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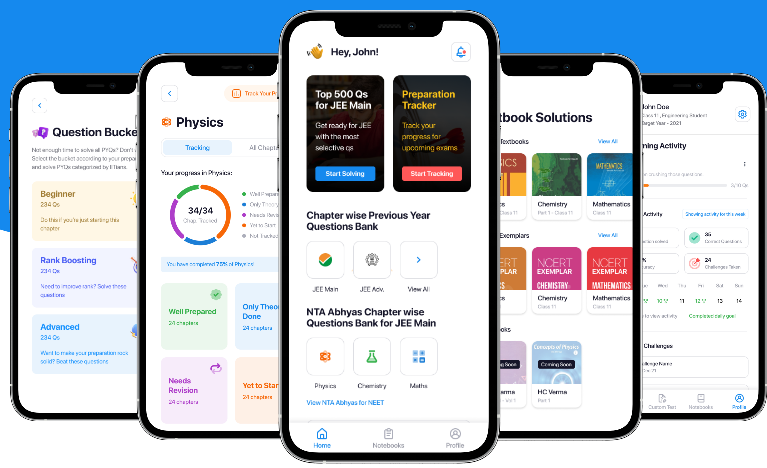

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WAVE OPTICS

NATURE OF LIGHT :

Light is an electromagnetic wave which is sinusoidally varying electric and magnetic fields propagated from one part to another part. The electric and the magnetic field are given by

$$E_y = E_0 \sin(kx \pm wt)$$

$$B_z = B_0 \sin(kx \pm wt)$$

It propagates as transverse non mechanical wave in a medium at a speed given by $V = \dfrac{1}{\sqrt{\mu \in}}$;

The electric and magnetic fields are related as $E_0 = VB_0$

REFRACTIVE INDEX OF A MEDIUM :

Refractive index of a medium is defined as

$$\mu = \frac{\text{speed of light in vaccum}}{\text{speed of light in med}} \Rightarrow \mu = \frac{c}{v}$$

INTERFERENCE :

The modification in the distribution of intensity of light in the region of superposition of two coherent light waves is called interference. At some points the waves superpose in such a way that the resultant intensity is greater than the sum of the intensities due to separate waves (constructive interference) while at some other points intensity is less than the sum of the separate intensities (destructive interference).

YOUNG' DOUBLE SLIT EXPERIEMENT :

Let the two waves each of angular frequency ω from sources S_1 and S_2 reach the point P. Equations are given by $y_1 = A_1 \sin[\omega t - kx]$ and, $y_2 = A_2 \sin[\omega t - k(x + \Delta x)]$

So the resultant wave at P by principle of superposition will be

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx - \phi);\ \text{where } \phi \text{ is initail phase difference}$$

$$y = A \sin(\omega t - kx - \alpha) \quad \text{where,} \quad A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$$

$$\text{and,} \quad \alpha = \tan^{1}\left[A_2 \sin\phi / (A_1 + A_2 \cos\phi)\right]$$

Superposition of two waves of equal frequencies propagating almost in the same direction, results in harmonic wave of same frequency ω and wavelength $(\lambda = 2\pi / k)$ but amplitude A. The intensity of resultant wave

$$I = K\left[A_1^2 + A_2^2 + 2A_1A_2 \cos\phi\right]$$

$$I = I_1 + I_2 + 2\left(\sqrt{I_1 I_2}\right)\cos\phi$$

The resultant intensity at P is not just the sum of intensities due to separate waves $(I_1 + I_2)$ but different and depends on phase difference ϕ and the position of the point P.

CONDITION FOR INTERFERENCE

(a) Intensity will be maximum when :

$$\cos\phi = \max. = 1$$

or $\phi = \pm 2\pi n$ with n = 0, 1, 2

$$\frac{2\pi}{\lambda}(\Delta x) = \pm 2\pi n$$

$$\Delta x = \pm n\lambda$$

$$I_{max} = \left(I_1 + I_2 + 2\sqrt{I_1 I_2}\right)$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \propto (A_1 + A_2)^2$$

Intensity will be maximum at those points where path difference is an integral multiple of wavelength λ and maximum intensity is greater than the sum of two intensities (I_1 + I_2). These points are called points of constructive interference or interference maxima.

(b) Intensity will be minimum when :

$$\cos\phi = \min. = -1;\ \phi = \pm\pi, \pm 3\pi, \pm 5\pi;\ \phi = \pm(2n-1)\pi;\ \frac{2\pi}{\lambda}(\Delta x) = \pm(2n-1)\pi;\ \Delta x = \pm(2n-1)\lambda/2$$

$$I_{min} = \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\right) \Rightarrow I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \propto (A_1 - A_2)^2$$

Intensity will be minimum at those points where path difference is an odd integral multiple of $(\lambda/2)$ and minimum intensity is less than the sum of two intensities (I_1 + I_2). These points are called points of destructive interference or interference minima.

