ASSIGNMENT-2 : ICSE-2019, 12th GRADE

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PROBLEM 5-B : (b).Verify the Lagrange's mean value theorem for the function: f(x) = x + 1/x in the interval [1, 3].

SOLUTION :
f(x) = x + $\frac{1}{x}$ be in closed interval $1 \leq x \leq 3$, i.e [1,3]
f$'$(x) = 1 - $\frac{1}{x^2}$ is existing in the open interval $1 < x < 3$, i.e (1,3).
Since, f(x) is a polynomial function, therefore, it is continuous and derivable in (1, 3).
The conditions of lagrange's mean value theorem are satisfied.
f(1) = 1+1 = 2 f(3) = 3 + $\frac{1}{3}$ = $\frac{10}{3}$.
To verify further, need to show that there exists a 'c' $\in$ (1,3) such that,

$$\implies f'(c) = \frac{f(b) - f(a)}{b - a} \tag{1}$$

$$\implies 1 - \frac{1}{x^2} = \frac{\frac{10}{3} - 2}{3 - 1} \tag{2}$$

$$\implies \frac{4}{3} \times \frac{1}{2} = \frac{2}{3} \tag{3}$$

$$\implies 1 - \frac{1}{x^2} = \frac{2}{3} \tag{4}$$

$$\implies \frac{1}{x^2} = 1 - \frac{2}{3} = \frac{1}{3} \tag{5}$$

$$\implies x^2 = 3 \tag{6}$$

$$\implies \boxed{x = \pm\sqrt{3}} \tag{7}$$

$\therefore$+$\sqrt{3}$ (or) +1.732 lies in the open interval (1,3) or c in (1,3)
Hence, Mean Value theorem for the given function is verified in the given interval.