Universitat Politècnica de Catalunya

Masters Thesis

Implementation of a 3D pose estimation algorithm

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Abstract

Escola Tècnica Superior d'Enginyeria Industrial de Barcelona (ETSEIB)
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Master's Degree in Automatic Control and Robotics

Implementation of a 3D pose estimation algorithm

by Edgar Riba Pi

In this project, I present the implementation of a 3D pose estimation algorithm for rigid objects considering a single monocular camera. The algorithm based on the matching between natural feature points and a textured 3D model, recovers in an efficient way the 3D pose of a given object using a PnP method. Furthermore, during this project a *C++* implementation of the UPnP [1] approach published by the supervisors of this project has been done.

Both the algorithm and the UPnP source codes have been included in the OpenCV library. The first as a tutorial explaining how developers could implement this kind of algorithms, and the second as an extension of the *Camera Calibration and 3D Reconstruction* module. In order to ensure the quality of the code, both passed the accuracy and performance tests imposed by the organization before merging the code in the new released version.

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And finally, the guys from Aldebaran Robotics, to accept me as one of them during my internship, in special to Vincent and Karsten for their support.

Contents

Chapter 1

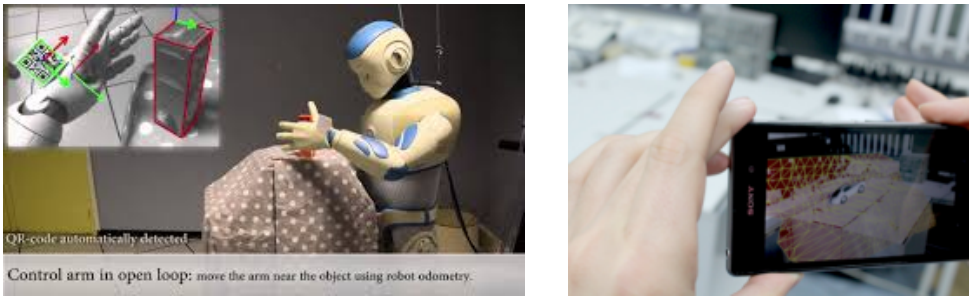
Introduction

The current chapter presents the main motivations for the realisation of this project as well as the objectives to achieve in its finalization. A brief description of the main topics in 3D Computer Vision related to this work are provided in order to outline the applied methodology.

1.1 Poject Motivation

Nowadays augmented reality (AR) is one of the most interesting research topic in computer vision and robotics areas. The most elemental problem in AR is the estimation of the camera pose respect to an object (See Fig.1.1). However, with the current technology, Computer Vision still has a lack regarding to the large computational cost of applying algorithms in order to achieve simple tasks that humans we complete immediately.

Computer Vision is a scientific discipline which comprises a huge range of methods to process and analyse digital images in order to extract numerical or symbolic information. We can find many definitions of Computer Vision in the literature. In [2], it is defined



(A) Robot grasping an object (B) Augmented reality

FIGURE 1.1: Pose estimation applications

as a branch of artificial intelligence and image processing concerned with computer processing of images from the real world.

Nonetheless, the real emergence of Computer Vision came with the necessity to automatically analyse images in order to copy the human behaviour and still today is a constant challenge to replicate generic methods such for depth estimation, relative pose estimation or even classify perceived objects.

1.2 Focus and Thesis Organization

Objects can be textured, non textured, transparent, articulated, etc. In this project, we will focus on giving a solution to objects recognition in addition to estimate its 3D pose given an image sequence.

Until today there is no unique method to solve this essential task since many variables will define the problem formulation. The most important actor to take into account is the application goal, followed by the number of degrees of freedom of the object and the camera, in addition to the camera sensor type. However, in this project we will focus on the use of a monocular perspective camera to recover the six degrees of freedom that define the relative position and orientation between the scene and the camera. The proposed algorithm, based on a tracking by detection methodology [3], recovers the six degrees of freedom of the camera considering natural feature points and a previously registered 3D textured model.

We will first introduce in Chapter 2 the previous background and the premises about the pose estimation problem in order to understand the applied methodology to solve the problem. Chapter 3 will deal with the geometric and mathematics techniques used to solve the pose estimation problem. Chapter 4 will introduce the designed software architecture model. Finally, Chapter 5 and 6 will show the results and contributions obtained during this project, and the proposed future work.

Chapter 2

Background

In this chapter, is presented the current state of the art related to pose estimation algorithms, from the origin with the Direct Linear Transform (DLT) algorithm, to the PnP problem for calibrated and uncalibrated cameras, in addition to robust estimation.

2.1 Calibrated Cameras

The camera pose estimation from n 3D-to-2D points correspondences is a fundamental and already solved problem in geometric computer vision area. The main goal is to estimate the six degrees of freedom of the camera pose and the camera calibration parameters: the focal length, the principal point, the aspect ratio and the skew. A first approach to solve the problem using a minimum of 6 pair of correspondences can be done using the well-known Direct Linear Transform (DLT) algoritm [4].

Since the DLT algorithm requires the camera parameters, numerous simplifications to the problem have been achieved in order to improve the solution accuracy and forming in consequence a large set of new different algorithms. A variant of the previous algorithm is the very called Perspective-n-Point problem, which assumes that the camera intrinsic parameters are known. In its minimal version only three point correspondences are needed to recover the camera pose [5]. There also exist many iterative solutions to the over-constrained problem with $n > 3$ point correspondences [6–8] . However, the non-iterative solutions are strongly differentiate for its computational complexity and accuracy from $O(n^8)$ [9] to $O(n^2)$ [10] down to $O(n)$ [11]. On the other hand, we can differentiate between algebraic and geometric solutions considering as a representative case Gao's solution [5], also known as P3P, in which propose a triangular decomposition of the equations system. Many other iterative methods for large values of n exist [6–8],

in this case, considering Lu's method [8] as the most representative one since its good performance in terms of speed and accuracy, produces slightly better results than the non-iterative methods such as EPnP algorithm [11]. In some cases, in order to improve the method accuracy, some non-linear optimizations such as Gauss-Newton are applied with negligible cost.

In most of the cases, iterative methods get trapped in local minima, however, the aim to find a global optimal solution encourage the researchers to reformulate the problem as positive semidefinite quadratic optimization problem [12].

2.2 Uncalibrated Cameras

For the uncalibrated case, we can find many solutions assuming that the intrinsic parameters are unknown, the pixel size is squared, and the principal point is close to the image center [4, 13], which then the problem is simplified to estimate only the focal length. For this case, exist solutions to solve the minimal problem assuming unknown focal lenght [14–17], as well as for the case with unknown focal length plus unknown radial distortion [13, 17–19].

Groebner *et al* [16], bases the computation of the 5- and 6-point relative pose problem with unknown focal length to solve large systems of polynomial equations [15], in both, solving the problem computing the polynomial eigenvalues. Furthermore, Bujnak *et al* [14] propose a general solution to the P4P Problem for cameras with unknown focal length reducing the equation system to 5 with 4 unknowns and 20 monomials. More recently, other solutions for the P4P Problem have been proposed for unknown focal length and radial distortion [13, 18, 19] which always are followed by a robust estimation method.

2.3 Robust Estimation

Since we are dealing with camera sensors, the presence of noise is unavoidable and the methods to find the correspondences may give us some mismatches or *outliers*, then, due to that fact, the solutions to the problem become unstable and not reliable.

In front of this problem, and the need to establish a robust pairing between features, the most common solution is to include an extra iterative step using the RANSAC [20] algorithm for outliers removal or even Least Median Squares [21]. Unfortunately, even taking its minimal or non-minimal subsets [22], a supplementary high computational

load is always assured which brings the method to rely on a random sampling. Recent attempts to reformulate the problem as a quasi-convex optimization problem have been carried out in order to guarantee the estimation of global minima [23–25].

The original RANSAC algorithm has a lot of variations such as Guided-MLESAC [26] and PROSAC [27] which are aimed to avoid sampling unlikely correspondences by using appearance based scores. Additionally, GroupSAC [28] uses image segmentation in order to sample the data more efficiently. Other techniques, such as Preemptive RANSAC [29] or ARRSAC [30] are used in limited scenarios increasing then probability to obtain the better estimation as possible. In [31], is presented MultiGS, which speeds up the search strategy guided by the sampling information obtained from a residual sorting in addition to be able to account for multiple structures in the scene.

May happen the absence of robust appearance information in graphs, which then the problem can be solved as is proposed in [32]. Nonetheless, due to the high computational cost, this graph methods are useless for large graphs.

During the next chapter, will be explained in detail all the mathematical methodology and tools introduced in this chapter in order to deeply understand how the built application works.

Chapter 3

Problem Formulation

The current chapter is aimed to explain from a geometric point of view the pose estimation problem formulation using in this case a pinhole camera model.

3.1 Camera Representation

In this section we will focus on the camera used for this project, a standard pinhole model. This type of cameras are very popular and commonly used in this type of projects since have hyperbolics or parabolic mirrors which allow to achieve very wide field of views.

FIGURE 3.1: Example of two standard pinhole cameras. A simple webcam *Creative HD 720P, 10MP* (left), and a digital compact camera *Nikon D3100* (right)

3.1.1 The Perspective Projection Model

In order to compute the 3D pose we need a set of 3D-to-2D correspondences between n reference points $M_1, ..., M_n$ where $\mathbf{M_i} = [X, Y, Z]^T$ are expressed in a Euclidean world coordinate system w and their pairing 2D projections $m_1, ..., m_n$ where $\mathbf{m_i} = [u, v]^T$ in the image plane (See Fig. 3.2).

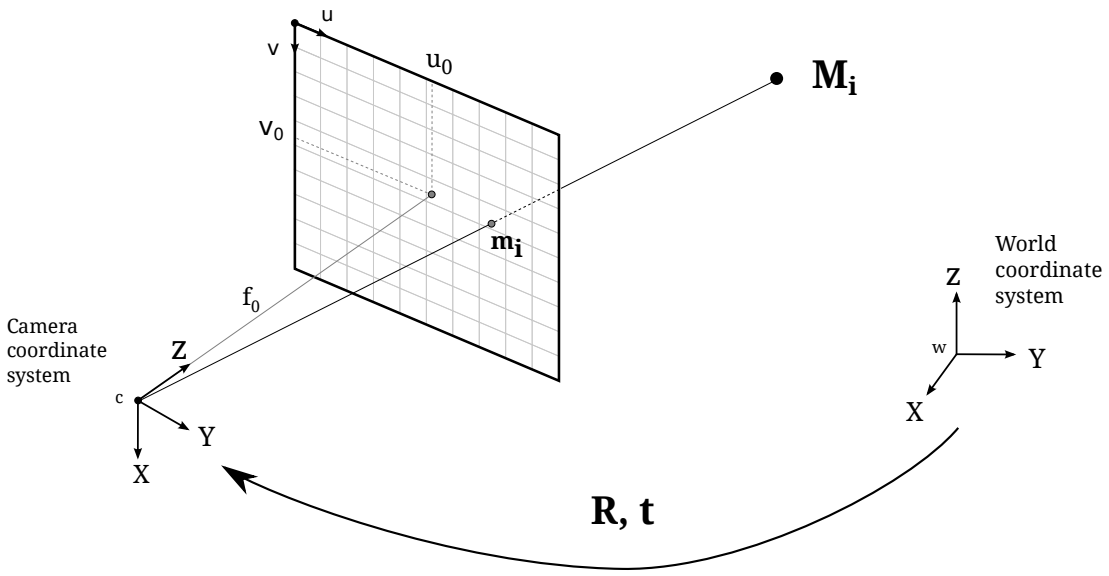


FIGURE 3.2: **Problem Formulation:** Given a 3D point $\mathbf{M_i}$ expressed in a world reference frame, and its 2D projection $\mathbf{m_i}$ onto the image, we seek to retrieve the pose ($\mathbf{R}$ and $\mathbf{t}$) of the camera w.r.t. the world.

Thus, the defined projection can be expressed within the equation 3.1

$$s\tilde{\mathbf{m}}_\mathbf{i} = \mathbf{P}\tilde{\mathbf{M}}_\mathbf{i}, \tag{3.1}$$

where s is a scale factor, $\tilde{\mathbf{m}}_\mathbf{i} = [u, v, 1]^T$ and $\tilde{\mathbf{M}}_\mathbf{i} = [X, Y, Z, 1]^T$ are the homogeneous coordinates of points $\mathbf{m_i}$ and $\mathbf{M_i}$, and $\mathbf{P}$ is a 3 x 4 projection matrix.

Is known that $\mathbf{P}$ is defined by a scale factor which also depends on 11 parameters. The perspective matrix can be decompose as:

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}] \tag{3.2}$$

where:

- $\mathbf{K}$ is a 3x3 matrix which contains the camera calibration parameters such as the focal length, the scale factor and the optical center point coordinates.
- $[\mathbf{R}|\mathbf{t}]$ is a 3x4 matrix which corresponds to the Euclidean transformation from a world coordinate system to the camera coordinate system. $\mathbf{R}$ is the rotation matrix, and $\mathbf{t}$ the translation vector.

3.1.2 The Intrinsic Parameters

In the previous section we defined **K** as a matrix with the internal camera calibration parameters, usually referred as camera calibration matrix, can be expressed by the following equation:

$$\mathbf{K} = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3.3}$$

where:

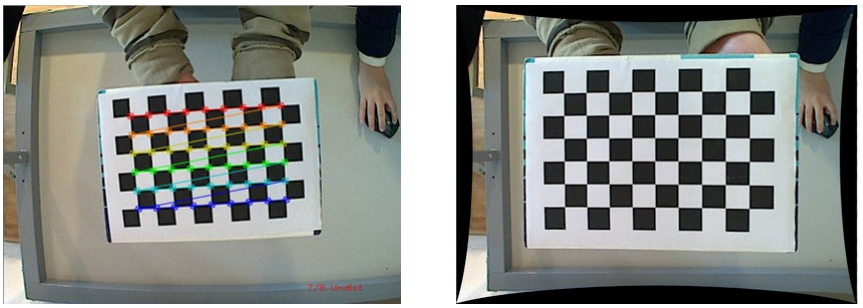
- $\alpha_\mathbf{u}$ and $\alpha_\mathbf{v}$ are the scale factor defined in each coordinate direction u and v. This scale factors are proportional to the camera focal length: $\alpha = k_u f$ and $\alpha = k_v f$, where k_u and k_v are the total number of pixels per unit in the u and v directions.
- $\mathbf{c} = [u_0, v_0]^T$ represents the principal point coordinates, which it is the intersection of the optical axis and the image plane.
- **s**, referred as the skew angle, is the ratio which defines the perpendicularity of the u and v directions. In modern cameras usually this value is zero.

In order to simplify the problem, often a common approximation is to set the principal point **c** at the image center. Furthermore, in modern cameras we assume that the pixels have a squared shape, which lead us then to take α_u and α_v with equal values. In geometric computer vision it is said that when the camera calibration matrix or the intrinsic parameters are known, the camera is calibrated.

3.1.3 The Extrinsic Parameters

Previously we defined $[\mathbf{R}|\mathbf{t}]$ as a 3x4 matrix which corresponds to the Euclidean transformation from a world coordinate system to the camera coordinate system. In fact this matrix is the horizontal concatenation of the rotation matrix and the translation vector which is often referred as the *camera pose*. (See eq. 3.4)

$$[\mathbf{R}|\mathbf{t}] = \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_1 \\ R_{21} & R_{22} & R_{23} & t_2 \\ R_{31} & R_{32} & R_{33} & t_3 \end{bmatrix}, \tag{3.4}$$



(A) Pattern detection (B) Image rectification

FIGURE 3.3: Screenshots of the calibration process using the OpenCV framework. On the left, the pattern detection. On the right, the rectified image after the camera calibration process using the estimated intrinsic parameters.

Almost all pose estimation algorithms assume that the $\mathbf{K}$ calibration matrix is known and are focused on minimizing $\mathbf{R}$ and $\mathbf{t}$ (See eq. 3.5), which in other words are the orientation and position of the object respect to the camera.

$$\min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} \|\mathbf{u_i} - \tilde{\mathbf{u}}_\mathbf{i}\|^2, \tag{3.5}$$

Another conventional way to express that is assuming $\mathbf{R}$ and $\mathbf{t}$ as the Euclidean transformation from a world coordinate system w to the camera coordinate system. Then, a 3D point represented by the vector $\mathbf{M_i^w}$ in world coordinates will be represented by the vector $\mathbf{M_i^c} = \mathbf{R}\mathbf{M_i^w} + \mathbf{t}$ in the camera coordinate system. From this previous assumption, the *camera center*, or *optical center* $\mathbf{C}$ can be recovered in the world coordinate system satisfying $\mathbf{0} = \mathbf{R}\mathbf{C} + \mathbf{t}$, and then $\mathbf{C} = -\mathbf{R^{-1}}\mathbf{t} = -\mathbf{R^T}\mathbf{t}$.

3.1.4 The Camera Calibration Estimation

Since in most of the 3D pose estimation algorithms it is assumed that the intrinsic parameters are known and fixed, the camera zoom must be disabled in order to preserve the same focal length during all the procedure. Besides, the camera parameters must be computed in an offline process using images taken with the same camera that will be used later for detection.

The default methodology to perform this parameters estimation is using what is called a calibration object with a pattern of known size. The most common objects are black-white chessboards, symmetrical or asymmetrical circle patterns (See fig. 3.3). The goal is to find the 2D-3D correspondences between the grid patterns centres to finally compute the projection matrix. Nowadays, it is easy to find several toolbox as in *OpenCV* or *Matlab* which provide user friendly automated applications to carry on this process.

3.2 Camera Pose Parametrization

In order to estimate the camera pose, first an appropriated parametrization of its translation vector and rotation matrix is needed.

A rotation matrix has only three degrees of freedom in $\mathbb{R}^3$, which means that it will directly depend on the nine elements of the 3x3 rotation matrix. Since this matrix must be orthonormal, are needed six additional non-linear constrains - three to force all three columns to be of unit length, and three to ensure the orthogonality between them.