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \qquad I_{min} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}\ ;\ \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$$

- All maxima are equally spaced (as path difference between two consecutive maxima is λ) and equally bright the two waves from S_1 and S_2 have same frequency and start in the same phase at P they have a constant phase difference $\phi = (2\pi/\lambda)\Delta x$, developed due to different paths traversed by them. Such waves are said to be 'Coherent' and produce sustained interference effects.

If d is the separation between the slits and D (>>d) is the distance of screen from the plane of slits as

$\Delta x = d\sin\theta \quad \sin\theta = (\Delta x / d)$

for small θ, $\sin\theta = \tan\theta = \theta = (y/D)$

$$\frac{y}{D} = \frac{\Delta x}{d} \qquad y = \frac{D}{d}(\Delta x)$$

- If the point P is nth bright fringe, $\Delta x = n\lambda$ and hence

$$(y_n)_{Bright} = \frac{D}{d}(n\lambda) \qquad n = 0, 1, 2, \text{etc.}$$

- If the point P is nth minima

$$(y_n)_{Dark} = \frac{D}{d}(2n-1)\frac{\lambda}{2} \qquad n = 1, 2, ... \text{etc.}$$

- Fringe-width β is defined as the distance between two consecutive maxima (or minima) on the screen

$$\beta = \Delta y \qquad \Delta x = \lambda$$

$$\beta = \frac{D}{d}(\lambda)$$

- As linear position y is related to the angular position θ by $\theta = (y/D)$, i.e., $\Delta\theta = (\Delta y/D)$, the so called angular fringe-width

$$\theta_0 = \frac{\beta}{D} = \frac{\lambda}{d}$$ Fringe-width is independent of n.

- If the transparent sheet of refractive index μ and thickness t is introduced in one of the paths of interfering waves, optical path will become μt instead of t for the portion in which glass is inserted so the optical path will increase by $(\mu-1)t$. Due to this, a given fringe from its present position y will shift to a new position y', the lateral shift of the fringe is

$$y_0 = y' - y = \frac{D}{d}(\mu-1)t = \frac{\beta}{\lambda}(\mu-1)t$$

entire fringe-pattern is displaced by y_0 towards the side in which the thin sheet is introduced without any change in fringe width.

DIFFRACTION :

The flaring out or encroachment of light in the shadow zone as it passes around obstacles or through small apertures is called diffraction.

As a result of diffraction, the edges of the shadow do not remain well defined and sharp but become blurred and fringed. If the width of the aperture is comparable to the wavelength of light, most of the incident wavefront will be obstructed and in accordance with Huygens' wave theory a cylindrical (or spherical) wavefront depending on the aperture (slit or hole) will originate from it as the direction of wave motion is normal to the wavefront, after passing through the aperture light will flare out. This flaring out or spreading of light is the so called diffraction.

In case of diffraction at single slit theory shows that intensity at a point on the screen is given by:

$$I_{(\theta)} = I_m\left[\frac{\sin\alpha}{\alpha}\right]^2; \alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}(d\sin\theta)$$

From this it is clear that I will be minimum when for $\alpha \neq 0$,

$$\sin\alpha = 0, \quad \alpha = n\pi \qquad n = 1, 2, ...$$

- Angular position of minima in case of diffraction at single slit is given by:

$$\frac{\pi}{\lambda}(d\sin\theta) = n\pi \qquad d\sin\theta = n\lambda$$

And as central maximum extends between first minima on either side, for small θ, the angular width of central maximum will be: $\theta_0 = 2\theta_1 = (2\lambda / d)$

- At centre as $\theta = 0$ and hence $(\sin\alpha / \alpha) \to 1$. This in turn means that intensity at centre is always maximum and equal to I_m. This maximum of intensity is called central maximum.
- At the position of a minima, wavelets from the two ends of the slit reach in phase differeing by an integral multiple of 2π as in this situation path difference $(d\sin\theta)$ condition of minima is $n\lambda$.
- Subsidiary maxima are approximately midway between two consecutive minima and of decreasing inteisty. The position of nth subsidiary maxima will therefore be given by:

$$[d\sin\theta]_{max} = \frac{n\lambda + (n+1)\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$$

- The angular width of subsidiary maximum $\theta_s = (\theta_n - \theta_{n-1}) = (\lambda / d)$ is half that of central maximum $[\theta_0 = 2(\lambda / d)]$.
- Due to diffraction at a circular aperture, a converging lens can never form a point image of an object but it produces a bright disc called Airy disc surrounded by dark and bright concentric rings. The minimum radius of the image disc is given by

$$r = 1.22\frac{\lambda}{d}(f)$$

- Diffraction limits the ability of optical instruments to form distinct images of objects when they are close to each other. According to Rayleigh (called rayleigh's criterion), two objects of equal intensity will be just resolved (i.e., distinctly visible) by an optical instrument if the central diffraction maximum of one lies at the first minimum of the other. So the angular limit of resolution will be equal to the angular separation between the centre of central maximum and first minimum, which for a single slit will be

$$\theta_R = \frac{\lambda}{d} - 0, \qquad \theta_R = \frac{\lambda}{d}$$

for circular aperture such as lens, θ_R is found to be $1.22(\lambda / d)$; so two objects at a distance D with separation y will be distinctly visible only if

$$\theta \geq \theta_R, \qquad (y / D) \geq 1.22(y / d)$$

- A diffraction-grating consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the direction of principal maxima (PM) is given by

$$d\sin\theta = n\lambda$$

where d is the distance between two consecutive slits and is called grating element and n order of principle maxima.

