STATISTICAL THINKING IN PYTHON I

Introduction to Exploratory Data Analysis

Exploratory data analysis

- The process of organizing, plotting, and summarizing a data set

“Exploratory data analysis can never be the whole story, but nothing else can serve as the foundation stone.”

—John Tukey

2008 US swing state election results

2008_swing_states.csv

```
state,county,total_votes,dem_votes,rep_votes,dem_share
PA,Erie County,127691,75775,50351,60.08
PA,Bradford County,25787,10306,15057,40.64
PA,Tioga County,17984,6390,11326,36.07
PA,McKean County,15947,6465,9224,41.21
PA,Potter County,7507,2300,5109,31.04
PA,Wayne County,22835,9892,12702,43.78
PA,Susquehanna County,19286,8381,10633,44.08
PA,Warren County,18517,8537,9685,46.85
OH,Ashtabula County,44874,25027,18949,56.94
OH,Lake County,121335,60155,59142,50.46
PA,Crawford County,38134,16780,20750,44.71
OH,Lucas County,219830,142852,73706,65.99
OH,Fulton County,21973,9900,11689,45.88
OH,Geauga County,51102,21250,29096,42.23
OH,Williams County,18397,8174,9880,45.26
PA,Wyoming County,13138,5985,6983,46.15
PA,Lackawanna County,107876,67520,39488,63.10
PA,Elk County,14271,7290,6676,52.20
PA,Forest County,2444,1038,1366,43.18
PA,Venango County,23307,9238,13718,40.24
OH,Erie County,41229,23148,17432,57.01
```

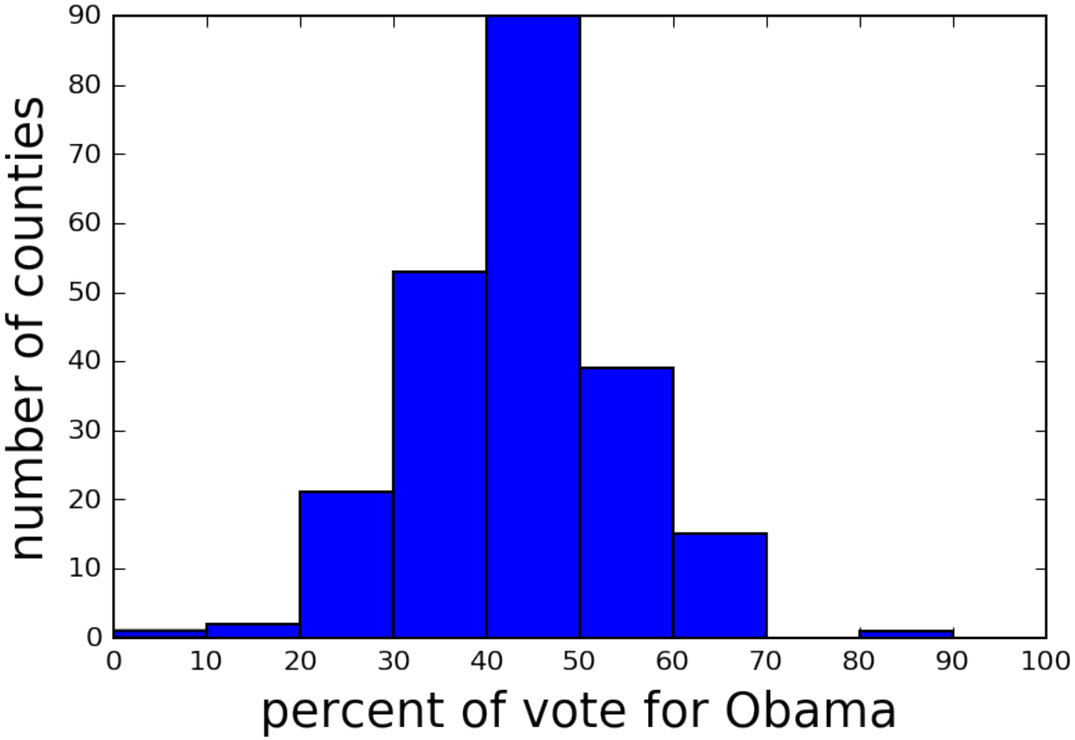
Data retrieved from Data.gov (https://www.data.gov/)

2008 US swing state election results

```
In [1]: import pandas as pd

In [2]: df_swing = pd.read_csv('2008_swing_states.csv')

In [3]: df_swing[['state', 'county', 'dem_share']]
Out[3]:
    state              county  dem_share
0      PA          Erie County      60.08
1      PA     Bradford County      40.64
2      PA        Tioga County      36.07
3      PA       McKean County      41.21
4      PA       Potter County      31.04
5      PA        Wayne County      43.78
6      PA  Susquehanna County      44.08
7      PA       Warren County      46.85
8      OH    Ashtabula County      56.94
```

Data retrieved from Data.gov (https://www.data.gov/)

2008 US swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

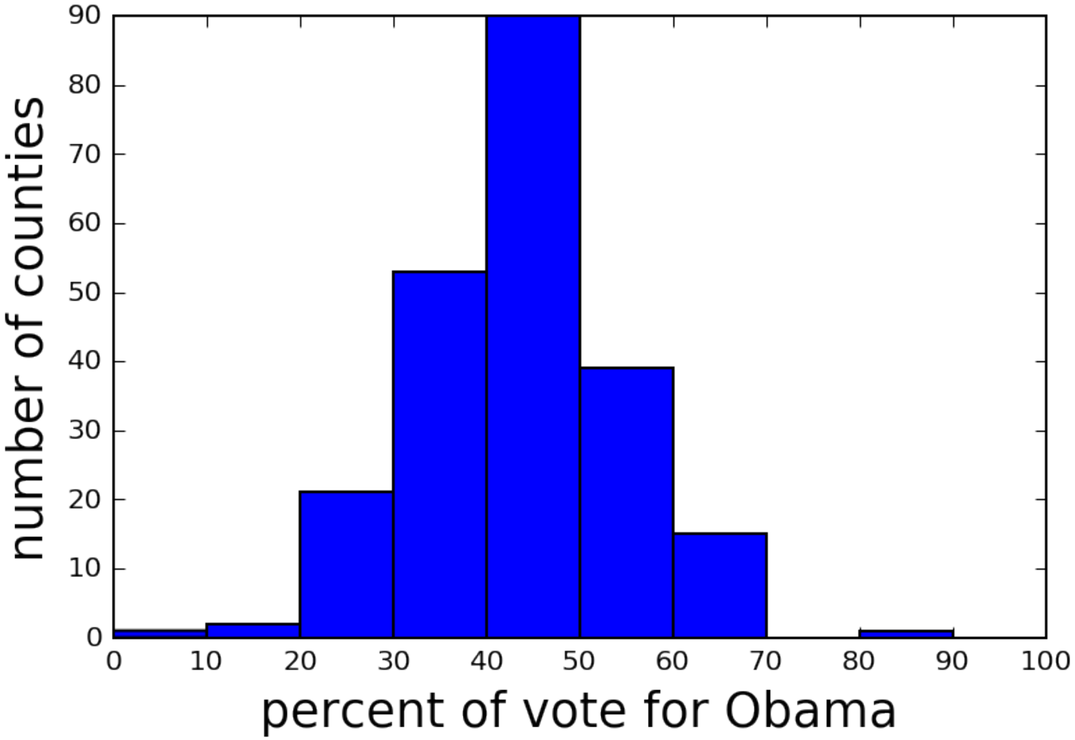
STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

Plotting a histogram

2008 US swing state election results



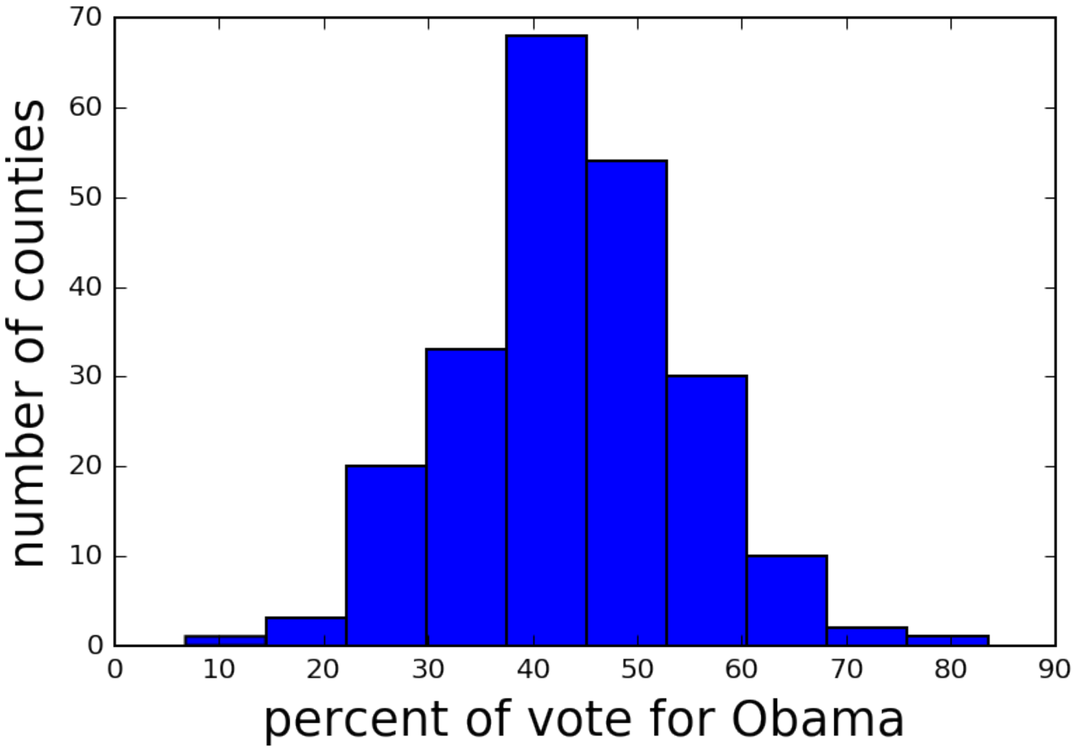
Data retrieved from Data.gov (https://www.data.gov/)

Generating a histogram

```
In [1]: import matplotlib.pyplot as plt

In [2]: _ = plt.hist(df_swing['dem_share'])

In [3]: _ = plt.xlabel('percent of vote for Obama')

In [4]: _ = plt.ylabel('number of counties')

In [5]: plt.show()
```

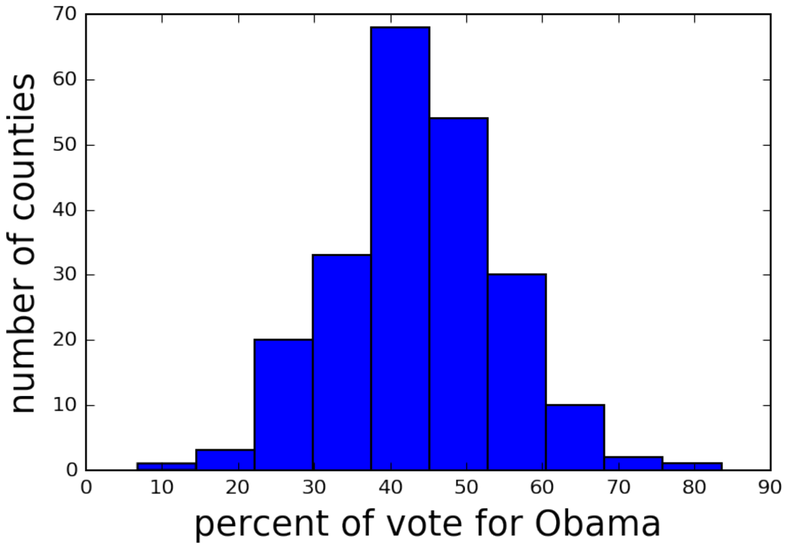
- Always label your axes

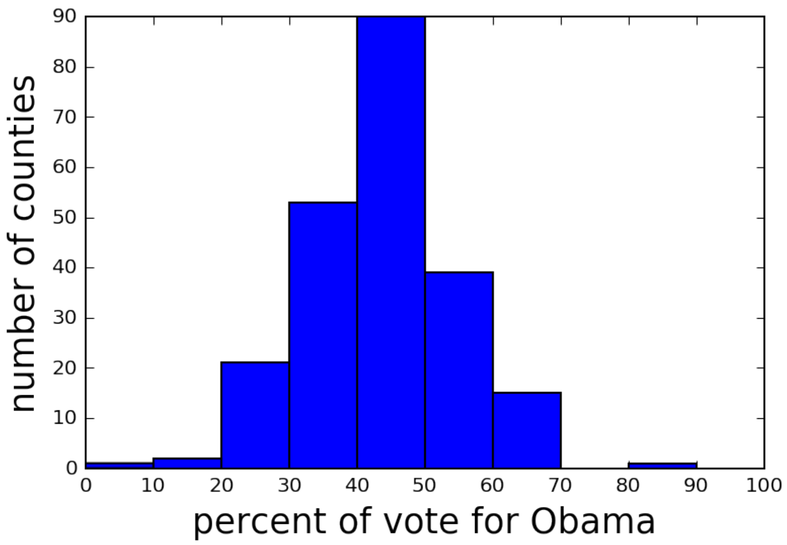
2008 US swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

Histograms with different binning

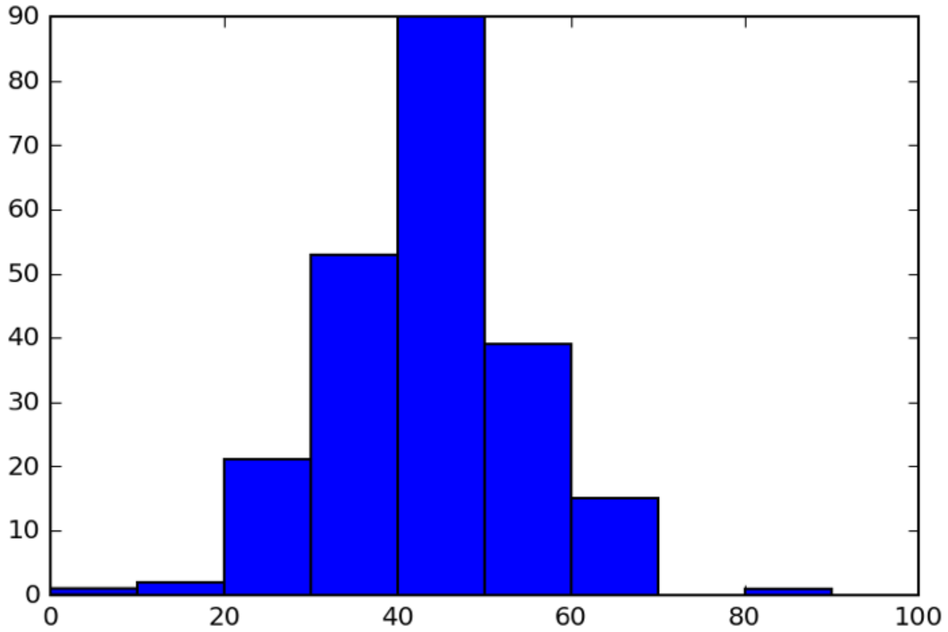




Data retrieved from Data.gov (https://www.data.gov/)

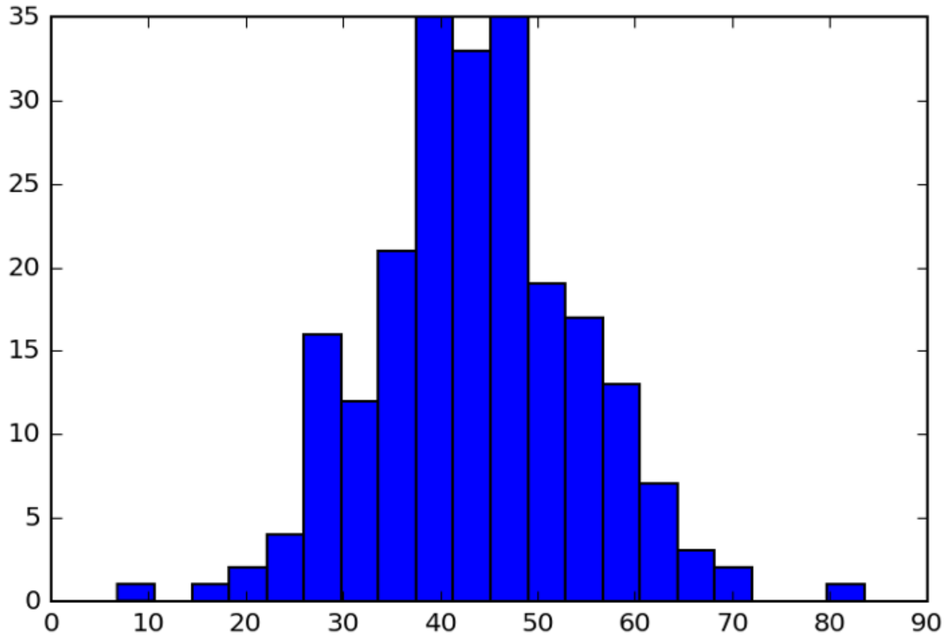
Setting the bins of a histogram

```
In [1]: bin_edges = [0, 10, 20, 30, 40, 50,
   ...:              60, 70, 80, 90, 100]

In [2]: _ = plt.hist(df_swing['dem_share'], bins=bin_edges)

In [3]: plt.show()
```



Setting the bins of a histogram

```
In [1]: _ = plt.hist(df_swing['dem_share'], bins=20)

In [2]: plt.show()
```



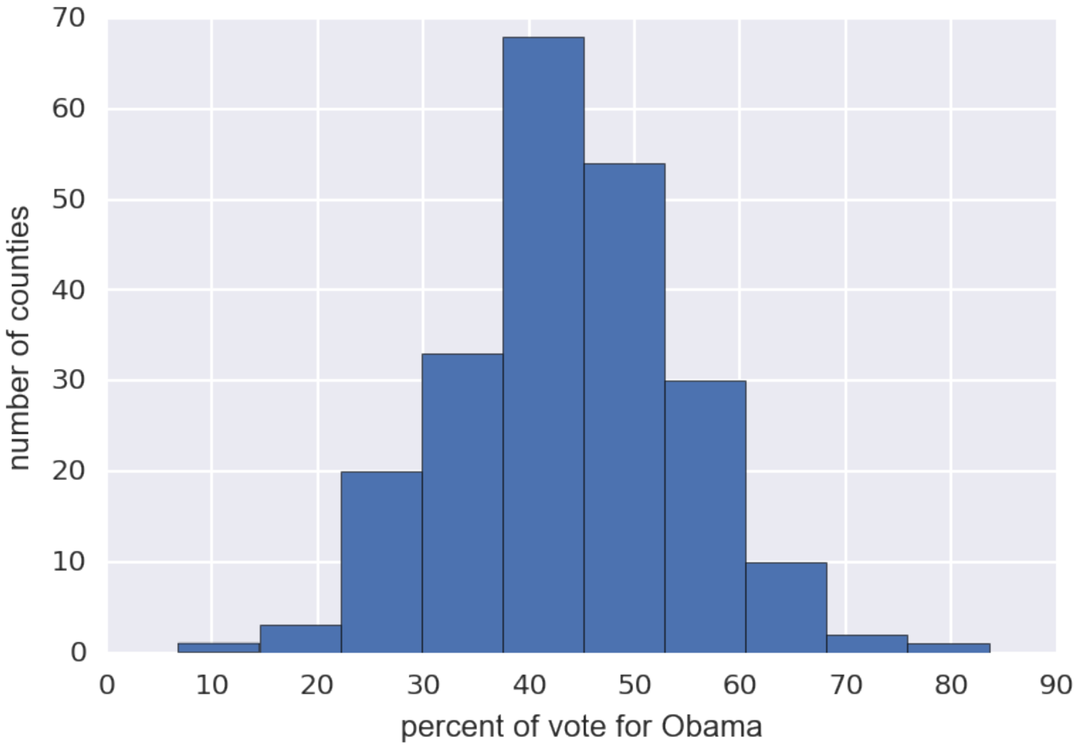
Seaborn

- An excellent Matplotlib-based statistical data visualization package written by Michael Waskom

Setting Seaborn styling

```
In [1]: import seaborn as sns

In [2]: sns.set()

In [3]: _ = plt.hist(df_swing['dem_share'])

In [4]: _ = plt.xlabel('percent of vote for Obama')

In [5]: _ = plt.ylabel('number of counties')

In [6]: plt.show()
```

A Seaborn-styled histogram



Data retrieved from Data.gov (https://www.data.gov/)

