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# Tangents and Normals

## 1. GEOMETRICAL INTERPRETATION OF THE DERIVATIVE

Let  $y = f(x)$  be a given function. Its derivative  $f'(x)$  or  $\frac{dy}{dx}$  is equal to the trigonometrical tangent of the angle which tangent to the graph of the function at the point  $(x, y)$  makes with the positive direction of  $x$ -axis. Therefore  $\frac{dy}{dx}$  is the slope of the tangent.

Thus 
$$f'(x) = \frac{dy}{dx} = \tan \Psi$$

Hence at any point of a curve  $y = f(x)$

- (i) Inclination of the tangent (with  $x$ -axis) =  $\tan^{-1}\left(\frac{dy}{dx}\right)$
- (ii) Slope of the tangent =  $\frac{dy}{dx}$
- (iii) Slope of the normal =  $-\frac{1}{\left(\frac{dy}{dx}\right)} = -\left(\frac{dx}{dy}\right)$
- (iv) Slope of the tangent at  $(x_1, y_1)$  is denoted by  $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$
- (v) Slope of the normal at  $(x_1, y_1)$  is denoted by  $\left(-\frac{dx}{dy}\right)_{(x_1, y_1)}$

## 2. EQUATION OF TANGENT

- (i) The equation of tangent to the curve  $y = f(x)$  at  $(x_1, y_1)$  is  $(Y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (X - x_1)$   
Slope of tangent =  $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

- (ii) If a tangent is parallel to the axis of  $x$  then  $\Psi = 0$

$$\therefore \frac{dy}{dx} = \tan \Psi = \tan 0 = 0 \Rightarrow \boxed{\frac{dy}{dx} = 0}$$

- (iii) If the equation of the curve be given in the parametric form say  $x = f(t)$  and  $y = g(t)$ , then

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) / \left(\frac{dx}{dt}\right) = \frac{g'(t)}{f'(t)}$$

The equation of tangent at any point 't' on the curve is given by 
$$\boxed{y - g(t) = \frac{g'(t)}{f'(t)}(x - f(t))}$$

(v) If the tangent is perpendicular to the axis of  $x$ , then  $\Psi = \frac{\pi}{2}$

$$\therefore \frac{dy}{dx} = \tan \Psi = \tan \frac{\pi}{2} = \infty \Rightarrow \boxed{\frac{dx}{dy} = 0}$$

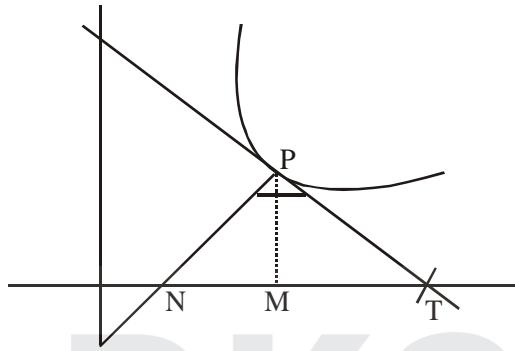
(vi) Q The value of  $\Psi$  always lie in  $(-\pi, \pi]$

(vii) If the tangent at any point on the curve is equally inclined to both the axes then  $\left(\frac{dy}{dx}\right) = \pm 1$

#### 4. LENGTH OF THE TANGENT

$$PT = y \cosec \Psi = y \sqrt{1 + \cot^2 \Psi}$$

$$= \left| y \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \right| \quad \text{or} \quad \left| \frac{y \sqrt{1 + (dy/dx)^2}}{(dy/dx)} \right|$$



#### 5. LENGTH OF SUB-TANGENT

$$TM = y \cot \Psi = \frac{y}{\tan \Psi} = \left| y \frac{dx}{dy} \right| \quad \text{or} \quad \frac{y}{(dy/dx)}$$

#### 6. EQUATION OF NORMAL

The equation of normal

(i) at the point  $P(x_1, y_1)$  on the curve  $Y = f(x)$  is

$$y - y_1 = -\frac{1}{\left( \frac{dy}{dx} \right)_{(x_1, y_1)}} (x - x_1)$$

$$\boxed{y - y_1 = -\left( \frac{dx}{dy} \right)_{(x_1, y_1)} (x - x_1)}$$

$$\text{Slope of normal} = -\frac{1}{\text{slope of tangent}} = -\frac{1}{\left( \frac{dy}{dx} \right)}$$

