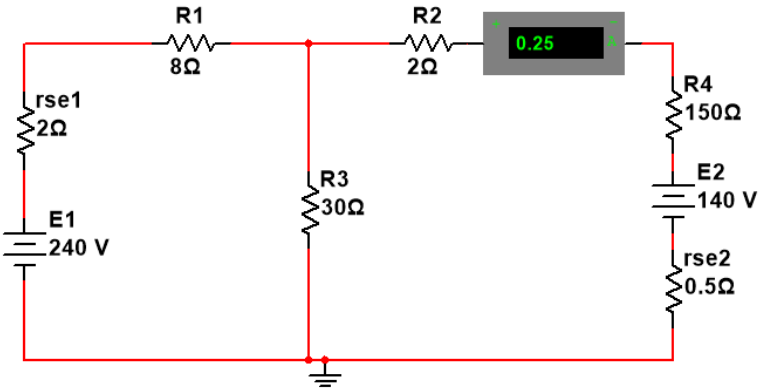
Example: 8 <u>Superposition Theorem</u>



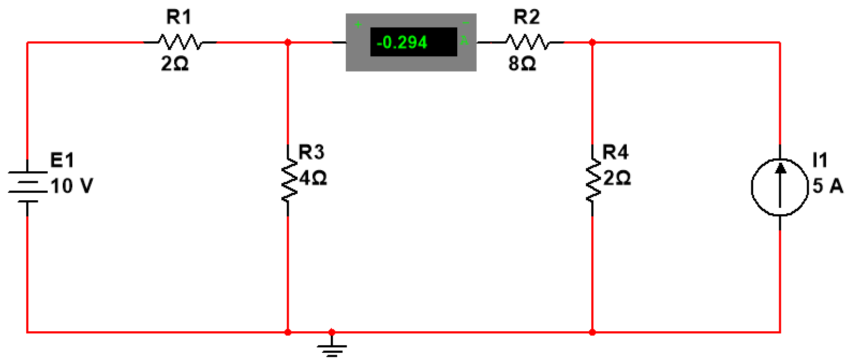
Example: 9 & 10 <u>Superposition Theorem</u>



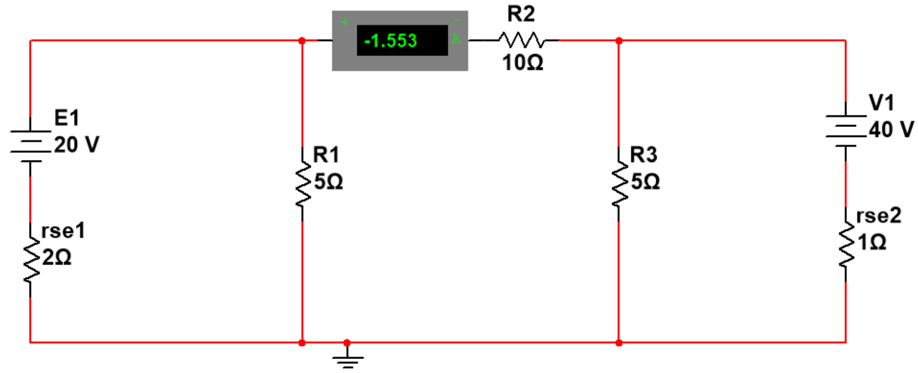
Example: 11 & 12 <u>Superposition Theorem</u>



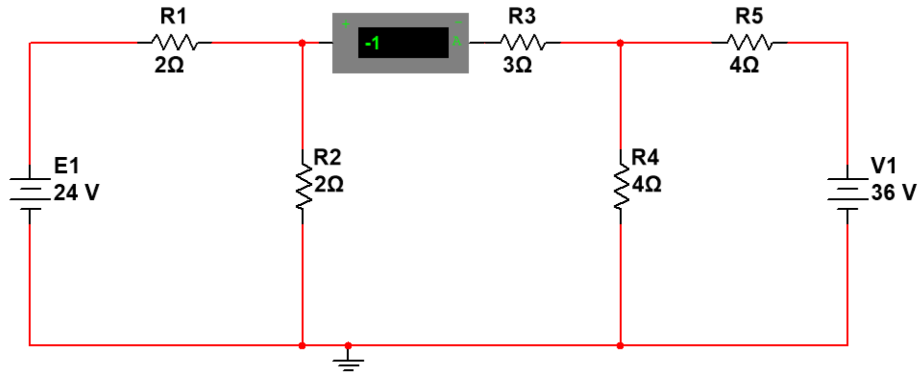
Example: 13 & 14 <u>Superposition Theorem</u>



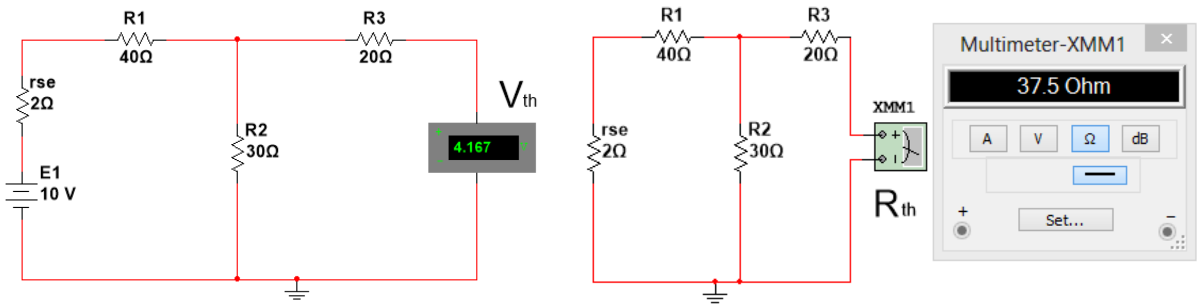
Example: 15 Superposition Theorem



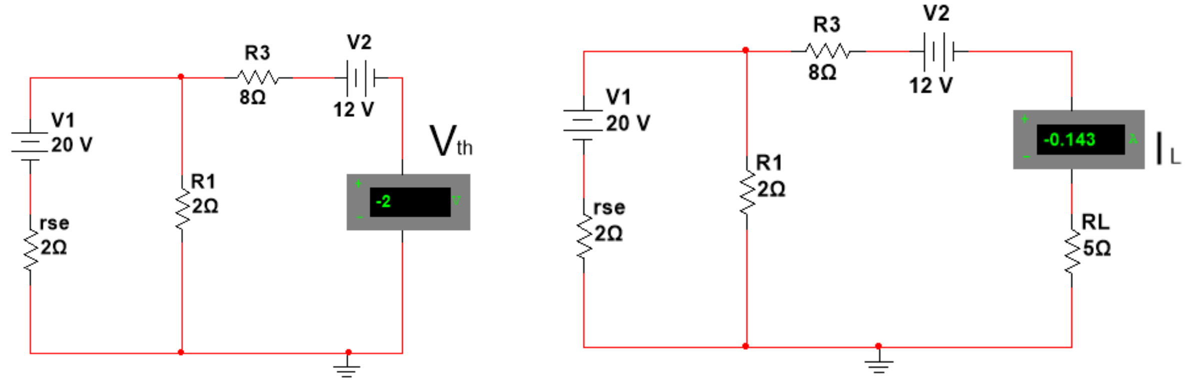
Example: 16 Superposition Theorem



Example: 17 Thevenin's Theorem

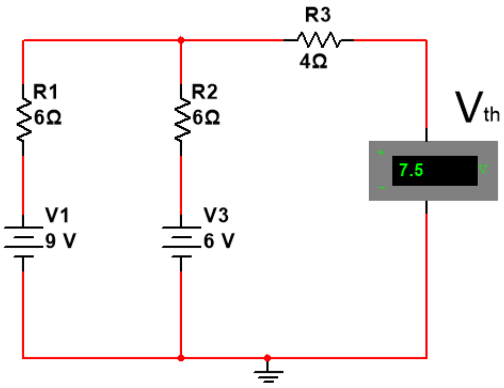


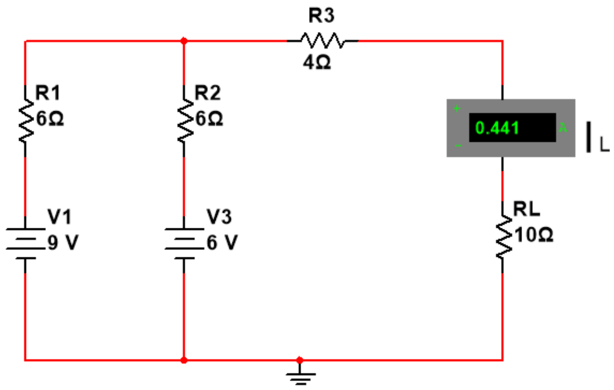
Example: 18 Thevenin's Theorem



Example: 19

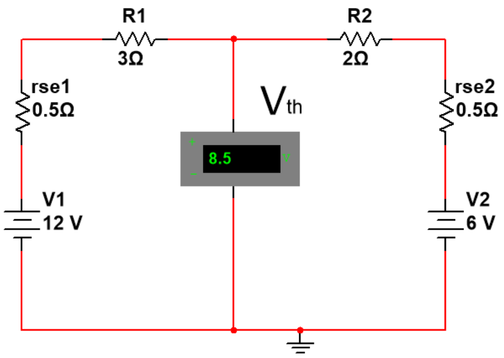
Thevenin's Theorem

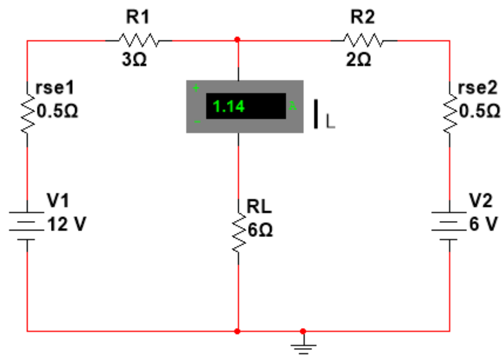




Example: 20

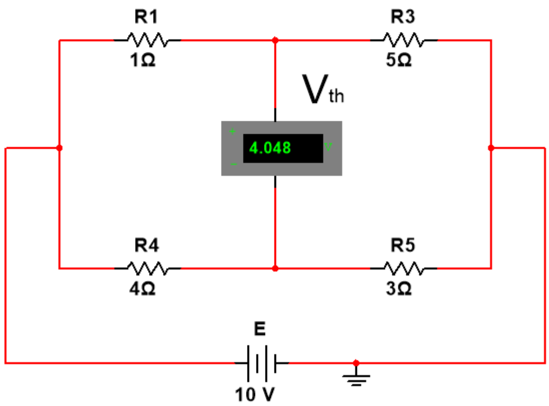
Thevenin's Theorem

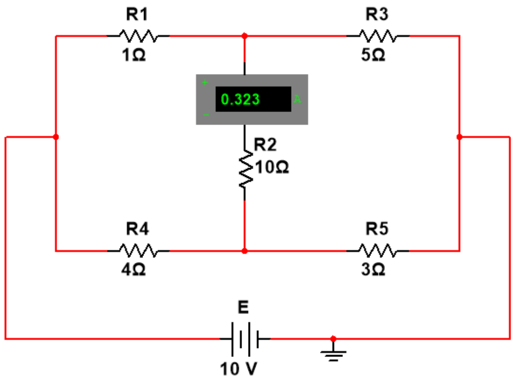




Example: 21

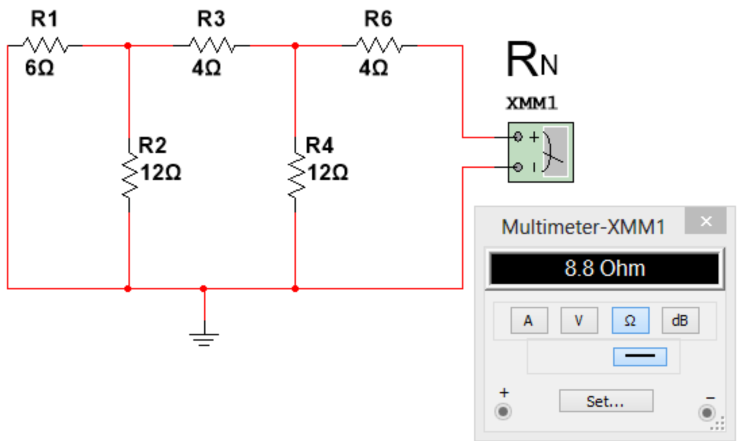
Thevenin's Theorem

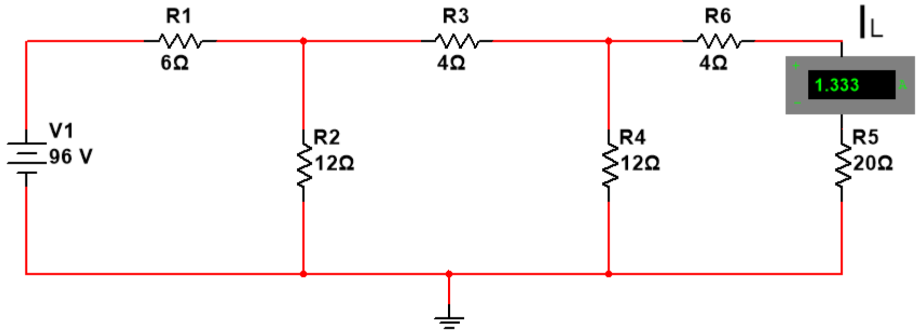




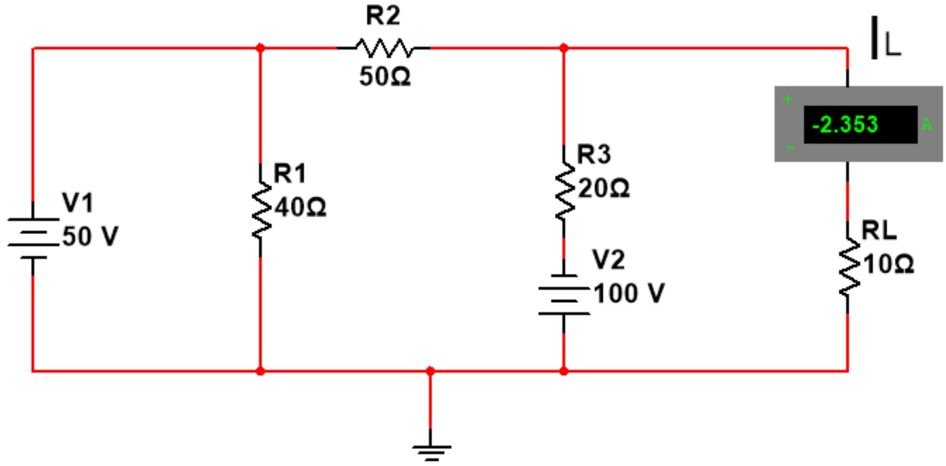
Example: 22

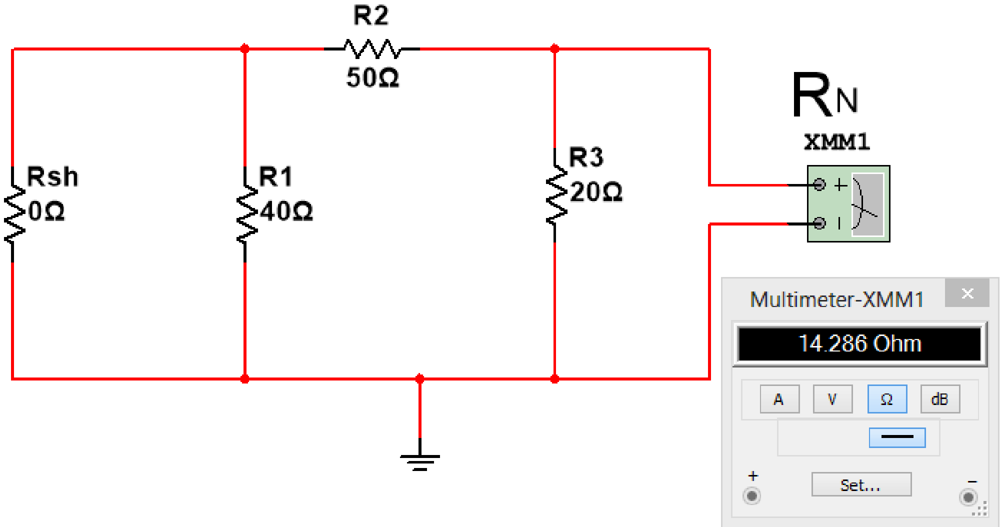
Norton's Theorem



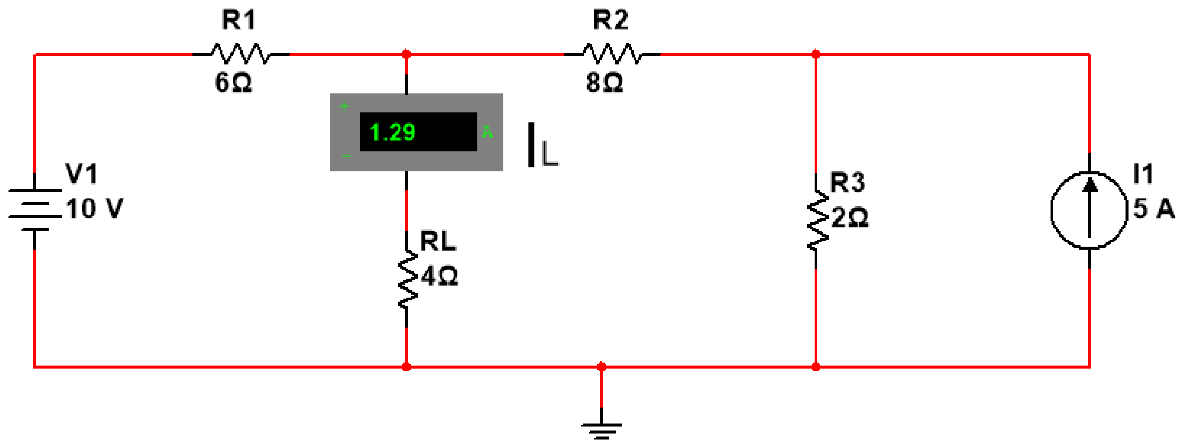


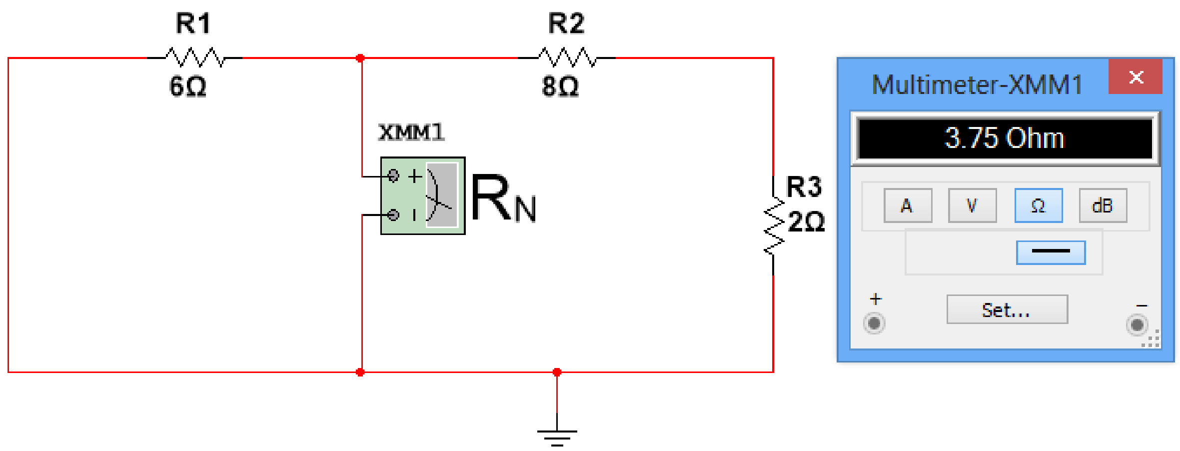
Example: 23 Norton's Theorem



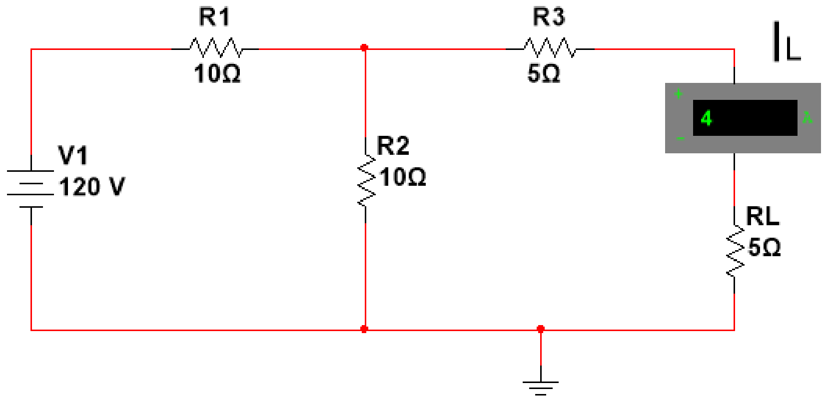


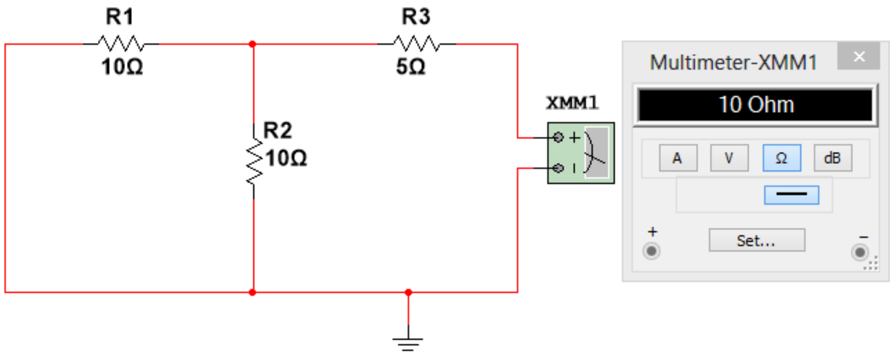
Example: 24 Norton's Theorem





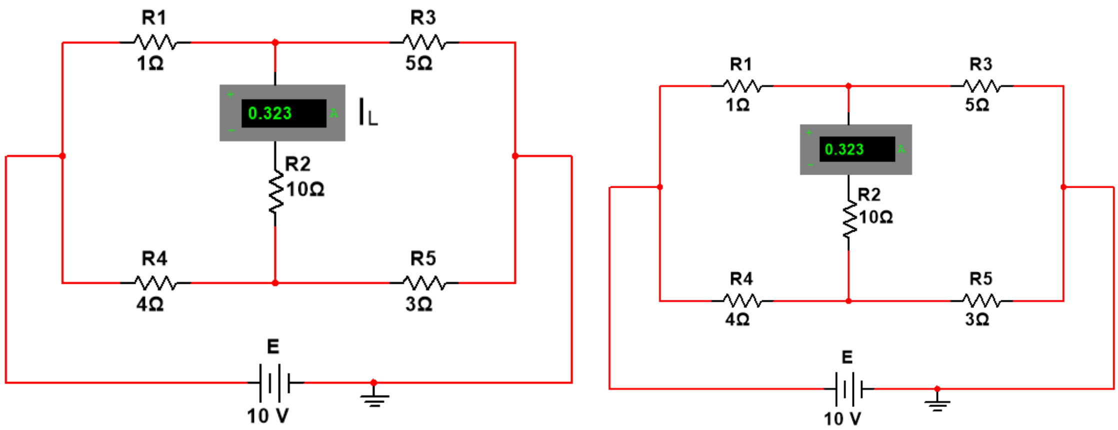
Example: 25 Norton's Theorem





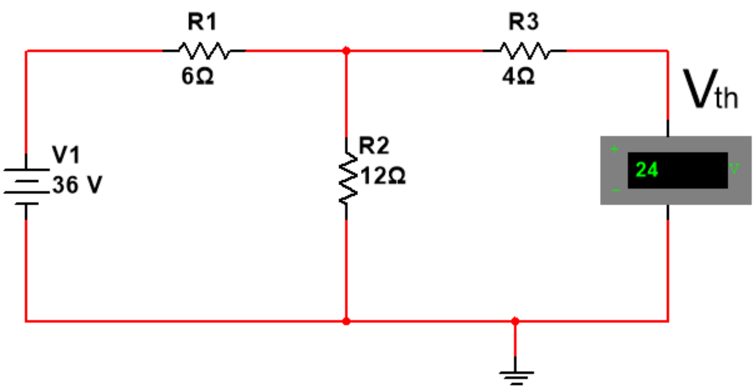
Example: 26

<u>Norton's Theorem</u>



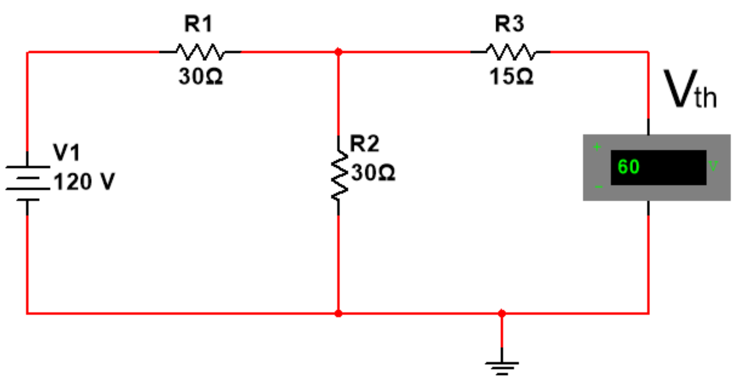
Example: 27

<u>Maximum Power Transfer Theorem</u>



Example: 28

<u>Maximum Power Transfer Theorem</u>



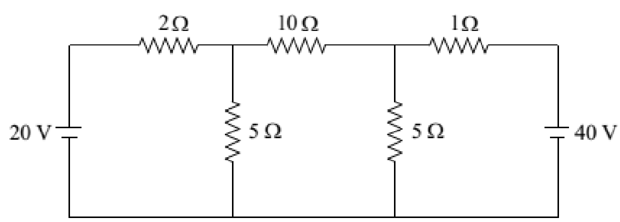
REVIEW QUESTIONS
UNIT : II NETWORK THEOREMS

PART – A : 2 Mark Questions

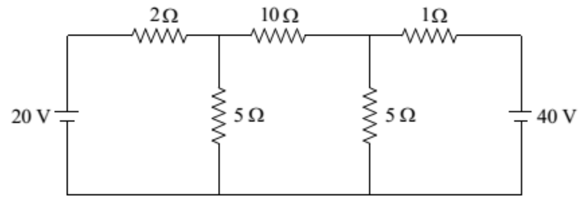
1. Define circuit element and state the elements.
2. Define electric circuit and network.
3. Define node or junction in a network.
4. Define simple node and principal node.
5. Define branch of a network.
6. Define mesh in a circuit.
7. Define loop in a network.
8. Define active network.
9. What is meant by a bilateral circuit?
10. What is meant by linear circuit?
11. State the name of the law applied in mesh current analysis.
12. State the name of the law applied in node voltage analysis.
13. What is the potential of reference node in node voltage analysis?
14. How many equations are required for mesh analysis?
15. How many equations are required for node voltage analysis?
16. A 10A current source has an internal resistance of 100Ω. Draw the equivalent voltage source.
17. A Voltage Source has a terminal voltage of 12V with an internal resistance of 0.1 Ohms. What is its equivalent current source?
18. Convert a current source of 100A with an internal resistance of 0.2 ohm into an equivalent voltage source.
19. State the condition for maximum power transfer theorem.
20. A source has an internal resistance of 8Ω. What must be the value of load resistance to transfer maximum power in it?
21. 3 identical resistors of 12Ω each are in delta. Find the equivalent value of resistors in star.
22. Three identical resistors of 8Ω each are in star. Find the equivalent value of resistor in delta.

PART – B : 3 Mark Questions

1. Draw a network and show atleast two principal nodes.
2. Write the mesh equation in matrix form for the network shown below.



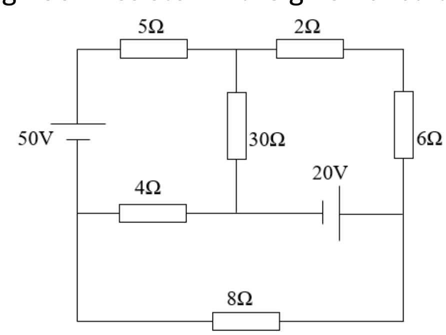
3. Write the node equation in matrix form for the network shown below.



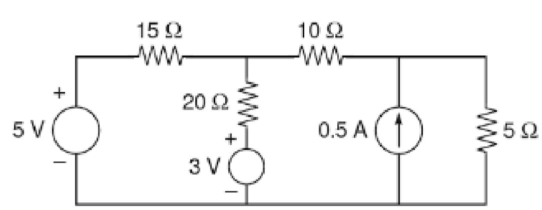
4. Write the equations to transfer Y to Δ.
5. Write the equation to transfer Δ to Y.
6. State Superposition Theorem.
7. State Thevenin's Theorem.
8. State Norton's Theorem.
9. Show how a Thevenin's circuit can be obtained from Norton's form.
10. Show how a Norton's circuit can be obtained from Thevenin's form.
11. State maximum power transfer theorem.

PART – C : 10 Mark Questions

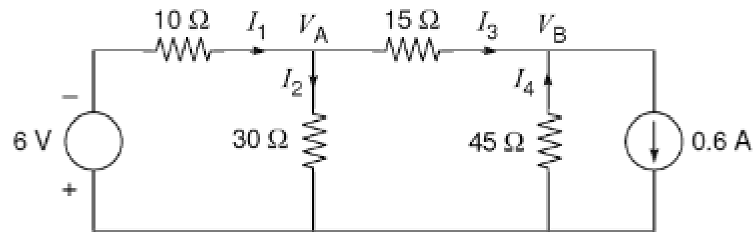
1. Discuss mesh current analysis with an example.
2. Discuss node voltage analysis with an example.
3. Derive the equations needed for star to delta and delta to star transformations.
4. State and explain Thevenin's theorem.
5. State and explain Norton's Theorem.
6. State and explain Superposition theorem with an example.
7. State Maximum power transfer theorem and derive the conditions for maximum power transfer in a single source circuit.
8. Determine the current through 30Ω resistor in the given circuit by using mesh analysis.



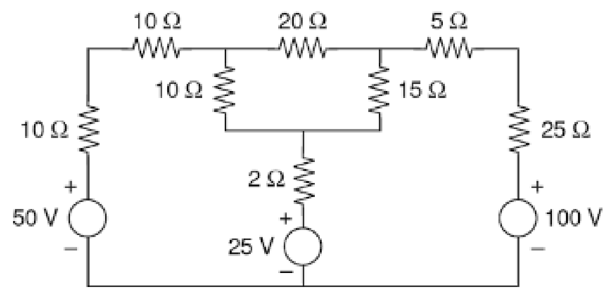
9. Use the mesh analysis method to find the current through the 20Ωresistance. Also find the voltage drop it.



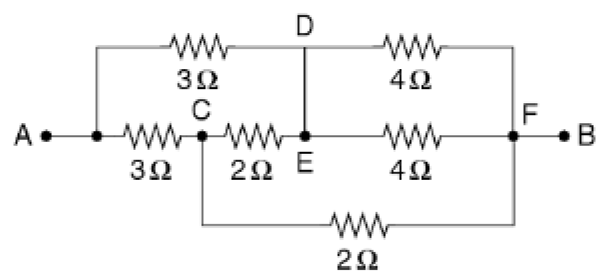
10. Use nodal analysis method to find current through 30 ohm resistor in the circuit given below.



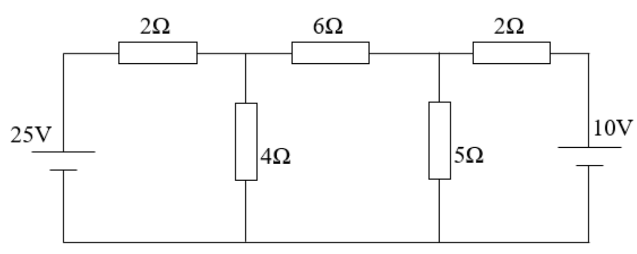
11. Determine the current through the 5Ω resistance using Nodal analysis.



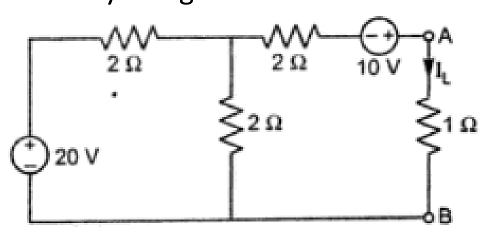
12. Calculate the equivalent resistance of the network shown in figure across terminals A and B using star delta transformation where necessary.



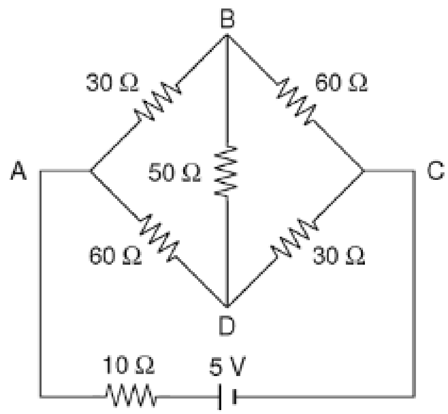
13. Determine the current through 6Ω resistor in the given circuit by using superposition theorem.



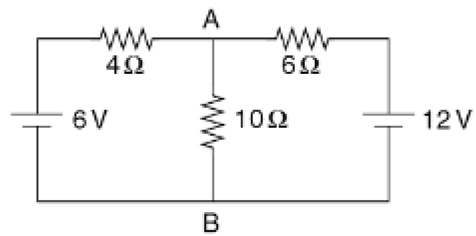
14. Determine current through load by using Thevenin's Theorem.



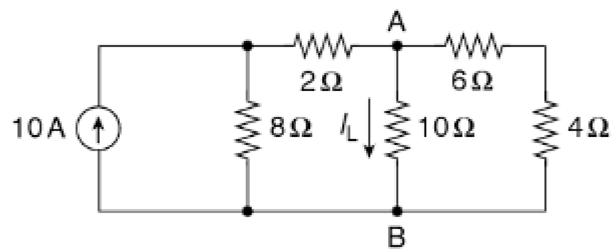
15. Determine the current through 50Ω Resistor by using Thevenin's Theorem.



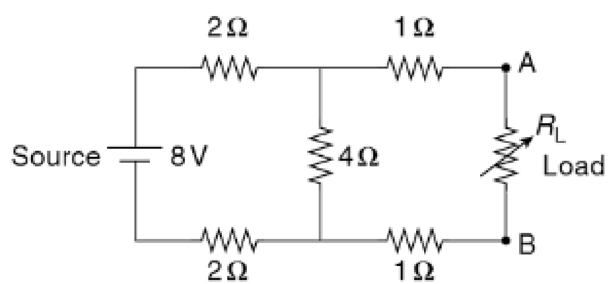
16. Using Norton's theorem, calculate the current through 10Ω resistor in the given circuit.



17. Using Norton's theorem, calculate the current through 10Ω resistor in the given circuit.



18. Calculate the value of Load Resistance, at which the power transfer will be maximum in the given circuit.



UNIT III – SINGLE PHASE CIRCUITS

Syllabus:

'j' notations – rectangular and polar coordinates – Sinusoidal voltage and current – instantaneous, peak, average and effective values – form factor and peak factor(derivation for sine wave) – pure resistive, inductive and capacitive circuits – RL,RC, RLC series circuits – impedance –phase angle – phasor diagram – power and power factor – power triangle – apparent power, active and reactive power – parallel circuits (two branches only) - Conductance, susceptance and admittance –problems on all above topics.

3.0 Introduction:

A.C means Alternating Current. The current or voltage which alternates its direction and magnitude every time. In dc circuits, Power = Voltage x Current. Everything was constant, so there was no problem between instantaneous power and average power. Resistance dissipates power, converting it to heat, light, sound or motion. It was simple and straight forward.

However in a,c circuits the voltage and current vary continuously. So taking the product VxI involves multiplying two sine waves at different angles and results in a sinusoid at twice the frequency and shifted in dc level.

3.1 'j' notation:

A complex number represents a point in a two-dimensional plane located with reference to two distinct axes. The horizontal axis is called the real axis, while the vertical axis is called the imaginary axis. In the complex plane, the horizontal or real axis represents all positive numbers to the right of the imaginary axis and all negative numbers to the left of the imaginary axis. All positive imaginary numbers are represented above the real axis, and all negative imaginary numbers, below the real axis.

In electrical circuit, a ±j prefix is used to designate numbers that lie on the imaginary axis in order to distinguish them from numbers lying on the real axis. The prefix is known as j operator.

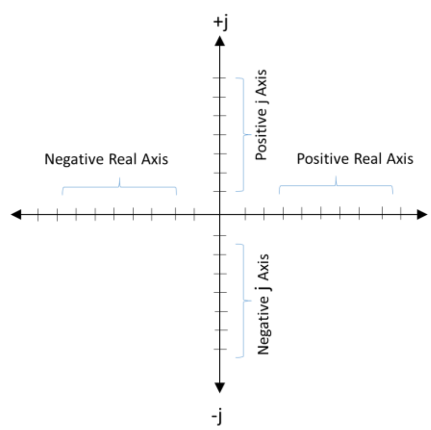
$$\boxed{j = \sqrt{-1}}$$

$$\boxed{j^2 = -1}$$

$$j^3 = j^2 j = -1j = -j$$

$$j^4 = j^2 j^2 = (-1)(-1) = +1$$

$$j^5 = j$$



3.2 Rectangular and Polar Coordinates:

Rectangular and polar are two forms of complex numbers that are to represent phasor quantities. A phasor quantities contains both magnitude and angular position or phase.