In the literature we can find some parametrizations which demonstrates that are effective for pose estimation: Euler angles, quaternions, and exponential maps [33]. In the subsections below we will see in detail some properties about Euler angles and exponential maps. Quaterions will not be explained since have not been used for the completion of this project.

3.2.1 Euler Angles

The rotation matrix $\mathbf{R}$ can be written as the product of three matrices representing rotations around the X, Y, and Z axis, where we can find several conventions that assures that. The most common one, takes α, β, γ respectively as a rotation angles around the Z, Y, and X axis

$$\mathbf{R} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}, \tag{3.6}$$

The extraction of the Euler angles for a given rotation matrix can be easily realized by identifying the matrix coefficient as well as its analytical expression.

However, Euler angles have a well known drawback called gimbal lock, which it is caused when two of the three rotations axis are aligned, producing no effect in one rotation axis. For this reason, Eulers angles are not used anymore for pose estimation algorithms.

3.2.2 Exponential Map

The exponential map needs only three parameters to describe a rotation and does not stumble in the gimbal lock problem.

Given a 3D vector $\vec{\omega} = [\omega_x, \omega_y, \omega_z]^T$ and $\theta = \|\vec{\omega}\|$ its norm, then an angular rotation θ around an axis of direction $\bar{\omega}$ can be represented as the infinite series

$$exp(\mathbf{\Omega}) = I + \mathbf{\Omega} + \frac{1}{2!}\mathbf{\Omega}^2 + \frac{1}{3!}\mathbf{\Omega}^3 + \cdots \tag{3.7}$$

where $\mathbf{\Omega}$ is the skew-symmetric matrix

$$\mathbf{\Omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}, \tag{3.8}$$

In eq. 3.7 we can see that the exponential map representation can be rewritten as the series expansion of an exponential. Then, it can be evaluated using Rodrigues formula

$$\mathbf{R}(\mathbf{\Omega}) = exp(\mathbf{\Omega}) = \mathbf{I} + \sin\theta\hat{\mathbf{\Omega}} + (1 - \cos\theta)\hat{\mathbf{\Omega}}^2, \tag{3.9}$$

To summarize, with the exponential map we can represent a rotation as a 3-vector which each axis will have an associated magnitude. Moreover, using this representation the gimbal lock problem of Euler angles is avoided.

3.3 External Parameters Matrix Estimation

In this section are presented the evolution over the time of some methods to estimate the external camera parameters without any prior knowledge of camera position. It is assumed that some correspondences between 3D points in the world coordinate system and their projections in the image plane and the camera parameters are known.

The main objective of pose estimation is to find the perspective projection matrix $\mathbf{P}$ which projects the 3D points $\mathbf{M_i}$ on $\mathbf{m_i}$ given a set of n correspondences. We can rewrite that in terms of $\mathbf{P}\tilde{\mathbf{M}}_\mathbf{i} \equiv \tilde{\mathbf{m}}_\mathbf{i}$ for all i, where $\equiv$ represents the equality up to a scale factor.

The number of correspondences will depend on the used approach to recover the pose. However, when the intrinsic parameters are known and $n = 3$, this known correspondences $\mathbf{M_i} \leftrightarrow \mathbf{m_i}$ produce 4 possible solutions. In the case of $n = 4$ or $n = 5$ pairings,in [20] is shown that there are at least two solutions in general configurations. For $n \geq 4$ and the points are coplanar and there is no triplets of collinear points, the solution is unique. When $n \geq 6$, the solution is unique.

3.3.1 The Direct Linear Transformation

The Direct Linear Transform (DLT) was the starting point in pose recovering. First used by photogrammetrists, and then introduced in the computer vision community, this algorithm is able to estimate the projection matrix $\mathbf{P}$ by solving a linear equations system with unknown camera parameters. For each pairing $\mathbf{M_i} \leftrightarrow \mathbf{m_i}$, two linearly independent equations can be written as follows:

$$\begin{aligned} \frac{\mathbf{P}_{11}X_i + \mathbf{P}_{12}Y_i + \mathbf{P}_{13}Z_i + \mathbf{P}_{14}}{\mathbf{P}_{31}X_i + \mathbf{P}_{32}Y_i + \mathbf{P}_{33}Z_i + \mathbf{P}_{34}} &= u_i, \\ \frac{\mathbf{P}_{21}X_i + \mathbf{P}_{22}Y_i + \mathbf{P}_{23}Z_i + \mathbf{P}_{24}}{\mathbf{P}_{31}X_i + \mathbf{P}_{32}Y_i + \mathbf{P}_{33}Z_i + \mathbf{P}_{34}} &= v_i, \end{aligned}$$

This system can be written in the form of $\mathbf{Ap} = \mathbf{0}$, where $\mathbf{p}$ is a vector composed by the coefficients $\mathbf{P}_{ij}$. The solution to this system can be found from the Singular Value Decomposition (SVD) of $\mathbf{A}$ taking the *eigenvector* with the minimal *eigenvalue*.

To recover the camera pose, then the camera calibration matrix $\mathbf{K}$ is needed in order to be extracted from $\mathbf{P}$ up to a scale factor as follows: $[\mathbf{R} \mid \mathbf{t}] \sim \mathbf{K}^{-1}\mathbf{P}$. Finally, the 3x3 rotation matrix can be computed from the first three columns applying a correction step [34].

Is known that pixel locations $\mathbf{m_i}$ are often noisy, this method should be refined by an iterative optimization step in order to minimize the non-linear reprojection error. We will see this topic in subsection 3.3.3.

3.3.2 The Perspective-*n*-Point Problem

Since the DLT estimates 11 parameters of the projection matrix without knowing the camera intrinsics and using only a single point, then the internal parameters must be estimated in an offline process. However, this information can be used in addition to introduce extra points in the system, which will make more robust the camera pose estimation process.

When more than one point is introduced to solve the system, the problem becomes to what it is called a Perspective n Point Problem, PnP in short, which determines the position and orientation given a set of n pairings between 3D points and their 2D projections (See fig. 3.4). As mentioned in Chapter 1, we can find in the literature many

methods which uses 3 to n points to estimate the camera pose and can be classified as iterative or non-iterative, and with known or unknown camera parameters.

A reference method is the perspective-3-point problem (P3P) [5], which it is a non-iterative method that using the cosines law give up to 4 solutions to the problem. Due to that fact, the solution needs a refined process by a non-linear estimation and since the pose is only estimated with 3 points, then the solution may be inaccurate. A solution to this problem is to add more correspondences to the system.

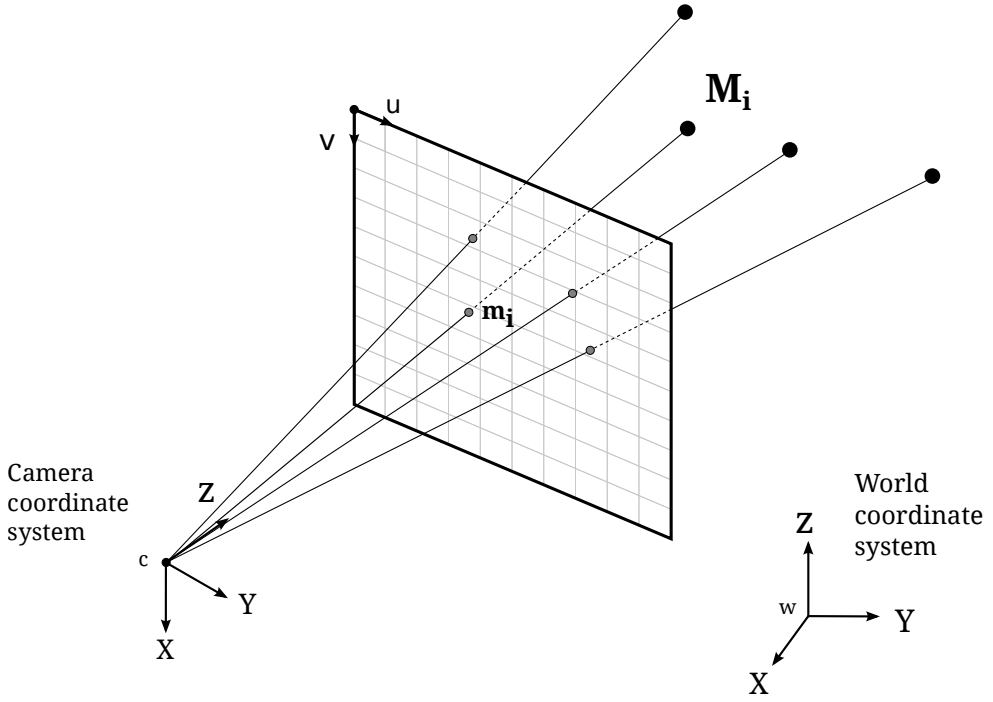


FIGURE 3.4: **PnP Problem scheme:** Given a set of 3D points $\mathbf{M_i}$ expressed in a world reference frame, and their 2D projections $\mathbf{m_i}$ onto the image, we seek to retrieve the pose ($\mathbf{R}$ and $\mathbf{t}$) of the camera w.r.t. the world.

3.3.2.1 EPnP

In this section we will focus on a detailed explanation of a method to estimate the camera pose using a set of n correspondences and known parameters. Introduced by F.Moreno *et al.'s* [11], the "EPnP: An Accurate Non-Iterative $O(n)$ Solution to the PnP Problem" solves the camera pose in an efficient way assuming that the camera parameters and a set of n correspondences whose 3D coordinates in the world coordinate system and its 2D image projections are known, therefore expressing the coordinates as a weighted sum

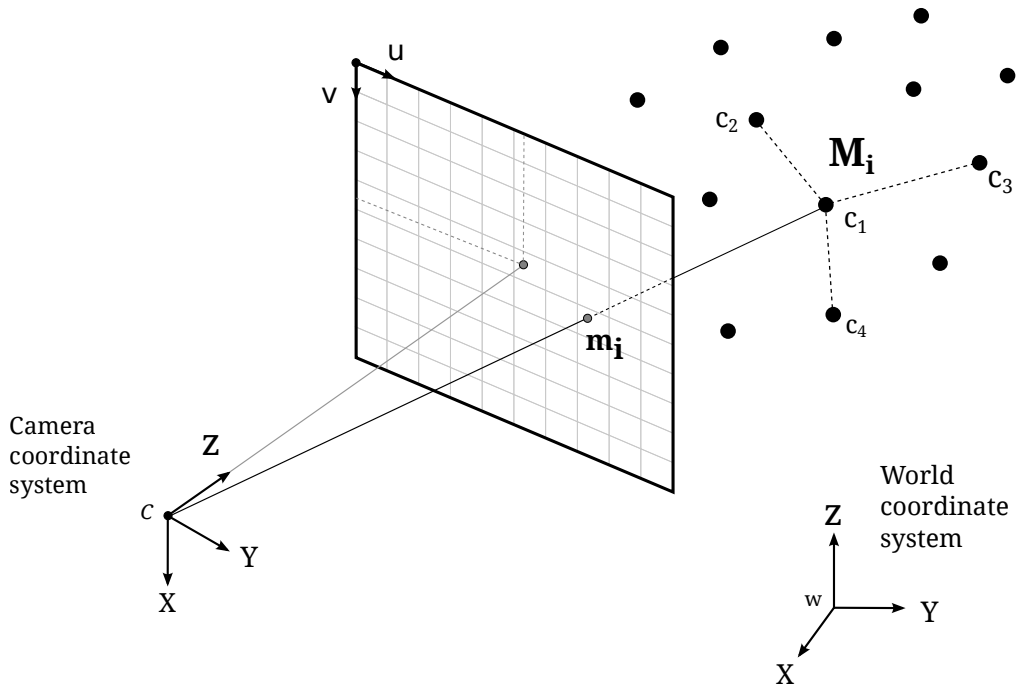


FIGURE 3.5: **EP*n*P Problem formulation:** Given a set of 3D points $\mathbf{M_i}$ expressed in a world reference frame, and their 2D projections $\mathbf{m_i}$ onto the image, the points $\mathbf{c_{1..4}}$ form the base which represents all the set by a linear combination in order to retrieve the pose ($\mathbf{R}$ and $\mathbf{t}$) of the camera w.r.t. the world.

of 4 non-coplanar virtual *control points* (See fig. 3.5). The problem is reformulated as follows

$$\mathbf{p}_i^w = \sum_{i=1}^{4} \alpha_{ij} \mathbf{c}_j^w \tag{3.10}$$

where $\mathbf{p}_i^w = [X^w, Y^w, Z^w]^T$ is a 3D point in world coordinates system, α_{ij} are the homogeneous barycentric coordinates and $\mathbf{c}_j^w = [X^w, Y^w, Z^w]^T$ is a 3D control point in world coordinates. Then, the 4 control points in camera coordinates $\mathbf{c}_j^c$ become the unknown of the problem, giving a total of 12 unknowns.

Similar to the DLT, is needed to build a linear system in the control points reference frame:

$$\forall i, w_i \begin{bmatrix} \mathbf{u}_i \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{p}_i^c = \mathbf{K} \sum_{i=1}^{4} \alpha_{ij} \mathbf{c}_j^c \tag{3.11}$$

where w_i are the scalar projective parameters, $\mathbf{u}_i$ are the 2D coordinates $[u_i, v_i]^T$, $\mathbf{K}$ is the camera parameters matrix. This expression can be rewritten as follows:

$$\forall i, w_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix} \sum_{i=1}^{4} \alpha_{ij} \mathbf{c}_j^c \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix} \tag{3.12}$$

From 3.12 we can obtain two linearly independent equations:

$$\begin{aligned} \sum_{i=1}^{4} \alpha_{ij} f_u x_j^c + \alpha_{ij}(u_c - u_i) z_j^c &= 0, \\ \sum_{i=1}^{4} \alpha_{ij} f_v y_j^c + \alpha_{ij}(v_c - v_i) z_j^c &= 0, \end{aligned}$$

Hence, a linear system is generated with the following form:

$$\mathbf{M}\mathbf{x} = \mathbf{0} \tag{3.13}$$

where $\mathbf{M}$ is a $2n$x12 matrix with known coefficients and $\mathbf{x} = [\mathbf{c}_1^c, \mathbf{c}_2^c, \mathbf{c}_3^c, \mathbf{c}_4^c]^T$ is a 12-vector made of the unknowns.

The solution to this system lies on the null space, or kernel, of $\mathbf{M}$, expressed as:

$$\mathbf{x} = \sum_{i=1}^{N} \beta_i \mathbf{v}_i \tag{3.14}$$

where $\mathbf{v}_i$ are the right eigenvectors of $\mathbf{M}$, corresponding to the N null eigenvalues of $\mathbf{M}$. The efficiency of EPnP remains in the transformation of $\mathbf{M}$ into a small constant matrix $\mathbf{M^T M}$ of size 12x12 before computing the eigenvectors.

From 3.14, in theory the solution will be given by $\beta_i' s$ for each N. Nonetheless, in practice the solution will be obtained only for $N = 1, ..., 4$

$$
\begin{aligned}
N=1: \ \mathbf{x} &= \beta_1 \mathbf{v}_1 \\
N=2: \ \mathbf{x} &= \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 \\
N=3: \ \mathbf{x} &= \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 + \beta_3 \mathbf{v}_3 \\
N=4: \ \mathbf{x} &= \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 + \beta_3 \mathbf{v}_3 + \beta_4 \mathbf{v}_4
\end{aligned}
$$

In order to find the correct betas, a geometric constraint must be added. The method assumes that the distances between control points in the camera coordinate system should be equal to the ones computed in the world coordinate system:

$$\|\mathbf{c}_i^c - \mathbf{c}_j^c\|^2 = \|\mathbf{c}_i^w - \mathbf{c}_j^w\|^2, \tag{3.15}$$

For simplicity, I will show only the case for $N = 1$, where:

$$\|\beta \mathbf{v}^{[i]} - \beta \mathbf{v}^{[j]}\|^2 = \|\mathbf{c}_i^w - \mathbf{c}_j^w\|^2, \tag{3.16}$$

Then the beta can be computed as follows

$$\beta = \frac{\sum_{\{i,j\} \in [1;4]} \|\mathbf{v}^{[i]} - \mathbf{v}^{[j]}\| \cdot \|\mathbf{c}_i^w - \mathbf{c}_j^w\|}{\sum_{\{i,j\} \in [1;4]} \|\mathbf{v}^{[i]} - \mathbf{v}^{[j]}\|^2}, \tag{3.17}$$

Once the betas are computed, in order to get the camera pose is needed to do the inverse process. Firstly, compute the control points coordinates in the camera frame reference. Secondly, compute the coordinates of all 3D points in camera frame reference and finally, as shown in [35], extract the rotation matrix $\mathbf{R}$ and the translation vector $\mathbf{t}$.

3.3.2.2 UP*n*P

The following method, "Exhaustive Linearization for Robust Camera Pose and Focal Length Estimation" or Uncalibrated PnP (UPnP), introduced by A. Peñate *et al.'s* [1], is an extension of the EPnP for the case of uncalibrated cameras. The method, allows to estimate the camera pose in addition to the camera focal length in bounded time. Although the solution is non-minimal, it becomes robust in front of several noise coming from the input data.

Similar to the EPnP algorithm, the solution to the problem belongs to the kernel of a matrix derived from the set of 2D-to-3D correspondences, which can be expressed as

a linear combination of its eigenvectors. Again, the weights of this linear combination become the unknown of the problem, solved by applying additional distance constraints.

It is assumed that a set of 2D-to-3D correspondences are given between n reference points $\mathbf{p}_1^w, ..., \mathbf{p}_n^w$ respect to a world coordinate system w, and their 2D projections $\mathbf{u}_1, ..., \mathbf{u}_n$ in the image plane. Furthermore, it is expected a squared pixel size camera with the principal point (u_0, v_0) at the center of the image. Under these assumptions, the problem is formulated to retrieve the focal length f, the rotation matrix $\mathbf{R}$ and the translation vector $\mathbf{t}$ by minimizing an objective function based on the reprojection error:

$$\min_{f,\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} \|\mathbf{u}_i - \tilde{\mathbf{u}}_i\|^2, \tag{3.18}$$

where $\tilde{\mathbf{u}}_i$ is the projection of point $\mathbf{p}_i^w$:

$$k_i \begin{bmatrix} \tilde{\mathbf{u}}_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{p}_i^w \\ 1 \end{bmatrix}, \tag{3.19}$$

with a k_i scalar projective parameter.

