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# QUADRATIC EQUATIONS & EXPRESSIONS

## 1. QUADRATIC EQUATION

An equation  $ax^2 + bx + c = 0$  (where  $a \neq 0$ , and  $a, b, c \in C$ ), is called a quadratic equation. Here  $a$ ,  $b$  and  $c$  are called coefficients of the equation. This equation always has two roots. Let the roots be  $\alpha$  and  $\beta$ .

$$\text{Then } \alpha + \beta = -\frac{b}{a} \quad (\text{sum of the roots}) \quad \alpha\beta = \frac{c}{a} \quad (\text{product of the roots})$$

The quantity  $D = b^2 - 4ac$  is called discriminant of the equation.

$$\text{Roots of the equation are given by } x = \frac{-b \pm \sqrt{D}}{2a}$$

## 2. NATURE OF ROOTS

- (i) If  $D > 0$ , roots of the equation are real and distinct.
- (ii) If  $D = 0$ , roots of the equation are real and equal.
- (iii) If  $D < 0$ , roots of the equation are imaginary.
- (iv) If  $D$  is square of a rational number, roots of the equation are rational. (Provided  $a$ ,  $b$  and  $c$  are rational numbers)
- (v) If  $D > 0$  and  $D$  is not a perfect square, then roots are irrational

## 3. FORMATION OF AN EQUATION WITH GIVEN ROOTS

A quadratic equation whose roots are  $\alpha$  and  $\beta$  is given by

$$(x - \alpha)(x - \beta) = 0$$

$$\Rightarrow x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e.,  $x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$

## TRANSFORMATION OF AN EQAUTION

If  $\alpha, \beta$  are roots of the equation  $ax^2 + bx + c = 0$ , then the equation whose roots are

- (i)  $\frac{1}{\alpha}, \frac{1}{\beta}$  is  $cx^2 + bx + a = 0$  (Replace  $x$  by  $\frac{1}{x}$ )
- (ii)  $-\alpha, -\beta$  is  $ax^2 - bx + c = 0$  (Replace  $x$  by  $-x$ )
- (iii)  $k + \alpha, k + \beta$  is  $a(x - k)^2 + b(x - k) + c = 0$  {Replace  $x$  by  $(x - k)$ }
- (iv)  $\alpha^n, \beta^n$  ( $n \in N$ ) is  $a(x^{1/n})^2 + b(x^{1/n}) + c = 0$  (Replace  $x$  by  $x^{1/n}$ )

(v)  $\alpha^{1/n}, \beta^{1/n}$  ( $n \in N$ ) is  $a(x^n)^2 + b(x^n) + c = 0$  (Replace  $x$  by  $x^n$ )

(vi)  $k\alpha, k\beta$  is  $ax^2 + kbx + k^2c = 0$  (Replace  $x$  by  $\frac{x}{k}$ )

(vii)  $\frac{\alpha}{k}, \frac{\beta}{k}$  is  $k^2ax^2 + kbx + c = 0$  (Replace  $x$  by  $kx$ )

## 5. ROOTS IN SPECIAL CASES

$ax^2 + bx + c \equiv a(x - \alpha)(x - \beta)$  (If  $\alpha$  and  $\beta$  are roots of the equation)

| If $b^2 - 4ac > 0$    | Then                                  |   |
|-----------------------|---------------------------------------|---|
| $a > 0, b > 0, c > 0$ | $\alpha + \beta < 0, \alpha\beta > 0$ | Both roots are negative   |
| $a > 0, b > 0, c < 0$ | $\alpha + \beta < 0, \alpha\beta < 0$ | Roots are opposite in sign. Magnitude of negative root is more than the magnitude of positive root. |
| $a > 0, b < 0, c > 0$ | $\alpha + \beta > 0, \alpha\beta > 0$ | Both roots are positive   |
| $a > 0, b < 0, c < 0$ | $\alpha + \beta > 0, \alpha\beta < 0$ | Roots are opposite in sign. Magnitude of positive root is more than the magnitude of negative root. |

## 6. CONDITION FOR COMMON ROOT(S)

- (i) Let  $ax^2 + bx + c = 0$  and  $dx^2 + ex + f = 0$  have a common root  $\alpha$  (say). Then  $a\alpha^2 + b\alpha + c = 0$  and  $d\alpha^2 + e\alpha + f = 0$ . Then

$$\Rightarrow (dc - af)^2 = (bf - ce)(ae - bd)$$

which is the required condition for the two equations to have a common root.

