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PROGRESSIONS

SEQUENCE

A succession of numbers $a_1, a_2, a_3, \dots, a_n$ formed according to some definite rule is called a sequence.

ARITHMETIC PROGRESSION (A.P.)

A sequence of number $\{a_n\}$ is called an arithmetical progression, if there is a number d , such that $d = a_n - a_{n-1}$ for all n ; and d is called as common difference (c.d).

Useful Formulae

If a = first term, d = common difference and n is the number of terms, then

(a) n th term is denoted by t_n and is given by

$$t_n = a + (n - 1)d.$$

(b) Sum of first n terms is denoted by S_n and is given by

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

or $S_n = \frac{n}{2} (a + l)$, where l = last term in the series i.e. $l = t_n = a + (n - 1)d$.

(c) Arithmetic mean A of any two numbers a and b is given by

$$A = \frac{a+b}{2}$$

Also $A = \frac{1}{n} (a_1 + a_2 + \dots + a_n)$ is arithmetic mean of n numbers a_1, a_2, \dots, a_n

(d) Sum of first n natural numbers ($\sum n$)

$$\Sigma n = \frac{n(n+1)}{2} \text{ where, } n \in N.$$

(e) Sum of first n odd natural numbers $\Sigma(2n-1)$

$$\Sigma(2n-1) = 1 + 3 + 5 + \dots + (2n-1) = n^2$$

(f) Sum of first n even natural numbers ($\Sigma 2n$)

$$\Sigma 2n = 2 + 4 + 6 + \dots + 2n = n(n+1)$$

(g) Sum of squares of first n natural numbers (Σn^2)

$$\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$

(h) Sum of cubes of first n natural numbers (Σn^3)

$$\Sigma n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

(i) Sum of fourth powers of first n natural numbers (Σn^4)

$$\Sigma n^4 = 1^4 + 2^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

- (j) If terms are given in A.P., and their sum and product are known, then the terms must be picked up in following way in solving certain problem.
- For three terms $(a - d), a, (a + d)$
 - For four terms $(a - 3d), (a - d), (a + d), (a + 3d)$
 - For five terms $(a - 2d), (a - d), a, (a + d), (a + 2d)$

USEFUL PROPERTIES

- (a) If $t_n = an + b$, then the series so formed is an A.P.
- (b) If $S_n = an^2 + bn + c$, then series so formed is an A.P.
- (c) If every term of an A.P. is increased or decreased by the same quantity, the resulting terms will also be in A.P.
- (d) If every term of an A.P. is multiplied or divided by the same non-zero quantity, the resulting terms will also be in A.P.
- (e) If terms $a_1, a_2, \dots, a_n, a_{n+1}, \dots, a_{2n+1}$ are in A.P. Then sum of these terms will be equal to $(2n + 1)a_{n+1}$. Here total number of terms in the series is $(2n + 1)$ and middle term is a_{n+1} .
- (f) In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to sum of first and last terms.
- (g) Sum and difference of corresponding terms of two A.P.'s will form a series in A.P.
- (h) If terms $a_1, a_2, \dots, a_{2n-1}, a_{2n}$ are in A.P. The sum of these terms will be equal to $(2n) \left(\frac{a_n + a_{n+1}}{2} \right)$, where $\frac{a_n + a_{n+1}}{2} = \text{A.M. of middle terms.}$
- (i) n th term of a series is $a_n = S_n - S_{n-1}$ ($n \geq 2$)

GEOMETRIC PROGRESSION (G.P.)

The sequence $\{a_n\}$ in which $a_1 \neq 0$ is termed a geometric progression if there is a number $r \neq 0$ such that $\frac{a_n}{a_{n-1}} = r$ for all n , then r is called common ratio.

Useful Formulae

If a = first term, r = common ratio and n is the number of term, then

- (a) n^{th} term denoted by t_n is given by

$$t_n = ar^{n-1}$$
- (b) Sum of first n terms denoted by S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r} \text{ or } \frac{a(r^n - 1)}{r-1} \text{ corresponding to } r < 1 \text{ (or) } r > 1, \text{ (or) } S_n = \frac{ar}{1-r}$$

where l is the last term in the series.

- (c) Sum of infinite terms (S_∞)

$$S_n = \frac{a}{1-r} \text{ (For } |r| < 1\text{)}$$

- (d) Geometric mean (G)

(i) $G = \sqrt{ab}$ where a, b are two positive numbers.

(ii) $G = (a_1 a_2 \dots a_n)^{1/n}$ is geometric mean of n positive numbers $a_1, a_2, a_3, \dots, a_n$.

- (e) If terms are given in G.P. and their product is known, then the terms must be picked up in following way.

