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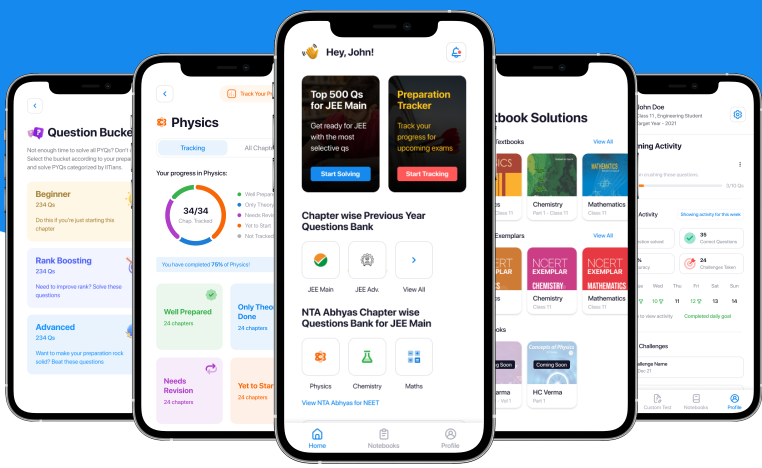


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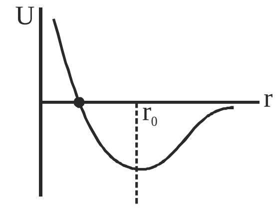
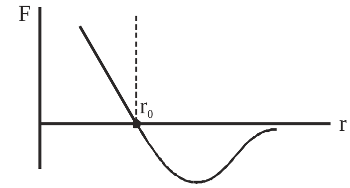
# ELASTICITY

## INTERMOLECULAR / INTERATOMIC FORCES :

In a solid in equilibrium the separation between atom or molecules is equal to $r_0$. At this separation the force between atoms or molecules is zero and their potential energy is minimum. If atoms or molecules are brought closer than $r_0$, their PE increases and repulsive force comes into play.




If atoms/molecules are taken farther than $r_0$, their PE increases and attractive force comes into play. Molecules/Atoms in a solid therefore behave as if they are joined by stiff springs.

If separation between atom/molecules is changed from $r_0$, a restoring force comes into play which tends to bring them back to their equilibrium separation.

## SOLID :

The state of matter which has s definite shape and size. The constituent atoms/molecules are so closely packed that interatomic forces maintain the relative separation between atoms/molecules thereby giving it a definite shape and size.

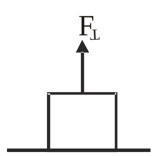
## ELASTICITY :

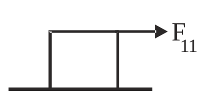
When a solid is acted upon by a system of external forces in equilibrium, the intermolecular separations change giving rise to internal restoring forces.

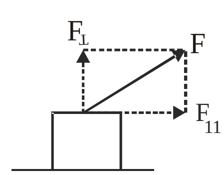
- The change in intermolecular separation results in change in shape or size of body i.e., deformation of body.
- The system of external forces, which tend to change shape or size of body is called deforming forces.
- The system of internal forces, which tend to bring the body back to original shape or size is called restoring force.
- If the body regains its original shape or size on removal of deforming forces (i.e., internal restoring forces bring body back to original shape or size) the body is said to be elastic. Otherwise the body is said to be plastic.

## STRESS :

The restoring force set up per unit cross sectional area of body is called stress. For small and slow deformation deforming force is equal to restoring force.




$$\text{Stress} = \frac{\text{Restoring force}}{\text{Cross sectional area}} = \frac{\text{Deforming force}}{\text{Cross sectional area}} = \frac{F}{A}$$

Units and Dimensions of stress are :

(i) $Nm^{-2}$

(ii) $ML^{-1}T^{-2}$

- Stress is not a vector
- Deforming force acting perpendicular to cross sectional area gives rise to normal stress.

$$\text{Normal stress} = \frac{F_{\perp}}{A}$$

- Deforming force acting the parallel to cross sectional area produces tangential stress.

$$\text{Tangential stress} = \frac{F_{||}}{A}.$$

## STRAIN :

A body subjected to stress suffers change in dimensions or shape. The fractional or relative change in dimension or shape is called strain.

$$\text{Strain} = \frac{\text{change in dimension/shape/size}}{\text{original dimension/shape/size}}$$

- Strain has no unit and is a scalar.

