ASSIGNMENT-3

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QUESTION

EXAMPLE 2-13(papoulis chapter):
A box contains white and black balls. When two balls are drawn without replacement, suppose the probability that both are white is $1/3$. (a) Find the smallest number of balls in the box.
(b) How small can the total number of balls be if black balls are even in number?

SOLUTION

(a) Let a and b denote the number of white and black balls in the box, and Wk the event.

Wk = "a white ball is drawn at the kth draw."

We are given that P(WI n W2) = 1/3. But,

$$P((W_1)(W_2)) = P((W_2)(W_1)) = P(W_2|W_1)P(W_1) \tag{1}$$

$$= \left(\frac{a-1}{a+b-1}\right).\left(\frac{a}{a+b}\right) = \frac{1}{3} \tag{2}$$

Because, $(\frac{a}{a+b}) > (\frac{a-1}{a+b-1})$; b > 0

SOLUTION

we can rewrite (2) as; $(\frac{a-1}{a+b-1})^2 < (\frac{1}{3}) < (\frac{a}{a+b})^2$
This gives the inequalities

$$[(\sqrt{3}+1)b/2] < [a] < [1+(\sqrt{3}+1)b/2] \tag{3}$$

For b = 1, this gives 1.36 < a < 2.36, or a = 2, and we get

$$\boxed{P((W_1)(W_2)) = \frac{2}{3}.\frac{1}{2} = \frac{1}{3}}$$

Thus the smallest number of balls required is 3.

SOLUTION

b	a from(2)	$P(W_1 W_2)$
2	3	$\frac{3}{4} \cdot \frac{2}{4} = \frac{3}{10} \neq \frac{1}{3}$
4	6	$\frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$

Table-1

(b) For even value of b, we can use (2) with b = 2, 4, ... as shown in Table-1. From the table, when b is even, 10 is the smallest number of balls (a = 6, b = 4) that gives the desired probability.