- For three terms $\frac{a}{r}, a, ar$
- For four terms $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
- For five terms $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

USEFUL PROPERTIES

- (a) The product of the terms equidistant from the beginning and end is constant. And it is equal to the product of first and last terms.
- (b) If every term of G.P. is increased or decreased by the same non-zero quantity, the resulting series may not be in G.P.
- (c) If every term of G.P. is multiplied or divided by the same non-zero quantity, the resulting series is in G.P.
- (d) If a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots be two G.P.'s of common ratio r_1 and r_2 respectively, then

a_1b_1, a_2b_2, \dots and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots$ will also form G.P. common ratio will be r_1r_2 and $\frac{r_1}{r_2}$ respectively.

- (e) If a_1, a_2, a_3, \dots be a G.P. of positive terms, then $\log a_1, \log a_2, \log a_3, \dots$ will be in A.P. and conversely.

Let $b = ar, c = ar^2$ and $d = ar^3$. Then, a, b, c, d are in G.P.

HARMONIC PROGRESSION (H.P.)

A sequence is said to be a harmonic progression, if and only if the reciprocal of its terms form an arithmetic progression.

SOME USEFUL FORMULAE & PROPERTIES

(a) n^{th} term of H.P. = $\frac{1}{n^{\text{th}} \text{term of AP}}$

- (b) Harmonic mean H of any two numbers a and b is given by

$$H = \frac{2ab}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b} \text{ where } a, b \text{ are two non-zero numbers.}$$

$$\text{Also } H = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}} = \frac{n}{\sum_{j=1}^n \frac{1}{a_j}}$$

for the harmonic mean of n non-zero numbers $a_1, a_2, a_3, \dots, a_n$.

- (c) If terms are given in H.P. then the terms could be picked up in the following way

- For three terms

$$\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$$

- For four terms

$$\frac{1}{a-3d}, \frac{1}{a-d}, \frac{1}{a+d}, \frac{1}{a+3d}$$

- For five terms

$$\frac{1}{a-2d}, \frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$$

INSERTION OF MEANS BETWEEN TWO NUMBERS

If a and b are two given numbers.

ARITHMETIC MEANS

Let $a, A_1, A_2, \dots, A_n, b$ be in A.P. then A_1, A_2, \dots, A_n are n A.M.'s between a and b . If d is common difference, then

$$b = a + (n + 2 - 1)d \Rightarrow d = \frac{b-a}{n+1}$$

$$A_1 = a + d = a + \frac{b-a}{n+1} = \frac{an+b}{n+1}$$

$$A_2 = a + 2d = a + 2 \frac{b-a}{n+1} = \frac{a(n-1)+2b}{n+1}$$

$$A_3 = a + 3d = a + 3 \frac{b-a}{n+1} = \frac{a(n-2)+3b}{n+1}$$

: : : :

$$A_n = a + nd = a + n \frac{(b-a)}{n+1} = \frac{a+nb}{n+1}$$

Note : The sum of n A.M.'s, $A_1 + A_2 + \dots + A_n = \frac{n}{2}(a+b)$.

GEOMETRIC MEANS

Let $a, G_1, G_2, \dots, G_n, b$ be in G.P., then G_1, G_2, \dots, G_n are n G.M.s between a and b . If r is a common ratio, then

$$b = ar^{n+1} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{(n+1)}}$$

$$G_1 = ar = a \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = a^{\frac{n}{n+1}} b^{\frac{1}{n+1}}$$

$$G_2 = ar^2 = a \left(\frac{b}{a}\right)^{\frac{2}{n+1}} = a^{\frac{n-1}{n+1}} b^{\frac{2}{n+1}}$$

$$G_3 = ar^3 = a \left(\frac{b}{a}\right)^{\frac{3}{n+1}} = a^{\frac{n-2}{n+1}} b^{\frac{3}{n+1}}$$

: : : :

$$G_n = ar^n = a \left(\frac{b}{a}\right)^{\frac{n}{n+1}} = a^{\frac{1}{n+1}} b^{\frac{n}{n+1}}$$

Note : The product of n G.M.'s $G_1 G_2 \dots G_n = (\sqrt{ab})^n$

HARMONIC MEANS

If a, H_1, H_2, \dots, H_n are in H.P., then H_1, H_2, \dots, H_n are the n H.M.'s between a and b . If d is the common difference of the corresponding A.P. then

$$\frac{1}{b} = \frac{1}{a} + (n+2-1)d \Rightarrow d = \frac{a-b}{ab(n+1)}$$

$$\frac{1}{H_1} = \frac{1}{a} + d = \frac{1}{a} + \frac{a-b}{ab(n+1)} = \frac{bn+a}{ab(n+1)}$$

$$\frac{1}{H_2} = \frac{1}{a} + 2d = \frac{1}{a} + \frac{2(a-b)}{ab(n+1)} = \frac{b(n-1)+2a}{ab(n+1)}$$

$$\frac{1}{H_3} = \frac{1}{a} + 3d = \frac{1}{a} + \frac{3(a-b)}{ab(n+1)} = \frac{b(n-2)+3a}{ab(n+1)}$$

$$\begin{matrix} : & : & : & : \\ : & : & : & : \end{matrix}$$

$$\frac{1}{H_n} = \frac{1}{a} + nd = \frac{1}{a} + \frac{n(a-b)}{ab(n+1)} = \frac{b(1)+na}{ab(n+1)}$$

