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# BINOMIAL THEOREM

## 1. STATEMENT OF BINOMIAL THEOREM

$$(x + a)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} a + {}^n C_2 x^{n-2} a^2 + \dots + {}^n C_n a^n \quad (\text{where } n \in \mathbb{N})$$

•  ${}^n C_0, {}^n C_1, {}^n C_2, \dots, {}^n C_n$  are binomial coefficients  ${}^n C_r = \frac{n!}{r!(n-r)!}$

$$\text{General Term} = T_{r+1} = {}^n C_r x^{n-r} a^r$$

- There are  $(n+1)$  terms in the expansion of  $(x + a)^n$ .
- The sum of powers of  $a$  and  $x$  in each term of expansion is  $n$ .
- The binomial coefficients in the expansion of  $(x + a)^n$  equidistant from the beginning and the end are equal.

## 2. GREATEST BINOMIAL COEFFICIENT

- If  $n$  is even : When  $r = \frac{n}{2}$  i.e.  ${}^n C_{n/2}$  takes maximum value.
- If  $n$  is odd :  $r = \frac{n-1}{2}$  or  $\frac{n+1}{2}$  i.e.  ${}^n C_{\frac{n-1}{2}} = {}^n C_{\frac{n+1}{2}}$  and take maximum value.

## 3. MIDDLE TERM OF THE EXPANSION

- If  $n$  is even  $T_{\left(\frac{n+1}{2}\right)}$  is the middle term. So the middle term  $T_{\left(\frac{n+1}{2}\right)} = {}^n C_{n/2} x^{n/2} y^{n/2}$
- If  $n$  is odd  $T_{\left(\frac{n+1}{2}\right)}$  and  $T_{\left(\frac{n+3}{2}\right)}$  are middle terms. So the middle terms are

$$T_{\left(\frac{n+1}{2}\right)} = {}^n C_{\left(\frac{n-1}{2}\right)} x^{\frac{n+1}{2}} y^{\frac{n-1}{2}} \text{ and } T_{\left(\frac{n+3}{2}\right)} = {}^n C_{\left(\frac{n+1}{2}\right)} x^{\frac{n-1}{2}} y^{\frac{n+1}{2}}$$

## 4. TO DETERMINE A PARTICULAR TERM IN THE EXPANSION

In the expansion of  $\left(x \pm \frac{1}{x}\right)^n$ , if  $x^m$  occurs in  $T_{r+1}$ , then  $r$  is given by  
$$r = \frac{n - m}{+}$$

The term which is independent of  $x$ , occurs in  $T_{r+1}$ , then  $r$  is  $r = \frac{n}{+}$

## 5. BINOMIAL COEFFICIENT PROPERTIES

$$(1) \quad C_0 + C_1 + C_2 + \dots + C_n = 2^n$$

$$(2) \quad C_0 - C_1 + C_2 - C_3 + \dots + (-1)^n C_n = 0$$

$$(3) \quad C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$$

$$(4) \quad C_0 C_r + C_1 C_{r+1} + \dots + C_{n-r} C_n = {}^{2n}C_{n-r} = \frac{2n!}{(n+r)!(n-r)!}$$

$$(\text{if } r=0) \quad C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{2n!}{n! n!}$$

$$(\text{if } r=1) \quad C_0 C_1 + C_1 C_2 + C_2 C_3 + \dots + C_{n-1} C_n = {}^{2n}C_{n-1} = \frac{2n!}{(n+1)!(n-1)!}$$

$$(5) \quad C_1 + 2C_2 + 3C_3 + \dots + nC_n = n \cdot 2^{n-1}$$

$$(6) \quad C_1 - 2C_2 + 3C_3 - \dots + (-1)^n \cdot nC_n = 0$$

$$(7) \quad C_0 + 2C_1 + 3C_2 + \dots + (n+1)C_n = 2^{n-1} (n+2)$$

$$(8) \quad C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \frac{C_3}{4}x^3 + \dots + \frac{C_n}{n+1}x^n = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

$$\Rightarrow C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1} \quad (x=1)$$

$$\Rightarrow C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \dots + \frac{(-1)^n \cdot C_n}{n+1} = \frac{1}{(n+1)} \quad (x=-1)$$

$$(9) \quad C_0^2 - C_1^2 + C_2^2 - C_3^2 + \dots + (-1)^n C_n^2 = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^{n/2} n_{C_{n/2}} & \text{if } n \text{ is even} \end{cases}$$