- (ii) Condition for both the roots to be common is  $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

## 8. GREATEST AND LEAST VALUES OF QUADRATIC EXPRESSION

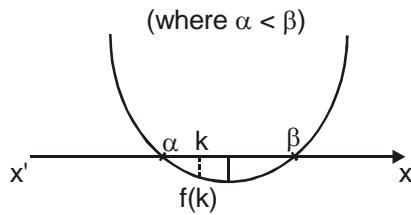
- (i) If  $a > 0$ , then the quadratic expression  $ax^2 + bx + c$  has least value  $\frac{4ac - b^2}{4a}$  at  $x = -\frac{b}{2a}$

- (ii) If  $a < 0$ , then the quadratic expression  $ax^2 + bx + c$  has greatest value  $\frac{4ac - b^2}{4a}$  at  $x = -\frac{b}{2a}$

## 10. LOCATION OF ROOTS (INTERVAL IN WHICH ROOTS LIE)

In some problems we want the roots  $\alpha$  and  $\beta$  of the equation  $ax^2 + bx + c = 0$  to lie in a given interval. For this we impose conditions on  $a$ ,  $b$  and  $c$ . Since  $a \neq 0$ , we can take  $f(x) = ax^2 + bx + c$ .

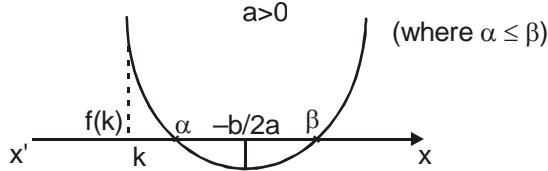
- (i) A given number  $k$  will lie between the roots if  $f(k) < 0$ ,  $D > 0$ .



In particular, the roots of the equation will be of opposite signs if 0 lies between the roots  $\Rightarrow f(0) < 0$ .

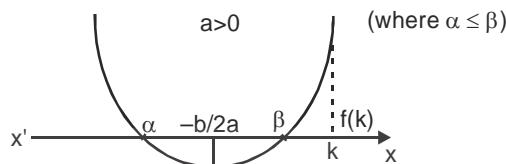
- (ii) Both the roots are greater than given number k if the following three conditions are satisfied

$$D \geq 0, -\frac{b}{2a} > k \text{ and } f(k) > 0$$



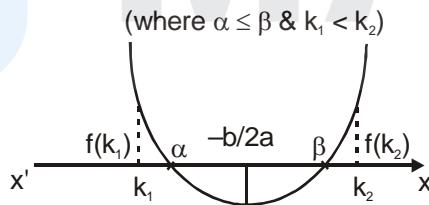
- (iii) Both the roots will be less than a given number k if the following conditions are satisfied:

$$D \geq 0, -\frac{b}{2a} < k \text{ and } f(k) > 0$$

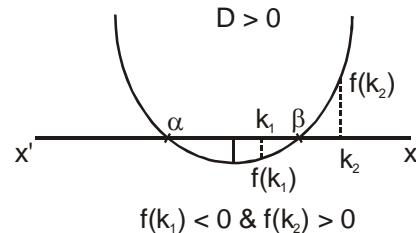
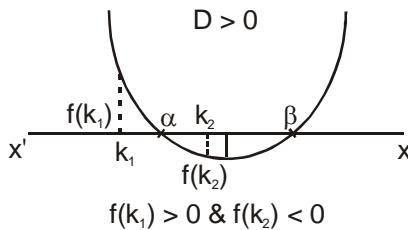


- (iv) Both the roots will lie in the given interval  $(k_1, k_2)$  if the following conditions are satisfied:

$$D \geq 0, k_1 < -\frac{b}{2a} < k_2 \text{ and } f(k_1) > 0, f(k_2) > 0$$



- (v) Exactly one of the root lies in the given interval  $(k_1, k_2)$  if  $f(k_1)f(k_2) < 0$



## 11. THEORY OF EQUATIONS

- (i) If  $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ , ( $a_0, \dots, a_n \in \mathbb{R}$  and  $a_n \neq 0$ ) then  $p(x) = 0$  has n roots. (real /complex)
- (ii) A polynomial equation in x of odd degree has at least one real root ( it has odd no. of real roots).
- (iii) If  $x_1, \dots, x_n$  are the roots of  $p(x) = 0$ , then  $p(x)$  can be written in the form  $p(x) \equiv a_n(x - x_1)(x - x_2) \dots (x - x_n)$ .
- (iv) If  $\alpha$  is a root of  $p(x) = 0$ , then  $(x - \alpha)$  is a factor of  $p(x)$  and vice - versa.