3.2.1 Rectangular Form:

A phasor quantity is represented in rectangular form by the algebraic sum of the real value (A) of the coordinate and j value (B) of the coordinate, expressed in the following general form:

$$C = A + jB$$

3.2.2 Polar Form:

Phasor quantities can also be expressed in polar form, which consists of the phasor magnitude (A) and the angular relative to the positive real axis (θ), expressed in the following general form:

$$C = A\angle \pm \theta^{\circ}$$

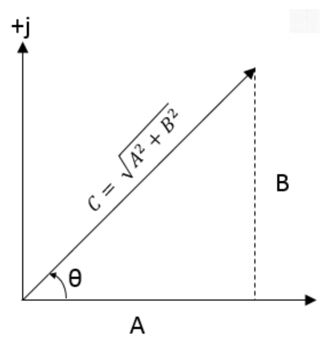
3.2.3 Rectangular to Polar Conversion:

A phasor can be visualized as forming a right triangle in the complex plane as shown in figure below. The horizontal side of the triangle is the real value A and the vertical side is the j value B. The hypotenuse of the triangle is the length of the phasor C, representing the magnitude and can be expressed as follows:

$$C = A + jB$$

$$Magnitude: \quad C = \sqrt{A^2 + B^2}$$

$$Phase\ angle \quad \theta = \tan^{-1}\left(\frac{\pm B}{A}\right)$$



3.2.4 Polar to Rectangular Conversion:

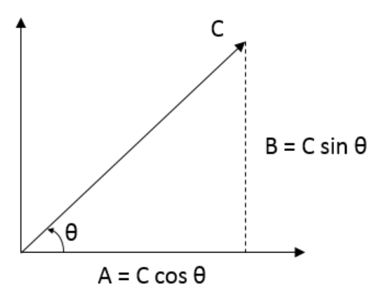
The polar form gives the magnitude and angle of a phasor quantity as shown in figure.

$$A = C\cos\theta$$

$$B = C\sin\theta$$

$$C\angle \pm \theta^{\circ} = A + jB$$

$$C\angle \pm \theta^{\circ} = C\cos\theta + jC\sin\theta$$



Example: 1

Express the following in j form.

a) 15∠30˚ b) 15∠-30˚ c) 20∠225˚ d) 5∠-200˚

Solution:

a) 15∠30˚ :

$$\text{Where, } C = 15$$
$$\theta = 30^\circ$$
$$15\angle 30^\circ = C\cos\theta + jC\sin\theta$$
$$= 15\cos 30 + j15\sin 30$$
$$= 15\ (0.86) + j15\ (0.5)$$
$$15\angle 30^\circ = 12.9 + j7.5 \quad (\text{'j' form})$$

b) 15∠-30˚ :

$$\text{Where, } C = 15$$
$$\theta = -30^\circ$$
$$15\angle\text{-}30^\circ = C\cos\theta + jC\sin\theta$$
$$= 15\cos(-30) + j15\sin(-30)$$
$$= 15(0.86) + j15(-0.5)$$
$$15\angle\text{-}30^\circ = 12.9 - j7.5 \quad (\text{'j' form})$$

c) 20∠225˚ :

$$\text{Where, } C = 20$$
$$\theta = 225^\circ$$
$$20\angle 225^\circ = C\cos\theta + jC\sin\theta$$
$$= 20\cos 225 + j20\sin 225$$
$$= 20(-0.707) + j15(-0.707)$$
$$20\angle 225^\circ = -14.14 - j10.6 \quad (\text{'j' form})$$

d) 5∠-200˚ :

$$\text{Where, } C = 5$$
$$\theta = -200^\circ$$
$$5\angle\text{-}200^\circ = C\cos\theta + jC\sin\theta$$
$$= 5\cos(-200) + j5\sin(-200)$$
$$= 5(-0.93) + j5(0.34)$$
$$5\angle\text{-}200^\circ = -4.65 + j1.7 \quad (\text{'j' form})$$

Example: 2

Express the following in polar form.

a) $3 + j4$ b) $8 - j6$ c) $8 + j6$ d) $-6 + j10$

<u>Solution:</u>

a) $3 + j4$:

$$\begin{aligned}
\text{Where, A} &= 3 \\
\text{B} &= j4 \\
\text{Magnitude} &= \sqrt{3^2 + 4^2} \\
\text{Magnitude} &= 5 \\
\text{Phase angle} \quad \theta &= \tan^{-1}\left(\frac{\pm B}{A}\right) \\
&= \tan^{-1}\left(\frac{4}{3}\right) \\
\theta &= 53.13^\circ \\
\text{In polar form} &= 5\angle 53.13^\circ
\end{aligned}$$

b) $8 - j6$:

$$\begin{aligned}
\text{Where, A} &= 8 \\
\text{B} &= -j6 \\
\text{Magnitude} &= \sqrt{8^2 + (-6)^2} \\
\text{Magnitude} &= 10 \\
\text{Phase angle} \quad \theta &= \tan^{-1}\left(\frac{\pm B}{A}\right) \\
&= \tan^{-1}\left(\frac{-6}{8}\right) \\
\theta &= -36.86^\circ \\
\text{In polar form} &= 10\angle -36.86^\circ
\end{aligned}$$

c) $8 + j6$:

$$\begin{aligned}
\text{Where, A} &= 8 \\
\text{B} &= j6 \\
\text{Magnitude} &= \sqrt{8^2 + 6^2} \\
\text{Magnitude} &= 10 \\
\text{Phase angle} \quad \theta &= \tan^{-1}\left(\frac{\pm B}{A}\right) \\
&= \tan^{-1}\left(\frac{6}{8}\right) \\
\theta &= 36.86^\circ \\
\text{In polar form} &= 10\angle 36.86^\circ
\end{aligned}$$

d) $-6 + j10$:

$$\begin{aligned}
\text{Where, A} &= -6 \\
\text{B} &= j10 \\
\text{Magnitude} &= \sqrt{(-6)^2 + 10^2} \\
\text{Magnitude} &= 11.66 \\
\text{Phase angle} \quad \theta &= \tan^{-1}\left(\frac{\pm B}{A}\right) \\
&= \tan^{-1}\left(\frac{-10}{6}\right)
\end{aligned}$$

$$\theta = -59.03^\circ$$

$$\text{In polar form} = 11.66\angle{-59.03^\circ}$$

Example: 3

Add the vectors A and B and give result in polar form.

A= 16 + j12 and B= -6 + j10

Solution:

$$A+B = (16 + j12) + (-6 + j10)$$

$$= 10 + j22$$

$$= 10+j22 \quad \text{(Vector form)}$$

$$\text{Magnitude} = \sqrt{10^2 + 22^2}$$

$$\text{Magnitude} = 24.16$$

$$\text{Phase angle} \quad \theta = \tan^{-1}\left(\frac{\pm B}{A}\right)$$

$$= \tan^{-1}\left(\frac{22}{10}\right)$$

$$\theta = 65.55^\circ$$

$$\text{In polar form } A+B = 24.16\angle 65.55^\circ$$

Example: 4

Multiply the two phasor A and B. A= 40 + j30 and B= 4 – j3

Solution:

$$A.B = (40 + j30).(4 - j3)$$

$$= (40x4) - (40 \text{ x } j3) + j(30x4) - (j30xj3)$$

$$= 160\text{-}j120\text{+}j120 + 90$$

$$A.B = 250 + j0$$

Example: 5

Find the ratio of the two phasor A and B. (A/B) A= 100 + j60 and B= 8 – j6

Solution:

$$Ratio \quad \frac{A}{B} = \frac{100 + j60}{8 - j6}$$

$$= \frac{100 + j60}{8 - j6} \, x \, \frac{8 + j6}{8 + j6}$$

$$= \frac{(100x8) + j(100x6) + j(60x8) + (j60xj6)}{(8x8) + j(8x6) - j(6x8) - (j6xj6)}$$

$$= \frac{800 + j600 + j480 - 360}{64 + j48 - j48 + 36}$$

$$= \frac{440 + j1080}{100}$$

$$= \frac{440}{100} + j\frac{1080}{100}$$

$$\frac{A}{B} = 4.4 + j10.8$$

Example: 6

Two phasors are given by A=5+j8 and B =7-j6.5. Find: i) A.B ii) A/B

Solution:

$$A = 5 + j8$$

$$\text{Magnitude} = \sqrt{5^2 + 8^2}$$

$$\text{Magnitude} = 9.43$$

$$\text{Phase angle} \quad \theta = \tan^{-1}\left(\frac{\pm B}{A}\right)$$

$$= \tan^{-1}\left(\frac{8}{5}\right)$$

$$= 57.99°$$

$$\text{In polar form:} \quad A = 9.43\angle 57.99^\circ$$

$$B = 7 - j6.5$$

$$\text{Magnitude} = \sqrt{7^2 + 6.5^2}$$

$$\text{Magnitude} = 9.55$$

$$\text{Phase angle} \quad \theta = \tan^{-1}\left(\frac{\pm B}{A}\right)$$

$$= \tan^{-1}\left(\frac{-6.5}{7}\right)$$

$$= -42.88°$$

$$\text{In polar form:} \quad B = 9.55\angle -42.88^\circ$$

i) Product

$$A.B = (9.43\angle 57.06^\circ) \text{ x } (9.55\angle -42.88^\circ)$$

$$= (9.43 \text{ x } 9.55) \angle 57.06^\circ + (-42.88^\circ)$$

$$A.B = 90.05 \angle 14.18^\circ$$

ii) Ratio

$$\frac{A}{B} = \frac{9.43 \angle 57.06^\circ}{9.55\angle -42.88^\circ}$$

$$= \frac{9.43}{9.55}\angle 57.06^\circ - (-42.88^\circ)$$

$$= 0.987\angle 99.94^\circ$$

$$\frac{A}{B} = 0.987\angle 99.94^\circ$$

3.4 A.C Voltage: (Sinusoidal Voltage)

A.C voltage or alternating voltage changes its polarity periodically and also changes its magnitude at every instant.

3.5 A.C Current: (Sinusoidal current)

An alternating current is a current which reverses its direction periodically and changes its magnitude at every instant.

3.6 Alternator:

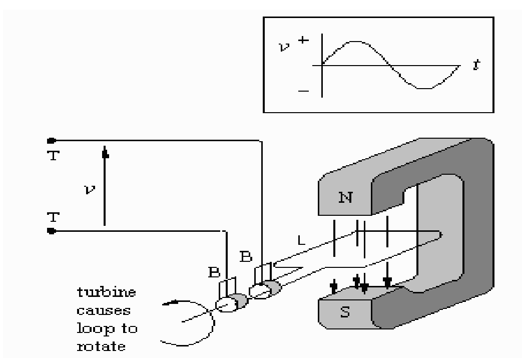
An alternating current generator is called alternator. It works on the principle of electromagnetic induction. An alternator is also called as AC Generator or synchronous generator.

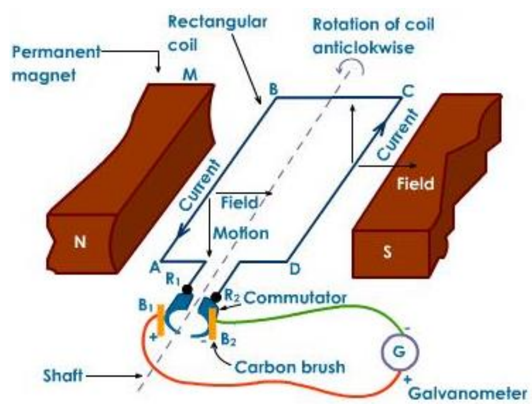
<u>Working principle of alternator:</u>

An alternator works on the principle of faraday's law of electromagnetic induction. It states that when there is a relative motion between magnetic field and a conductor an e.m.f is induced in the conductor.

Alternating voltages / current may be generated by two ways:

- ❖ By rotating a coil in a magnetic field.
- ❖ By rotating a magnetic field within a stationary coil.





The quantity of voltages / current generated depends upon:

- ❖ The number of turns in the coil.
- ❖ Strength of the magnetic field
- ❖ The speed at which the coil of magnetic fields rotates.

It is seen that the induced E.M.F. varies as sine function of the time angle ωt. This curve is known as sine wave and the E.M.F which varies in this manner is known as sinusoidal E.M.F.

$$e = E_m \sin \omega t$$

Where, e = Instantaneous voltage

E_m = Maximum voltage

ωt = Angular velocity of the coil

The induced sinusoidal voltages produce sinusoidal current. These values are $I_m \sin \omega t$.

$i = I_m \sin \omega t$ Where, I_m = Maximum value of alternating current.

3.7 A.C Waveform:

The shape obtained by plotting the instantaneous ordinate values of either voltage or current against time is called an AC Waveform. An AC waveform is constantly changing its polarity every half cycle and alternating between a positive maximum value and a negative maximum value respectively with respect to time.

The sinusoidal waveform or sine wave is the fundamental type of alternating current and alternating voltage. It is also referred to as a sinusoidal wave or simply sinu soid. The electrical service provided by the power company (Electricity Board) is in the form of sinusoidal voltage or current.

AC waveforms can also take the shape of either *Complex Waves*, *Square Waves* or *Triangular Waves* and these are shown below.

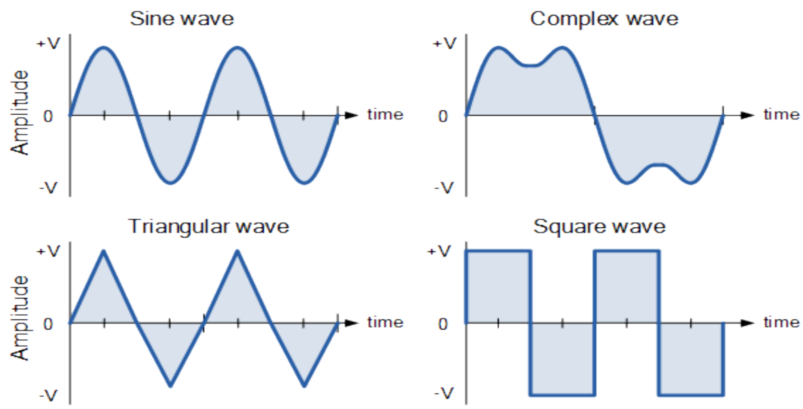


Fig. Types of periodic waveform

3.8 Instantaneous value:

The value of an alternating quantity at any instant is called instantaneous value. They are represented by small letters, i, *v*, e etc.,

Expression for instantaneous value of current:

$$i = I_m \sin \theta$$
$$i = I_m \sin \omega t$$
$$i = I_m \sin 2\pi f t$$

Expression for instantaneous value of voltage:

$$v = V_m \sin \theta$$
$$v = V_m \sin \omega t$$
$$v = V_m \sin 2\pi f t$$

3.9 Amplitude:

It is the highest value attained by the current or voltages in a half cycle either positive or negative half cycle of an alternating quantity.

3.10 Cycle:

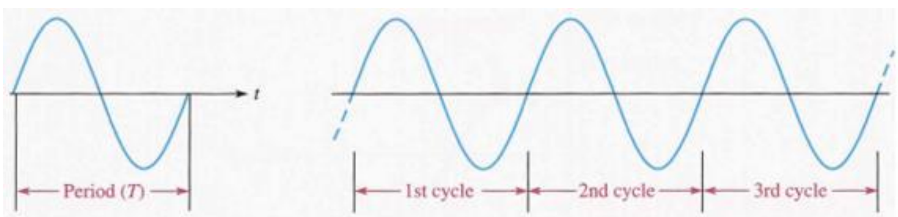
One complete set of positive and negative value of an alternating quantity is called a cycle.

3.11 Time period (T):

The time required for a sine wave to complete one full cycle is called Time period.

T = 1/F sec (or) m sec. F = Frequency.

The period of a sine wave can be measured from a zero crossing to the next corresponding zero crossing as shown in figure. The period can be also being measured from any peak in a given cycle to the corresponding peak in next cycle.



3.12 Frequency (F):

Frequency is the number of cycles that a wave completes per second. It is represented by F. Its unit is Hertz.

One hertz = 1 cycles per second

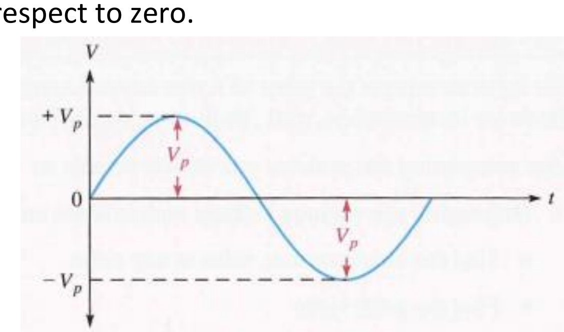
3.12.1 Relationship between frequency and Time:

$$\text{Frequency} = \frac{1}{\text{Time Period}} \qquad \text{Time Period} = \frac{1}{\text{Frequency}}$$

$$F = \frac{1}{T} \qquad T = \frac{1}{F}$$

3.13 Peak value or Maximum value:

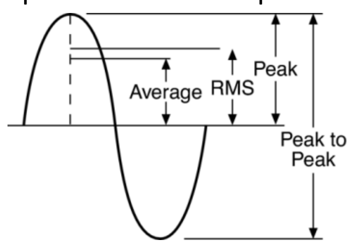
The peak value of a sine wave is the value of voltage (or current) at the positive or the negative maximum with respect to zero.



Since the positive and negative peak values are equal in magnitude, a sine wave is characterized by a single peak value. It is represented by V_p or I_p.

3.14 Peak to Peak value:

The peak to peak value of a sine wave is the voltage or current from the positive peak to negative peak. It is always twice the peak value. It is represented by V_{pp} or I_{pp}.



3.15 Average value or Mean value:

This is the average of the instantaneous values of an alternating quantity over one complete cycle of the a.c waveform. It is also known as Mean value.

To obtain the average, divide the period into 'n' equal intervals. At these individual intervals let the current are i_1, i_2, i_3 …… i_n.

Thus the average value of the current and voltage are as follows:

$$I_{av} = \frac{i_1 + i_2 + i_3 + \cdots \ldots . + i_n}{n}$$

Where, i_1, i_2 …. - instantaneous values

$$V_{av} = \frac{v_1 + v_2 + v_3 + \cdots \ldots + v_n}{n}$$

Where, v_1, v_2 ….. - instantaneous values

Average value can also be defined as the ratio of the total area under one half cycle to the base period.

$$Average\ \text{Value} = \frac{\text{Area Under the curve of half cycle}}{\text{Base of half cycle}}$$

3.15.1 Expression for Average Value of Sine wave:

Instantaneous value of current $i = I_m\, Sin\, \theta \qquad Where, \theta = \omega t$

Consider an elementary strip of width dθ in the first half cycle of current wave. Let i be the mid-ordinate of this strip. Then,

$$\text{Area of strip} = \text{i}\, d\theta$$

$$\text{Area under the curve for half cycle} = \int_0^{\pi} I_m \,\text{Sin}\, \theta\, d\theta$$

$$= I_m \int_0^{\pi} \text{Sin}\, \theta\, d\theta$$

$$= I_m \, [-\cos\theta]_0^{\pi}$$
$$= I_m \, [-\cos\pi - (-\cos 0)]$$
$$= I_m \, [-(-1) + 1]$$
$$= I_m \, [1 + 1]$$
$$\text{Area under the curve for half cycle} = 2I_m$$

$$\text{Average Value} \quad I_{av} = \frac{\text{Area of Half Cycle}}{\text{Base length of half cycle}}$$
$$I_{av} = \frac{2\, I_m}{\pi}$$
$$I_{av} = 0.637\, I_m$$
$$I_{av} = 63.7\%\, I_m$$

Similarly,

$$\text{Average Voltage,} \quad V_{av} = \frac{2\, V_m}{\pi}$$
$$V_{av} = 0.637\, V_m$$
$$V_{av} = 63.7\%\, V_m$$

3.16 RMS Value:

Root Mean Square value of an alternating current is given by the steady D.C current, which produce the same heat as that produced by the alternating current in a given time and given resistance.

To obtain the average, divide the period into 'n' equal intervals. At these individual intervals let the current are i_1, i_2, i_3 …… i_n.

Thus the r.m.s value of the current and voltage are as follows:

$$I_{rms} = \frac{{i_1}^2 + {i_2}^2 + {i_3}^2 + \cdots \ldots + {i_n}^2}{n}$$

Where, i_1, i_2 …. - instantaneous values

$$V_{rms} = \frac{{v_1}^2 + {v_2}^2 + {v_3}^2 + \cdots \ldots + {v_n}^2}{n}$$

Where, v_1, v_2 …. - instantaneous values

R.M.S value is also defined as follows:

$$\text{RMS Value} = \sqrt{\frac{\text{Area under the squared curve}}{\text{Base length or period}}}$$

3.16.1 Expression for RMS Value:

Instantaneous value of current $i = I_m \, Sin\, \theta \qquad Where, \theta = \omega t$

Consider an elementary strip of width $d\theta$ in the first half cycle of current wave. Let i^2 be the mid-ordinate of this strip.

$$\text{Area of strip} = i^2\, d\theta$$

Area under squared curve for full cycle $= \int_0^{2\pi} (I_m \sin\theta)^2 \, d\theta$

$$= \int_0^{2\pi} I_m{}^2 \sin^2\theta \, d\theta$$

$$= I_m{}^2 \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= I_m{}^2 \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta$$

$$= I_m{}^2 \int_0^{2\pi} \left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right) d\theta$$

$$= I_m{}^2 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$= I_m{}^2 \left[\frac{2\pi}{2} - \frac{\sin 2.2\pi}{4} - \frac{0}{2} + \frac{\sin 0}{4}\right]$$

$$= I_m{}^2 [\pi - 0 - 0 + 0]$$

Area under squared curve for full cycle $= I_m{}^2 \pi$

RMS Value of current $I_{rms} = \sqrt{\frac{\text{Area under the squared curve}}{\text{Base length or period}}}$

$$I_{rms} = \sqrt{\frac{I_m{}^2 \pi}{2\pi}}$$

$$= \sqrt{\frac{I_m{}^2}{2}}$$

$$= \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = 0.707 \, I_m$$

$$I_{rms} = 70.7\% \, I_m$$

Similarly,

RMS Value of voltage $V_{rms} = \frac{V_m}{\sqrt{2}}$

$$V_{rms} = 0.707 \, V_m$$

$$V_{rms} = 70.7\% \, V_m$$

3.17 Form factor:

It is defined as the ratio of RMS value to the average value of alternating current or voltage.

$$\text{Form Factor} = \frac{\textbf{RMS Value}}{\textbf{Average Value}}$$

$$= \frac{\frac{I_m}{\sqrt{2}}}{\frac{2I_m}{\pi}} = \frac{I_m}{\sqrt{2}} * \frac{\pi}{2I_m} = \frac{\pi}{2\sqrt{2}} = 1.11$$

3.18 Peak factor: (Crest factor)

It is defined as the ratio of maximum value to the RMS value of alternating current or voltage.

$$\text{Peak Factor} = \frac{\textbf{Maximum Value}}{\textbf{RMS Value}}$$

$$= \frac{I_m}{\frac{I_m}{\sqrt{2}}} = I_m * \frac{\sqrt{2}}{I_m} = \sqrt{2} = 1.414$$

Example: 7

The alternating current passing through a circuit is being by i = 141.4 sin 314.2t. Calculate (a) maximum value of current (b) r.m.s value of current (c) the frequency and (d) the instantaneous value of the current when t=0.02 sec.

Given Data:

instantaneous value (i) = 141.4 sin 314.2t

To Find:

i. maximum value of current (I_m)

ii. RMS Current (I_{rms})

iii. Frequency (F)

iv. ‘I’ when t=0.02sec

Solution:

$$\text{Maximum value:} \quad I_m = 141.4\text{ A}$$

$$\text{RMS Value:} \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$= \frac{141.4}{\sqrt{2}}$$

$$I_{rms} = 100\text{ A}$$

$$\text{Angular velocity:} \quad \omega = 2\pi f$$

$$= 314.2$$

$$\text{Frequency:} \quad F = \frac{314.2}{2\pi}$$

$$F = 50\text{ Hz}$$

When $t = 0.02$ sec

$$\text{Instantaneous value of current:} \quad i = 141.4 \sin 314 \times 0.02$$

$$= 141.4 \sin\left(\frac{180}{\pi}\right) \times 314.2 \times 0.02$$

$$= \quad 141.4 \sin 360$$

$$i \quad = \quad 0$$

Answer:

i) Maximum value $I_m = 141.4$ A ii) RMS Value $I_{rms} = 100$ A

iii) Frequency (f) = 50 Hz iv) Instantaneous value of current i = 0

Example: 8

What is the equation for a sinusoidal current of 25 Hz frequency having an RMS value of 40 Ampere?

Given Data:

RMS value of current (I_{rms}) = 40 A

Frequency (F) = 25 Hz

To Find:

Equation of sinusoidal current.

Solution:

Instantaneous value of current: $i = I_m \sin 2\pi ft$

$$I_m = I_{rms} \times \sqrt{2}$$

$$= 40 \times \sqrt{2}$$

$$I_m = 56.57 \text{ A}$$

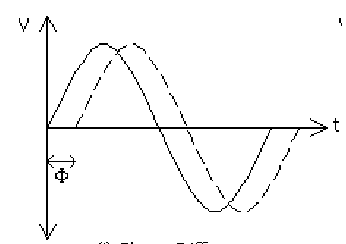
$$i = 56.57 \sin 157t$$

3.19 Phase:

The phase of an alternating quantity at any time 't' is defined by the angle by which the phasor makes with reference value.

3.19.1 Phase Difference:

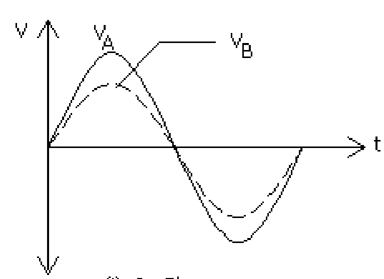
The difference in angle (ϕ) between two voltages or currents is known as *phase difference*.



(i) Phase Difference

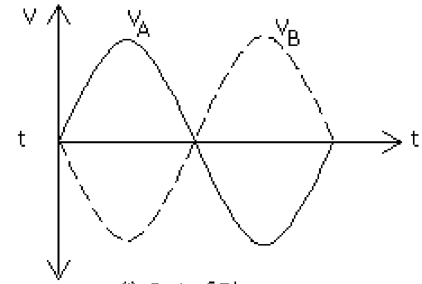
3.19.2 In-phase:

If the phase difference between the two voltages is zero, then that are said to be in-phase. In fig, V_A and V_B start at the same point and reach the maximum value at the same time. The angle between V_A and V_B is equal to zero.



(i) In Phase

3.19.3 Out of phase:



(i) Out of Phase

If the phase difference between two voltages is 180° then they are said to be out of phase. From the figure V_A and V_B are out of phase. Their starting points are same, but when voltage V_A reaches its positive maximum value, V_B reaches its negative maximum value.

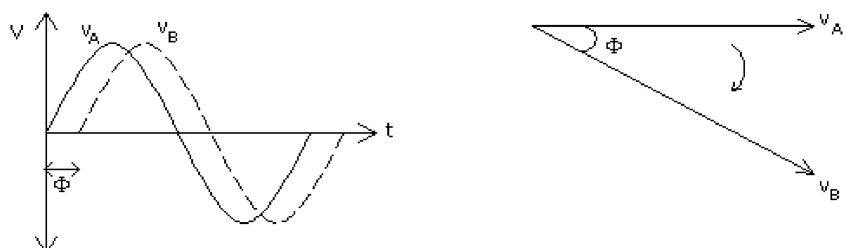
3.20 Phase angle (θ):

It is defined as the angle between the voltage and current. It is represented by **'θ'**.

3.20.1 Phase lag:

A lagging alternating quantity is one, which reaches its maximum (or zero) value later as compared to another alternating quantity. From the figure the alternating voltage V_B reaches its maximum value (or zero value) later by ϕ degrees when compared to V_A.

$V_A = V_m \sin \omega t$ and $V_B = V_m \sin(\omega t - \phi)$



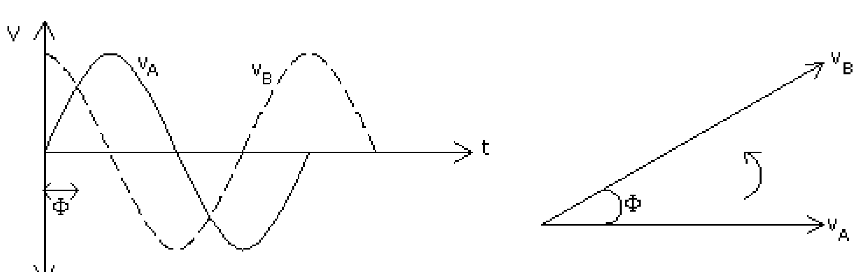
(i) Phase lag

The phasor diagram clearly shows that V_B lags V_A by an angle ϕ. A negative sign is used to denote the lagging angle.

3.20.2 Phase lead:

A leading quantity is one which reaches its maximum (or zero) value earlier than the other alternating quantity. From the figure, the alternating voltage V_B reaches its maximum or zero value earlier by ϕ degrees when compared to V_A. (V_B is leading than V_A)

$V_A = V_m \sin \omega t$, and $V_B = V_m \sin(\omega t+\phi)$ Where, '+' sign indicates leading



(ii) Phase Lead

3.20.3 Phasor diagram:

So, far we have used phasor diagrams to solve problems on a.c circuits. A phasor diagram is a graphical representation of the phasors (i.e voltages and currents) of an a.c circuits and may not yield quick results in case of complex circuits. Engineers have developed techniques to represent a phasor i an algebraic (i.e mathematical) form. Such a technique is known as *phasor algebra or complex algebra*. Phasor algebra has provided a relatively simple but powerful tool for obtaining quick solution of a.c circuits. It simplifies the mathematical manipulation of phasors to a great extent.

In this chapter, we shall discuss the various methods of representing phasors in a mathematical form and their applications to a.c circuits.

Consider a phasor V lying along OX axis. If we multiply this phasor by –1, the phasor is reversed i.e, it is rotated through 180° in the counter clockwise (CCW) direction.

Suppose this factor is j multiplying the phasor by j^2 rotates the phasor through 180° in CCW direction. This means that multiplying the phasor by j is the same as multiply by –1.

$$j^2 = -1$$

We arrive at a very important conclusion that when a phasor is multiplied by j, the phasor is rotated through 90° in the CCW direction. Each successive multiplication by j rotates the phasor through an additional 90° in the CCW direction. It is easy to see that multiplying a phasor by,

$\mathrm{j} = \sqrt{-1}$ ……… 90° CCW direction from OX axis

$\mathrm{j}^2 = -1$ ……… 180° CCW direction from OX axis

$\mathrm{j}^3 = \mathrm{j}^2.\mathrm{j} = -\mathrm{j}$ ……….270°CCW direction from OX axis

$\mathrm{j}^4 = \mathrm{j}^2.\mathrm{j}^2 = 1$ ………..360° CCW direction from OX axis

3.20.4 Reference Phasor:

The reference phasor is normally drawn horizontally along x-axis.

3.20.5 Angular velocity (ω):

The angle of one full cycle is 2π radians. When one second is covered in ‘f’ cycles, the angle covered per second is 2πf radians.

Angular velocity, ω = 2πf radians / second.

3.21 Impedance (Z):

It is the opposition offered by an AC circuit to the flow of AC current. It is denoted by the letter 'Z'. Its unit is Ohm (Ω).

$$\text{Impedance (Z)} = \frac{V}{I}$$

Impedance in complex quantity: $Z = R + jX$

Where, Real part : R - Resistance

Imaginary part: X - Reactance.

3.22 Inductive Reactance (X_L):

The opposition offered by an inductance to current flow is called inductive reactance. It is denoted by the letter X_L and its unit is Ohm (Ω).

$$\text{Inductive Reactance } X_L = 2\pi fL$$

3.23 Capacitive Reactance (X_C):

The opposition offered by a capacitance to current flow is called capacitive reactance. It is denoted by the letter X_C and its unit is ohm (Ω).

$$Xc = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

3.24 Admittance (Y):

It is defined as the reciprocal of impedance. It is represented by the letter 'Y' and its unit is mho (℧) or siemen.

$$Y = \frac{1}{Z}$$

Admittance (Y) in complex quantity $Y = G + jB$

Where, Real part : G - Conductance

Imaginary part : B - Susceptance

3.25 Conductance (G):

It is defined as the reciprocal of resistance. It is represented by the letter G and its unit is mho (℧) or siemen.

$$G = \frac{1}{R}$$

3.26 Susceptance (B):

Susceptance is defined as the reciprocal of reactance. Its unit is also mho. It is denoted by the letter 'B' and its unit is mho (℧) or siemen.

$$B = \frac{1}{X}$$

3.27 Power (P):

Power is generally defined as the rate at which the work is done.

3.27.1 Real power (P):

It is a resistive power that is dissipated as heat. Its unit is Watts. It is always positive.

$$P = VI\ \cos\theta$$

3.27.2 Reactive Power (Q):

It is the product of the power developed in reactance of the circuit. Its unit is Volt-Ampere Reactive (VAR).

$$Q = VI\ \sin\ \theta$$

3.27.3 Apparent Power (S):

It is defined as the product of magnitude of voltage and magnitude of current. It is measured in Volt-Ampere (VA).

$$S = VI$$

3.28 Power Factor (cos θ):

Power actor is defined as the cosine value of phase angle between the voltage and current.

Or

Power factor is defined as the ratio of real power to apparent power. It has no unit. It is always less than unity.

$$\text{Power Factor} = \frac{\text{Real Power}}{\text{Apparent Power}} = \frac{VI\cos\theta}{VI}$$

or

It is the ratio of resistance to the impedance of the a.c circuit.

$$\text{Power Factor} = \frac{R}{Z}$$

3.29 Voltage and Current Relationship in Pure Resistance:

Consider a circuit having a resistor of resistance R ohm and connected across a.c source.

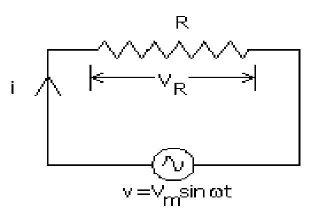


Fig. (i) Circuit Arrangement

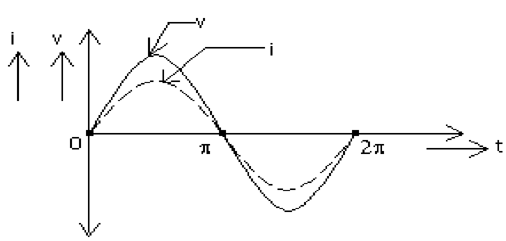


Fig. (ii) Wave form representation

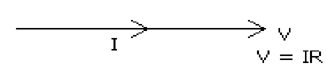


Fig. (iii) Phasor representation

Let 'v' be the AC voltage applied across the pure resistance 'R' and 'i' be the current flowing through the resistor.

According to ohm's law :

$$v = I.R$$

$$i = \frac{v}{R}$$

$$v = V_m \sin\omega t \qquad \text{--------- (1)}$$

$$i = \frac{V_m \sin\omega t}{R} = \frac{V_m}{R}.\sin\omega t \qquad \text{--------- (2)}$$

$$\text{where,} \quad i_m = \frac{V_m}{R}$$

$$\text{Hence,} \quad i = i_m.\sin\omega t \qquad \text{--------- (3)}$$

From equation (1): Instantaneous voltage $(v) = V_m \sin\omega t$

From equation (3): Instantaneous current $(i) = I_m \sin\omega t$

$I = 0$ when $v = 0$, so $i \; \alpha \; v$

From equation (1) & (3): Phase angle between v & i $= 0$

Power factor $(\cos\theta) = \cos\theta = \cos 0 = 1$ (unity)

Relationship between v & i = in phase

Instantaneous power : $p = v \text{ x } i$

Instantaneous power : $p = V_m \sin\omega t \text{ x } I_m \sin(\omega t \pm \theta)$

The average power over one complete cycle $= P_{av}$

Average power :

$$P_{av} = \frac{1}{\pi}\int_0^{\pi} p \, d(\omega t)$$

$$P_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \text{ x } I_m \sin\omega t \, d(\omega t)$$

$$P_{av} = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\omega t \, d(\omega t)$$

$$P_{av} = \frac{V_m I_m}{\pi}\left[\frac{1-\cos 2\omega t}{2}\right]_0^{\pi} d(\omega t)$$

$$P_{av} = \frac{V_m I_m}{2\pi}[1-\cos 2\omega t]_0^{\pi} d(\omega t)$$

$$P_{av} = \frac{V_m I_m}{2\pi}\left[\omega t - \frac{\sin 2\,\omega t}{2}\right]_0^{\pi}$$

$$P_{av} = \frac{V_m I_m}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$P_{av} = \frac{V_m I_m}{2\pi}[\pi]$$

$$P_{av} = \frac{V_m I_m}{2}$$

$$P_{av} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

$$\textit{Average Power } P_{av} = V \; I$$

Where, V and I are rms values

3.30 Voltage and Current Relationship in Pure Inductance:

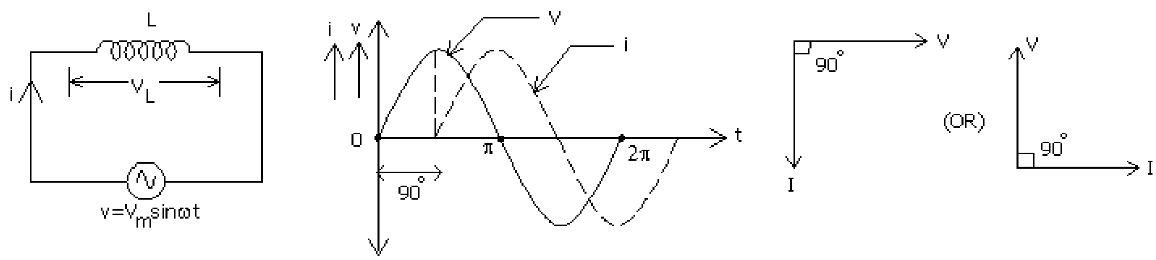


Fig. (i) Circuit Arrangement Fig. (ii) Wave form representation Fig. (iii) Phasor representation

When an alternating voltage is applied to an inductive coil, the back emf produced due to self-inductance opposes the rise or fall of current through it. Since there is no ohmic drop, the applied voltage has to overcome this induced e.m.f only.

Let, $v = V_m \sin \omega t$ --------- (1)

The voltage across the coil, $v = L\frac{di}{dt}$

$v\,dt = L\,di$ --------- (2)

Where, L = Self inductance of the coil

$\frac{di}{dt}$ = rate of change of current

By integrating the equation (2),

$$\int v\,dt = \int L\,di$$

$$\frac{1}{L}\int v\,dt = \int di$$

$$\frac{1}{L}\int v\,dt = i$$

$$\boxed{i = \frac{1}{L}\int v\,dt}$$

we know that $v = V_m \text{Sin } \omega t$

$$\because \quad i = \frac{1}{L}\int V_m \text{Sin } \omega t\, dt$$

$$i = \frac{V_m}{L}\int \text{Sin } \omega t\, dt$$

$$i = \frac{V_m}{L}\left[\frac{-\text{Cos } \omega t}{\omega}\right]$$

$$i = \frac{V_m}{\omega L}[-\text{Cos } \omega t]$$

Let $\omega L = X_L$ = Inductive Reactance

$$\therefore i = \frac{V_m}{X_L}[-\text{Cos } \omega t]$$

Since $\text{Sin } (\theta - 90^\circ) = -\text{Cos } \theta$, we can write,

$$i = \frac{V_m}{X_L}\text{Sin } (\omega t - 90^\circ)$$

Let, $\frac{V_m}{X_L} = I_m$ = Maximum value of current

$$\therefore i = I_m \text{Sin}(\omega t - 90^\circ) \quad \dots\dots\dots\dots\dots(3)$$

From the equations (1)and (3)it is very clear that the current lags the voltage by 90°, in a pure inductance.

From equation (1):	Instantaneous voltage (v)	=	$V_m \sin \omega t$
From equation (3):	Instantaneous current (i)	=	$I_m \sin(\omega t - 90^\circ)$
From equation (1) & (3):	Phase angle between v & i	=	90°
	Power factor ($\cos\theta$)	=	$\cos\theta = \cos 90 = 0$ (lagging)
	Relationship between v & i	=	Current lags the voltage by 90°
Average power (P)		=	v x i
		=	$V_m \sin \omega t \times I_m \sin(\omega t - 90^\circ)$

$$\text{Average Power, } P = \frac{1}{\pi}\int_0^{\pi} p\,d(\omega t)$$

$$P = \frac{1}{\pi}\int_0^{\pi} -\frac{V_m I_m}{2}\text{Sin } 2\omega t\, d(\omega t) = \frac{-V_m I_m}{2\pi}\int_0^{\pi}\text{Sin } 2\omega t\, d(\omega t)$$

$$P = \frac{-V_m I_m}{2\pi}\left[\frac{-\text{Cos } 2\omega t}{2}\right]_0^{\pi} = \frac{-V_m I_m}{2\pi}\left[\frac{-\text{Cos } 2\pi}{2} + \frac{\text{Cos } 0}{2}\right]$$

$$P = \frac{-V_m I_m}{2\pi}\left[\frac{-1}{2} + \frac{1}{2}\right] = -\frac{V_m I_m}{2\pi}[0] = 0$$

$$\boxed{\text{Average Power, } P = 0}$$

Thus the average power in a pure inductive circuit is zero.

