Leaving Spatial-Temporal Surrounding-Aware Correlation Filter with Mutation Sensitive Regularization via Adaptive Hybrid Label.

① Baseline Tracker (BACF):-

$$E(h,r) = \frac{1}{2}\sum_{k=1}^{c}\left\| r^k - \sum_{k=1}^{c}(Ph_t^k)\odot x_t^k \right\|_{2,1} + \lambda_1/2 \sum_{k=1}^{c}\left\| h_t^k \right\|_F^2 .$$

② Enhanced Spatial Feature Detection:-

$$E(h,r) = \frac{1}{2}\sum_{k=1}^{c}\left\| r^k - \sum_{k=1}^{c}(Ph_t^k)\odot x_t^k \right\|_{2,1} + \lambda_1/2 \sum_{i=1}^{M}\left\| h_{i:} \right\|_F + \lambda_2/2 \sum_{j=1}^{N}\left\| h_{:j} \right\|_F .$$

③ Surrounding-Aware:-

$$E(h,r) = \frac{1}{2}\sum_{k=1}^{c}\left\| r^k - \sum_{k=1}^{c}(Ph_t^k)\odot x_t^k \right\|_{2,1} + \lambda_1/2 \sum_{i=1}^{M}\left\| h_{i:} \right\|_F + \lambda_2/2 \sum_{j=1}^{N}\left\| h_{:j} \right\|_F + \frac{1}{2}\sum_{k=1}^{K}\left\| \sum_{d=1}^{D}\alpha^k x_{dt}^k \odot h_t^k \right\|_F^2$$

④ Temporal Regularization:-

$$E(h,r) = \frac{1}{2}\sum_{k=1}^{c}\left\| r^k - \sum_{k=1}^{c}(Ph_t^k)\odot x_t^k \right\|_{2,1} + \lambda_1/2 \sum_{i=1}^{M}\left\| h_{i:} \right\|_F + \lambda_2/2 \sum_{j=1}^{N}\left\| h_{:j} \right\|_F + \frac{1}{2}\sum_{k=1}^{K}\left\| \sum_{d=1}^{D}\alpha^k x_{dt}^k \odot h_t^k \right\|_F^2 + \lambda_3/2 \sum_{k=1}^{c}\left\| h_t^k - h_{t-1}^k \right\|_F^2$$

⑤ Optimal Adaptive Hybrid Label:-

$$E(h,r) = \frac{1}{2}\sum_{k=1}^{c}\left\| r^k - \sum_{k=1}^{c}(Ph_t^k)\odot x_t^k \right\|_{2,1} + \lambda_1/2 \sum_{i=1}^{M}\left\| h_{i:} \right\|_F + \lambda_2/2 \sum_{j=1}^{N}\left\| h_{:j} \right\|_F + \frac{1}{2}\sum_{k=1}^{K}\left\| \sum_{d=1}^{D}\alpha^k x_{dt}^k \odot h_t^k \right\|_F^2 + \lambda_3/2 \sum_{k=1}^{c}\left\| h_t^k - h_{t-1}^k \right\|_F^2$$
$$+ \frac{\lambda_4}{2}\sum_{k=1}^{c}\left\| \Omega_t^k - r_t^k \right\|_F^2 .$$

⑥ Adaptive Hybrid Label with Temporal Regularization:-

$$E(h,r) = \frac{1}{2}\sum_{k=1}^{c}\left\| r^k - \sum_{k=1}^{c}(Ph_t^k)\odot x_t^k \right\|_{2,1} + \lambda_1/2 \sum_{i=1}^{M}\left\| h_{i:} \right\|_F + \lambda_2/2 \sum_{j=1}^{N}\left\| h_{:j} \right\|_F + \frac{1}{2}\sum_{k=1}^{K}\left\| \sum_{d=1}^{D}\alpha^k x_{dt}^k \odot h_t^k \right\|_F^2 + \lambda_3/2 \sum_{k=1}^{c}\left\| h_t^k - h_{t-1}^k \right\|_F^2$$
$$+ \lambda_4/2 \sum_{k=1}^{c}\left\| \Omega_t^k - r_t^k \right\|_F^2 + \phi/2 \sum_{k=1}^{c}\left\| r_t^k - r_{t-1}^k \right\|_F^2 .$$

