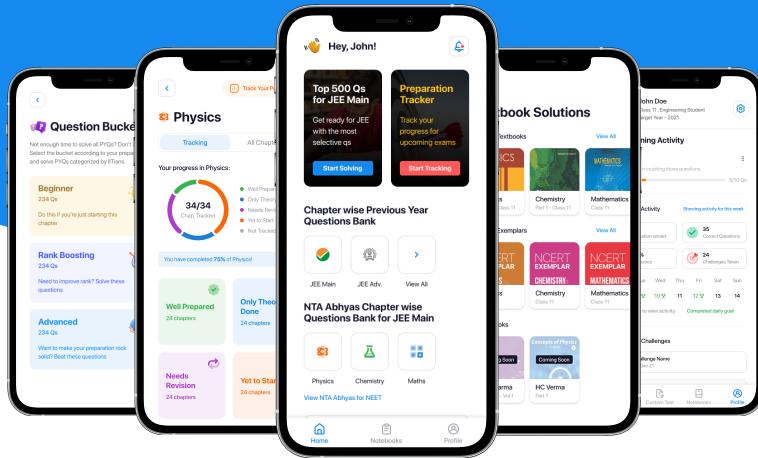




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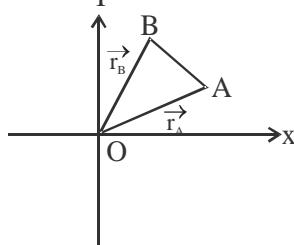


KINEMATICS-2D

- When motion of a body/particle is analysed by a moving system, then motion is said to be a relative motion.
- Relative velocity of A w.r. to B is defined as the time rate of change of relative displacement of A w.r. to B, which is given by

$$\mathbf{V}_{AB} = \frac{d\mathbf{r}_{AB}}{dt} = \frac{d\mathbf{r}_{BA}}{dt} = \frac{d}{dt}(\mathbf{OA} - \mathbf{OB}) = \frac{d\mathbf{r}_A}{dt} - \frac{d\mathbf{r}_B}{dt}$$

$$\mathbf{V}_{AB} = \mathbf{V}_A - \mathbf{V}_B \text{ or } \mathbf{V}_{BA} = \mathbf{V}_B - \mathbf{V}_A$$



\mathbf{r}_A = position vector of A at time t

\mathbf{r}_B = position vector of B at time t

Relative velocity is simply the vector difference of two velocities.

- For one dimension $\mathbf{V}_{AB} = \mathbf{V}_A - \mathbf{V}_B$
 - $\rightarrow A ; |\mathbf{V}_{AB}| = |\mathbf{V}_A - \mathbf{V}_B|$ when motions are along parallel lines
 $\rightarrow B$
 - $\leftarrow B ; |\mathbf{V}_{AB}| = \mathbf{V}_A + \mathbf{V}_B$ when motion are along antiparallel lines.
 $\rightarrow A$

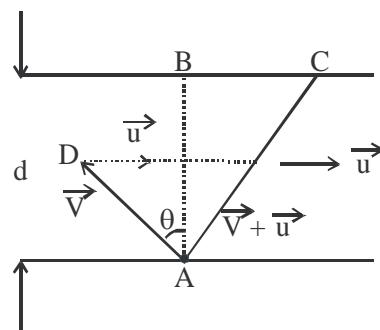
SWIMMER'S PROBLEMS

When boat/swimmer heads in the river to cross from one bank to another. Then motion of boat/swimmer in the direction of resultant of velocity of flow in the river and velocity of boat/swimmer in still water.

$$\mathbf{V}_{s,g} = \mathbf{V}_{s,w} + \mathbf{V}_{w,g} ; \quad \mathbf{V}_{s,g} = \text{velocity of swimmer w.r to ground.}$$

Let $\mathbf{V}_{s,w} = \mathbf{V}$ = velocity of swimmer in still water

$\mathbf{V}_{w,g} = \mathbf{u}$ velocity of water flow.



Swimmer heads along AD making angle θ with vertical in the direction of upstream so as while it crosses the river it has less drift along the direction of river flow.

- Time to cross the opposite bank = $\frac{d}{V \cos \theta}$

Minimum time to cross the river = $\frac{d}{v}$ for which $\theta = 0^\circ$ i.e. For minimum time to cross the river swimmer should head perpendicular to flow of stream.