Identically to the EPnP, each 3D point is rewritten in terms of barycentric coordinates respect to 4 control points, turning then the problem in finding the solution of a $2n$ equations with 12 unknowns linear system. The main difference of this method persists on the perspective projection equations construction, which now the focal length is taken into account:

$$\begin{aligned} \sum_{j=1}^{4} \alpha_{ij} x_j^c + \alpha_{ij}(u_0 - u_i)\frac{z_j^c}{f} &= 0, \\ \sum_{j=1}^{4} \alpha_{ij} y_j^c + \alpha_{ij}(v_0 - v_i)\frac{z_j^c}{f} &= 0, \end{aligned}$$

These equations can be expressed as the following linear system

$$\mathbf{M}\mathbf{x} = \mathbf{0}, \tag{3.20}$$

where $\mathbf{M}$ is a $2n$x12 matrix containing the coefficients α_{ij}, the 2D points $\mathbf{u}_i$ and the principal point. Therefore, the $\mathbf{x}$ vector contains the 12 unknowns: the control points 3D coordinates respect to the camera frame and the focal length dividing the z terms:

$$\mathbf{x} = [x_1^c, y_1^c, z_1^c/f, ..., x_4^c, y_4^c, z_4^c/f]^T, \tag{3.21}$$

In order to solve the system efficiently, the solution remains into the the null space, or kernel, of $\mathbf{M}$, expressed as:

$$\mathbf{x} = \sum_{i=1}^{N} \beta_i \mathbf{v}_i \tag{3.22}$$

where $\mathbf{v}_i$ are the right eigenvectors of $\mathbf{M}$ corresponding to the N null eigenvalues of $\mathbf{M}$. The efficiency of UPnP remains in the transformation of $\mathbf{M}$ into a small constant matrix $\mathbf{M^T M}$ of size 12x12 before computing its eigenvectors.

From 3.22, the solution remains finding the $\beta's$ values in this case for $N = 1, ..., 3$ while distance constraints are introduced

$$\|\mathbf{c}_i^c - \mathbf{c}_j^c\|^2 = d_{ij}^2, \tag{3.23}$$

where d_{ij}^2 is the Euclidean distance between both control points.

As an example and for simplicity, will be shown how is solved for the case $N = 1$, where only it is needed the value of β_1 and f. Applying the six distance constraints from 3.23 the following linear system is constructed

$$\mathbf{Lb} = \mathbf{d}, \tag{3.24}$$

where $\mathbf{b} = [\beta_{11}, \beta_{ff11}]^T = [\beta_1^2, f^2\beta_1^2]^T$, and $\mathbf{L}$ is a 6x2 matrix made from the known elements of the first eigenvector column $\mathbf{v_1}$, and $\mathbf{d}$ is a 6-vector with the squared distances between the control points. Lastly, the solution will be given using the least squares approach to estimate the values of β_1 and f by substitution:

$$\beta_1 = \sqrt{\beta_{11}}, \qquad f = \sqrt{\mid \beta_{ff11} \mid} / \mid \beta_1 \mid, \tag{3.25}$$

Once with the betas computed, the camera pose is estimated doing the inverse process. Firstly, computing the control points coordinates in the camera frame reference. Secondly, computing the coordinates of all 3D points in camera frame reference. Finally, as shown in [35], doing the extraction of the rotation matrix $\mathbf{R}$ and the translation vector $\mathbf{t}$.

3.4 Robust Estimation

Previously we mentioned the possibility to have noisy measurements and how this would affect the pose estimation algorithms. Robust estimation is a methodology to compute the camera pose removing as much as possible the noisy data introduced by gross errors. There are two popular methods to solve that problem, the RANSAC algorithm and M-estimators. Despite of the effectiveness of both algorithms, in this project we will only focus on RANSAC that minimizes what is called the reprojection error.

3.4.1 Non-Linear Reprojection Error

In section 3.3.1 we talked about the sensitivity in front of the noise and the lack precision of measurements $\mathbf{m_i}$. A proposed methodology to improve results is refining the estimated camera pose by doing a minimization of the sum of the reprojection errors, which it is the accumulated squared distance between the projection of a 3D point $\mathbf{M}_i$ and its measured 2D coordinates. It can therefore be written by

$$[\mathbf{R} \mid \mathbf{t}] = \min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} dist^2(\mathbf{P}\tilde{\mathbf{M}}_i, \mathbf{m}_i), \tag{3.26}$$

which can be assumed as optimal due to the fact that each measurement is independent and Gaussian. This minimization must be done into an iterative optimization scheme which usually requires an initial estimation of the camera pose.

3.4.2 RANSAC

The Random Sample Consensus or RANSAC [20] is a non-deterministic iterative method which estimates parameters of a mathematical model from observed data producing an approximate result as the number of iterations increase. (See fig. 3.6)

In the context of camera pose estimation, it is very simple to implement since an initial guess of the parameters is not needed. From the set of correspondences, the algorithm

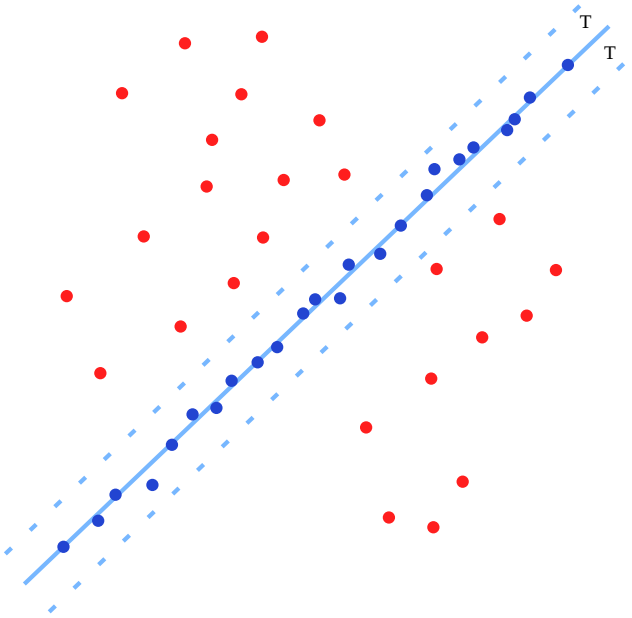


FIGURE 3.6: RANSAC applied to fit a simple line model

randomly extracts small subsets of points to generate what is called the hypothesis. For each hypothesis a PnP approach is used to recover a camera pose which then is used to compute the reprojection error. Those points which its reprojection is close enough to their 2D points are called inliers.

RANSAC depends on some parameters such as the tolerance error, which decides whether a point will be considered an inlier based on the reprojection error. In [20], it is proposed a formula to compute the number of iterations the algorithm should do given a desired probability p that at least one of the hypothesis succeed as a consistent solution. The mentioned formula is the following:

$$k = \frac{\log(1-p)}{\log(1-w^n)}, \tag{3.27}$$

where w is the ratio between inliers and the number of points. The value k tends to increase as the size of the subsets does.

3.5 Bayesian Tracking

Tracking algorithms, are useful in order to estimate the density of successive state $\mathbf{s}_t$ in the space of possible camera poses. Depending on the model used, the $\mathbf{s}_t$ state vectors include the rotation and translation parameters and often the additional parameters such as the translation and angular velocities. Bayesian trackers can be reformulated as

what is called the propagation rule (Equation 3.28) which is a recursive equation that relates over time t, the current with the previous density function of a process

$$p(\mathbf{s}_t \mid \mathbf{z}_{t-1} \dots \mathbf{z}_0) = \int_{\mathbf{s}_{t-1}} p(\mathbf{s}_t \mid \mathbf{s}_{t-1}) p_{t-1}(\mathbf{s}_{t-1}), \tag{3.28}$$

where $\int_{\mathbf{s}_{t-1}}$ is the integration over the set of possible values for the previous state $\mathbf{s}_{t-1}$. The term $p(\mathbf{s}_t \mid \mathbf{z}_{t-1} \dots \mathbf{z}_0)$ can be interpreted as a prediction on $p_t(\mathbf{s}_t)$ made by applying the motion model on the previous density state $p_{t-1}(\mathbf{s}_{t-1}$. The name of "Bayesian tracking" comes from the fact that Equation 3.28 is an equivalent reformulation of Baye's rule for the discrete time varying case.

Many tracking systems ignore the probability density function and retain only one single hypothesis for the camera pose, usually the maximum-likelihood. Nevertheless, the Bayesian formulation it is useful to make robust the tracking algorithms in front of bad estimations. The most common Bayesian algorithms are Particle Filters and Kalman Filters, even though in this project we will focus on the second one.

3.5.1 Kalman Filter

The Kalman Filter is a recursive method for estimate the state of a process that can be applied in many areas, however, the purpose will be the application to 3D tracking. In the literature we can find two main formulations of the problem, the Linear and non-Linear cases, that using one or the other will depend on the complexity of the problem and the expected accuracy. Nevertheless, in this project we will only focus on the simple case, the Linear Kalman Filter.

The successive states $\mathbf{s}_t \in R^n$ of a discrete-time controlled process are assumed to envolve according to a dynamics model written as follows

$$\mathbf{s}_t = \mathbf{A}\mathbf{s}_{t-1} + \mathbf{w}_t, \tag{3.29}$$

where A is called the *state transition matrix*, and $\mathbf{w}_t$ represents the process noise which is assumed to be normally distributed with zero mean. For tracking purposes, the state vector will be comprised by the 6 parameters of the camera pose, plus the translational and angular velocities.

The measurements $\mathbf{z}_t$ such as the the camera pose at time t, are assumed to be related to the state $\mathbf{s}_t$ by a linear measurement model

$$\mathbf{z}_t = \mathbf{C}\mathbf{s}_t + \mathbf{v}_t, \tag{3.30}$$

where $\mathbf{v}_t$ represents the measurement noise.

At each time step, the Kalman Filter makes a fist estimation of the current state called the *a priori* state estimate $\mathbf{s}_t^-$, which is refined by incorporating the measurements to yield the *a posteriori* estimate $\mathbf{s}_t$. $\mathbf{s}_t^-$ and its covariance matrix $\mathbf{S}_t^-$, are computed during the prediction stage and can be written as

$$\mathbf{s}_t^- = \mathbf{A}\mathbf{s}_{t-1}, \tag{3.31}$$

$$\mathbf{S}_t^- = \mathbf{A}\mathbf{S}_{t-1}\mathbf{A}^T + \mathbf{\Lambda}_\mathbf{w}, \tag{3.32}$$

where $\mathbf{S}_{t-1}$ is the *a posteriori* estimate error covariance for the previous time step, and $\mathbf{\Lambda}_\mathbf{w}$ is the process covariance noise that measures quality of the motion model respect to the reality. Next, the Kalman Filter does a "measurement update" or correction. The *a posteriori* state estimate $\mathbf{s}_t$ and its covariance matrix $\mathbf{S}_t$ are now generated by adding the measurements $\mathbf{z}_t$

$$\mathbf{s}_t = \mathbf{s}_t^- + \mathbf{G}_t(\mathbf{z}_t - \mathbf{G}\mathbf{s}_t^-), \tag{3.33}$$

$$\mathbf{S}_t = \mathbf{S}_t^- - \mathbf{G}_t\mathbf{C}\mathbf{S}_t^-, \tag{3.34}$$

where the Kalman gain $\mathbf{G}_t$ is computed as

$$\mathbf{G}_t = \mathbf{S}_t^-\mathbf{C}^T(\mathbf{C}\mathbf{S}_t^-\mathbf{C}^T + \mathbf{\Lambda}_\mathbf{v})^{-1}, \tag{3.35}$$

with $\mathbf{\Lambda}_\mathbf{v}$ being the measurements covariance matrix.

In the context of 3D tracking, the *a priori* state estimate $\mathbf{s}_t^-$ can be used to predict the camera extrinsic parameters, therefore, the predicted measurement vector $\mathbf{z}_t^-$ is the following

$$\mathbf{z}_t^- = \mathbf{C}\mathbf{s}_t^-, \tag{3.36}$$

The uncertainty on the prediction is represented by the covariance matrix $\mathbf{\Lambda_z}$ estimated by propagating the uncertainty

$$\mathbf{\Lambda_z} = \mathbf{C}\mathbf{S}_t^{-}\mathbf{C}^T + \mathbf{\Lambda_v}, \tag{3.37}$$

Chapter 4

Software Architecture

The aim of this chapter is to explain the algorithm implementation by using the well known modelling language Unified Modeling Language (UML) [36]. To do that, an activities diagram, an use of cases diagram and class diagrams are provided in order to understand the application structure.

4.1 Use Cases Design

The current section pretends to illustrate the use cases model by an use cases diagram, Figure 4.1, which represents all the application requirements including its internal and external influences. The uses cases are considered as a high level requirements to achieve the final task, in this case estimate an object pose. Each use case has its relationships and dependencies represented with arrows. From Fig. 4.1 we can see that the application will be composed by two main use cases: the model registration and the object detection, which at the same time will share other use cases.

In the next sections, each use case is explained in detail attached to some visual illustrations with the expected results.

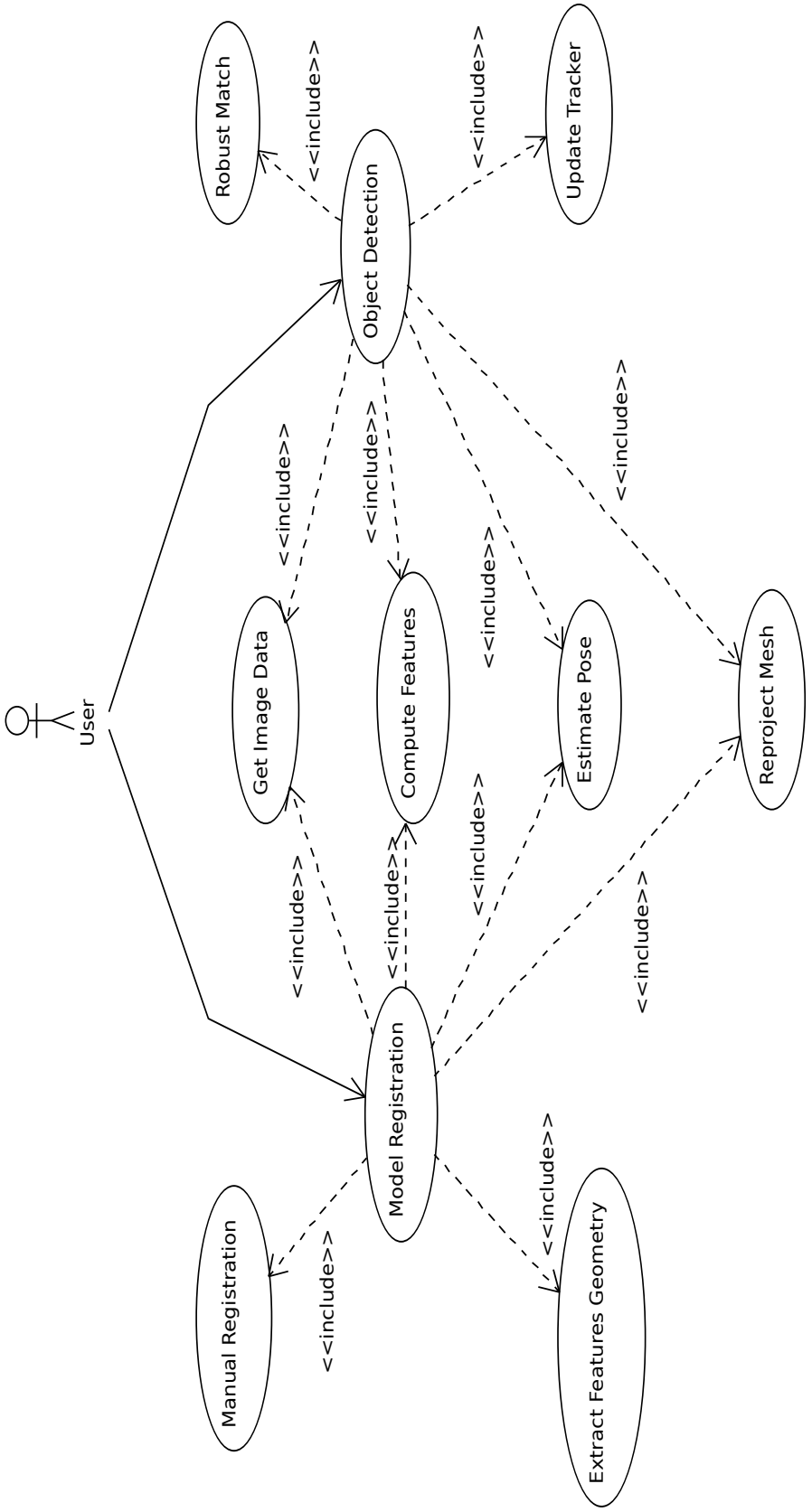


FIGURE 4.1: The use case diagram showing the different relations and dependencies between each use case.

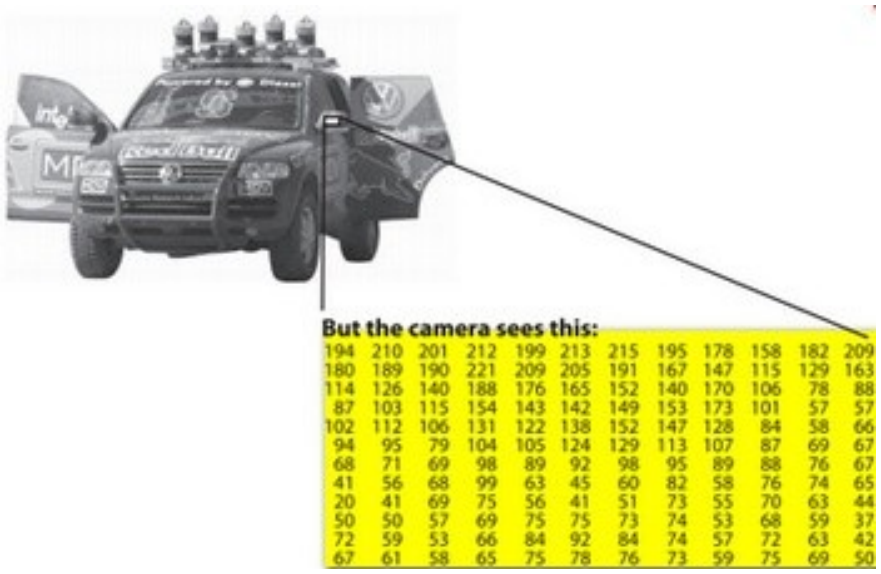


FIGURE 4.2: Screenshot of an image obtained from a standard camera and its matrix representation. Image extracted from OpenCV official documentation.