Objective Function :-

$$E(h,r) = \frac{1}{2}\sum_{k=1}^{C}\left\| r^k - \sum_{k=1}^{C}(Ph_t^k)\odot x_t^k \right\|_{2,1} + \frac{\lambda_1}{2}\sum_{i=1}^{M}\|h_{i:}\|_F + \frac{\lambda_2}{2}\sum_{j=1}^{C}\|h_{:j}\|_F + \frac{1}{2}\sum_{k=1}^{C}\left\|\sum_{d=1}^{D}\alpha^k x_{dt}^k \odot h_t^k\right\|_F^2 + \frac{\lambda_3}{2}\sum_{k=1}^{C}\|h_t^k - h_{t-1}^k\|_F^2$$

$$+ \frac{\lambda_4}{2}\sum_{k=1}^{C}\|\Omega_t^k - r_t^k\|_F^2 + \frac{\phi}{2}\sum_{k=1}^{C}\|r_t^k - r_{t-1}^k\|_F^2.$$

We Introduced the auxiliary Variable $\hat{g}^k = \sqrt{T}\,(I_D \otimes FP^T)\,h^k$.

① Subproblem - g :-

$$\hat{g} = \frac{1}{2}\sum_{k=1}^{C}\left\|\hat{r}^k - \sum_{k=1}^{C}\hat{g}_t^k \odot \hat{x}_t^k\right\|_{2,1} + \frac{1}{2}\sum_{k=1}^{C}\left\|\sum_{d=1}^{D}\alpha^k \hat{x}_{dt}^k \odot \hat{g}_t^k\right\|_F^2 + \frac{\lambda_3}{2}\sum_{k=1}^{C}\|\hat{g}_t^k - \hat{g}_{t-1}^k\|_F^2 + \frac{\mu}{2}\left\|\hat{g}_t^k - \hat{h}_t^k + \frac{\hat{\Gamma}^k}{\mu}\right\|_F^2.$$

② Sub Problem - h :-

$$\hat{h}_i = \frac{\lambda_1}{2}\sum_{i=1}^{M}\|h_{i:}\|_F + \frac{\mu}{2}\sum_{k=1}^{M}\left\|g_{i:}^k - h_{i:}^k + \frac{\Gamma_{i:}^k}{\mu}\right\|_F.$$

$$h_j = \frac{\lambda_2}{2}\sum_{j=1}^{N}\|h_{:j}\|_F + \frac{\mu}{2}\sum_{j=1}^{N}\left\|g_{:j}^k - h_{:j}^k + \frac{\Gamma_{:j}^k}{\mu}\right\|_F.$$

③ Subproblem - r :-

$$r = \frac{1}{2}\sum_{k=1}^{C}\left\|\hat{r}^k - \sum_{k=1}^{C}\hat{g}_t^k \odot \hat{x}_t^k\right\|_{2,1} + \frac{(1+\varphi^2)\lambda_4}{2}\|\hat{\Omega}_t^k - \hat{r}_t^k\|_2^2 + \frac{(1-\varphi^2)\phi}{2}\|r_t^k - r_{t-1}^k\|_2^2$$

$\hat{\Omega}^k \rightarrow$ Optimal Adaptive Hybrid Label

$\hat{r}_t^k \rightarrow$ Adaptive Hybrid Label.