3.31 Voltage and current relationship in capacitance:

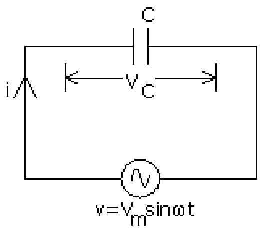


Fig. (i) Circuit Arrangement

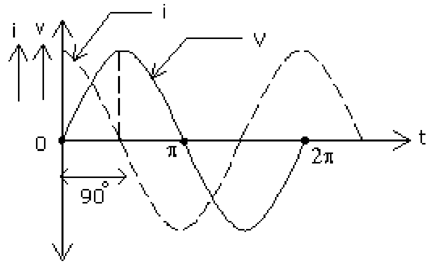


Fig. (ii) Wave form representation

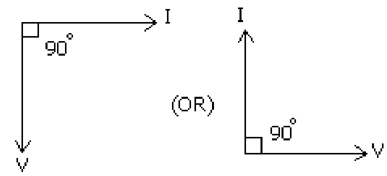


Fig. (iii) Phasor representation

Let 'v' is the instantaneous voltage applied across the capacitance 'C' and 'i' is the AC current flowing through it.

$$\text{Let, } v = V_m \sin \omega t \qquad \text{--------- (1)}$$

$$\text{The current through capacitor, } i = C\frac{dv}{dt} \qquad \text{--------- (2)}$$

$$\text{Where, } C = \text{Capacitance}$$

$$\frac{dv}{dt} = \text{rate of change of voltage}$$

Substitue the value of v in equation (2),

$$i = C\frac{dv}{dt}$$

$$i = C\frac{dV_m \sin \omega t}{dt}$$

$$i = CV_m\frac{d \sin \omega t}{dt}$$

$$i = CV_m \cos \omega t \,.\, \omega$$

$$i = C\omega\, V_m \cos \omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \cos \omega t$$

$$\text{Let, } \frac{1}{\omega C} = X_C = \text{Capacitive Reactance}$$

$$i = \frac{V_m}{X_C} \cos \omega t$$

$$\text{Since, } \sin(\theta + 90^\circ) = \cos\theta$$

$$\text{or } \sin(\omega t + 90^\circ) = \cos \omega t$$

$$i = \frac{V_m}{X_C} \sin(\omega t + 90^\circ)$$

$$\text{Let, } I_m = \frac{V_m}{X_C}$$

$$i = I_m \sin(\omega t + 90^\circ) \qquad \text{--------- (3)}$$

From the equations (1)and (3)it is very clear that the current leads the voltage by 90°, in a pure capacitive circuit.

From equation (1):	Instantaneous voltage (v)	=	$V_m \sin \omega t$
From equation (3):	Instantaneous current (i)	=	$I_m \sin(\omega t + 90^\circ)$
From equation (1) & (3):	Phase angle between v & i	=	90°
	Power factor ($\cos\theta$)	=	$\cos\theta = \cos 90 = 0$ (leading)
	Relationship between v & i	=	Current leads the voltage by 90°
	Average power (P)	=	v x i
	P	=	$V_m \sin \omega t \times I_m \sin(\omega t + 90^\circ)$

$$\text{Average Power, } P = \frac{1}{\pi}\int_0^{\pi} p\, d(\omega t)$$

$$P = \frac{1}{\pi}\int_0^{\pi} \frac{V_m I_m}{2} \sin 2\omega t\, d(\omega t) = \frac{V_m I_m}{2\pi}\int_0^{\pi} \sin 2\omega t\, d(\omega t)$$

$$P = \frac{V_m I_m}{2\pi}\left[\frac{-\cos 2\omega t}{2}\right]_0^{\pi} = \frac{V_m I_m}{2\pi}\left[\frac{-\cos 2\pi}{2} + \frac{\cos 0}{2}\right]$$

$$P = \frac{V_m I_m}{2\pi}\left[\frac{-1}{2} + \frac{1}{2}\right] = \frac{V_m I_m}{2\pi}[0] = 0$$

$$\boxed{\text{Average Power, } P = 0}$$

Thus the average power in a pure capacitive circuit is zero.

3.32 SERIES RL CIRCUITS:

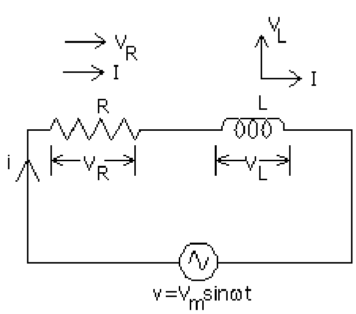


Fig. (i) RL Series Circuit

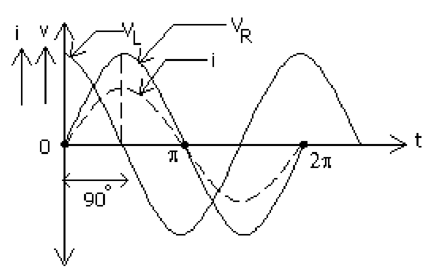


Fig. (ii) Wave form representation

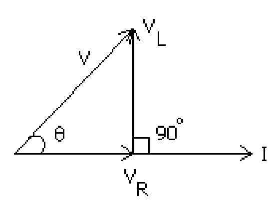


Fig. (iii) Phasor Diagram

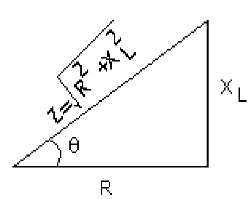


Fig. (iv) Impedance Triangle

Consider RL series circuit is shown in the figure (i), a pure resistance R and a pure inductive coil of inductance L are connected in series.

$$\text{r.m.s value of applied voltage} = V$$
$$\text{Current through R and L} = I \qquad \text{(Same current)}$$
$$\text{Reference phasor} = I \qquad \text{(Same current)}$$
$$\text{Voltage drop across the resistance} = V_R = I\,R \qquad \text{(in − phase with current)}$$
$$\text{Voltage drop across the inductive coil} = V_L = I\,X_L \qquad \text{(I lags the V by } 90^\circ\text{)}$$
$$\text{Voltage V} = \text{Vector sum of } V_R \text{ and } V_L$$
$$\text{Phase angle difference between V \& I} = \theta$$
$$\text{From phasor diagram,} \quad V^2 = {V_R}^2 + {V_L}^2$$
$$V = \sqrt{{V_R}^2 + {V_L}^2} = \sqrt{(IR)^2 + (IX_L)^2}$$
$$= I\sqrt{R^2 + {X_L}^2}$$
$$\frac{V}{I} = \sqrt{R^2 + {X_L}^2}$$
$$Z = \sqrt{R^2 + {X_L}^2}$$
$$\tan\theta = \frac{X_L}{R}$$
$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$
$$\tan\theta = \frac{X_L}{R}$$

Average Power:

$$\text{Instantaneous voltage (v)} = V_m \sin\omega t$$
$$\text{Instantaneous current (i)} = I_m \sin(\omega t - \theta)$$
$$\text{Phase angle between v \& i} = \theta$$
$$\text{Power factor } (\cos\theta) = \cos\left[\tan^{-1}\left(\frac{X_L}{R}\right)\right]$$
$$\text{Relationship between v \& i} = \text{Current lags the voltage by } \theta$$
$$\text{Instantaneous power:} \quad p = v \times i$$
$$\text{Instantaneous power:} \quad p = V_m \sin\omega t \times I_m \sin(\omega t - \theta)$$
$$p = V_m I_m [\sin\omega t \times \sin(\omega t - \theta)]$$
$$p = V_m I_m \left[\frac{\cos\theta - \cos(2\omega t - \theta)}{2}\right]$$
$$p = \frac{V_m I_m}{2}\cos\theta - \frac{V_m I_m}{2}\cos(2\omega t - \theta)$$

The average power over one complete cycle $= P_{av}$

Average power : $$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m\, I_m}{2}\cos\theta - \frac{V_m\, I_m}{2}\cos(2\omega t - \theta)\, d\theta$$

$$P_{av} = \frac{1}{2\pi}.\frac{V_m\, I_m}{2}\int_0^{2\pi} \cos\theta - \cos(2\omega t - \theta)\, d\theta$$

$$P_{av} = \frac{1}{2\pi}.\frac{V_m\, I_m}{2}\left[\int_0^{2\pi} \cos\theta\ d\theta \ - \int_0^{2\pi} \cos(2\omega t - \theta)\, d\theta\right]$$

$$P_{av} = \frac{V_m\, I_m}{2}\cos\theta$$

$$P_{av} = \frac{V_m}{\sqrt{2}}\ \frac{I_m}{\sqrt{2}}\cos\theta$$

$$P_{av} = V\ \ I\cos\theta$$

Where, V and I are rms values

3.33 SERIES RC CIRCUITS:

Consider RC series circuit is shown in the figure (i), a pure resistance R and a pure capacitance C are connected in series.

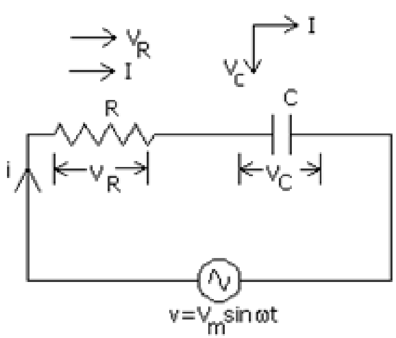


Fig. (i) RC Series Circuit

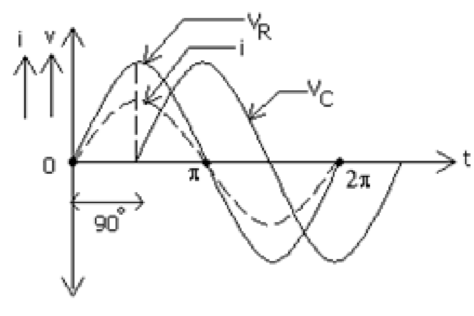


Fig. (ii) Wave form representation

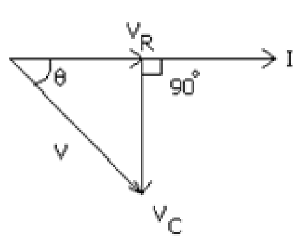


Fig. (iii) Phasor Diagram

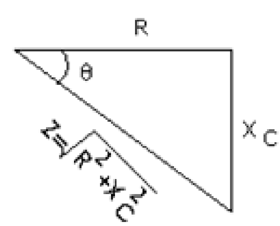


Fig. (iv) Impedance Triangle

Consider RC series circuit is shown in the figure (i), a pure resistance R and a pure capacitor of capacitance C are connected in series.

r.m.s value of applied voltage $= V$

Current through R and L $= I$ (Same current)

Reference phasor $= I$ (Same current)

Voltage drop across the resistance $= V_R = I\,R$ (in − phase with current)

Voltage drop across the capacitor $= V_C = I\,X_C$ (I leads the V by 90°)

Voltage V $=$ Vector sum of V_R and V_C

Phase angle difference between V & I $= \theta$

From phasor diagram, $V^2 = {V_R}^2 + {V_C}^2$

$$V = \sqrt{{V_R}^2 + {V_C}^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + {X_C}^2}$$

$$\frac{V}{I} = \sqrt{R^2 + {X_C}^2}$$

$$Z = \sqrt{R^2 + {X_C}^2}$$

$$\tan\theta = \frac{X_C}{R}$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$\tan\theta = \frac{X_C}{R}$$

Average Power:

Instantaneous current (v) $= V_m \sin\omega t$

Instantaneous voltage (i) $= I_m \sin(\omega t + \theta)$

Phase angle between v & i $= \theta$

Power factor ($\cos\theta$) $= \cos\left[\tan^{-1}\left(\frac{X_C}{R}\right)\right]$

Relationship between v & i $=$ Current leads the voltage by θ

Instantaneous power : $p = v \times i$

Instantaneous power : $p = V_m \sin\omega t \times I_m \sin(\omega t + \theta)$

$$p = V_m\, I_m\, [\sin\omega t \times \sin(\omega t + \theta)]$$

$$p = V_m\, I_m \left[\frac{\cos(-\theta) + \cos(2\omega t + \theta)}{2}\right]$$

$$p = \frac{V_m\, I_m}{2}\cos\theta - \frac{V_m\, I_m}{2}\cos(2\omega t + \theta)$$

The average power over one complete cycle $= P_{av}$

Average power :

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m\, I_m}{2}\cos\theta - \frac{V_m\, I_m}{2}\cos(2\omega t+\theta)\, d\theta$$

$$P_{av} = \frac{1}{2\pi}.\frac{V_m\, I_m}{2}\int_0^{2\pi} \cos\theta - \cos(2\omega t+\theta)\, d\theta$$

$$P_{av} = \frac{1}{2\pi}.\frac{V_m\, I_m}{2}\left[\int_0^{2\pi} \cos\theta\; d\theta \;-\; \int_0^{2\pi} \cos(2\omega t+\theta)\, d\theta\right]$$

$$P_{av} = \frac{V_m\, I_m}{2}\cos\theta$$

$$P_{av} = \frac{V_m}{\sqrt{2}}\;\frac{I_m}{\sqrt{2}}\cos\theta$$

$$P_{av} = V\;\; I\cos\theta$$

Where, V and I are rms values

3.34 SERIES RLC CIRCUIT:

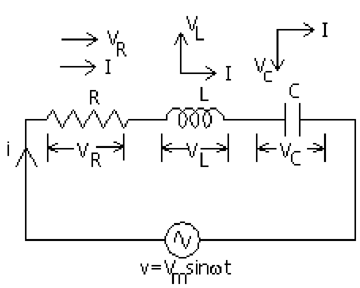


Fig. (i) RLC Series Circuit

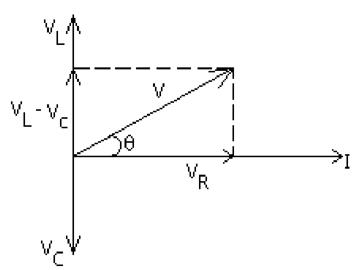


Fig. (ii) Phasor diagram ($X_L > X_C$)

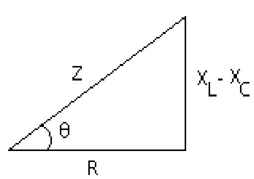


Fig. (iii) Impedance Triangle ($X_L > X_C$)

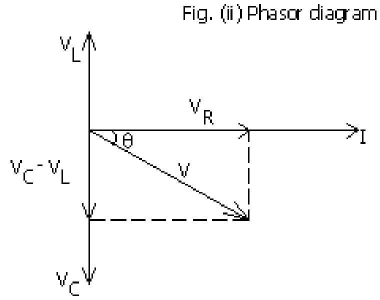


Fig. (iv) Phasor diagram ($X_C > X_L$)

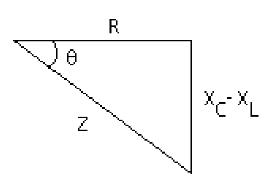


Fig. (v) Impedance Triangle ($X_C > X_L$)

Consider RLC series circuit is shown in the figure (i), a pure resistance R, a pure inductance L and a pure capacitance C are connected in series.

r.m.s value of applied voltage $= V$

Current through R and L $= I$ (Same current)

Reference phasor $= I$ (Same current)

Voltage drop across the resistance $= V_R = I\,R$ (in – phase with current)

Voltage drop across the inductor $= V_L = I\,X_L$ (I lags the V by 90°)

Voltage drop across the capacitor $= V_C = I\,X_C$ (I leads the V by 90°)

Voltage V $=$ Vector sum of V_R , V_L and V_C

Phase angle difference between V & I $= \theta$

Case – I: $X_L > X_C$

The net reactnace $X = X_L - X_C$

From phasor diagram, $V^2 = V_R{}^2 + (V_L - V_C)^2$

$$V = \sqrt{V_R{}^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\theta = \frac{(X_L - X_C)}{R}$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\tan\theta = \frac{X_L - X_C}{R}$$

Instantaneous voltage (v) $= V_m \sin\omega t$

Instantaneous current (i) $= I_m \sin(\omega t - \theta)$

Phase angle between v & i $= \theta$

Power factor $(\cos\theta)$ $= \cos\left[\tan^{-1}\left(\frac{X_L - X_C}{R}\right)\right]$

Relationship between v & i $=$ Current lags the voltage by θ

Case – II: $X_C > X_L$

The net reactnace $X = X_C - X_L$

From phasor diagram, $V^2 = V_R{}^2 + (V_C - V_L)^2$

$$V = \sqrt{V_R{}^2 + (V_C - V_L)^2} = \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$= I\sqrt{R^2 + (X_C - X_L)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_C - X_L)^2}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\tan\theta = \frac{(X_C - X_L)}{R}$$

$$\theta = \tan^{-1}(\frac{X_C - X_L}{R})$$

$$\tan\theta = \frac{X_C - X_L}{R}$$

Instantaneous current (v) = $V_m \sin\omega t$

Instantaneous voltage (i) = $I_m \sin(\omega t + \theta)$

Phase angle between v & i = θ

Power factor ($\cos\theta$) = $\cos[\tan^{-1}(\frac{X_C - X_L}{R})]$

Relationship between v & i = Current leads the voltage by θ

Average Power:

Instantaneous power : $p = v \times i$

Instantaneous power : $p = V_m \sin\omega t \times I_m \sin(\omega t \pm \theta)$

The average power over one complete cycle = P_{av}

Average power : $P_{av} = \frac{1}{2\pi}\int_0^{2\pi} p \, d\theta$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \times I_m \sin(\omega t \pm \theta)\, d\theta$$

$$P_{av} = \frac{V_m I_m}{2}\cos\theta$$

$$P_{av} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\theta$$

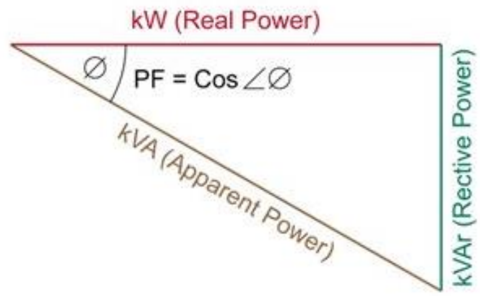
$$P_{av} = V\ I\cos\theta$$

Where, V and I are rms values

3.35 Power triangle:

The relationship among active power, reactive power and apparent power is illustrated by a right angled triangle called the power triangle. Reactive power is the vertical axis of the triangle, active power is the horizontal axis and apparent power or total power is the hypotenuse.

$$\text{Power Factor (pf)} = \frac{\text{kW (Real Power)}}{\text{kVA (Total Power)}}$$



3.35.1 Active Power:

When an active component of current is multiplied with circuit voltage, it results in active or true power. This power produces torque in motors, heat in heaters, light in lamps etc., Further wattmeter indicates this power.

3.35.2 Reactive Power:

When a reactive component of current is multiplied with circuit voltage, it results in reactive power. This power flows back and forth without doing any work. This power determines the power factor of the circuit.

3.35.3 Apparent Power:

When the circuit current is multiplied with circuit voltage, it results in apparent power. i.e product of voltage and current. In a.c circuit, there is phase difference between voltage and current so that product of voltage and current does not give real power. To avoid confusion, it is measured in volt-ampere.

3.35.4 Power Factor:

From power triangle, the power factor may also be determined by taking the ratio of true power to apparent power.

$$\text{Power Factor} = \frac{\text{True Power or Active Power}}{\text{Apparant Power}}$$

Example: 9

Determine the impedance of a RL series circuit with R=20 Ω and L=0.1H when connected to a 230V, 50Hz supply.

Given Data:

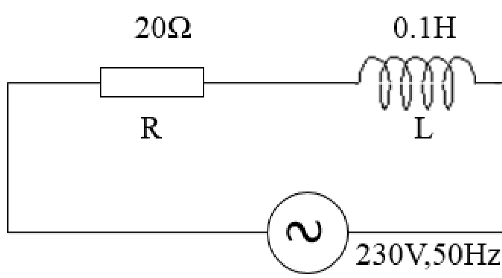
Circuit	=	R.L. Series
Resistance (R)	=	20 Ω
Inductance (L)	=	0.1 H
Frequency (F)	=	50 Hz
Voltage (V)	=	230V

To Find:

I. Inductive reactance (X_L)

ii. Impedance (Z)

Solution:



$Z = R + j X_L$

Inductive Reactance (X_L) = $2\pi fL$; Ω

= $2 \times 3.14 \times 50 \times 0.1$; Ω

X_L = 31.42 Ω

Impedance (Z) = $\sqrt{R^2 + (X_L)^2}$ Ω

Impedance (Z) = $\sqrt{20^2 + (31.42)^2}$

Impedance (Z) = 37.24 Ω

Impedance (Z) = $20 + j37.24$; Ω

Answer:

i) Inductive Reactance (X_L) = 31.42 Ω ii) Impedance (Z) = 37.24 Ω

Example: 10

Find the impedance, current and phase angle of the series circuit having a resistance of 10Ω and inductance of 10millihenry. The applied voltage is 200V, 50Hz.

Given Data:

Circuit	=	R.L. Series
Resistance (R)	=	10 Ω
Inductance (L)	=	10 mH
	=	$10 \times 10^{-3} H$
Frequency (F)	=	50 Hz
Voltage (V)	=	200V

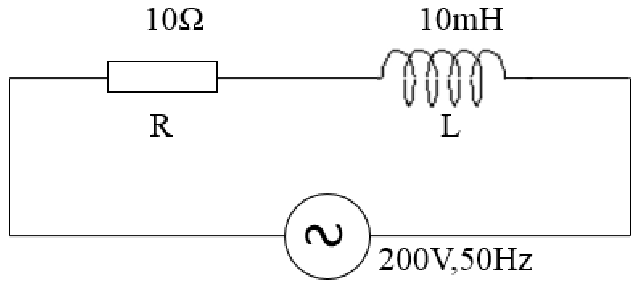
To Find:

i. Impedance (Z)

ii. Current (I)

iii. Phase angle (θ)

Solution:



Inductive Reactance (X_L) = $2\pi fL$; Ω

= $2 \times 3.14 \times 50 \times 10 \times 10^{-3}$; Ω

X_L = $3.14\ \Omega$

Impedance (Z) = $\sqrt{R^2 + (X_L)^2}\ \Omega$

Impedance (Z) = $\sqrt{10^2 + (3.14)^2}$

Impedance (Z) = $10.48\ \Omega$

Current (I) = $\frac{V}{Z}$

$I = \frac{200}{10.48}$

$I = 19.08$ Amps

Power Factor (Cos θ) = $\frac{R}{Z}$

$\cos\theta = \frac{10}{10.48} = 0.954$ lag

Phase angle θ = $\cos^{-1}0.954 = 17.44°$

Answer:

i) Impedance (Z) = 10.48 Ω

ii) Current (I) = 19.08$Amps$

iii) Phase angle (θ) = 17.44°

Example: 11

A voltage of 125V at 60Hz is applied across a non-inductive resistor connected in series with an inductance. The current is 2.2A. The power loss in the resistor is 96.8W and that the inductor is negligible calculate the resistance and the inductance.

Given Data:

Circuit	=	R.L. Series
Voltage (V)	=	125 V
Frequency (F)	=	60 Hz
Current (I)	=	2.2A
Power loss (P)	=	96.8W

To Find:

i. Resistance (R)

ii. Inductance (L)

Solution:

$$\text{Power} = I^2R$$

$$\text{Resistance R} = \frac{P}{I^2}$$

$$R = \frac{96.8}{2.2^2} = 20\Omega$$

$$\text{Power} = \text{V I Cos } \theta$$

$$\text{Cos } \theta = \frac{P}{V.I}$$

$$= \frac{96.8}{125.2.2} = 0.352$$

$$\theta = \cos^{-1} 0.325$$

$$\theta = 69.39^{\circ} \text{ lag}$$

$$\text{Impedance (Z)} = \frac{V}{I}$$

$$Z = \frac{125}{2.2} = 56.8\Omega$$

$$\text{Sin } \theta = \frac{X_L}{Z}$$

$$X_L = \sin\theta \text{ x } Z$$

$$= \sin 69.39 \text{ x } 56.8$$

$$X_L = 53.16\Omega$$

$$\text{Inductive Reactance } (X_L) = 2\pi fL \text{ ; } \Omega$$

$$L = \frac{X_L}{2\pi f}$$

$$= \frac{53.16}{2\text{x}3.14\text{x } 60}$$

$$L = 0.141 \text{ H}$$

Answer:

i) Resistance (R) = 20 Ω ii) Inductance (L) = 0.141 H

Example: 12

A coil of power factor 0.8 is in series with a 100μF capacitor and the combination is put across a 50Hz supply. The potential across the coil is found equal to that across the capacitor. Find the resistance and inductance of the coil.

Given Data:

Circuit	=	R.L.C Series
Power Factor (Cos θ)	=	0.8
Frequency (F)	=	50 Hz
Capacitor (C)	=	100μF

To Find:

i. Resistance (R)
ii. Inductance (L)

Solution:

Voltage across coil = Voltage across capacitor

Also $Z_L = Z_C$

Capacitive Reactance $(X_C) = \frac{1}{2\pi fC}$; Ω

$= \frac{1}{2 \times \pi \times 50 \times 100 \times 10^{-6}} = 31.84\ \Omega$

$Z_C = X_C = 31.84\ \Omega$

$Z_{Coil} = Z_C = 31.84\ \Omega$

Resistance of the coil (R) = Z. Cos θ

$= 31.48 \times 0.8 = 25.18\Omega$

Inductive Reactance (X_L) = Z. Sin θ

$= 31.48 \times 0.6 = 18.89\Omega$

Inductive Reactance $(X_L) = 2\pi fL$; Ω

Inductance $L = \frac{X_L}{2\pi f}$

$= \frac{18.89}{2 \times 3.14 \times 50}$

$L = 0.06\ H$

Example: 13

A current of 10A flows in a circuit with a 45° angle of lag when the applied voltage is 100V. Find the resistance, reactance and impedance of the circuit.

Given Data:

Circuit	=	R.L. Series
Current (A)	=	10A
Phase Angle (ϕ)	=	45°
Voltage (V)	=	230V

To Find:

I. resistance (R)
ii. Impedance (Z)
iii. inductive reactance (X_L)

Solution:

Impedance (Z) $= \frac{V}{I}$

$= \frac{100}{10}$

$Z = 10\ \Omega$

Power factor = Cos θ

= Cos 45°

= 0.707

Also $\cos\theta = \frac{R}{Z}$

$R = Z \cos\theta$

$$= 10 \times 0.707$$

$$R = 7.07\ \Omega$$

$$\text{Impedance (Z)} = \sqrt{R^2 + (X_L)^2}\ \Omega$$

$$X_L = \sqrt{Z^2 - R^2}\Omega$$

$$= \sqrt{10^2 - 7.07^2}\Omega$$

$$X_L = 7.07\ \Omega$$

Answer:

I. Resistance (R) = 7.07 Ω   II. Impedance (Z) = 10 Ω

III. Reactance (X_L) = 7.07 Ω

Example: 14

A resistor of 100Ω is connected in series with 50μF capacitor to supply of 200V, 50Hz. Find (i) impedance (ii) current (iii) power factor (iv) voltage across the resistor and (v) voltage across capacitor.

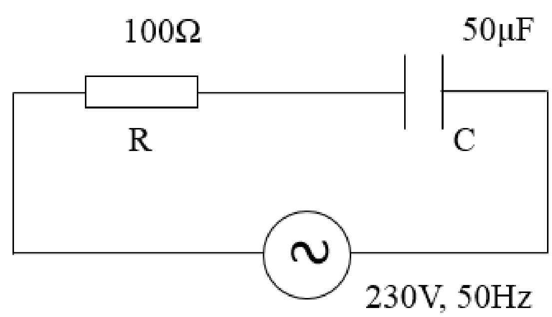
Given Data:

Circuit	=	R.C Series
Resistance (R)	=	100 Ω
Capacitance (C)	=	50 μF
	=	$50\text{x}10^{-6}$F
Supply Voltage (V)	=	200 Volts
Frequency (F)	=	50 Hz

To Find:

i) Impedance (Z)
ii) Current (I)
iii) Power Factor (P.F)
iv) P.D across R
v) P.D across C

Solution:



$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC}\ ;\Omega$$

$$= \frac{1}{2 \times \pi \times 50 \times 50\times10^{-6}}$$

$$X_C = 63.66\ \Omega$$

$$\text{Impedance (Z)} = \sqrt{R^2 + X_C{}^2}\ \Omega$$

$$\text{Impedance (Z)} = \sqrt{100^2 + (63.66)^2}$$

$$\text{Impedance (Z)} = 118.6\Omega$$

$$\text{Current (I)} = \frac{V}{Z}$$

$$= \frac{200}{118.6}$$

$$I = 1.69 \text{ Amps}$$

$$\text{Power Factor } (\cos\theta) = \frac{R}{Z}$$

$$= \frac{100}{118.6} = 0.843 \text{ lead}$$

$$\text{Voltage across coil } (V_R) = I\,R$$

$$= 1.69 \times 100$$

$$= 169 \text{ Volts}$$

$$\text{Voltage across capacitor } (V_L) = I\,X_C$$

$$= 1.69 \times 63.66$$

$$= 107.6 \text{ Volts}$$

Answer:

i) Capacitive Reactance (X_C) = 63.66 Ω
ii) Impedance (Z) = 118.6Ω
iii) Current (I) = 1.69 Amps
iv) Power Factor = 0.843
v) P.D across R (V_R) = 169 Volts
vi) P.D across coil (V_C) = 107.6 Volts

Example: 15

A circuit consisting of a resistor in series with a capacitor takes 80 watts at a power factor of 0.4 from a 100V, 50Hz supply. Find the resistance and capacitance. Take the current as 2 Amps

Given Data:

Circuit	=	R.C. Series
Frequency (F)	=	50 Hz
Voltage (V)	=	100 V
Current	=	2 A
Power	=	80 W
Power factor	=	0.4

To Find:

I. Resistance (R)
ii. Capacitance (C)

Solution:

$$P = V\,I\cos\theta$$

$$I = \frac{P}{V\cos\theta} = \frac{80}{100 \times 0.4}$$

$$= 2A$$

$$\text{Also } P = I^2 R$$

$$R = \frac{P}{I^2} = \frac{80}{2^2} = 20 \text{ ohm}$$

$$\text{Impedance } z = \frac{v}{I} = \frac{100}{2} = 50 \text{ ohm}$$

$$Z = \sqrt{R^2 + {X_c}^2}$$

$$X_c = \sqrt{Z^2 - R^2}$$

$$= \sqrt{50^2 - 20^2} = 45.83 \text{ ohm}$$

$$X_c = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi f X_c}$$

$$= \frac{1}{45.83 \times 2\pi \times 50}$$

$$69.45\times 10^{-6} \text{farad}$$

$$C = 69.45 \text{ mfd}$$

Answer:

i) Resistance (R) = 20 ohm ii) Capacitance (C) = 69.45 mfd

Example: 16

A resistance of 10 ohms and a capacitance of 400mfd are connected in series to a 60V sinusoidal supply. If the current is 5A, find the frequency of the supply. Also determine the phase angle between the current and the applied voltage and the power factor of the circuit.

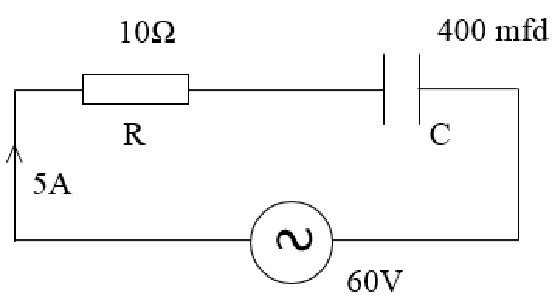
Given Data:

Circuit	=	R.C. Series
Resistance (R)	=	10 Ω
Capacitance (C)	=	400 mfd
Current (I)	=	5 A
Voltage (V)	=	60V

To Find:

i. Phase angle
ii. Power factor

Solution:



$$\text{Impedance } Z = \frac{v}{I} = \frac{60}{5} = 12\Omega$$

$$Z = \sqrt{R^2 + {X_c}^2}$$

$$X_c = \sqrt{Z^2 - R^2}$$

$$= \sqrt{12^2 - 10^2} = 6.63\Omega$$

$$X_c = \frac{1}{2\pi fC}$$

$$f = \frac{1}{2\pi CX_c}$$

$$= \frac{1}{6.63 \times 2\pi \times 400 \times 10^{-6}} = 60\text{Hz}$$

Power factor $\cos\theta = \frac{R}{Z} = \frac{10}{12} = 0.833$

Phase angle $\theta = \cos^{-1}0.833 = 33.56°$

Answer:

i) Power factor = 0.833 ii) Phase angle = 33.56°

Example: 17

A coil of resistance 10Ω and inductance of 0.1H is connected in series with a 150µF capacitor across a 200V, 50Hz. Calculate (i) inductive reactance (ii) capacitive reactance (iii) impedance (iv) current (v) power factor (vi) power in the circuit.

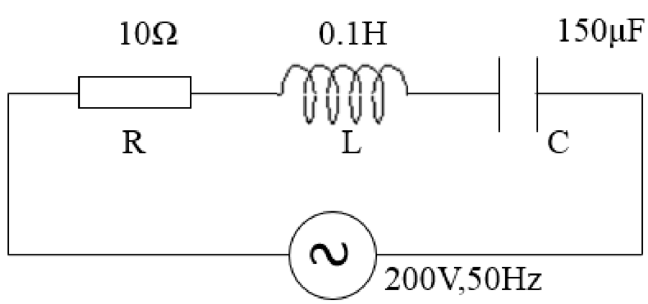
Given Data:

Circuit	=	R.L.C Series
Resistance (R)	=	10 Ω
Inductance (L)	=	0.1 H
Capacitance (C)	=	150 µF
	=	150×10^{-6}F
Supply Voltage (V)	=	200 Volts
Frequency (F)	=	50 Hz

To Find:

i) Inductive reactance (X_L)
ii) Capacitive reactance (X_C)
iii) Impedance (Z)
iv) Line Current (I) = ?
v) Power Factor (P.F) = ?
vi) Power (P) = ?

Solution:



Inductive Reactance (X_L) $= 2\pi fL$; Ω

$= 2 \times 3.14 \times 50 \times 0.1$; Ω

$X_L = 31.42$ Ω

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC} \; ; \; \Omega$$

$$= \frac{1}{2 \times \pi \times 50 \times 150 \times 10^{-6}}$$

$$X_C = 21.22\ \Omega$$

$$\text{Impedance } (Z) = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$$

$$\text{Impedance } (Z) = \sqrt{10^2 + (31.42 - 21.22)^2}$$

$$= \sqrt{10^2 + (10.2)^2} = \sqrt{100 + 104.04}$$

$$= \sqrt{204.04}$$

$$Z = 14.28\Omega$$

$$\text{Current } (I) = \frac{V}{Z}$$

$$= \frac{200}{14.28}$$

$$I = 14 \text{ Amps}$$

$$\text{Power Factor } (\cos\theta) = \frac{R}{Z} = \frac{10}{14.28} = 0.7$$

$$\text{Power } (P) = V\,I\cos\theta$$

$$= 200 \times 14 \times 0.7$$

$$= 1960 \text{ Watts}$$

Answer:

i) Inductive Reactance (X_L) = 31.42 Ω
ii) Capacitive Reactance (X_C) = 21.22 Ω
iii) Impedance (Z) = 14.28Ω
iv) Current (I) = 14 A
v) Power Factor = 0.7
vi) Power (P) = 1960 W

Example: 18

A coil of resistance 8Ω and an inductance of 0.1 H is connected in series with a capacitance of 75 µF with a voltage of 240V, 50Hz. Calculate (i) inductive reactance (ii) capacitive reactance (iii) impedance (iv) current (v) power factor (vi) power in the circuit.

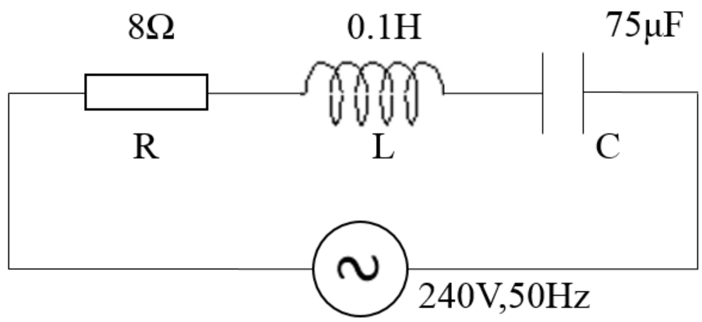
Given Data:

Circuit	=	R.L.C Series
Resistance (R)	=	8 Ω
Inductance (L)	=	0.1 H
Capacitance (C)	=	75µF=75x10^{-6}F
Supply Voltage (V)	=	240 Volts
Frequency (F)	=	50 Hz

To Find:

i) Inductive reactance (X_L)
ii) Capacitive reactance (X_C)
iii) Impedance (Z)
iv) Line Current (I)
v) Power (P)

Solution:



$$\text{Inductive Reactance } (X_L) = 2\pi fL \;;\; \Omega$$

$$= 2 \text{ x } 3.14 \text{ x } 50 \text{ x } 0.1 \;;\; \Omega$$

$$X_L = 31.42\ \Omega$$

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC} \;;\Omega$$

$$= \frac{1}{2 \text{ x } \pi \text{ x } 50 \text{ x} 75\text{x}10^{-6}}$$

$$X_C = 42.44\ \Omega$$

$$\text{Impedance } (Z) = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$$

$$\text{Impedance } (Z) = \sqrt{8^2 + (31.42 - 42.44)^2}$$

$$Z = \sqrt{8^2 + (-11.02)^2}$$

$$= \sqrt{64 + 121.44}$$

$$= \sqrt{185.44}$$

$$\text{Impedance } (Z) = 13.61\Omega$$

$$\text{Current } (I) = \frac{V}{Z}$$

$$= \frac{240}{13.61}$$

$$I = 17.6 \text{ Amps}$$

$$\text{Power Factor } (\text{Cos } \theta) = \frac{R}{Z} = \frac{8}{13.61} = 0.58$$

$$\text{Power } (P) = \text{V I Cos } \theta$$

$$= 240 \text{ x } 17.6 \text{ x } 0.58$$

$$= 2450 \text{ Watts}$$

Answer:

i) Inductive Reactance (X_L) = 31.42 Ω

ii) Capacitive Reactance (X_C) = 42.44 Ω

iii) Impedance (Z) = 13.61Ω

iv) Current (I) = 17.6 Amps

v) Power Factor = 0.58

vi) Power (P) = 2450 Watts

Example: 19

A coil of resistance 10Ω and an inductance of 0.1 H is connected in series with a capacitance of 150μF with a voltage of 200V, 50Hz supply. Calculate (i) inductive reactance (ii) capacitive reactance (iii) impedance (iv) current (v) power factor (vi) voltage across the coil and capacitor.

