TYPESETTING EXAMS

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Maths Assignment IITB #

problem 1. Show that there exists no non trivial unramified extension of Q

solution : If K/Q is a non-trivial number field then $|disk \quad k| >$. But then disk k has a prime factor so that so same prime ramifies in k

Problem 2. complete the following :

(a) how does one prove a cot theorem ?

(b) compute $\int cosxdx$

(c) how does one square $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$?

solutions :

(a) use rollaries

(b) we have

$$\int cosxdx = sinx + c \tag{1}$$

we can check (1) $\frac{d}{dx}(sinx + c) = cosx$

(c) This is routin. $\square$

Problem 3. Prove that $\sqrt{2}$ is irrational.

Proof : . Asuume that $\sqrt{2}=\frac{a}{b}$, Where a,b ϵ Z . Without loss of generality , we may assume gcd (a, b)=1 . Then we have

$$\sqrt{2}=\frac{a}{b}$$

$$\sqrt{2}^2 = \begin{pmatrix} a \\ b \end{pmatrix}^2 \tag{2}$$

$$2= \frac{a^2}{b^2}$$

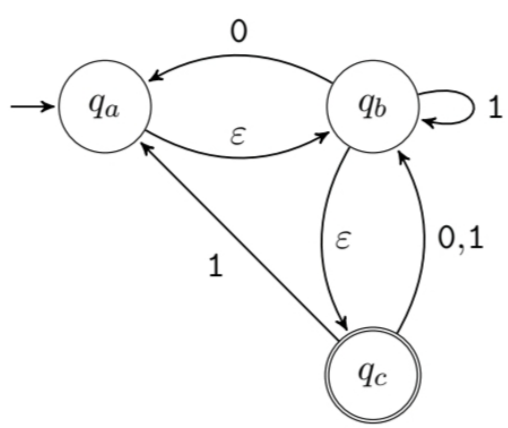
$$a^2 = 2b^2 \tag{3}$$

But then from (3),we know that a^2 is even so that a is even . But then we must have .

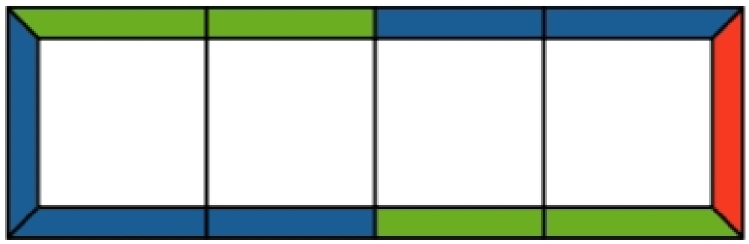
$$2a^2 = b^2$$

so that b^2 is even , implying b is even.But then gcd$(a, b) \geq 2$, a contradicion . $\square$

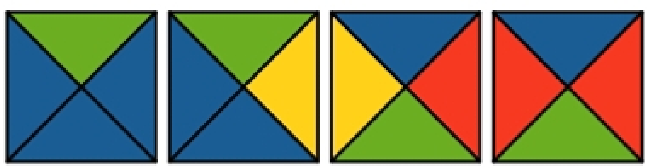
(b)



4.Solving Puzzles # 1 IN clas we did three puzzles, the first one which is equivalent to finiteautomata. In general , a puzzle of this type has a frame like (but possibly with more/fewer squares and different colors) :



and a finite set of tiles like this (but possibly with more/fewer tiles and different colors): The tiles must be arranged so that adjastment areas have



matching colors.there is an unlimited number of copies of each tile

(a) show how every puzzle of this type can be converted in to a finite automation M and a string w that M accepts w if and only if the puzzle has a solution.

(b) Apply your construction to the above instance.

(c) Briefly describe how this gives an o(n) algorithm for solving puzzles of this type