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Continuity

1. DEFINITION

If the graph of a function has no break or jump, then it is said to be continuous function. A function which is not continuous is called a discontinuous function.

2. CONTINUITY OF A FUNCTION AT A POINT

A Function $f(x)$ is said to be continuous at some point $x=a$ of its domain if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

i.e., If $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$

i.e., If $f(a - 0) = f(a + 0) = f(a)$

i.e., If $\{ \text{LHL at } x=a \} = \{ \text{RHL at } x = a \} = \{ \text{value of the function at } x = a \}$.

3. CONTINUITY FROM LEFT AND RIGHT

Function $f(x)$ is said to be

(i) Left Continuous at $x=a$ if $\lim_{x \rightarrow a^-} f(x) = f(a)$ i.e. $f(a - 0) = f(a)$

(ii) Right Continuous at $x=a$ if $\lim_{x \rightarrow a^+} f(x) = f(a)$ i.e. $f(a + 0) = f(a)$

Thus a function $f(x)$ is continuous at a point $x=a$ if it is left continuous as well as right continuous at $x=a$.

4. CONTINUITY IN AN INTERVAL

(1) A function $f(x)$ is continuous in an open interval (a, b) if it is continuous at every point of the interval.

(2) A function $f(x)$ is continuous in a closed interval $[a, b]$ if it is

- (i) continuous in (a, b)
- (ii) right continuous at $x=a$
- (iii) left continuous at $x=b$

5. CONTINUOUS FUNCTIONS

A function is said to be continuous function if it is continuous at every point in its domain. Following are examples of some continuous functions:

- (i) $f(x) = x$ (Identify function)
- (ii) $f(x) = c$ (Constant function)
- (iii) $f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a^n$ (Polynomial function)
- (iv) $f(x) = \sin x, \cos x$ (Trigonometric function)
- (v) $f(x) = a^x, e^x, e^{-x}$ (Exponential function)
- (vi) $f(x) = \log x$ (Logarithmic function)

- (vii) $f(x) = \sinh x, \cosh x, \tanh x$ (Hyperbolic function)
- (viii) $f(x) = |x|, x + |x|, x - |x|, x|x|$ (Absolute value functions)

6. DISCONTINUOUS FUNCTIONS

A function is said to be a discontinuous function if it is discontinuous at atleast one point in its domain. Following are examples of some discontinuous functions:

No.	Functions	Points of discontinuity
(i)	$[x]$	Every Integers
(ii)	$x - [x]$	Every Integers
(iii)	$\frac{1}{x}$	$x = 0$
(iv)	$\tan x, \sec x$	$x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$
(v)	$\cot x, \operatorname{cosec} x$	$x = 0, \pm \pi, \pm 2\pi, \dots$
(vi)	$\sin \frac{1}{x}, \cos \frac{1}{x}$	$x = 0$
(vii)	$e^{1/x}$	$x = 0$
(viii)	$\coth x, \operatorname{cosech} x$	$x = 0$

7. PROPERTIES OF CONTINUOUS FUNCTIONS

The sum, difference, product, quotient (if $D_g \neq 0$) and composite of two continuous functions are always continuous functions. Thus if $f(x)$ and $g(x)$ are continuous functions then following are also continuous functions:

- | | |
|--|---|
| (i) $f(x) + g(x)$ | (ii) $f(x) - g(x)$ |
| (iii) $f(x).g(x)$ | (iv) $\lambda f(x)$, where λ is a constant |
| (v) $\frac{f(x)}{g(x)}$, if $g(x) \neq 0$ | (vi) $f[g(x)]$ |

8. IMPORTANT POINT

The discontinuity of a function $f(x)$ at $x = a$ can arise in two ways

- (i) If $\lim_{x \rightarrow a^-} f(x)$ exist but $\neq f(a)$ or $\lim_{x \rightarrow a^+} f(x)$ exist but $\neq f(a)$, then the function $f(x)$ is said to have a removable discontinuity.
- (ii) The function $f(x)$ is said to have an unremovable discontinuity when $\lim_{x \rightarrow a} f(x)$ does not exist.

i.e. $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$

Differentiability at a point

Let $f(x)$ be a real valued function defined on an open interval (a, b) and let $c \in (a, b)$. Then $f(x)$ is said to be

differentiable or derivable at $x = c$, iff $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ exists finitely.

This limit is called the derivative or differential coefficient of the function $f(x)$ at $x = c$, and is denoted by $f'(c)$ or

$$Df(c) \text{ or } \left\{ \frac{d}{dx} f(x) \right\}_{x=c}$$

$$\lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{-h} = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{-h}$$

is called the left hand derivative of $f(x)$ at $x = c$ and is denoted by $f'(c^-)$ or $Lf'(c)$ while.

$$\lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c} \text{ or } \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

is called the right hand derivative off (x) at $x = c$ and is denoted by $f'(c^+)$ or $Rf'(c)$.

Thus, $f(x)$ is differentiable at $x = c \Leftrightarrow Lf'(c) = Rf'(c)$.

If $Lf'(c) \neq Rf'(c)$ we say that $f(x)$ is not differentiable at $x = c$.

Differentiability in a set

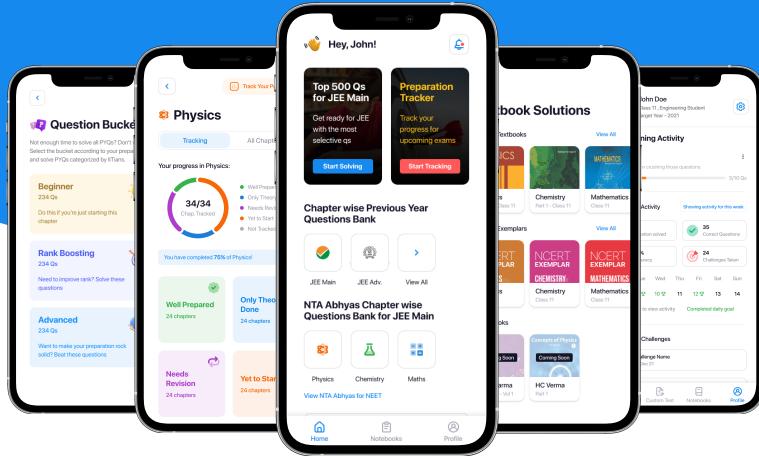
A fuction $f(x)$ defined on an open interval (a, b) is said tobe differentiable or derivable inopen interval (a, b) if it is differentiable at each point of (a, b)



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