Analog Integrated Circuits Lab

A Report on

Practical integrator using operational amplifier

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Contents

1 Aim

To design, simulate, implement and test a μA741-based practical voltage integrator.

Components required

- μA741 op-amp
- Resistors R = 120kΩ, 3.3kΩ, 4.7kΩ
- Capacitor C = 0.01 μF
- Signal generator
- Digital Storage Oscilloscope
- Breadboard
- Jumper wires

2 Circuit diagram

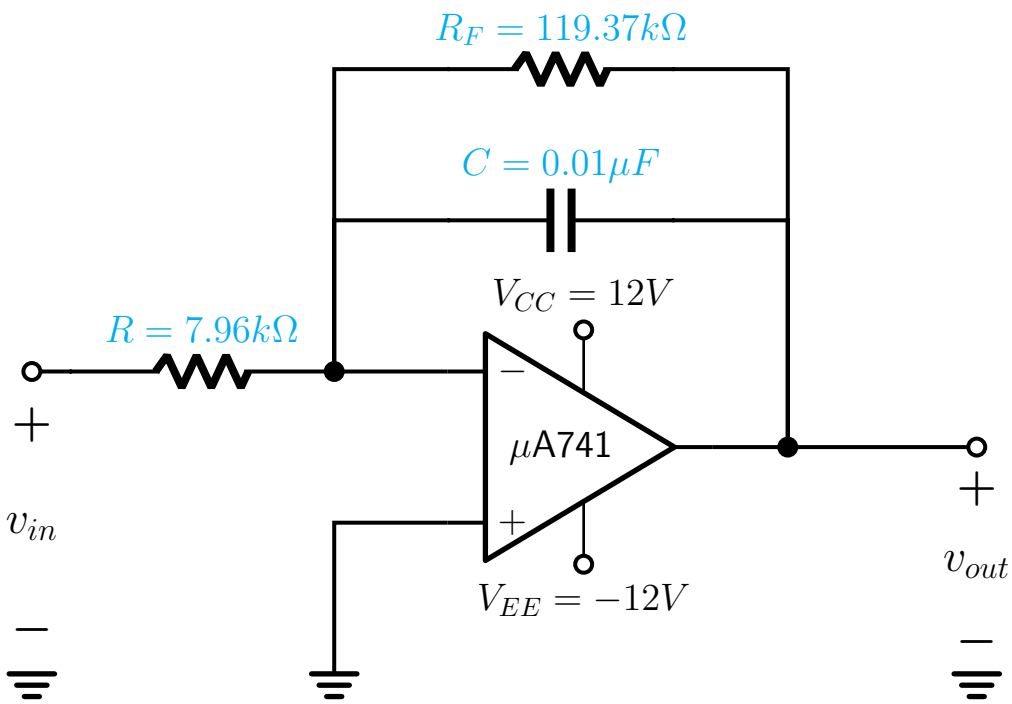

Figure 1: Circuit showing the final design

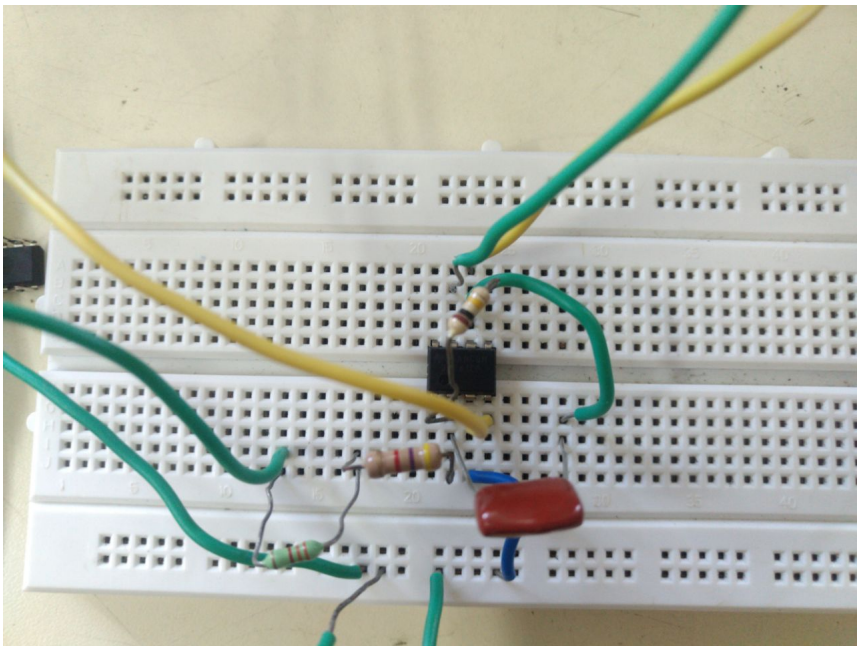

Figure 2: Circuit as implemented in the Lab using physical components

3 Theory