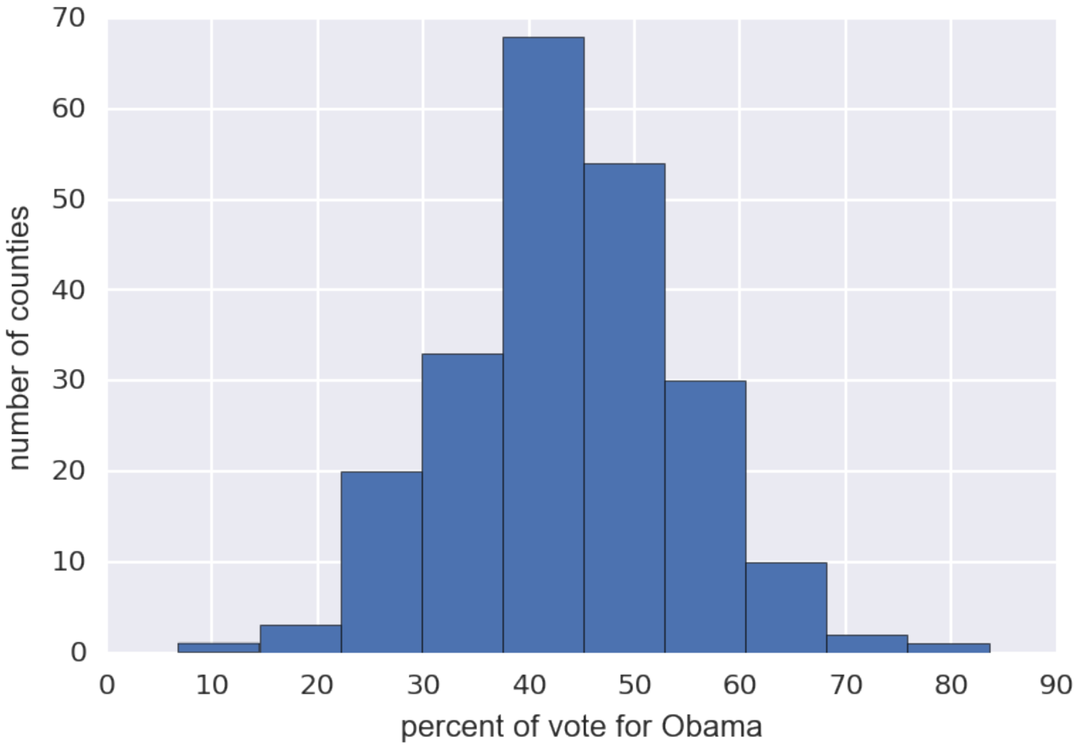
STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

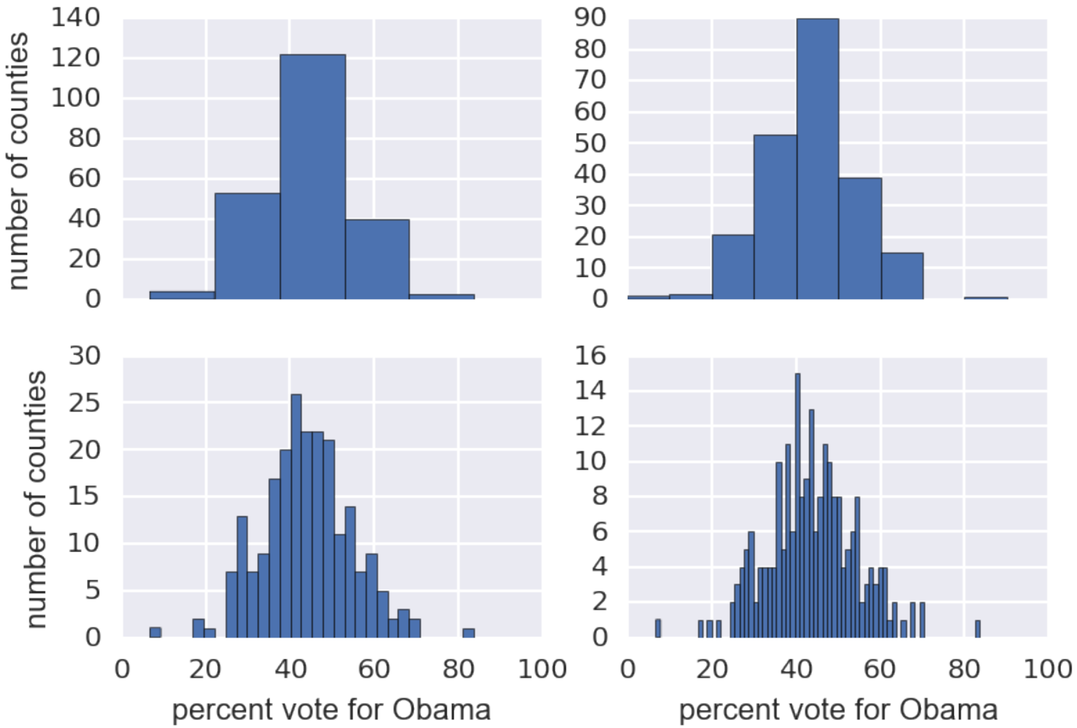
Plot all of your data: Bee swarm plots

2008 US swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

2008 US swing state election results

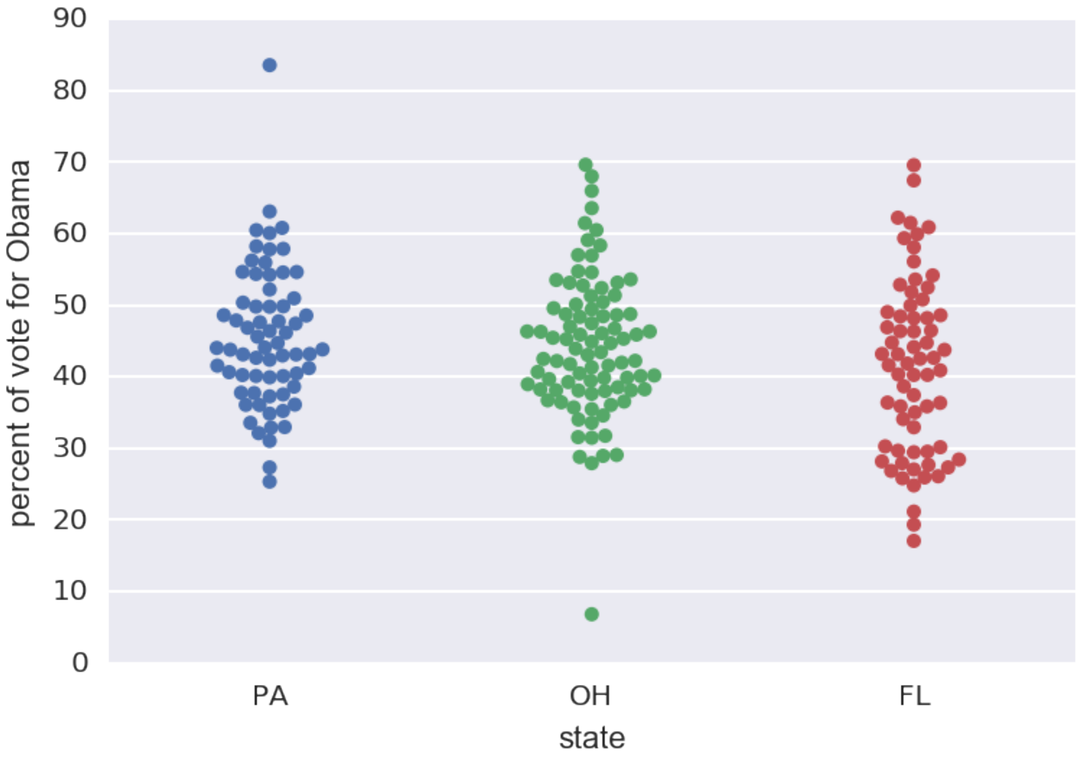


Data retrieved from Data.gov (https://www.data.gov/)

Binning bias

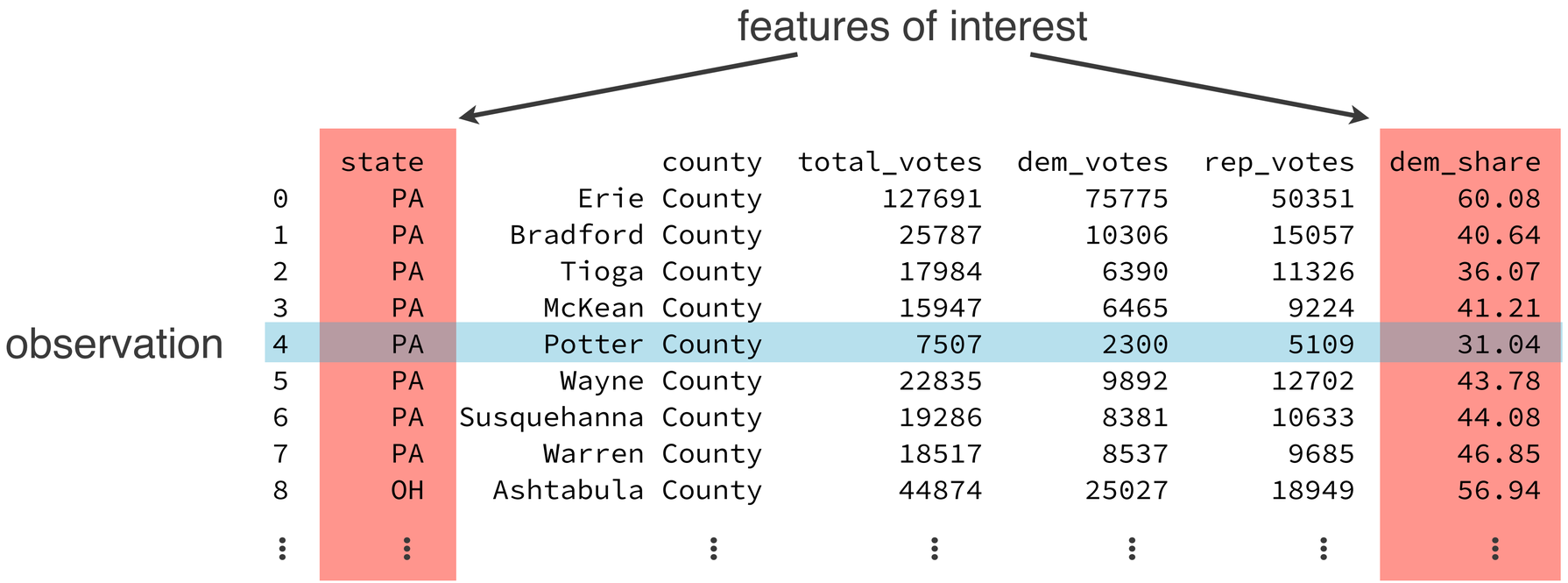
- The same data may be interpreted differently depending on choice of bins

Bee swarm plot



Data retrieved from Data.gov (https://www.data.gov/)

Organization of the data frame



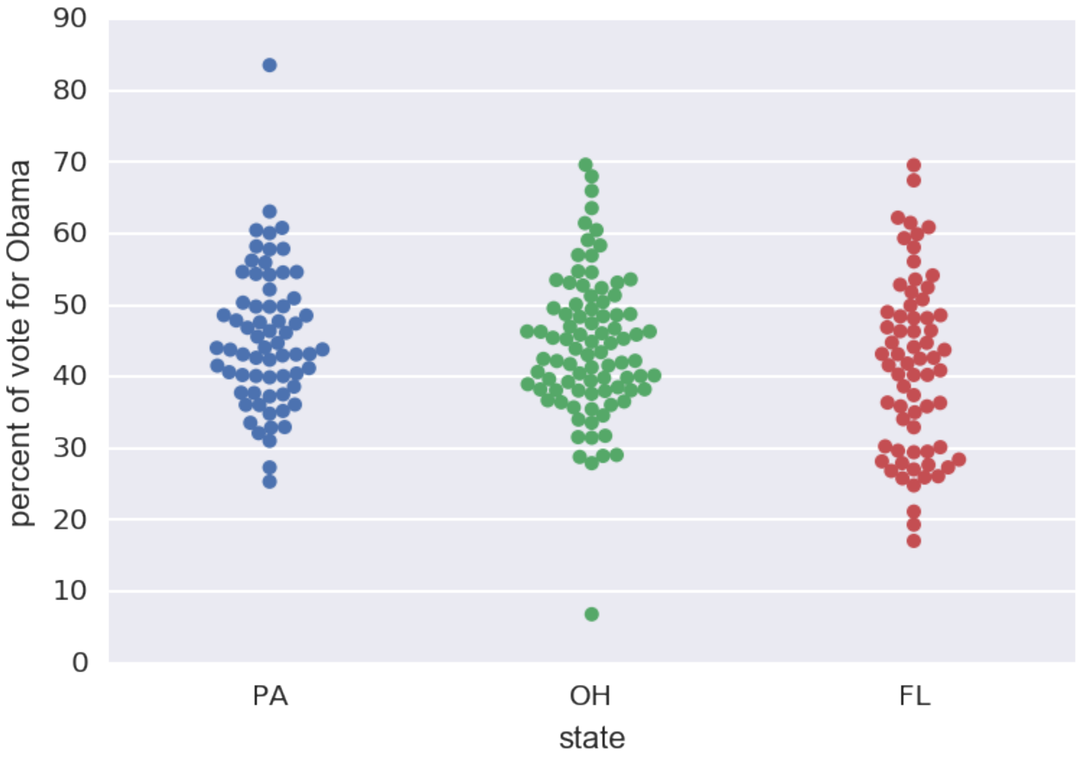
	state	county	total_votes	dem_votes	rep_votes	dem_share
0	PA	Erie County	127691	75775	50351	60.08
1	PA	Bradford County	25787	10306	15057	40.64
2	PA	Tioga County	17984	6390	11326	36.07
3	PA	McKean County	15947	6465	9224	41.21
4	PA	Potter County	7507	2300	5109	31.04
5	PA	Wayne County	22835	9892	12702	43.78
6	PA	Susquehanna County	19286	8381	10633	44.08
7	PA	Warren County	18517	8537	9685	46.85
8	OH	Ashtabula County	44874	25027	18949	56.94
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Data retrieved from Data.gov (https://www.data.gov/)

Generating a bee swarm plot

```
In [1]: _ = sns.swarmplot(x='state', y='dem_share', data=df_swing)

In [2]: _ = plt.xlabel('state')

In [3]: _ = plt.ylabel('percent of vote for Obama')

In [4]: plt.show()
```

2008 US swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

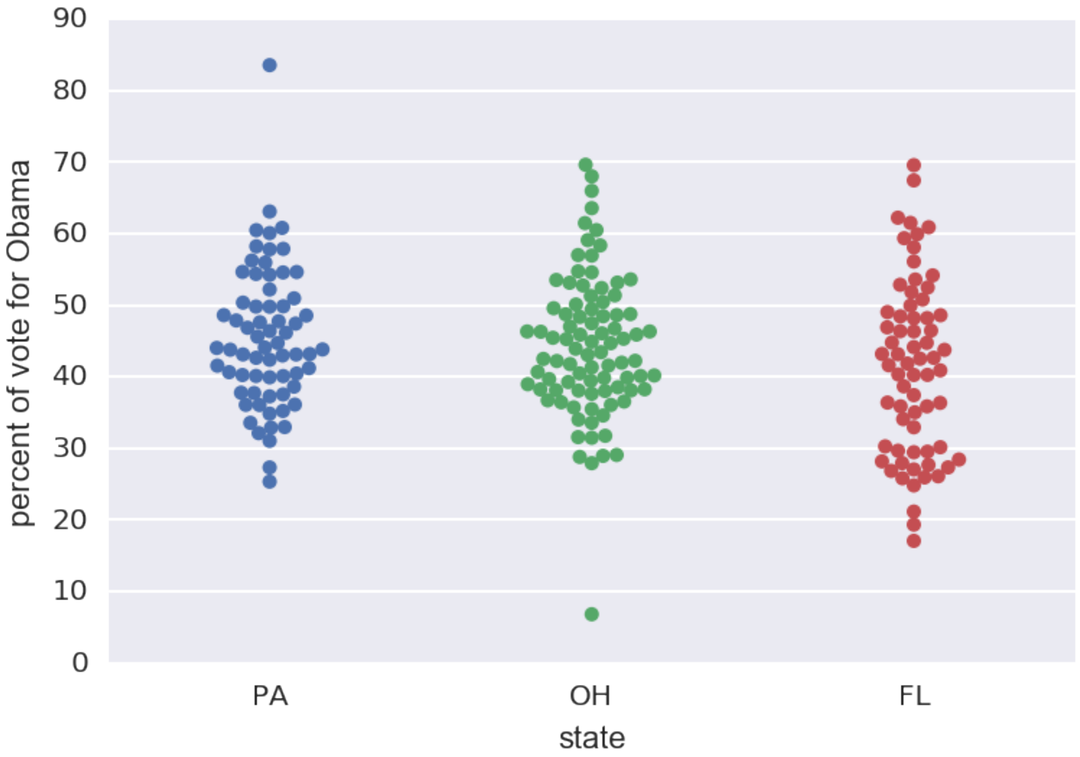
STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

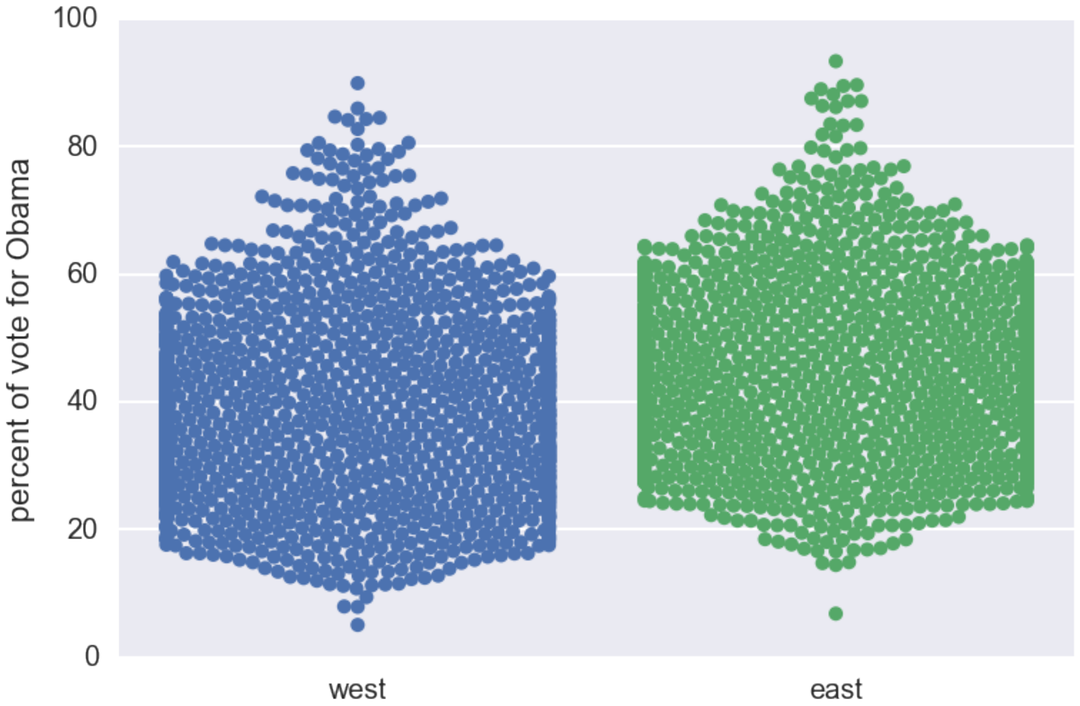
Plot all of your data: ECDFs

2008 US swing state election results



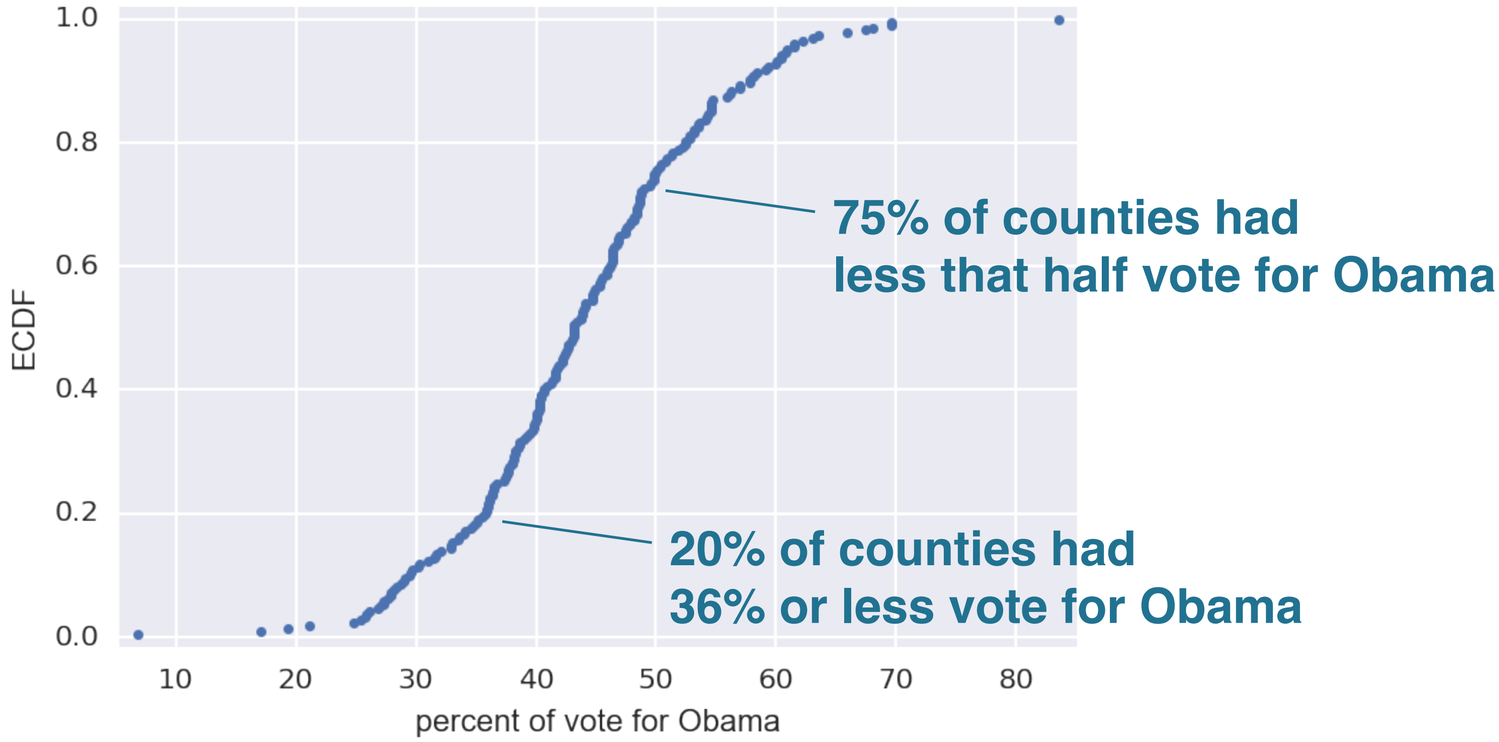
Data retrieved from Data.gov (https://www.data.gov/)