- (v) If  $x_1, \dots, x_n$  are the roots of  $p(x) \equiv a_n x^n + \dots + a_0 = 0$ ,  $a_n \neq 0$ .

$$\text{Then } \sum_{i=1}^n x_i = -\frac{a_{n-1}}{a_n}, \quad \sum_{1 \leq i < j \leq n} x_i x_j = \frac{a_{n-2}}{a_n}, \quad x_1 x_2 \dots x_n = (-1)^n \frac{a_0}{a_n}.$$

- (vi) If equation  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$  has more than  $n$  distinct roots then  $f(x)$  is identically zero.
- (vii) If  $p(a)$  and  $p(b)$  ( $a < b$ ) are of opposite signs, then  $p(x) = 0$  has odd number of real roots in  $(a, b)$ , i.e. it has at least one real root in  $(a, b)$  and if  $p(a)$  and  $p(b)$  are of same sign then  $p(x) = 0$  has even number of real roots in  $(a, b)$ .
- (ix) If coefficients of  $p(x)$  (polynomial in  $x$  written in descending order) have ' $m$ ' changes in signs, then polynomial equation of  $n^{\text{th}}$  degree  $p(x) = 0$  have at the most ' $m$ ' positive real roots and if  $p(-x)$  have ' $t$ ' changes in sign, then  $p(x) = 0$  have at most ' $t$ ' negative real roots. By this we can find maximum number of real roots.  
No. of complex roots =  $n - (m + t)$  where  $m + t < n$ .
- (x) Imaginary roots of a quadratic equation always occur in conjugate pair.
- (xi) Irrational roots of a quadratic equation always occur in conjugate pair.

## 12. QUADRATIC EXPRESSION IN TWO VARIABLE

The general form of a quadratic expression in two variables  $x, y$  is

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c$$

The condition that this expression may be resolved into two linear rational factors is

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

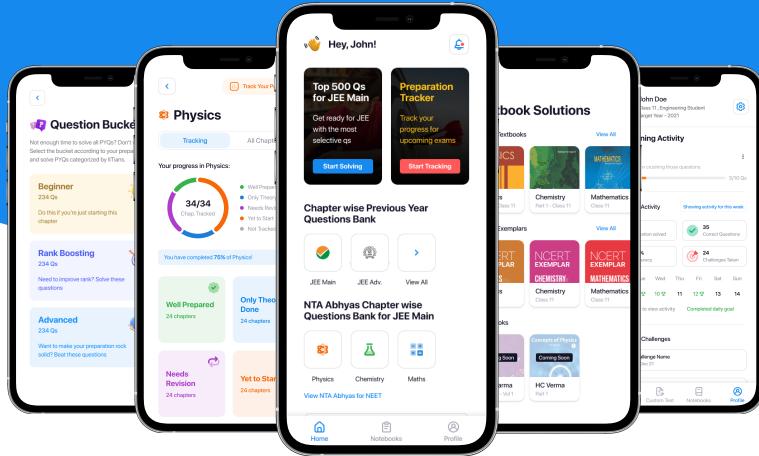
or

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

- (i) If roots of quadratic equations  $a_1 x^2 + b_1 x + c_1 = 0$  and  $a_2 x^2 + b_2 x + c_2 = 0$  are in the same ratio  
 $\left( \text{i.e. } \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} \right)$  then  $\frac{b_1^2}{b_2^2} = \frac{a_1 c_1}{a_2 c_2}$
- (ii) If one root is  $k$  times the other root of quadratic equation  $a_1 x^2 + b_1 x + c_1 = 0$  then  $\frac{(k+1)^2}{k} = \frac{b_1^2}{a_1 c_1}$



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