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# INCREASING & DECREASING FUNCTIONS AND MAXIMA & MINIMA

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## STRICTLY INCREASING FUNCTION :

A function  $f(x)$  is said to be a strictly increasing function on  $(a, b)$  if

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2) \text{ for all } x_1, x_2 \in (a, b)$$

Strictly Decreasing Function : A function  $f(x)$  is said to be a strictly decreasing function on  $(a, b)$  if

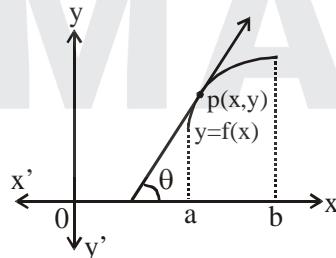
$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2) \text{ for all } x_1, x_2 \in (a, b)$$

By an increasing or a decreasing function we shall mean a strictly increasing or a strictly decreasing function.

Monotonic Function : A function  $f(x)$  is said to be monotonic on an interval  $(a, b)$ . it is either increasing or decreasing on  $(a, b)$

Definiton : A function  $f(x)$  is said to be increasing on  $[a, b]$  if it is increasing on  $(a, b)$  and it is also increasing at  $x = a$  and  $x = b$ .

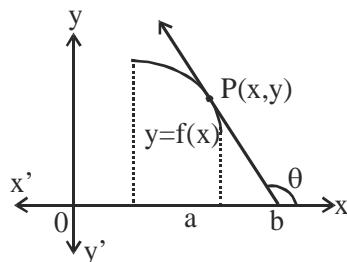
Necessary Condition : We observe that if  $f(x)$  is an increasing function on  $(a, b)$  then tangent at every point on the curve  $y = f(x)$  makes an acute angle  $\theta$  with the positive direction of  $x$ -axis.



$$\therefore \tan \theta > 0 \Rightarrow \frac{dy}{dx} > 0 \text{ or } f'(x) > 0 \text{ for all } x \in (a, b)$$

If  $f(x)$  is a decreasing function on  $(a, b)$ , then tangent at every point on the curve  $y = f(x)$  makes an obtuse angle  $\theta$  with the positive direction of  $x$ -axis.

$$\therefore \tan \theta < 0 \Rightarrow \frac{dy}{dx} < 0 \text{ or } f'(x) < 0 \text{ for all } x \in (a, b)$$



## SUFFICIENT CONDITION

**THEOREM** : Let  $f$  be a differentiable real function defined on an open interval  $(a, b)$

- (i) If  $f'(x) > 0$  for all  $x \in (a, b)$  then  $f(x)$  is increasing on  $(a, b)$
- (ii) If  $f'(x) < 0$  for all  $x \in (a, b)$ , then  $f(x)$  is decreasing on  $(a, b)$ .

#### Properties of Monotonic Function :

- (i) If  $f(x)$  is strictly increasing function on an interval  $[a, b]$ , then  $f^{-1}$  exists and it is also a strictly increasing function.
- (ii) If  $f(x)$  is strictly increasing function on an interval  $[a, b]$  such that it is continuous, then  $f^{-1}$  is continuous on  $[f(a), f(b)]$ .
- (iii) If  $f(x)$  is continuous on  $[a, b]$  such that  $f'(c) \geq 0$  ( $f'(c) > 0$ ) for each  $c \in (a, b)$ , then  $f(x)$  is monotonically (strictly) increasing function on  $[a, b]$
- (iv) If  $f(x)$  and  $g(x)$  are monotonically (or strictly) increasing (or decreasing) functions on  $[a, b]$ , then  $gof(x)$  is a monotonically (or strictly) increasing function on  $[a, b]$
- (v) If one of the two functions  $f(x)$  and  $g(x)$  is strictly (or monotonically) increasing and other a strictly (monotonically) increasing and other a strictly (monotonically) decreasing, then  $gof(x)$  is strictly (monotonically) decreasing on  $[a, b]$ .

### MAXIMA AND MINIMA

Let  $f(x)$  be a function with domain  $D \subset \mathbb{R}$ . Then  $f(x)$  is said to attain the maximum value at a point  $a \in D$  if  
 $f(x) \leq f(a)$  for all  $x \in D$

In such a case,  $a$  is called the point of maximum and  $f(a)$  is known as the maximum value or the greatest value.

Local Maximum : A function  $f(x)$  is said to attain a local maximum at  $x = a$  if there exists a neighbourhood .

$(a - \delta, a + \delta)$  of  $a$  such that

$f(x) < f(a)$  for all  $x \in (a - \delta, a + \delta), x \neq a$

or       $f(x) - f(a) < 0$  for all  $x \in (a - \delta, a + \delta), x \neq a$

In such a case  $f(a)$  is called the local maximum value of  $f(x)$  at  $x = a$ .

Local Minimum : A function  $f(x)$  is said to attain a local minimum at  $x = a$  if there exists a neighbourhood

$(a - \delta, a + \delta)$  of  $a$  such that  $f(x) > f(a)$  for all  $x \in (a - \delta, a + \delta), x \neq a$

or       $f(x) - f(a) > 0$  for all  $x \in (a - \delta, a + \delta), x \neq a$

The value of the function at  $x = a$  i.e.  $f(a)$  is called the local minimum value of  $f(x)$  at  $x = a$

Theorem : A necessary condition for  $f(a)$  to be an extreme value of a function  $f(x)$  is that  $f'(a) = 0$  in case it exists. A function may however attain an extreme value at a point without being derivable thereat. For example, the function  $f(x) = |x|$  attains the minimum value at the origin even though it is not derivable at  $x = 0$ .