4.1.1 Capture Image Data

The aim of this use case is capture an image from a digital camera or a sequence of images and transform the information that we (humans) see into numerical values for each of the points of the image. In Figure 4.2 we can see that the mirror of the car is represented by a matrix containing all the intensity values of the pixel points.

4.1.2 Compute Features

In Chapter 3 was explained that for fulfil the equations, first is necessary to find the pairings between the 2D image and the 3D model. For that reason, is needed to detect what is called natural features from the image data. This natural features, or *Keypoints*, are very singular locations in the image plane usually obtained from the computation of the image gradients. In addition, for each found Keypoint a *local descriptor* is computed, which is a vector of a fixed size that depending on the technique used for the extraction will provide some information about the particular location where was found such as gradients orientation, brightness, etc. In Figure 4.3 we can see the result to apply a features detection algorithm to a simple picture where the *green* points are the found features.

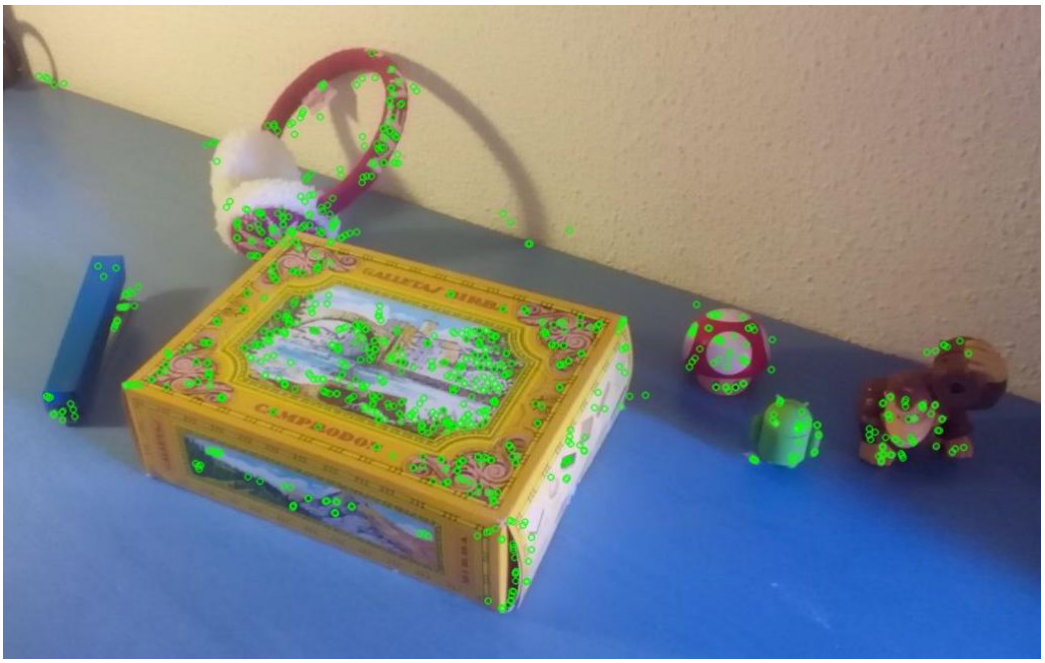


FIGURE 4.3: Screenshot with 2D features computation from a single image. In *green* the found Keypoints representing the interest points of this scene.

4.1.3 Model Registration

The model registration is an essential part to succeed in this algorithm. Since we need a model to recognize an specific object, the first step is its generation. For that reason, the registration must be done *offline* and previously to the detection stage (we will see in the Activities Diagram).

The model will be composed by a set of n 2D feature descriptors containing specific information about the object, which will be the base to discriminate between different objects. In addition, each descriptor will have an associated 3D coordinate respect to the object reference frame that will be used later to create the set of 2D-3D correspondences needed to recover the camera pose.

4.1.4 Manual Registration

In order to create the object model, it is needed a piece of software to compute its 2D features and for each found feature, its 3D position. For complex objects this will require a reconstruction algorithm using any *Structure From Motion* technique, however, it is not the focus of this project. For that reason and simplicity, a custom implementation has been developed for objects with planar surfaces which requires a 3D mesh and one or more perspective pictures of the object. The application loads the 3D mesh and requires

to provide the 2D positions of the vertices by clicking by hand using the mouse (See Figure 4.4).

FIGURE 4.4: Screenshots of the object registration process clicking the points by hand. In *red* the clicked points, in *green* the estimated points position.

4.1.5 Extract Features Geometry

Once the object vertices are defined, we will have the sufficient set of 2D-3D correspondences to apply the PnP in order to estimate the camera pose (See Section 4.1.7). The next step is to detect the 2D features (See Section 4.1.2) in order to find which of them lie onto the object surface.

In relation to the 3D coordinates extraction, *the Möller-Trumbore intersection* [37] algorithm has been applied which given a ray direction, computes the intersection point with a defined 3D plane. In Figures 4.5 and 4.6 we can see the visual result of the manual registration: in green, the obtained 2D features onto the object surface, which at the same time its 3D coordinates and descriptors were computed.

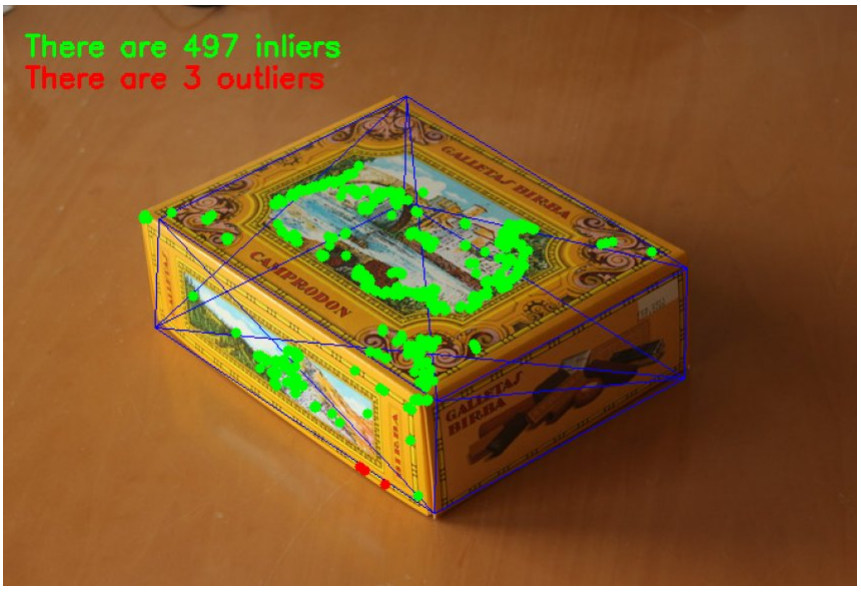


FIGURE 4.5: Texture extraction of a squared box without background features.

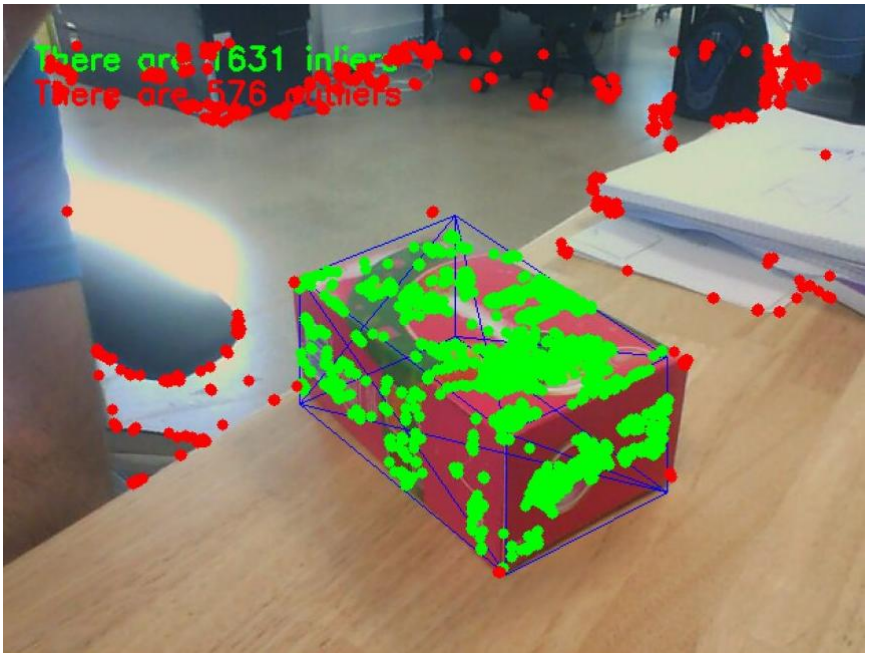


FIGURE 4.6: Texture extraction of a squared box with background features.

4.1.6 Robust Match

From the previous use case, a set of local descriptors is obtained which for each of them, a 2D position in the image plane is associated. However, in order to extract the pairings, the local descriptors containing its 3D position are needed. In Fig. 4.7, we can see an example on how a simple descriptors matching between two images looks like.

The most common technique to perform the descriptors matching with a high reliability is by brute force, which means that each descriptor of the set is compared to all the

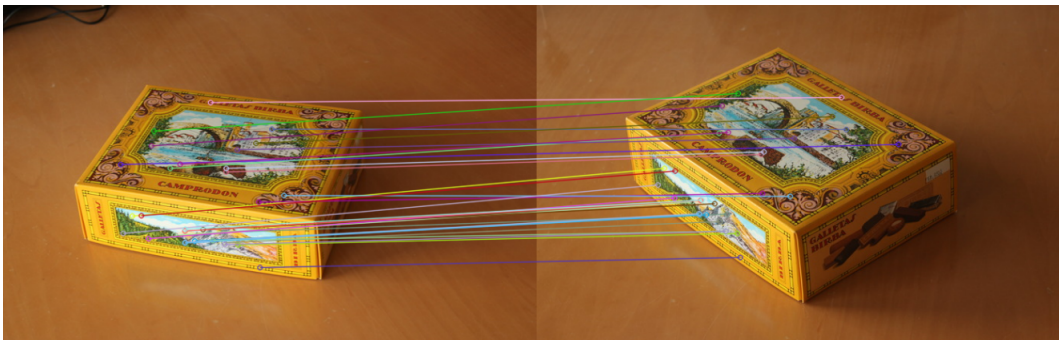


FIGURE 4.7: Screenshot with the result of a 2D features matching algorithm beetween two similar images.

descriptors in the other set. Nevertheless, in the literature we can find other searching methods such as the *Fast Library for Approximate Nearest Neighbors* (FLANN) [38], which it is a library that contains a collection of algorithms optimized for fast nearest neighbor search in large datasets and for high dimensional features that gives the closest metric d given two sets of features:

$$\{(P_A^i, P_B^j) : j = \min_k d(desc(P_A^i), desc(P_B^k))\}, \tag{4.1}$$

Moreover, sometimes the matching between local descriptors is not accurate and for that reason exist some techniques to refines the matching stage: the Symmetric distance or *Left-Right check*, and the *Nearest Neighbor Distance Ratio* or *ratio test* proposed in [39]. The first, keeps only mutual correspondences, which means that a match is kept if it is the same in the $A \rightarrow B$ and $B \rightarrow A$ order (See Eq. 4.2). The second, checks the distance between matches in order to remove repetitive elements if the ratio of a distance is inferior to a threshold δ the match is kept, otherwise it is rejected. δ is often chosen between 0.6 and 0.8 (See Eq. 4.3).

$$\{(P_A^i, P_B^j) : j = \min_k d(desc(P_A^i), desc(P_B^k)), i = \min_k d(desc(P_A^i), desc(P_B^k))\}, \tag{4.2}$$

$$\{(P_A^i, P_B^j) : j = \min_k d(desc(P_A^i), desc(P_B^k)) < \delta \min_{k \neq j} d(desc(P_A^i), desc(P_B^k))\}, \tag{4.3}$$

4.1.7 Estimate Pose

After the matches filtering we have to subtract the 2D and 3D correspondences from the found scene keypoints and our 3D model using an obtained matches list. In the example code 16 is shown in brief how to extract the 2D-3D correspondences which will be later used to recover the camera pose.

```
vector<Point3f> points3d;
vector<Point2f> points2d;

for(size_t match_idx = 0; match_idx < matches.size(); ++match_idx)
{
    // 3D point from model
    Point3f point3d =
        points3d_model[ matches[match_idx].trainIdx ];
    // 2D point from the scene
    Point2f point2d_scene =
        keypoints_scene[ matches[match_idx].queryIdx ].pt;

    points3d.push_back(point3d_model);     // add 3D point
    points2d.push_back(point2d_scene);     // add 2D point
}
```

Listing 4.1: Snippet for correspondences extraction.

4.1.8 Update Tracker

It is common in computer vision or robotics to use Bayesian tracking algorithms for results improvements. In this project a Linear Kalman Filter has been applied in order to keep tracking the object pose in those cases whether the number of inliers is lower than a given threshold. The defined state vector is the following:

$$X = \left(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z}, \psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}\right) \tag{4.4}$$

where the X vector contains the positional data (x, y, z) with its first and second derivatives(velocity and acceleration), the orientation data in Euler angles representation (ψ, θ, ϕ) with its first and second derivatives(velocity and acceleration). Then, the dynamic (Eq. 4.5) and measurement (Eq. 4.6) models are the followings:

$$\begin{pmatrix} x_k \\ y_k \\ z_k \\ \dot{x}_k \\ \dot{y}_k \\ \dot{z}_k \\ \ddot{x}_k \\ \ddot{y}_k \\ \ddot{z}_k \\ \psi_k \\ \theta_k \\ \phi_k \\ \dot{\psi}_k \\ \dot{\theta}_k \\ \dot{\phi}_k \\ \ddot{\psi}_k \\ \ddot{\theta}_k \\ \ddot{\phi}_k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{1}{2}(\Delta t)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{1}{2}(\Delta t)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{1}{2}(\Delta t)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{1}{2}(\Delta t)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{1}{2}(\Delta t)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{1}{2}(\Delta t)^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{k-1} \\ y_{k-1} \\ z_{k-1} \\ \dot{x}_{k-1} \\ \dot{y}_{k-1} \\ \dot{z}_{k-1} \\ \ddot{x}_{k-1} \\ \ddot{y}_{k-1} \\ \ddot{z}_{k-1} \\ \psi_{k-1} \\ \theta_{k-1} \\ \phi_{k-1} \\ \dot{\psi}_{k-1} \\ \dot{\theta}_{k-1} \\ \dot{\phi}_{k-1} \\ \ddot{\psi}_{k-1} \\ \ddot{\theta}_{k-1} \\ \ddot{\phi}_{k-1} \end{pmatrix}, \tag{4.5}$$

$$\begin{pmatrix} x_k \\ y_k \\ z_k \\ \psi_k \\ \theta_k \\ \phi_k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{k-1} \\ y_{k-1} \\ z_{k-1} \\ \psi_{k-1} \\ \theta_{k-1} \\ \phi_{k-1} \end{pmatrix}, \tag{4.6}$$

4.1.9 Reproject Mesh

Once the camera pose is known, with a small Augmented Reality application we can visualize the obtained results. In this case, applying the Projective Projection Model formula (Eq. 3.1) and knowing the 3D coordinates of the object, the mesh is backprojected into the image plane. In Figures 4.8 and 4.9 we can appreciate that the camera pose is well estimated since the object mesh (in *green*) fits correctly with the reality.

4.2 Activities Diagram

In order to visualize the application flow, an activities diagram has been designed. In the diagram (Figure 4.10), it is possible to appreciate the two main parts of the application: the *training* and *detection*. On the top, the training stage referred as model registration, which must be done *offline* and for simplicity is represented by its main process. On the bottom, the detection stage, which is represented with a closed loop behaviour and where it is possible to appreciate all its use cases.

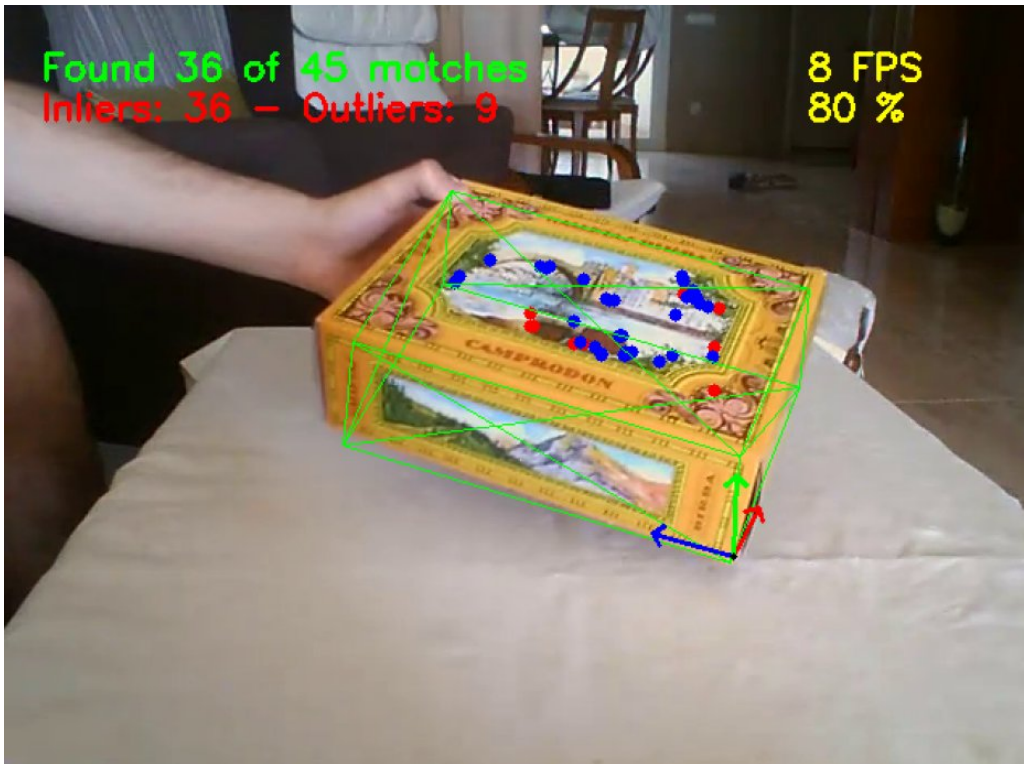


FIGURE 4.8: Pose estimation and mesh reprojection

4.3 Algorithm Implementation

In this section we will present the application implementation. Firstly, with a classes diagram is shown the big picture of the software structure. Secondly a brief introduction about OpenCV. Finally, a detailed explanation about each module and how OpenCV is integrated into the application.