The operational amplifier based integrator performs the mathematical operation of integration with respect to time, i.e. its output is proportional to the input voltage integrated over time. The integrator circuit is mostly used in analog computers, analog-to-digital converters and wave-shaping circuits such as charge amplifiers.

The response of an op-amp circuit with feedback reflects the characteristics of the feedback elements. Thus, in order to achieve integration, the feedback network is constructed using a capacitor.

An operational amplifier based integrator can ideally be constructed as shown in figure 3.

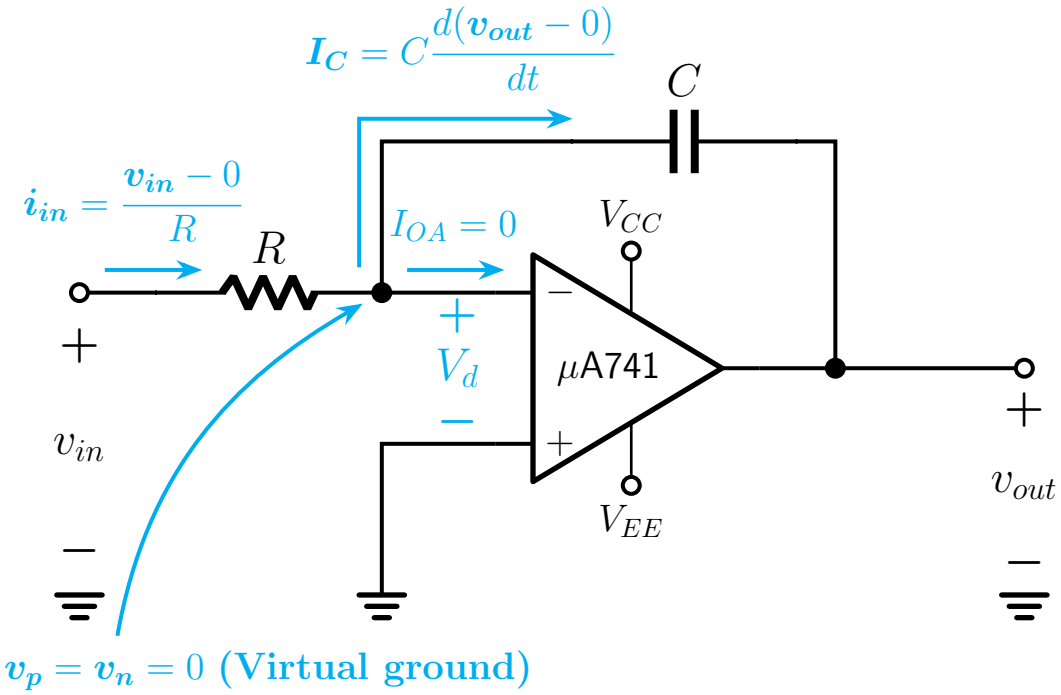

Figure 3: Ideal opamp integrator

The circuit can be analysed by applying Kirchhoff's current law at the node V_n, keeping ideal op-amp behaviour in mind.

