

Note that we have found the roots of $2x^2 - 5x + 3 = 0$ by factorising $2x^2 - 5x + 3$ into two linear factors and equating each factor to zero.

Example 4 : Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Solution : We have

$$\begin{aligned} 6x^2 - x - 2 &= 6x^2 + 3x - 4x - 2 \\ &= 3x(2x + 1) - 2(2x + 1) \\ &= (3x - 2)(2x + 1) \end{aligned}$$

The roots of $6x^2 - x - 2 = 0$ are the values of x for which $(3x - 2)(2x + 1) = 0$
Therefore, $3x - 2 = 0$ or $2x + 1 = 0$,

$$\text{i.e., } x = \frac{2}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

Therefore, the roots of $6x^2 - x - 2 = 0$ are $\frac{2}{3}$ and $-\frac{1}{2}$.

We verify the roots, by checking that $\frac{2}{3}$ and $-\frac{1}{2}$ satisfy $6x^2 - x - 2 = 0$.

Example 5 : Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$.

Solution :

$$\begin{aligned} 3x^2 - 2\sqrt{6}x + 2 &= 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 \\ &= \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) \\ &= (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) \end{aligned}$$

So, the roots of the equation are the values of x for which

$$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

Now, $\sqrt{3}x - \sqrt{2} = 0$ for $x = \frac{\sqrt{2}}{\sqrt{3}}$

So, this root is repeated twice, one for each repeated factor $\sqrt{3}x - \sqrt{2}$.

Therefore, the roots of $3x^2 - 2\sqrt{6}x + 2 = 0$ are $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$.