(ii) If the normal is parallel to the axis of  $y$ , then  $\Rightarrow \Psi = 0$

$$\therefore \frac{dy}{dx} = \tan \Psi = \tan 0 = 0$$

(iii) If the normal is parallel to the axis of  $x$ , then  $\therefore \frac{dx}{dy} = 0$

## 7. LENGTH OF NORMAL

$$PN = y \sec \Psi = y \sqrt{1 + \tan^2 \Psi} = \left| y \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \right|$$

## 8. LENGTH OF SUB-NORMAL

$$MN = y \tan \Psi = \left| y \frac{dy}{dx} \right|$$

## 9. ANGLE OF INTERSECTION OF TWO CURVES

If two curves  $y = f_1(x)$  and  $y = f_2(x)$  intersect at a point p, the angle between their tangents at p is defined as the angle between these two curves at p. But slopes of tangents at P are  $\left( \frac{dy}{dx} \right)_1$  and  $\left( \frac{dy}{dx} \right)_2$

So at p their angle of intersection  $\psi$  is given by

$$\tan \psi = \left| \frac{\left( \frac{dy}{dx} \right)_1 - \left( \frac{dy}{dx} \right)_2}{1 + \left( \frac{dy}{dx} \right)_1 \left( \frac{dy}{dx} \right)_2} \right| \quad \text{or} \quad \boxed{\tan \psi = \pm \frac{m_1 - m_2}{1 + m_1 m_2}}$$

1. If two curves cut perpendicular then  $\psi = \frac{\pi}{2}$

$$\left( \frac{dy}{dx} \right)_1 \left( \frac{dy}{dx} \right)_2 = -1 \text{ or } m_1 m_2 = -1$$

2. If two curves are parallel  $\psi = 0^\circ$

$$\left( \frac{dy}{dx} \right)_1 = \left( \frac{dy}{dx} \right)_2 \text{ or } m_1 = m_2$$

## 10. ROLLE'S THEOREM

If a function  $f(x)$  is defined on  $[a,b]$  satisfying

- (i)  $f$  is continuous on  $[a,b]$
- (ii)  $f$  is differentiable on  $(a,b)$
- (iii)  $f(a) = f(b)$  then there exists  $c \in (a,b)$ ; Such that  $f'(c) = 0$

## 11. LANGRAGE'S MEAN VALUE THEOREM

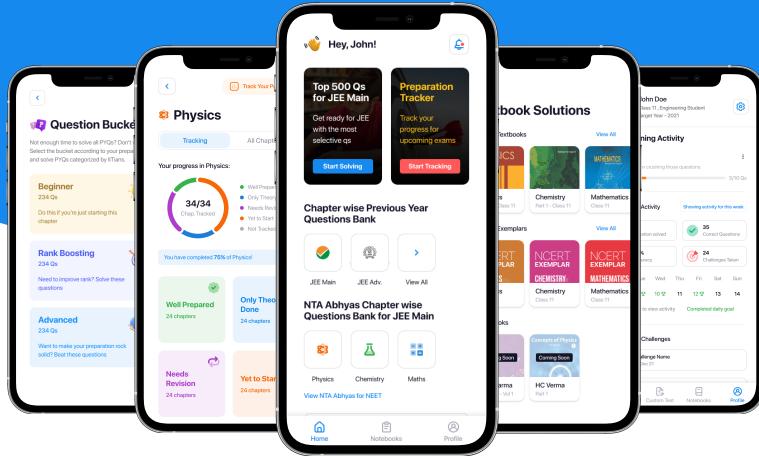
If a function  $f(x)$  is defined on  $[a,b]$  satisfying

- (i)  $f$  is continuous on  $[a,b]$

- (ii)  $f$  is differentiable on  $(a,b)$  then there exists  $c \in (a,b)$  Such that  $f'(c) = \frac{f(b)-f(a)}{b-a}$



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