$$i_{in} = I_C = \frac{V_{in}}{R} \tag{1}$$

The relationship between between the capacitor's voltage and current is modelled as:

$$I_C = C\frac{dV_C}{dt} \tag{2}$$

Substituting eq. 1 in eq. 2:

$$\frac{v_{in}-0}{R}=C\frac{d(0-v_{out})}{dt} \implies \frac{v_{in}}{R}=-C\frac{dv_{out}}{dt}$$

Integrating both sides with respect to time:

$$\int_0^t \frac{v_{in}}{R}=-\int_0^t \frac{dv_{out}}{dt}dt$$

$$v_{out}=-\frac{1}{RC}\int_0^t v_{in}dt$$

Thus the output of the circuit shown in figure 3 is the inverted, integrated input with a gain of $\frac{1}{RC}$.

The ideal integrator suffers from two main limitations. One comes from the fact that the output volatage of the op-amp cannot exceed the supply voltage. The output of the integrator is inversely proportional to the time constant $\tau = RC$. The larger the time constant τ ,the longer it takes to saturate the integrator. The second limitation is a consequence of the offset voltage present even for zero input. It may be only a few milivolts, but this gets integrated over time and it eventually drives the op-amp to saturation.

One can overcome the limitations of an ideal integrator by adding a resistor R_f in parallel with capacitor C as shown in figure 4. R_f prevents the op-amp from going into open loop configuration at low frequencies.

The resulting circuit is an inverting amplifier with $R_F||\frac{1}{Cs}$ acting as the feedback resistance. The gain of such an amplifier is given by:

$$\frac{V_{out}(s)}{V_{in}(s)}=-\frac{R_F||\frac{1}{Cs}}{R}$$

$$=-\frac{R_F}{R}\frac{1}{(1+R_FCs)}$$

Thus, the frequency response of the practical integrator is given by,

$$H(s)=\frac{-R_F||\frac{1}{Cs}}{R} \tag{3}$$

$$H(j\omega)=\frac{-R_F}{R}\left[\frac{1}{1+R_FCj\omega}\right] \tag{4}$$

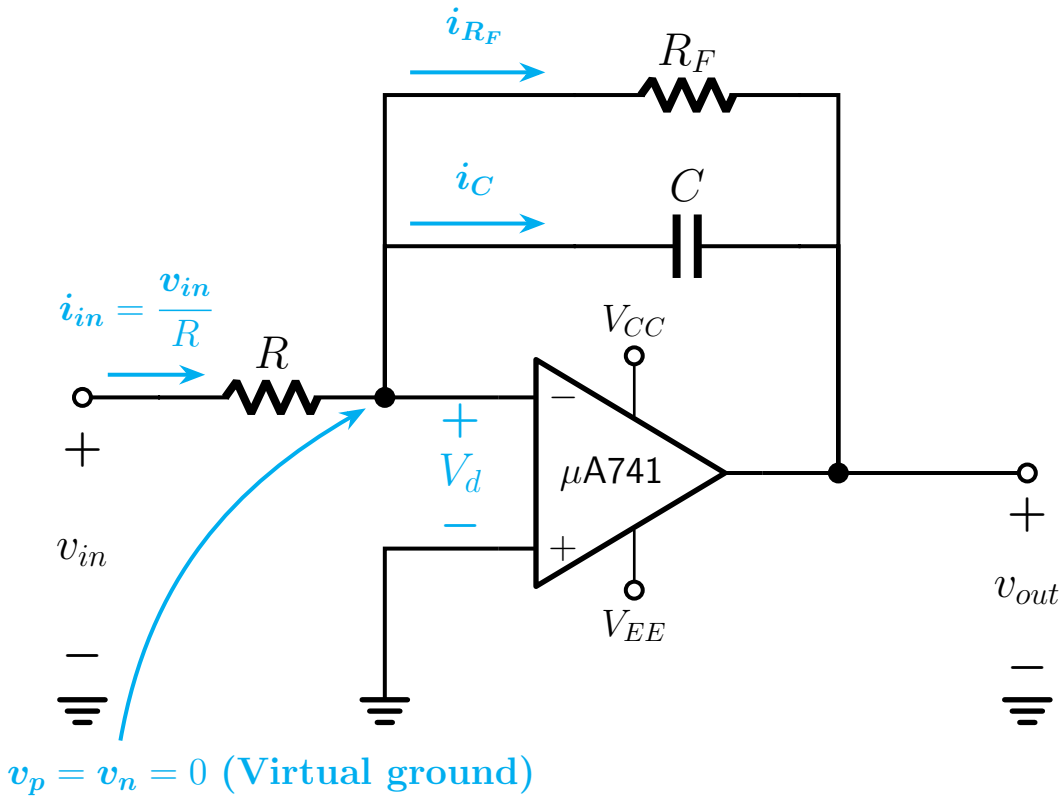

Figure 4: Practical op-amp integrator

The magnitude and phase response are given by,

$$|H(j\omega)| = \frac{R_F}{R}\frac{1}{\sqrt{1+\omega^2 R_F^2 C^2}}$$

$$\angle H(j\omega) = \pi - tan^{-1}(\omega R_F C)$$

DC gain is obtained by putting $\omega = 0$ in equation 4.

$$\text{DC gain} = H(j0) = -\frac{R_f}{R} \equiv -20\log_{10}(\frac{R_F}{R})\ \text{dB} \tag{5}$$

Phase shift $\Delta\phi$ between the input and output signals is given by

$$\Delta\phi = \pi - \tan^{-1}(\omega R_F C) \tag{6}$$

The practical integrator acts as a first order filter with corner frequency (also cut-off or pole frequency) at $f = -1/R_F C$. Thus, f_{-3dB} or cutoff-frequency is given by

$$f_{-3dB} = \frac{1}{2\pi R_F C} \tag{7}$$

Unity gain bandwidth ω_u is obtained from $|H(j\omega)| = 1$:

$$H(j\omega) = 1 = \frac{R_F}{R}\frac{1}{\sqrt{1+\omega^2 R_F^2 C^2}}$$

$$\left(\frac{R_F}{R}\right)^2 = 1 + (\omega R_F C)^2$$

$$\left(\frac{R_F}{R}\right)^2 \approx (\omega R_F C)^2 \qquad \left(\text{for } \frac{R_F}{R} \gg 1\right)$$

$$\implies \omega_u = \frac{1}{RC} \text{ or } f_u = \frac{1}{2\pi RC} \tag{8}$$

Roll-off rate for the frequency response of a first order low-pass filter is theoretically -20 dB/decade. It can be obtained by subtracting the decibel values of magnitude response $|H(j\omega)|$ at 2 frequencies ω_1 and ω_2 such that $\frac{\omega_2}{\omega_1} = 10$.

$$|H(j\omega)| = \frac{R_F}{R}\frac{1}{\sqrt{1+\omega^2 R_F^2 C^2}}$$

By amplitude and frequency scaling, the above can be written as

$$|H(j\omega)|^2 = \frac{1}{1+\omega^2}$$

In decibels, this becomes

$$|H(j\omega)|^2 = 10\log_{10}\left(\frac{1}{1+\omega^2}\right) \approx -10\log_{10}\left(\omega^2\right)$$

Roll-off is given by

$$\Delta = -20\log_{10}\left(\frac{\omega_2}{\omega_1}\right)$$

From the condition on ω_1 and ω_2, this gives

$$\text{Roll-off} = -20\text{dB/decade}$$

4 Design

The practical integrator is designed for the following constraints

(a) Unity gain

(b) $v_{in} = 2sin(4000\pi t)$

(c) $f_{-3dB} = \dfrac{f_{in}}{15}$

(d) Phase error tolerance of 5%

The provided input is $v_{in} = 2sin(4000\pi t)$ which can be compared with the general equation for sinusoidal waveforms $Asin(2\pi ft)$ to obtain $\underline{f_{in} = 2000\text{Hz}}$.