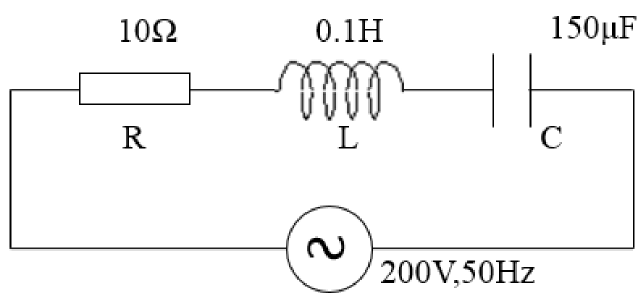
Given Data:

Circuit	=	R.L.C Series
Resistance (R)	=	10 Ω
Inductance (L)	=	0.1 H
Capacitance (C)	=	150 μF
	=	150×10^{-6}F
Supply Voltage (V)	=	200 Volts
Frequency (F)	=	50 Hz

To Find:

i) Inductive reactance (X_L)
ii) Capacitive reactance (X_C)
iii) Impedance (Z)
iv) Line Current (I)
v) Power Factor (P.F)
vi) P.D across L and C

Solution:



$$\text{Inductive Reactance } (X_L) = 2\pi fL \;;\; \Omega$$
$$= 2 \times 3.14 \times 50 \times 0.1 \;;\; \Omega$$
$$X_L = 31.42\ \Omega$$

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi fC} \;;\Omega$$
$$= \frac{1}{2 \times \pi \times 50 \times 150 \times 10^{-6}}$$
$$X_C = 21.22\ \Omega$$

$$\text{Impedance (Z)} = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$$
$$\text{Impedance (Z)} = \sqrt{10^2 + (31.42 - 21.22)^2}$$
$$= \sqrt{10^2 + (10.2)^2}$$
$$= \sqrt{100 + 104.04}$$
$$= \sqrt{221.44}$$
$$\text{Impedance (Z)} = 14.28\Omega$$

$$\text{Current (I)} = \frac{V}{Z}$$
$$= \frac{200}{14.28}$$

$$I = 14 \text{ Amps}$$

$$\text{Power Factor (Cos } \theta) = \frac{R}{Z}$$

$$= \frac{10}{14.28} = 0.7$$

$$\text{Voltage across coil } (V_L) = I X_L$$

$$= 14 \times 31.42$$

$$= 439.88 \text{ Volts}$$

$$\text{Voltage across capacitor } (V_L) = I X_C$$

$$= 14 \times 21.22$$

$$= 297.08 \text{ Volts}$$

Answer:

i) Inductive Reactance (X_L) = 31.42 Ω
ii) Capacitive Reactance (X_C) = 21.22 Ω
iii) Impedance (Z) = 14.28Ω
iv) Current (I) = 14 Amps
v) Power Factor = 0.7
vi) P.D across coil (V_L) = 439.88 Volts
vii) P.D across coil (V_C) = 297.08 Volts

Example: 20

An inductor having an inductance of 0.4H and a resistance of 5Ω is connected in series with a capacitor across 50Hz Supply. Calculate the capacitance required to give the circuit power factor 0.5 lagging. Assume supply voltage to be sinusoidal.

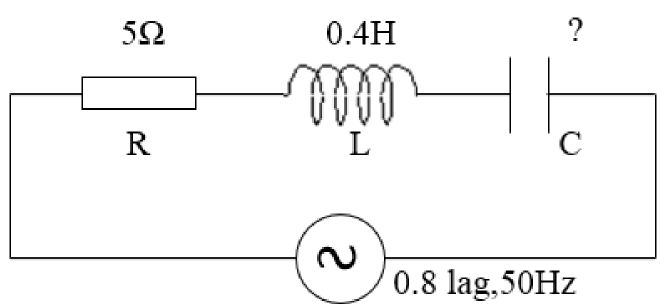
Given Data:

Circuit	=	R.L.C Series
Resistance (R)	=	5 Ω
Inductance (L)	=	0.4 H
Frequency (F)	=	50 Hz
Power factor	=	0.5 lagging

To Find:

I. Capacitance

Solution:



$$X_L = 2\pi f L$$

$$= 2\pi \times 50 \times 0.4 = 125.66\ \Omega$$

$$\text{Impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{Power Factor (Cos }\theta) = \frac{R}{Z}$$

$$0.5 = \frac{5}{Z}$$

$$Z = \frac{5}{0.5} = 10\ \Omega$$

$$\text{i.e., } 10 = \sqrt{R^2 + (X_L - X_C)^2}$$

$$(X_L - X_C) = \sqrt{10^2 - R^2} = \sqrt{10^2 - 5^2}$$

$$125.66 - X_C = 8.66\ \text{ohm}$$

$$X_C = 125.66 - 8.66 = 117\ \text{ohm}$$

$$\text{But, } X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_c}$$

$$= \frac{1}{117 \times 2\pi \times 50} = 27.2\ \text{mfd}$$

Answer:

i) Capacitance = 27.2 mfd

Example: 21

A circuits takes a current of a 3 amps at a p.f of 0.6 lag, When connected to a 120V, 50Hz source and another circuit takes a current of 5 amps at p.f of 0.707 lead when connected to the same source. If the two circuits are connected in series to a 240V, 50Hz supply. Calculate the current and power factor of the circuit?.

Given Data:

Circuit	=	R.L. Series
Current (A)	=	3 A
Power factor (cosθ)	=	0.6 lag
Voltage (V)	=	120V
Current (A)	=	5 A
Power factor (cosθ)	=	0.707 lag
Voltage (V)	=	120V
Voltage (V)	=	240V

To Find:

I. resistance (R)
ii. Impedance (Z)
iii. inductive reactance (X_L)

Solution:

$$\text{Impedance } (Z_A) = \frac{V}{I} = \frac{120}{3} = 40\ \Omega$$

$$R_A = Z_A \cos\theta = 40 \times 0.6 = 24\ \Omega$$

$$\text{Impedance } (Z_B) = \frac{V}{I} = \frac{120}{5} = 24\ \Omega$$

$$R_B = Z_B \cos\theta = 24 \times 0.707 = 16.97\ \Omega$$

$X_L = Z_A \sin\theta$

$= 40 \times 0.8$

$= 32\ \Omega$

$X_L = Z_B \sin\theta$

$= 24 \times 0.707$

$= 16.97\ \Omega$

$$\text{Impedance (Z)} = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$$

$$\text{Impedance (Z)} = \sqrt{(24 + 16.97)^2 + (32 - 16.97)^2}$$

$$= \sqrt{40.97^2 + (15.03)^2}$$

$$\text{Impedance (Z)} = 43.64\Omega$$

$$\text{Voltage (V)} = 240\text{ V}$$

$$I = \frac{V}{Z} = \frac{240}{43.64} = 5.5\text{ Amps}$$

$$\cos\theta = \frac{R}{Z} = \frac{40.97}{43.64} = 0.94$$

Answer:

I. Current (I) = 5.5 Amps   II. Cos = 0.94

Example: 22

A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250V, 50Hz. If the voltage across the resistance is 125V and across the coil is 200V, calculate (a) the impedance, reactance, and resistance of the coil, (b) the power observed by the coil and (c) the total power. Draw the vector diagram. Take the phase angle of the circuit as 52.4° lag.

Given Data:

Current (I)	=	5 A
Voltage (V)	=	250 V
Frequency (f)	=	50Hz
Voltage across resistance V_R	=	125 V
Voltage across coil V_{coil}	=	200 V
Phase angle	=	52.4° lag

To Find:

i). resistance (R)

ii). reactance (X_L)

iii). Impedance (Z)

iv). power absorbed by the coil

v). total power

Solution:

AB = V_R - voltage drop across the non-inductive resistor R

BC = V_r - voltage drop across the resistance of the coil

CD = V_L – voltage drop across the reactance of the coil

BD = 200V – voltage across the choking coil

AD = 250V – applied voltage to the series circuit

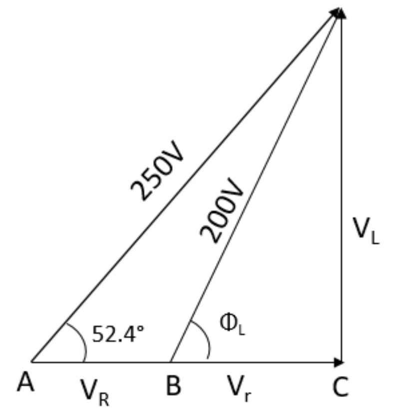
$$V_R = IR = 125\text{ V}$$

$$\text{Impedance of the coil } Z_L = \frac{\text{Voltage across the coil}}{I}$$

$$= \frac{200}{5}$$

$$= 40\ \Omega$$

Power factor of the circuit is:



$$\text{Cos } \theta = \cos 52.4° = 0.61$$

$$\text{Cos } \theta = \frac{AC}{AD}$$

$$AC = AD \cos \theta$$

$$\text{But } AC = AB + BC$$

$$BC = AC - AB$$

$$= 152.54 - 125 = 27.54$$

$$BC = IX_r$$

$$X_r = \frac{BC}{I} = \frac{27.54}{5} = 5.51\Omega$$

$$\text{Sin } \theta = \frac{CD}{AD}$$

$$CD = AD \times \sin \theta$$

$$= 250 \times \sin 52.4° = 198$$

$$\text{But } CD = IX_L$$

$$X_L = \frac{CD}{I} = \frac{198}{5} = 39.6\Omega$$

Power observed by the coil:

$$I^2 r = 5^2 \times 5.51 = 137.75 \text{ watts}$$

Power of resistor R:

$$= I^2 R$$

$$= 5^2 \times 25 = 626 \text{ watts}$$

$$\text{Total power} = 137.75 + 625 = 762.75 \text{ watts}$$

Answer:

I. Resistance of the coil = 5.51Ω

II. Reactance of the coil = 39.6Ω

III. Impedance of the coil Z_L = 40 Ω

IV. Power observed by the coil = 137.75 watts

V. Total power = 762.75 watts

Example: 23

When a voltage of 100V at 50 Hz is applied to chocking coil 'A' the current is 8A and the power is 120 watts. When applied to a coil B the current is 10A and the power is 500W. What current and power will be taken when 100V at 50Hz is applied to the two coils connected in series.

Given Data:

Circuit	=	RL –RL Series
Voltage V	=	100 V
Frequency	=	50 Hz
Coil A		
Current	=	8 A
Power	=	120 W

To Find:

i. Current

ii. Power

Coil B

Current = 10 A

Power = 500 W

Solution:

Coil A

V= 100V, 50Hz

$I_A = 8A$
$P_A = 120watts$

i. e, $I^2R_A = 120$

$R_A = \frac{120}{I^2} = \frac{120}{8^2}$

$R_A = 1.875$ ohm

$Z_A = \frac{V}{I_A} = \frac{100}{8} = 12.5\ \Omega$

$X_{LA} = \sqrt{{Z_A}^2 - {R_A}^2}$

$X_{LA} = \sqrt{12.5^2 - 1.875^2}$

$X_{LA} = 12.36$ ohm

Coil B

V = 100V, 50Hz

$I_B = 10A$
$P_B = 500$ watts

${I_B}^2R_B = 500$

$R_B = \frac{500}{10^2}$

$R_R = 5$ ohm

$Z_B = \frac{V}{I_B} = \frac{100}{10} = 10$ ohm

$X_{LB} = \sqrt{{Z_B}^2 - {R_B}^2}$

$X_{LB} = \sqrt{10^2 - 5^2}$

$X_{LB} = 8.66$ ohm

When two coils are connected in series

$$
\begin{aligned}
\text{Total R} &= R_A + R_B \\
&= 1.875 + 5 = 6.875 \text{ ohm} \\
X_L &= X_{LA} + X_{LB} \\
&= 12.36 + 8.66 = 21.02 \text{ ohm} \\
Z &= \sqrt{R^2 + {X_L}^2} \\
&= \sqrt{6.875^2 + 21.02^2} \\
&= 22.12 \text{ ohm} \\
\text{Circuit current I} &= \frac{v}{Z} = \frac{100}{22.12} = 4.52A \\
\text{Power} &= I^2R = 4.52^2 \times 6.875 = 140.46 \text{ watts}
\end{aligned}
$$

Answer:

i) Circuit current = 4.52 A ii) Power = 140.46 W

Example: 24

A two element series circuit of R= 10Ω and =15Ω has an effective voltage of 230Volts at 50Hz. Determine the active power, apparent power and reactive power.

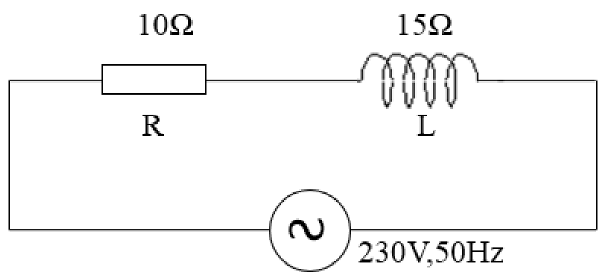
Given Data:

Circuit	=	R.L. Series
Resistance (R)	=	10 Ω
Inductive Reactance (X_L)	=	15Ω
Frequency (F)	=	50 Hz
Voltage (V)	=	230V

To Find:

i. Active Power (P)
ii. Apparent Power (S)
iii. Reactive Power (Q)

Solution:



$$\text{Impedance } (Z) = \sqrt{R^2 + (X_L)^2}\ \Omega$$

$$\text{Impedance } (Z) = \sqrt{10^2 + 15^2}$$

$$\text{Impedance } (Z) = 18.03\ \Omega$$

$$\text{Current } (I) = \frac{V}{Z}$$

$$= \frac{230}{18.03} = 12.76\ Amps$$

$$\text{Phase angle} \quad \theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{15}{10} = 56.3^{\circ}$$

$$\cos\theta = \cos 56.3 = 0.55$$

$$\text{Active Power } (P) = V\, I \cos\theta$$

$$= 230 \times 12.76 \times 0.55$$

$$= 1614 \text{ Watts}$$

$$\text{Apparent Power } (S) = V\, I$$

$$= 230 \times 12.76$$

$$= 2935 \text{ VA}$$

$$\text{Reactive Power } (Q) = V\, I \sin\theta$$

$$= 230 \times 12.76 \times 0.83$$

$$= 2436 \text{ VAR}$$

Example: 25

What is the equation for a sinusoidal current of 25 Hz frequency having an RMS value of 40 Amps.

<u>Given Data:</u>

RMS current I_{RMS} = 40 A

Frequency = 25 Hz

<u>To Find:</u>

Equation of sinusoidal current.

<u>Solution:</u>

Standard form of sinusoidal current is $i = i_m \sin\omega t$

$$i = I_m \sin 2\pi ft$$

$$I_m = I_{rms} \times \sqrt{2}$$

$$= 40 \times \sqrt{2}$$

$$I_m = 56.57 \text{ A}$$

$$i = I_m \sin 2\pi ft$$

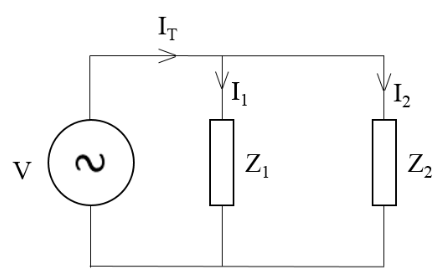
$$i = 56.57 \sin 157t$$

<u>Answer:</u>

i) $i = 56.57 \sin 157t$

PARALLEL AC CIRCUITS:

In an AC circuit, when the impedances are connected in parallel, it is said to be a parallel AC circuit. A parallel circuit can be solved by impedance method or Admittance method.



Impedance Method: A two branch parallel circuit is shown in fig. Let V be the applied voltage to the circuit and I is the total current supplied by the source. I_1, and I_2 are the currents flowing through impedances Z_1 and Z_2 respectively.

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z_T} = \frac{Z_1 + Z_2}{Z_1 . Z_2}$$

$$\text{Total Impedance: } Z_T = \frac{Z_1 . Z_2}{Z_1 + Z_2}$$

$$\text{Total Current}: \quad I = \frac{V}{Z_T} = \frac{V}{R + jX_L} \text{ ; Amps}$$

$$\text{Total Current}: \quad I = \frac{V}{Z_T}$$

$$I = V.\left[\frac{Z_1 + Z_2}{Z_1 . Z_2}\right] \text{ ; Amps}$$

$$\text{Current in Branch 1}: \quad I_1 = \frac{V}{Z_1}$$

$$I_1 = I.\left[\frac{Z_2}{Z_1 + Z_2}\right] \text{ ; Amps}$$

$$\text{Current in Branch 2}: \quad I_2 = \frac{V}{Z_2}$$

$$I_2 = I.\left[\frac{Z_1}{Z_1 + Z_2}\right] \text{ ; Amps}$$

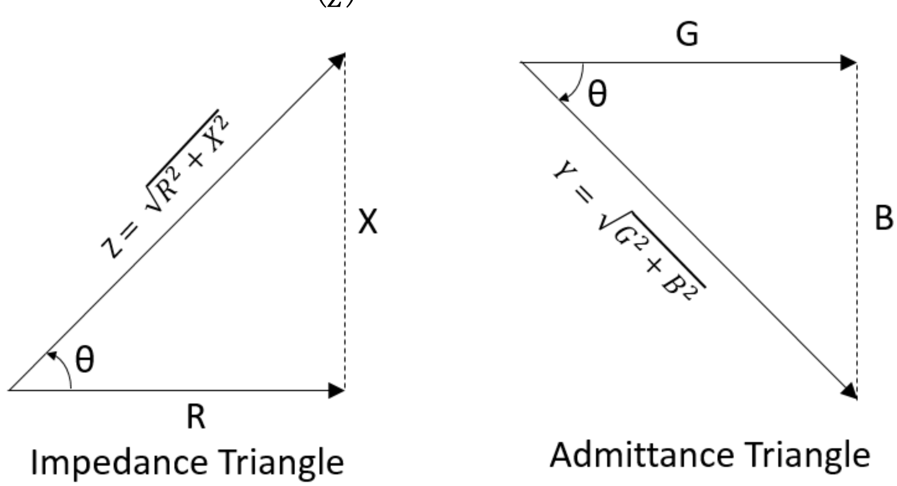
$$\text{Also Total Current}: \quad I = I_1 + I_2$$

Admittance Method:

$$\text{Conductance (G)} = \left(\frac{1}{R}\right) ; \mathit{Siemens}$$

$$\text{Susceptance (B)} = \left(\frac{1}{X}\right) ; \mathit{Siemens}$$

$$\text{Admittance (G)} = \left(\frac{1}{Z}\right) ; \mathit{Siemens}$$



Admittance has in-phase component as well as quadrature components. It can be represented by a right angled triangle called admittance triangle as shown in figure.

$$\text{Conductance (G)} = Y.\cos\theta$$
$$= \left(\frac{1}{Z}\right).\left(\frac{R}{Z}\right) \quad = \left(\frac{R}{Z^2}\right) \text{ siemens}$$
$$\text{Conductance (G)} = \left(\frac{R}{Z^2}\right) \text{ siemens}$$

If a parallel circuit consists of several branches, then the total conductance of the parallel circuit can be written as the sum of the individual conductances.

$$\text{Total Conductance (G)} = G_1 + G_2 + G_3 + \ldots\ldots + G_n$$
$$\text{Where } G_1, G_2 \ldots G_n = \text{Conductance of the individual branches}$$

$$\text{Susceptance (B)} = Y.\sin\theta$$
$$= \left(\frac{1}{Z}\right).\left(\frac{X}{Z}\right) \quad = \left(\frac{X}{Z^2}\right) \text{ siemens}$$
$$\text{Susceptance (B)} = \left(\frac{X}{Z^2}\right) \text{ siemens}$$

If a parallel circuit consists of several branches, then the total susceptance of the parallel circuit can be written as the sum of the individual susceptance.

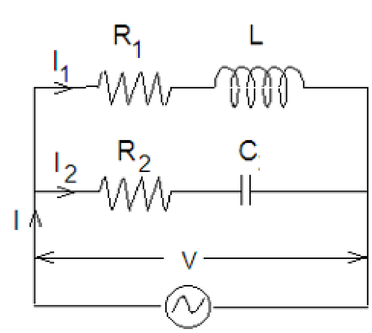
$$\text{Total Susceptance (B)} = B_1 + B_2 + B_3 + \ldots\ldots + B_n$$
$$\text{Where } B_1, B_2 \ldots B_n = \text{Susceptance of the individual branches}$$

Note: In a parallel circuit the inductive susceptance is considered as negative($-B_L$) and capacitive susceptance is consider as positive($+B_c$).

$$\text{Admittance (Y)} = \sqrt{\text{Conductance}^2 + \text{Susceptance}^2}$$
$$Y = \sqrt{G^2 + B^2}$$
$$\text{Phase angle : } \theta = \tan^{-1}\frac{B}{G}$$

Power in AC Parallel Circuit:

$$\text{Admittance : } Y = \frac{1}{Z_1} = \frac{1}{V/I} = \frac{I}{V}$$
$$\text{Total Current : } I = V.Y$$
$$\text{Branch Current : } I_1 = V.Y_1$$
$$\text{Branch Current : } I_2 = V.Y_2$$
$$\text{Total Current : } I = I_1 + I_2$$
$$V.Y = V.Y_1 + V.Y_2$$
$$Y = Y_1 + Y_2$$



Parallel A.C. circuit

In branch I:

$$\text{Admittance}: \quad Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L}$$

$$= \frac{1}{R_1 + jX_L} \cdot \frac{R_1 - jX_L}{R_1 - jX_L}$$

$$= \frac{R_1 - jX_L}{R_1^2 + X_L^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$$

$$= \frac{R_1}{Z_1^2} - j\frac{X_L}{Z_1^2}$$

$$Y_1 = G_1 - jB_1$$

In branch II:

$$\text{Admittance}: \quad Y_2 = \frac{1}{Z_2} = \frac{1}{R_1 - jX_C}$$

$$= \frac{1}{R_2 - jX_C} \cdot \frac{R_2 + jX_C}{R_2 + jX_C}$$

$$= \frac{R_2 + jX_C}{R_2^2 + X_C^2}$$

$$= \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$$

$$= \frac{R_2}{Z_2^2} - j\frac{X_L}{Z_2^2}$$

$$Y_1 = G_2 + jB_2$$

$$\text{Total Admittance}: \quad Y = Y_1 + Y_2$$

$$= (G_1 - jB_1) + (G_2 + jB_2)$$

$$= (G_1 + G_2) + j(-B_1 + B_2)$$

$$= \sqrt{(G_1 + G_2)^2 + (-B_1 + B_2)^2}$$

$$Y = \sqrt{G^2 + B^2}$$

$$\text{Total Current}: \quad I = V.Y$$

$$\text{Phase angle}: \quad \theta = \tan^{-1}\frac{B}{G}$$

$$\theta = \tan^{-1}\frac{(-B_1 + B_2)}{(G_1 + G_2)}$$

$$\text{Power}: \quad P = V.I.\cos\theta$$

Example: 26

Two impedances Z_1 = 8+j6 ohm and Z_2 = 5-j12 ohm are connected in parallel across 200V, 50Hz supply. Find the total impedance.

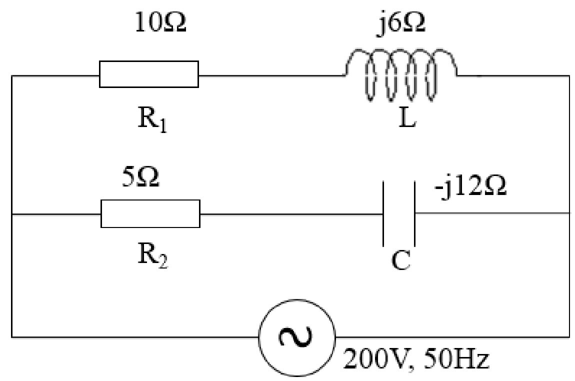
Given Data:

Impedance (Z_1)	=	8+j6
Impedance (Z_2)	=	5-j12
Supply Voltage (V)	=	200 Volts
Frequency (F)	=	50 Hz

To Find:

i) Total Impedance (Z) = ?

Solution:



$$\text{Total Impedance (Z)} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(8 + j6)x(5 - j12)}{(8 + j6) + (5 - j12)}$$

$$= \frac{40 - j96 + j30 - j^2 72}{(13 - j6)}$$

$$= \frac{40 - j96 + j30 + 72}{13 - j6}$$

$$= \frac{112 - j66}{13 - j6}$$

$$= \frac{112 - j66}{13 - j6} x \frac{13 + j6}{13 + j6}$$

$$= \frac{1456 + j672 - j858 - j^2 396}{13^2 - j^2 6^2}$$

$$= \frac{1852 - j186}{205}$$

$$= 9.034 - j0.907$$

$$= 9.08\angle - 5.73^\circ$$

Answer:

i) Total Impedance (Z) = $9.08\angle - 5.73^\circ$ Ohm

Example: 27

In a coil of resistance 8 Ω and reactance of 6 Ω is connected in parallel with a resistance of 12 Ω and a capacitive reactance of 16 Ω across 200V mains. Calculate the current in each branch, total current, total power factor and total power.

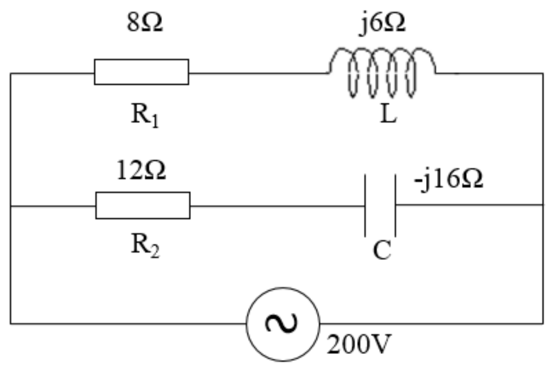
Given Data:

Circuit	=	Parallel
Impedance (Z_1)	=	8+j6
Impedance (Z_2)	=	12 - j16
Supply Voltage (V)	=	200 Volts

To Find:

i) Branch Current (I_1& I_2)
ii) Total Current (I)
iii) Power Factor (P.F)
iv) Power (P)

Solution: Method-I



$$\text{Supply Voltage (V)} = 200+j0$$

$$Z_1 = 8 + j6$$

$$Z_2 = 12 - j16$$

$$\text{Current } (I_1) = \frac{V}{Z_1} = \frac{200}{8 + j6}$$

$$= \frac{200}{8 + j6} \text{ x } \frac{8 - j6}{8 - j6}$$

$$= \frac{1600 - j1200}{8^2 + 6^2}$$

$$= \frac{1600 - j1200}{100}$$

$$= 16 - j12$$

$$\text{Current } (I_1) = 20\angle - 36.87^\circ$$

$$\text{Current } (I_2) = \frac{V}{Z_2} = \frac{200}{12 - j16}$$

$$= \frac{200}{12 - j16} \text{ x } \frac{12 + j16}{12 + j16}$$

$$= \frac{2400 + j3200}{12^2 + 16^2}$$

$$= \frac{2400 + j3200}{400}$$

$$\text{Current } (I_2) = 6 + j8$$

$$\text{Current } (I_2) = 10\angle 53.13^\circ$$

$$\text{Total Current } (I) = I_1 + I_2$$

$$I = (16 - j12) + (6 + j8)$$

$$I = 22 - j4$$

$$I = 22.36\angle -10.30^\circ \text{Amps}$$

$$\text{Power Factor } (\cos\theta) = \cos 10.3$$

$$= 0.98$$

$$\text{Power } (P) = V\, I \cos\theta$$

$$= 200 \text{ x } 22.36 \text{ x } 0.98$$

$$= 4382.5 \text{ Watts}$$

Answer:

i) Current (I_1) = $10\angle 53.13^\circ$ Amps

ii) Current (I_2) = $10\angle 53.13^\circ$ Amps

iii) Total Current (I) = $22.36\angle -10.30^\circ$ Amps

iv) Power Factor (P.F) = 0.98

v) Power (P) = 4382.5 Watts

Solution: Method-II

$$\text{Supply Voltage } (V) = 200 + j0 = 200\angle 0^\circ$$

$$Z_1 = 8 + j6 = 10\angle 36.8^\circ$$

$$Z_2 = 12 - j16 = 20\angle -53.13^\circ$$

$$\text{Current } (I_1) = \frac{V}{Z_1} = \frac{200\angle 0^\circ}{10\angle 36.8^\circ} = 20\angle -36.8^\circ$$

$$\text{Current } (I_1) = 20\angle -36.87^\circ$$

$$\text{Current } (I_2) = \frac{V}{Z_2} = \frac{200\angle 0^\circ}{20\angle -53.13^\circ} = 10\angle 53.13^\circ$$

$$\text{Current } (I_2) = 10\angle 53.13^\circ$$

$$\text{Total Current } (I) = I_1 + I_2$$

$$I = 20\angle -36.8^\circ + 10\angle 53.13^\circ$$

$$I = (16 - j12) + (6 + j8)$$

$$I = 22 - j4$$

$$I = 22.36\angle -10.30^\circ \text{Amps}$$

$$\text{Power Factor } (\cos\theta) = \cos 10.3$$

$$= 0.98$$

$$\text{Power } (P) = V\, I \cos\theta$$

$= \quad 200 \text{ x } 22.36 \text{ x } 0.98$

$= \quad 4382.5 \text{ Watts}$

Answer:

i) Current (I_1) = $10\angle 53.13^\circ$ Amps
ii) Current (I_2) = $10\ \angle 53.13^\circ$ Amps
iii) Total Current (I) = $22.36\ \angle -10.30^\circ$ Amps
iv) Power Factor (P.F) = 0.98
v) Power (P) = 4382.5 Watts

Example: 28

A capacitor of 80 µF is connected in parallel with a coil that has a resistance of 20 Ω and inductance of 0.08H. If this combination is connected across 230V, 50Hz supply calculate current, P.F and power.

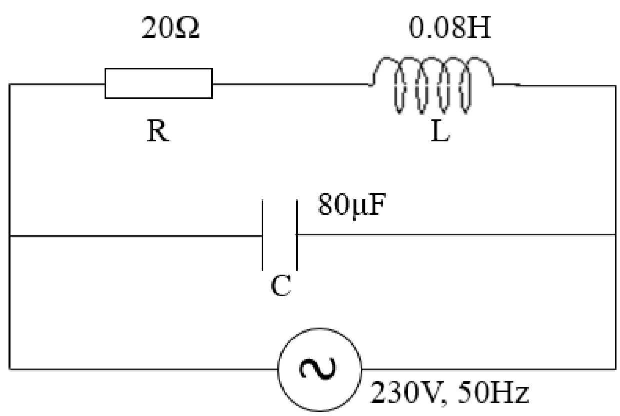
Given Data:

Circuit	=	Parallel
Resistance (R)	=	20 Ω
Inductance (L)	=	0.08 H
Capacitance (C)	=	80 µF
	=	$80\text{x}10^{-6}$F
Supply Voltage (V)	=	230 Volts
Frequency (F)	=	50 Hz

To Find:

v) Total Current (I)
vi) Power Factor (P.F)
vii) Power (P)

Solution:



Inductive Reactance (X_L) $= 2\pi fL\ ;\ \Omega$

$= 2 \text{ x } 3.14 \text{ x } 50 \text{ x } 0.08\ ;\ \Omega$

$X_L = 25.13\ \Omega$

Capacitive Reactance (X_C) $= \dfrac{1}{2\pi fC}\ ;\Omega$

$= \dfrac{1}{2 \text{ x } \pi \text{ x } 50 \text{ x}80\text{x}10^{-6}}$

$X_C = 39.79\ \Omega$

$$\text{Current } (I_1) = \frac{V}{Z_1} = \frac{230\angle 0}{20 + j25.13} = \frac{230\angle 0}{32.12\angle 51.49^\circ}$$

$$I_1 = 7.16\angle -51.49^\circ$$

$$\text{Current } (I_2) = \frac{V}{Z_2} = \frac{230\angle 0}{0 - j39.79}$$

$$I_2 = 5.78\angle 90^\circ \text{Amps}$$

$$I = I_1 + I_2$$

$$I = 7.16\angle -51.49^\circ + 5.78\angle 90^\circ$$

$$= 4.45 - j5.49 + 0 + j5.78$$

$$= 4.45 - j0.29$$

$$I = 4.46\angle -3.72^\circ \text{Amps}$$

$$\text{Power Factor } (\text{Cos}\,\theta) = 0.9$$

$$\text{Power } (P) = V\,I\,\text{Cos}\,\theta$$

$$= 230 \times 4.46 \times 0.9$$

$$= 923.2 \text{ Watts}$$

Answer:

i) Current (I) = 4.46 Amps   ii) Power Factor = 0.9

iii) Power (P) = 923.2 Watts

Example: 29

Two impedances Z_1 = 8+j6 and Z_2 = 3-j4 are connected in parallel across 230V, 50Hz supply. Calculate (a) Current in each branch (b) the total current of the circuit (c) power factor (d) Power taken by the circuit.

Given Data:

Impedance (Z_1)	=	8+j6
Impedance (Z_2)	=	3-j4
Supply Voltage (V)	=	230 Volts
Frequency (F)	=	50Hz

To Find:

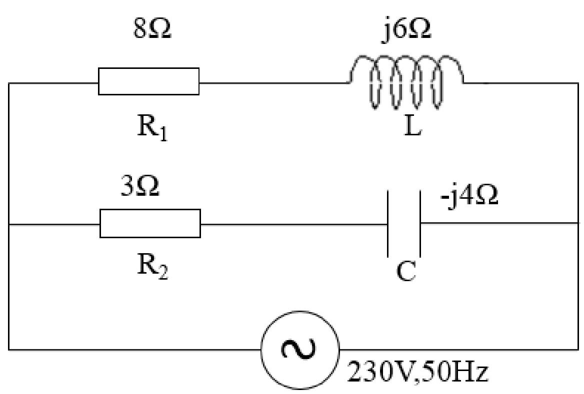
i) Current in each branch=?

ii) Total current = ?

iii) Power factor (P.F) = ?

iv) Power (P) = ?

Solution:



$$Z_1 = \sqrt{{R_1}^2 + {X_L}^2}$$
$$= \sqrt{8^2 + 6^2} = 10\Omega$$
$$Z_2 = \sqrt{{R_2}^2 + {X_C}^2}$$
$$= \sqrt{3^2 + 4^2} = 5\Omega$$
$$\text{Conductance } (G_1) = \frac{8}{{Z_1}^2}$$
$$= \frac{8}{10^2} = 0.08\mho$$
$$\text{Conductance } (G_2) = \frac{R_2}{{Z_2}^2}$$
$$= \frac{3}{5^2} = 0.12\mho$$
$$\text{Susceptance } (B_1) = \frac{-X_1}{{Z_1}^2}$$
$$= \frac{-6}{10^2} = -0.06\mho$$
$$\text{Susceptance } (B_2) = \frac{X_2}{{Z_2}^2}$$
$$= \frac{4}{5^2} = 0.16\mho$$
$$\text{Admittance } (Y_1) = \sqrt{{G_1}^2 + {B_1}^2}$$
$$= \sqrt{0.08^2 + 0.06^2} = 0.1\mho$$
$$\text{Admittance } (Y_2) = \sqrt{{G_2}^2 + {B_2}^2}\ \Omega$$
$$= \sqrt{0.12^2 + 0.16^2} = 0.2\mho$$

Current in branch 1 : $I_1 = V.Y_1$
$$= 230x.0.1 = 23\text{Amps}$$
Current in branch 2 : $I_2 = V.Y_2$
$$= 230x.0.2 = 46\text{Amps}$$
$$\text{Total Conductance } (G) = G_1 + G_2$$
$$= 0.08 + 0.12 = 0.2\ \mho$$
$$\text{Total Susceptance } (B) = B_1 + B_2$$
$$= -0.06 + 0.16 = 0.1\mho$$
$$\text{Total Admittance } (Y) = \sqrt{G^2 + B^2}\ \Omega$$
$$= \sqrt{0.2^2 + 0.1^2} = 0.22\ \mho$$

$$\text{Phase angle} = \tan^{-1}\frac{B}{G}$$

$$= \tan^{-1}\frac{0.1}{0.2} = 26.57$$

$$\text{Power Factor : Cos }\theta = \text{Cos } 26.57$$

$$= 0.89$$

$$\text{Total Current (I)} = V.Y$$

$$= 230x0.22 = 66.7 \text{ Amps}$$

$$\text{Power (P)} = V I \text{ Cos } \theta$$

$$= 230 \text{ x } 66.7 \text{ x } 0.89$$

$$= 13653.5 \text{ Watts}$$

Answer:

i) Current in branch 1 (I_1) = 23 A

ii) Current in branch 2 (I_2) = 46 A

iii) Total current (I) = 66.7 Amps

iv) Power factor = 0.89

v) Power (P) = 13653.5 Watts

Example: 30

An impedance (6+j8) is connected across 220V, 50Hz mains in parallel with another circuit having an impedance of (8-j6) ohm. Calculate (a) the admittance, the conductance and the susceptance of the combined circuit (b) the total current taken from the mains (c) power factor (d) Total Power

Given Data:

Impedance (Z_1)	=	6+j8
Impedance (Z_2)	=	8-j6
Supply Voltage (V)	=	220 Volts
Frequency (F)	=	50Hz

To Find:

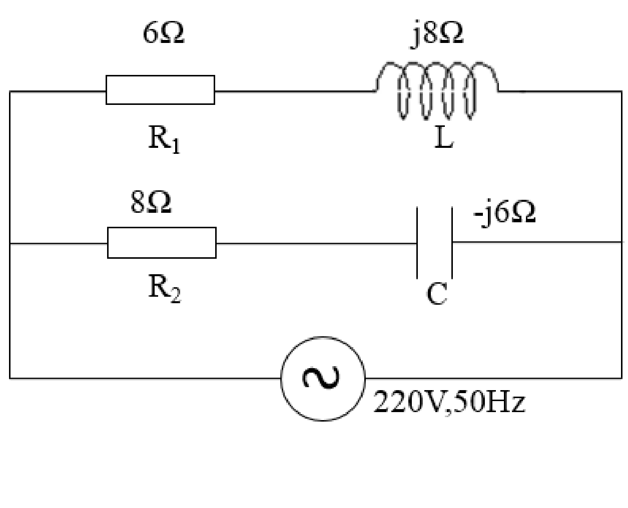
i) Admittance, Conductance & Susceptance = ?

ii) Total current = ?

iii) Power factor (P.F) = ?

iv) Power (P) = ?

Solution:



$$Z_1 = \sqrt{{R_1}^2 + {X_L}^2}$$
$$= \sqrt{6^2 + 8^2} = 10\Omega$$
$$Z_2 = \sqrt{{R_2}^2 + {X_C}^2}$$
$$= \sqrt{8^2 + 6^2} = 10\Omega$$
$$\text{Conductance } (G_1) = \frac{R_1}{{Z_1}^2} = \frac{6}{10^2} = 0.06℧$$
$$\text{Conductance } (G_2) = \frac{R_2}{{Z_2}^2} = \frac{8}{10^2} = 0.08℧$$
$$\text{Susceptance } (B_1) = \frac{-X_1}{{Z_1}^2} = \frac{-8}{10^2} = -0.08℧$$
$$\text{Susceptance } (B_2) = \frac{X_2}{{Z_2}^2} = \frac{6}{10^2} = 0.06℧$$
$$\text{Admittance } (Y_1) = \sqrt{{G_1}^2 + {B_1}^2}$$
$$= \sqrt{0.06^2 + 0.08^2} = 0.1℧$$
$$\text{Admittance } (Y_2) = \sqrt{{G_2}^2 + {B_2}^2}$$
$$= \sqrt{0.08^2 + 0.06^2} = 0.1℧$$
Current in branch 1 : $I_1 = V.Y_1$
$$= 220x.0.1 = 22\text{Amps}$$
Current in branch 2 : $I_2 = V.Y_2$
$$= 220x.0.1 = 22\text{Amps}$$
$$\text{Total Conductance } (G) = G_1 + G_2$$
$$= 0.06 + 0.08 = 0.14℧$$
$$\text{Total Susceptance } (B) = B_1 + B_2$$
$$= -0.08 + 0.06 = -0.02℧$$
$$\text{Total Admittance } (Y) = \sqrt{G^2 + B^2}\ \Omega$$
$$= \sqrt{0.14^2 + 0.02^2} = 0.14\ ℧$$
$$\text{Phase angle} = \tan^{-1}\frac{B}{G}$$
$$= \tan^{-1}\frac{0.02}{0.14} = 8.13$$
$$\text{Power Factor : } \cos\theta = \cos 8.13$$
$$= 0.98$$
$$\text{Total Current } (I) = V.Y$$

$$= 220 \times 0.14 = 30.8 \text{ Amps}$$

$$\text{Power (P)} = V\, I \cos\theta$$

$$= 220 \times 30.8 \times 0.98$$

$$= 6640 \text{ Watts}$$

Answer:

i) G =0.14Ʊ, B = 0.02Ʊ and Y = 0.14Ʊ

ii) Total current (I) = 30.8 Amps

iii) Power factor = 0.98

iv) Power (P) = 6640 Watts

Example: 31

Two impedances Z_1 = (10+j5) ohm and Z_2 = (8+j6) ohm are connected in parallel across 200V. Find the total current taken and also the power factor of the circuit.