- The dispersive and resolving power of a grating are given by

$$DP = \frac{d\theta}{d\lambda} = \frac{n}{d\cos\theta} \qquad RP = \frac{\lambda}{d\lambda} = nN$$

closely spaced lines on a grating give greater dispersion while greater number of lines increase its resolving power.

POLARISATION :

An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ Because all directions of vibrations of $\vec{E}$ are equally probable the resultant electromagnetic wave is a superposition of waves produced by the individual atomic sources. This resultant wave is called unpolarised light and is symmetrical about the direction of wave propagation. If somehow (say using polaroids or Nicol-prism) we confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion the light is said to be plane polarised or linearly-polarised and the phenomenon of confining the vibrations of a wave in a specific direction perpendicular to the direction of wave motion is called polarisation. The plane containing the direction of vibration and wave motion is called plane of polarisation.

All the vibrations of an unpolarised light at a given instant can be resolved in two mutually perpendicular directions and hence, an unpolarised light is equivalent to the superposition of two mutually perpendicular identical plane polarised lights.

- If in case of unpolarised light, electric vector in some plane is either more or less, than in its perpendicular plane, the light is said to be 'partially polarised'
- If an unpolarised light is converted into plane polarised light, its intensity reduces to half.
- A part from partially polarised and plane (i.e., linearly) polarised, light can also be circularly or elliptically polarised, that too left-handed or right handed. Elliptically and circularly polarised lights result due to superposition of two mutually perpendicular plane polarised lights differing in phase by $(\pi/2)$ with.

Methods of Obtaining Plane Polarised Light :

By Reflection :

Brewster discovered that when light is incident at a particular angle on a transparent substance, the reflected light is completely plane polarised with vibrations in a plane perpendicular to the plane of incidence. This specific angle of incidence is called polarising angle θ_p and is related to the refractive index μ of the material through the relation: $\tan\theta_p = \mu$. This is known as Brewster's law.

By Scattering :

When light is incident on atoms and molecules, the electrons absorb the incident light and re-radiate it in all directions. This process is called scattering. it is found that scattered light in directions perpendicular to the direction of incident light is completely plane polarised while transmitted light is unpolarised. Light in all other directions is partially polarised.

- If plane polarised light of intensity $I_0\left(= KA^2\right)$ is incident on a polaroid and its vibrations of amplitude A make an angle θ with the transmission axis, then the component of vibrations parallel to transmission axis will be A cos θ while perpendicular to it A sin θ. Now, as polaroid will pass only those vibrations which are parallel to its transmission axis, i.e., A cos θ, so the intensity of emergent light will be

$$I = K\left(A\cos\theta\right)^2 = KA^2\cos^2\theta$$

$$I = I_0\cos^2\theta$$

This law is called Malus law.

- If an unpolarised light is converted into plane polarised light (say by passing it through a polaroid or a Nicol-prism), its intensity becomes half.
- If light of intensity I_1 emerging from one polaroid called polariser is incident on a second polaroid (usually called analyser) the intensity of the light emerging from the second polaroid in accordance

with Malus law will be given by $I_2 = I_1 \cos^2 \theta'$

where θ' is the angle between the transmission axis of the two polaroids.

- Optically activity of a substance is measured with the help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 cm (1 dm) and of unit concentration (i.e., 1 g/cc) for a given wavelength of light at a given temperature,

$$[\alpha]_{t^\circ C}^{\lambda} = \frac{\theta}{L \times C}$$

where θ is the rotation in length L at concentration C.

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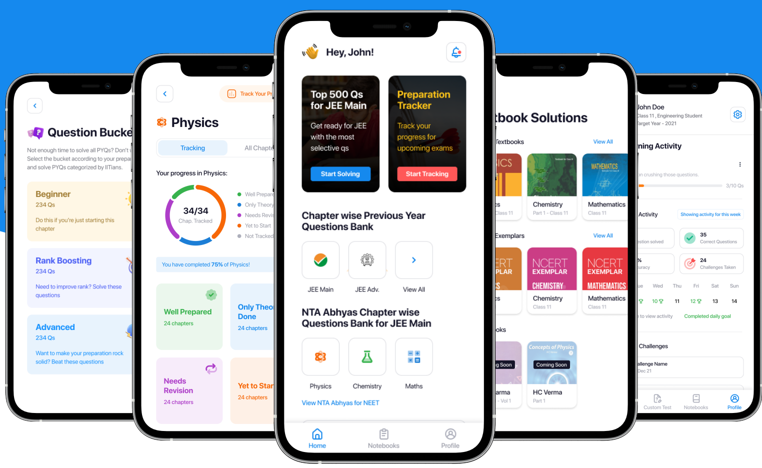

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