2008 US election results: East and West



Data retrieved from Data.gov (https://www.data.gov/)

Empirical cumulative distribution function (ECDF)

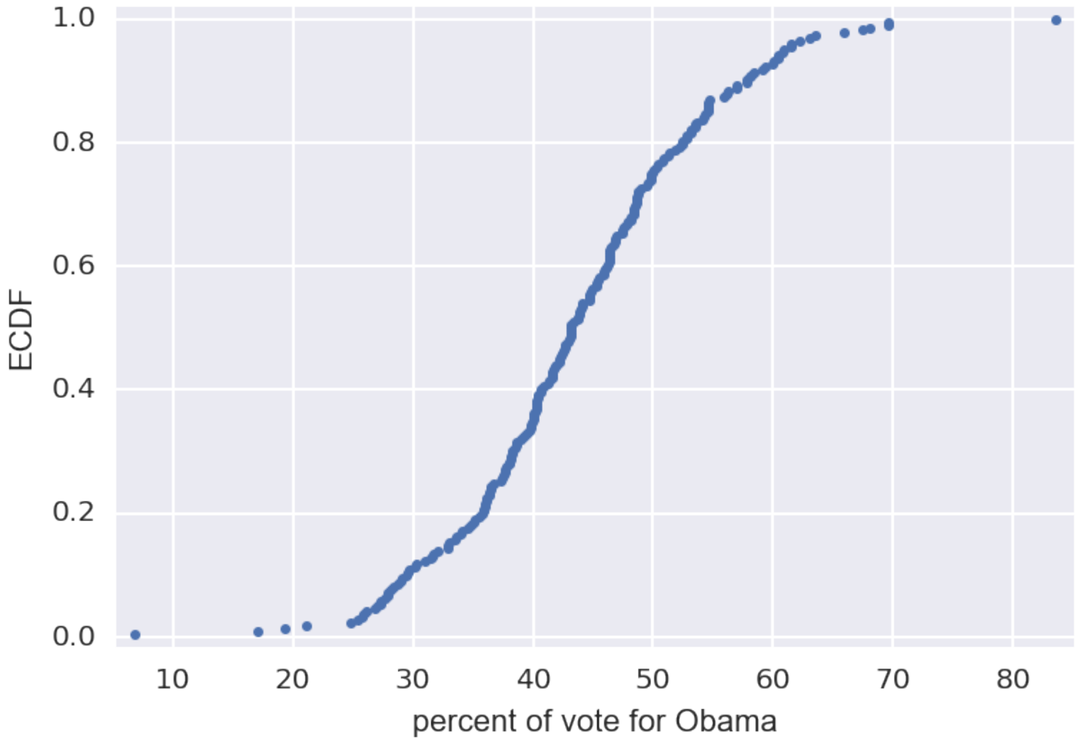


Data retrieved from Data.gov (https://www.data.gov/)

Making an ECDF

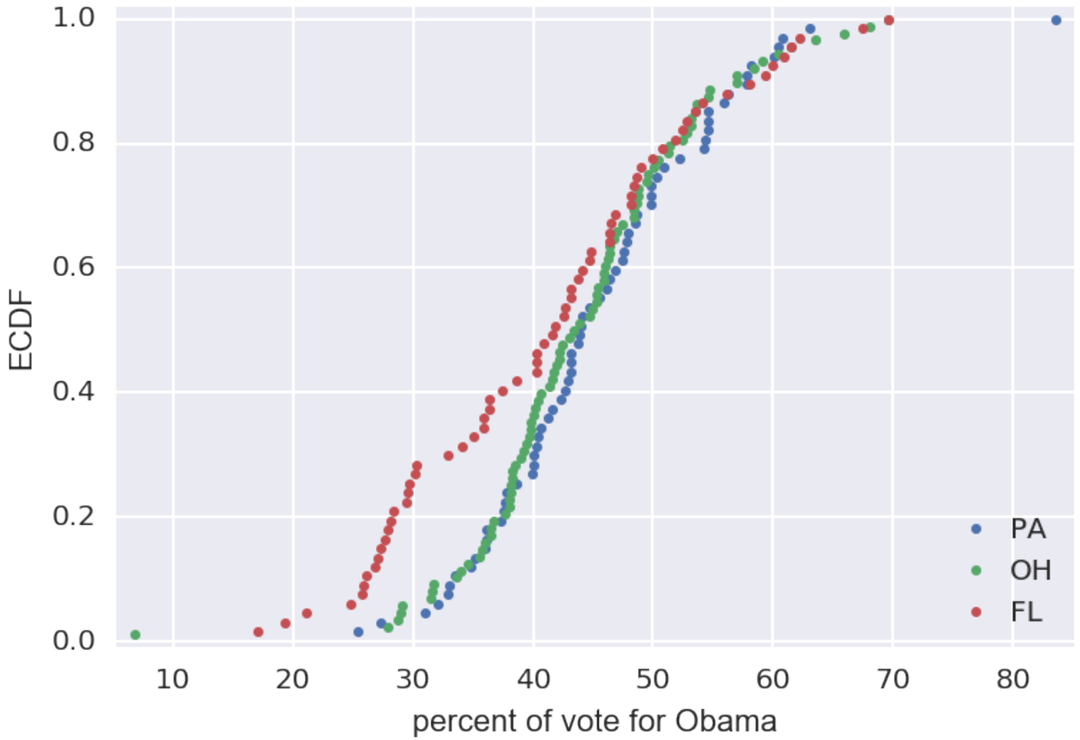
```
In [1]: import numpy as np

In [2]: x = np.sort(df_swing['dem_share'])

In [3]: y = np.arange(1, len(x)+1) / len(x)

In [4]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [5]: _ = plt.xlabel('percent of vote for Obama')

In [6]: _ = plt.ylabel('ECDF')

In [7]: plt.margins(0.02) # Keeps data off plot edges

In [8]: plt.show()
```

2008 US swing state election ECDF



Data retrieved from Data.gov (https://www.data.gov/)

2008 US swing state election ECDFs



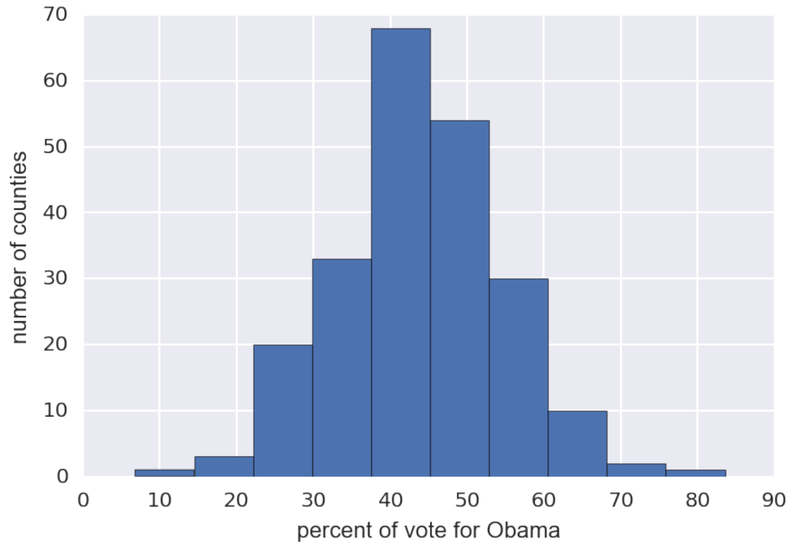
Data retrieved from Data.gov (https://www.data.gov/)

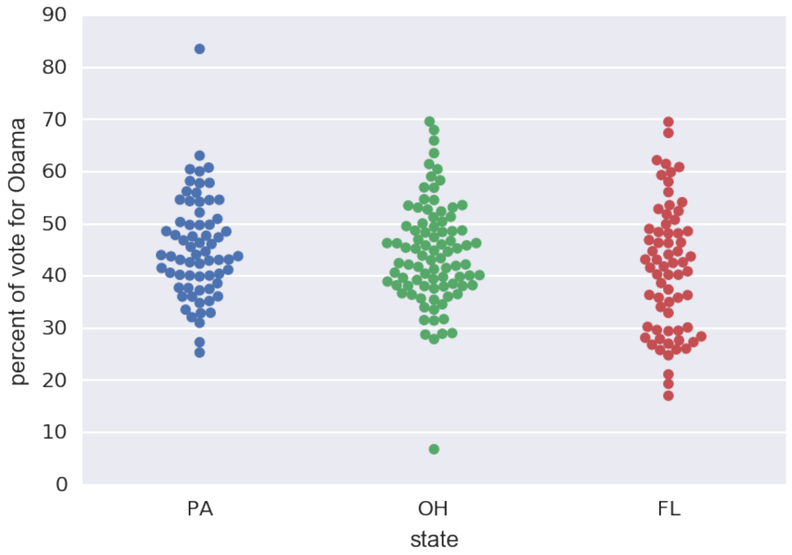
STATISTICAL THINKING IN PYTHON I

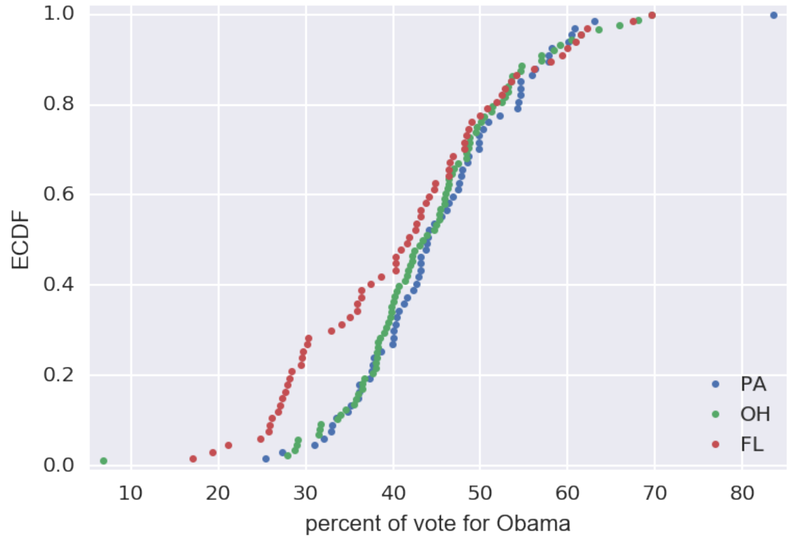
Let’s practice!

STATISTICAL THINKING IN PYTHON I

Onward toward the whole story!


number of counties
percent of vote for Obama
0
10
20
30
40
50
60
70
80
90


percent of vote for Obama
state
PA
OH
FL


ECDF
percent of vote for Obama
PA
OH
FL

“Exploratory data analysis can never be the whole story, but nothing else can serve as the foundation stone.”

—John Tukey

Coming up...

- Thinking probabilistically
- Discrete and continuous distributions
- The power of hacker statistics using `np.random()`

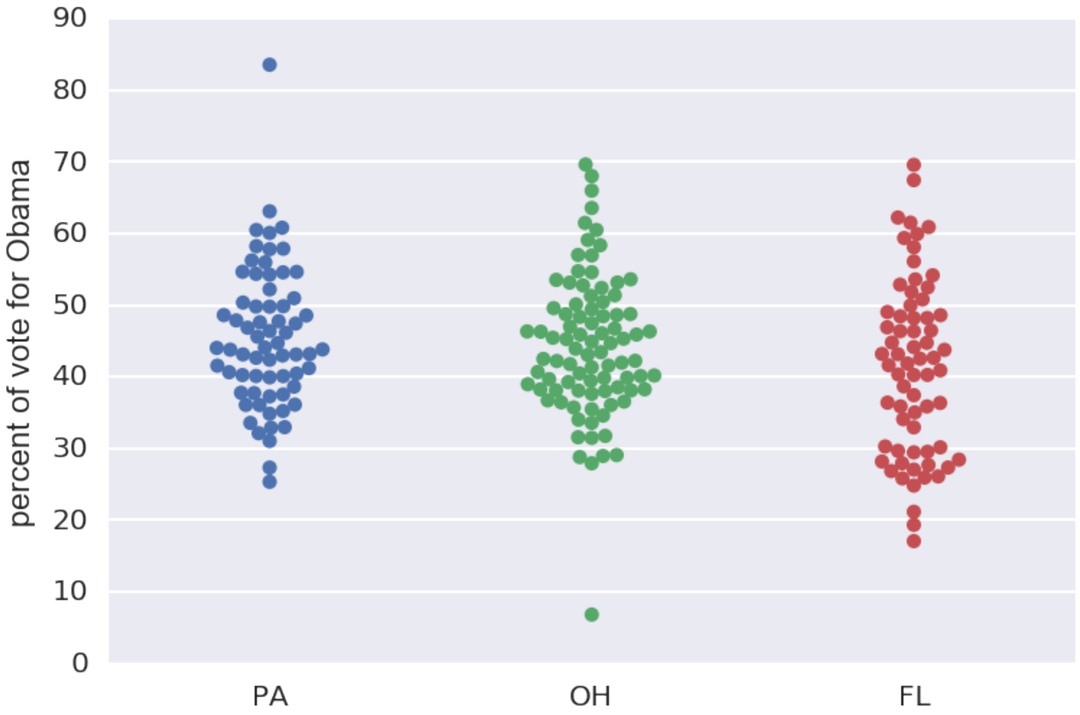
STATISTICAL THINKING IN PYTHON I

Let’s get to work!

STATISTICAL THINKING IN PYTHON I

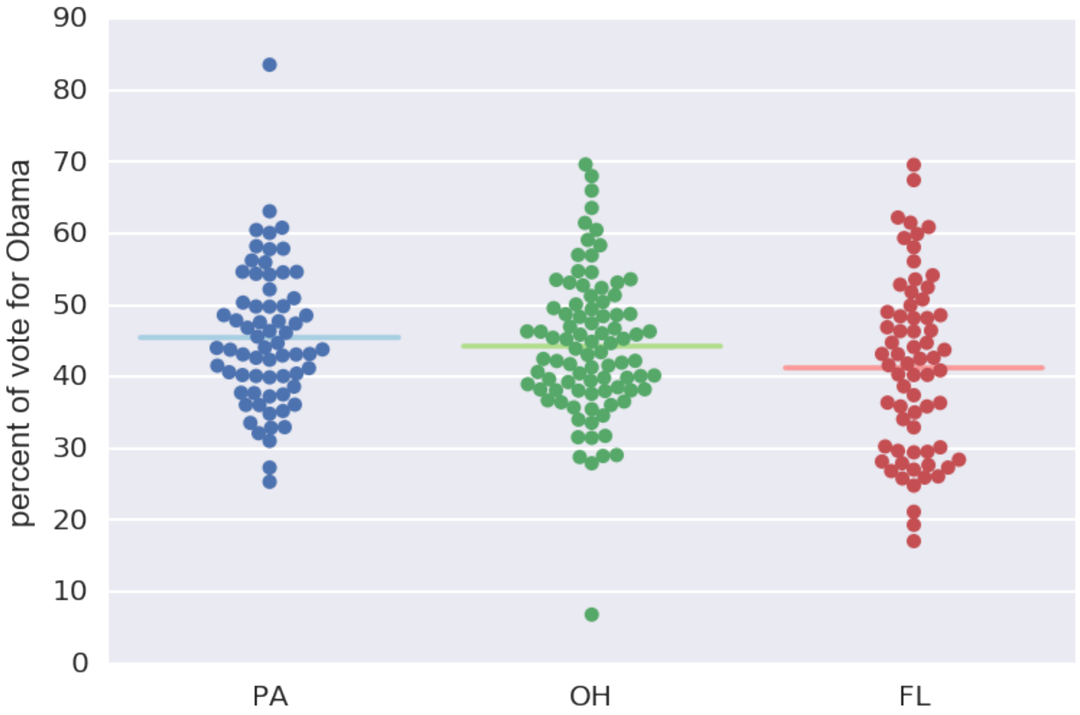
Introduction to summary statistics: The sample mean and median

2008 US swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

2008 US swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

Mean vote percentage

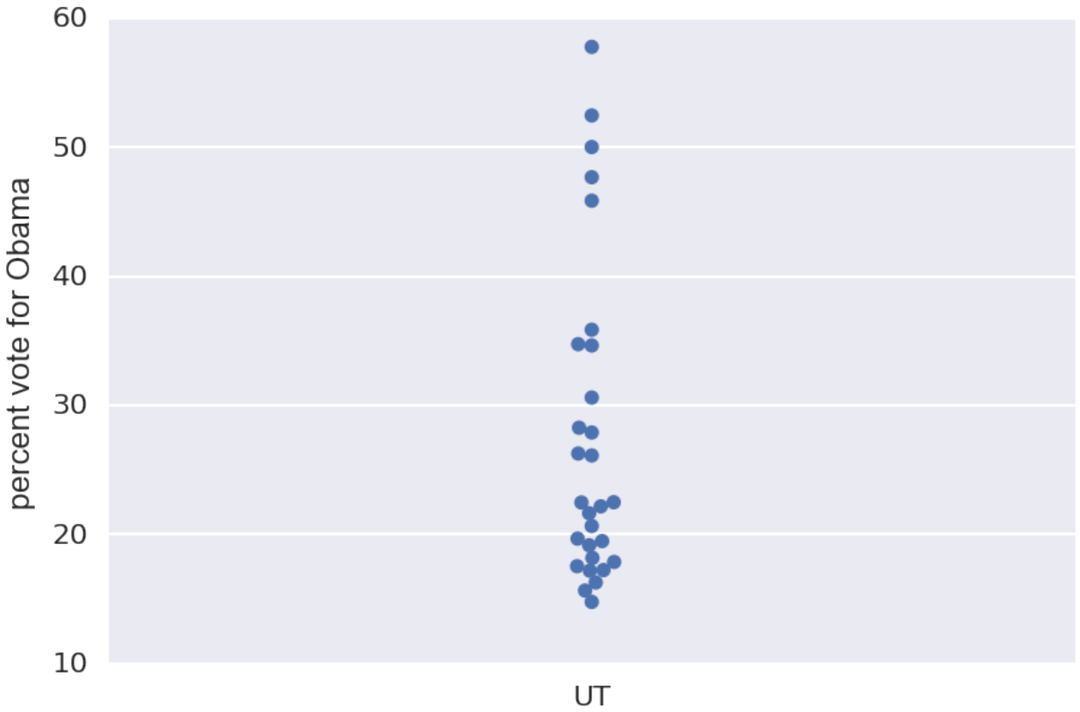
```
In [1]: import numpy as np

In [2]: np.mean(dem_share_PA)
Out[2]: 45.476417910447765
```

$$\text{mean} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Outliers

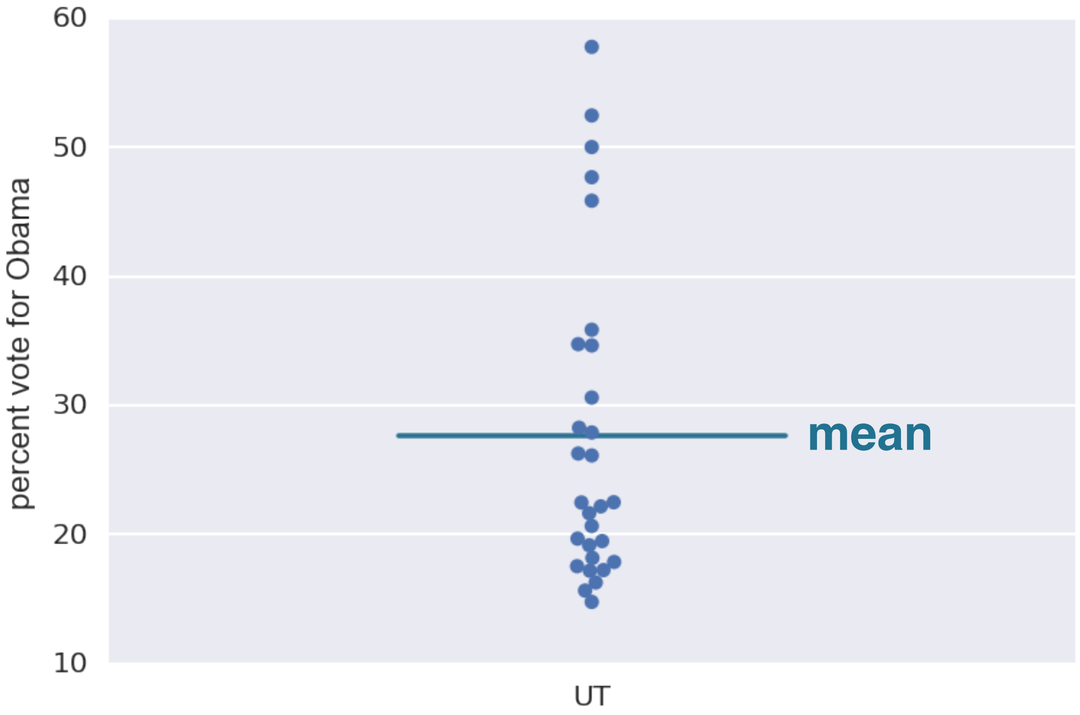
- Data points whose value is far greater or less than most of the rest of the data