Given Data:

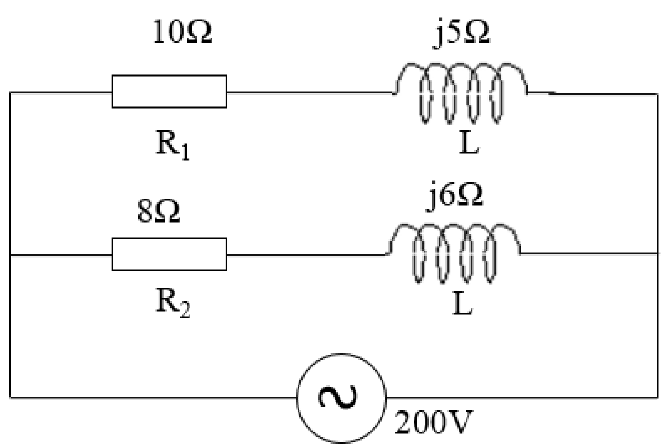
Impedance (Z_1)	=	10+j5
Impedance (Z_2)	=	8+j6
Supply Voltage (V)	=	200 Volts

To Find:

i) Total current = ?

ii) Power factor (P.F) = ?

Solution:



$$Z_1 = \sqrt{{R_1}^2 + {X_L}^2}$$

$$= \sqrt{10^2 + 5^2} = 11.18\Omega$$

$$Z_2 = \sqrt{{R_2}^2 + {X_C}^2}$$

$$= \sqrt{8^2 + 6^2} = 10\Omega$$

$$\text{Conductance } (G_1) = \frac{R_1}{{Z_1}^2} = \frac{10}{11.18^2} = 0.08\mho$$

$$\text{Conductance } (G_2) = \frac{R_2}{{Z_2}^2} = \frac{8}{10^2} = 0.08\mho$$

$$\text{Susceptance } (B_1) = \frac{-X_1}{{Z_1}^2} = \frac{-5}{11.18^2} = -0.04℧$$

$$\text{Susceptance } (B_2) = \frac{-X_2}{{Z_2}^2} = \frac{-6}{10^2} = -0.06℧$$

$$\text{Admittance } (Y_1) = \sqrt{{G_1}^2 + {B_1}^2}$$

$$= \sqrt{0.08^2 + 0.04^2} = 0.08℧$$

$$\text{Admittance } (Y_2) = \sqrt{{G_2}^2 + {B_2}^2}$$

$$= \sqrt{0.08^2 + 0.06^2} = 0.1℧$$

Current in branch 1 : $I_1 = V.Y_1$

$= 200x.0.08 = 16\text{Amps}$

Current in branch 2 : $I_2 = V.Y_2$

$= 200x.0.1 = 20\text{Amps}$

Total Conductance (G) $= G_1 + G_2$

$= 0.08 + 0.08 = 0.16℧$

Total Susceptance (B) $= B_1 + B_2$

$= -0.04 - 0.06 = -0.1℧$

Total Admittance (Y) $= \sqrt{G^2 + B^2}\ \Omega$

$= \sqrt{0.16^2 + 0.1^2} = 0.18\ ℧$

Phase angle $= \tan^{-1}\frac{B}{G}$

$= \tan^{-1}\frac{0.1}{0.16} = 32$

Power Factor : Cos θ = Cos 32

$= 0.84$

Total Current (I) $= V.Y$

$= 200x0.18 = 36\ Amps$

Power (P) $= V\ I\ \text{Cos}\ \theta$

$= 200\ x\ 36\ x\ 0.84$

$= 6077$ Watts

Answer:

i) Total current (I) = 36 Amps ii) Power factor = 0.84

Example: 32

A coil of resistance of 8 ohm and a reactance of 10 ohm are connected in parallel with a resistor of 10 ohm. If the voltage across the combination is 200 volts a.c. Find the total current taken from the mains. Also find the power factor of the circuit.

Given Data:

Impedance (Z_1) = 8+j10

Impedance (Z_2) = 10+j0

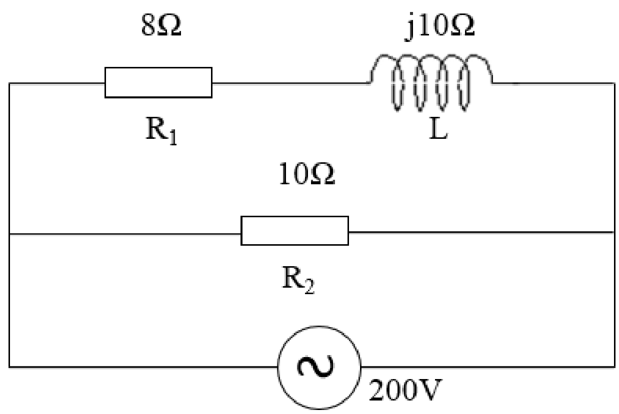
Supply Voltage (V) = 200 Volts

To Find:

i) Total current = ?

ii) Power factor (P.F) = ?

Solution:



$$Z_1 = \sqrt{{R_1}^2 + {X_L}^2}$$

$$= \sqrt{8^2 + 10^2} = 12.8\Omega$$

$$Z_2 = \sqrt{{R_2}^2 + {X_C}^2}$$

$$= \sqrt{10^2} = 10\Omega$$

$$\text{Conductance } (G_1) = \frac{R_1}{{Z_1}^2} = \frac{8}{12.8^2} = 0.04℧$$

$$\text{Conductance } (G_2) = \frac{R_2}{{Z_2}^2} = \frac{10}{10^2} = 0.1℧$$

$$\text{Susceptance } (B_1) = \frac{-X_1}{{Z_1}^2} = \frac{-10}{12.8^2} = -0.06℧$$

$$\text{Susceptance } (B_2) = 0$$

$$\text{Admittance } (Y_1) = \sqrt{{G_1}^2 + {B_1}^2}$$

$$= \sqrt{0.04^2 + 0.06^2} = 0.07℧$$

$$\text{Admittance } (Y_2) = \sqrt{{G_2}^2 + {B_2}^2}$$

$$= \sqrt{0.1^2 + 0^2} = 0.1℧$$

Current in branch 1 : $I_1 = V.Y_1$

$= 200x.0.07 = 14Amps$

Current in branch 2 : $I_2 = V.Y_2$

$= 200x.0.1 = 20Amps$

Total Conductance (G) $= G_1 + G_2$

$= 0.04 + 0.1 = 0.14℧$

Total Susceptance (B) $= B_1 + B_2$

$= -0.06℧$

Total Admittance (Y) $= \sqrt{G^2 + B^2}\ \Omega$

$= \sqrt{0.14^2 + 0.06^2} = 0.15\ ℧$

Phase angle $= \tan^{-1}\frac{B}{G}$

$= \tan^{-1}\frac{0.06}{0.14} = 23.19$

Power Factor : Cos θ $=$ Cos 23.19

$= 0.91$

Total Current (I) $= V.Y$

$= 200x0.15 = 30$ Amps

Power (P) $=$ V I Cos θ

$=$ 200 x 30 x 0.91

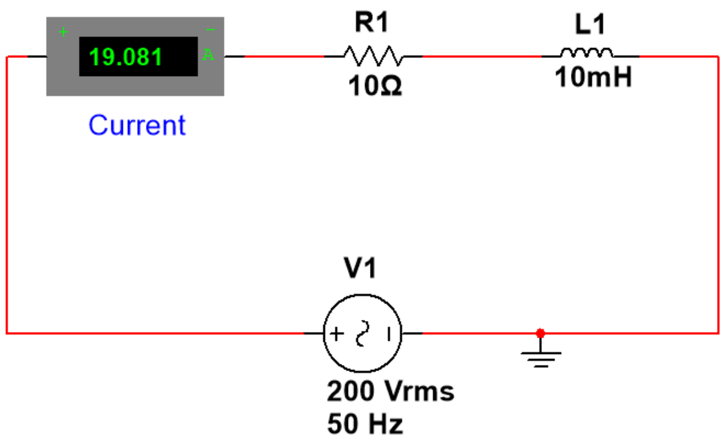
$=$ 5460 Watts

<u>Answer:</u> i) Total current (I) = 30 Amps

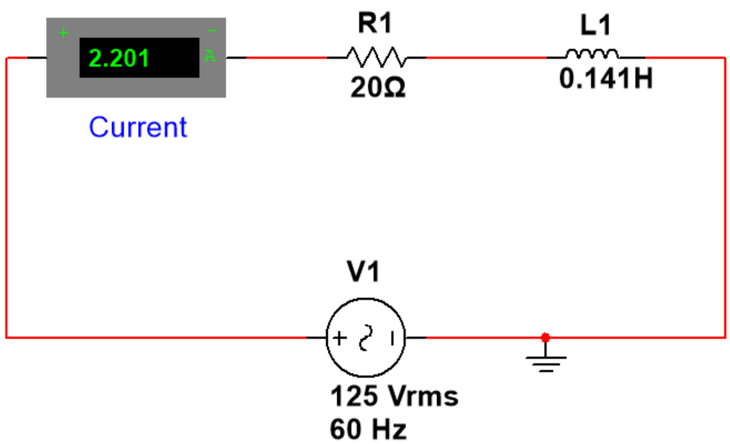
ii) Power factor = 0.91

RESULT OF SIMULATION OF PROBLEMS IN UNIT III

Problem : 10

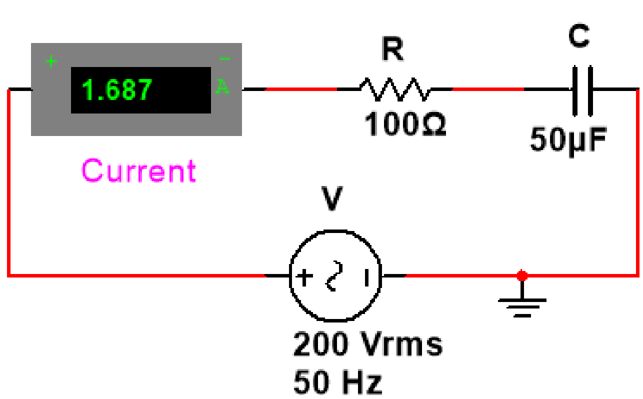


Problem : 11

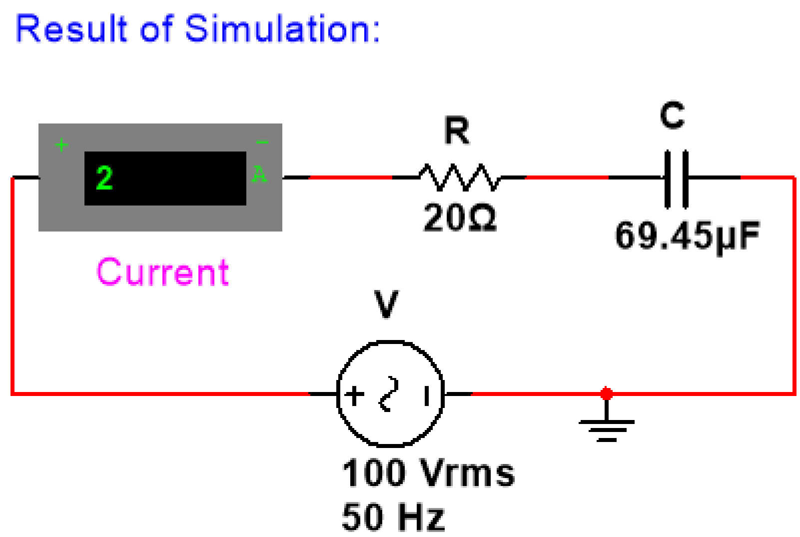


Problem: 14

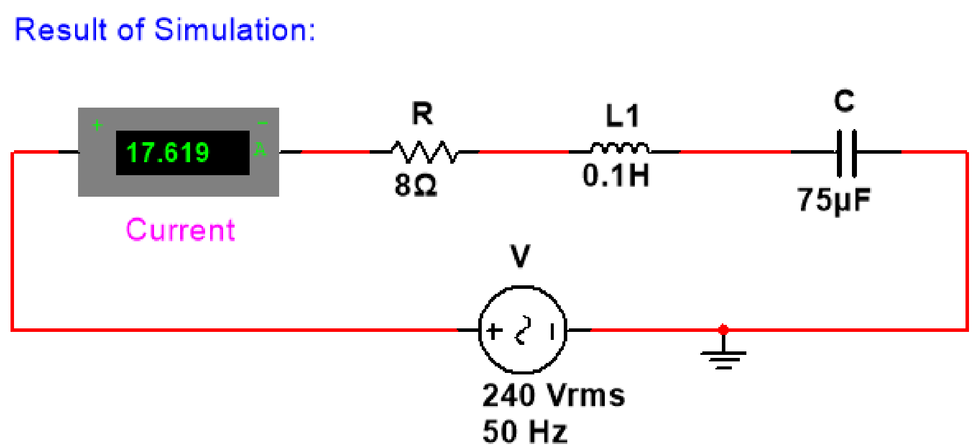
Result of Simulation:



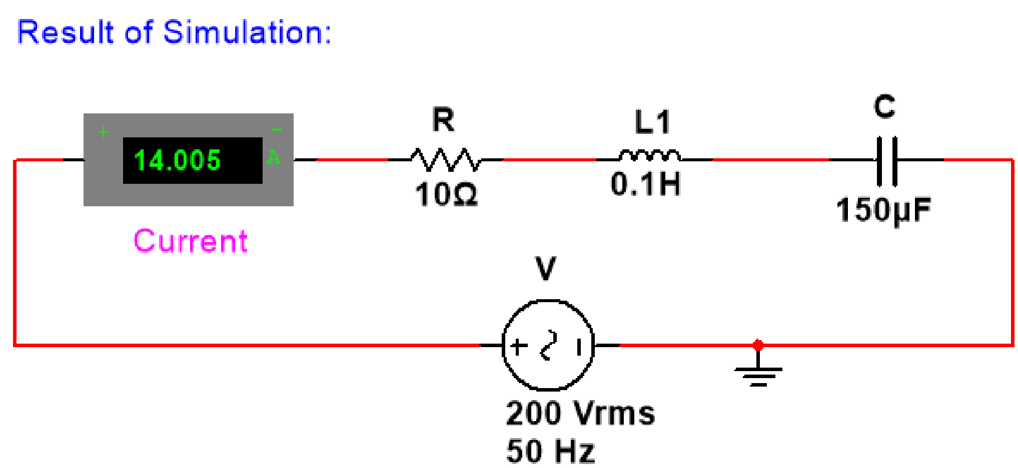
Problem: 15



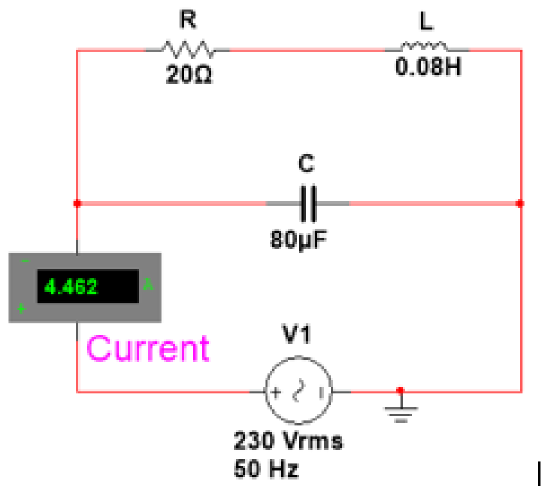
Problem: 18



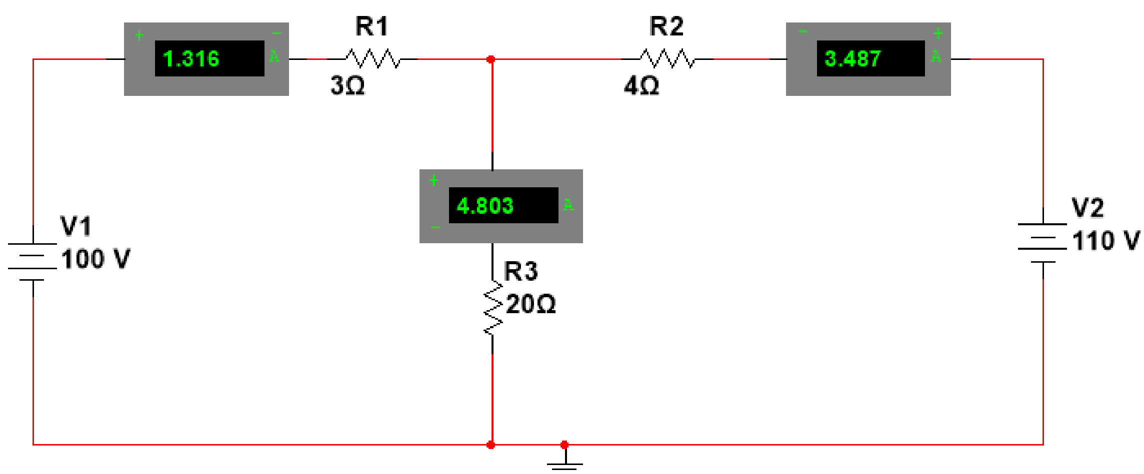
Problem: 19



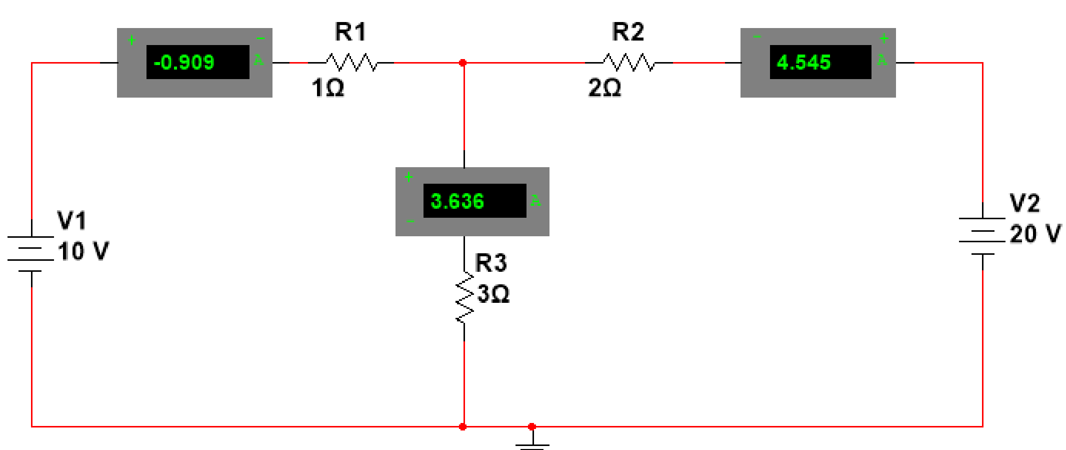
Problem: 28



Problem 30:



Problem 31:



REVIEW QUESTIONS
UNIT : III SINGLE PHASE AC CIRCUITS

PART – A : 2 Mark Questions

1. Define A.C Voltage.
2. Define Cycle.
3. Define frequency and state its unit.
4. Define time period.
5. State the relationship between frequency and period.
6. Define instantaneous value of a.c voltage.
7. Define maximum or peak value or crest value?
8. Why average value for a symmetrical wave is computed for half cycle only?
9. Define form factor.
10. Define peak factor or Amplitude or crest factor.
11. Define power factor.
12. State the value of form factor and peak factor for sinusoidal a.c. quantity?
13. Define Phase difference?
14. What is meant by reference phasor?
15. Define in-phase quantities.
16. Define inductive reactance.
17. State the expression for inductance reactance.
18. What is the unit of inductive reactance?
19. Draw the phasor diagram for a pure inductor.
20. Define capacitive reactance.
21. State the expression for capacitive reactance and its unit.
22. Convert the following current in polar into rectangular form. 5 I30°
23. Define admittance and state its unit.
24. What do you mean by conductance, admittance and susceptance.

PART – B : 3 Marks Questions

1. Define alternating quantity, amplitude and cycle.
2. Draw a sinusoidal voltage waveform and mark the cycle, time period and peak value.
3. Define: (i) Cycle (ii) Frequency (iii) Time period
4. Define : (i) Instantaneous value (ii) time period (iii) frequency
5. Define average value and RMS value of a.c quantity.
6. Derive an expression for average value of a.c quantity in terms of maximum value.
7. Define Instantaneous value of a.c current and write the equations in 3 different forms.

8. Explain the terms: (i) Form factor (ii) Peak factor
9. Prove that power drawn by a pure inductor connected across AC supply is zero.
10. Show that active power in pure capacitor connected across AC supply is zero
11. Draw impedance triangle and phasor diagram for RC series circuit.
12. Draw impedance triangle and phasor diagram for RL series circuit.
13. Draw the impedance triangle for a RLC series circuit and write the expression for impedance.
14. Define Power factor.
15. Explain what is meant by leading and lagging p.f?
16. Briefly explain about power triangle.
17. Explain the terms: (i) apparent power (ii) active power (iii) reactive power.
18. Draw admittance triangle and phasor diagram for RL Parallel circuit.
19. Draw admittance triangle and phasor diagram for RC Parallel circuit.

PART – C : 10 Mark Questions

1. Define and derive average value of alternating voltage.
2. Define and derive RMS value of alternating current.
3. Draw the vector diagram of a series R-L circuit and derive the impedance.
4. Draw the vector diagram of a series R-C circuit and derive the impedance.
5. Draw the vector diagram of a series R-L-C circuit and derive the impedance.
6. Show that the power in an R-L series circuit is P= V.I.cosΦ watts.
7. Show that the power in an R-C series circuit is P= V.I.cosΦ watts.
8. Show that the power in an R-L-C series circuit is P= V.I.cosΦ watts.
9. An alternating voltage is given by e = 100 sin314 t. Find a) Max value b) Frequency c) Time period and d) value of current after t=0.01 sec.

10. The equation for a voltage is written as E = 100 sin 314 t. find a) Frequency b) max value c) Average value d) RMS value and e) voltage at time 1/200 sec after passing first zero.

11. An ac voltage of 50 Hz frequency has a peak value of 200 V. a) Write an equation to calculate the instantaneous value of the voltage b) write an equation for a current having a max value of 20 A and lagging the voltage by 45° and e) Find average effective values of voltage and current.

12. An alternating voltage v = 200 sin 314 t is applied to a device which offers an ohmic resistance of 25Ω to the flow of current in one direction while entirely preventing the flow of current in the opposite direction. Calculate the RMS value, average value.

13. A 50 Hz alternating voltage of 250 V sends a current of 2.5A through the pure inductive coil. Find a) inductive reactance of the coil b) Inductance of the coil c) power absorbed and d) write down the equation for voltage and current.

14. An alternating current of maximum value 10A flows through a capacitor of 31.8 mfd. Calculate a) capacitive reactance b) RMS value of applied voltage and c) RMS value of current.

15. A current 20A flows in a circuit with 45° angle of lag. When the applied voltage is 200V 50Hz. Find a) Resistance b) Inductance and c) Impedance of the circuit.

16. A coil has a resistance of 20Ω and an inductance of 0.2 H, connected 230 V, 50 Hz supply. Calculate a) the circuit current b) phase angle power factor & d) power consumed.
17. A coil of resistance of 8 Ω, an inductance of 0.1 and a capacitance of 75 mfd across a 230 V 50 Hz supply. Find a) current in the circuit b) power factor c) power and d) voltage across coil and capacitor.

18. A resistance of 15Ω and a coil of inductance 30 mH and negligible resistance are connected in parallel across 240V 50Hz supply. Find a) the line current b) power factor and c) power consumed by the circuit.

19. A coil having a resistance of 5Ω and an inductance of 0.02H is connected in parallel with another coil having a resistance of 1 Ω and an inductance 0.08H. Calculate the current through the combination and the power absorbed when a voltage of 100V 50 Hz is applied. Draw the phasor diagram.

20. A circuit having a resistance of 6 Ωand inductive reactance of 8 Ω is connected across 230 V 50 Hz mains in parallel with another circuit having a resistance of 8 Ω and a capacitive reactance of 6 Ω. Find a) Total current b) phase angle between current supply voltage.

21. A voltage of 200 + j 100 volts applied to a circuit causes a current of 10 + j2 ampere to flow. Find the impedance and power factor of the circuit.

22. A coil of resistance 15 Ω and inductance 0.1 H is in parallel with a resistor of 2.5 Ω. A voltage of 220 V at 50 Hz is applied to the parallel combination. Determination the total current and the power factor.

23. A 10 Ω resistor, 12.5 mH inductor and 150 mfd capacitor are connected in parallel to a 230 V 50 Hz source. Calculate the supply current and power factor.

24. A chocking coil of resistance 5 and inductance 0.6 H is in series with a capacitor of 10 mfd. If a voltage of 200 V is applied and the frequency is adjusted to resonance, find the current and the voltage across inductance and capacitor.

25. A circuit having a resistance of 5 Ω an inductance of 0.5 H and a variable capacitor in series is connected across 200V 50Hz supply. Calculate a) the capacitance to give resonance b) voltage across inductance and capacitance c) 'Q' factor of the circuit.

26. A coil 10 Ω resistance and 0.2 H inductance is connected in parallel with a variable capacitor across a 230V 50Hz supply. Determine a) capacitance required for parallel resonance b) Effective impedance of the circuit and c) Power absorbed.

UNIT IV – RESONANT CIRCUITS

Syllabus:

Series resonance – parallel resonance (R,L &C; RL&C only) – quality factor – dynamic resistance – comparison of series and parallel resonance – Problems in the above topics - Applications of resonant circuits.

4.1 Introduction:

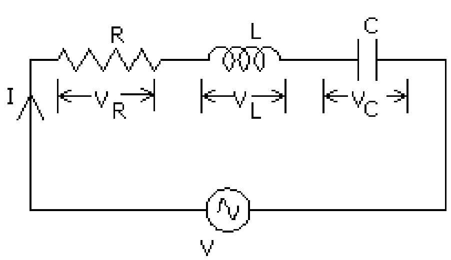
When introducing a.c. circuits in unit- III, a supply was defined by its voltage and frequency. For many applications, they are constant; for example, the source of supply to our homes. However, many communications systems involve circuits in which either the supply voltage or input signal operates with a varying frequency.

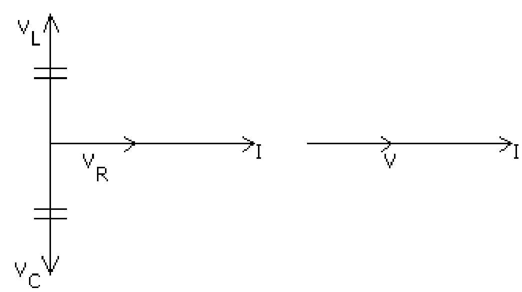
Circuits with both inductance and capacitance can exhibit the property of resonance, which is important in many types of applications. Resonance is the basis for frequency selectivity in communication systems. For example, the ability of a radio or television receiver to select a certain frequency that is transmitted by a particular station and, at the same time to eliminate frequencies from other stations is based on the principle of resonance. The conditions in RLC circuits that produce resonance and characteristics of resonance circuits are covered in this unit.

4.1 Resonance:

Inductive reactance increases as the frequency is increased, but capacitive reactance decreases with higher frequencies. Because of these opposite characteristics, for any LC combination there must be a frequency at which the X_L equals the X_C as one increases while other decreases. This case of equal and opposite reactance is called resonance and this circuit is called as resonance circuit.

4.2 R.L.C Series Resonance Circuit:





Resonance is a condition in a series RLC circuit in which the capacitive reactance and inductive reactance are equal in magnitude. When they are equal, they cancel each other and total reactance is zero and result is a purely resistive impedance. And also the p.f is unity.

$$\text{Impedance (Z)} = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$$

When, $X_L = X_C$

Impedance (Z) $= \sqrt{R^2} = R$

4.3 Resonance frequency:

The frequency at which the inductive reactance and capacitive reactance are equal is the resonant frequency.

At resonance:

$$\text{Inductive Reactance} = \text{Capacitive Reactance}$$

$$X_L = X_C$$

$$2\,\pi\, f_r\, L = \frac{1}{2\pi\, f_r\, C}$$

$$f_r^{\,2} = \frac{1}{4\,\pi^2\, L\, C}$$

$$f_r = \frac{1}{2\,\pi\sqrt{L\,C}} \quad ;\ \text{Hz}$$

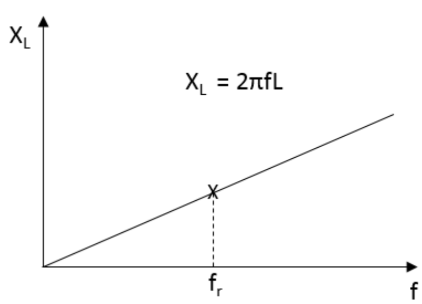
Where, f_r – Resonance frequency

Resonance Frequency:

$$f_r = \frac{1}{2\,\pi\sqrt{L\,C}} \quad ;\ \text{Hz}$$

At the resonance frequency (f_r), the voltage across C and L are equal in magnitude but 180°out of phase and they cancel each other.

4.4 Effect of varying frequency on Inductive reactance (X_L) :



Inductive Reactance (X_L) $= 2\,\pi\, f L$

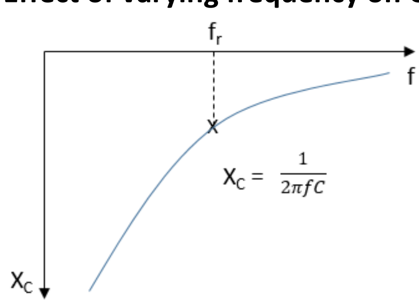
When frequency = 0; X_L = ∝

When frequency increase = X_L also increases

When frequency = ∝; X_L = 0

Hence, X_L varies linearly with frequency and it is a straight line passing through origin.

4.5 Effect of varying frequency on Capacitive reactance (X_C) :



Capacitive Reactance $X_C = \frac{1}{2\pi\, f\, C}$

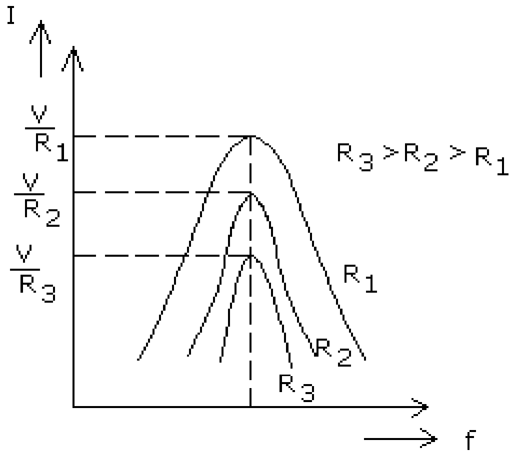
When frequency = 0; X_C = 0

When frequency increase = X_C decreases

When frequency = ∝; X_L = ∝

Hence, X_C varies indirectly with frequency and it is a rectangular hyperbola.

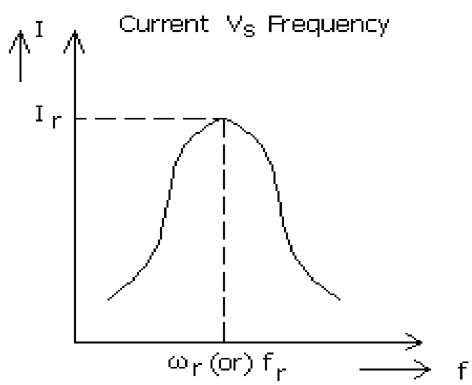
4.6 Effect of varying circuit resistance:

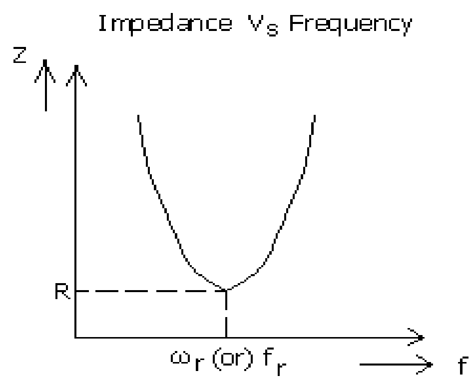


1. The impedance at resonance frequency is minimum, current is maximum.
2. Above f_r, $X_L > X_C$ and the circuit behaves as R_L circuit with lagging power factor.
3. At frequency less than f_r, $X_C < X_L$ and the circuit behaves as RC circuit drawing a leading current.

The resistance value does not affect the frequency at which resonance occurs, but the magnitude of current is decided by the resistance value. When R is increased, the current becomes less.

4.7 Characteristics of series resonance:

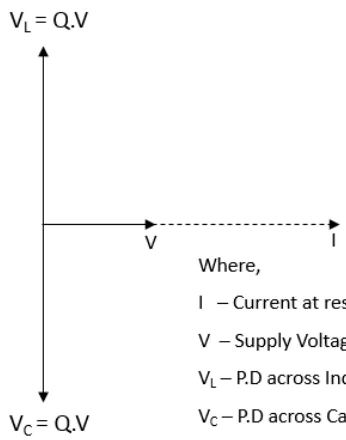




Resonance frequency (f_r)	Effect on reactance	Effect on impedance	Behavior of circuit	Effect on current	Effect of power factor
Below resonance frequency	$X_C > X_L$	Z > R	Capacitive	$I < I_{max}$	Less than unity (leading)
At resonance frequency	$X_C = X_L$	Z = R	Resistive	$I = I_{max}$	p.f = 1 (unity)
Above resonance frequency	$X_C < X_L$	Z > R	Inductive	$I < I_{max}$	Less than unity (lagging)

4.8 Q-factor (or) Quality factor:

At series resonance, the p.d across L or C builds up to a value many times greater than the applied voltage. The voltage magnification produced by series resonance is termed as Q-factor of the series resonance circuit. It is also called as voltage magnification.



Where,

I – Current at resonance

V – Supply Voltage

V_L – P.D across Inductor

V_C – P.D across Capacitor

Q - Q factor

$$\text{Voltage magnification (or) Q-factor} = \frac{\text{Voltage across L or C}}{\text{Supply Voltage}}$$

$$\text{At resonance,} \quad I = \frac{V}{R}$$

$$\text{P.D across L or C} = I.X_L \quad \text{(or)} \quad I.X_C$$

$$\text{Supply Voltage} \quad V = I.R$$

$$\text{Q-factor} = \frac{I.X_L}{I.R}$$

$$= \frac{X_L}{R}$$

$$\text{Q-factor} = \frac{2\pi\, f_r L}{R}$$

$$\text{Where,} \quad f_r = \frac{1}{2\,\pi\sqrt{L\,C}}\ ; \text{Hz}$$

$$\text{Q-factor} = \frac{2\pi\, L}{R\, 2\,\pi\sqrt{L\,C}}$$

$$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

(or)

$$\text{Q-factor} = \frac{\text{Resonance Frequency}}{\text{Bandwidth}}$$

Q-factor is large only when the value of L is large.

The value of Q –factor depends upon the design of the coil because resistance arises in this rather in a capacitor. With a well-designed coil, the quality factor can be 200 or more.

4.8.1 Importance of Q-Factor:

The Q-factor of series a.c circuit indicates how many times the p.d across L or C is greater than the applied voltage at resonance. For example, consider an R.L.C series circuit connected to 240V a.c source. If Q-factor of the coil is 20, then voltage across L or C will be 20x240 = 4800V at resonance

i.e $V_C = V_L = Q\, V_R = 20 \times 240 = 4800$ V.

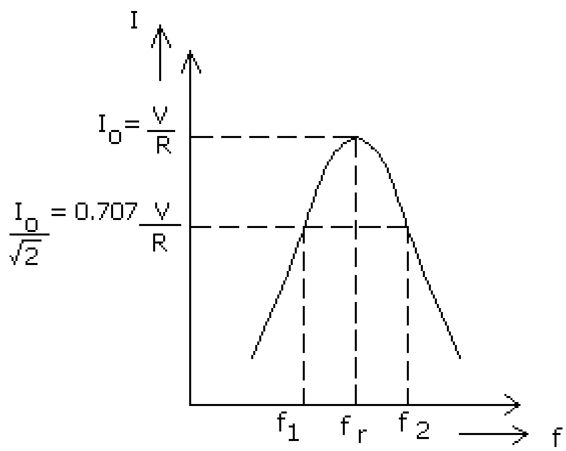
4.8.2 Q- Factor and Resonance curve:

At series resonance, the current is maximum and the current is limited by circuit resistance only. The smaller the circuit resistance, the greater is the circuit current and sharper will be resonance curve. Smaller circuit resistance means large value of Q-factor (=X_L/R). Therefore, the greater the Q-factor of resonance RLC circuit, the sharper is the resonance curve.

4.9 Bandwidth (BW):

The bandwidth is defined as the range of frequency at the limits of which the current is equal to or greater than 70.7% of maximum current. Thus

$$\text{Bandwidth} = f_2 - f_1$$



The frequency f_1 and f_2 are called as half power points or frequency. They are also referred to as cut-off frequencies. The bandwidth is an indication of sharpness or degree of tuning.

$$\text{Bandwidth} = \frac{\text{Resonance Frequency}}{\text{Q factor}}$$

$$= \frac{f_r}{Q}$$

4.9.1 Derive Q factor from Bandwidth and Resonant frequency:

$$\text{Q-factor} = \frac{\text{Resonance Frequency}}{\text{Bandwidth}}$$

$$\text{Q-factor} = \frac{f_r}{BW}$$

$$\text{Q-factor} = \frac{f_r}{f_2 - f_1}$$

At smaller bandwidth, there is high Q-factor

4.10 Selectivity:

The ability of resonant circuit to reject the frequency other than the resonant frequency is known as *selectivity*. It is also defined as the ratio of bandwidth and resonant frequency.

$$\text{Selectivity} = \frac{\text{Bandwidth}}{\text{Resonance Frequency}}$$

$$= \frac{\text{BW}}{f_r}$$

$$\text{Selectivity} = \frac{f_2 - f_1}{f_r}$$

A circuit is said to be selective if the response has a sharp peak and narrow bandwidth and is achieved with a high Q factor. Q is therefore a measure of selectivity.

4.10.1 Importance of Selectivity:

1. Fine-tuning
2. To avoid the interference of adjacent channel
3. To reject the image frequency

4.11 Acceptor Circuit:

An RLC series circuit accepts maximum current from the source at resonance and for that reason is called an acceptor circuit.

4.12 Properties of Series Resonance:

1. Impedance Z = R
2. Voltage across L and C are equal (i.e.) $V_L = V_C$
3. Current in the resonance circuit is maximum.
4. It magnifies voltage.
5. It also called as acceptor circuit.
6. Power factor is unity.(i.e.) cos θ=1

Example: 1

A coil having a resistance of 8 Ω and an inductance of 20mH is connected in series with a 10 MFD capacitor. Calculate i) Resonance frequency ii) Q-factor of the coil iii) Bandwidth and iv) Half power frequencies.

Given Data:			**To Find:**
Connection	=	Series	i) Resonance Frequency =?
Resistance (R)	=	8 Ω	ii) Q-Factor =?
Inductance (L)	=	20mH = $20\text{x}10^{-3}$	iii) Bandwidth =?
Capacitance (C)	=	10 MFD	iv) Half power frequencies =?