① Subproblem - g :

$$g = \frac{1}{2}\sum_{k=1}^{c} \| \hat{r}^k - \sum_{k=1}^{c} \hat{x}_t^k \circ \hat{g}_t^k \|_2^2 + \frac{1}{2}\sum_{k=1}^{c} \| \sum_{d=1}^{D} \alpha^k \hat{x}_{dt}^k \circ \hat{g}_t^k \|_2^2 + \frac{\lambda_3}{2}\sum_{k=1}^{c} \| \hat{g}_t^k - \hat{g}_{t-1}^k \|_2^2 + \frac{\mu}{2}\sum_{k=1}^{c} \| \hat{g}_t^k - \hat{h}_t^k + \frac{\hat{\Gamma}^k}{\mu} \|_2^2 .$$

Solution :

$$\Rightarrow \frac{1}{2} \| \hat{r}^k - \hat{x}_t^k \hat{g}_t^k \|_2^2 + \frac{1}{2} \| \alpha^k \hat{x}_{dt}^k \circ \hat{g}_t^k \|_2^2 + \frac{\lambda_3}{2} \| \hat{g}_t^k - \hat{g}_{t-1}^k \|_2^2 + \frac{\mu}{2} \| \hat{g}_t^k - \hat{h}_t^k + \hat{\Gamma}^k / \mu \|_2^2 .$$

Taking derivative w.r.t $\hat{g}_t^k$ & equal to zero.

$$\Rightarrow \frac{1}{\cancel{2}} \hat{x}_t^T \cancel{2} (\hat{x}_t^k \hat{g}_t^k - \hat{r}_t^k) + \frac{1}{\cancel{2}} \cancel{2} (\alpha^k \hat{x}_{dt}^k \hat{g}_t^k) + \frac{\lambda_3}{\cancel{2}} \cancel{2} (\hat{g}_t^k - \hat{g}_{t-1}^k) + \frac{\mu}{\cancel{2}} \cancel{2} (\hat{g}_t^k - \hat{h}_t^k + \hat{\Gamma}^k / \mu) = 0$$

$$\Rightarrow \hat{x}_t^T (\hat{x}_t^k \hat{g}_t^k - \hat{r}_t^k) + (\alpha^k \hat{x}_{dt}^k \hat{g}_t^k) + \lambda_3 (\hat{g}_t^k - \hat{g}_{t-1}^k) + \mu (\hat{g}_t^k - \hat{h}_t^k + \hat{\Gamma}^k / \mu) = 0$$

$$\Rightarrow \hat{x}_t^{kT} \hat{x}_t^k \hat{g}_t^k - \hat{x}_t^{kT} \hat{r}_t^k + \alpha^k \hat{x}_{dt}^k \hat{g}_t^k + \lambda_3 \hat{g}_t^k - \lambda_3 \hat{g}_{t-1}^k + \mu \hat{g}_t^k - \mu \hat{h}_t^k + \hat{\Gamma}^k = 0$$

Take the $\hat{g}_t^k$ term as common.

$$\Rightarrow \hat{g}_t^k (\hat{x}_t^{kT} \hat{x}_t^k + \alpha^k \hat{x}_{dt}^k + \lambda_3 + \mu \cancel{\hat{g}_t^k}) = \hat{x}_t^{kT} \hat{r}_t^k + \lambda_3 \hat{g}_{t-1}^k + \mu \hat{h}_t^k - \hat{\Gamma}^k$$

$$\boxed{\hat{g}_t^k = \frac{\hat{x}_t^{kT} \hat{r}_t^k + \lambda_3 \hat{g}_{t-1}^k + \mu \hat{h}_t^k - \hat{\Gamma}^k}{\hat{x}_t^{kT} \hat{x}_t^k + \alpha^k \hat{x}_{dt}^k + \lambda_3 + \mu}}$$

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② Subproblem – h_i:

(i)

$$h_i = \lambda_1/2 \sum_{i=1}^{M} \|h_{i:}\|_2 + \mu/2 \sum_{i=1}^{M} \| g^k_{i:} - h^k_{i:} + \frac{\Gamma^k_{i:}}{\mu} \|_2^2.$$

Solution:

$$\Rightarrow \quad h_i = \frac{\lambda_1}{2} \|h_{i:}\|_2 + \mu/2 \| g^k_{i:} - h^k_{i:} + \frac{\hat{\Gamma}^k_{i:}}{\mu} \|_2^2.$$

Now taking derivative w.r.t h_i & equal to zero.