4.3.1 Classes Diagram

In Figure 4.11 we can see that the application has been split in four main modules: the Core, the Input/Output, the Visualization and the Tracking. Each module is provided by several classes which are used by the two main programs (the training and the detection) in order to fulfil all the use cases explained in Section 4.1.

4.3.2 OpenCV

OpenCV (Open Source Computer Vision Library: http://opencv.org) is an open-source BSD-licensed library that includes several hundreds of computer vision algorithms. OpenCV is released under a BSD license and hence it's free for both academic and

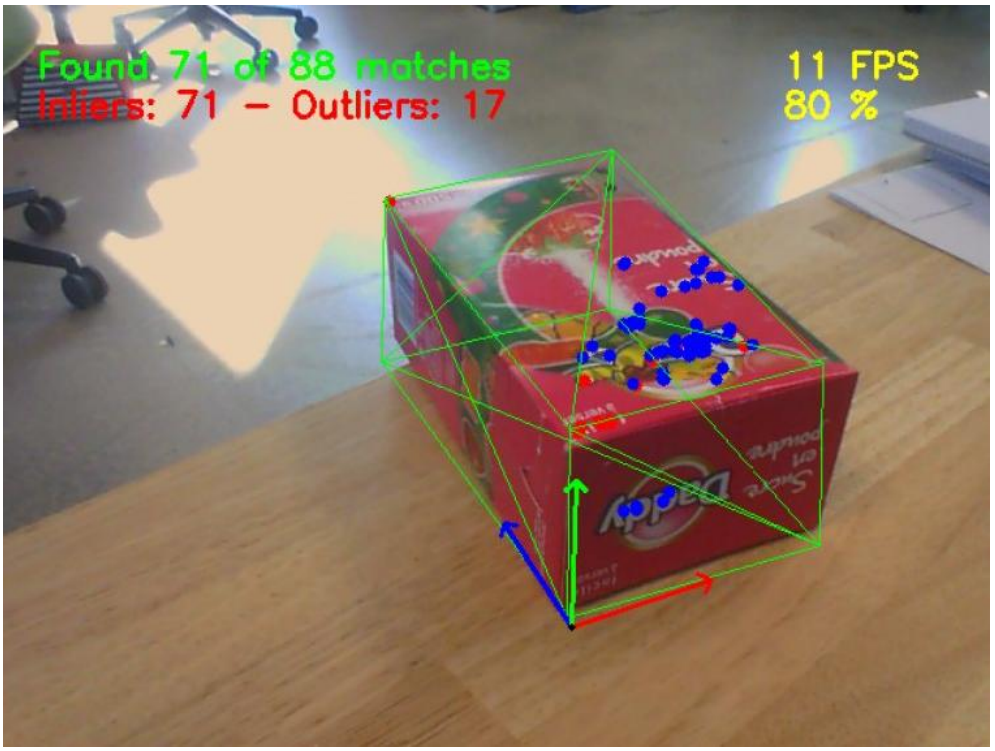


FIGURE 4.9: Pose estimation and mesh reprojection

commercial use. It has C++, C, Python and Java interfaces and supports Windows, Linux, Mac OS, iOS and Android. OpenCV was designed for computational efficiency and with a strong focus on real-time applications. Written in optimized C/C++, the library can take advantage of multi-core processing. Enabled with OpenCL, it can take advantage of the hardware acceleration of the underlying heterogeneous compute platform.

OpenCV provides a module for Camera Calibration and 3D Reconstruction which has several functions for basic multiple-view geometry algorithms, single and stereo camera calibration, object pose estimation, stereo correspondence algorithms, and elements of 3D reconstruction. During the next section, we will see which module are used in order to cover the application requirements.

4.3.3 The Input/Output module

The Input/Output module is devoted to load and save the obtained information such as the object mesh and the generated model files. This module contains the *CsvReader* and *CsvWritter* classes which as their name indicate, read and write files in the CSV format. In Figure 4.12 we can see the structure and the main methods that composes this module. The *CsvWritter* is used to create the generated model file given a list of 2D-3D points and its descriptors. The *CsvReader* is devoted to load the 3D objects

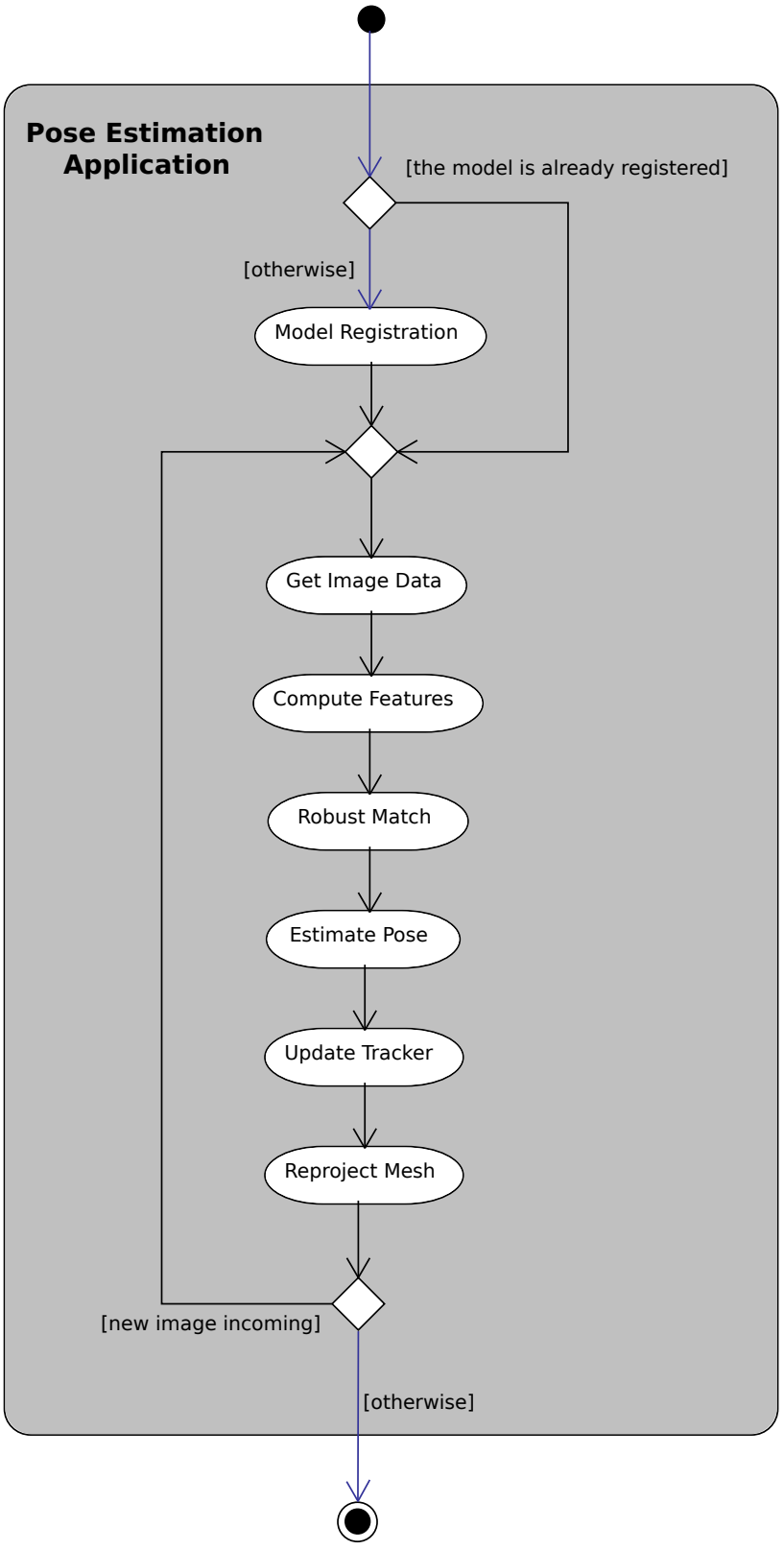


FIGURE 4.10: The Activities Diagram showing the flow chart of the complete application including the traning and detection processes.

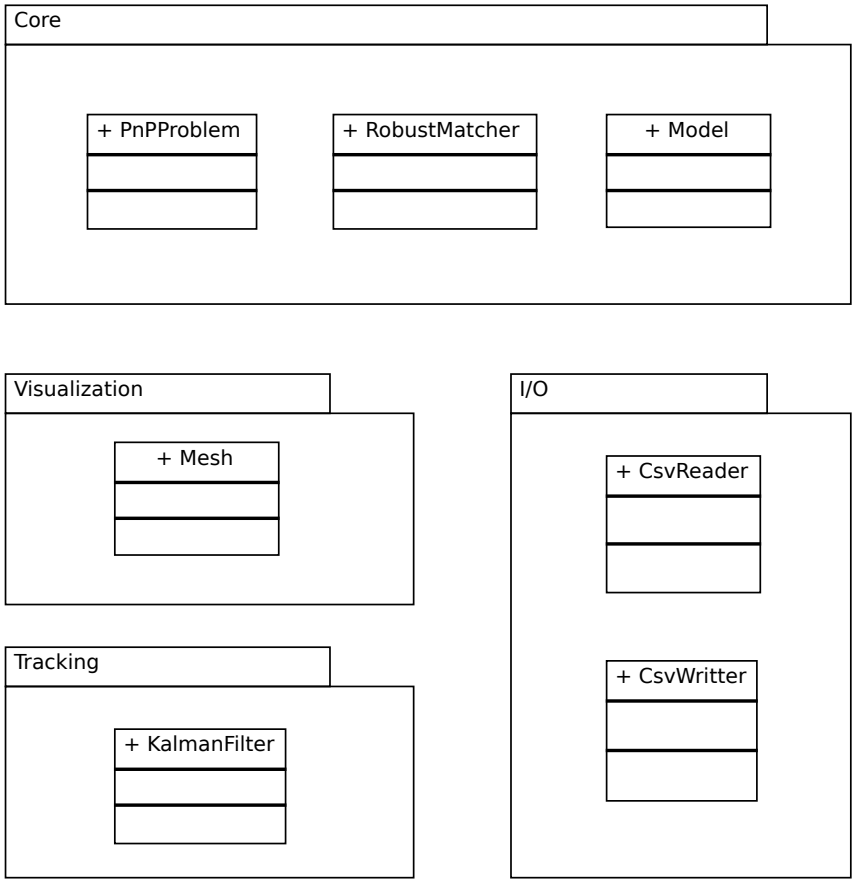


FIGURE 4.11: The classes diagram showing the application structure and its modules.

models from a *PLY* format file returning the vertices and the triangles that compose the mesh.

4.3.4 The Visualization module

The visualization module is devoted to manage the object mesh. It is composed by a single class which stores the loaded mesh. In Figure 4.13 we can see the *Mesh* class diagram which is defined by a set of points and indices that relates the mesh triangles. This class has a main method to load the data from a given file, which is internally done by an instantiation of the *CsvReader* class.

4.3.5 The Core module

The core module is composed by the *Model*, the *RobustMatcher* and the *PnPProblem* classes. The *Model* class represents the generated model in the registration process. In Figure 4.14 we can see the class diagram which show that is composed by a list of 2D and

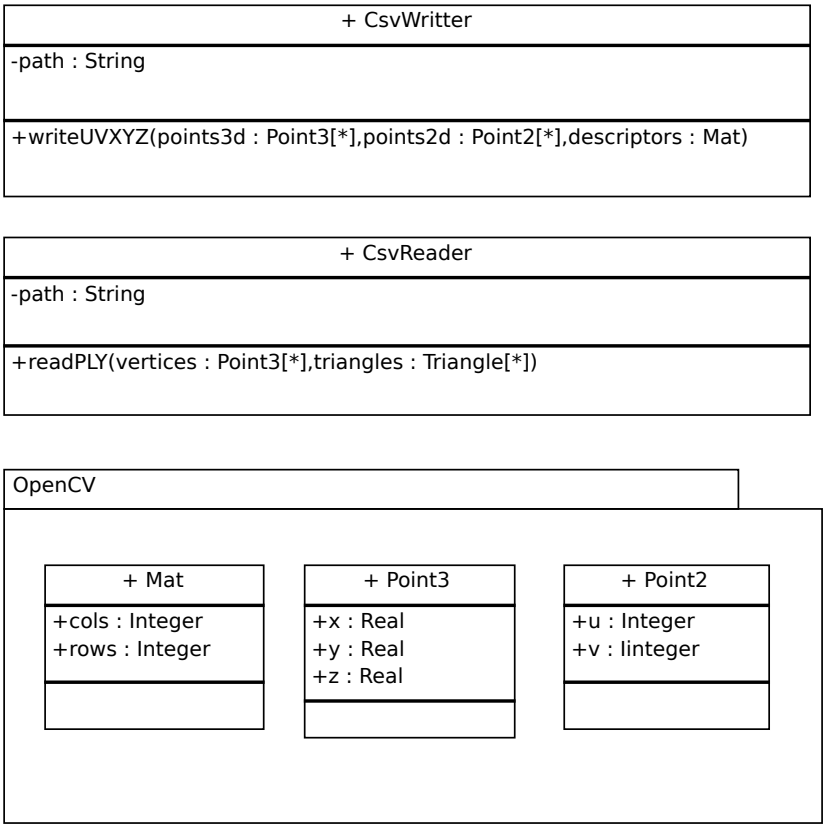


FIGURE 4.12: The classes diagram describing the I/O module.

3D points, a list of descriptors embedded into an OpenCV matrix, and for visualization the Keypoints information are also stored.

The *RobustMatcher* class, described in Figure 4.15, is devoted to detect and compute the keypoints and descriptors as well as to perform the descriptors matching. *OpenCV* provides a large set of functionalities in the *features2d. 2D Features Framework* module to detect and compute the points of interest and its descriptors given an image. In brief, we can find most of the common descriptors such as SIFT [39], SURF [40], AKAZE [41], KAZE [42], ORB [43] or FAST [44]. For this project ORB[43] has been used since is based on FAST [44] to detect the keypoints and BRIEF [45] to extract the descriptors which means that is fast in terms of speed and robust to rotations. For the matching use case, *OpenCV* provides some interfaces for searching methods in the *features2d. 2D Features Framework* module. Since, in this project we are using the binary ORB[43] descriptors, the authors recommend to use a FlannBasedMatcher [38] with an index created by a *Multi-Probe LSH: Efficient Indexing for High-Dimensional Similarity Search* [46] in order to speed up the computation time.

The *PnPProblem* class is devoted to compute the camera pose. In Figure 4.16 is shown its class diagram where we can easily see that it contains three main methods. Using the

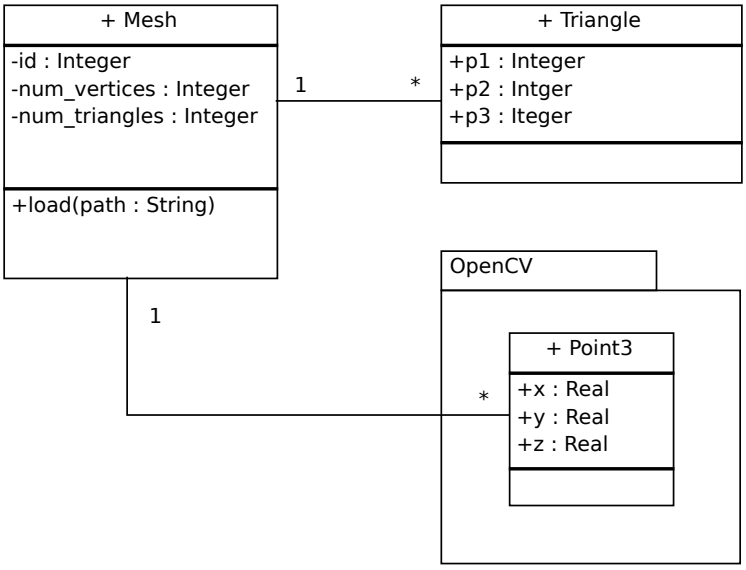


FIGURE 4.13: The classes diagram describing the Mesh class.