From the constraint list the value of $\underline{f_{-3dB}}$ is obtained as:

$$f_{-3dB} = \frac{f_{in}}{15} \implies f_{-3dB} = 133.33\text{Hz}$$

Using equation 7, and assuming $\underline{C = 0.01\mu F}$, the value of $\underline{R_F}$ comes out to be:

$$f_{-3dB} = \frac{1}{2\pi R_F C}$$

$$R_F = \frac{1}{2\pi f_{-3dB} C}$$

$$= \frac{1}{2\pi \times 133.33 \times 0.01 \times 10^{-6}}$$

$$= 119.37k\Omega \approx 120k\Omega$$

From the unity gain constraint, the output has to have the same amplitude as the input at $f_in = 2000$Hz. Using equation 8, the value of $\underline{R}$ comes out to be:

$$f_u = \frac{1}{2\pi RC}$$

$$R = \frac{1}{2\pi f_u C}$$

$$= \frac{1}{2\pi \times 2000 \times 0.01 \times 10^{-6}}$$

$$= 7.96k\Omega \approx 8k\Omega$$

5 LTSpice simulation and results[1]

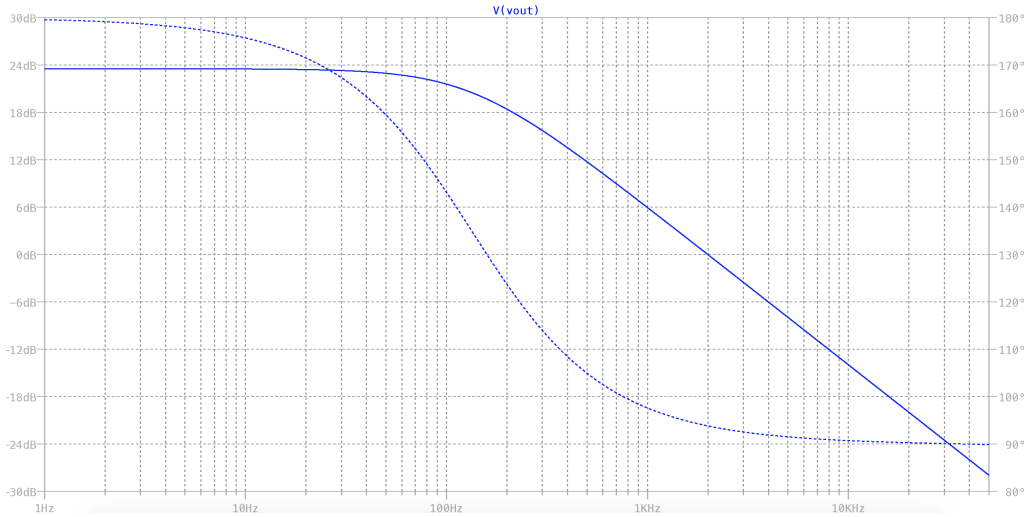

Figure 5: Phase and magnitude response of the practical integrator output