$$\Rightarrow \lambda_1/\cancel{2}\, \cancel{2}(h_{i:}) + \mu/\cancel{2}\, \cancel{2}(g^k_{i:} - h^k_{i:} + \Gamma^k_{i:}/\mu)(-1) = 0$$

$$\Rightarrow \lambda_1 h^k_{i:} - \mu g^k_{i:} + \mu h^k_{i:} - \Gamma^k_{i:} = 0$$

Now take the $h^k_{i:}$ term as common.

$$\Rightarrow \mu h^k_{i:} = \mu g^k_{i:} + \Gamma^k_{i:} - \lambda_1$$

$$\Rightarrow h_{i:} = \frac{\cancel{\mu} g^k_{i:}}{\cancel{\mu}} + \frac{\Gamma^k_{i:}}{\mu} - \frac{\lambda_1}{\mu}$$

$$\Rightarrow h_{i:} = \left(g^k_{i:} + \frac{\Gamma^k_{i:}}{\mu} \right) \left(1 - \frac{\lambda_1}{\mu (g^k_{i:} + \Gamma^k_{i:}/\mu)} \right).$$

$$\Rightarrow \boxed{h_i = \max \left(0, 1 - \frac{\lambda_1}{\mu (g^k_{i:} + \Gamma^k_{i:}/\mu)} \right) \left(g^k_{i:} + \frac{\Gamma^k_{i:}}{\mu} \right)}$$

② Subproblem- h_j :

(ii)

$$h_j = \lambda_2/2 \sum_{j=1}^{N} \| h_{:j} \|_2 + \mu/2 \sum_{j=1}^{N} \| g^k_{:j} - h^k_{:j} + \frac{\Gamma^k_{:j}}{\mu} \|_2^2 .$$

Solution:

$$\Rightarrow \quad h_j = \lambda_2/2 \| h_{:j} \|_2 + \mu/2 \| g^k_{:j} - h^k_{:j} + \frac{\Gamma^k_{:j}}{\mu} \|_2^2 .$$

Now taking derivative with.r.t h_j equal to zero.

$$\Rightarrow \lambda_2/\cancel{2}\,\cancel{2}(h_{:j}) + \mu/\cancel{2}\,\cancel{2}\left(g^k_{:j} - h^k_{:j} + \Gamma^k_{:j}/\mu\right)(-1) = 0$$

$$\Rightarrow \lambda_2 (h_{:j}) + \mu \left(g^k_{:j} - h^k_{:j} + \Gamma^k_{:j}/\mu\right)(-1) = 0$$

$$\Rightarrow \lambda_2 h_{:j} \overline{+} \mu g^k_{:j} + \mu h^k_{:j} - \Gamma^k_{:j} = 0$$

Now take the $h^k_{:j}$ term as Common.

$$\Rightarrow \mu h^k_{:j} = \mu g^k_{:j} + \Gamma^k_{:j} - \lambda_2$$

$$\Rightarrow h_{:j} = \frac{\cancel{\mu} g^k_{:j}}{\cancel{\mu}} + \frac{\Gamma^k_{:j}}{\mu} - \frac{\lambda_2}{\mu}$$

$$\Rightarrow h_{:j} = \left(g^k_{:j} + \frac{\Gamma^k_{:j}}{\mu}\right)\left(1 - \frac{\lambda_2}{\mu\left(g^k_{:j} + \frac{\Gamma^k_{:j}}{\mu}\right)}\right)$$

$$\Rightarrow \boxed{h_j = \max\left(0, 1 - \frac{\lambda_2}{\left(g^k_{:j} + \Gamma^k_{:j}/\mu\right)}\right)\left(g^k_{:j} + \frac{\Gamma^k_{:j}}{\mu}\right)}$$

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(3) Sub problem - $\hat{r}$!

$$\frac{1}{2}\|\hat{r}_t-\hat{X}_t^k\hat{g}_t^k\|_2^2+\frac{(1+\psi^2)\lambda_4}{2}\|\hat{\Omega}-\hat{r}\|_2^2+\frac{(1-\psi^2)\phi}{2}\|\hat{r}_t^k-\hat{r}_{t-1}^k\|_2^2.$$