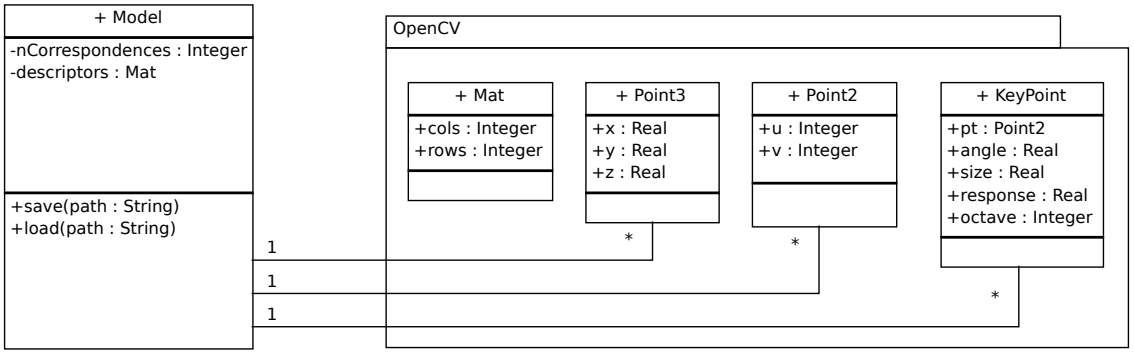


FIGURE 4.14: The classes diagram describing the Model class.

estimatePose() and *estimatePoseRansac()* with the found correspondences, internally calls the *cv::solvePnP()* and *cv::solvePnPRansac()* from the *calib3d. Camera Calibration and 3D Reconstruction* module in order to estimate the camera pose. For the model registration is only needed the *cv::solvePnP()* since we manually set the set correspondences. However, for the object detection we might use *cv::solvePnPRansac()* due to the fact that after the matching not all the found correspondences are reliable and, as like as not, there are outliers which will be removed by the RANSAC algorithm embed into this function. The *solvePnPRansac* which has as inputs the camera calibration parameters, the distortion coefficients and the list of 2D-3D pairings, will return as outputs the translation vector and the rotation vector in its exponential map representation, which later we can be used the *Rodrigues* formula [47] in order to transform it into a rotation matrix. Additionally, the function returns a list with the inlier points used to compute the camera pose.

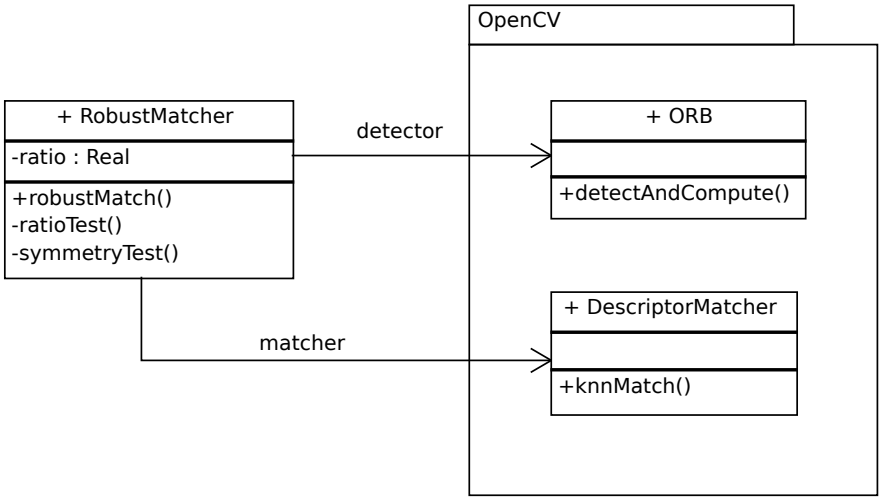


FIGURE 4.15: The classes diagram describing the RobustMatcher class.

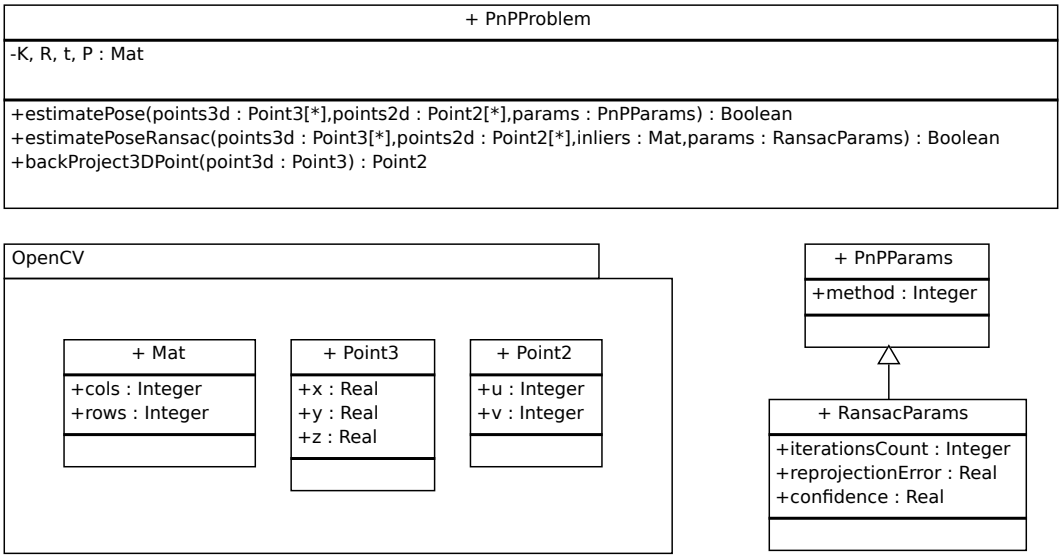


FIGURE 4.16: The classes diagram describing the PnPProblem class.

4.3.6 The Tracking module

The present module, used in the detection process, is composed by the *KalmanFilter* class which is a custom interface to the OpenCV Kalman Filter. Shown in Figure 4.17, it is seen that there is a public method which predicts the pose using the defined dynamic model given a measurement. This method it is only called when the number of found inliers is higher of a given threshold *minInliersKalman*.

OpenCV provides a complete implementation of the Kalman Filter in the *video. Video Analysis* module. The current implementation allows to the define the dynamic and

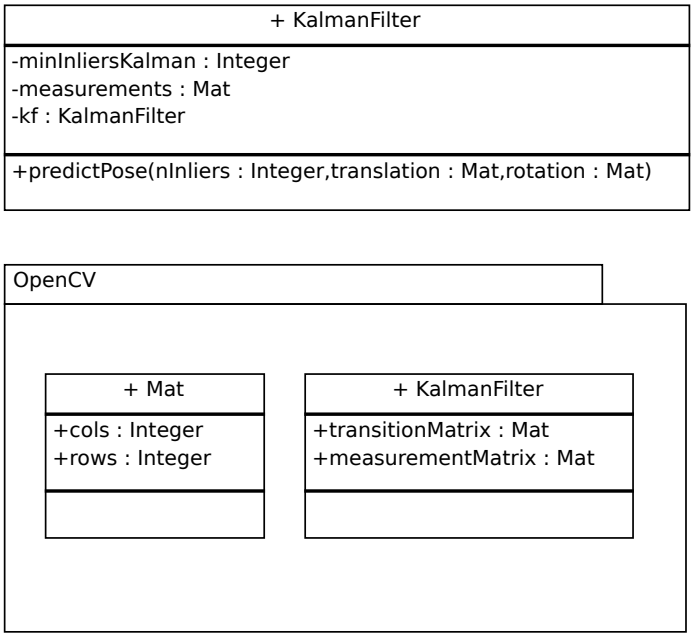


FIGURE 4.17: The classes diagram describing the KalmanFilter class.

measurement matrices each time, which it means that is possible to define more complex models such as an Extended Kalman Filter.

Chapter 5

Results and Contributions

In this chapter we will see the obtained results and contributions: an application for objects pose estimation, the creation of an OpenCV tutorial and finally, the implementation and inclusion of the UPnP approach into the OpenCV libraries under the scope of the *OpenSource* community.

5.1 Pose Estimation Application

The first contribution of this project is an application for pose estimation written in *C++* which is divided by two main modules: the model registration and the object detection.

The first, registers an object extracting its 2D features and computes the 3D coordinates of each feature given a 3D model of the object in *PLY* format. This module writes in a custom made format the result of the object registration in order to be loaded by the detection module. The detection module, loads a given object model generated by the registration module and from a given a image sequence detects and tracks the object reprojecting its mesh in order to visualise the obtained results.

The complete application has been included in the OpenCV repositories as a complementary resource for the tutorial which explains step by step the complete application from the point of view of a Computer Vision developer.

5.2 OpenCV Tutorial

We created an tutorial addressed to those developers interested in creating a pose estimation application. The tutorial, attached in the end of this document (Appendix A), is focused in explain step by step the code implementation of each use of case defined in the previous sections. Moreover, the document is in the OpenCV tutorials standard format and has been included in the new OpenCV 3.0.0 documentation [48].

5.3 UPnP Implementation

An implementation in *C++* of the UPnP approach has been developed and included in OpenCV. The source code, based on the *Matlab* implementation from the authors, follows the same structure as the EPnP and can be used inside the *cv::solvePnP()* function from the *calib3d. Camera Calibration and 3D Reconstruction module.*

5.3.1 Method validation

In order to be included into the library, the algorithm passed an unit test in terms of accuracy in front of generated synthetic data. Explicitly, the method must guarantee a rotation and translation error lower than $10e^{-3}$ after 1000 consecutive tests. For the data generation, was used the OpenCV testing framework which simulates the 3D-to 2D correspondences creating many set of points with different size uniformly distributed in the cube $[-1,1]$x$[-1,1]$x$[5,10]$, and projected onto a $[10^{-3},100]$x$[10^{-3},100]$ image using a virtual calibrated camera with non squared pixels (except for UPnP). For any given ground truth camera pose, $\mathbf{R}_{true}$ and $\mathbf{t}_{true}$ and corresponding estimates $\mathbf{R}$ and $\mathbf{t}$, the relative rotation error was computed $E_{rot} = \|\mathbf{r}_{true} - \mathbf{r}\|/\|\mathbf{r}\|$ where $\mathbf{r}$ and $\mathbf{r}_{true}$ are the exponential vector representation of $\mathbf{R}$ and $\mathbf{R}_{true}$ respectively; the relative translation vector error was computed with $E_{trans} = \|\mathbf{t}_{true} - \mathbf{t}\|/\|\mathbf{t}\|$; and the error in the estimation of the focal length was determined by $E_f = \|f_{true} - f\|/f$. All the errors reported in this section correspond to the median values errors estimated over 100 experiments with random positions of the 3D points.

In the following Figures 5.1 and 5.2 are shown the obtained results from the accuracy test, comparing the rotation and translation errors between the different PnP methods provided by OpenCV: ITERATIVE, P3P, EPNP, DLS and UPNP. The ITERATIVE method, based on Levenberg-Marquardt optimization, which minimizes the reprojection error. The P3P method, based on the paper of X.S. Gao [5], which in this case is required exactly four object and image points. The EPNP method, based on the approach

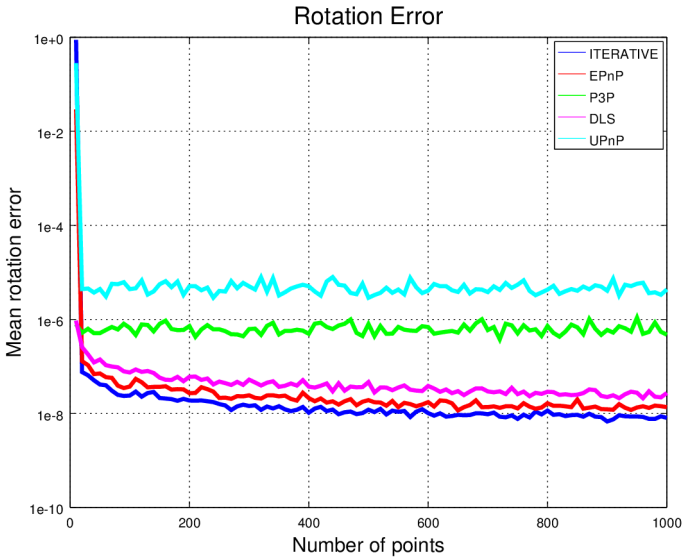


FIGURE 5.1: Rotation errors from synthetic data for non-planar distributions of points by increasing the number of 2D-3D correspondences

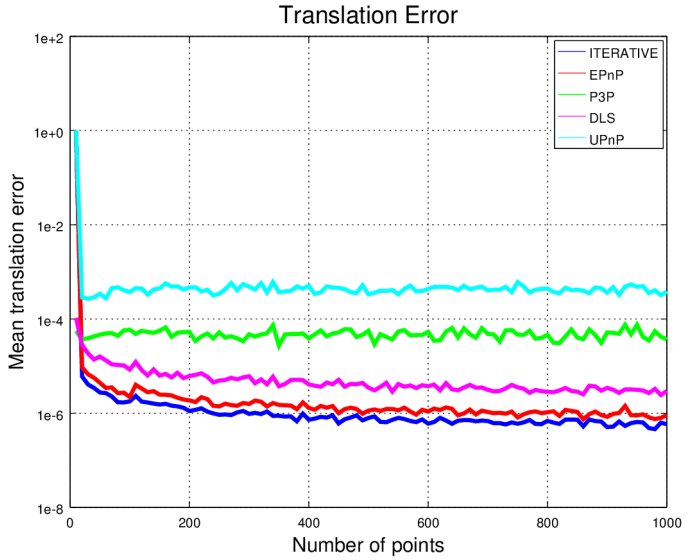


FIGURE 5.2: Translation errors from synthetic data for non-planar distributions of points by increasing the number of 2D-3D correspondences

presented by F.Moreno-Noguer *et al's* [11]. The DLS method, based based on the paper of Joel A. *et al's* [49]. And the UPNP method, based on the paper of A.Penate-Sanchez *et al's* [1]. In addition, in Figure 5.4 it is shown the comparison of the computation obtained for each method. Finally, in Figure 5.3 it is shown the focal length error obtained form the UPnP method which we can appreciate that similar to the rotation and translation errors, is lower than the unit test requirement.

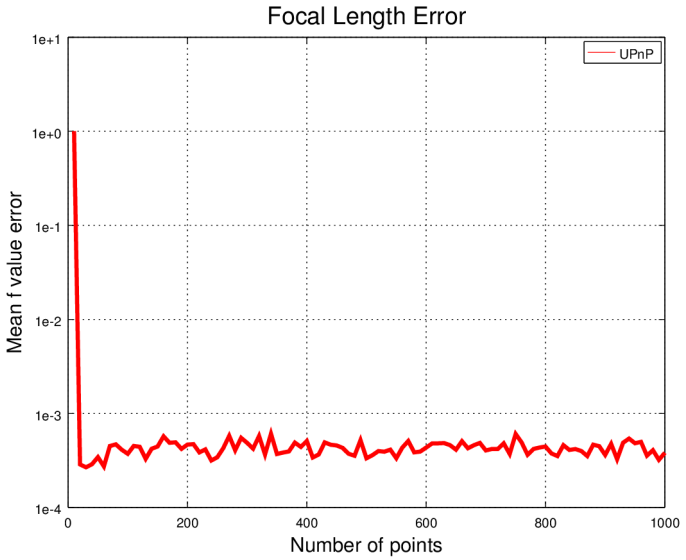


FIGURE 5.3: Focal Length error from synthetic data for non-planar distributions of points by increasing the number of 2D-3D correspondences

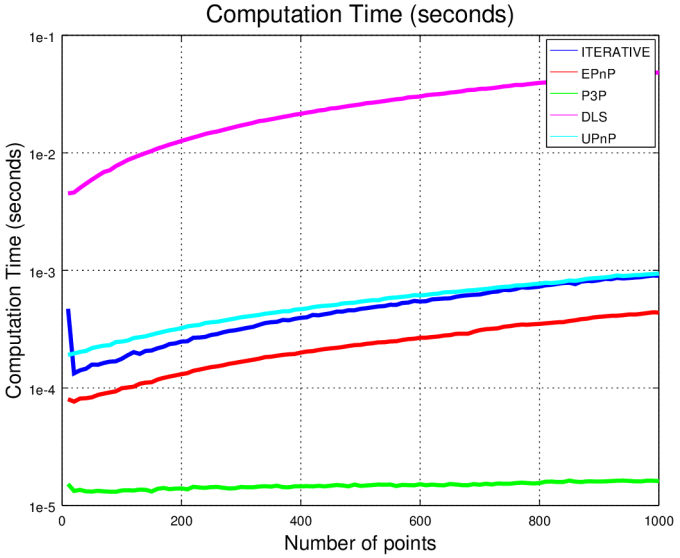


FIGURE 5.4: Computation Time from synthetic data for non-planar distributions of points by increasing the number of 2D-3D correspondences

Chapter 6

Future Work

In this chapter is explained a proposed future work after this project. Since the developed algorithm is a custom made implementation, the idea is make it more user-friendly and integrate it into an object recognition framework.

Objects Recognition Kitchen

The Objects Recognition Kitchen (ORK) [50] is an Open Source project started in Willow Garage for object recognition purpose. The framework, designed to run simultaneously several object recognition techniques, takes care of all the non-aspects of the problem such as the database management, inputs/outputs handlings and robot/ROS integration. In addition, it is built on top of Ecto, which it is a lightweight hybrid C++/Python framework for organizing computations as directed acyclic graphs.

Since for objects recognition is needed the model registration, ORK provides a reconstruction module that allows to create 3D models of objects with a RGBD sensor using a calibration pattern. (See Figure 6.1)

Moreover, ORK also has several object recognition pipelines such as the LINE-MOD [51], which it is one of the best methods in the literature for generic rigid object recognition due to its fast template matching; the TableTop, which does a segmentation in order to find a dominant plane in the point cloud based on analysis of 3D normal vectors, and therefore doing a point cloud clustering followed by an iterative fitting technique finds an object from the database; the Recognition for Transparent Objects that can detect and estimate poses of transparent objects given a point cloud model; and finally, the Textured Object Detection (TOD) which it is based on a standard bag of features technique and the algorithm for detection, it is the same as the presented in this project.

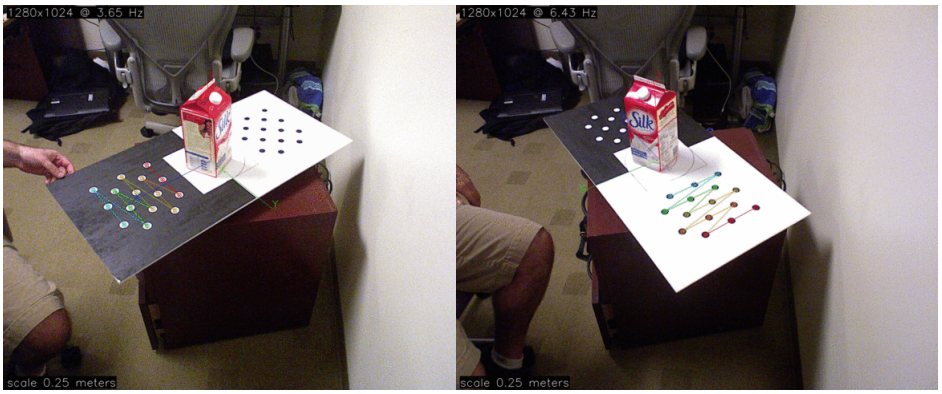


FIGURE 6.1: View of the model registration process

The current TOD implementation assumes that the detection stage is done with a RGBD input and the descriptors are checked with the nearest neighbors (descriptor-wise) for analogous 3D configuration followed by a 3D to 3D comparison in order to recover the camera pose. Hence, the challenge here is to integrate in this framework the PnP algorithm using as an input a single monocular camera in the detection stage. Currently it is Work in Progress.