(i) Longitudinal Strain

$$= \frac{\text{Change in linear dimension}}{\text{Original linear dimension}}$$

$$= \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

Longitudinal strain is produced by normal stress in solids only.

(ii) Volumetric Strain :

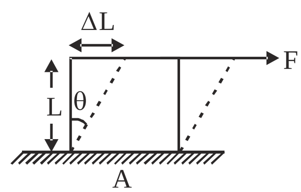
$$= \frac{\text{Change in volume}}{\text{Original volume}}$$

$$= \frac{\Delta V}{V_0} = \frac{V - V_0}{V_0}$$

It is produced by normal stress in all states of matter.

(iii) Shearing Strain :

$$= \frac{\text{Displacement of free surace of body over which tangential deforming force is applied}}{\text{Distance of this free surface from fixed surface}}$$




$$= \frac{\Delta L}{L} = \tan\theta \approx \theta.$$

It is also called the angle of shear.

It is produced by tangential stress in solids only. It refers to change in shape of body without any change in volume.

## HOOKE'S LAW :

For small deformations, the stress developed in a body is directly proportional to strain produced.

$$\text{Stress} \propto \text{Strain}$$

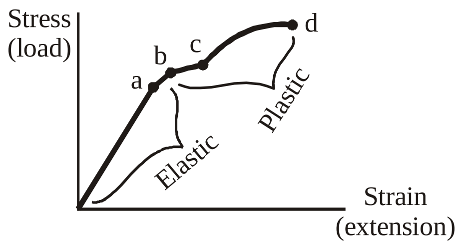
$$\text{Stress} = E \times \text{Strain}$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \text{Modulus of elasticity.}$$

Hooke's Law can also be expressed as : The tension developed in a body is directly proportional to extension produced. (i.e., Load is directly proportional to extension). Modulus of elasticity depends on the nature of stress and strain, material of body and state of body. It has units $Nm^{-2}$ and is a scalar.

The range of deforming forces over which Hooke's Law holds is called proportional range. The limit beyond which body does not remain elastic is called elastic limit.

## Variation of Stress and Strain




| | | |
|---|---|---|
| a : | Proportionate limit | (Hooke's law is valid) |
| b : | Ealstic limit | (body remains elastic) |
| c : | Fracture point | (body breaks) |

If large deformation takes place between the elastic limit and fracture point, the material is ductile. If the material breaks soon after the elastic limit is crossed, it is brittle.

The stress corresponding to fracture point is called breaking stress.

## MODULII OF ELASTICITY

Young's Modulus (Y) : When a normal stress produces a longitudinal strain in a solid, the modulus of elasticity is called Young's modulus.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L}$$

Area perpendicular to L or F is taken to be the area of cross section.

$$\Delta L = \frac{FL}{YA}$$

$$F = YA\left(\frac{\Delta L}{L}\right) = \frac{YA}{L}\Delta L$$

Comparing with $K\Delta x$ yields

$$K = \frac{YA}{L} \qquad \Rightarrow \qquad K \propto \frac{1}{L}.$$

Bulk Modulus (B) : When normal stress produces volumetric strain modulus of elasticity is called Bulk Modulus.

$$B = \frac{\text{Normal stress}}{\text{Volume strain}}$$

$$= -\frac{F/A}{\Delta V/V} = -\frac{\Delta p}{\Delta V/V} = -\frac{V\Delta p}{\Delta V}.$$

Negative sign is put to keep B positive (volume decreases on increasing pressure).

Modulus of Rigidity $(\eta)$ : When tangential stress produces change in shape without change in volume in solid the modulus of elasticity is called modulus of rigidity.

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$= \frac{F/A}{\Delta L/L} = \frac{F/A}{\theta}.$$

Compressibility (C) : The reciprocal of bulk modulus of a material is called its compressibility.

Poisson's Ratio ($\sigma$) :

The extension in length is called longitudinal strain.