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \text{ x } 3.14\sqrt{LC}}$$

$$= \frac{1}{2 \text{ x } 3.14\sqrt{20\text{x}10^{-3} \text{ x } 10\text{x}10^{-6}}}$$

$$= \frac{1}{2 \text{ x } 3.14 \text{ x } 4.47\text{x}10^{-4}}$$

$$\text{Resonance Frequency } (F_r) = 356.06 \text{ Hz}$$

$$\text{Q- Factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{8}\sqrt{\frac{20\text{x}10^{-3}}{10\text{x}10^{-6}}}$$

$$= \frac{1}{8} \text{ x } 44.72$$

$$\text{Q- Factor} = 5.59$$

Half Power Frequencies:

$$\text{Lower cut-off frequencies } (F_1) = F_r - \frac{R}{4\pi L}$$

$$= 356.06 - \frac{8}{4\pi \text{ x } 20\text{x}10^{-3}}$$

$$= 356.06 - 31.84$$

$$= 324.22 \text{ Hz}$$

$$\text{Upper cut-off frequencies } (F_2) = F_r + \frac{R}{4\pi L}$$

$$= 356.06 + \frac{8}{4\pi \text{ x } 20\text{x}10^{-3}}$$

$$= 356.06 + 31.84$$

$$= 387.9 \text{ Hz}$$

$$\text{Bandwidth} = F_2 - F_1$$

$$= 387.9 - 324.22$$

$$= 63.68 \text{ Hz}$$

Answer:

i) Resonance Frequency (F_r) = 356.06 Hz

ii) Q- Factor = 5.59

iii) Half power frequencies:
F_1 = 324.22Hz and F_2 = 387.9Hz

iv) Bandwidth = 63.68 Hz

Example: 2

A series RLC circuit has resistance of 5 Ohm, inductance of 10mH and capacitance of 1mfd with an applied voltage of 100V, variable frequency. Calculate the resonant frequency, circuit current and voltage across inductor and capacitor. Also find the quality factor.

Given Data:

Connection	=	Series
Resistance (R)	=	5 Ω
Inductance (L)	=	10mH = $10x10^{-3}$
Capacitance (C)	=	1 MFD
Voltage (V)	=	100 V

To Find:

i) Resonance Frequency =?
ii) Circuit Current (I) =?
iii) P.D across L and C
iv) Q-Factor =?

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \text{ x } 3.14\sqrt{LC}}$$

$$= \frac{1}{2 \text{ x } 3.14\sqrt{10x10^{-3} \text{ x } 1x10^{-6}}}$$

$$= \frac{1}{2 \text{ x } 3.14 \text{ x } 1x10^{-4}}$$

$$\text{Resonance Frequency } (F_r) = 1592 \text{ Hz}$$

$$\text{Circuit Current (I)} = \frac{V}{R} = \frac{100}{5} = 20 \text{ Amps}$$

$$I = 20 \text{ Amps}$$

$$\text{Q- Factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{5}\sqrt{\frac{10x10^{-3}}{1x10^{-6}}}$$

$$= \frac{1}{5} \text{ x } 100$$

$$\text{Q- Factor} = 20$$

$$\text{P.D across inductor } (V_L) = I.\, X_L$$

$$= I \text{ x } 2\pi f_r L$$

$$= 20 \text{ x } 2 \text{ x } 3.14 \text{ x } 1592 \text{ x } 10x10^{-3}$$

$$= 2000 \text{ Volts}$$

$$\text{P.D across Capacitor } (V_c) = V_L \qquad (\because V_L = V_C)$$

$$= 2000 \text{ Volts}$$

Example: 3

A series circuit contains a resistance of 4 Ohms and inductance of 0.5H and a variable capacitor across 100V, 50Hz supply. Find (i) the capacitance for getting resonance (b) the p.d across inductance and capacitance (iii) the Q- factor of the series circuit.

<u>Given Data:</u>

Connection	=	Series
Resistance (R)	=	4 Ω
Inductance (L)	=	0.5H
Voltage (V)	=	100V
Frequency (F)	=	50Hz

<u>To Find:</u>

i) Capacitance (C) =?

ii) P.D across L and C

iii) Q-Factor =?

<u>Solution:</u>

At resonance:

$$X_L = X_C$$

$$2\pi FL = \frac{1}{2\pi\, fC}$$

$$2\pi FC = \frac{1}{2\pi\, fL}$$

$$C = \frac{1}{4\,\pi^2\, f^2\, L}$$

$$= \frac{1}{4 \times 3.14^2 \times 50^2 \times 0.5}$$

$$C = 20.28\mu F$$

$$\text{Circuit Current (I)} = \frac{V}{R} = \frac{100}{4} = 25\text{ Amps}$$

$$\text{Q- Factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{4}\sqrt{\frac{0.5}{20.28 \times 10^{-6}}}$$

$$= \frac{1}{4} \times 157$$

$$\text{Q- Factor} = 39.25$$

$$\text{P.D across inductor } (V_L) = I.\, X_L$$

$$= I \times 2\pi f_r L$$

$$= 25 \times 2 \times 3.14 \times 50 \times 0.5$$

$$= 3925\text{ Volts}$$

$$\text{P.D across Capacitor } (V_c) = V_L$$

$$= 3925\text{ Volts}$$

Example: 4

In a RLC series resonance circuit a resistance of 10 Ω and an inductance of 20mH is connected in series with a 0.5 MFD capacitor. Calculate i) Resonance frequency ii) Q-factor of the coil iii) Half power frequencies iv) Bandwidth and v) Power consumed at resonance if applied voltage to circuit is 200V a.c.

<u>Given Data:</u>

Connection	=	Series
Resistance (R)	=	10 Ω
Inductance (L)	=	20mH = 20×10^{-3}
Capacitance (C)	=	0.5 MFD
Voltage (V)	=	200V

<u>To Find:</u>

i) Resonance Frequency =?
ii) Q-Factor =?
iii) Bandwidth =?
iv) Half power frequencies =?
v) Power consumed =?

<u>Solution:</u>

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \text{ x } 3.14\sqrt{LC}}$$

$$= \frac{1}{2 \text{ x } 3.14\sqrt{20\text{x}10^{-3} \text{ x } 0.5\text{x}10^{-6}}}$$

$$= \frac{1}{2 \text{ x } 3.14 \text{ x } 1\text{x}10^{-4}}$$

$$\text{Resonance Frequency } (F_r) = 1592 \text{ Hz}$$

$$\text{Q- Factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{10}\sqrt{\frac{20\text{x}10^{-3}}{0.5\text{x}10^{-6}}}$$

$$= \frac{1}{10} \text{x } 200$$

$$\text{Q- Factor} = 20$$

Half Power Frequencies:

$$\text{Lower cut-off frequencies } (F_1) = F_r - \frac{R}{4\pi L}$$

$$= 1592 - \frac{10}{4\pi \text{ x } 20\text{x}10^{-3}}$$

$$= 1592 - 39.8$$

$$= 1552 \text{ Hz}$$

$$\text{Upper cut-off frequencies } (F_2) = F_r + \frac{R}{4\pi L}$$

$$= 1592 + \frac{10}{4\pi \text{ x } 20\text{x}10^{-3}}$$

$$= 1592 + 39.8$$

$$= 1632 \text{ Hz}$$

$$\text{Bandwidth} = F_2 - F_1$$

$$= 1632 - 1552$$

$$= 80 \text{ Hz}$$

$$\text{Power (P)} = V \times I \times \cos\theta$$

$$= 200 \times \frac{V}{R} \times 1$$

$$= 200 \times \frac{200}{10} \times 1$$

$$= 200 \times 20 \times 1$$

$$P = 4000 \text{ Watts}$$

Answer:

i) Resonance Frequency (F_r) = 1592 Hz

ii) Q- Factor = 20

iii) Half power frequencies:
F_1 = 1552Hz and F_2 = 1632Hz

iv) Bandwidth = 80

v) Power (P) = 4000 Watts

Example: 5

An R.L.C series circuit consists of a resistance of 10 Ω an inductance of 100mH and a capacitance of 10μF. If a voltage of 100V is applied across the combines find i) Resonance frequency ii) Q-factor of the circuit and iii) Bandwidth iv) Half power points v) Power.

Given Data:

Connection	=	Series
Resistance (R)	=	10 Ω
Inductance (L)	=	100mH = 100×10^{-3}
Capacitance (C)	=	10μF = 10×10^{-6}
Voltage (V)	=	100V

To Find:

i) Resonance Frequency =?
ii) Q-Factor =?
iii) Bandwidth =?
iv) Half power frequencies
v) Power consumed

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14\sqrt{100 \times 10^{-3} \times 10 \times 10^{-6}}}$$

$$= \frac{1}{2 \times 3.14 \times 1 \times 10^{-3}}$$

$$\text{Resonance Frequency } (F_r) = 159 \text{ Hz}$$

$$\text{Q- Factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{10}\sqrt{\frac{100\text{x}10^{-3}}{10\text{x}10^{-6}}}$$

$$= \frac{1}{10}\text{ x }100$$

$$\text{Q- Factor} = 10$$

Half Power Frequencies:

$$\text{Lower cut-off frequencies } (F_1) = F_r - \frac{R}{4\pi L}$$

$$= 159 - \frac{10}{4\pi \text{ x } 100\text{x}10^{-3}}$$

$$= 159 - 7.96$$

$$= 151 \text{ Hz}$$

$$\text{Upper cut-off frequencies } (F_2) = F_r + \frac{R}{4\pi L}$$

$$= 159 + \frac{10}{4\pi \text{ x } 100\text{x}10^{-3}}$$

$$= 159 + 7.96$$

$$= 167 \text{ Hz}$$

$$\text{Bandwidth} = F_2 - F_1$$

$$= 167 - 151$$

$$= 16 \text{ Hz}$$

$$\text{Power (P)} = \text{V x I x Cos}\theta$$

$$= 100 \text{ x} \frac{V}{R} \text{ x } 1$$

$$= 100 \text{ x} \frac{100}{10} \text{ x } 1$$

$$= 100\text{x } 10 \text{ x } 1$$

$$P = 1000 \text{ Watts}$$

Answer:

i) Resonance Frequency (F_r) = 159 Hz

ii) Q- Factor = 10

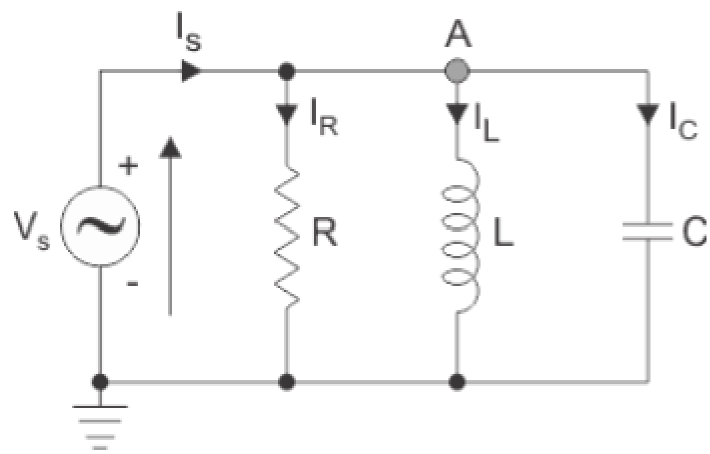
iii) Bandwidth = 16 Hz

iv) Half power frequencies:
F_1 = 151Hz and F_2 = 167Hz

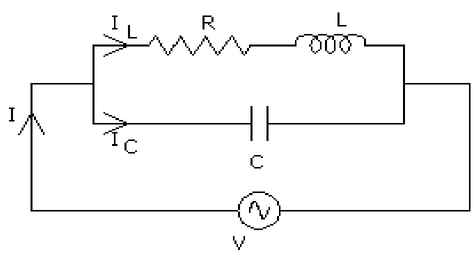
v) Power (P) = 1000 Watts

4.13 PARALLEL RESONANCE:

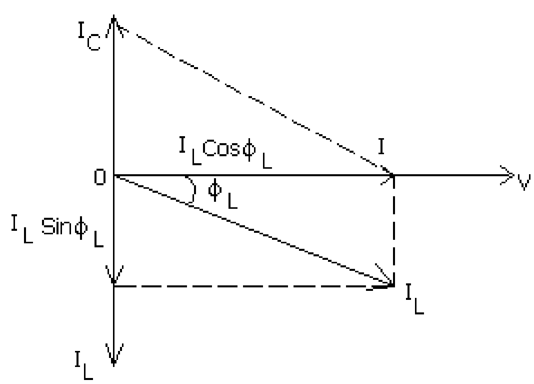
Consider a RLC circuit in which resistor, inductor and capacitor are connected in parallel to each other. This parallel combination is supplied by voltage supply. In parallel resonance circuit, the voltage across each element remains the same and the current gets divided in each component depending upon the impedance of each component. The total current, I_S drawn from the supply is equal to the vector sum of the resistive, inductive and capacitive current.



Consider coil of inductance L and a low resistance R shunted across a pure capacitor C.



4.13.1 Vector diagram



A variable frequency is applied across the circuit. At particular frequency, the reactive component of the line current becomes zero. This frequency is called *resonant frequency*.

When total current is in phase with supply voltage parallel resonance is said to occur. This is true, only if reactive component of inductive branch current is equal to capacitive branch current.

4.14 Q-factor:

At resonance, the current in the branches of the parallel circuit can be many times greater than the supply current. The factor of magnification in the parallel circuit, is called as Q factor. It is also called as the current magnification.

$$\mathrm{Q - Factor\ or\ Current\ Magnification} = \frac{\mathrm{Current\ in\ the\ inductive\ or\ Capacitive\ \ branch}}{\mathrm{Current\ in\ supply\ at\ resonance}}$$

$$\mathrm{Q\ \text{-}\ Factor} = \frac{I_C}{I}$$

$$I_C = \frac{V}{X_C} = \frac{1}{X_C}.V = (2.\pi.f.c).V = \omega.CV$$

$$I = \frac{V}{Z_0} = \frac{V}{\frac{L}{CR}} = \frac{VCR}{L}$$

$$\mathrm{Q\ \text{-}\ Factor} = \frac{I_C}{I}$$

$$\mathrm{Q\ \text{-}\ Factor} = \frac{\omega.CV}{\frac{VCR}{L}} = \frac{\omega.L}{R} = \frac{2.\pi.f_o.L}{R}$$

$$\mathrm{Substitute,}\quad f_o = \frac{1}{2.\pi\sqrt{LC}}$$

$$\mathrm{Q\ \text{-}\ Factor} = \frac{2.\pi.L}{2.\pi\sqrt{LC}\,R}$$

$$= \frac{1}{R}\frac{L}{\sqrt{LC}}$$

$$= \frac{1}{R}\frac{\sqrt{L}\,\sqrt{L}}{\sqrt{LC}}$$

$$\mathrm{Q\ \text{-}\ Factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

4.15 Condition for Parallel Resonance:

Reactive component of inductive branch current is equal to capacitive branch current.

4.16 Resonance Frequency:

$$\mathrm{At\ resonance,}\ I_C = I_L \sin \Phi_L$$

$$\mathrm{We,\ Know\ that,}\ I_C = \frac{V}{X_C}$$

$$I_L = \frac{V}{Z_L}$$

$$\text{Sin } \Phi_L = \frac{X_L}{Z_L}$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \cdot \frac{X_L}{Z_L}$$

$$X_L\, X_C = {Z_L}^2$$

$$2.\pi.f_r.L \;.\; \frac{1}{2.\pi f_r.C} = {Z_L}^2$$

$$\frac{L}{C} = {Z_L}^2$$

$${Z_L}^2 = \frac{L}{C}$$

$$R^2 + {X_L}^2 = \frac{L}{C}$$

$$R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$(2\pi f_r)^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\text{Resonance Frequency:} \qquad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

4.17 Impedance at Resonance:

The effective resistance of the RLC parallel circuit at resonance is called dynamic impedance or dynamic resistance.

$$\text{At resonance, } I_r = I_L \text{ Cos } \Phi_L$$

$$\text{We, Know that, } I_r = \frac{V}{Z_r}$$

$$I_L = \frac{V}{Z_L}$$

$$\text{Cos } \Phi_L = \frac{R}{Z_L}$$

$$\frac{V}{Z_r} = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$

$$\frac{1}{Z_r} = \frac{R}{{Z_L}^2}$$

$$\frac{1}{Z_r} = \frac{R}{L/C}$$

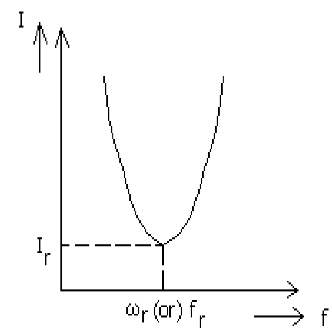
$$Z_r = \frac{L/C}{R}$$

$$Z_r = \frac{L}{CR}$$

The entire circuit behaves as a non-reactive resistor of L/CR ohms under resonance. The quantity L/CR ohm is often called dynamic impedance or dynamic resistance.

4.18 Graphical Representation:

Line current Vs Frequency curve



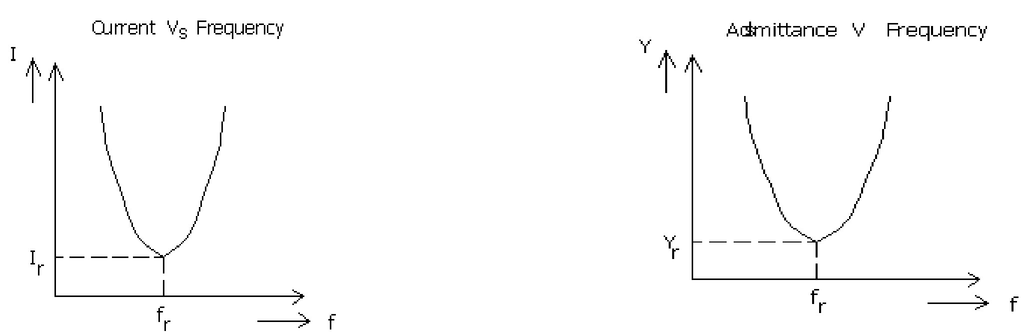
At parallel resonance, the line current I_r is minimum and is given by:

$$I_r = \frac{V}{Z_r}$$

$$\text{Where,} \quad Z_r = \frac{L}{CR}$$

The small current is only the amount needed to supply the resistance losses in the circuit.

4.18.1 Characteristics of Parallel Resonance:



Characteristic of Parallel Resonance

4.19 Rejector circuit:

The lowest current from the source occurs at the resonant frequency of a parallel circuit hence it is called a rejector circuit.

4.20 Properties of parallel resonance

1. At resonance the net reactance is zero.
2. At resonance the impedance is maximum
3. At resonance the current is minimum
4. At resonance the power factor of the circuit is unity
5. It magnifies current.
6. Line current is in phase with voltage

4.21 Comparison of Series and Parallel Resonant circuit:

Criterion for comparison	Series circuit	Parallel circuit
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum	Minimum
Effective impedance at resonance	R	L/CR
Resonant frequency (f_r)	$\frac{1}{2.\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$
It magnifies	Voltage	Current
Power factor at resonance	1 (unity)	1 (unity)
Q-factor	ωL/R	ωL/R
Other name	Acceptor circuit	Rejector circuit

4.22 Application of Resonance

1. Used in oscillator circuit to provide different frequencies.
2. Used in tuning circuit of Radio and TV to obtain the required station.

Example: 6

A coil of 10 Ohms resistance and 0.1H inductance is connected in parallel with a capacitor of 100mfd capacitance. Calculate the frequency at which the circuit will act as a non-inductive resistance of R ohm. Find also the dynamic resistance.

Given Data:

Connection	=	Parallel Circuit
Resistance (R)	=	10 Ω
Inductance (L)	=	0.1 H
Capacitance (C)	=	100mfd = 100x10^{-6}

To Find:

i) Resonance Frequency =?

ii) Dynamic Resistance =?

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \text{x} \ 100 \text{ x } 10^{-6}}-\frac{10^2}{0.1^2}}$$

$$= \frac{1}{2\pi}\sqrt{100000 - 10000}$$

$$= \frac{1}{2\pi}\sqrt{90000}$$

$$= \frac{1}{2\pi} \text{ x } 300$$

$$= 47.77 \text{ Hz}$$

$$\text{Resonance Frequency } (F_r) = 47.77 \text{ Hz}$$

$$\text{Dynamic Resistance } (R_D) = \frac{L}{CR}$$

$$= \frac{0.1}{100 \text{ x } 10^{-6} \text{ x } 10}$$

$$\text{Dynamic Resistance } (R_D) = 100\ \Omega$$

Answer:

i) Resonance Frequency (F_r) = 47.77Hz ii) Dynamic Resistance (R_D) = 100 Ω

Example: 7

A coil of 20 Ohms resistance and 0.2H inductance is connected in parallel with a capacitor of 100mfd capacitance. Calculate the frequency at which the circuit will act as a non-inductive resistance of R ohm. Also find the dynamic resistance.

Given Data:

Connection	=	Parallel Circuit
Resistance (R)	=	20 Ω
Inductance (L)	=	0.2 H
Capacitance (C)	=	100mfd = 100x10^{-6}

To Find:

i) Resonance Frequency =?

ii) Dynamic Resistance =?

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2x\,100\,x\,10^{-6}} - \frac{20^2}{0.2^2}}$$

$$= \frac{1}{2\pi}\sqrt{50000 - 10000}$$

$$= \frac{1}{2\pi}\sqrt{40000}$$

$$= \frac{1}{2\pi}\,x\,200$$

$$\text{Resonance Frequency } (F_r) = 31.84\text{ Hz}$$

$$\text{Dynamic Resistance } (R_D) = \frac{L}{CR}$$

$$= \frac{0.2}{100\,x\,10^{-6}\,x\,20}$$

$$\text{Dynamic Resistance } (R_D) = 100\,\Omega$$

Answer:

i) Resonance Frequency (F_r) = 31.84Hz ii) Dynamic Resistance (R_D) = 100 Ω

Example: 8

A parallel circuit consists of a 2.5µF capacitor and a coil whose resistance and inductance are 15Ω and 260mH respectively. Determine (i) Resonant frequency (ii) Q-Factor of the circuit at resonance (iii) Dynamic resistance of the circuit.

Given Data:

Connection	=	Parallel Circuit
Resistance (R)	=	15 Ω
Inductance (L)	=	260mH = $260x10^{-3}$ H
Capacitance (C)	=	2.5 µf = $2.5x10^{-6}$F

To Find:

i) Resonance Frequency =?
ii) Q-Factor =?
iii) Dynamic Resistance =?

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{260\,x\,10^{-3}x\,2.5x\,10^{-6}} - \frac{15^2}{(260x\,10^{-3})^2}}$$

$$= \frac{1}{2\pi}\sqrt{1538461.5 - 3328.4}$$

$$= \frac{1}{2\pi}\,x\,1239$$

Resonance Frequency (F_r) = 197.3 Hz

$$\text{Q- Factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{15}\sqrt{\frac{260 x 10^{-3}}{2.5 x 10^{-6}}}$$

$$= \frac{1}{15} \text{ x } 322.5$$

Q- Factor = 21.49

$$\text{Dynamic Resistance } (R_D) = \frac{L}{CR}$$

$$= \frac{260 \text{ x } 10^{-3}}{2.5 \text{ x } 10^{-6} \text{ x } 15}$$

Dynamic Resistance (R_D) = 6933 Ω

Answer:

i) Resonance Frequency (F_r) = 197.3Hz ii) Q-Factor = 21.49

iii) Dynamic Resistance (R_D) = 6933 Ω

Example: 9

A coil of resistance 12Ω and inductance 0.12 H is connected in parallel with a 60µF capacitor to a 100 Volt variable frequency supply. Calculate (i) the frequency at which the circuit will act as a non-inductive resistor (ii) the value of dynamic impedance at resonance.

Given Data:

Connection	=	Parallel Circuit
Resistance (R)	=	12 Ω
Inductance (L)	=	0.12 H
Capacitance (C)	=	60mfd = 60x10^{-6}F

To Find:

i) Resonance Frequency =?

ii) Dynamic Resistance =?

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.12 x \; 60 \; x \; 10^{-6}} - \frac{12^2}{0.12^2}}$$

$$= \frac{1}{2\pi}\sqrt{138889 - \; 10000}$$

$$= \frac{1}{2\pi} \; x \; 359$$

Resonance Frequency (F_r) = $57.16 \; Hz$

$$\text{Dynamic Resistance } (R_D) = \frac{L}{CR}$$

$$= \frac{0.12}{60 \; x \; 10^{-6} \; x \; 12}$$

$$\text{Dynamic Resistance } (R_D) = 167\ \Omega$$

Answer:

i) Resonance Frequency (F_r) = 57.16Hz ii) Dynamic Resistance (R_D) = 167 Ω

Example: 10

An inductive circuit of resistance 2 Ohms and inductance of 0.01H is connected to a 250V, 50Hz supply. What capacitance placed in parallel will produce resonance? Find the total current taken from the supply and the current in each branch circuits.

Given Data:

Connection	=	Parallel Circuit
Resistance (R)	=	2 Ω
Inductance (L)	=	0.01 H
Voltage (V)	=	250V
Frequency (F)	=	50Hz

To Find:

i) Capacitance =?
ii) Total Current =?
iii) Branch Current = ?

Solution:

$$\text{Resonance Frequency } (F_r) = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{0.01x\ C} - \frac{2^2}{0.01^2}}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{0.01x\ C} - 40000}$$

$$50 \text{ x } 2\pi = \sqrt{\frac{1}{0.01x\ C} - 40000}$$

$$(50 \text{ x } 2\pi)^2 = \frac{1}{0.01x\ C} - 40000$$

$$98596 + 40000 = \frac{1}{0.01x\ C}$$

$$138596 = \frac{1}{0.01x\ C}$$

$$C = \frac{1}{0.01x\ 138596}$$

$$C = 721\mu F$$

$$\text{Dynamic Resistance } (R_D) = \frac{L}{CR}$$

$$= \frac{0.01}{721\ x\ 10^{-6}\ x\ 2}$$

$$\text{Dynamic Resistance } (R_D) = 6.93\ \Omega$$

$$\text{Total Current } (I_T) = \frac{V}{R_D} = \frac{250}{6.93} = 36 \text{ Amps}$$

Branch Current:

$$\text{Inductive Reactance } (X_L) = 2\pi FL$$

$$= 2 \text{ x } 3.14 \text{ x } 50 \text{ x } 0.01$$

$$X_L = 3.14\ \Omega$$

$$\text{Capacitive Reactance } (X_C) = \frac{1}{2\pi f C}$$

$$= \frac{1}{2x\ 3.14\ x\ 50\ x\ 721x\ 10^{-6}}$$

$$X_C = 4.42\ \Omega$$

$$\text{Current through coil} = \frac{V}{Z_L}$$

$$\text{Impedance } (Z_L) = \sqrt{R^2 + {X_L}^2}$$

$$= \sqrt{2^2 + 3.14^2}$$

$$= 3.72\ \Omega$$

$$\text{Current through coil} = \frac{V}{Z_L} = \frac{250}{3.72} = 67.2 \text{ Amps}$$

$$\text{Current through Capacitor} = \frac{V}{X_C}$$

$$= \frac{250}{4.42}$$

$$= 56.56 \text{ Amps}$$

<u>Answer:</u>

i) Capacitance (C) = 721 µF

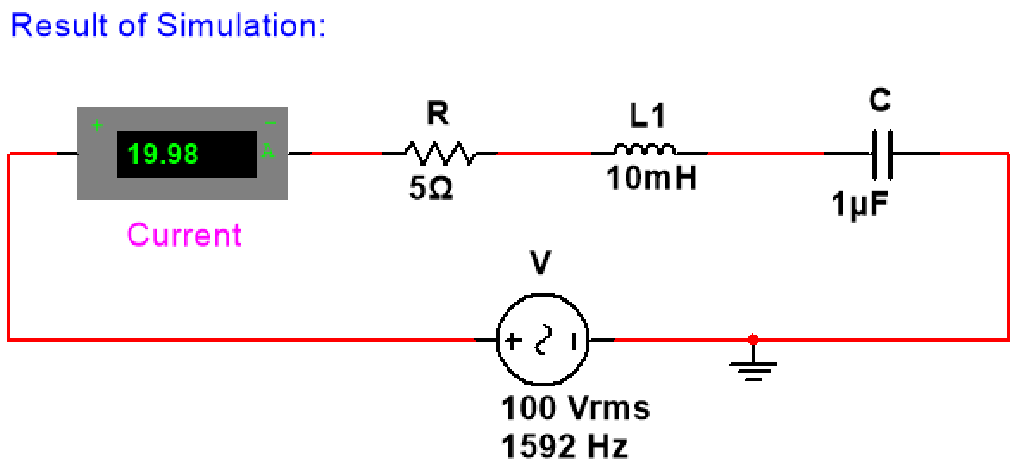
ii) Total Current (I) = 36 Amps

iii) Current through coil = 67.2Amps

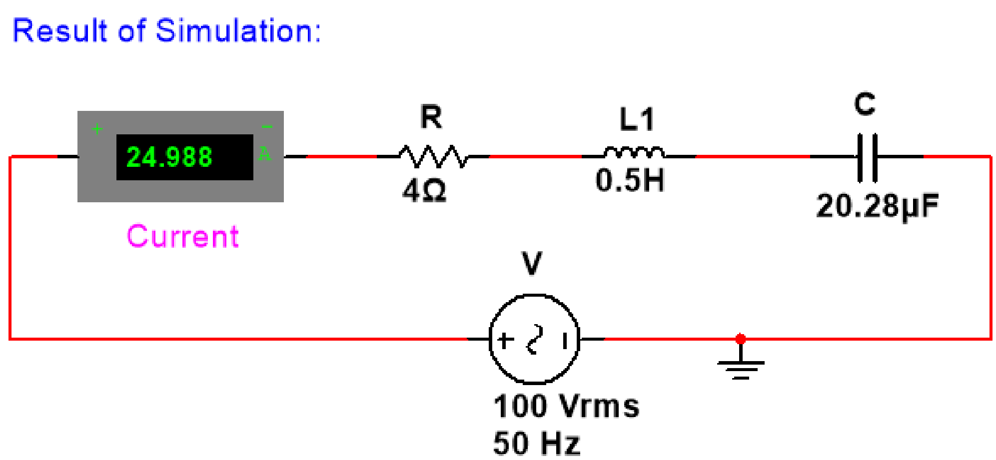
iv) Current through capacitor = 56.56A

RESULT OF SIMULATION OF PROBLEMS IN UNIT IV

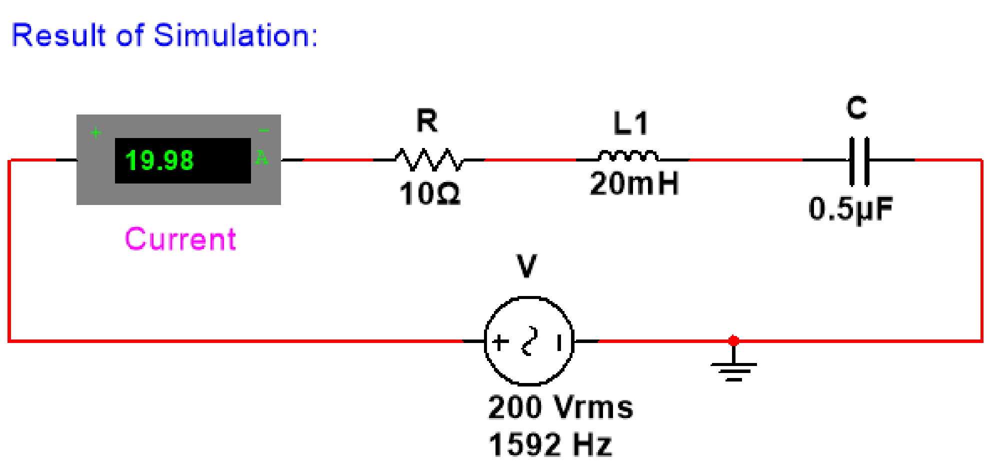
Problem : 2



Problem : 3

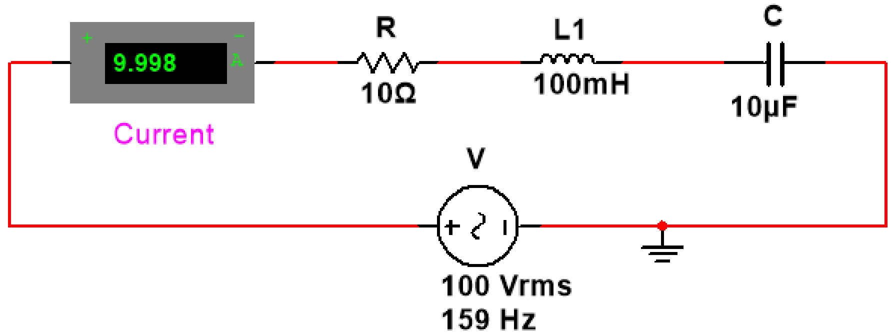


Problem : 4



Problem : 5

Result of Simulation:



REVIEW QUESTIONS

UNIT : IV RESONANT CIRCUITS

PART – A : 2 Mark Questions

1. What is resonance?
2. State the condition for resonance in R.L.C series circuit.
3. What is resonance frequency?
4. Write the expression for resonance frequency in R.L.C series circuit.
5. What is the power factor of the R.L.C series circuit at resonance?
6. What is meant by voltage magnification in R.L.C series resonance circuit?
7. Define voltage magnification factor in R.L.C series resonance circuit.
8. Why the series resonance circuit is called as acceptor circuit?
9. Define Q - factor in series resonance.
10. Write the expression for quality factor of a series RLC circuit.
11. Define half power frequency.
12. Define band width of an RLC series circuit.
13. Write the expression for band width of an RLC series circuit.
14. Define Selectivity.
15. What are half power frequencies?
16. Write the expression for lower cut off frequency.
17. Write the expression for upper cut off frequency.
18. Write the expression for resonance frequency in the circuit R.L parallel with C.
19. Draw the frequency response of an RLC parallel circuit.
20. Define Q factor in parallel resonance circuit.
21. Write the expression for quality factor of a parallel RLC circuit.
22. What is dynamic resistance? Write the expression for dynamic resistance of an RL circuit parallel with C.
23. Draw a curve showing the relationship between current and frequency in a parallel resonance circuit.
24. Draw the curve for impedance versus frequency in parallel resonance circuit.

PART – B : 3 Mark Questions

1. Draw the phasor diagram and state the condition for series resonance.
2. Derive an expression for resonance frequency in RLC series circuit.
3. Draw the frequency response of RLC series circuit.
4. Sketch the response of RLC series circuit and mark (i) Bandwidth (ii) Both cut off frequencies (iii) resonance frequency.
5. Define Selectivity and state its importance
6. Define "Q" factor in series resonant circuit and derive its expression.
7. Define half power frequencies and band width.
8. Write the expression for half power frequencies of an RLC series circuit.
9. Write the properties of series resonance.
10. Draw the frequency response of RLC parallel circuit.
11. Draw the phasor diagram and state the condition for parallel resonance.
12. Define "Q" factor in parallel resonant circuit and derive its expression.
13. Define dynamic impedance and derive its expression for parallel resonance.
14. Write the properties of parallel resonance.
15. A circuit has the resonant frequency of 60Hz and lower half power frequency of 40 Hz. What is the band width?
16. A coil having an inductance of 0.5mH and resistance 10Ω is connected in series with 10μF capacitor across a 200V AC supply. Calculate the resonance frequency of the circuit.
17. A coil having an inductance of 50mH and resistance 10Ω is connected in series with 25μF capacitor across a 200V AC supply. Calculate the resonance frequency of the circuit.
18. A coil having an inductance of 5H and resistance 90Ω is connected in series with 100μF capacitor across a 10V AC supply. Calculate the Q factor.
19. An RLC series circuit consists of Resistance of 16 Ω¸ an inductance of 5mH and capacitance of 2 μF. Calculate the quality factor.
20. Parallel circuit consists of a coil of 50mH and 0.01μF capacitor are connected across a 100V AC supply. Calculate the resonance frequency of the circuit.

PART – C : 10 Mark Questions

1. What is resonance frequency. Derive the expression to find the resonance frequency in series RLC circuit.
2. Compare series and parallel resonance.
3. Define the following terms:
   (i) Q – factor (ii) selectivity (iii) half power frequency (iv) band width.
4. Derive the expression to find the resonance frequency in parallel RLC circuit.
5. Define quality factor. Derive the expression to find the quality factor in parallel resonance circuit.
6. A series RLC circuit consists of R=5Ω, L=40mH and C = 1μF. Calculate the Resonance frequency, Q factor, Bandwidth and half power frequencies.

7. A series RLC circuit consists of R=100Ω, L=10mH and C = 1μF is connected to a 20V a.c supply. Find the the Resonance frequency, Q factor, Bandwidth and half power frequencies.

8. An inductive coil having a resistance of 20Ω and an inductance of 0.02H is connected in series with 0.01μF capacitor. Calculate (q) Q –Factor (b) Resonant frequency and (c) Bandwidth.

9. A series RLC circuit has an impedance of 40 Ω at a frequency of 200rad/sec. When the circuit is made to resonate by connecting a 10V source of variable frequency the current at resonance is 0.5A and the quality factor at resonance is 10. Determine the circuit parameters.

10. A series circuit consists of a 10Ω resistor, a 30 mH inductor and a 1 μF capacitor, and is supplied from a 10V variable-frequency source. Find the frequency for which the voltage developed across the capacitor is a maximum and calculate the magnitude of this voltage.

11. A coil of 5 Ω resistance and 15mH inductance is connected in parallel with a capacitor of 160 μF. Calculate the frequency at which resonance occurs. Also calculate the dynamic impedance of the circuit.

12. A coil of 1 kΩ resistance and 0.15 H inductance is connected in parallel with a variable capacitor across a 2.0V, 10 kHz a.c. supply. Calculate:(a) the capacitance of the capacitor when the supply current is minimum; (b) the effective impedance Zr of the network at resonance; (c) the supply current.

13. A coil of resistance 1Ω and inductance 0.12 H is connected in parallel with a 60μF capacitor to a 100V variable-frequency supply. Calculate the frequency at which the circuit will behave as a non-reactive resistor, and also the value of the dynamic impedance.

UNIT V – THREE PHASE CIRCUITS

Syllabus:

Three phase systems-phase sequence – necessity of three phase system – concept of balanced and unbalanced load - balanced star & delta connected loads – relation between line and phase voltages and currents – phasor diagram – three phase power and power factor measurement by single wattmeter and two wattmeter methods – Problems in all above topics.

5.1 Three Phase Systems:

In general, generation, transmission and distribution of electrical energy are in three phase. Three phase system can be viewed as the combination of three single phase systems with a phase difference of 120° between each winding. Hence, a three phase generating system is formed by three separate windings with 120° phase difference between them. As the windings are made to rotate in a common magnetic field in a three phase generator, three voltages of the same magnitude and frequency but 120° phase difference between each other are produced. The convention adopted to identify each of the phase voltages is: R-red, Y-yellow, and B-blue.

Consider three coils RR_1, YY_1 and BB_1 placed in a magnetic field of maximum value of flux Φ_m Weber is shown in figure (i). Let all the coils rotate in the anticlockwise direction at an angular velocity ω.

Fig. (i) Generation of three phase emf

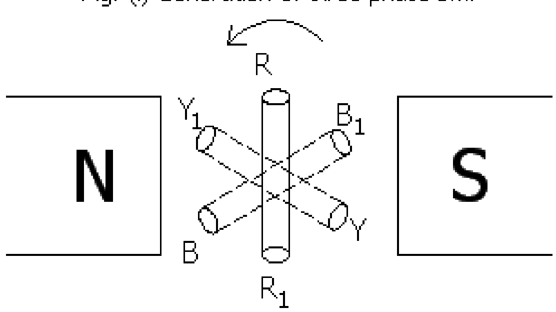
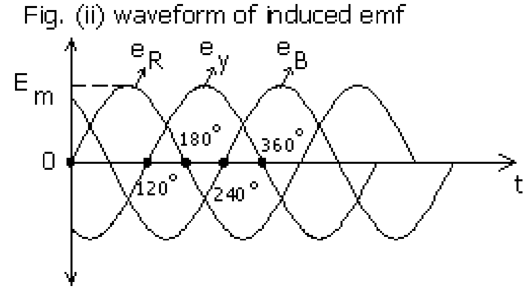


Fig. (ii) waveform of induced emf



According to Faradays law of electromagnetic induction, emfs are induced in the coils RR_1, YY_1 and BB_1. The induced emf in coil YY_1 lags behind the induced emf in coil RR_1 by 120° and the induced emf in coil BB_1 lags behind that in coil by 240°.