Solution :

$$\Rightarrow \frac{1}{2}\|\hat{r}_t^k-\hat{x}_t^k\hat{g}_t^k\|_2^2+\frac{(1+\psi^2)\lambda_4}{2}\|\hat{\Omega}-\hat{r}\|_2^2+\frac{(1-\psi^2)\phi}{2}\|\hat{r}_t^k-\hat{r}_{t-1}^k\|_2^2.$$

Now we taking derivative w.r.t $\hat{r}_t^k$ & equal to Zero.

$$\Rightarrow \frac{1}{2}(\hat{r}-\hat{X}\hat{g})^T(\hat{r}-\hat{X}\hat{g})+\frac{(1+\psi^2)\lambda_4}{2}(\hat{\Omega}-\hat{r})^T(\hat{\Omega}-\hat{r})+\frac{(1-\psi^2)\phi}{2}(\hat{r}_t-\hat{r}_{t-1})^T(\hat{r}_t-\hat{r}_{t-1})=0$$

$$\Rightarrow \frac{1}{2}(\hat{r}^T-\hat{x}^T\hat{g}^T)(\hat{r}-\hat{x}\hat{g})+\frac{(1+\psi^2)\lambda_4}{2}(\hat{\Omega}^T-\hat{r}^T)(\hat{\Omega}-\hat{r})+\frac{(1-\psi^2)\phi}{2}(\hat{r}_t^T-\hat{r}_{t-1}^T)(\hat{r}_t-\hat{r}_{t-1})=0$$

$$\Rightarrow \frac{1}{2}(\hat{r}^T\hat{r}-\hat{r}^T\hat{x}\hat{g}-\hat{x}^T\hat{g}^T\hat{r}+\underbrace{\cancel{\hat{x}^T\hat{g}^T\hat{x}\hat{g}}}_{0})+\frac{(1+\psi^2)\lambda_4}{2}(\underbrace{\cancel{\hat{\Omega}^T\hat{\Omega}}}_{0}-\hat{\Omega}^T\hat{r}-\hat{r}^T\hat{\Omega}+\hat{r}^T\hat{r})+\frac{(1-\psi^2)\phi}{2}(\hat{r}_t^T\hat{r}_t-\cancel{2}\hat{r}_t^T\hat{r}_{t-1}-\hat{r}_{t-1}^T\hat{r}_t+\underbrace{\cancel{\hat{r}_{t-1}^T\hat{r}_{t-1}}}_{0}).$$

$$\Rightarrow \frac{1}{\cancel{2}}\left[\hat{r}^2-\cancel{2}\hat{x}^T\hat{g}^T\cancel{\hat{r}}\right]+\frac{(1+\psi^2)\lambda_2}{2}\left[r^2-2\Omega^T\cancel{r}\right]+\frac{(1-\psi^2)\phi}{2}\left[r^2-2\hat{r}_{t-1}^T\cancel{r}_t\right]=0 \quad \left\{\begin{array}{l}\text{Take derivative}\\ \text{w.r.t } \hat{r}_t\end{array}\right\}$$

$$\Rightarrow \hat{r}-\hat{x}\hat{g}+(1+\psi^2)\lambda_2(r^2-\hat{\Omega}^T\cancel{r})+(1-\psi^2)\phi(\cancel{r}-\hat{r}_{t-1}^T)=0$$

$$\Rightarrow \hat{r}(1+(1+\psi^2)\lambda_2+(1-\psi^2)\phi)=\hat{x}\hat{g}+(1+\psi^2)\lambda_2\hat{\Omega}+(1-\psi^2)\phi\,\hat{r}_{t-1}$$

$$\Rightarrow \boxed{\hat{r}=\frac{\hat{x}\hat{g}+(1+\psi^2)\lambda_2\hat{\Omega}+(1-\psi^2)\phi\,\hat{r}_{t-1}}{1+(1+\psi^2)\lambda_2+(1-\psi^2)\phi}}$$

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