Appendix A

OpenCV tutorial

Real Time pose estimation of a textured object

Nowadays, augmented reality is one of the top research topic in computer vision and robotics fields. The most elemental problem in augmented reality is the estimation of the camera pose respect of an object in the case of computer vision area to do later some 3D rendering or in the case of robotics obtain an object pose in order to grasp it and do some manipulation. However, this is not a trivial problem to solve due to the fact that the most common issue in image processing is the computational cost of applying a lot of algorithms or mathematical operations for solving a problem which is basic and immediateley for humans.

Goal

In this tutorial is explained how to build a real time application to estimate the camera pose in order to track a textured object with six degrees of freedom given a 2D image and its 3D textured model.

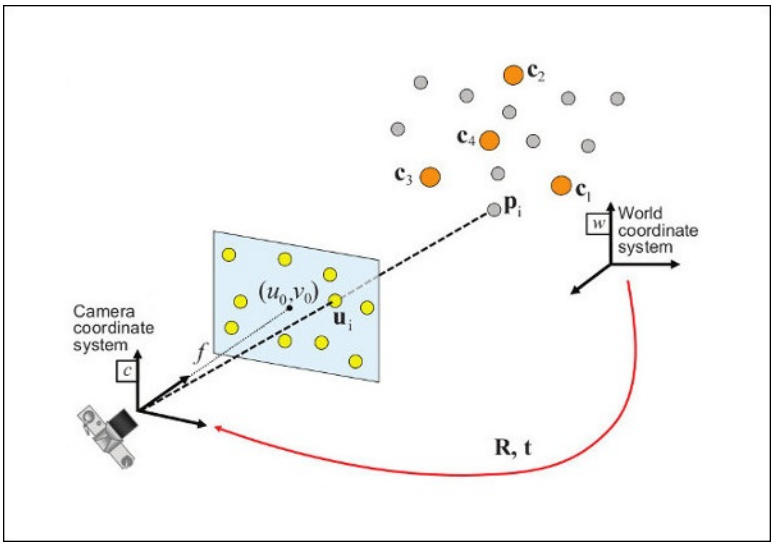
The application will have the followings parts:

- Read 3D textured object model and object mesh.
- Take input from Camera or Video.
- Extract ORB features and descriptors from the scene.
- Match scene descriptors with model descriptors using Flann matcher.
- Pose estimation using PnP + Ransac.
- Linear Kalman Filter for bad poses rejection.

Theory

In computer vision estimate the camera pose from *n* 3D-to-2D point correspondences is a fundamental and well understood problem. The most general version of the problem requires estimating the six degrees of freedom of the pose and five calibration parameters: focal length, principal point, aspect ratio and skew. It could be established with a minimum of 6 correspondences, using the well known Direct Linear Transform (DLT) algorithm. There are, though, several simplifications to the problem which turn into an extensive list of different algorithms that improve the accuracy of the DLT.

The most common simplification is to assume known calibration parameters which is the so-called Perspective-*n*-Point problem:



Problem Formulation: Given a set of correspondences between 3D points p_i expressed in a world reference frame, and their 2D projections u_i onto the image, we seek to retrieve the pose (R and t) of the camera w.r.t. the world and the focal length f.

OpenCV provides four different approaches to solve the Perspective-*n*-Point problem which return R and t. Then, using the following formula it's possible to project 3D points into the image plane:

s\ \left [\begin{matrix} u \\ v \\ 1 \end{matrix} \right] = \left [\begin{matrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{matrix} \right] \left [\begin{matrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{matrix} \right] \left [\begin{matrix} X \\ Y \\ Z \\ 1 \end{matrix} \right]

The complete documentation of how to manage with this equations is in **Camera Calibration and 3D Reconstruction** .

Source code

You can find the source code of this tutorial in the `samples/cpp/tutorial_code/calib3d/real_time_pose_estimation/` folder of the OpenCV source library.

The tutorial consists of two main programs:

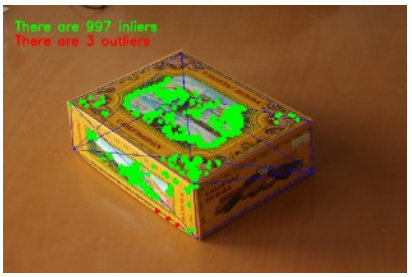
1. **Model registration**

   This applicaton is exclusive to whom don't have a 3D textured model of the object to be detected. You can use this program to create your own textured 3D

model. This program only works for planar objects, then if you want to model an object with complex shape you should use a sophisticated software to create it.

The application needs an input image of the object to be registered and its 3D mesh. We have also to provide the intrinsic parameters of the camera with which the input image was taken. All the files need to be specified using the absolute path or the relative one from your application's working directory. If none files are specified the program will try to open the provided default parameters.

The application starts up extracting the ORB features and descriptors from the input image and then uses the mesh along with the Möller–Trumbore intersection algorithm to compute the 3D coordinates of the found features. Finally, the 3D points and the descriptors are stored in different lists in a file with YAML format which each row is a different point. The technical background on how to store the files can be found in the **File Input and Output using XML and YAML files** tutorial.



2. **Model detection**

   The aim of this application is estimate in real time the object pose given its 3D textured model.

   The application starts up loading the 3D textured model in YAML file format with the same structure explained in the model registration program. From the scene, the ORB features and descriptors are detected and extracted. Then, is used **cv::FlannBasedMatcher** with **cv::flann::GenericIndex** to do the matching between the scene descriptors and the model descriptors. Using the found matches along with **cv::solvePnPRansac** function the `R` and `t` of the camera are computed. Finally, a KalmanFilter is applied in order to reject bad poses.

   In the case that you compiled OpenCV with the samples, you can find it in opencv/build/bin/cpp-tutorial-pnp_detection`. Then you can run the application and change some parameters:

```
This program shows how to detect an object given its 3D textured model. You can choose to use a recorded video or
       the webcam.
Usage:
  ./cpp-tutorial-pnp_detection -help
Keys:
  'esc' - to quit.
--------------------------------------------------------------------------

Usage: cpp-tutorial-pnp_detection [params]

  -c, --confidence (value:0.95)
      RANSAC confidence
  -e, --error (value:2.0)
      RANSAC reprojection errror
  -f, --fast (value:true)
      use of robust fast match
  -h, --help (value:true)
      print this message
  --in, --inliers (value:30)
      minimum inliers for Kalman update
  --it, --iterations (value:500)
      RANSAC maximum iterations count
  -k, --keypoints (value:2000)
      number of keypoints to detect
  --mesh
      path to ply mesh
  --method, --pnp (value:0)
      PnP method: (0) ITERATIVE - (1) EPNP - (2) P3P - (3) DLS
  --model
      path to yml model
  -r, --ratio (value:0.7)
      threshold for ratio test
  -v, --video
      path to recorded video
```

   For example, you can run the application changing the pnp method:

```
./cpp-tutorial-pnp_detection --method=2
```

Explanation

Here is explained in detail the code for the real time application:

1. **Read 3D textured object model and object mesh.**

   In order to load the textured model I implemented the *class* **Model** which has the function *load()* that opens a YAML file and take the stored 3D points with its corresponding descriptors. You can find an example of a 3D textured model in `samples/cpp/tutorial_code/calib3d/real_time_pose_estimation/Data/cookies_ORB.yml`.

```
/* Load a YAML file using OpenCV */
void Model::load(const std::string path)
{
    cv::Mat points3d_mat;
```

```
    cv::FileStorage storage(path, cv::FileStorage::READ);
    storage["points_3d"] >> points3d_mat;
    storage["descriptors"] >> descriptors_;

    points3d_mat.copyTo(list_points3d_in_);

    storage.release();

}
```

In the main program the model is loaded as follows:

```
Model model;                // instantiate Model object
model.load(yml_read_path); // load a 3D textured object model
```

In order to read the model mesh I implemented a *class* **Mesh** which has a function *load()* that opens a *.ply file and store the 3D points of the object and also the composed triangles. You can find an example of a model mesh in `samples/cpp/tutorial_code/calib3d/real_time_pose_estimation/Data/box.ply`.

```
/* Load a CSV with *.ply format */
void Mesh::load(const std::string path)
{

    // Create the reader
    CsvReader csvReader(path);

    // Clear previous data
    list_vertex_.clear();
    list_triangles_.clear();

    // Read from .ply file
    csvReader.readPLY(list_vertex_, list_triangles_);

    // Update mesh attributes
    num_vertexs_ = list_vertex_.size();
    num_triangles_ = list_triangles_.size();

}
```

In the main program the mesh is loaded as follows:

```
Mesh mesh;                // instantiate Mesh object
mesh.load(ply_read_path); // load an object mesh
```

You can also load different model and mesh:

```
./cpp-tutorial-pnp_detection --mesh=/absolute_path_to_your_mesh.ply --model=/absolute_path_to_your_model.yml
```

2. **Take input from Camera or Video**

   To detect is necessary capture video. It's done loading a recorded video by passing the absolute path where it is located in your machine. In order to test the application you can find a recorded video in `samples/cpp/tutorial_code/calib3d/real_time_pose_estimation/Data/box.mp4`.

```
cv::VideoCapture cap;               // instantiate VideoCapture
cap.open(video_read_path);          // open a recorded video

if(!cap.isOpened())                 // check if we succeeded
{
   std::cout << "Could not open the camera device" << std::endl;
   return -1;
}
```

   Then the algorithm is computed frame per frame:

```
cv::Mat frame, frame_vis;

while(cap.read(frame) && cv::waitKey(30) != 27)    // capture frame until ESC is pressed
{

    frame_vis = frame.clone();                     // refresh visualisation frame

    // MAIN ALGORITHM

}
```

   You can also load different recorded video:

```
./cpp-tutorial-pnp_detection --video=/absolute_path_to_your_video.mp4
```

3. **Extract ORB features and descriptors from the scene**

   The next step is to detect the scene features and extract it descriptors. For this task I implemented a *class* **RobustMatcher** which has a function for keypoints detection and features extraction. You can find it in `samples/cpp/tutorial_code/calib3d/real_time_pose_estimation/src/RobusMatcher.cpp`. In your *RobusMatch* object you can use any of the 2D features detectors of OpenCV. In this case I used **cv::ORB** features because is based on **cv::FAST** to detect the keypoints and **cv::xfeatures2d::BriefDescriptorExtractor** to extract the descriptors which means that is fast and robust to rotations. You can find more detailed information about *ORB* in the documentation.

   The following code is how to instantiate and set the features detector and the descriptors extractor:

```
RobustMatcher rmatcher;                                                    // instantiate RobustMatcher

cv::FeatureDetector * detector = new cv::OrbFeatureDetector(numKeyPoints);       // instatiate ORB feature
      detector
cv::DescriptorExtractor * extractor = new cv::OrbDescriptorExtractor();          // instatiate ORB descriptor
      extractor
```

```
rmatcher.setFeatureDetector(detector);                                  // set feature detector
rmatcher.setDescriptorExtractor(extractor);                             // set descriptor extractor
```

The features and descriptors will be computed by the *RobustMatcher* inside the matching function.

4. **Match scene descriptors with model descriptors using Flann matcher**

   It is the first step in our detection algorithm. The main idea is to match the scene descriptors with our model descriptors in order to know the 3D coordinates of the found features into the current scene.

   Firstly, we have to set which matcher we want to use. In this case is used **cv::FlannBasedMatcher** matcher which in terms of computational cost is faster than the **cv::BFMatcher** matcher as we increase the trained collectction of features. Then, for FlannBased matcher the index created is *Multi-Probe LSH: Efficient Indexing for High-Dimensional Similarity Search* due to *ORB* descriptors are binary.

   You can tune the *LSH* and search parameters to improve the matching efficiency:

```
cv::Ptr<cv::flann::IndexParams> indexParams = cv::makePtr<cv::flann::LshIndexParams>(6, 12, 1); // instantiate LSH
       index parameters
cv::Ptr<cv::flann::SearchParams> searchParams = cv::makePtr<cv::flann::SearchParams>(50);       // instantiate
       flann search parameters

cv::DescriptorMatcher * matcher = new cv::FlannBasedMatcher(indexParams, searchParams);         // instantiate
       FlannBased matcher
rmatcher.setDescriptorMatcher(matcher);                                                         // set matcher
```

   Secondly, we have to call the matcher by using *robustMatch()* or *fastRobustMatch()* function. The difference of using this two functions is its computational cost. The first method is slower but more robust at filtering good matches because uses two ratio test and a symmetry test. In contrast, the second method is faster but less robust because only applies a single ratio test to the matches.

   The following code is to get the model 3D points and its descriptors and then call the matcher in the main program:

```
// Get the MODEL INFO

std::vector<cv::Point3f> list_points3d_model = model.get_points3d();  // list with model 3D coordinates
cv::Mat descriptors_model = model.get_descriptors();                  // list with descriptors of each 3D
       coordinate
```

```
// -- Step 1: Robust matching between model descriptors and scene descriptors

std::vector<cv::DMatch> good_matches;       // to obtain the model 3D points  in the scene
std::vector<cv::KeyPoint> keypoints_scene;  // to obtain the 2D points of the scene

if(fast_match)
{
    rmatcher.fastRobustMatch(frame, good_matches, keypoints_scene, descriptors_model);
}
else
{
    rmatcher.robustMatch(frame, good_matches, keypoints_scene, descriptors_model);
}
```

   The following code corresponds to the *robustMatch()* function which belongs to the *RobustMatcher* class. This function uses the given image to detect the keypoints and extract the descriptors, match using *two Nearest Neighbour* the extracted descriptors with the given model descriptors and vice versa. Then, a ratio test is applied to the two direction matches in order to remove these matches which its distance ratio between the first and second best match is larger than a given threshold. Finally, a symmetry test is applied in order the remove non symmetrical matches.

```
void RobustMatcher::robustMatch( const cv::Mat& frame, std::vector<cv::DMatch>& good_matches,
                                 std::vector<cv::KeyPoint>& keypoints_frame,
                                 const std::vector<cv::KeyPoint>& keypoints_model, const cv::Mat&
       descriptors_model )
{

    // 1a. Detection of the ORB features
    this->computeKeyPoints(frame, keypoints_frame);

    // 1b. Extraction of the ORB descriptors
    cv::Mat descriptors_frame;
    this->computeDescriptors(frame, keypoints_frame, descriptors_frame);

    // 2. Match the two image descriptors
    std::vector<std::vector<cv::DMatch> > matches12, matches21;

    // 2a. From image 1 to image 2
    matcher_->knnMatch(descriptors_frame, descriptors_model, matches12, 2); // return 2 nearest neighbours

    // 2b. From image 2 to image 1
    matcher_->knnMatch(descriptors_model, descriptors_frame, matches21, 2); // return 2 nearest neighbours

    // 3. Remove matches for which NN ratio is > than threshold
    // clean image 1 -> image 2 matches
    int removed1 = ratioTest(matches12);
    // clean image 2 -> image 1 matches
    int removed2 = ratioTest(matches21);

    // 4. Remove non-symmetrical matches
    symmetryTest(matches12, matches21, good_matches);

}
```

   After the matches filtering we have to subtract the 2D and 3D correspondences from the found scene keypoints and our 3D model using the obtained *DMatches* vector. For more information about **cv::DMatch** check the documentation.

```
// -- Step 2: Find out the 2D/3D correspondences

std::vector<cv::Point3f> list_points3d_model_match;    // container for the model 3D coordinates found in the
       scene
```

```
std::vector<cv::Point2f> list_points2d_scene_match;    // container for the model 2D coordinates found in the
      scene

for(unsigned int match_index = 0; match_index < good_matches.size(); ++match_index)
{
    cv::Point3f point3d_model = list_points3d_model[ good_matches[match_index].trainIdx ];   // 3D point from
       model
    cv::Point2f point2d_scene = keypoints_scene[ good_matches[match_index].queryIdx ].pt;    // 2D point from the
       scene
    list_points3d_model_match.push_back(point3d_model);                                      // add 3D point
    list_points2d_scene_match.push_back(point2d_scene);                                      // add 2D point
}
```

You can also change the ratio test threshold, the number of keypoints to detect as well as use or not the robust matcher:

```
./cpp-tutorial-pnp_detection --ratio=0.8 --keypoints=1000 --fast=false
```

5. **Pose estimation using PnP + Ransac**

Once with the 2D and 3D correspondences we have to apply a PnP algorithm in order to estimate the camera pose. The reason why we have to use **cv::solvePnPRansac** instead of **cv::solvePnP** is due to the fact that after the matching not all the found correspondences are correct and, as like as not, there are false correspondences or also called *outliers*. The Random Sample Consensus or *Ransac* is a non-deterministic iterative method which estimate parameters of a mathematical model from observed data producing an aproximate result as the number of iterations increase. After appyling *Ransac* all the *outliers* will be eliminated to then estimate the camera pose with a certain probability to obtain a good solution.

For the camera pose estimation I have implemented a *class* **PnPProblem**. This *class* has 4 atributes: a given calibration matrix, the rotation matrix, the translation matrix and the rotation-translation matrix. The intrinsic calibration parameters of the camera which you are using to estimate the pose are necessary. In order to obtain the parameters you can check **Camera calibration with square chessboard** and **Camera calibration With OpenCV** tutorials.