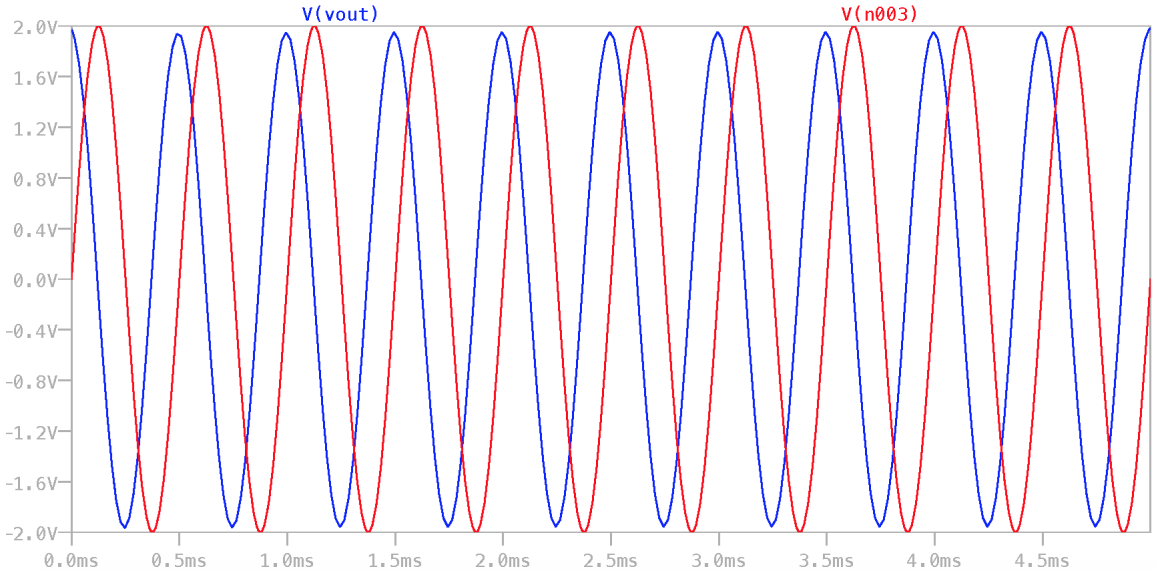

Figure 6: V_{out} and V_{in} in time domain

[1]The .asc file containing the LTSpice schematic can be found at github.com/rshwndsz/aic-report.

6 Observations

A. The practical integrator is first designed based on the following constraints: Unity gain, $f_{-3dB} = f_{in}/15$ for a sinusoidal input $v_{in} = 2sin(4000\pi t)$ and a phase error below 5%.

A leading phase sinusoidal wave appears at the output of the integrator as shown in figure 7. The DC gain, -3dB frequency f_{3dB}, unity gain frequency f_u, roll-off rate and phase shift are noted from the waveforms obtained on the DSO and the phase error is verified to be below 5%.

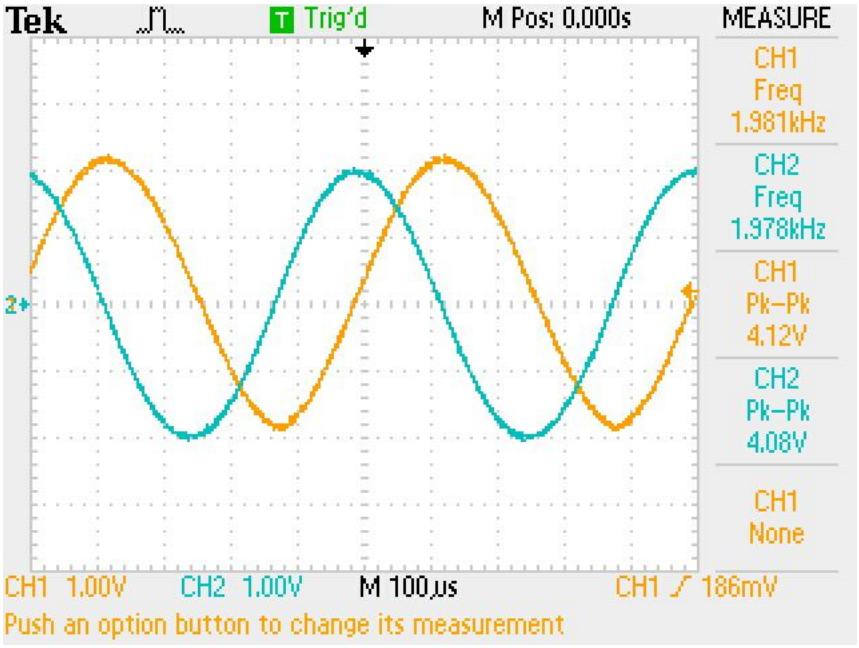

Figure 7: Result V_{out} of passing a sinusoidal input V_{in} through the integrator.

The observed parameters are listed below.