2008 Utah election results



Data retrieved from Data.gov (https://www.data.gov/)

2008 Utah election results

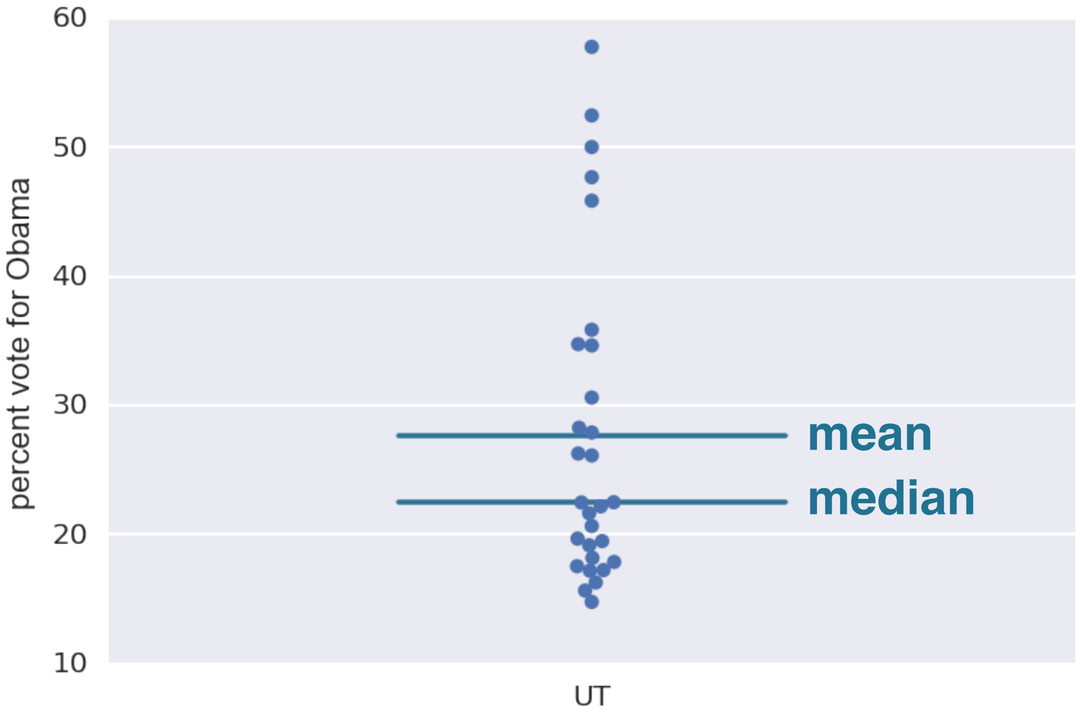


Data retrieved from Data.gov (https://www.data.gov/)

The median

- The middle value of a data set

2008 Utah election results



Data retrieved from Data.gov (https://www.data.gov/)

Computing the median

```
In [1]: np.median(dem_share_UT)
Out[1]: 22.469999999999999
```

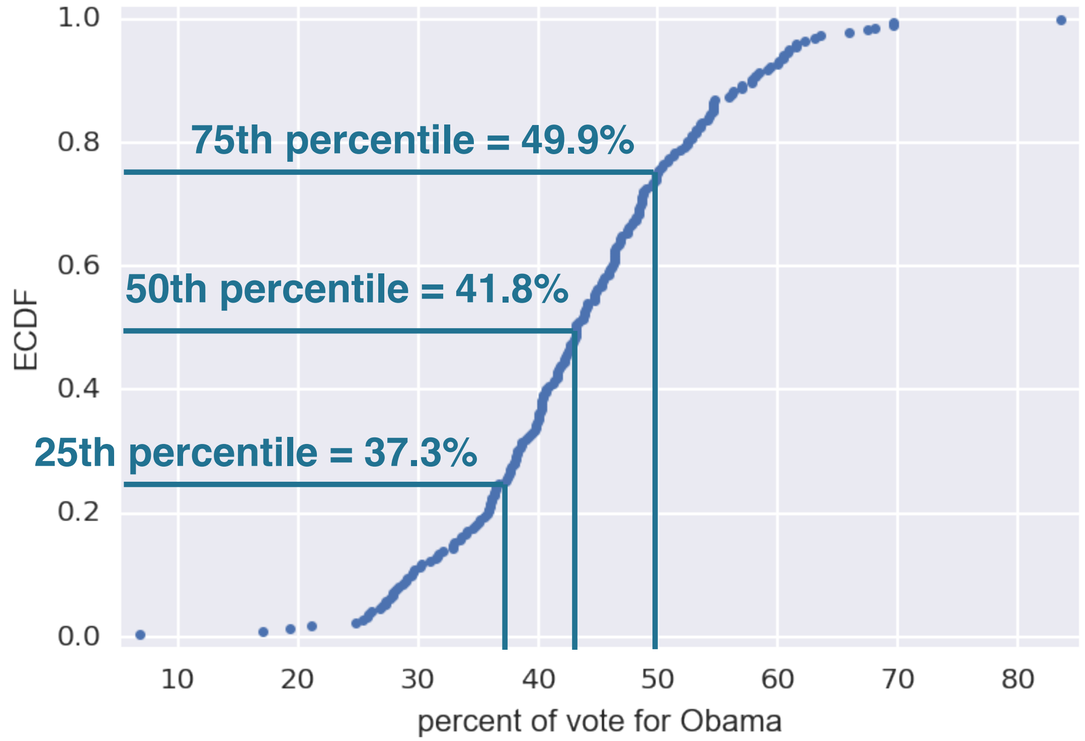
STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

Percentiles, outliers, and box plots

Percentiles on an ECDF

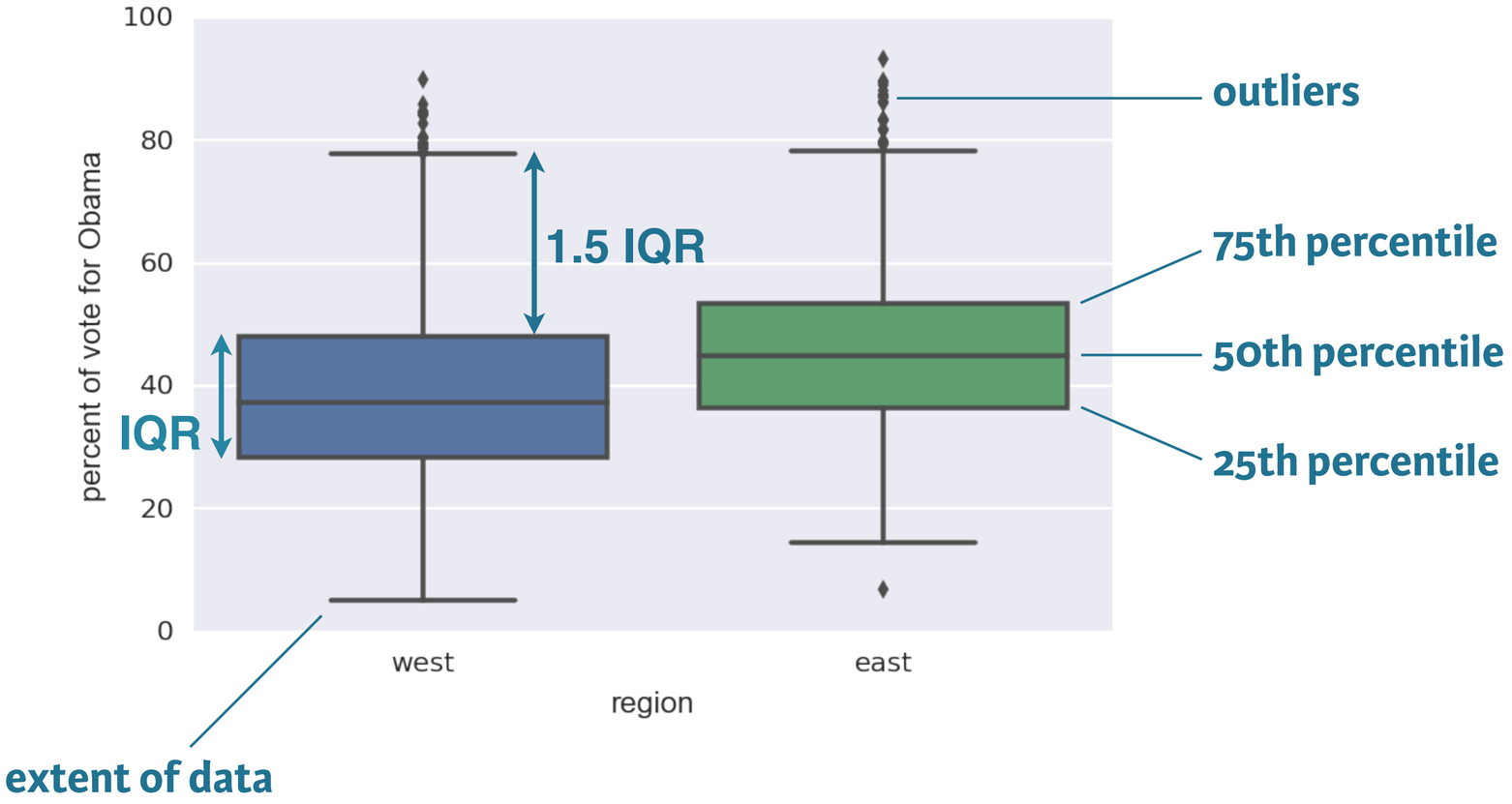


Data retrieved from Data.gov (https://www.data.gov/)

Computing percentiles

```
In [1]: np.percentile(df_swing['dem_share'], [25, 50, 75])
Out[1]: array([ 37.3025,  43.185 ,  49.925 ])
```

2008 US election box plot



Data retrieved from Data.gov (https://www.data.gov/)

Generating a box plot

```
In [1]: import matplotlib.pyplot as plt

In [2]: import seaborn as sns

In [3]: _ = sns.boxplot(x='east_west', y='dem_share',
   ...:                 data=df_all_states)

In [4]: _ = plt.xlabel('region')

In [5]: _ = plt.ylabel('percent of vote for Obama')

In [6]: plt.show()
```

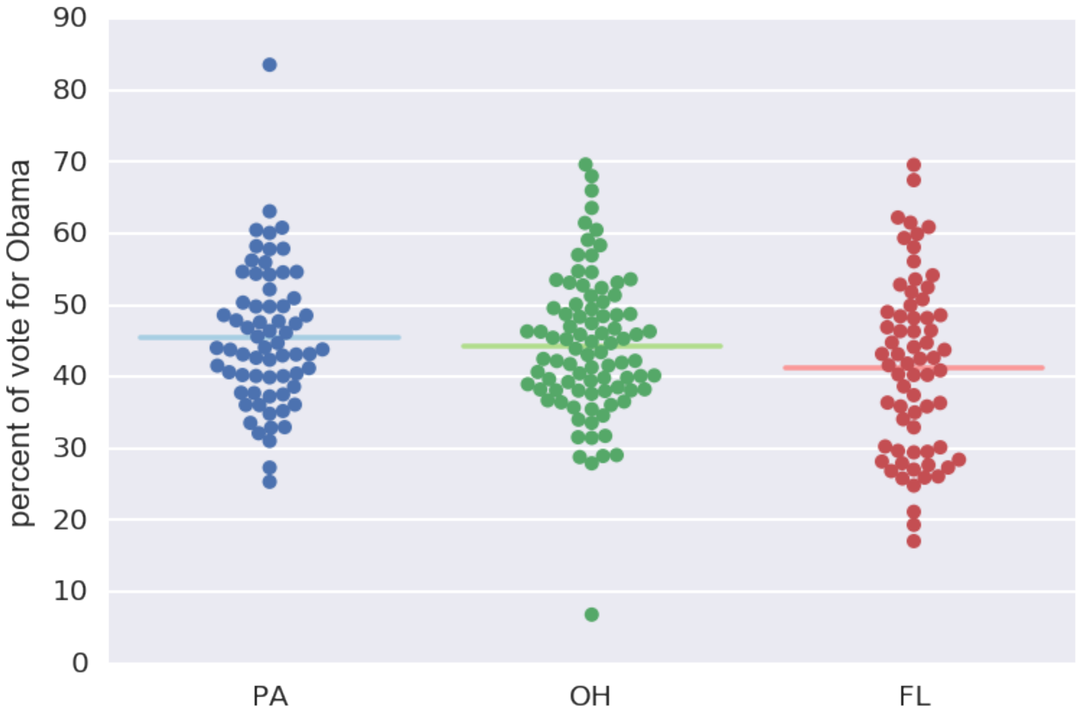
STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

Variance and standard deviation

2008 US swing state election results

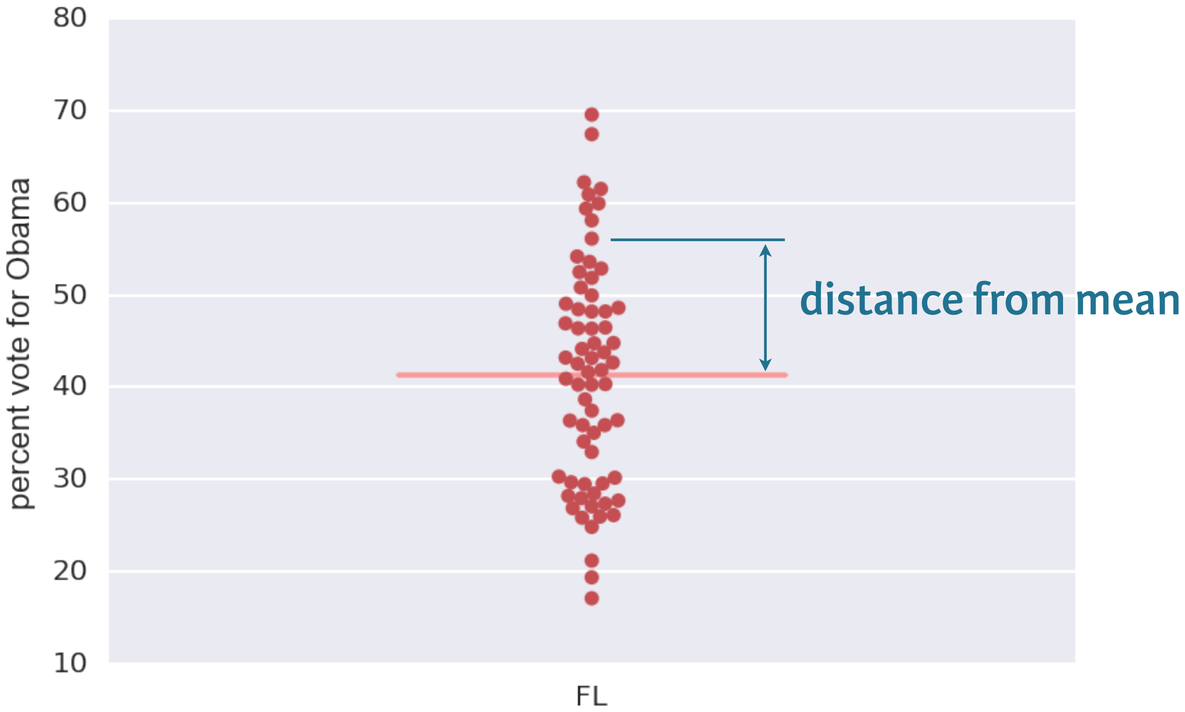


Data retrieved from Data.gov (https://www.data.gov/)

Variance

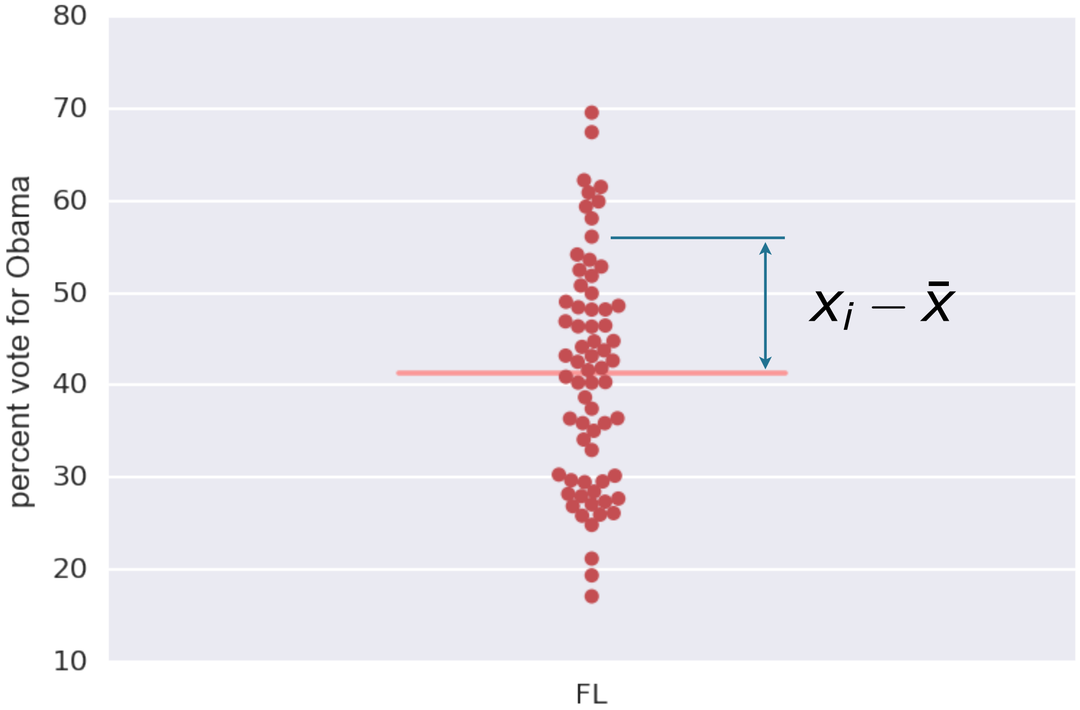
- The mean squared distance of the data from their mean
- Informally, a measure of the spread of data

2008 Florida election results



Data retrieved from Data.gov (https://www.data.gov/)

2008 Florida election results



$$\text{variance} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Data retrieved from Data.gov (https://www.data.gov/)

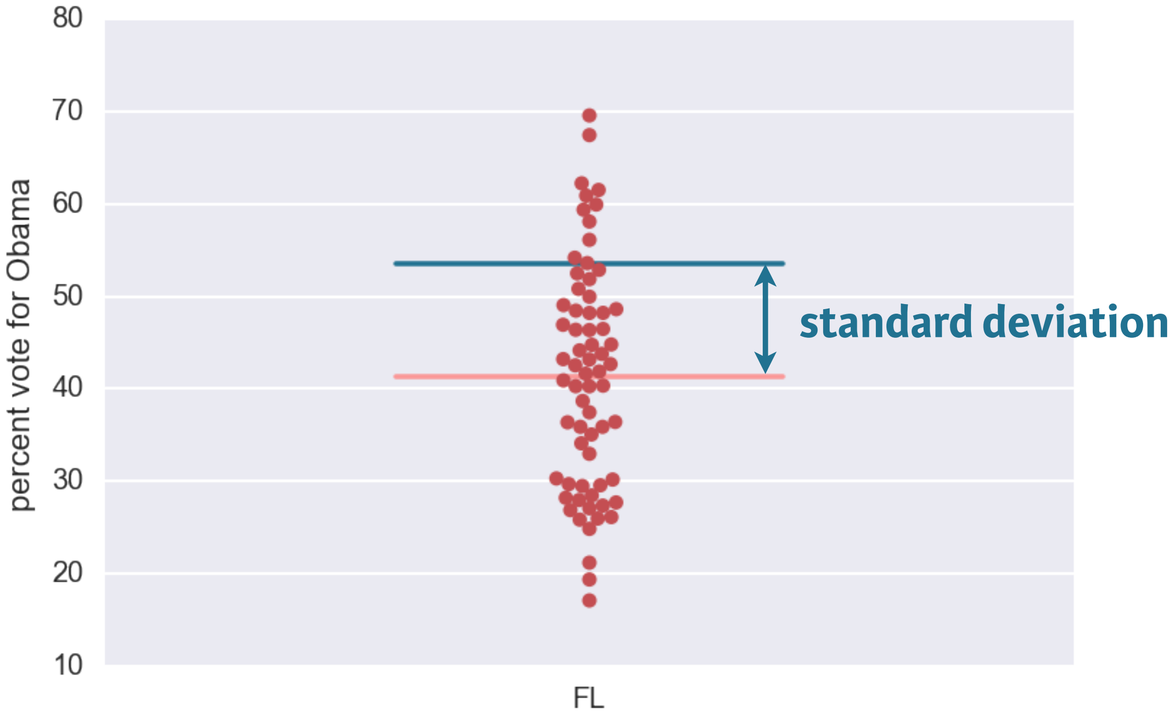
Computing the variance

```
In [1]: np.var(dem_share_FL)
Out[1]: 147.44278618846064
```

Computing the standard deviation

```
In [1]: np.std(dem_share_FL)
Out[1]: 12.142602117687158

In [2]: np.sqrt(np.var(dem_share_FL))
Out[2]: 12.142602117687158
```

2008 Florida election results



Data retrieved from Data.gov (https://www.data.gov/)

STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

Covariance and the Pearson correlation coefficient

2008 US swing state election results

Data retrieved from Data.gov (https://www.data.gov/)