- Time to reach just. opposite back (only for $v > u$)

$$u = v \sin \theta$$

i.e. $\theta = \sin^{-1} \frac{u}{v}$ and time to reach opposite bank = $\frac{d}{V \sqrt{1 - \left(\frac{u}{v}\right)^2}} = \frac{d}{\sqrt{V^2 - u^2}}$

- For $v < u$ then swimmer heads to reach the opposite bank for minimum drift or through shortest path and hence

$$\frac{dBC}{d\theta} = 0 \text{ where } BC = (u - V \sin \theta) \cdot \frac{d}{V \cos \theta}$$

$$\Rightarrow \sin \theta = \frac{V}{u} \text{ or } \theta = \sin^{-1} \left(\frac{V}{u} \right)$$

Time to reach the opposite bank through shortest path = $\frac{d}{\sqrt{1 - \left(\frac{V}{u}\right)^2}} = \frac{du}{v \sqrt{u^2 - v^2}}$

PROJECTILE MOTION

An oblique projection of a body from surface of earth the following motion of the body is said to be projectile motion and body itself is called projectile θ is the angle of projection u is velocity of projection. After time t the projectile reaches at P with velocity V.

Then from equation of projectile

$$\ddot{a}_x = \frac{d^2 r_x}{dt^2} = 0 \text{ and } \ddot{a}_y = \frac{d^2 r_y}{dt^2} = g(-\hat{j})$$

We have $v_x = u_x = u \cos \theta$ and $v_y = u_y - gt = u \sin \theta - gt$

Hence $v = \sqrt{u_x^2 + v_y^2} = \sqrt{u^2 - 2u \sin 2\theta gt + g^2 t^2}$

$$\text{and } \alpha = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \left(\frac{u \sin \theta - gt}{u \cos \theta} \right)$$

Equation of trajectory or path of projectile is given by $x = u \cos \theta \cdot t$ and $y = u \sin \theta \cdot t - \frac{1}{2} gt^2$

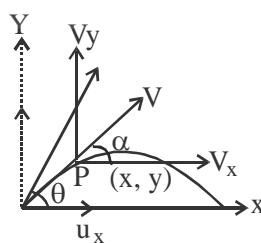
Hence we have the equation by eliminating t.

$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}. \text{ Hence trajectory is a parabolic path.}$$

Range is the horizontal distance from point of projection to the point in the same plane where projectile strikes which is given by

$$R = u \cos \theta \times T; T = \text{time of flight}$$

$$\text{Since } T = \frac{2u \sin \theta}{g} (\text{Sy} = 0 = u_y - \frac{1}{2} g T^2) \Rightarrow u \sin \theta \cdot T - \frac{1}{2} g T^2 = 0 \Rightarrow T = \frac{2u \sin \theta}{g}$$



$$R = \frac{u^2 \sin 2\theta}{g} . \text{ If } \theta \text{ is replaced by } 90^\circ - \theta, R \text{ does not change.}$$

Hence for given initial velocity R remains the same for two possible values of angle of projections if one is θ then other is $\pi/2 - \theta$.

- Equation of trajectory in terms of range $y = x \tan \theta (1 - x/R)$
- Time of ascent = time of descent = $\frac{u \sin \theta}{g} = \frac{u_y}{g}$
- Maximum height – attained by the projectile from plane from where projectile is projected.

$$H = \frac{u^2 \sin^2 \theta}{g} = \frac{u_y^2}{2} \quad (\text{At maximum height } v_y^2 = 0 = u_y^2 - 2gH \Rightarrow (u \sin \theta)^2 - 2gH = 0)$$

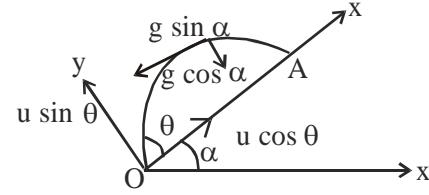
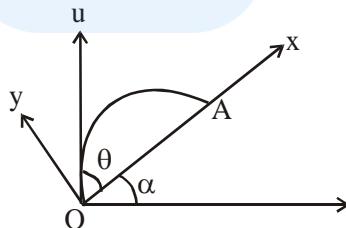
- Along motion of projectile path horizontal velocity remains the same and at highest point it directs horizontally as no vertical velocity at highest point.
- Every elementary section of projectile path is considered as on curve and the necessary centripetal force required to keep a body on the curve path is pointed along radial direction towards centre of elementary curve path, which is provided by component of weight.
- Time after which the velocity of projectile becomes perpendicular to initial velocity.

$$\vec{u} \cdot \vec{v} = 0 \Rightarrow (u \cos \theta \hat{i} + u \sin \theta \hat{j}) \cdot [u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}] = 0$$

$$\Rightarrow u^2 - u \sin \theta gt = 0 \text{ or } t = \frac{u}{g \sin \theta}$$