RELATION BETWEEN A, G AND H

If A, G and H are A. M., G.M. and H.M. of two positive numbers a and b , then

$$(i) \quad G^2 = AH, \quad (ii) \quad A \geq G \geq H$$

Note :

- (1) For given n positive numbers $a_1, a_2, a_3, \dots, a_n$, A.M. \geq GM. \geq H.M. . The equality holds when the numbers are equal.
- (2) If sum of the given n positive numbers is constant then their product will be maximum if numbers are equal.

ARITHMETICO-GEOMETRIC SERIES

The series whose each term is formed by multiplying corresponding terms of an A.P. and G.P. is called the Arithmetico-geometric series.

For Examples

$$\begin{aligned} \cdot & \quad 1 + 2x + 4x^2 + 6x^3 + \dots \\ \cdot & \quad a + (a+d)r + (a+2d)r^2 + \dots \end{aligned}$$

SUMMATION OF n TERMS OF ARITHMETICO-GEOMETRIC SERIES

Let $S = a + (a+d)r + (a+2d)r^2 + \dots$

$$(i) \quad t_n = [a + (n-1)d]r^{n-1}$$

$$(ii) \quad S_n = a + (a+d)r + (a+2d)r^2 + \dots + [a + (n-1)d]r^{n-1}$$

Multiply by ' r ' and rewrite the series in following way.

$$rS_n = ar + (a+d)r^2 + (a+2d)r^3 + \dots + [a + (n-2)d]r^{n-1} + [a + (n-1)d]r^n$$

On subtraction,

$$S_n(1 - r) = a + d(r + r^2 + \dots + r^{n-1}) - [a + (n-1)d]r^n$$

$$\text{or, } S_n(1 - r) = a + \frac{dr(1 - r^{n-1})}{1 - r} - [a + (n - 1)d] \cdot r^n$$

$$\text{or, } \boxed{S_n = \frac{a}{1 - r} + \frac{dr(1 - r^{n-1})}{(1 - r)^2} - \frac{[a + (n - 1)d] \cdot r^n}{1 - r}}$$

SUMMATION OF INFINITE TERMS SERIES :

$$\begin{aligned} S &= a + (a + d)r + (a + 2d)r^2 + \dots \infty \\ rS &= \quad \quad \quad a r + (a + d) r^2 + \dots \text{to } \infty \end{aligned}$$

On subtraction

$$S(1 - r) = a + d(r + r^2 + r^3 + \dots \infty)$$

$$S = \frac{a}{1 - r} + \frac{dr}{(1 - r)^2}$$

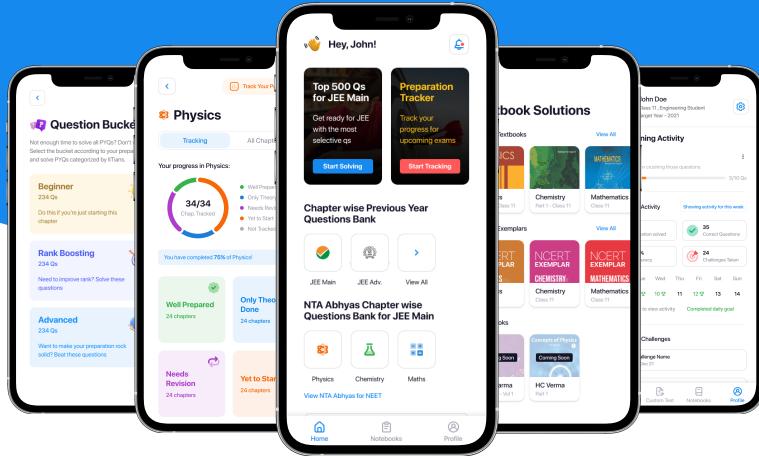
DIFFERENCE METHOD

Let $T_1, T_2, T_3, \dots, T_n$ are the terms of sequence, then

- (i) If $(T_2 - T_1), (T_3 - T_2), \dots, (T_n - T_{n-1})$ are in A.P. then, the sum of the such series may be obtained by using summation formulae in nth term,
- (ii) If $(T_2 - T_1), (T_3 - T_2), \dots, (T_n - T_{n-1})$ are found in G.P. then the sum of the such series may be obtained by using summation formulae of a G.P.



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