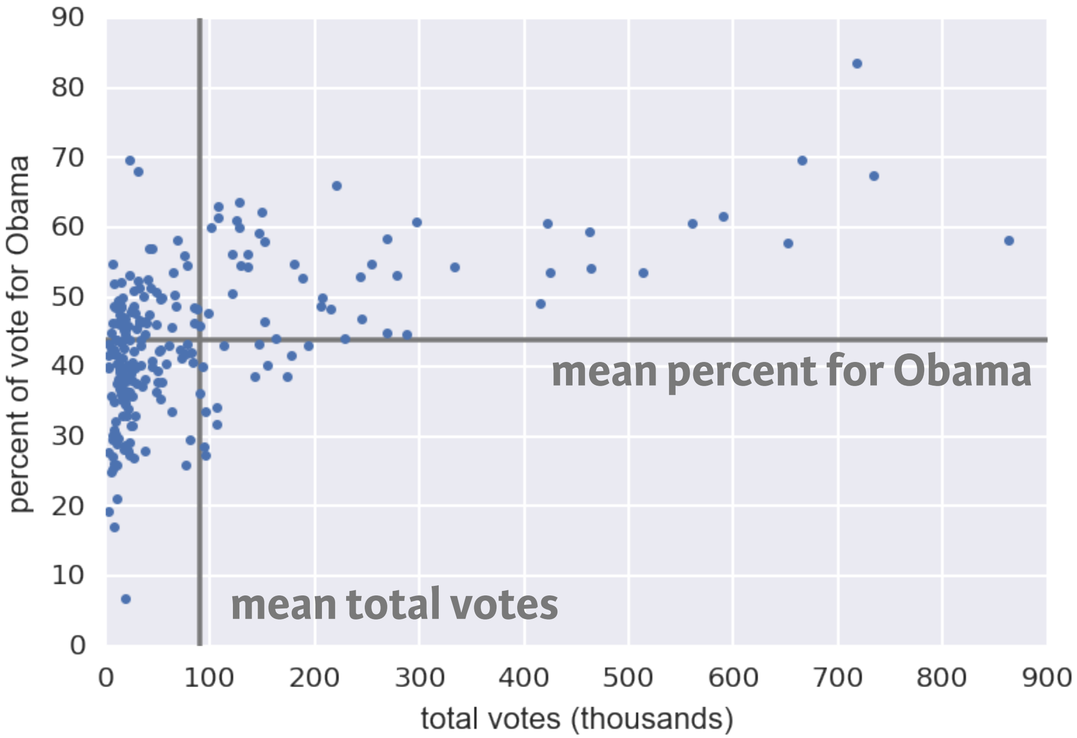
Generating a scatter plot

```
In [1]: _ = plt.plot(total_votes/1000, dem_share,
   ...:                marker='.', linestyle='none')

In [2]: _ = plt.xlabel('total votes (thousands)')

In [3]: _ = plt.ylabel('percent of vote for Obama')
```

Covariance

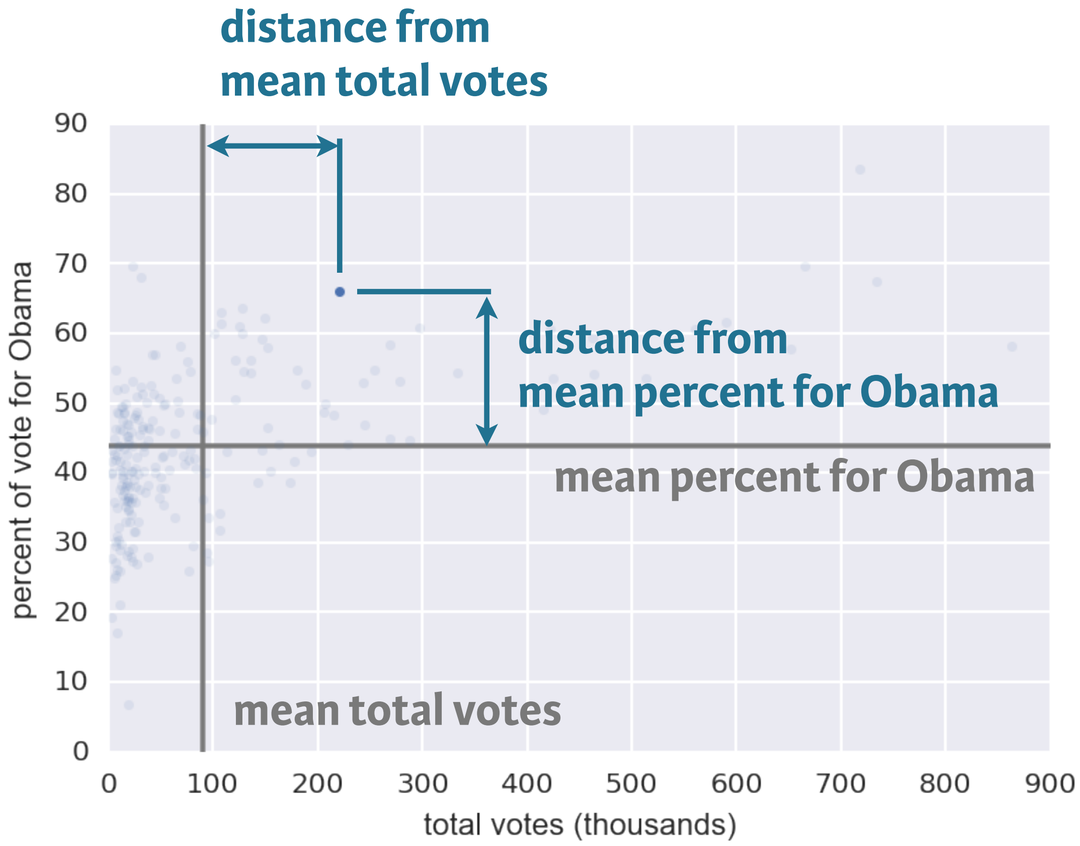
- A measure of how two quantities vary *together*

Calculation of the covariance



Data retrieved from Data.gov (https://www.data.gov/)

Calculation of the covariance



$$\text{covariance} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

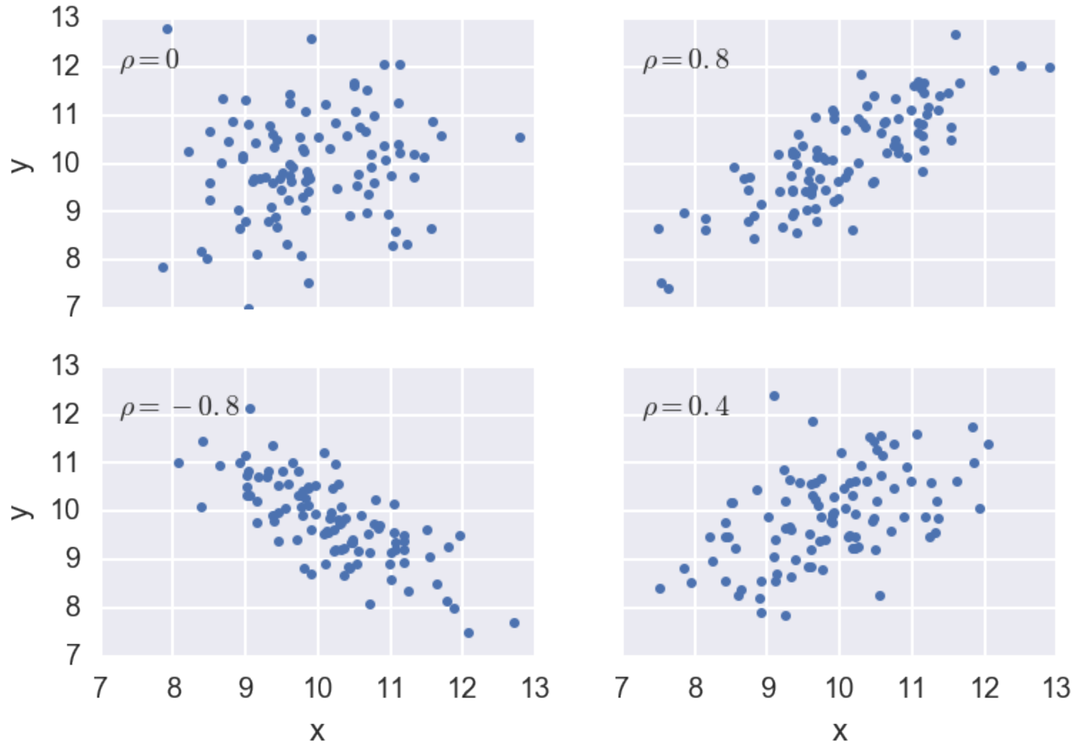
Data retrieved from Data.gov (https://www.data.gov/)

Pearson correlation coefficient

$$\rho = \text{Pearson correlation} = \frac{\text{covariance}}{(\text{std of } x)\,(\text{std of } y)}$$

$$= \frac{\text{variability due to codependence}}{\text{independent variability}}$$

Pearson correlation coefficient examples



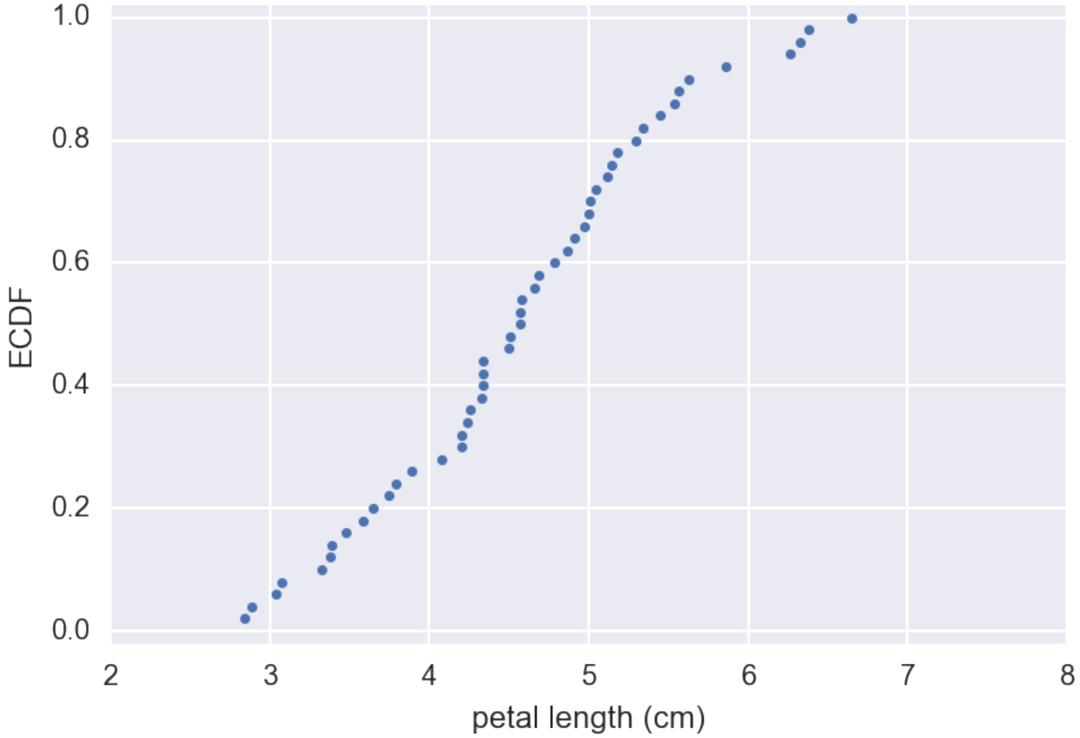
STATISTICAL THINKING IN PYTHON I

Let’s practice!

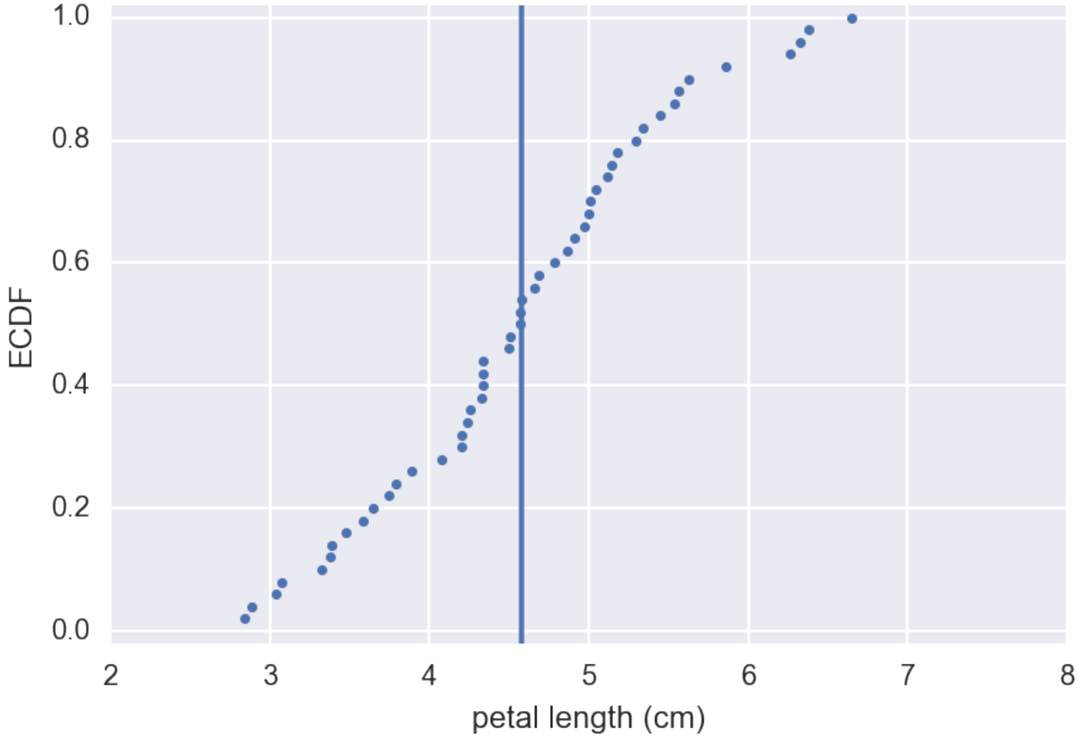
STATISTICAL THINKING IN PYTHON I

Probabilistic logic and statistical inference

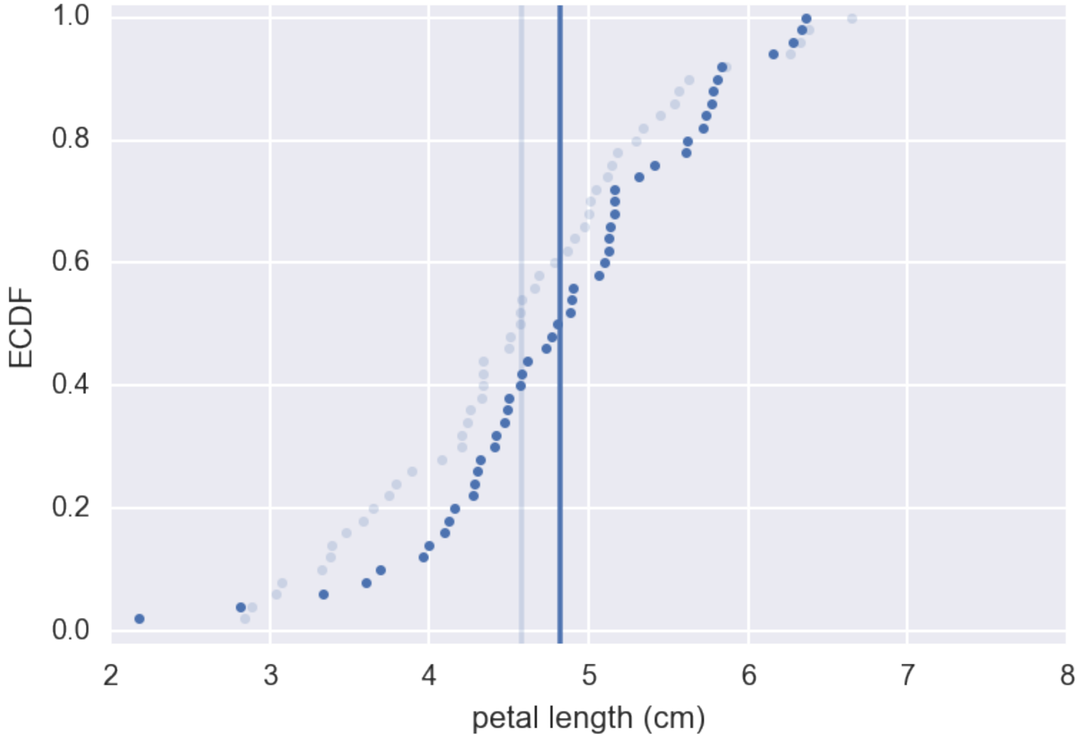
50 measurements of petal length



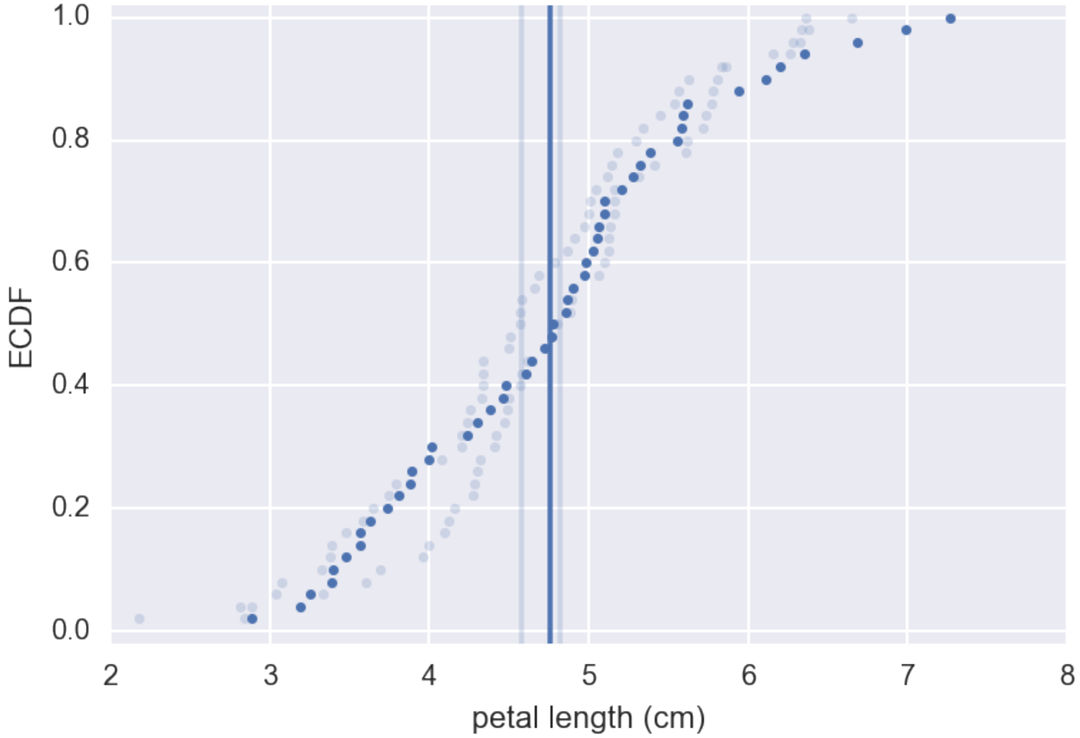
50 measurements of petal length



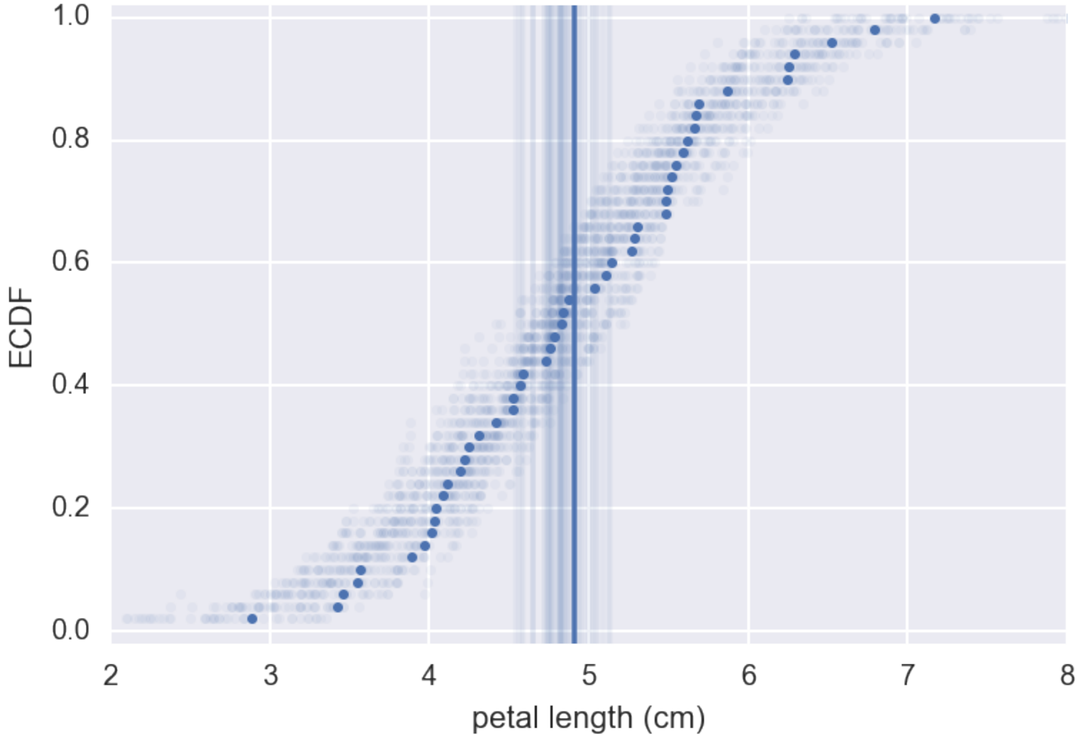
50 measurements of petal length



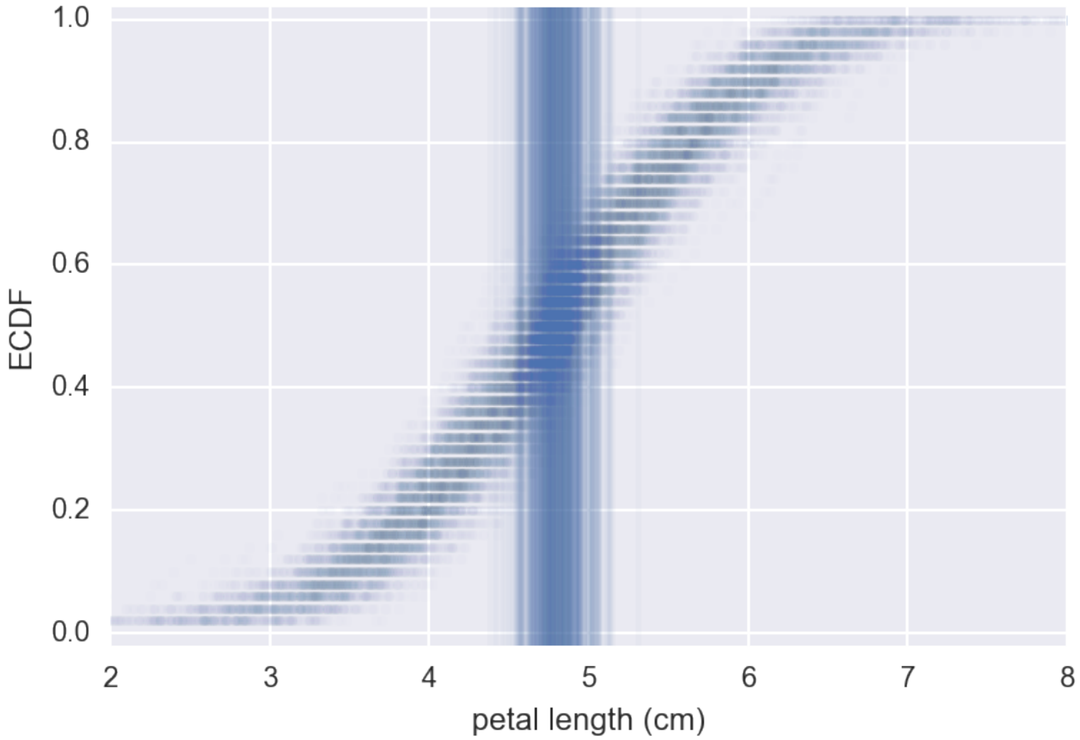
50 measurements of petal length



50 measurements of petal length



Repeats of 50 measurements of petal length



STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

Random number generators and hacker statistics

Hacker statistics

- Uses simulated repeated measurements to compute probabilities.