The following code is how to declare the *PnPProblem class* in the main program:

```
// Intrinsic camera parameters: UVC WEBCAM

double f = 55;                           // focal length in mm
double sx = 22.3, sy = 14.9;             // sensor size
double width = 640, height = 480;        // image size

double params_WEBCAM[] = { width*f/sx,   // fx
                           height*f/sy,  // fy
                           width/2,      // cx
                           height/2};    // cy

PnPProblem pnp_detection(params_WEBCAM); // instantiate PnPProblem class
```

The following code is how the *PnPProblem class* initialises its atributes:

```
// Custom constructor given the intrinsic camera parameters

PnPProblem::PnPProblem(const double params[])
{
  _A_matrix = cv::Mat::zeros(3, 3, CV_64FC1);   // intrinsic camera parameters
  _A_matrix.at<double>(0, 0) = params[0];       //      [ fx   0  cx ]
  _A_matrix.at<double>(1, 1) = params[1];       //      [  0  fy  cy ]
  _A_matrix.at<double>(0, 2) = params[2];       //      [  0   0   1 ]
  _A_matrix.at<double>(1, 2) = params[3];
  _A_matrix.at<double>(2, 2) = 1;
  _R_matrix = cv::Mat::zeros(3, 3, CV_64FC1);   // rotation matrix
  _t_matrix = cv::Mat::zeros(3, 1, CV_64FC1);   // translation matrix
  _P_matrix = cv::Mat::zeros(3, 4, CV_64FC1);   // rotation-translation matrix

}
```

OpenCV provides four PnP methods: ITERATIVE, EPNP, P3P and DLS. Depending on the application type, the estimation method will be different. In the case that we want to make a real time application, the more suitable methods are EPNP and P3P due to that are faster than ITERATIVE and DLS at finding an optimal solution. However, EPNP and P3P are not especially robust in front of planar surfaces and sometimes the pose estimation seems to have a mirror effect. Therefore, in this this tutorial is used ITERATIVE method due to the object to be detected has planar surfaces.

The OpenCV Ransac implementation wants you to provide three parameters: the maximum number of iterations until stop the algorithm, the maximum allowed distance between the observed and computed point projections to consider it an inlier and the confidence to obtain a good result. You can tune these paramaters in order to improve your algorithm performance. Increasing the number of iterations you will have a more accurate solution, but will take more time to find a solution. Increasing the reprojection error will reduce the computation time, but your solution will be unaccurate. Decreasing the confidence your arlgorithm will be faster, but the obtained solution will be unaccurate.

The following parameters work for this application:

```
// RANSAC parameters

int iterationsCount = 500;        // number of Ransac iterations.
float reprojectionError = 2.0;    // maximum allowed distance to consider it an inlier.
float confidence = 0.95;          // ransac successful confidence.
```

The following code corresponds to the *estimatePoseRANSAC()* function which belongs to the *PnPProblem class*. This function estimates the rotation and translation matrix given a set of 2D/3D correspondences, the desired PnP method to use, the output inliers container and the Ransac parameters:

```
// Estimate the pose given a list of 2D/3D correspondences with RANSAC and the method to use

void PnPProblem::estimatePoseRANSAC( const std::vector<cv::Point3f> &list_points3d,        // list with model 3D
      coordinates
                                     const std::vector<cv::Point2f> &list_points2d,        // list with scene 2D
      coordinates
```

```
                                    int flags, cv::Mat &inliers, int iterationsCount,     // PnP method; inliers
      container
                                    float reprojectionError, float confidence )           // Ransac parameters
{
    cv::Mat distCoeffs = cv::Mat::zeros(4, 1, CV_64FC1);    // vector of distortion coefficients
    cv::Mat rvec = cv::Mat::zeros(3, 1, CV_64FC1);          // output rotation vector
    cv::Mat tvec = cv::Mat::zeros(3, 1, CV_64FC1);          // output translation vector

    bool useExtrinsicGuess = false;   // if true the function uses the provided rvec and tvec values as
                                      // initial approximations of the rotation and translation vectors

    cv::solvePnPRansac( list_points3d, list_points2d, _A_matrix, distCoeffs, rvec, tvec,
                        useExtrinsicGuess, iterationsCount, reprojectionError, confidence,
                        inliers, flags );

    Rodrigues(rvec,_R_matrix);                  // converts Rotation Vector to Matrix
    _t_matrix = tvec;                           // set translation matrix

    this->set_P_matrix(_R_matrix, _t_matrix);    // set rotation-translation matrix

}
```

In the following code are the 3th and 4th steps of the main algorithm. The first, calling the above function and the second taking the output inliers vector from Ransac to get the 2D scene points for drawing purpose. As seen in the code we must be sure to apply Ransac if we have matches, in the other case, the function **cv::solvePnPRansac** crashes due to any OpenCV *bug*.

```
if(good_matches.size() > 0) // None matches, then RANSAC crashes
{

    // -- Step 3: Estimate the pose using RANSAC approach
    pnp_detection.estimatePoseRANSAC( list_points3d_model_match, list_points2d_scene_match,
                                      pnpMethod, inliers_idx, iterationsCount, reprojectionError, confidence );


    // -- Step 4: Catch the inliers keypoints to draw
    for(int inliers_index = 0; inliers_index < inliers_idx.rows; ++inliers_index)
    {
    int n = inliers_idx.at<int>(inliers_index);         // i-inlier
    cv::Point2f point2d = list_points2d_scene_match[n]; // i-inlier point 2D
    list_points2d_inliers.push_back(point2d);           // add i-inlier to list
}
```

Finally, once the camera pose has been estimated we can use the R and t in order to compute the 2D projection onto the image of a given 3D point expressed in a world reference frame using the showed formula on *Theory*.

The following code corresponds to the *backproject3DPoint()* function which belongs to the *PnPProblem class*. The function backproject a given 3D point expressed in a world reference frame onto a 2D image:

```
// Backproject a 3D point to 2D using the estimated pose parameters

cv::Point2f PnPProblem::backproject3DPoint(const cv::Point3f &point3d)
{
    // 3D point vector [x y z 1]'
    cv::Mat point3d_vec = cv::Mat(4, 1, CV_64FC1);
    point3d_vec.at<double>(0) = point3d.x;
    point3d_vec.at<double>(1) = point3d.y;
    point3d_vec.at<double>(2) = point3d.z;
    point3d_vec.at<double>(3) = 1;

    // 2D point vector [u v 1]'
    cv::Mat point2d_vec = cv::Mat(4, 1, CV_64FC1);
    point2d_vec = _A_matrix * _P_matrix * point3d_vec;

    // Normalization of [u v]'
    cv::Point2f point2d;
    point2d.x = point2d_vec.at<double>(0) / point2d_vec.at<double>(2);
    point2d.y = point2d_vec.at<double>(1) / point2d_vec.at<double>(2);

    return point2d;
}
```

The above function is used to compute all the 3D points of the object *Mesh* to show the pose of the object.

You can also change RANSAC parameters and PnP method:

```
./cpp-tutorial-pnp_detection --error=0.25 --confidence=0.90 --iterations=250 --method=3
```

6. **Linear Kalman Filter for bad poses rejection**

Is it common in computer vision or robotics fields that after applying detection or tracking techniques, bad results are obtained due to some sensor errors. In order to avoid these bad detections in this tutorial is explained how to implement a Linear Kalman Filter. The Kalman Filter will be applied after detected a given number of inliers.

You can find more information about what Kalman Filter is. In this tutorial it's used the OpenCV implementation of the **cv::KalmanFilter** based on Linear Kalman Filter for position and orientation tracking to set the dynamics and measurement models.

Firstly, we have to define our state vector which will have 18 states: the positional data (x,y,z) with its first and second derivatives (velocity and acceleration), then rotation is added in form of three euler angles (roll, pitch, jaw) together with their first and second derivatives (angular velocity and acceleration)

$$X = (x,y,z,\dot x,\dot y,\dot z,\ddot x,\ddot y,\ddot z,\psi,\theta,\phi,\dot \psi,\dot \theta,\dot \phi,\ddot \psi,\ddot \theta,\ddot \phi)^T$$

Secondly, we have to define the number of measuremnts which will be 6: from R and t we can extract (x,y,z) and (\psi,\theta,\phi). In addition, we have to define the number of control actions to apply to the system which in this case will be *zero*. Finally, we have to define the differential time between measurements which in this case is 1/T, where *T* is the frame rate of the video.

```
cv::KalmanFilter KF;         // instantiate Kalman Filter

int nStates = 18;            // the number of states
int nMeasurements = 6;       // the number of measured states
int nInputs = 0;             // the number of action control

double dt = 0.125;           // time between measurements (1/FPS)

initKalmanFilter(KF, nStates, nMeasurements, nInputs, dt);    // init function
```

The following code corresponds to the *Kalman Filter* initialisation. Firstly, is set the process noise, the measurement noise and the error covariance matrix. Secondly, are set the transition matrix which is the dynamic model and finally the measurement matrix, which is the measurement model.

You can tune the process and measurement noise to improve the *Kalman Filter* performance. As the measurement noise is reduced the faster will converge doing the algorithm sensitive in front of bad measurements.

```
void initKalmanFilter(cv::KalmanFilter &KF, int nStates, int nMeasurements, int nInputs, double dt)
{

  KF.init(nStates, nMeasurements, nInputs, CV_64F);                 // init Kalman Filter

  cv::setIdentity(KF.processNoiseCov, cv::Scalar::all(1e-5));       // set process noise
  cv::setIdentity(KF.measurementNoiseCov, cv::Scalar::all(1e-4));   // set measurement noise
  cv::setIdentity(KF.errorCovPost, cv::Scalar::all(1));             // error covariance


                /* DYNAMIC MODEL */

  //  [1 0 0 dt  0  0 dt2   0   0 0 0 0  0  0  0   0   0   0]
  //  [0 1 0  0 dt  0   0 dt2   0 0 0 0  0  0  0   0   0   0]
  //  [0 0 1  0  0 dt   0   0 dt2 0 0 0  0  0  0   0   0   0]
  //  [0 0 0  1  0  0  dt   0   0 0 0 0  0  0  0   0   0   0]
  //  [0 0 0  0  1  0   0  dt   0 0 0 0  0  0  0   0   0   0]
  //  [0 0 0  0  0  1   0   0  dt 0 0 0  0  0  0   0   0   0]
  //  [0 0 0  0  0  0   1   0   0 0 0 0  0  0  0   0   0   0]
  //  [0 0 0  0  0  0   0   1   0 0 0 0  0  0  0   0   0   0]
  //  [0 0 0  0  0  0   0   0   1 0 0 0  0  0  0   0   0   0]
  //  [0 0 0  0  0  0   0   0   0 1 0 0 dt  0  0 dt2   0   0]
  //  [0 0 0  0  0  0   0   0   0 0 1 0  0 dt  0   0 dt2   0]
  //  [0 0 0  0  0  0   0   0   0 0 0 1  0  0 dt   0   0 dt2]
  //  [0 0 0  0  0  0   0   0   0 0 0 0  1  0  0  dt   0   0]
  //  [0 0 0  0  0  0   0   0   0 0 0 0  0  1  0   0  dt   0]
  //  [0 0 0  0  0  0   0   0   0 0 0 0  0  0  1   0   0  dt]
  //  [0 0 0  0  0  0   0   0   0 0 0 0  0  0  0   1   0   0]
  //  [0 0 0  0  0  0   0   0   0 0 0 0  0  0  0   0   1   0]
  //  [0 0 0  0  0  0   0   0   0 0 0 0  0  0  0   0   0   1]

  // position
  KF.transitionMatrix.at<double>(0,3) = dt;
  KF.transitionMatrix.at<double>(1,4) = dt;
  KF.transitionMatrix.at<double>(2,5) = dt;
  KF.transitionMatrix.at<double>(3,6) = dt;
  KF.transitionMatrix.at<double>(4,7) = dt;
  KF.transitionMatrix.at<double>(5,8) = dt;
  KF.transitionMatrix.at<double>(0,6) = 0.5*pow(dt,2);
  KF.transitionMatrix.at<double>(1,7) = 0.5*pow(dt,2);
  KF.transitionMatrix.at<double>(2,8) = 0.5*pow(dt,2);

  // orientation
  KF.transitionMatrix.at<double>(9,12) = dt;
  KF.transitionMatrix.at<double>(10,13) = dt;
  KF.transitionMatrix.at<double>(11,14) = dt;
  KF.transitionMatrix.at<double>(12,15) = dt;
  KF.transitionMatrix.at<double>(13,16) = dt;
  KF.transitionMatrix.at<double>(14,17) = dt;
  KF.transitionMatrix.at<double>(9,15) = 0.5*pow(dt,2);
  KF.transitionMatrix.at<double>(10,16) = 0.5*pow(dt,2);
  KF.transitionMatrix.at<double>(11,17) = 0.5*pow(dt,2);


       /* MEASUREMENT MODEL */

  //  [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
  //  [0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
  //  [0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
  //  [0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
  //  [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
  //  [0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]

  KF.measurementMatrix.at<double>(0,0) = 1;  // x
  KF.measurementMatrix.at<double>(1,1) = 1;  // y
  KF.measurementMatrix.at<double>(2,2) = 1;  // z
  KF.measurementMatrix.at<double>(3,9) = 1;  // roll
  KF.measurementMatrix.at<double>(4,10) = 1; // pitch
  KF.measurementMatrix.at<double>(5,11) = 1; // yaw

}
```

In the following code is the 5th step of the main algorithm. When the obtained number of inliers after *Ransac* is over the threshold, the measurements matrix is filled and then the *Kalman Filter* is updated:

```
// -- Step 5: Kalman Filter

// GOOD MEASUREMENT
if( inliers_idx.rows >= minInliersKalman )
{

    // Get the measured translation
    cv::Mat translation_measured(3, 1, CV_64F);
    translation_measured = pnp_detection.get_t_matrix();

    // Get the measured rotation
    cv::Mat rotation_measured(3, 3, CV_64F);
    rotation_measured = pnp_detection.get_R_matrix();

    // fill the measurements vector
    fillMeasurements(measurements, translation_measured, rotation_measured);
```

```
}

// Instantiate estimated translation and rotation
cv::Mat translation_estimated(3, 1, CV_64F);
cv::Mat rotation_estimated(3, 3, CV_64F);

// update the Kalman filter with good measurements
updateKalmanFilter( KF, measurements,
                translation_estimated, rotation_estimated);
```

The following code corresponds to the *fillMeasurements()* function which converts the measured Rotation Matrix to Eulers angles and fill the measurements matrix along with the measured translation vector:

```
void fillMeasurements( cv::Mat &measurements,
                   const cv::Mat &translation_measured, const cv::Mat &rotation_measured)
{
    // Convert rotation matrix to euler angles
    cv::Mat measured_eulers(3, 1, CV_64F);
    measured_eulers = rot2euler(rotation_measured);

    // Set measurement to predict
    measurements.at<double>(0) = translation_measured.at<double>(0); // x
    measurements.at<double>(1) = translation_measured.at<double>(1); // y
    measurements.at<double>(2) = translation_measured.at<double>(2); // z
    measurements.at<double>(3) = measured_eulers.at<double>(0);      // roll
    measurements.at<double>(4) = measured_eulers.at<double>(1);      // pitch
    measurements.at<double>(5) = measured_eulers.at<double>(2);      // yaw
}
```

The following code corresponds to the *updateKalmanFilter()* function which update the Kalman Filter and set the estimated Rotation Matrix and translation vector. The estimated Rotation Matrix comes from the estimated Euler angles to Rotation Matrix.

```
void updateKalmanFilter( cv::KalmanFilter &KF, cv::Mat &measurement,
                     cv::Mat &translation_estimated, cv::Mat &rotation_estimated )
{

    // First predict, to update the internal statePre variable
    cv::Mat prediction = KF.predict();

    // The "correct" phase that is going to use the predicted value and our measurement
    cv::Mat estimated = KF.correct(measurement);

    // Estimated translation
    translation_estimated.at<double>(0) = estimated.at<double>(0);
    translation_estimated.at<double>(1) = estimated.at<double>(1);
    translation_estimated.at<double>(2) = estimated.at<double>(2);

    // Estimated euler angles
    cv::Mat eulers_estimated(3, 1, CV_64F);
    eulers_estimated.at<double>(0) = estimated.at<double>(9);
    eulers_estimated.at<double>(1) = estimated.at<double>(10);
    eulers_estimated.at<double>(2) = estimated.at<double>(11);

    // Convert estimated quaternion to rotation matrix
    rotation_estimated = euler2rot(eulers_estimated);

}
```

The 6th step is set the estimated rotation-translation matrix:

```
// -- Step 6: Set estimated projection matrix
pnp_detection_est.set_P_matrix(rotation_estimated, translation_estimated);
```

The last and optional step is draw the found pose. To do it I implemented a function to draw all the mesh 3D points and an extra reference axis:

```
// -- Step X: Draw pose

drawObjectMesh(frame_vis, &mesh, &pnp_detection, green);                // draw current pose
drawObjectMesh(frame_vis, &mesh, &pnp_detection_est, yellow);           // draw estimated pose

double l = 5;
std::vector<cv::Point2f> pose_points2d;
pose_points2d.push_back(pnp_detection_est.backproject3DPoint(cv::Point3f(0,0,0)));    // axis center
pose_points2d.push_back(pnp_detection_est.backproject3DPoint(cv::Point3f(l,0,0)));    // axis x
pose_points2d.push_back(pnp_detection_est.backproject3DPoint(cv::Point3f(0,l,0)));    // axis y
pose_points2d.push_back(pnp_detection_est.backproject3DPoint(cv::Point3f(0,0,l)));    // axis z
draw3DCoordinateAxes(frame_vis, pose_points2d);                                       // draw axes
```

You can also modify the minimum inliers to update Kalman Filter:

```
./cpp-tutorial-pnp_detection --inliers=20
```

Results

The following videos are the results of pose estimation in real time using the explained detection algorithm using the following parameters:

```
// Robust Matcher parameters

int numKeyPoints = 2000;      // number of detected keypoints
float ratio = 0.70f;          // ratio test
bool fast_match = true;       // fastRobustMatch() or robustMatch()


// RANSAC parameters

int iterationsCount = 500;    // number of Ransac iterations.
int reprojectionError = 2.0;  // maximum allowed distance to consider it an inlier.
float confidence = 0.95;      // ransac successful confidence.
```

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