Remark : Above condition is only a necessary condition for the point.  $x = a$  to be an extreme point. It is not sufficient i.e.  $f'(a) = 0$  does not necessarily imply that  $x = a$  is an extreme point. For example for the function  $f(x) = x^3$ ,  $f'(0) = 0$  but at  $x = 0$  the function does not attain an extreme value.

Remark : The value of  $x$  for which  $f'(x) = 0$  are called stationary values or critical values of  $x$  and the corresponding values of  $f(x)$  are called stationary or turning values of  $f(x)$ .

Theorem : (First derivative test for local maximum and minima) Let  $f(x)$  be a function differentiable at  $x = a$ . Then,

(A)  $x = a$  is a point of local maximum of  $f(x)$ , if

(i)  $f'(a) = 0$  and

(ii)  $f'(x)$  changes sign from positive to negative as  $x$  passes through  $a$  i.e.  $f'(x) > 0$  at every point in the left nbd  $(a - \delta, a)$  and  $f'(x) < 0$  at every point in the right nbd  $(a, a + \delta)$  of  $a$ .

(B)  $x = a$  is a point of local minimum of  $f(x)$ , if

(i)  $f'(a) = 0$  and

(ii)  $f'(x)$  changes sign from negative to positive as  $x$  passes through  $a$  i.e.  $f'(x) < 0$  at every point in the left nbd  $(a - \delta, a)$  of  $a$  and  $f'(x) > 0$  at every point in the right nbd  $(a, a + \delta)$  of  $a$ .

(C) If  $f'(a) = 0$  but  $f'(x)$  does not change sign, that is  $f'(a)$  has the same sign in the complete nbd of  $a$ , then  $a$  is neither a point of local maximum nor a point of local minimum.

Theorem : (Higher order derivative test). Let  $f$  be a differentiable function on an interval  $I$  and let  $c$  be an interior point of  $I$  such that

(i)  $f'(c) = f''(c) = f'''(c) = \dots = f^{n-1}(c) = 0$  and

(ii)  $f^n(c)$  exists and is non-zero

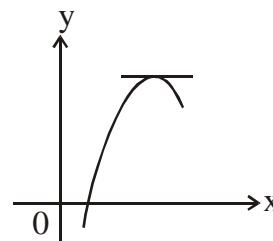
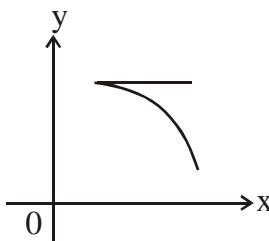
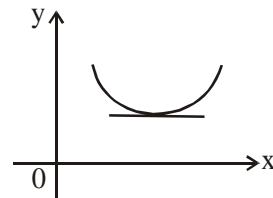
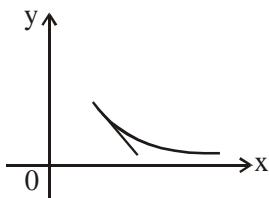
Then,

(a) If  $n$  is even and  $f^n(c) < 0 \Rightarrow x = c$  is a point of local maximum.

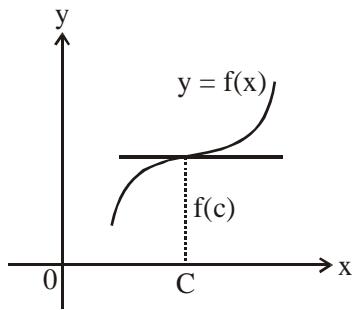
(b) If  $n$  is even and  $f^n(c) > 0 \Rightarrow x = c$  is a point of local minimum

(c) If  $n$  is odd  $\Rightarrow x = c$  is neither a point of local maximum nor a point of local minimum.

Point of inflection : An arc of a curve  $y = f(x)$  is called concave upward if, at each of its points, the arc lies above the tangent at the point. An arc of a curve  $y = f(x)$  is called concave downward if, at each of its points, the arc lies below the tangent at the point.



Definition : A point of inflection is a point at which a curve is changing concave upward to concave downward, or vice-versa.



A curve  $y = f(x)$  has one of its points  $x = c$  as an inflection point

If  $f''(c) = 0$  or is not defined and

If  $f''(x)$  changes sign as  $x$  increases through  $x = c$ .

The later condition may be replaced by  $f''(c) \neq 0$  when  $f''(c)$  exists.

Thus,  $x = c$  is a point of inflection if  $f''(c) = 0$  and  $f''(c) \neq 0$ .

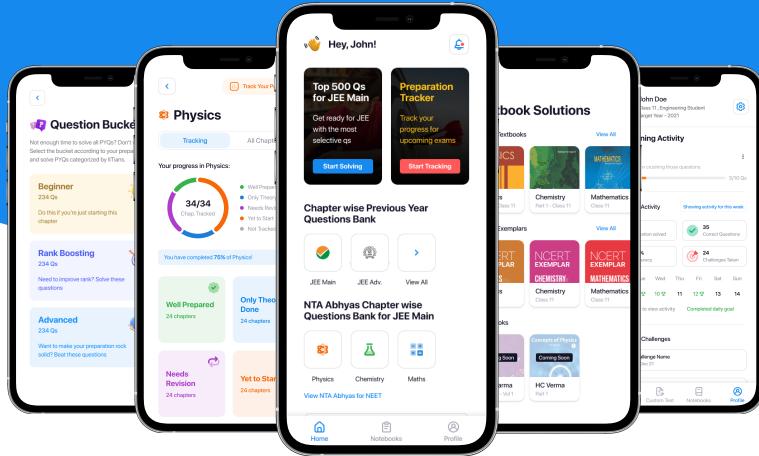
Critical point : A point  $x = \alpha$  is a critical point of a function  $f(x)$  if

$f'(\alpha) = 0$  or  $f'(\alpha)$  does not exist.





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