Image: artist unknown

Image: Heritage Auction

The `np.random` module

- Suite of functions based on *random number generation*
- `np.random.random()`:

  draw a number between 0 and 1

The `np.random` module

- Suite of functions based on *random number generation*
- `np.random.random()`:

  draw a number between 0 and 1

< 0.5 ⟶ 

≥ 0.5 ⟶ 

Images: Heritage Auction

Bernoulli trial

- An experiment that has two options, "success" (True) and "failure" (False).

Random number seed

- Integer fed into random number generating algorithm
- Manually seed random number generator if you need reproducibility
- Specified using `np.random.seed()`

Simulating 4 coin flips

```
In [1]: import numpy as np

In [2]: np.random.seed(42)

In [3]: random_numbers = np.random.random(size=4)

In [4]: random_numbers
Out[4]: array([ 0.37454012,  0.95071431,  0.73199394,
0.59865848])

In [5]: heads = random_numbers < 0.5

In [6]: heads
Out[6]: array([ True, False, False, False], dtype=bool)

In [7]: np.sum(heads)
Out[7]: 1
```

Simulating 4 coin flips

```
In [1]: n_all_heads = 0  # Initialize number of 4-heads trials

In [2]: for _ in range(10000):
    ...:     heads = np.random.random(size=4) < 0.5
    ...:     n_heads = np.sum(heads)
    ...:     if n_heads == 4:
    ...:         n_all_heads += 1
    ...:
    ...:

In [3]: n_all_heads / 10000
Out[3]: 0.0621
```

Hacker stats probabilities

- Determine how to simulate data
- Simulate many many times
- Probability is approximately fraction of trials with the outcome of interest

STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

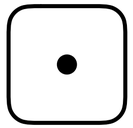
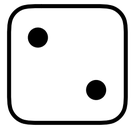
Probability distributions and stories: The Binomial distribution

Probability mass function (PMF)

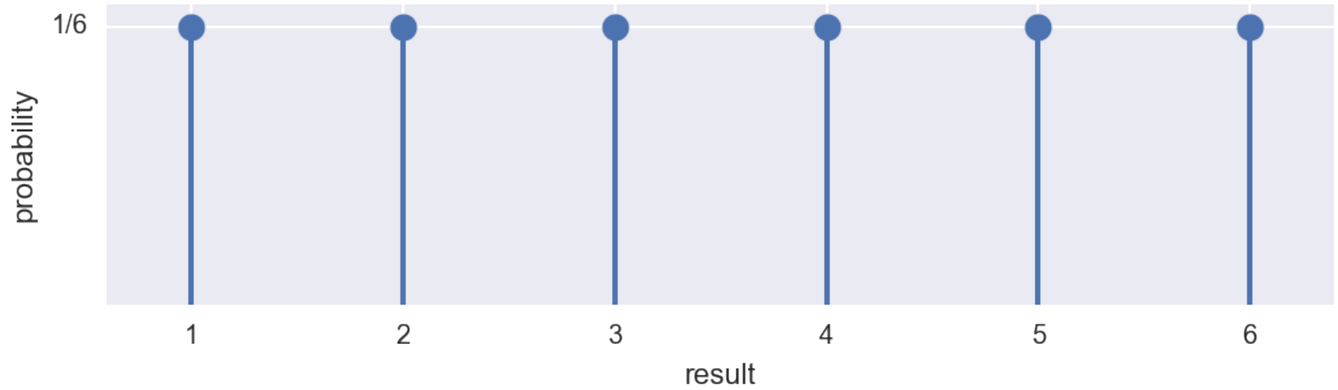
- The set of probabilities of discrete outcomes

Discrete Uniform PMF

Tabular

					
1/6	1/6	1/6	1/6	1/6	1/6

Graphical



Probability distribution

- A mathematical description of outcomes

Discrete Uniform distribution: the story

- The outcome of rolling a single fair die is Discrete Uniformly distributed.

Binomial distribution: the story

- The number *r* of successes in *n* Bernoulli trials with probability *p* of success, is Binomially distributed

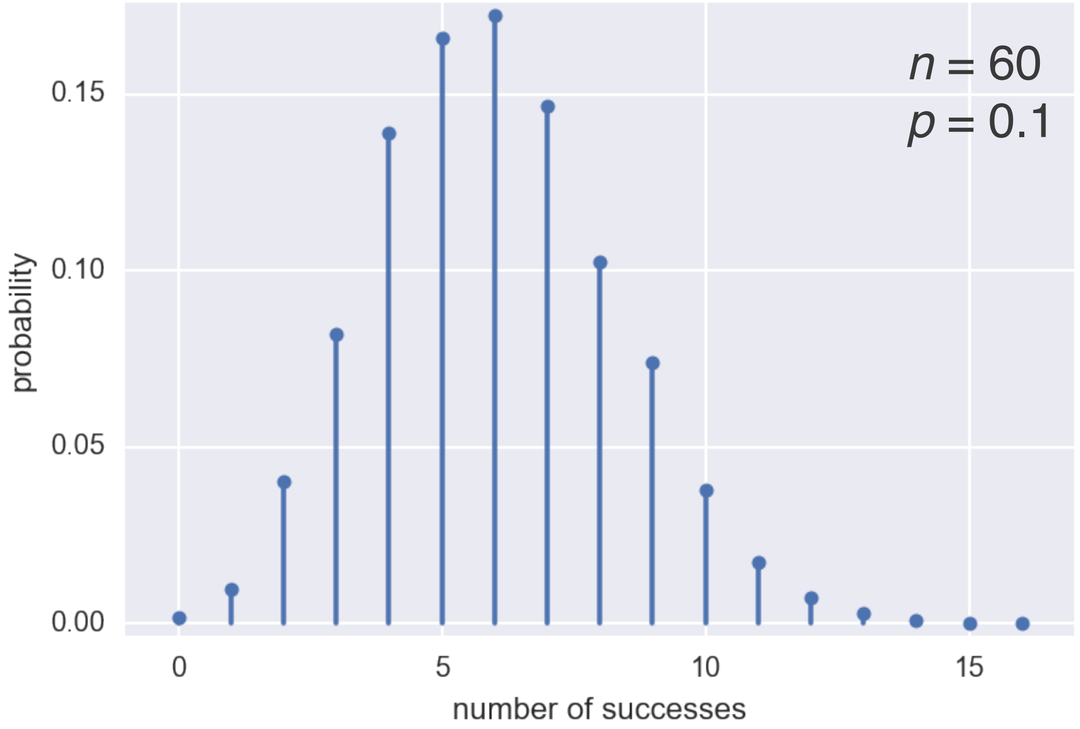
- The number *r* of heads in 4 coin flips with probability 0.5 of heads, is Binomially distributed

Sampling from the Binomial distribution

```
In [1]: np.random.binomial(4, 0.5)
Out[1]: 2

In [2]: np.random.binomial(4, 0.5, size=10)
Out[2]: array([4, 3, 2, 1, 1, 0, 3, 2, 3, 0])
```

The Binomial PMF

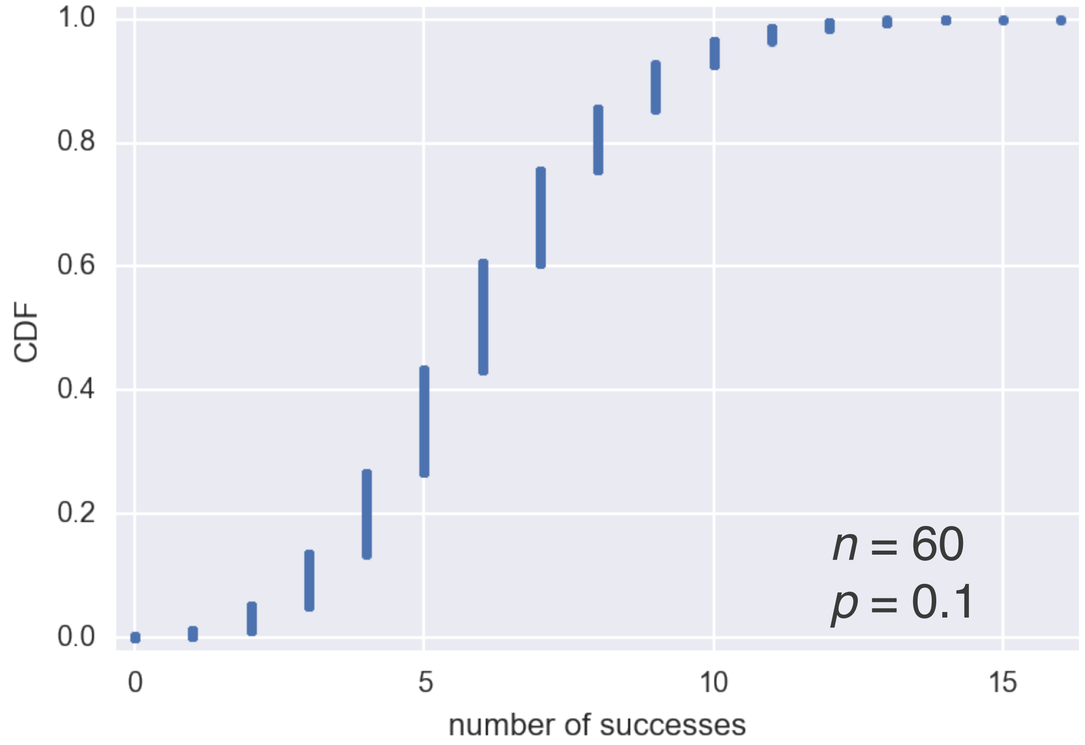
```
In [1]: samples = np.random.binomial(60, 0.1, size=10000)
```



The Binomial CDF

```
In [1]: import matplotlib.pyplot as plt

In [2]: import seaborn as sns

In [3]: sns.set()

In [4]: x, y = ecdf(samples)

In [5]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [6]: plt.margins(0.02)

In [7]: _ = plt.xlabel('number of successes')

In [8]: _ = plt.ylabel('CDF')

In [9]: plt.show()
```

The Binomial CDF



STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

Poisson processes and the Poisson distribution

Poisson process

- The timing of the next event is completely independent of when the previous event happened

Examples of Poisson processes

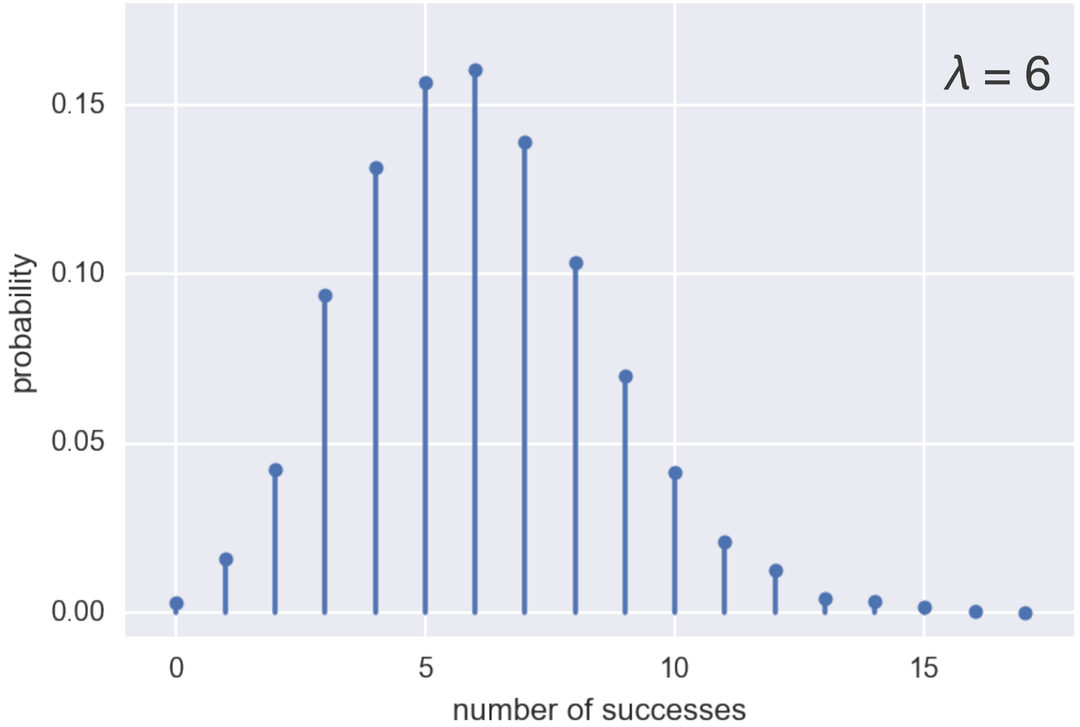
- Natural births in a given hospital
- Hit on a website during a given hour
- Meteor strikes
- Molecular collisions in a gas
- Aviation incidents
- Buses in Poissonville

Poisson distribution

- The number *r* of arrivals of a Poisson process in a given time interval with average rate of λ arrivals per interval is Poisson distributed.

- The number *r* of hits on a website in one hour with an average hit rate of 6 hits per hour is Poisson distributed.

Poisson PMF


$\lambda = 6$
0.15
0.10
0.05
0.00
probability
0
5
10
15
number of successes

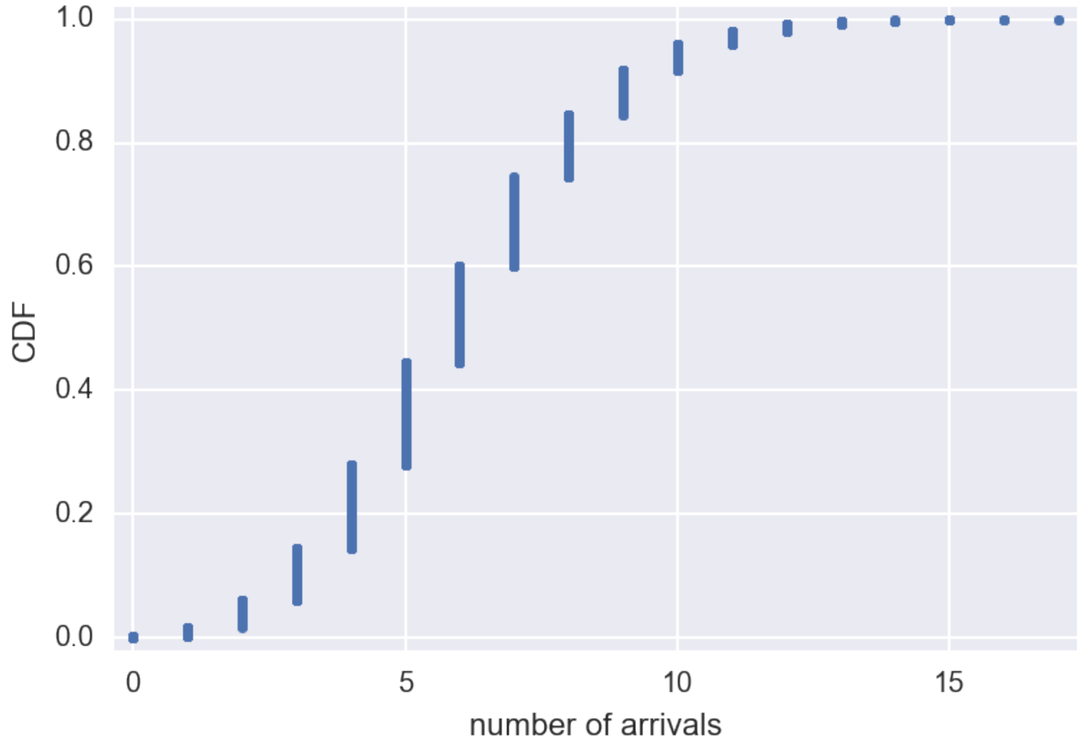
Poisson Distribution

- Limit of the Binomial distribution for low probability of success and large number of trials.
- That is, for rare events.

The Poisson CDF

```
In [1]: samples = np.random.poisson(6, size=10000)

In [2]: x, y = ecdf(samples)

In [3]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [4]: plt.margins(0.02)

In [5]: _ = plt.xlabel('number of successes')

In [6]: _ = plt.ylabel('CDF')

In [7]: plt.show()
```

The Poisson CDF



STATISTICAL THINKING IN PYTHON I

Let’s practice!