All the three induced emfs have the same amplitude, same period and frequency. Thus, the above sets of voltages are called three phase-balanced system of voltages. The waveforms of the induced voltages are shown in figure.

5.1.1 Expressions for Voltage and Current in a three phase system:

$E_R = E_m \sin\omega t$
$E_Y = E_m \sin(\omega t\text{-}120°)$
$E_B = E_m \sin(\omega t\text{-}240°)$

5.1.1 Expressions for Voltage and Current in a three phase system:

$I_R = I_m \sin\omega t$
$I_Y = I_m \sin(\omega t\text{-}120°)$
$I_B = I_m \sin(\omega t\text{-}240°)$

5.2 Necessity of Three Phase system:

For resistive loads (Lamps and Heater etc.,) single phase supply works satisfactorily. However, when a.c motors were developed, it was found that the single phase a.c supply did not work properly as it was not able to produce the starting torque. Hence three phase system is necessary to power large motors and other heavy loads. A three-phase system is usually more economical than an equivalent single-phase at the same line to ground voltage because it uses less conductor material to transmit three phase electrical power.

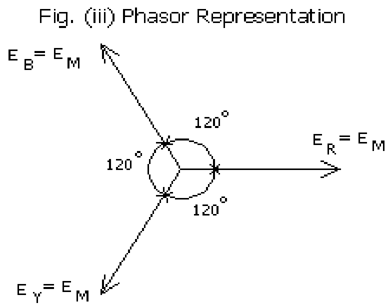
5.2.1 Advantages of three phase system:

The three phase system has the following advantages compared to a single phase system:

i) The amount of copper or aluminium wires required to transfer the given amount of power is minimum in a three phase system than that is required in a single phase system.
ii) A 3 phase machine gives more output compared to a single phase machine of the same size.
iii) Three phase motors have uniform torque whereas most of the single phase motors have pulsating torque.
iv) A three phase motor produces more torque as compared to a single phase motor.
v) Domestic power and industrial or commercial power can be supplied from the same source.
vi) Three phase motors are self-starting whereas single phase induction motors are not.
vii) In three phase system has better voltage regulation.
viii) Three phase machines have better power factor and efficiency.
ix) Generation, transmission and utilisation of power is more economical in three phase systems compared to single phase system.
x) In a three phase system power never falls to zero.

5.3 Terms and Definition:

5.3.1 Phasor Representation:



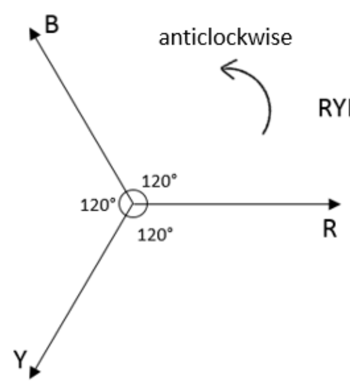
Let the emf induced in R phase, E_R be taken as reference. E_Y lags ER by 120° and E_B lags ER by 240°. The three phasors are represented in figure (iii).

5.3.2 Phase sequence:

In three phase system the order in which the three phase emfs or currents attain their maximum value is called Phase Sequence. The three phases are generally represented as R (Red), Y (Yellow) and B (Blue).

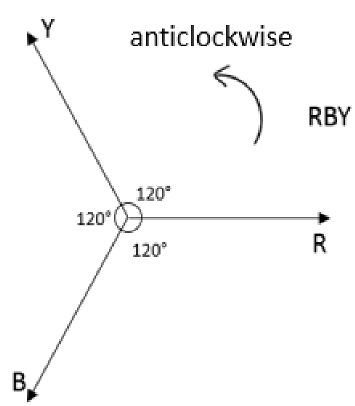
5.3.3 Positive sequence:

If the phase sequence is given as RYB then the convention is R phase reaches its maximum value first, Y phase follows 'R' and 'B' phase follows Y in reaching the maximum value. The RYB sequence in the anticlockwise direction defines the positive sequence.



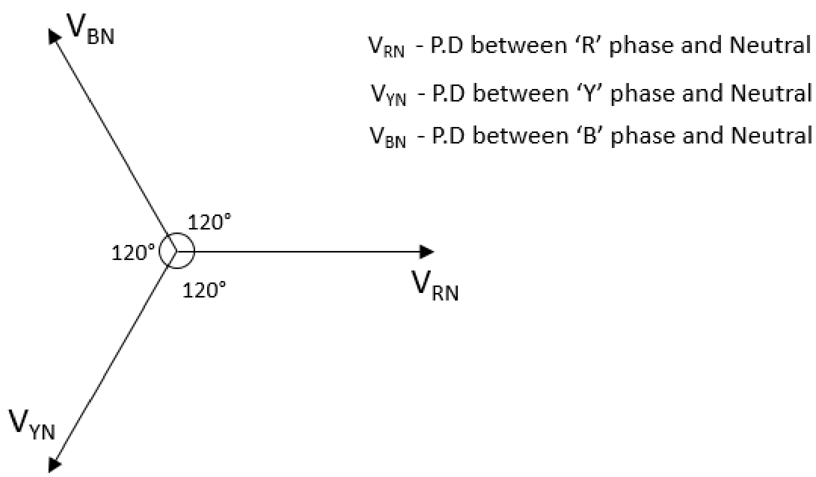
5.3.4 Negative sequence:

A three-phase system in which the voltages and currents in each of the three phases reach their maximum values in the reverse order to conventional phase sequence, ie R,B Y as opposed to R, Y,B is called as negative sequence.



5.3.5 Phase voltage:

The voltage between one of the phase terminal and the neutral terminal is known as phase voltage and is represented by V_{ph}.



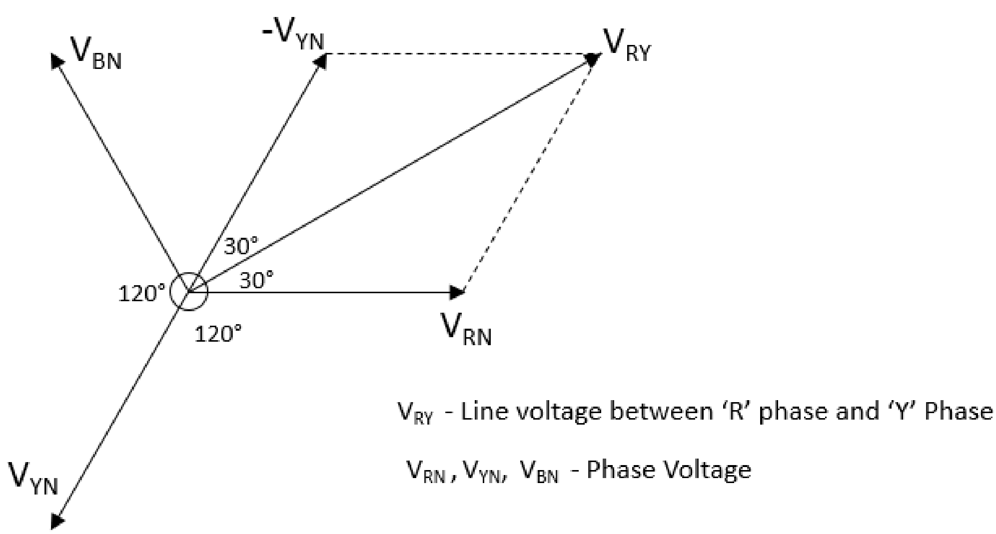
V_{RN}, V_{YN} and V_{BN} are the phase voltages.

5.3.6 Line voltage:

A line voltage is the phasor difference between the appropriate pair of phase voltage. Thus V_{RY} is the phasor difference between V_{RN} and V_{YN}.

(or)

The voltage between any two phase terminals of a three phase system is known as line voltage and is represented by V_L.



5.3.7 Phase current:

The current flowing through any one of the phase windings of the system is called phase current and is denoted by I_{ph}

5.3.8 Line current:

Line current means the current flowing through the AC supply lines and it is denoted by I_L

5.4 Concept of balanced and unbalanced load:

5.4.1 Balanced load:

The balanced load is a load in which each phase have identical impedances. i.e each impedance has the same magnitude and phase angle. Hence each impedances draws equal current and power factor.

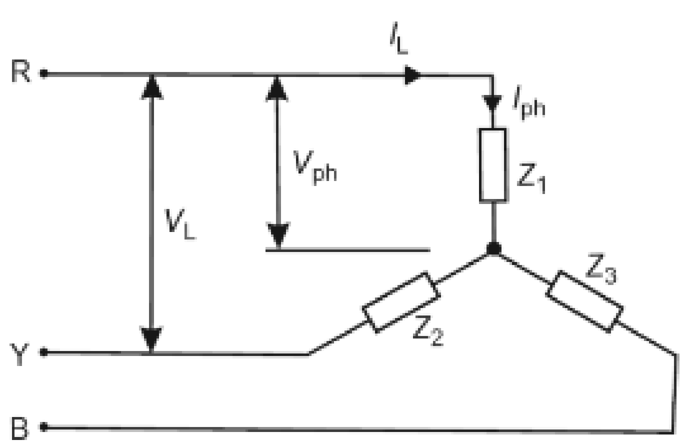
5.42 Unbalanced load:

The unbalanced load is a load in which each phase have unequal impedances. i.e each impedance has the different magnitude and phase angle. Hence each impedances draws unequal current and power factor.

5.5 Methods of three phase connection:

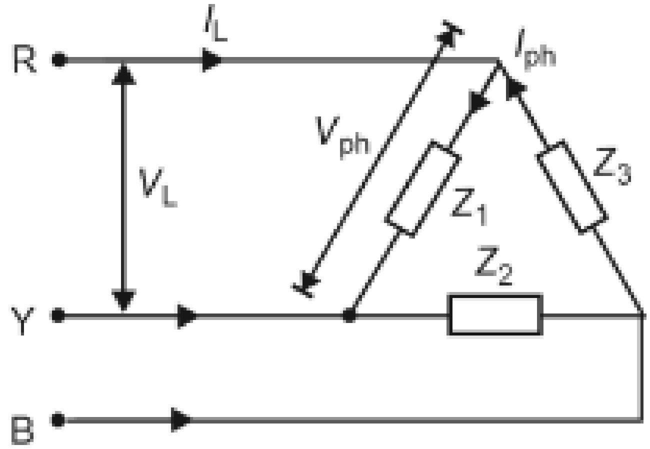
In a three phase alternator, there are three independent phase windings or coils. Each phase or coil has two terminals, viz start and finish. The coil ends are interconnected to form a star (Y) or delta (Δ) connected three phase system.

5.5.1 Star or Why (Y) connection:



In star connection, similar ends of the three phase windings are jointed together within the alternator and three lines are run from the other free ends as shown in fig. The common terminal so farmed is referred as Neutral point (N) or Neutral terminal. The terminals R, Y and B are called the line terminals. The voltage between any line and neutral point is called the phase voltage (V_{RN}, V_{YN} and V_{BN}), while the voltage between any two lines is called line voltage (V_{RY}, V_{YB} and V_{BR}). The current flowing through the phases are called the phase currents, while those flowing in the lines are called the line currents.

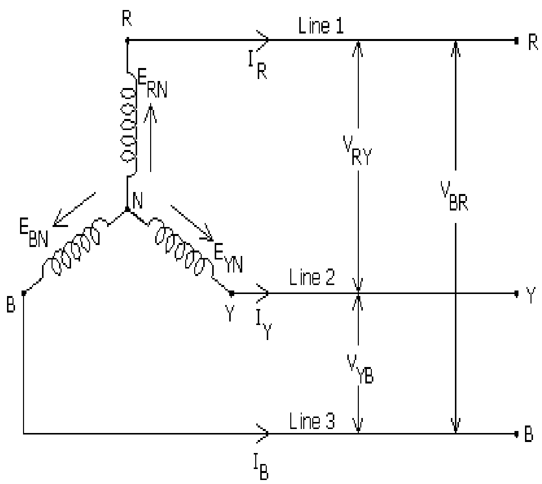
5.5.2 Delta (Δ) connection:

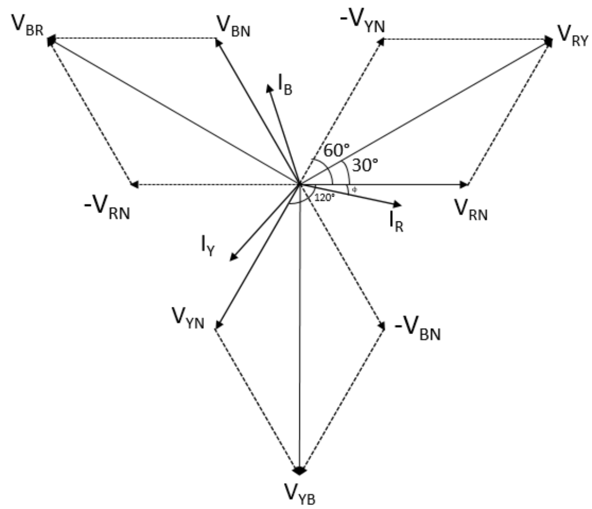


If the start end of one winding is connected to the finish end of the next, and so on until all three windings are interconnected, the result is the delta or mesh connection. Hence in delta connection, dissimilar ends of the phases are jointed to each other to form a closed mesh. Here there is no common terminal. Hence only three line voltages (V_{RY}, V_{YB} and V_{BR}) are available.

5.6 Relationship between line current and phase current, line voltage and phase voltage in a star connected system.

In star connection, the three phases are joined together to form a common junction N. N is the star point or neutral point. When three phases supply feeds a balanced load, the current in three phase conductors will be equal in magnitude and displaced 120° from each other.





Phasor Diagram

Line Voltage and Phase Voltage:

The potential difference between any two line terminals is the phasor difference between the potentials of these terminals w.r.t neutral point.

$$V_{RN} = V_{YN} = V_{BN} = V_{Ph} \quad ; \text{ Phase Voltage}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L \quad ; \text{ Line Voltage}$$

P.D between lines R and Y: $V_{RY} = V_{RN} + V_{NY} = V_{RN} - V_{YN}$

P.D between lines Y and B: $V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

P.D between lines B and R: $V_{BR} = V_{BN} + V_{NR} = V_{BN} - V_{RN}$

From Phasor Diagram: $V_{RY} = V_{RN} - V_{YN}$

$$V_{RY} = 2 \text{ x } V_{ph} \text{ x Cos } 30°$$

$$V_{RY} = 2 \text{ x } V_{ph} \text{ x } \frac{\sqrt{3}}{2}$$

$$V_{RY} = \sqrt{3} \text{ x } V_{ph}$$

Similarly $V_{YB} = V_{YN} - V_{BN} = \sqrt{3} \text{ x } V_{ph}$

$$V_{BR} = V_{BN} - V_{RN} = \sqrt{3} \text{ x } V_{ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L \quad ; \text{Line Voltage}$$

Hence in star connection, $\mathbf{V_L = \sqrt{3} \text{ x } V_{ph}}$

Line Current Vs Phase Current:

It is seen from Fig. the line current in each line is the same as the current in the phase winding to which the line is connected.

Current in line 1 = I_R

Current in line 2 = I_Y

Current in line 3 = I_B

Since $I_R = I_Y = I_B = I_{ph}$; Phase Current

Line Current = Phase Current

$$\mathbf{I_L = I_{ph}}$$

Power:

$$\text{Total Power} = \text{Sum of three phase powers}$$

$$\text{Total power} = 3 \text{ x Phase Power}$$

$$P = 3 \text{ x } V_{ph}\, I_{ph} \text{ Cos } \varphi$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} \quad \text{and} \quad V_L = \sqrt{3} \text{ x } V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} \quad \text{and } I_L = I_{ph}$$

$$\text{Total Power} = P = 3 \text{ x } \frac{V_L}{\sqrt{3}} \text{ x } I_L \text{ x Cos } \varphi$$

$$\mathbf{P = \sqrt{3} \text{ x } V_L \text{ x } I_L \text{ x Cos } \varphi}$$

5.7 Relationship between line current and phase current, line voltage and phase voltage in a delta connected system.

In delta connection, the three windings are joined in series to form a closed mesh as shown in fig. If the system is balanced then sum of the three voltages around the closed mesh is zero. It has no common point.

Line Voltage and Phase Voltage:

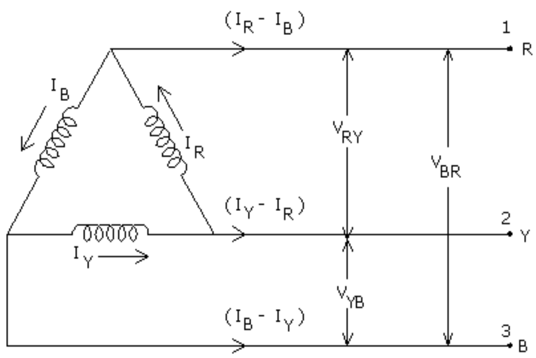
In delta connection, Line voltage is equal to phase voltage.

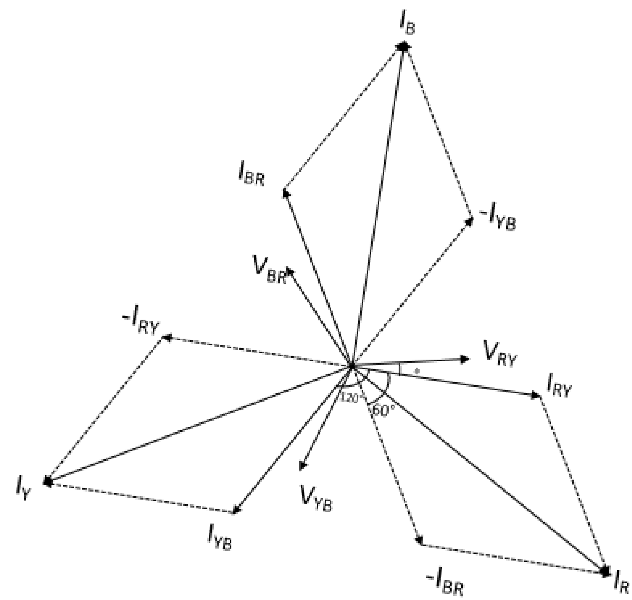
$V_{RY} = V_{YB} = V_{BR} = V_L$ = Line Voltage

$V_{RN} = V_{YN} = V_{BN} = V_{ph}$ = Phase Voltage

Line Voltage = Phase Voltage

$$\mathbf{V_L = V_{ph}}$$





__Line Current and Phase Current:__

The current in any line is equal to the phasor difference between the potentials of currents in the two phases attached to that lines.

Current in line 1: $\mathrm{I_R = I_{RY} - I_{BR}}$

Current in line 2: $\mathrm{I_Y = I_{YB} - I_{RY}}$

Current in line 3: $\mathrm{I_B = I_{BR} - I_{YR}}$

From Phasor Diagram:

$$\mathrm{I_R = I_{RY} - I_{BR}}$$

$$\mathrm{I_R = 2 \ x\ I_{ph} \ x\ Cos\ 30°}$$

$$\mathrm{I_R = 2 \ x\ I_{ph} \ x\ \frac{\sqrt{3}}{2}}$$

$$\mathrm{I_R = \sqrt{3} \ x\ I_{ph}}$$

Similarly

$$I_Y = I_{YB} - I_{RY} = \sqrt{3} \text{ x } I_{ph}$$

$$I_B = I_{BR} - I_{YR} = \sqrt{3} \text{ x } I_{ph}$$

$$I_R = I_Y = I_B = I_L \quad \text{; Line Current}$$

$$\textbf{Hence in delta connection}, \quad \mathbf{I_L = \sqrt{3} \text{ x } I_{ph}}$$

Power:

$$\text{Total Power} = \text{Sum of three phase powers}$$

$$\text{Total power} = 3 \text{ x Phase Power}$$

$$P = 3 \text{ x } V_{ph}\, I_{ph} \text{ Cos } \varphi$$

$$\text{Substitue} \quad I_{ph} = \frac{I_L}{\sqrt{3}} \quad \text{and} \quad V_{ph} = V_L$$

$$\text{Total Power} = P = 3 \text{ x } V_L \text{ x } \frac{I_L}{\sqrt{3}} \text{ x Cos } \varphi$$

$$\mathbf{P = \sqrt{3} \text{ x } V_L \text{ x } I_L \text{ x Cos } \varphi}$$

5.71 Advantages of star connection:

i) In star connection, phase voltage $V_{ph}=V_l/\sqrt{3}$. Hence a star connected alternator will require less number of turns than a Δ - connected alternator for the same line voltage.
ii) For the same line voltage, a star connected alternator requires less insulation than a delta connected alternator
iii) In star connection, we can get 3-phase 4-wire system. This permits to use two voltages viz., phase voltages as well as line voltages.
iv) Single phase loads can be connected between any one line and neutral wire while the 3-phase loads can be put across the three lines. Such flexibility is not available in Δ - connection.
v) In star connection, the neutral point can be earthed.

5.72 Advantages of delta connection:

1. Most of 3- phase induction motors are delta connected.
2. Delta connection is most suitable for rotary convertors.
3. High Reliability

5.73 Floating neutral point:

The isolated neutral (star) point of a load is called floating neutral point.

5.8 Power Measurement by single wattmeter method:

This method can be only used for balanced three phase load. Wattmeter must be connected in such a way that its current coil must carry I_{ph} and its voltage coil must be V_{ph}. When the load is balanced, total power can be calculated as :

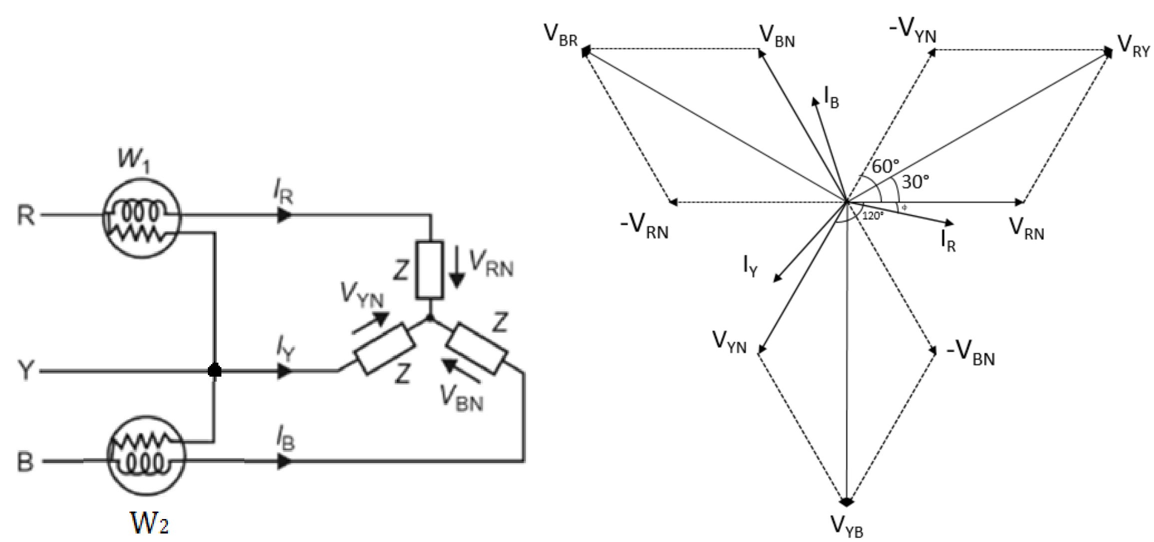
$$P = 3\ V_{ph}\ I_{ph}\ \text{Cos}\ \phi$$

$$P = 3 \times \text{Wattmeter reading}$$

Hence one wattmeter is to be used to measure single phase power and then reading is to be multiplied by 3.

5.9 Power Measurement by two wattmeter method:

The connection diagram (a) and phasor diagram (b) for a three phase balanced load is shown in figure. The three phase voltages V_{RN}, V_{YN} and V_{BN} displaced by an angle of 120˚ are shown in phasor diagram. The phase currents lags behind their respective phase voltage by an angle Φ.



Reading of Wattmeter W_1 :

Current through current coil of W_1 $= I_R$

P.D across potential coil of W_1 = V_{RY}

$$V_{RY} = V_{RN} - V_{YN}$$

From phasor diagram, phase angle between V_{RY} and I_R is $(30^\circ + \Phi)$

$$W_1 = V_{RY} \cdot I_R \cdot \text{Cos}\,(30^\circ + \Phi) \quad \ldots\ldots. (1)$$

Reading of Wattmeter W_2 :

Current through current coil of W_2 = I_B

P.D across potential coil of W_2 = V_{BY}

$$V_{BY} = V_{BN} - V_{YN}$$

From phasor diagram, phase angle between V_{BY} and I_B is $(30^\circ - \Phi)$

$$W_2 = V_{BY} \cdot I_B \cdot \text{Cos}\,(30^\circ - \Phi) \quad \ldots\ldots. (2)$$

Since the load is balanced

$$V_{RY} = V_{BY} = \text{Line Voltage } (V_L) \quad \text{and}$$

$$I_R = I_B = \text{Line Current } (I_L)$$

From equ. (1) and (2)

$$W_1 = V_L \cdot I_L \cdot \text{Cos}\,(30^\circ + \Phi)$$

$$W_2 = V_L \cdot I_L \cdot \text{Cos}\,(30^\circ - \Phi)$$

$$W_1 + W_2 = V_L \cdot I_L \cdot \text{Cos}\,(30^\circ + \Phi) + V_L \cdot I_L \cdot \text{Cos}\,(30^\circ - \Phi)$$

$$W_1 + W_2 = V_L \cdot I_L \cdot (2\,\text{Cos}\,30^\circ + \text{Cos}\,\Phi)$$

$$W_1 + W_2 = V_L \cdot I_L \cdot (2 \times \frac{\sqrt{3}}{2} \times \text{Cos}\,\Phi)$$

$$W_1 + W_2 = \sqrt{3}\, V_L \cdot I_L \cdot \text{Cos}\,\Phi$$

$W_1 + W_2$ = Total Power in the 3 phase load.

Now

$$W_2 - W_1 = [V_L \cdot I_L\, \text{Cos}\,(30^\circ - \Phi) - V_L \cdot I_L\, \text{Cos}\,(30^\circ + \Phi)]$$

$W_2 - W_1 = V_L \cdot I_L$ [Cos 30˚ Cos Φ + Sin 30˚ Sin Φ) –

(Cos 30˚ Cos Φ – Sin 30˚ Sin Φ)]

$W_2 - W_1 = V_L \cdot I_L \cdot (2 \cdot \sin 30^\circ \sin \Phi)$

$W_2 - W_1 = V_L \cdot I_L \cdot \sin \Phi$

Power Factor:

$W_2 + W_1 = \sqrt{3}\, V_L \cdot I_L \cdot \cos \Phi$

$W_2 - W_1 = V_L \cdot I_L \cdot \sin \Phi$

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{V_L\, I_L \sin \emptyset}{\sqrt{3}\, V_L\, I_L \cos \emptyset}$$

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{\tan \emptyset}{\sqrt{3}}$$

$$\tan \emptyset = \sqrt{3}\left\{\frac{W_2 - W_1}{W_2 + W_1}\right\}$$

$$\emptyset = \tan^{-1} \sqrt{3}\left\{\frac{W_2 - W_1}{W_2 + W_1}\right\}$$

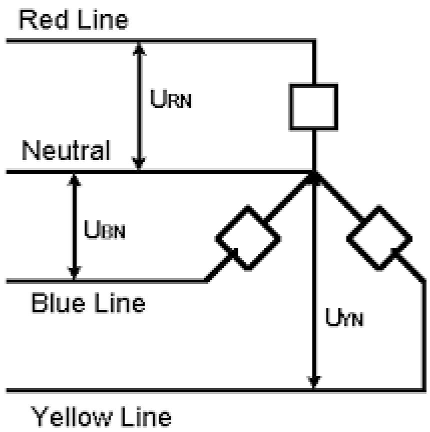
Advantages of two wattmeter method:

i) Access to the star point is not necessary
ii) The power dissipated in both balanced and unbalanced load is obtained, without any modification to the connections.
iii) For balanced loads, the power factor can be determined.

5.10 Effects of Load P.F on Wattmeter Readings:

Ø	0˚	60˚	More than 60˚	90˚
Cos Ø	1	0.5	< 0.5	0
W_2	Positive	Positive	Positive	Positive
W_1	Positive	0	Negative	Negative
Conclusion	$W_1 = W_2$ Total Power $= W_1 + W_2$	$W_1 = 0$ Total Power $= W_2$	$W_2 > W_1$ Total Power $= W_2 - W_1$	$W_2 = -W_1$ Total Power $= 0$

5.11 3 phase, 4 wire star connected system:



The three phase loads are directly connected across the three lines while single phase loads are connected between one of the lines and the neutral wire. Three phase loads are mostly balanced and the single phase loads introduces the imbalance or unbalance.

If the load is unbalanced, then the three lines currents will be unequal. Consequently the line currents I_R, I_Y and I_B will be different in magnitude and displaced from one another by unequal angles. In this case the neutral has to carry the resulting out-of-balance current. This current is simply obtained by calculating the phasor sum of the line currents.

Current in neutral wire, $I_N = I_R + I_Y + I_B$

Example: 1

A balanced delta connected load of (8+j6) ohms per phase is connected to a 3 phase 230V supply. Find the line current, power factor, power and total volt ampere.

Given Data:

Connection	=	DELTA
Resistance (R_{ph})	=	8 Ω
Inductive Reactance (X_{Lph})	=	6 Ω
Line Voltage	=	230 Volts

To Find:

Line Current (I_L) = ?
Power Factor (Cos θ) = ?
Power (P) = ?
Volt-Ampere (VA) = ?

Solution:

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{8^2 + 6^2}$$

$$= \sqrt{64 + 36}$$

$$= \sqrt{100}$$

$$\text{Impedance } (Z_{ph}) = 10\Omega$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}}$$

$$\text{Phase Voltage } (V_{ph}) = V_L = 230\text{V} \quad [\because \text{Delta Connection}]$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{230}{10} = 23\text{A}$$

$$I_{ph} = 23\text{A}$$

$$\text{Line Current } (I_L) = \sqrt{3}\, I_{ph}$$

$$= \sqrt{3} \text{ x } 23 = 39.84 \text{ A}$$

$$I_L = 39.84 \text{ A}$$

$$\text{Power Factor } (\text{Cos } \theta) = \frac{R_{ph}}{Z_{ph}} = \frac{8}{10} = 0.8$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos } \theta$$

$$= \sqrt{3} \text{ x } 230 \text{ x } 39.84 \text{ x } 0.8$$

$$= 12696.5 \text{ Watts}$$

$$\text{Volt –Ampere (VA)} = \sqrt{3}\, V_L\, I_L$$

$$= \sqrt{3} \text{ x } 230 \text{ x } 39.84$$

$$= 15870.6 \text{ VA}$$

Example: 2

Each branch of a delta connected load has an impedance of (16+j12) Ω. Calculate the line current and total power when connected to a 400V, 3 phase 50Hz mains.

Given Data:

Connection	=	DELTA
Resistance (R_{ph})	=	16 Ω
Inductive Reactance (X_{Lph})	=	12 Ω
Line Voltage (V_L)	=	400 Volts
Frequency (F)	=	50Hz

To Find:

Line Current (I_L) = ?

Power (P) = ?

Solution:

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{16^2 + 12^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400}$$

$$\text{Impedance } (Z_{ph}) = 20\Omega$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}}$$

$$\text{Phase Voltage } (V_{ph}) = V_L = 400V \quad [\because \text{Delta Connection}]$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{400}{20} = 20A$$

$$I_{ph} = 20A$$

$$\text{Line Current } (I_L) = \sqrt{3}\, I_{ph}$$

$$= \sqrt{3} \text{ x } 20 = 34.64 \text{ A}$$

$$I_L = 34.64 \text{ A}$$

$$\text{Power Factor } (\text{Cos } \theta) = \frac{R_{ph}}{Z_{ph}} = \frac{16}{20} = 0.8$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos } \theta$$

$$= \sqrt{3} \text{ x } 400 \text{ x } 34.64 \text{ x } 0.8$$

$$= 19198.8 \text{ Watts}$$

Answer:

i) Line Current (I_L) = 34.64 A   ii) Power = 19198.8 Watts

Example: 3

A balanced three phase load consists of three coils each of resistance 6 Ω and an inductive reactance of 8 Ω . Determine the line current and power absorbed when the coils are delta connected across a 400V 3 phase supply.

<u>Given Data:</u>

Connection	=	DELTA
Resistance (R_{ph})	=	6 Ω
Inductive Reactance (X_{Lph})	=	8 Ω
Line Voltage (V_L)	=	400 Volts

<u>To Find:</u>

i) Line Current (I_L) = ?

ii) Power (P) = ?

<u>Solution:</u>

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

$$\text{Impedance } (Z_{ph}) = 10\Omega$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}}$$

$$\text{Phase Voltage } (V_{ph}) = V_L = 400\text{V} \quad [\because \text{Delta Connection}]$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40\text{A}$$

$$I_{ph} = 40\text{A}$$

$$\text{Line Current } (I_L) = \sqrt{3}\, I_{ph}$$

$$= \sqrt{3} \text{ x } 40 = 69.28 \text{ A}$$

$$I_L = 69.28 \text{ A}$$

$$\text{Power Factor } (\text{Cos } \theta) = \frac{R_{ph}}{Z_{ph}} = \frac{6}{10} = 0.6$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos } \theta$$

$$= \sqrt{3} \text{ x } 400 \text{ x } 69.28 \text{ x } 0.6$$

$$= 28798.3 \text{ Watts}$$

<u>Answer:</u>

i) Line Current (I_L) = 69.28 A ii) Power = 28798.3 Watts

Example: 4

A load in each branch of delta connected balanced 3 phase circuit consists of an inductance of 0.0318H in series with a resistance of 10 Ohms. The line voltage is 400V at 50Hz. Calculate (i) the line current and (ii) the total power in the circuit

<u>Given Data:</u>

Connection	=	DELTA
Resistance (R_{ph})	=	10 Ω
Inductance (L_{ph})	=	0.0318 H
Line Voltage (V_L)	=	400 Volts
Frequency (F)	=	50 Hz

<u>To Find:</u>

i) Line Current (I_L) = ?

ii) Power (P) = ?

<u>Solution:</u>

$$\text{Inductive Reactance } (X_{Lph}) = 2\pi fL \ ; \ \Omega$$

$$= 2 \text{ x } 3.14 \text{ x } 50 \text{ x } 0.0318 \ ; \ \Omega$$

$$X_{Lph} = 9.98\ \Omega = 10\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{10^2 + 10^2}$$

$$= \sqrt{100 + 100} = \sqrt{200}$$

$$\text{Impedance } (Z_{ph}) = 14.14\Omega$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}}$$

$$\text{Phase Voltage } (V_{ph}) = V_L = 400\text{V} \quad [\because \text{Delta Connection}]$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{400}{14.14} = 28.28\text{A}$$

$$I_{ph} = 28.28\text{A}$$

$$\text{Line Current } (I_L) = \sqrt{3}\ I_{ph}$$

$$= \sqrt{3} \text{ x } 28.28 = 48.98 \text{ A}$$

$$I_L = 48.98 \text{ A}$$

$$\text{Power Factor } (\text{Cos}\ \theta) = \frac{R_{ph}}{Z_{ph}} = \frac{10}{14.14} = 0.7$$

$$\text{Power } (P) = \sqrt{3}\ V_L\ I_L\ \text{Cos}\ \theta$$

$$= \sqrt{3} \text{ x } 400 \text{ x } 48.98 \text{ x } 0.7$$

$$= 23753.3 \text{ Watts}$$

<u>Answer:</u> i) Line Current (I_L) = 48.98 A   ii) Power = 23753.3 Watts

Example: 5

Three similar coils each having a resistance of 15Ω and inductance of 0.5 H are connected in delta to a three phase 415V, 50Hz supply. Find the i) Line Current ii) P.F and iii) Power.

Given Data:

Connection	=	DELTA
Resistance (R_{ph})	=	15 Ω
Inductive Reactance (X_{Lph})	=	0.5 H
Line Voltage (V_L)	=	415 Volts
Frequency (F)	=	50 Hz

To Find:

i) Line Current (I_L) = ?

ii) Power Factor (Cos θ) = ?

iii) Power (P) = ?

Solution:

$$\text{Inductive Reactance } (X_{Lph}) = 2\pi fL\ ;\ \Omega$$

$$= 2 \text{ x } 3.14 \text{ x } 50 \text{ x } 0.5\ ;\ \Omega$$

$$X_{Lph} = 157\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{15^2 + 157^2}$$

$$= \sqrt{225 + 24649} = \sqrt{24874}$$

$$\text{Impedance } (Z_{ph}) = 157.7\Omega$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}}$$

$$\text{Phase Voltage } (V_{ph}) = V_L = 415V \quad [\because \text{Delta Connection}]$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{415}{157.7} = 2.63 \text{ A}$$

$$I_{ph} = 2.63 \text{ A}$$

$$\text{Line Current } (I_L) = \sqrt{3}\, I_{ph}$$

$$= \sqrt{3} \text{ x } 2.63 = 4.56 \text{ A}$$

$$I_L = 4.56 \text{ A}$$

$$\text{Power Factor } (\text{Cos } \theta) = \frac{R_{ph}}{Z_{ph}} = \frac{15}{157.7} = 0.09$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos } \theta$$

$$= \sqrt{3} \text{ x } 415 \text{ x } 4.56 \text{ x } 0.09$$

$$= 295 \text{ Watts}$$

Answer:

i) Line Current (I_L) = 4.56 A ii) P.F = 0.09 iii) Power = 295 Watts

Example: 6

A balanced star connected load of (8+j6) ohms per phase is connected to a 3 phase 230V supply. Find the line current, power factor, power and total volt ampere.

Given Data:

Connection	=	STAR
Resistance (R_{ph})	=	8 Ω
Inductive Reactance (X_{Lph})	=	6 Ω
Line Voltage	=	230 Volts

To Find:

Line Current (I_L) = ?

Power Factor (Cos θ) = ?

Power (P) = ?

Volt-Ampere (VA) = ?

Solution:

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{8^2 + 6^2}$$

$$= \sqrt{64 + 36}$$

$$= \sqrt{100}$$

$$\text{Impedance } (Z_{ph}) = 10\Omega$$

$$\text{Phase Voltage } (V_{ph}) = \frac{V_L}{\sqrt{3}} \qquad [\because \text{Star Connection}]$$

$$\text{Phase Voltage } (V_{ph}) = \frac{230}{\sqrt{3}} = 132.8\text{ V}$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{132.8}{10} = 13.28\text{A}$$

$$I_{ph} = 13.28\text{A}$$

$$\text{Line Current } (I_L) = I_{ph} = 13.28\text{ A} \qquad [\because \text{Star Connection}]$$

$$I_L = 13.28\text{ A}$$

$$\text{Power Factor } (\text{Cos }\theta) = \frac{R_{ph}}{Z_{ph}} = \frac{8}{10} = 0.8\text{ Lag}$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos }\theta$$

$$= \sqrt{3} \text{ x } 230 \text{ x } 13.28 \text{ x } 0.8$$

$$= 4232 \text{ Watts}$$

$$\text{Volt –Ampere (VA)} = \sqrt{3}\, V_L\, I_L$$

$$= \sqrt{3} \text{ x } 230 \text{ x } 39.84$$

$$= 5290 \text{ VA}$$

Answer:

i) Line Current (I_L) = 13.28 A

ii) Power Factor = 0.8

iii) Power = 4232 Watts

iv) Volt Ampere = 5290 VA

Example: 7

Each phase of a 3 phase wye connected load has an impedance of (100 – j120)Ω. It is connected to 440V, 3 phase, 50Hz supply. Calculate the line current and power factor.

