

Assignment-8 : Papoullis Chapter 10

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Question

Problem 10.23

(Cauchy inequality) Show that $|\sum_i a_i b_i|^2 \leq \sum_i |a_i|^2 \sum_i |b_i|^2$ with equality iff $a_j = k b_j^*$.

Solving a

(a)

Since $|\sum_i a_i b_i|^2 \leq \sum_i |a_i|^2 \sum_i |b_i|^2$, it suffices to assume that the numbers a_i and b_i are real. The quadratic

$$I(z) = \sum_i (a_i - z b_i)^2 = z^2 \sum_i b_i^2 - 2z \sum_i a_i b_i + \sum_i a_i^2$$

is nonnegative for every real z , hence, its discriminant cannot be positive. This yields (i).

Solving b

(b)

With $f[n]$ and $R_v[m] = S_o\delta[m]$.

$$y_f[n_o] = \sum h[n]f[n_o - n] \quad y_v[n] = \sum h[n]v[n]$$

$$Ey_v^2[n] = s_o p[o] = s_o \sum |h[n]|^2$$

From $S_{xx}(e^{jw} \geq 0)$ and (i) yields

$$\frac{y_f^2[n_o]}{Ey_v^2[n]} = \frac{\sum h[n]f[n_o-n]}{s_o \sum |h[n]|^2} \leq \frac{1}{s_o} \sum |h[n]|^2$$

with the equality iff $h[n] = kf^*[n_o - n]$.