When length of a wire is increased it's area of cross section or radius decreases i.e. Extension in length is accompanied by contraction in radius or area of cross section Contraction in area of cross sectionor radius gives rise to lateral (or perpendicular) strain.

The ratio of lateral strain to longitudinal strain is called Poisson's Ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\Delta D/D}{\Delta L/L} = -\frac{\Delta r/r}{\Delta L/L}.$$

Negative sign is put to keep $\sigma$ positive as radius decreases on increasing length.

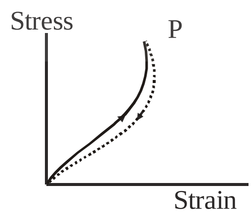
Value of $\sigma$ lies between 0.1 and 0.3 for many common materials.

$Y, B, \eta$ and $\sigma$ are related as :

$$Y = 3B(1-2\sigma) = 2\eta(1+\sigma).$$

## ELASTIC HYSTERSIS :

For many materials the lack of retraceability of stress and strain curve for increasing and decreasing stress is called elastic hystersts. The area enclosed by hystersis loop is proportional to the energy density dissipated in the material. Materials having large hysterisis loss are used as vibration absorbers in machines.




## ELASTIC POTENTIAL ENERGY :

In a uniform wire of length L and cross sectional area A, the restoring force acting when extension in it is x is

$$F = \frac{YA}{L}x.$$

Work done in a further very small extension dx.

$$dW = Fdx = \frac{YA}{L}xdx.$$

Total work done for extension from O to x.

$$W = \frac{1}{2}\frac{YA}{L}x^2 = \frac{1}{2}\times\frac{YAx}{L}\times x = \frac{1}{2}\times\frac{Yx}{L}\times\frac{x}{L}\times AL = \frac{1}{2}Yx.\left(\frac{x}{L}\right)^2\times AL$$

$$= \frac{1}{2}\times \text{ stretching force } \times \text{ stretch.}$$

This work is stored in the wire as Elastic potential energy

$$U = W = \frac{1}{2} \times \text{ stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times Y \times \text{ strain}^2 \times \text{volume.}$$

Energy density in stretched wire

$$u = \frac{U}{V} = \frac{1}{2} \times \text{ stress} \times \text{strain}$$

$$= \frac{1}{2} \times \text{ } Y \times \text{strain}^2.$$

- Variation of density with increase in pressure,

$$\rho = \rho_0 (1 + C\,\Delta P)$$

- In case of a rod of length L and radius r fixed at one end , the angle of shear $\phi$ is related to angle of twist $\theta$ by the relation, $L\phi = r\,\theta$
- In case of twisting of a cylinder of length L and radius r, elastic restoring couple per unit twist is given by,

$$C = \frac{\pi\eta r^4}{2L}$$

- In case of bending of a beam of length L, breadth b and thickness d, by a load Mg at the middle, the depression

$$\delta = \frac{MgL^3}{4bd^3Y}$$

and for a beam of circular cross section of radius r and length L.

$$\delta = \frac{MgL^3}{12\pi r^3 Y}$$

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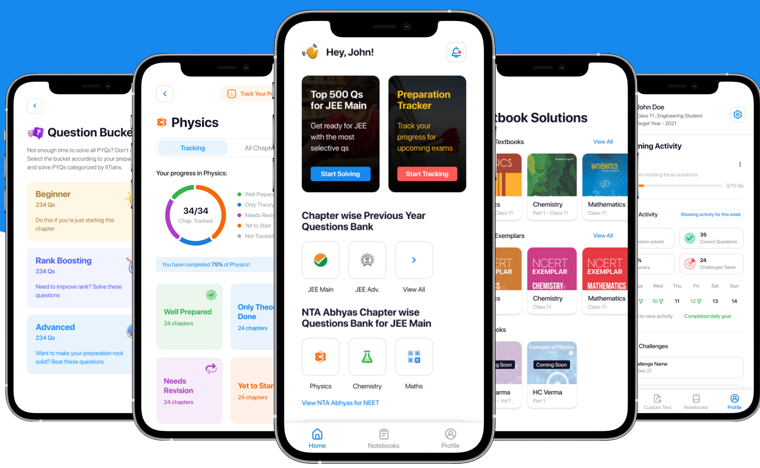


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