DC gain = 14.89

-3dB frequency f_{-3dB} = 132 Hz

Unity gain frequency f_u = 2.07 kHz

Roll-off rate = -18.35 dB/decade

Phase shift $\phi = 1.608^c$ or 92.13°, an error of 2.4%

B. The feedback resistor R_F is then removed and the effect on the op-amp integrator configuration is observed.

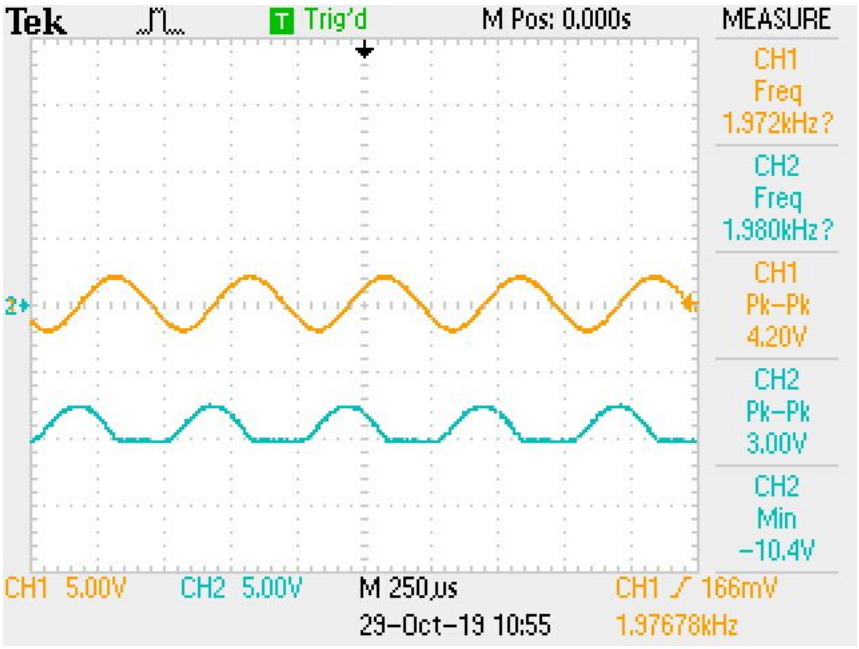

Figure 8: The output V_{out} saturates at low frequencies when R_F is removed

The output is given by $V_{out} = \pm V_{sat}$ as the capacitor acts as an open circuit at low frequencies as shown in figure 8. This sends the op-amp into an open loop configuraton.

C. The sinusoidal input is replaced by a square wave of $4V_{pk-pk}$ amplitude and a frequency of 2kHz to observe the integration operation of the configuration more clearly.

A triangular waveform is obtained as a result of the square wave being integrated, as shown in figure 9.

The peak-to-peak amplitude of the output can be obtained by using the relation

$$C\Delta V = I\Delta t$$

Substituting $C = 0.01\mu F$, $I = \dfrac{2}{8 \times 10^3}$, $\Delta t = \dfrac{0.5}{2000}$,

$$\Delta V = 6.25V$$

Thus, the triangular waveform at the output is supposed to have a peak-to-peak voltage of 6.25V and 6.32V is observed as shown in figure 9.

D. The frequency of the input is lowered to 130Hz and the output is observed again.

Using $C\Delta V = I\Delta t$ and substituting $C = 0.01\mu F$, $I = \dfrac{2}{8 \times 10^3}$, $\Delta t = \dfrac{0.5}{130}$, we get:

$$\Delta V = 96.15V$$

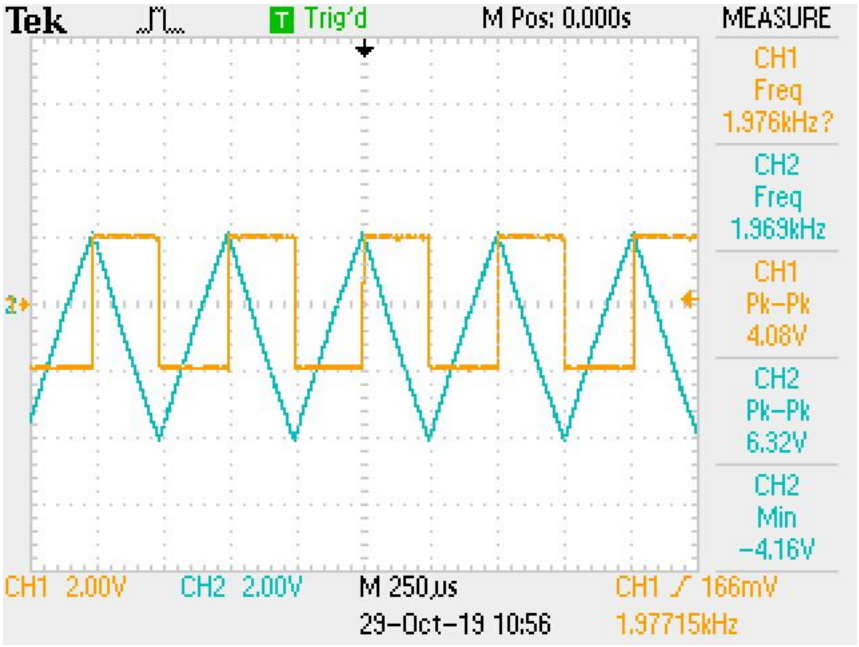

Figure 9: Result V_{out} of integrating a square wave V_{in}

Since $\Delta V > 2V_{sat}$, V_{out} gets clipped as shown in figure 10.

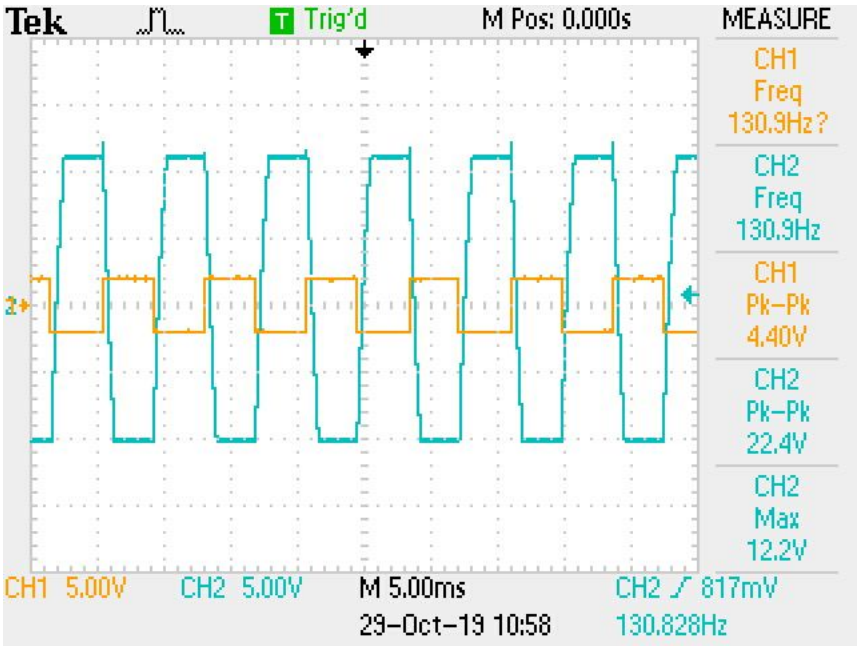

Figure 10: Effect of lowering the input frequency on the output V_{out}

7 Results & Conclusions

The op-amp based practical voltage integrator was designed subject to the given contraints, simulated in LTSpice and then implemented using the μA741 IC on a breadboard. The outputs were tested for sinusoidal and square wave at high and low input frequencies to understand the limitations of such a design.

All the observations made during the testing of the physical circuit are listed below in brief.

1. The output waveform for the given sinusoidal input was obtained with a phase of 92.13° resulting in a phase error of 2.4%, well within the limit of 5% specified in the given constraints.
2. The ouput voltage was observed to saturate at V_{sat} at low frequencies, when the resistor R_F was removed.
3. A triangular waveform was obtained when a square wave was given as input. This waveform was observed to be clipped at low frequencies.

It can therefore be seen that an op-amp based practical integrator, which has a resistor R_F in parallel with the capacitor C, overcomes the drawbacks of an ideal op-amp based integrator.