# CBCS SCHEME

USN



18AI56

## Fifth Semester B.E. Degree Examination, Jan./Feb. 2023
## Mathematics for Machine Learning

Time: 3 hrs. Max. Marks: 100

**Note:** ***Answer any FIVE full questions, choosing ONE full question from each module.***

### Module-1

1 a. Find the general and particular solution of the system of equations :
$-2x_1 + 4x_2 - 2x_3 - x_4 + 4x_5 = -3$
$4x_1 - 8x_2 + 8x_3 - 3x_4 + x_5 = 2$
$x_1 - 2x_2 + x_3 - x_4 + x_5 = 0$
$x_1 - 2x_2 - 3x_4 + 4x_5 = a.$ **(10 Marks)**

b. Define a basis, rank and inner product space with an example. **(06 Marks)**

c. Determine a basis for a vector subspace $U \subseteq R^5$, spanned by the vectors $\{x_1, x_2, x_3, x_4\} \in \mathbb{R}^5$, where $x_1 = [1, 2, -1, -1, -1]^T$, $x_2 = [2, -1, 1, 2, -2]^T$, $x_3 = [3, -4, 3, 5, -3]^T$, $x_4 = [-1, 8, -5, -6, 1]^T$. **(04 Marks)**

**OR**

2 a. If $u = [2, -5, -1]^T$ and $v = [-7, -4, 6]^T$ then :
i) $u \cdot v$ ii) $u + v$ iii) $||u||$ iv) $||v||$ v) $||u + v||$. **(10 Marks)**

b. For x, y $\in$ v where v is a inner product space then define distance between x and y. Also find the distance between (7, 1) and (3, 2). **(05 Marks)**

c. Define angle between two vectors form Cauchy – Schwartz inequality. Hence compete angle between $[1, 1]^T$ and $[1\ 2]^T$. **(05 Marks)**

### Module-2

3 a. For the matrix $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & 4 \\ -8 & 0 & -1 \end{bmatrix}$. Find |A| and trace (A). **(05 Marks)**

b. Find the Eigen values, Eigen vectors, and Eigen spaces for the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ **(06 Marks)**

c. Find the singular value decomposition of $A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$. **(09 Marks)**

**OR**

4 a. Explain the terms orthogonal and orthonomal vectors. **(04 Marks)**

b. If $a = [-2, 1]^T$, $b = [-3, 1]^T$, $c = \left[\frac{4}{3}, -1, \frac{2}{3}\right]^T$ and $d = [5, 6, -1]^T$, then compute.

i) $\left(\dfrac{a \cdot b}{a \cdot a}\right) \cdot a$

ii) Find a unit vector 'u' in the direction c

iii) Show that 'd' is orthogonal to 'c'. **(06 Marks)**

c. Compute the singular value decomposition of a matrix $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

## Module-3

5 a. Obtain the Maclaurin's series of $f(x)=\sqrt{1+\sin 2x}$. Draw the graphs of $f(x) = f(0)$, $f(x)=f(0)+\frac{x}{1!}f'(0)$ and $f(x)=f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)$. **(07 Marks)**

b. Define gradient of the vector valued functions $f(x_1, x_2)$ and hence find gradient of $f(x_1, x_2) = x_1^2 x_2 + x_1 x_2^3$. **(07 Marks)**

c. Compute the derivative of the function $h(x) = g[f(x)]$ where $g[f(x)] = [f(x)]^4$ and $f(x) = 2x + 1$. **(06 Marks)**

**OR**

6 a. If $f(x, y) = x^2 + 2y$ ; where $x_1 = \sin t$ and $x_2 = \cos t$, then find $\frac{df}{dt}$. **(06 Marks)**

b. Compute the gradient of h with respect to t, for the function $h : \mathbb{R} \to \mathbb{R}$, $h(t) = (f \circ g)(t)$ with $f : \mathbb{R}^2 \to R$, $g : \mathbb{R} \to \mathbb{R}^2$ defined by $f(x) = e^{xy^2}$ $x = \begin{bmatrix} x \\ y \end{bmatrix} = g(t) = \begin{bmatrix} t\cos t \\ t\sin t \end{bmatrix}$. **(06 Marks)**

c. For the linear regression model $f(x)=\sqrt{x^2+e^{x^2}}+\cos(x^2+e^{x^2})$. Compute the gradient and $\frac{\partial f}{\partial x}$ by working backward. **(08 Marks)**

## Module-4

7 a. Let x and y are two independent variables. If variance of $(2x - y) = 6$ and variance of $(x + 2y) = 9$. Find the variance of x and variance of y. **(06 Marks)**

b. A joint probability distribution is given by the following table.

| X \ Y | –3 | 2 | 4 |
|---|---|---|---|
| 1 | 0.1 | 0.2 | 0.2 |
| 3 | 0.3 | 0.1 | 0.1 |

Find variance of x and y and write the marginal distributions p(x) and p(y). **(07 Marks)**

c. State the following :

i) Sum rule for discrete and continuous random variables

ii) Product rule which relates the joint distribution to the conditional distribution.

iii) Baye's theorem

iv) Likelihood $P\left(y/x\right)$

v) Posterior $P\left(x/y\right)$. **(07 Marks)**

**OR**

8 a. A bag contain 2 mango and 3 apples and a bag B contains 4 mango and 5 apples. One fruit is drawn at random from one of the bags and it is found to be apple. Find the probability that the apple fruit is drawn from bag B. **(06 Marks)**

b. Given a real valued function $f(x) = a\,g(x) + b\,h(x)$ where $a, b \in R$ and $x \in R^D$, then prove that $E_x[f(x)] = a\,E_x[g(x)] + b\,E_x[h(x)]$. **(04 Marks)**

c. For any two random variable, prove that

i) $E(X + Y) = E(X) + E(Y)$

ii) $E(X - Y) = E(X) - E(Y)$

iii) $V(X + Y) = V(X) + V(Y) + COV(X, Y) + COV(Y, X)$

iv) $V(X - Y) = V(X) + V(Y) - COV(X, Y) - COV(Y, X)$. **(10 Marks)**

## Module-5

9 a. Distress for the function $f(x) = x^4 + 7x^3 + 5x^2 - 17x + 3$, the convexity and concavity in the interval $(-6, 2)$. Further graphically represent the negative gradients and global minimum. **(10 Marks)**

b. For a quadratic function in two dimensions $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 20 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x \\ y \end{bmatrix}$ with gradient $\begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 20 \end{bmatrix} - \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T$ initial location, $x_0 = [-3, -1]^T$ and $r = 0.085$. Apply gradient descent algorithm which converges to minimal value perform two iterations. **(10 Marks)**

**OR**

10 a. Define convex optimization problem, further discuss the convexity of $x \log_2 x$ between the points $x = 2$ and $x = 4$. Also plot the function and tangent at $x = 2$. **(10 Marks)**

b. Write a brief note on concavity and convexity with suitable examples. Further discuss the convexity of $y = 3x^2 - 5x + 2$ with graph. **(10 Marks)**

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