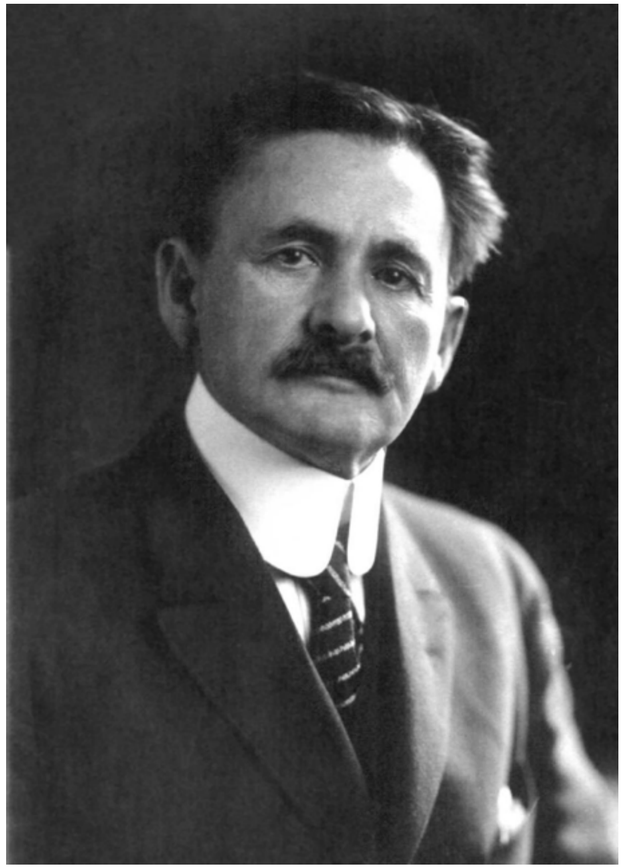
STATISTICAL THINKING IN PYTHON I

Probability density functions

Continuous variables

- Quantities that can take any value, not just discrete values

Michelson's speed of light experiment



measured speed of light (1000 km/s)

299.85	299.74	299.90	300.07	299.93
299.85	299.95	299.98	299.98	299.88
300.00	299.98	299.93	299.65	299.76
299.81	300.00	300.00	299.96	299.96
299.96	299.94	299.96	299.94	299.88
299.80	299.85	299.88	299.90	299.84
299.83	299.79	299.81	299.88	299.88
299.83	299.80	299.79	299.76	299.80
299.88	299.88	299.88	299.86	299.72
299.72	299.62	299.86	299.97	299.95
299.88	299.91	299.85	299.87	299.84
299.84	299.85	299.84	299.84	299.84
299.89	299.81	299.81	299.82	299.80
299.77	299.76	299.74	299.75	299.76
299.91	299.92	299.89	299.86	299.88
299.72	299.84	299.85	299.85	299.78
299.89	299.84	299.78	299.81	299.76
299.81	299.79	299.81	299.82	299.85
299.87	299.87	299.81	299.74	299.81
299.94	299.95	299.80	299.81	299.87

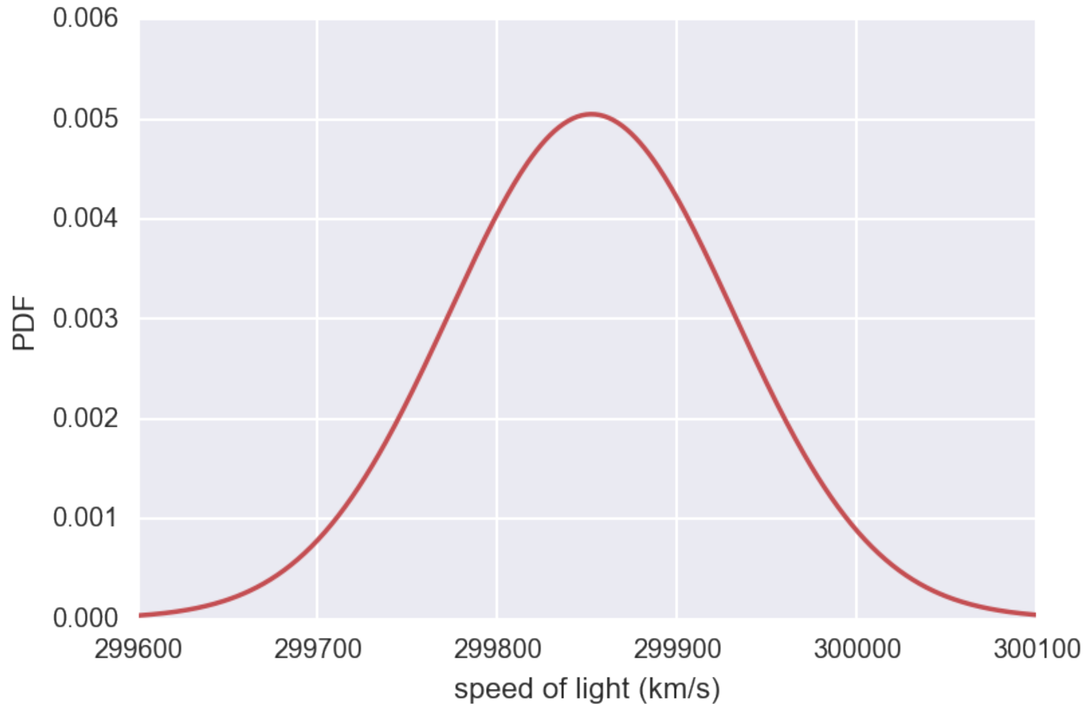
Image: public domain, Smithsonian

Data: Michelson, 1880

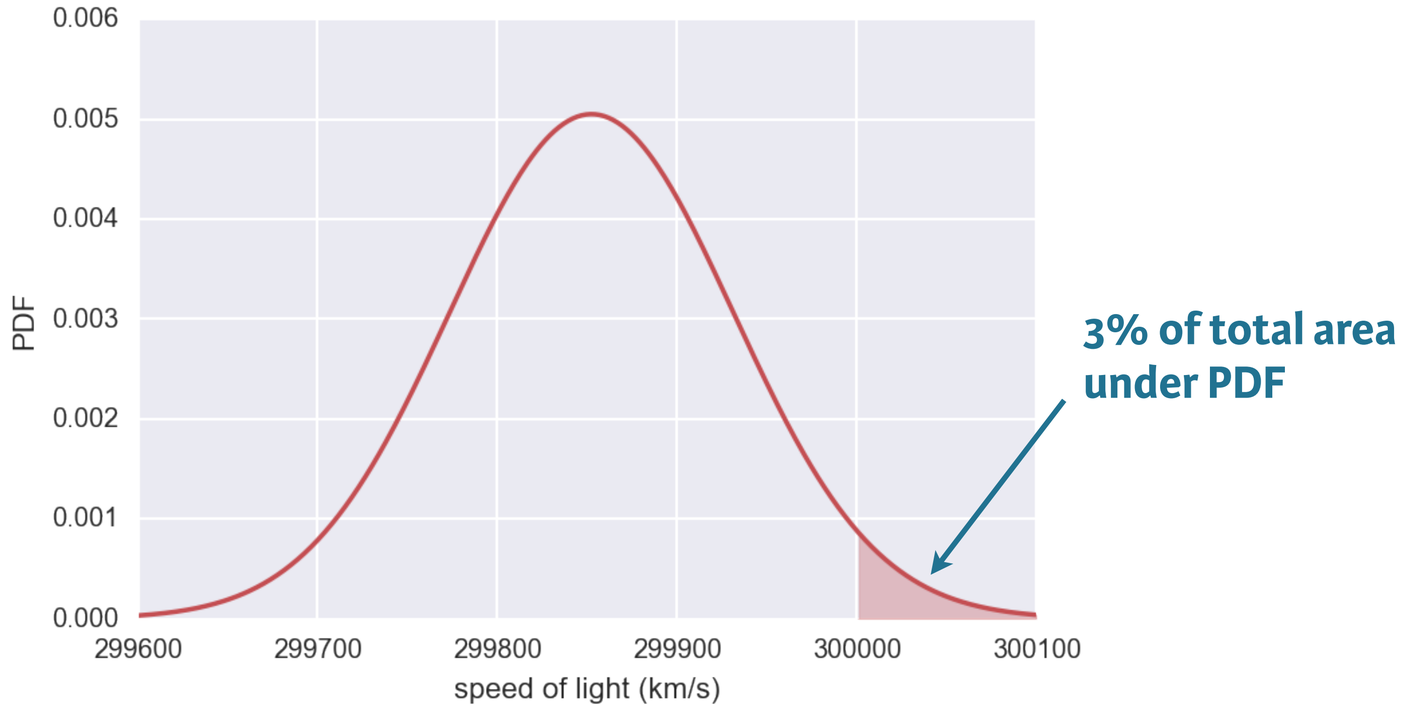
Probability density function (PDF)

- Continuous analog to the PMF
- Mathematical description of the relative likelihood of observing a value of a continuous variable

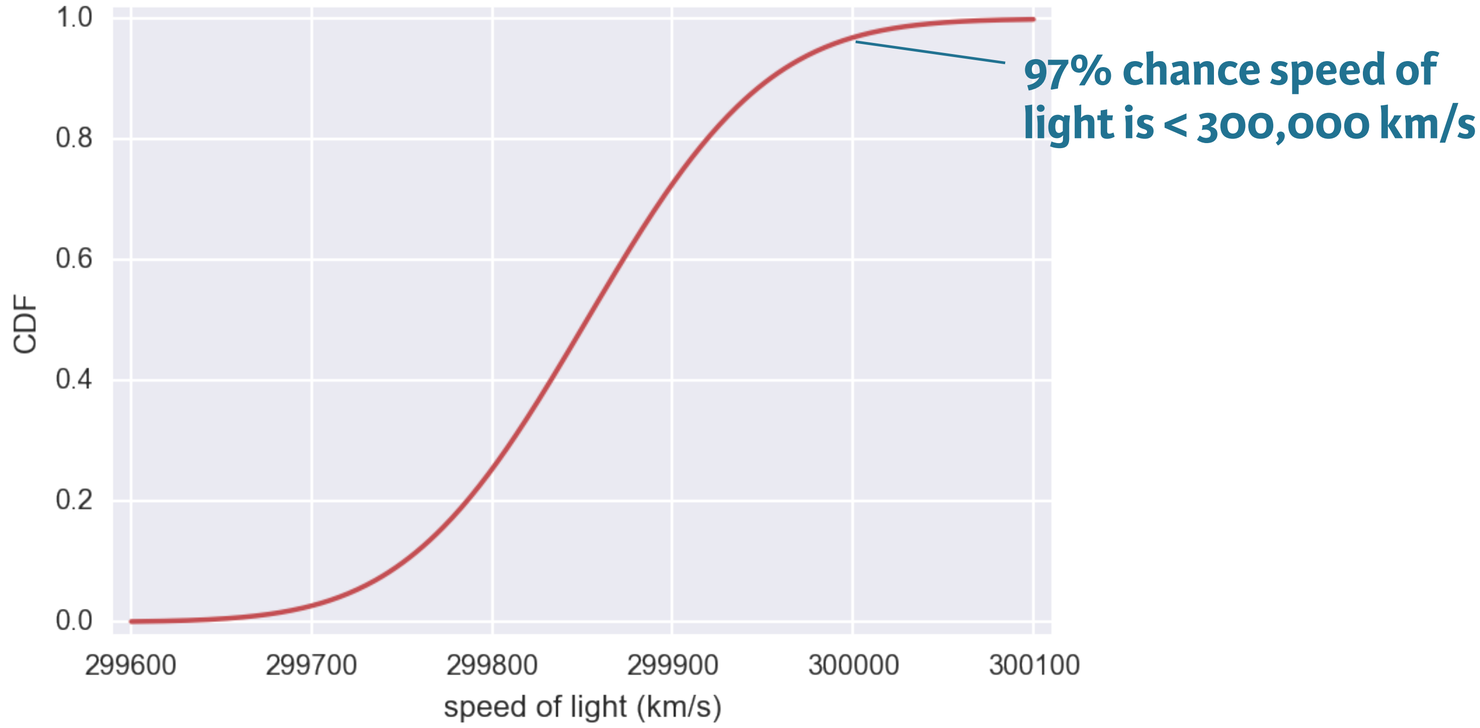
Normal PDF


0.006
0.005
0.004
0.003
0.002
0.001
0.000
PDF
299600
299700
299800
299900
300000
300100
speed of light (km/s)

Normal PDF


0.006
0.005
0.004
0.003
0.002
0.001
0.000
PDF
299600
299700
299800
299900
300000
300100
speed of light (km/s)
3% of total area
under PDF

Normal CDF


1.0
0.8
0.6
0.4
0.2
0.0
CDF
299600
299700
299800
299900
300000
300100
speed of light (km/s)
97% chance speed of
light is < 300,000 km/s

STATISTICAL THINKING IN PYTHON I

Let’s practice!

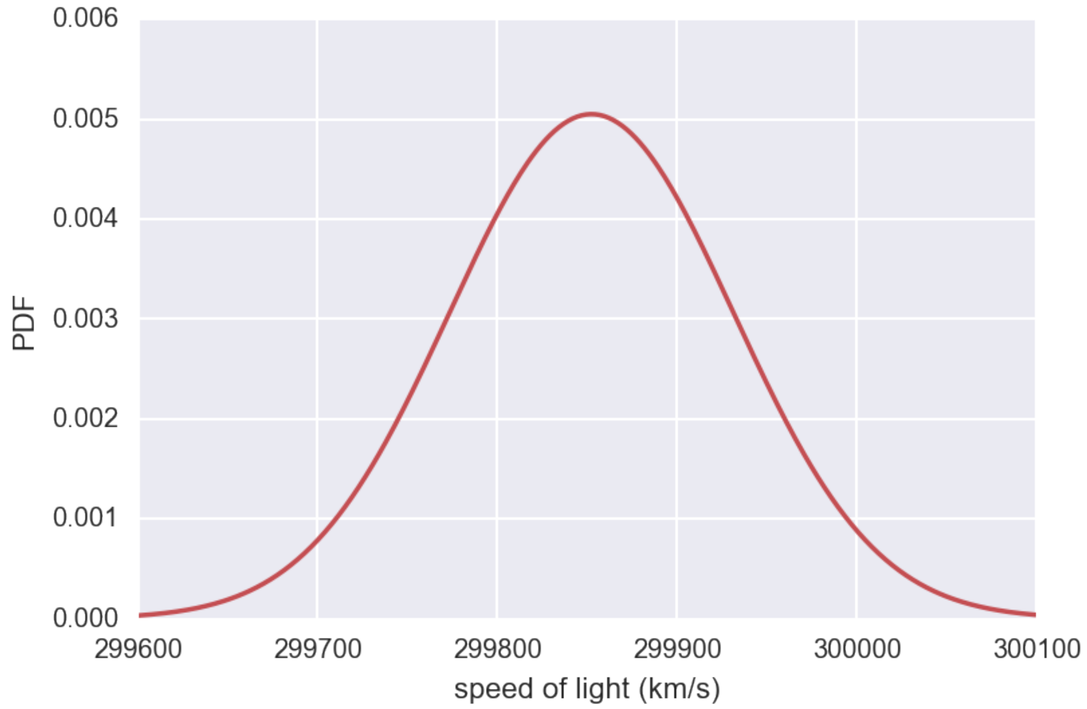
STATISTICAL THINKING IN PYTHON I

Introduction to the Normal distribution

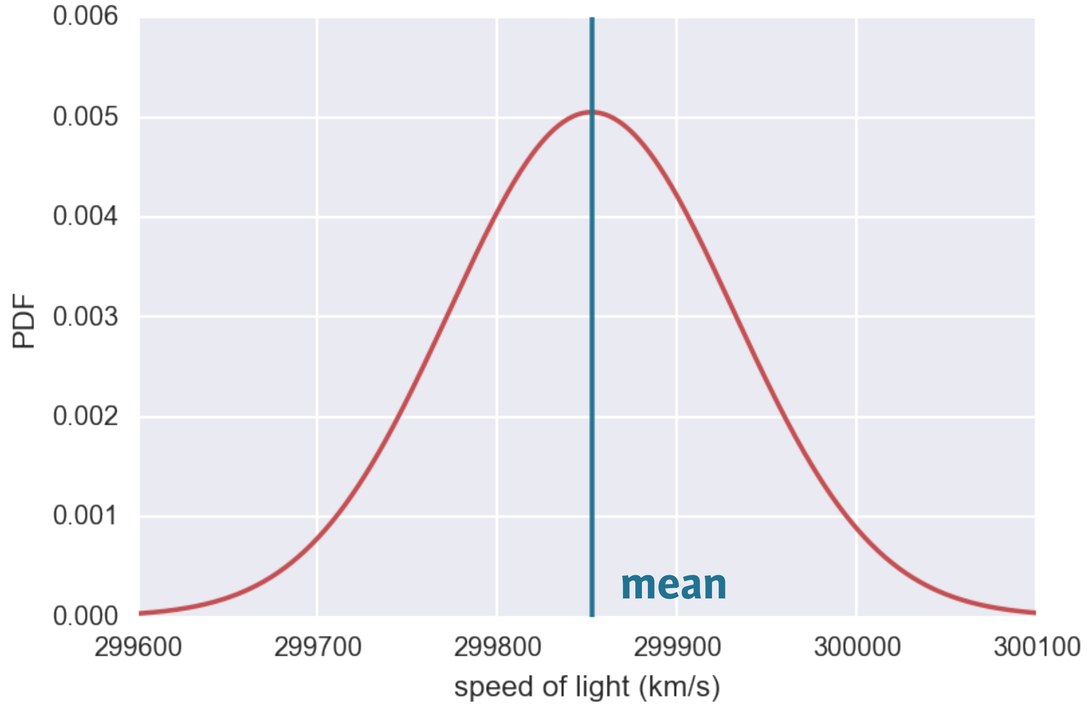
Normal distribution

- Describes a continuous variable whose PDF has a single symmetric peak.

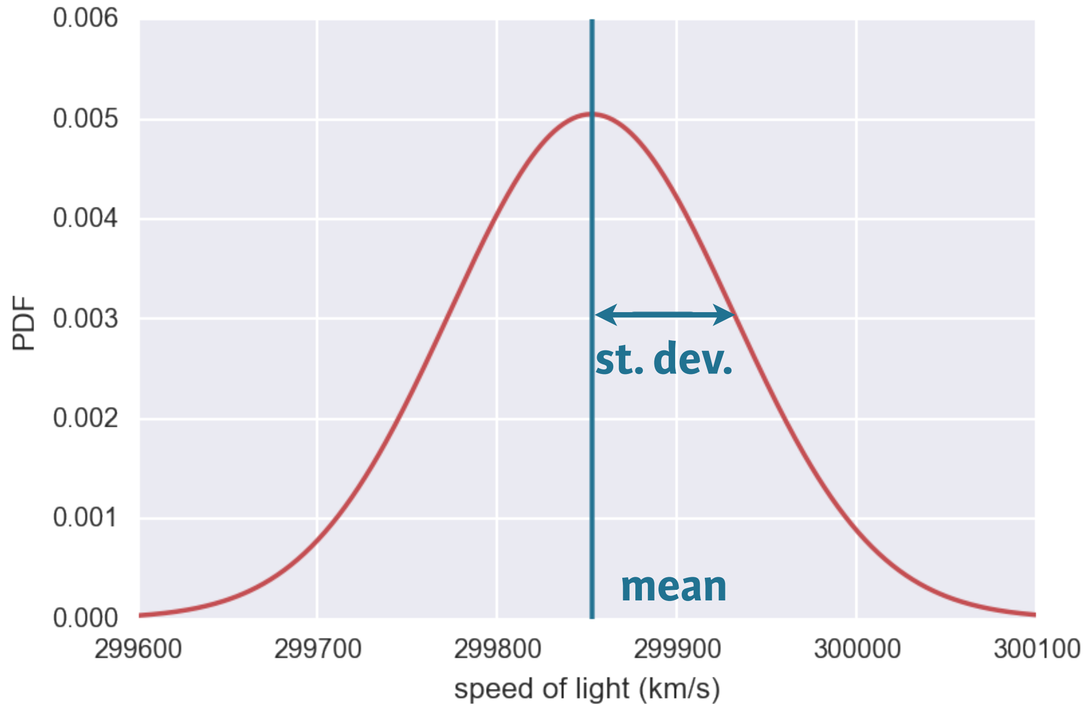
Normal distribution



Normal distribution

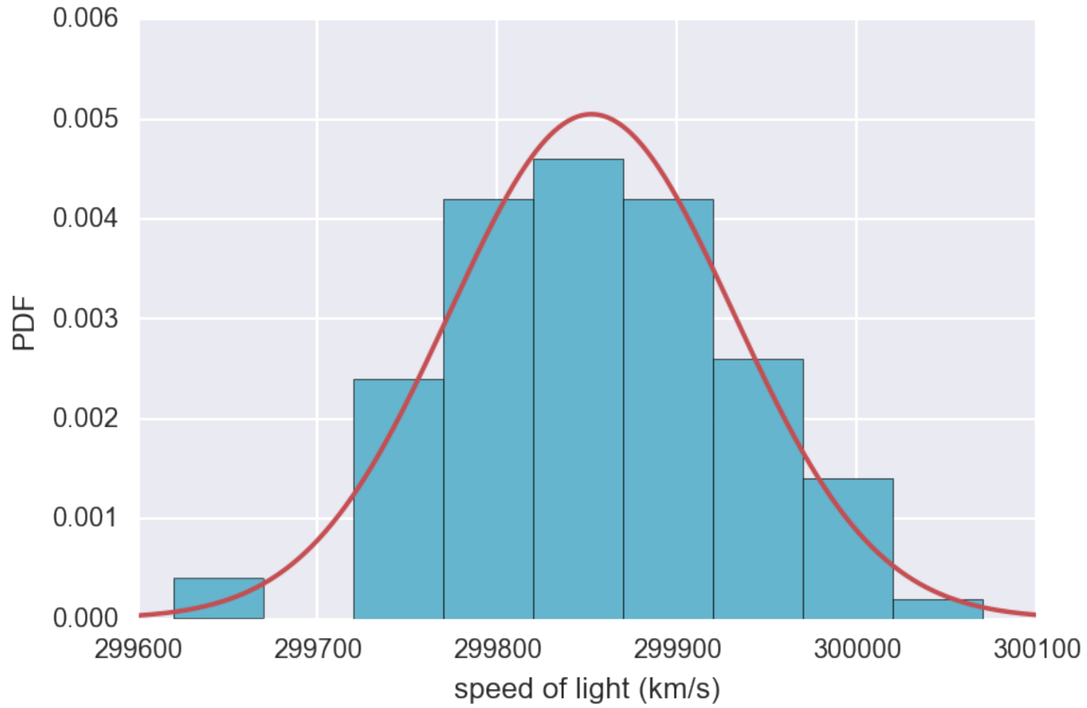


Normal distribution



Parameter		Calculated from data
mean of a Normal distribution	≠	mean computed from data
st. dev. of a Normal distribution	≠	standard deviation computed from data