## 6. NUMERICALLY GREATEST TERM OF BINOMIAL EXPANSION

$$(a+x)^n = C_0 a^n + C_1 a^{n-1} x + \dots + C_n x^n.$$

$$\left| \frac{T_{r+1}}{T_r} \right| = \left| \frac{{}^nC_r}{{}^nC_{r-1}} \right| \left| \frac{x}{a} \right| = \left| \frac{n-r+1}{r} \right| \left| \frac{x}{a} \right|$$

$$\text{Take } \left| \frac{n-r+1}{r} \right| \left| \frac{x}{a} \right| \geq 1 \quad \{ \text{As } |T_{r+1}| \geq |T_r|\}$$

$$r \leq \frac{n+1}{1 + \left| \frac{a}{x} \right|}$$

$$\text{So greatest term will be } T_{r+1} \text{ where } r = \left[ \frac{n+1}{1 + \left| \frac{a}{x} \right|} \right]$$

[ . ] denotes greatest integer function.

Note : If  $\frac{n+1}{1 + \left| \frac{a}{x} \right|}$  itself is a natural number, then  $T_r = T_{r+1}$  and both the terms are numerically greatest.

## 7. BINOMIAL THEOREM FOR ANY INDEX

If  $n \in \mathbb{Q}$ ,  $|x| < 1$ , then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r + \dots \infty$$

Note : In this case there are infinite terms in the expansion.

Some Important Expansions :

If  $|x| < 1$  and  $n \in \mathbb{Q}$  then

$$(a) (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \dots + \frac{n(n+1)\dots(n+r-1)}{r!}x^r + \dots$$

$$(b) (1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \dots + \frac{n(n+1)\dots(n+r-1)}{r!}(-x)^r + \dots$$

By putting  $n = 1, 2, 3$  in the above results, we get the following results-

- $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots + x^r + \dots$
- $(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots + (-x)^r + \dots$
- $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots + (r+1)x^r + \dots$
- $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots + (r+1)(-x)^r + \dots$
- $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \dots + \frac{(r+1)(r+2)}{2!}x^r + \dots$
- $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \dots + \frac{(r+1)(r+2)}{2!}(-x)^r + \dots$

## 8. SOME IMPORTANT RESULTS

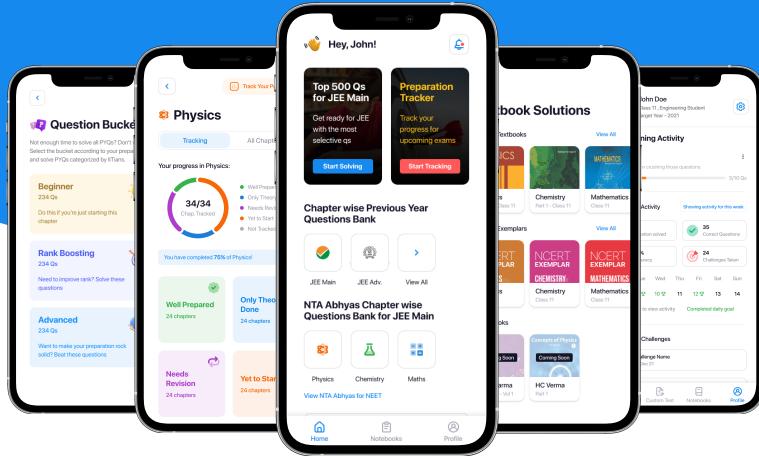
- If the coefficient of the  $r^{\text{th}}$ ,  $(r+1)^{\text{th}}$  and  $(r+2)^{\text{th}}$  terms in the expansion of  $(1+x)^n$  are in H.P. then  

$$n + (n-2r)^2 = 0$$
- If coefficient of  $r^{\text{th}}$ ,  $(r+1)^{\text{th}}$ , and  $(r+2)^{\text{th}}$  terms in the expansion of  $(1+x)^n$  are in A.P. then  

$$n^2 - n(4r+1) + 4r^2 - 2 = 0$$
- Number of terms in the expansion of  $(x_1 + x_2 + \dots + x_r)^n$  is  ${}^{(n+r-1)}C_{r-1}$



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