Given Data:

Connection	=	STAR (Y)
Resistance (R_{ph})	=	100 Ω
Inductive Reactance (X_{Lph})	=	120 Ω
Line Voltage (V_L)	=	440 Volts
Frequency (F)	=	50Hz

To Find:

Line Current (I_L) = ?

Power factor (Cos θ) = ?

Solution:

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{100^2 + 120^2}$$

$$= \sqrt{10000 + 14400}$$

$$= \sqrt{24400}$$

$$\text{Impedance } (Z_{ph}) = 156.2\Omega$$

$$\text{Phase Voltage } (V_{ph}) = \frac{V_L}{\sqrt{3}} \qquad [\because \text{Star Connection}]$$

$$\text{Phase Voltage } (V_{ph}) = \frac{440}{\sqrt{3}} = 254\text{ V}$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{254}{156.2} = 1.63\text{A}$$

$$I_{ph} = 1.63\text{ A}$$

$$\text{Line Current } (I_L) = I_{ph} = 1.63\text{ A} \qquad [\because \text{Star Connection}]$$

$$I_L = 1.63\text{ A}$$

$$\text{Power Factor } (\text{Cos }\theta) = \frac{R_{ph}}{Z_{ph}} = \frac{100}{156.2} = 0.64\text{ Lead}$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L\, \text{Cos }\theta$$

$$= \sqrt{3} \text{ x } 440 \text{ x } 1.63 \text{ x } 0.64$$

$$= 795\text{ Watts}$$

Answer:

i) Line Current (I_L) = 1.63 A

ii) Power Factor = 0.64

iii) Power = 795 Watts

Example: 8

Three similar coils each having a resistance of 20Ω and reactance of 15Ω are connected in star to a three phase 400V, 50Hz supply. Determine the i) Phase current ii) Line Current c) Power factor and d) Total power.

<u>Given Data:</u>

Connection	=	STAR
Resistance (R_{ph})	=	20 Ω
Inductive Reactance (X_{Lph})	=	15 Ω
Line Voltage (V_L)	=	400 Volts
Frequency (F)	=	50 Hz

<u>To Find:</u>

Phase Current (I_{ph}) = ?
Line Current (I_L) = ?
Power Factor (Cos θ) = ?
Power (P) = ?

<u>Solution:</u>

$$\text{Impedance } (Z_{ph}) = \sqrt{R_{ph}{}^2 + X_{Lph}{}^2}\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{20^2 + 15^2}$$

$$= \sqrt{400 + 225}$$

$$= \sqrt{625}$$

$$\text{Impedance } (Z_{ph}) = 25\Omega$$

$$\text{Phase Voltage } (V_{ph}) = \frac{V_L}{\sqrt{3}} \qquad [\because \text{Star Connection}]$$

$$\text{Phase Voltage } (V_{ph}) = \frac{400}{\sqrt{3}} = 231\text{ V}$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{231}{25} = 9.24\text{ A}$$

$$I_{ph} = 9.24\text{ A}$$

$$\text{Line Current } (I_L) = I_{ph} = 9.24\text{ A} \qquad [\because \text{Star Connection}]$$

$$I_L = 9.24\text{ A}$$

$$\text{Power Factor } (\text{Cos }\theta) = \frac{R_{ph}}{Z_{ph}} = \frac{20}{25} = 0.8\text{ Lag}$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos }\theta$$

$$= \sqrt{3} \text{ x } 400 \text{ x } 9.24 \text{ x } 0.8$$

$$= 5121\text{ Watts}$$

<u>Answer:</u>

i) Line Current (I_L) = 9.24 A   ii) Power Factor = 0.8

iii) Power = 5121 Watts

Example: 9

Three identical impedances are connected in delta to a 3 phase 400V supply. The line current is 34.65A and the total power taken from the supply is 14.4KW. Calculate the resistance and reactance values of each impedance.

Given Data:

Connection = DELTA

Line Current (I_L) = 34.65 A

Total Power (P) = 14.4 KW

Line Voltage (V_L) = 400 Volts

To Find:

Resistance (R_{ph}) = ?

Reactance (X_{ph}) = ?

Solution:

$$\text{Phase Current } (I_{ph}) = \frac{I_L}{\sqrt{3}} \quad [\because \text{Delta Connection}]$$

$$= \frac{34.65}{\sqrt{3}} \quad [\because \text{Delta Connection}]$$

$$= 20\text{ A}$$

$$\text{Phase Voltage } (V_{ph}) = V_L = 400\text{ V} \quad [\because \text{Delta Connection}]$$

$$\text{Impedance } (Z_{ph}) = \frac{V_{ph}}{I_{ph}}$$

$$Z_{ph} = \frac{400}{20} = 20\Omega$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos } \theta$$

$$\text{Cos } \theta = \frac{P}{\sqrt{3}\ V_L\ I_L}$$

$$= \frac{14.4 \times 10^3}{\sqrt{3} \times 400 \times 34.65}$$

$$= \frac{14400}{24005.5}$$

$$\text{Cos } \theta = 0.6$$

$$\text{Cos } \theta = \frac{R_{ph}}{Z_{ph}}$$

$$R_{ph} = Z_{ph} \times \text{Cos } \theta$$

$$= 20 \times 0.6$$

$$\text{Resistance } (R_{ph}) = 12\Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Reactance } (X_{ph}) = \sqrt{{Z_{ph}}^2 - {R_{ph}}^2}$$

$$= \sqrt{20^2 - 12^2}$$

$$= \sqrt{400 - 144}$$

$$X_{ph} = 16\ \Omega$$

Answer:

i) Resistance (R_{ph}) = 12 Ω ii) Reactance (X_{ph}) = 16 Ω

Example: 10

Three similar coils are connected in star taken at a total power of 1.5KW at a P.F of 0.2 lagging from a 3 phase 400V, 50Hz supply. Calculate the resistance and inductance of each phase.

Given Data:

Connection	=	STAR
Total Power (P)	=	1.5 KW
Power Factor (Cos θ)	=	0.2
Line Voltage (V_L)	=	400 Volts
Frequency (F)	=	50 Hz

To Find:

Resistance (R_{ph}) = ?

Inductance (L_{ph}) = ?

Solution:

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \cos\theta$$

$$\text{Line Current } (I_L) = \frac{P}{\sqrt{3}\ V_L \cos\theta}$$

$$= \frac{1.5 \times 10^3}{\sqrt{3} \times 400 \times 0.2}$$

$$= \frac{1500}{138.56}$$

$$I_L = 10.8\ A$$

$$\text{Phase Voltage } (V_{ph}) = \frac{V_L}{\sqrt{3}} \quad [\because \text{Star Connection}]$$

$$= \frac{400}{\sqrt{3}} = 231 \text{ Volts} \quad [\because \text{Star Connection}]$$

$$= 231 \text{ Volts}$$

$$\text{Impedance } (Z_{ph}) = \frac{V_{ph}}{I_{ph}}$$

$$= \frac{231}{10.8} = 21.3\ \Omega$$

$$\text{Impedance } (Z_{ph}) = 21.3\ \Omega$$

$$\text{Cos } \theta = \frac{R_{ph}}{Z_{ph}}$$

$$R_{ph} = Z_{ph} \text{ x Cos } \theta$$

$$= 21.3 \text{ x } 0.2$$

$$\text{Resistance } (R_{ph}) = 4.26\Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Reactance } (X_{ph}) = \sqrt{{Z_{ph}}^2 - {R_{ph}}^2}$$

$$= \sqrt{21.3^2 - 4.26^2}$$

$$= \sqrt{453.7 - 18.15}$$

$$= 20.87\ \Omega$$

$$\text{Inductive Reactance } (X_{Lph}) = 2\pi fL\ ;\ \Omega$$

$$\text{Inductance } (L_{ph}) = \frac{X_L}{2\pi f}$$

$$= \frac{20.87}{2 \text{ x } \pi \text{ x } 50}$$

$$L_{ph} = 0.06H$$

Answer:

i) Resistance (R_{ph}) = 4.26 Ω   ii) Inductance (L_{ph}) = 0.06 Ω

Example: 11

Three similar resistors are connected in star across 400V, 3 phase supply. The line current is 5A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors are in delta connected?

Given Data:

Connection = STAR To DELTA

Line Voltage (V_L) = 400 Volts

Line Current (I_L) = 5 Amps

To Find:

Line Voltage (V_L) = ?

Solution:

$$\text{Phase Current } (I_{ph}) = I_L = 5\text{ A} \qquad [\because \text{Star Connection}]$$

$$\text{Phase Voltage } (V_{ph}) = \frac{V_L}{\sqrt{3}} \qquad [\because \text{Star Connection}]$$

$$= \frac{400}{\sqrt{3}} = 231\text{ Volts} \qquad [\because \text{Star Connection}]$$

$$V_{ph} = 231\text{ Volts}$$

$$\text{Impedance } (Z_{ph}) = \text{Resistance } (R_{ph})$$

$$\text{Impedance } (Z_{ph}) = \frac{V_{ph}}{I_{ph}}$$

$$Z_{ph} = \frac{231}{5} = 46.2\Omega$$

$$= 46.2\ \Omega$$

$$\text{Impedance } (Z_{ph}) = 46.2\ \Omega$$

$$\text{Impedance } (Z_{ph}) = \text{Resistance } (R_{ph})$$

If same resistors are connected in delta

$$\text{Phase Voltage } (V_{ph}) = V_L = 400\text{ V} \qquad [\because \text{Delta Connection}]$$

$$V_{ph} = I_{ph} \times Z_{ph}$$

$$= \frac{I_L}{\sqrt{3}} \times 46.2$$

$$= \frac{5}{\sqrt{3}} \times 46.2$$

$$= 133.34\text{ V}$$

$$\text{Phase Voltage } (V_{ph}) = 133.34\text{ V}$$

$$\text{Line Voltage } (V_L) = 133.34\text{ V}$$

Answer:

i) Resistance (R_{ph}) = 4.26 Ω ii) Inductance (L_{ph}) = 0.06 Ω

iii) Line Voltage (V_L) = 133.34V

Example: 12

A balanced three phase star connected load of 150KW takes a leading current of 100A with a line voltage of 1100V, 50Hz. Find the circuit constants of the load per phase.

<u>Given Data:</u>

Connection	=	STAR
Line Current (I_L)	=	100 A
Total Power (P)	=	150 KW
Line Voltage (V_L)	=	1100 Volts

<u>To Find:</u>

Resistance (R_{ph}) = ?

Reactance (X_{ph}) = ?

<u>Solution:</u>

$$\text{Phase Current } (I_{ph}) = I_L = 100\text{ A} \qquad [\because \text{Star Connection}]$$

$$I_{ph} = 100\text{ A}$$

$$\text{Phase Voltage } (V_{ph}) = \frac{V_L}{\sqrt{3}} \qquad [\because \text{Star Connection}]$$

$$= \frac{1100}{\sqrt{3}} = 635\text{ Volts} \qquad [\because \text{Star Connection}]$$

$$= 635\text{ Volts}$$

$$\text{Impedance } (Z_{ph}) = \frac{V_{ph}}{I_{ph}}$$

$$Z_{ph} = \frac{635}{100} = 6.35\Omega$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos } \theta$$

$$\text{Cos } \theta = \frac{P}{\sqrt{3}\ V_L\ I_L}$$

$$= \frac{150 \text{ x } 10^3}{\sqrt{3} \text{ x } 1100 \text{ x } 100}$$

$$= \frac{150 \text{ x } 10^3}{190520}$$

$$\text{Cos } \theta = 0.79$$

$$\text{Cos } \theta = \frac{R_{ph}}{Z_{ph}}$$

$$R_{ph} = Z_{ph} \text{ x Cos } \theta$$

$$= 6.35 \text{ x } 0.79$$

$$\text{Resistance } (R_{ph}) = 5\Omega$$

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$

$$\text{Reactance } (X_{ph}) = \sqrt{{Z_{ph}}^2 - {R_{ph}}^2}$$

$$= \sqrt{6.35^2 - 5^2}$$

$$= \sqrt{40.3 - 25} = \sqrt{15.3}$$

$$X_{ph} = 3.9\ \Omega$$

$$\text{Capacitive Reactance } (X_{Cph}) = \frac{1}{2\pi fC}\ ;\Omega$$

$$\text{Capacitance } (C_{ph}) = \frac{1}{2\pi fX_C}$$

$$= \frac{1}{2 \times \pi \times 50 \times 3.9}$$

$$C_{ph} = 812 \text{ MFD}$$

Answer:

i) Resistance (R_{ph}) = 5 Ω ii) Capacitance (C) = 812 MFD

Example: 13

A balanced connected to a three phase supply comprises three identical coils in star. The line current is 25 Amps. KVA input is 20 and KW input is 11. Find KVAR input, phase voltage, line voltage, resistance and reactance of each coil of the load.

Given Data:

Connection	=	STAR
Line Current (I_L)	=	25 A
KW	=	11
KVA	=	20

To Find:

Resistance (R_{ph}) = ?

Reactance (X_{ph}) = ?

Solution:

$$\text{KVA} = \sqrt{\text{KW}^2 + \text{KVAR}^2}$$

$$\text{KVAR} = \sqrt{\text{KVA}^2 - \text{KW}^2}$$

$$\text{Reactance } (X_{ph}) = \sqrt{20^2 - 11^2}$$

$$= \sqrt{400 - 121}$$

$$= \sqrt{279} = 16.7$$

$$\text{KVAR} = 16.7$$

$$\text{KVA} = \frac{\sqrt{3}\ V_L\ I_L}{1000}$$

$$\text{Line Voltage } (V_L) = \frac{KVA \times 1000}{\sqrt{3}\ V_L}$$

$$= \frac{20 \times 1000}{\sqrt{3}\ I_L}$$

$$= \frac{20 \times 1000}{\sqrt{3} \times 25}$$

$$= 461.89 \text{ Volts}$$

$$\text{Phase Voltage } (V_{ph}) = \frac{V_L}{\sqrt{3}} \qquad [\because \text{Star Connection}]$$

$$= \frac{461.89}{\sqrt{3}} = 266.6 \text{ Volts} \qquad [\because \text{Star Connection}]$$

$$V_{ph} = 266.6 \text{ Volts}$$

$$\text{Phase Current } (I_{ph}) = I_L = 25\text{ A} \qquad [\because \text{Star Connection}]$$

$$I_{ph} = 25\text{ A}$$

$$\text{Impedance } (Z_{ph}) = \frac{V_{ph}}{I_{ph}}$$

$$Z_{ph} = \frac{266.6}{25} = 10.67\ \Omega$$

$$\text{Cos } \theta = \frac{KW}{KVA}$$

$$= \frac{11}{20}$$

$$\text{Cos } \theta = 0.55$$

$$\text{Cos } \theta = \frac{R_{ph}}{Z_{ph}}$$

$$R_{ph} = Z_{ph} \times \text{Cos } \theta$$

$$= 10.67 \times 0.55$$

$$\text{Resistance } (R_{ph}) = 5.86\ \Omega$$

$$\text{Reactance } (X_{ph}) = \sqrt{{Z_{ph}}^2 - {R_{ph}}^2}$$

$$= \sqrt{10.67^2 - 5.86^2}$$

$$= \sqrt{113.8 - 34.3} = \sqrt{79.5}$$

$$X_{ph} = 8.9\ \Omega$$

<u>Answer:</u>

i) Resistance (R_{ph}) = 5.86 Ω   ii) Reactance (X_{ph}) = 8.9 Ω

Example: 14

A 440V, three phase delta connected induction motor has an output of 14.92KW at p.f of 0.82 and efficiency 85%. Calculate the readings on each of the two wattmeters connected to measure the input

Given Data:

Connection	=	DELTA
Line Voltage (V_L)	=	440 Volts
Output Power	=	14.92 KW
Power factor (Cos θ)	=	0.82
Efficiency (η)	=	85% = 0.85

To Find:

W_1 = ?

W_2 = ?

Solution:

$$W_2+W_1 = \frac{\text{Output Power}}{\text{Efficiency}}$$

$$W_2+W_1 = \frac{14.92 \text{ x } 10^3}{0.85}$$

$$W_2+W_1 = 17553 \text{ Watts} \quad \text{------ (1)}$$

$$\text{Power Factor (Cos } \varphi) = 0.82$$

$$\varphi = \text{Cos}^{-1}\, 0.82$$

$$= 35.92^0$$

$$\tan\varphi = \tan 35.92^0$$

$$= 0.698$$

$$\tan\varphi = \frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}$$

$$0.698 = \frac{\sqrt{3}\,(W_2 - W_1)}{17553}$$

$$0.698 \text{ x } 17553 = \sqrt{3}\,(W_2 - W_1)$$

$$\frac{0.698 \text{ x } 17553}{\sqrt{3}} = (W_2 - W_1)$$

$$W_2 - W_1 = \frac{0.698 \text{ x } 17553}{\sqrt{3}}$$

$$W_2 - W_1 = \frac{12252}{\sqrt{3}}$$

$$W_2 - W_1 = 7074 \text{ W} \quad \text{------ (2)}$$

$$W_2 + W_1 = 17553 \quad \text{------ (1)}$$

$$W_2 - W_1 = 7074 \quad \text{------ (2)}$$

By adding equ (1) and (2)

$$2W_2 = 24627$$

$$W_2 = \frac{24627}{2}$$

$$W_2 = 12313.5 \text{ Watts}$$

Substitute the value of W_2 in equ (1)

$$12313.5 + W_1 = 17553 \quad \text{------ (1)}$$

$$W_1 = 17553 - 12313.5 = 5239.5 \text{ W}$$

$$W_1 = 5239.5 \text{ Watts}$$

Answer:

i) W_1 = 5239.5 Watts ii) W_2 = 12313.5 Watts

Example: 15

A 500 volts, 3 phase motor has an output of 3.73KW and operates at a power factor of 0.85 with an efficiency of 90%. Calculate the reading on each of the two wattmeter connected to measure the input.

Given Data:

Line Voltage (V_L)	=	500 Volts
Output Power	=	3.73 KW
Power factor (Cos θ)	=	0.85
Efficiency (η)	=	90% = 0.9

To Find:

W_1 = ?

W_2 = ?

Solution:

$$W_2+W_1 = \frac{\text{Output Power}}{\text{Efficiency}}$$

$$W_2+W_1 = \frac{3.73 \text{ x } 10^3}{0.9}$$

$$W_2+W_1 = 4144 \text{ Watts} \quad \text{------ (1)}$$

$$\text{Power Factor (Cos } \varphi) = 0.85$$

$$\varphi = \text{Cos}^{-1}\, 0.85$$

$$= 31.79^0$$

$$\tan \varphi = \tan 31.79^0$$

$$= 0.619$$

$$\tan \varphi = \frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}$$

$$0.619 = \frac{\sqrt{3}\,(W_2 - W_1)}{17553}$$

$$0.619 \text{ x } 4144.4 = \sqrt{3}\,(W_2 - W_1)$$

$$\frac{0.619 \text{ x } 4144.4}{\sqrt{3}} = (W_2 - W_1)$$

$$W_2 - W_1 = \frac{0.619 \text{ x } 4144}{\sqrt{3}}$$

$$W_2 - W_1 = \frac{2565}{\sqrt{3}}$$

$$W_2 - W_1 = 1481 \text{ W} \quad \text{------ (2)}$$

$$W_2 + W_1 = 4144 \quad \text{-------- (1)}$$

$$W_2 - W_1 = 1481 \quad \text{------ (2)}$$

By adding equ (1) and (2)

$$2W_2 = 5625$$

$$W_2 = \frac{5625}{2}$$

$$W_2 = 2813 \text{ Watts}$$

Substitute the value of W_2 in equ (1)

$$2813 + W_1 = 4144 \quad \text{------ (1)}$$

$$W_1 = 4144 - 2813 = 5239.5 \text{ W}$$

$$W_1 = 1331 \text{ Watts}$$

Answer:

i) W_1 = 1331 Watts ii) W_2 = 2813 Watts

Example: 16

A 3 phase motor delivers an output of 46KW and operated within efficiency of 90% at 0.85 lagging power factor. Calculate the line current and total power drawn if the supply voltage is 400V.

Given Data:

Line Voltage (V_L)	=	400 Volts
Output Power	=	46 KW
Power factor (Cos θ)	=	0.85
Efficiency (η)	=	90% = 0.9

To Find:

W_1 = ?

W_2 = ?

Solution:

$$W_2 + W_1 = \frac{\text{Output Power}}{\text{Efficiency}}$$

$$W_2+W_1 = \frac{46 \times 10^3}{0.9}$$

$$W_2+W_1 = 51111 \text{ Watts} \quad \text{------ (1)}$$

$$\text{Power Factor } (\cos \varphi) = 0.85$$

$$\varphi = \cos^{-1} 0.85$$

$$= 31.79^0$$

$$\tan \varphi = \tan 31.79^0$$

$$= 0.619$$

$$\tan \varphi = \frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}$$

$$0.619 = \frac{\sqrt{3}\,(W_2 - W_1)}{51111}$$

$$0.619 \times 51111 = \sqrt{3}\,(W_2 - W_1)$$

$$\frac{0.619 \times 51111}{\sqrt{3}} = (W_2 - W_1)$$

$$W_2 - W_1 = \frac{0.619 \times 51111}{\sqrt{3}}$$

$$W_2 - W_1 = \frac{31637.7}{\sqrt{3}}$$

$$W_2 - W_1 = 18267 \text{ W} \quad \text{------ (2)}$$

$$W_2 + W_1 = 51111 \quad \text{------ (1)}$$

$$W_2 - W_1 = 18267 \quad \text{------ (2)}$$

By adding equ (1) and (2)

$$2W_2 = 69378$$

$$W_2 = \frac{69378}{2}$$

$$W_2 = 34689 \text{ Watts}$$

Substitute the value of W_2 in equ (1)

$$34689 + W_1 = 51111 \quad \text{------ (1)}$$

$$W_1 = 51111 - 34689 = 16422 \text{ W}$$

$$W_1 = 16422 \text{ Watts}$$

<u>Answer:</u>

i) W_1 = 16422 Watts   ii) W_2 = 34689 Watts

Example: 17

A three phase 440 volts motor operates with a power factor of 0.4. Two wattmeters are connected to measure the input power, and the total power taken from the mains is 30KW. Find the readings of each wattmeter.

Given Data:

Line Voltage (V_L)	=	440 Volts		
Output Power	=	30 KW		
Power factor (Cos θ)	=	0.4		
Efficiency (η)	=	90% = 0.9		

To Find:

$W_1 = ?$

$W_2 = ?$

Solution:

$$W_2 + W_1 = \frac{\text{Output Power}}{\text{Efficiency}}$$

$$W_2 + W_1 = \frac{30 \text{ x } 10^3}{0.9}$$

$$W_2 + W_1 = 33333 \text{ Watts} \quad \text{------ (1)}$$

$$\text{Power Factor } (\text{Cos } \varphi) = 0.4$$

$$\varphi = \text{Cos}^{-1}\, 0.4$$

$$= 66.42^0$$

$$\tan \varphi = \tan 66.42^0$$

$$= 2.29$$

$$\tan \varphi = \frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}$$

$$2.29 = \frac{\sqrt{3}\,(W_2 - W_1)}{33333}$$

$$2.29 \text{ x } 33333 = \sqrt{3}\,(W_2 - W_1)$$

$$\frac{2.29 \text{ x } 33333}{\sqrt{3}} = (W_2 - W_1)$$

$$W_2 - W_1 = \frac{2.29 \text{ x } 33333}{\sqrt{3}}$$

$$W_2 - W_1 = \frac{76332}{\sqrt{3}}$$

$$W_2 - W_1 = 44072 \text{ W} \quad \text{------ (2)}$$

$$W_2 + W_1 = 33333 \quad \text{------ (1)}$$

$$W_2 - W_1 = 44072 \quad \text{------ (2)}$$

By adding equ (1) and (2)

$$2W_2 = 77405$$

$$W_2 = \frac{77405}{2}$$

$$W_2 = 38702.5 \text{ Watts}$$

Substitute the value of W_2 in equ (1)

$$38702.5 + W_1 = 33333 \text{ ------ (1)}$$

$$W_1 = 33333 - 38702.5 = -5369.5 \text{ W}$$

$$W_1 = -5369.5 \text{ Watts}$$

Answer:

i) W_1 = -5369.5 Watts ii) W_2 = 38702.5 Watts

Example: 18

The power input to a 400 volts, 50Hz, 3 phase motor is measured by two wattmeters which indicate 300KW and 100KW respectively. Calculate (a) the input power (b) power factor and (c) the line current.

Given Data:

Wattmeter (W_2)	=	300KW
Wattmeter (W_1)	=	100KW
Line Voltage (V_L)	=	400 Volts
Frequency (F)	=	50 Hz

To Find:

i) The input Power (P) = ?
ii) Power factor (Cos ϕ) = ?
iii) Line Current (I_L) = ?

Solution:

$$\text{Input Power (P)} = W_2+W_1$$

$$= 300 + 100$$

$$W_2+W_1 = 400 \text{ KW}$$

Power Factor (Cos φ):

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_2 - W_1)}{(W_2 + W_1)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,(300 - 100)}{300 + 100)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3} \text{ x } 200}{400}\right]$$

$$= \tan^{-1}\left[\frac{346.4}{400}\right]$$

$$= \tan^{-1}[0.866]$$

$$\varphi = 40.89$$

$$\text{Cos } \varphi = \text{Cos } 40.89$$

$$\text{Cos } \varphi = 0.76$$

$$\text{Power (P)} = \sqrt{3}\, V_L I_L \text{ Cos } \theta$$

$$\text{Line Current } (I_L) = \frac{P}{\sqrt{3}\; V_L \text{ Cos } \theta}$$

$$= \frac{400 \text{ x } 10^3}{\sqrt{3} \text{ x } 400 \text{ x } 0.76}$$

$$= \frac{400 \text{ x } 10^3}{526.5}$$

$$I_L = 759.7 \text{ A}$$

Answer:

i) Input Power = 400 KW

ii) Power Factor (Cos ϕ) = 0.76

iii) Line Current (I_L) = 759.7 A

Example: 19

Two wattmeters are connected to measure the power of a 3 phase circuit indicated 2500W and 500 W respectively. Find the power factor of the circuit when (a) both the readings are positive and (b) the later reading is obtained after reversing the connection of the current coil of the wattmeter.

Given Data:

Wattmeter (W_2) = 2500 W

Wattmeter (W_1) = 500 W

To Find:

Power factor (Cos ϕ) when

i) Both the readings are positive

ii) Later reading is negative

Solution:

Power Factor (Cos φ): **When both the readings are positive**

$$\text{Wattmeter } (W_2) = 2500 \text{ W}$$

$$\text{Wattmeter } (W_1) = 500 \text{ W}$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_2 - W_1)}{(W_2 + W_1)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,(2500 - 500)}{(2500 + 500)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,(2000)}{(3000)}\right]$$

$$= \tan^{-1}[1.154]$$

$$\varphi = 49.08$$

$$\text{Cos } \varphi = \text{Cos } 49.09$$

$$\text{Cos } \varphi = 0.66$$

Power Factor (Cos φ): **<u>Later wattmeter reading is negative</u>**

$$\text{Wattmeter } (W_2) = 2500 \text{ W}$$

$$\text{Wattmeter } (W_1) = -500 \text{ W}$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_2 - W_1)}{(W_2 + W_1)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,(2500-(-500))}{(2500+500)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,(2500+500)}{(3000)}\right]$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3} \text{ x } 3000}{3000}\right]$$

$$= \tan^{-1}[2.59]$$

$$\varphi = 68.88$$

$$\text{Cos } \varphi = \text{Cos } 68.88$$

$$\text{Cos } \varphi = 0.36$$

<u>Answer:</u>

i) Power factor when both readings are positive = 0.66
ii) Power factor when later reading is negative = 0.36

Example: 20

Two wattmeter are connected to measure the power in a 3 phase balanced load. Determine the total power and power factor if the two wattmeters read 1000Watts each (i) both positive and (ii) Second reading is negative.

<u>Given Data:</u>

Wattmeter (W_2) = 1000 W
Wattmeter (W_1) = 1000 W

<u>To Find:</u>

Power factor (Cos ϕ) when
i) Both the readings are positive
ii) Second reading is negative

<u>Solution:</u>

Power Factor (Cos φ): **<u>When both the readings are positive</u>**

$$\text{Wattmeter } (W_2) = 1000 \text{ W}$$

$$\text{Wattmeter } (W_1) = 1000 \text{ W}$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_2 - W_1)}{(W_2 + W_1)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,(1000 - 1000)}{(1000 + 1000)}\right]$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3} \text{ x } 0}{2000}\right]$$

$$= \tan^{-1}[0]$$

$$\varphi = 0$$

$$\text{Cos } \varphi = \text{Cos } 0$$

$$\text{Cos } \varphi = 1$$

Power Factor (Cos φ): **<u>Later wattmeter reading is negative</u>**

$$\text{Wattmeter } (W_2) = 1000 \text{ W}$$

$$\text{Wattmeter } (W_1) = -1000 \text{ W}$$

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_2 - W_1)}{(W_2 + W_1)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,(1000 - (-1000))}{(1000 - 1000)}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3} \text{ x } 2000}{0}\right]$$

$$\varphi = \tan^{-1}\left[\sqrt{3} \text{ x } 2000\right]$$

$$= \tan^{-1}[3464]$$

$$\varphi = 89.98$$

$$\text{Cos } \varphi = \text{Cos } 89.98$$

$$\text{Cos } \varphi = 0.00035$$

<u>Answer:</u>

iii) Power factor when both readings are positive = 1

iv) Power factor when later reading is negative = 0.00035

Example: 21

The power input to a 3 phase induction motor is read by two wattmeters. The readings are 860W and 240W. What is the input power and power factor of the motor?

<u>Given Data:</u>

Wattmeter (W_2) = 860 W

Wattmeter (W_1) = 240 W

<u>To Find:</u>

i) The input Power (P) = ?

ii) Power factor (Cos ϕ) = ?

Solution:

$$\text{Input Power (P)} = W_2+W_1$$
$$= 860 + 240$$
$$W_2+W_1 = 1100 \text{ W}$$

Power Factor (Cos φ):

$$\varphi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_2 - W_1)}{(W_2 + W_1)}\right]$$
$$= \tan^{-1}\left[\frac{\sqrt{3}\,(860 - 240)}{(860 + 240)}\right]$$
$$\varphi = \tan^{-1}\left[\frac{\sqrt{3} \text{ x } 620}{1100}\right]$$
$$= \tan^{-1}\left[\frac{1074}{1100}\right]$$
$$= \tan^{-1}[0.976]$$
$$\varphi = 44.3$$
$$\text{Cos } \varphi = \text{Cos } 44.3$$
$$\text{Cos } \varphi = 0.72$$

Answer:

i) Input Power = 1100 W

ii) Power Factor (Cos ϕ) = 0.72

Example: 22

Three identical coils each having a resistance of 10 ohms and reactance of 10 ohms are connected in delta across 400 volts, 3 phase supply. Find the line current and the readings on each of the two wattmeters connected to measure the power.

Given Data:

Connection	=	DELTA
Resistance (R_{ph})	=	10 Ω
Inductive Reactance (X_{Lph})	=	10 Ω
Line Voltage	=	400 Volts

To Find:

Line Current (I_L) = ?

W_1 =?

W_2 =?

Solution:

$$\text{Impedance } (Z_{ph}) = \sqrt{{R_{ph}}^2 + {X_{Lph}}^2}\ \Omega$$
$$\text{Impedance } (Z_{ph}) = \sqrt{10^2 + 10^2}$$
$$= \sqrt{100 + 100}$$
$$= \sqrt{200}$$

$$\text{Impedance } (Z_{ph}) = 14.14\Omega$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}}$$

$$\text{Phase Voltage } (V_{ph}) = V_L = 400 \text{ V} \quad [\because \text{Delta Connection}]$$

$$\text{Phase Current } (I_{ph}) = \frac{V_{ph}}{Z_{ph}} = \frac{400}{14.14} = 28.28\text{A}$$

$$I_{ph} = 28.28\text{A}$$

$$\text{Line Current } (I_L) = \sqrt{3}\, I_{ph}$$

$$= \sqrt{3} \text{ x } 28.28 = 48.9 \text{ A}$$

$$I_L = 48.9 \text{ A}$$

$$\text{Power Factor } (\text{Cos } \theta) = \frac{R_{ph}}{Z_{ph}} = \frac{10}{14.14} = 0.707$$

$$\text{Power } (P) = \sqrt{3}\, V_L\, I_L \text{ Cos } \theta$$

$$= \sqrt{3} \text{ x } 400 \text{ x } 48.9 \text{ x } 0.707$$

$$= 23952 \text{ Watts}$$

$$W_2 + W_1 = 23952 \text{ Watts} \text{ ------ } (1)$$

$$\text{Power Factor } (\text{Cos } \varphi) = 0.707$$

$$\varphi = \text{Cos}^{-1}\, 0.707$$

$$= 45^0$$

$$\tan \varphi = \tan 45^0$$

$$= 1$$

$$\tan \varphi = \frac{\sqrt{3}\,(W_2 - W_1)}{(W_1 + W_2)}$$

$$1 = \frac{\sqrt{3}\,(W_2 - W_1)}{30 \text{ x } 10^3}$$

$$1 \text{ x } 23952 = \sqrt{3}\,(W_2 - W_1)$$

$$\frac{23952}{\sqrt{3}} = (W_2 - W_1)$$

$$W_2 - W_1 = \frac{23952}{\sqrt{3}}$$

$$W_2 - W_1 = 13829\text{ W} \quad \text{------ (2)}$$

$$W_2 + W_1 = 23952 \quad \text{------ (1)}$$

$$W_2 - W_1 = 13829 \quad \text{------ (2)}$$

By adding equ (1) and (2)

$$2W_2 = 37781$$

$$W_2 = \frac{37781}{2}$$

$$W_2 = 18891\text{ Watts}$$

Substitute the value of W_2 in equ (1)

$$34832.5 + W_1 = 23952 \quad \text{------ (1)}$$

$$W_1 = 23952 - 18891 = 5061\text{ W}$$

$$W_1 = 5061\text{ Watts}$$

Answer:

i) $W_1 = 5061$ Watts ii) $W_2 = 18891$ Watts

REVIEW QUESTIONS
UNIT : V THREE PHASE CIRCUITS

PART – A : 2 Mark Questions

1. What is phase sequence?
2. Define phase voltage.
3. Define Line voltage.
4. Define phase current.
5. Define Line current.
6. Define balanced load.
7. Define unbalanced load.
8. What is a star connection?
9. What is a Delta connection?
10. Write the expression for 3 phase power in Star connection.
11. State the relationship between line current and phase current in a balanced star connected load.
12. State the relationship between line voltage and phase voltage in a balanced star connected load
13. State the relationship between line current and phase current in a balanced delta connected load.
14. State the relationship between line voltage and phase voltage in a balanced delta connected load.
15. Write the expression for calculating real power in a three phase circuit.
16. Write the expression for calculating apparent power in a three phase circuit.
17. Write the expression for calculating reactive power in a three phase circuit.
18. Write down the expression of power factor in two wattmeter method of power measurement.
19. What is the p.f of 3 phase load if $W_1=W_2$.
20. What is the p.f of 3 phase load if $W_1 = -W_2$

PART – B : 3 Mark Questions

1. List the advantages of 3-phase system.
2. Write the voltage expressions of a three phase sinusoidal voltage.
3. Define positive sequence.
4. Define negative sequence.
5. Show that in a balanced 3 phase star connected system, the neutral current is zero.
6. What is meant by balanced and unbalanced load?
7. Show that the line voltage in a balanced star connected system is 3 times of phase voltage.
8. Derive an expression for line current in terms of phase current in a three phase delta connected system.
9. How can you determine the phase angle from two wattmeter readings?
10. Prove that 3 phase power is given by $\sqrt{3}$ VI cosΦ in delta connection.
11. Discuss the effect of unbalanced loads in star connected system.
12. Write the relation between the power factor and wattmeter readings in two wattmeter method of power measurement.

13. A star connected load has 6+j8Ω impedance per phase. Determine the line current if it is connected to 400V, 3 phase , 50Hz supply.
14. A delta connected load has 30-j40Ω impedance per phase. Determine the line current if it is connected to 415V, 3 phase , 50Hz supply.
15. A star connected balanced load draw a current of 35A per phase when connected to a 440V supply. Determine the apparent power.

PART – C : 10 Mark Questions

1. Establish the relationship between line current and phase current, line voltage and phase voltage in a star connected system.

2. Establish the relationship between line current and phase current, line voltage and phase voltage in a delta connected system.

3. Prove that the sum of the readings on two single-phase wattmeter's connected in 3-phase circuit give 3-phase power.

4. Explain the two wattmeter method of 3-phase power measurement in a 3 phase star or delta connected load.

5. A balanced star connected load of (4+j3)Ω per phase is connected to a balanced 3 phase 400V, AC supply. The phase current is 12A. Find (1) the total active power (2) reactive power and (3) apparent power.

6. A three phase, balanced delta connected load of (4+j8)Ω per phase is connected across a 400V, AC supply. Determine the phase current, line current and power consumed by the load.

7. A three phase, balanced star connected load of (15+j20)Ω per phase is connected across a 400V, 50Hz, AC supply. Determine the line current and power consumed by the load.

8. Each phase of a delta connected load consists of a resistance 10Ω and a capacitance 100 µF. A supply of 440V, 50Hz is applied to the load. Find (i) line current, (ii) Power factor (iii) Power consumed by the load.

9. The power consumed in a three phase balanced star connected load is 2KW at a power factor of 0.8 lagging. The supply voltage is 400V, 50Hz.Calculate the resistance and reactance of each phase.

10. A delta connected balanced 3 phase load is supplied from a 3 phase, 400V supply. The line current is 20A and the power taken by the load is 10,000W. Find (i) impedance in each branch, (ii) line current, power factor and power consumed if the same load is connected in star.

11. The input power to a three phase load is 10KW at 0.8 p.f. Two wattmeters are connected to measure the power. Find the individual readings of the wattmeters.

12. The readings of the two wattmeters used to measure power in a capacitive load are 3000W and 8000W respectively. Calculate (i) the input power and (ii) the power factor at the load.

13. The power input to a 2000V, 50Hz, three phase motor is measured by two wattmeters which indicate 300KW and 100KW respectively. Calculate the input power, power factor and line current.

14. A 500V, 3 phase induction motor has an output of 3.73KW and operates at a power factor of 0.85, with an efficiency of 90%. Calculate the reading on each wattmeter connected to measure the input power.

15. The input power to a three phase motor was measured by two wattmeter method. The readings are 5.2KW and 1.7KW. The later reading has been obtained after reversing the current coil connections. The line voltage was 400V. Calculate a) The total power b) Power factor and c) Line current.

16. Two wattmeters are used to measure power in a three phase load. The wattmeter readings are 400W and -35W. Calculate a) Total active power b) Power factor c) Reactive Power and d) Volt-ampere.

17. Calculate the total power input and reading of the two wattmeters connected to measure power in a three phase balanced load, if the reactive power input is 15KVAR and the load power factor is 0.8.

18. A 3v phase star connected load draws a line current of 25A. The load KVA and KW are 20 and 16 respectively. Find the readings on each of the two wattmeters used to measure the three phase power.

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