Comparing data to a Normal PDF



Data: Michelson, 1880

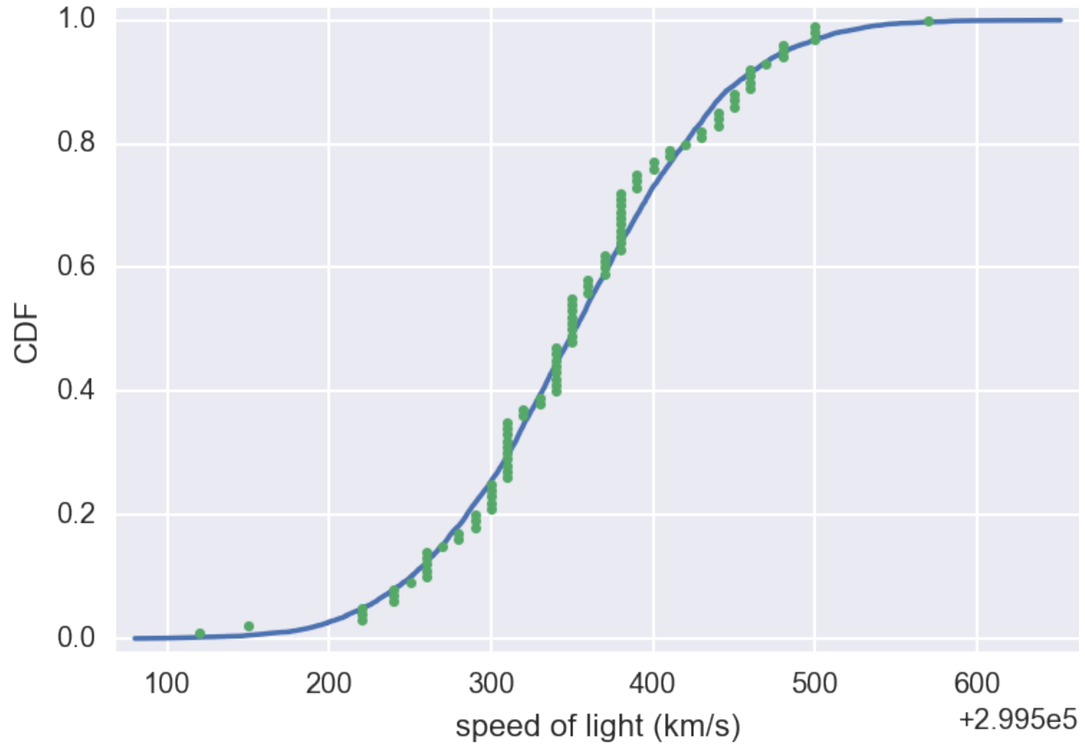
Checking Normality of Michelson data

```
In [1]: import numpy as np

In [2]: mean = np.mean(michelson_speed_of_light)

In [3]: std = np.std(michelson_speed_of_light)

In [4]: samples = np.random.normal(mean, std, size=10000)

In [5]: x, y = ecdf(michelson_speed_of_light)

In [6]: x_theor, y_theor = ecdf(samples)
```

Checking Normality of Michelson data

```
In [1]: import matplotlib.pyplot as plt

In [2]: import seaborn as sns

In [3]: sns.set()

In [4]: _ = plt.plot(x_theor, y_theor)

In [5]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [6]: _ = plt.xlabel('speed of light (km/s)')

In [7]: _ = plt.ylabel('CDF')

In [8]: plt.show()
```

Checking Normality of Michelson data



Data: Michelson, 1880

STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

The Normal distribution: Properties and warnings

Image: Deutsche Bundesbank

The Gaussian distribution

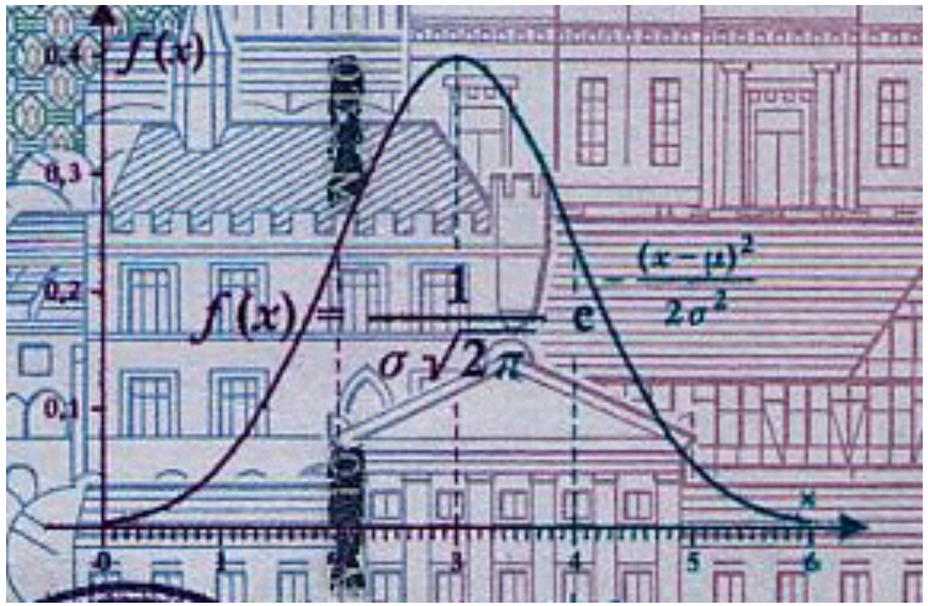
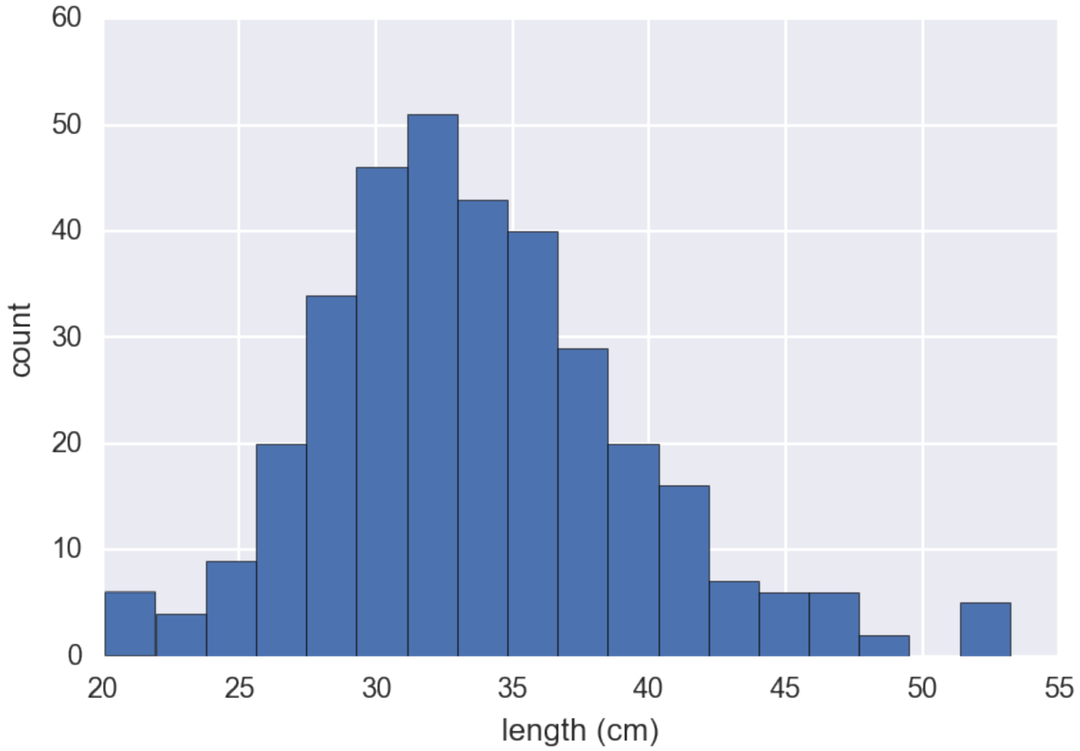


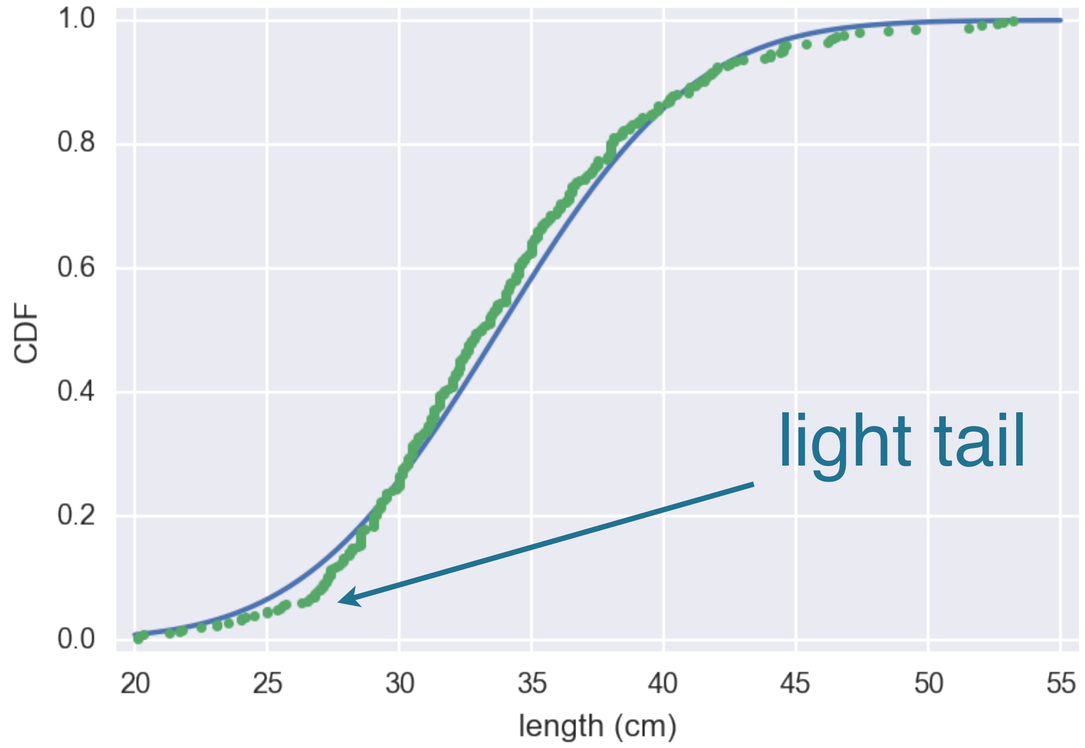
Image: Deutsche Bundesbank

Length of MA large mouth bass



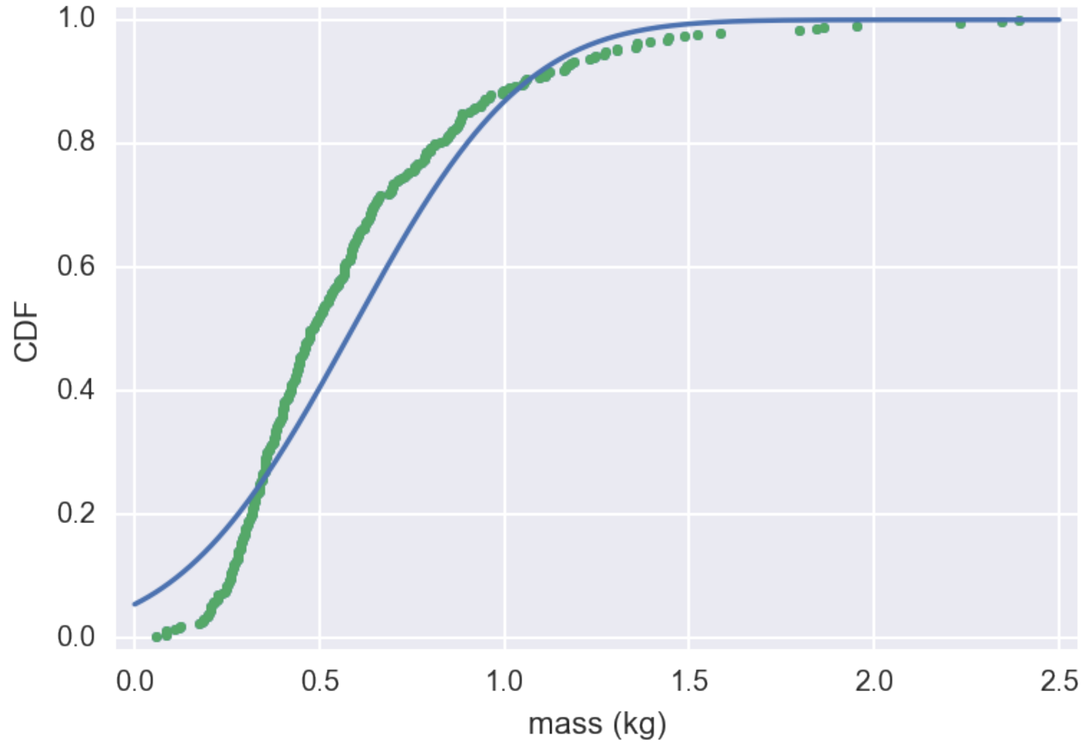
Source: Mass. Dept. of Environmental Protection

Length of MA large mouth bass



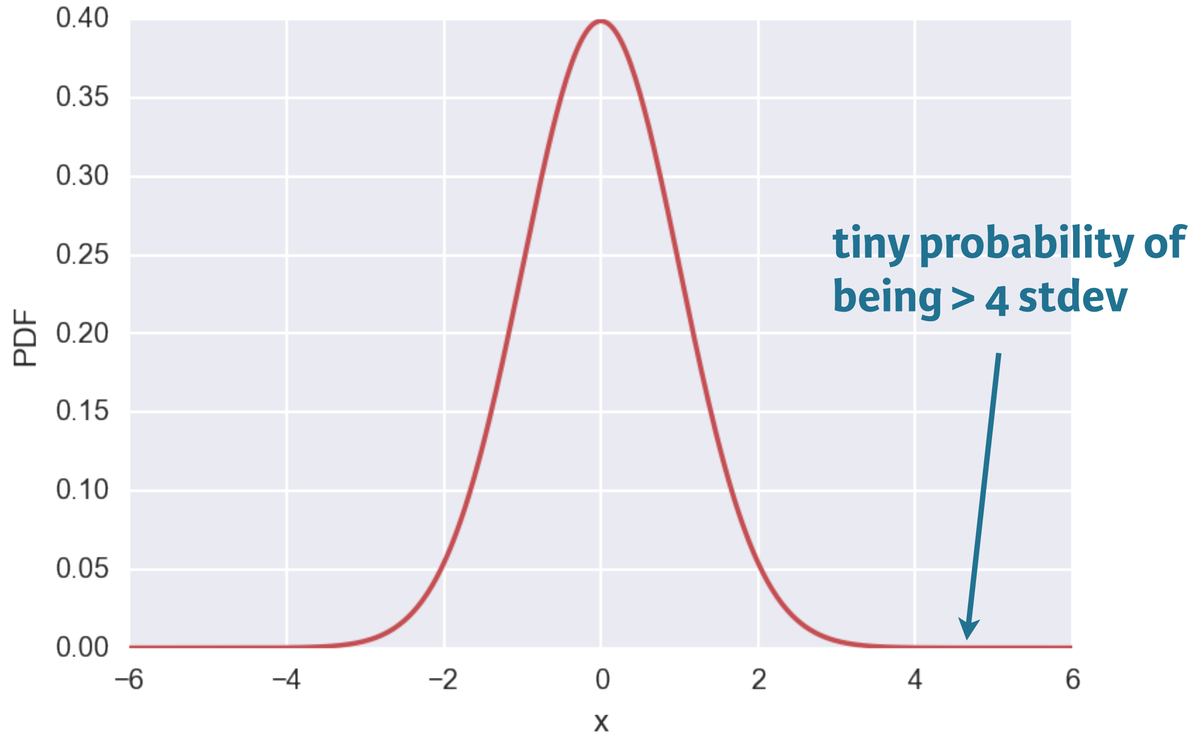
Source: Mass. Dept. of Environmental Protection

Mass of MA large mouth bass



Source: Mass. Dept. of Environmental Protection

Light tails of the Normal distribution



STATISTICAL THINKING IN PYTHON I

Let’s practice!

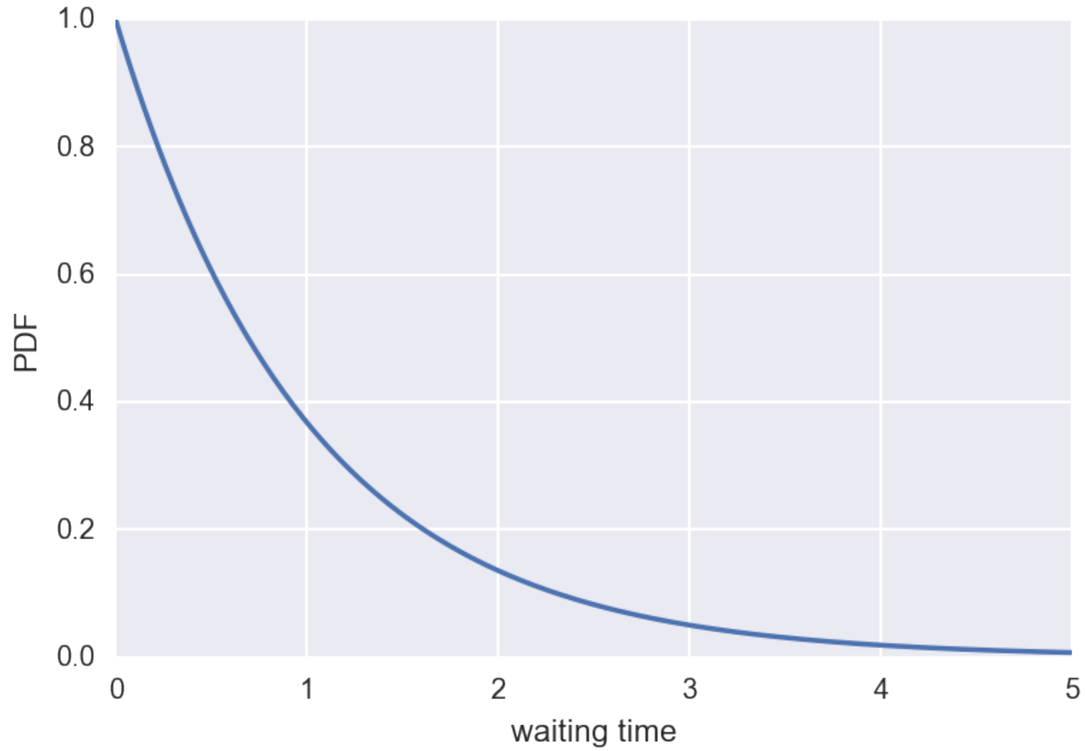
STATISTICAL THINKING IN PYTHON I

The Exponential distribution

The Exponential distribution

- The waiting time between arrivals of a Poisson process is Exponentially distributed

The Exponential PDF



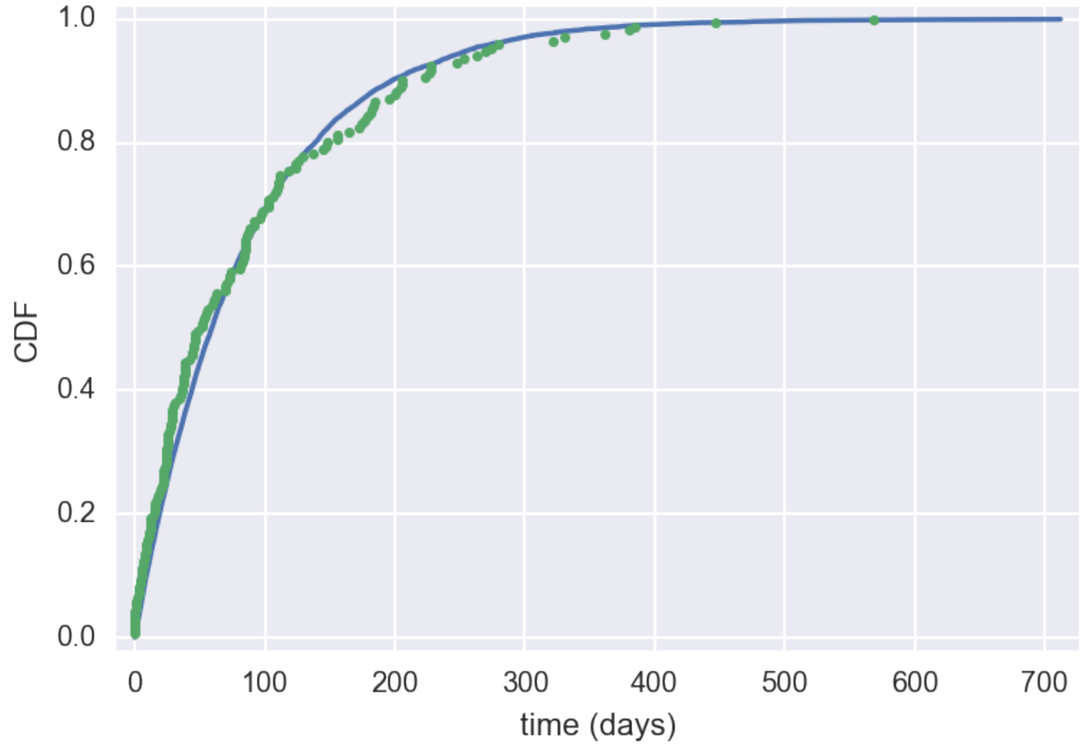
Possible Poisson process

- Nuclear incidents:
  - Timing of one is independent of all others

Exponential inter-incident times

```
In [1]: mean = np.mean(inter_times)

In [2]: samples = np.random.exponential(mean, size=10000)

In [3]: x, y = ecdf(inter_times)

In [4]: x_theor, y_theor = ecdf(samples)

In [5]: _ = plt.plot(x_theor, y_theor)

In [6]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [7]: _ = plt.xlabel('time (days)')

In [8]: _ = plt.ylabel('CDF')

In [9]: plt.show()
```

Exponential inter-incident times



Data Source: Wheatley, Sovacool, Sornette, Nuclear Events Database

STATISTICAL THINKING IN PYTHON I

Let’s practice!

STATISTICAL THINKING IN PYTHON I

Final thoughts

You now can...

- Construct (beautiful) instructive plots
- Compute informative summary statistics
- Use hacker statistics
- Think probabilistically

In the sequel, you will...

- Estimate parameter values
- Perform linear regressions
- Compute confidence intervals
- Perform hypothesis tests

STATISTICAL THINKING IN PYTHON I

See you in the sequel!