Projectile Motion on the inclined plane

(i) Projectile Motion up the plane



Taking x-axis along inclined plane and y-axis perpendicular to it at point O.

$$\vec{a}_x = \text{acceleration along x-axis} = g \sin \alpha (-\hat{i})$$

$$\vec{a}_y = g \cos \alpha (-\hat{j})$$

The time of flight is the time taken for projectile travel from O to A

$$\therefore \text{From } S_y = u_y t + \frac{1}{2} a_y t^2 \text{ for O to A, } S_y = 0$$

$$\therefore \Rightarrow t = \frac{2u \sin \theta}{g \cos \alpha}$$

As at $t = 0$, Projectile is at O.

Time of flight = $\frac{2u \sin \theta}{g \cos \alpha}$; Range = OA = R is given

$$\text{by } S_x = u_x \cdot t + \frac{1}{2} a_x t^2$$

$$S_x = R = u \cos \theta \left(\frac{2u \sin \theta}{g \cos \alpha} \right) - \frac{1}{2} g \sin \alpha \left(\frac{2u \sin \theta}{g \cos \alpha} \right)^2$$

$$= \frac{2u^2 \sin \theta}{g \cos^2 \alpha} [\cos \theta \cdot \cos \alpha - \sin \theta \cdot \sin \alpha] = \frac{2u^2 \sin \theta}{g \cos^2 \alpha} [\cos(\theta + \alpha)]$$

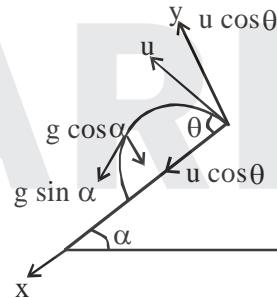
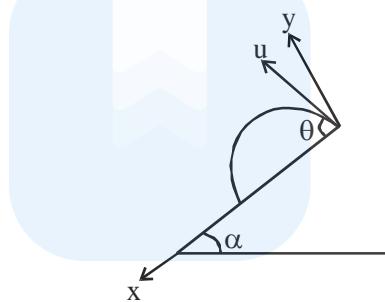
$$R = \frac{u^2}{g \cos^2 \alpha} [\sin(2\theta + \alpha) - \sin \alpha]$$

$$\text{For the maximum-range } \sin(2\theta + \alpha) = 1; \quad \theta = 45^\circ - \alpha/2$$

$$R_{\max} \text{ for projection inclined up to plane is } R_{\max} = \frac{u^2}{g(1 + \sin \alpha)}$$

(ii) Projectile Motion down the inclined plane

The equation of projectile



$$\vec{a}_x = g \sin \alpha \hat{i} \text{ & } \vec{a}_y = g \cos \alpha (-\hat{j}); \text{ Range down the plane} = \frac{2u^2 \sin \theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$$

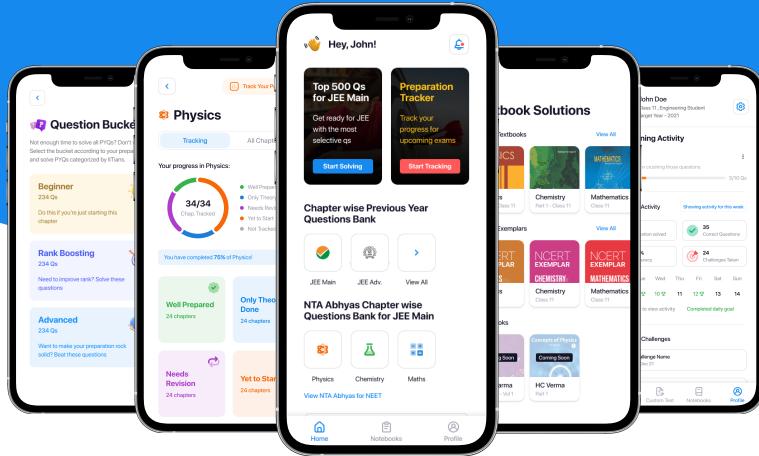
$$\text{Time of flight} = \frac{2u \sin \theta}{g \cos \alpha}$$

$$R_{\max} \text{ down the plane} = \frac{u^2 (1 + \sin \alpha)}{g \cos^2 \alpha} = \frac{u^2}{g(1 - \sin \alpha)}$$

It occurs when direction of projection bisects the angle between the vertical and downward slope of the plane.



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