Memory Sampled-Data Control for T–S Fuzzy-Based Permanent Magnet Synchronous Generator via an Improved Looped Functional

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***Abstract*—This study presents a memory sampled-data control design for the Takagi–Sugeno (T–S) fuzzy-based permanent magnet synchronous generator (PMSG) via an improved looped functional (ILF). To do this, first, the nonlinear PMSG is modeled as a T–S fuzzy system. Then, to derive the sufficient criteria, a novel ILF is proposed, which includes the available information about the sampling pattern characteristics from $x(t_k)$ to $x(t)$ and $x(t)$ to $x(t_{k+1})$ together with the signal transmission delay. In addition, the ILF introduces a fractional parameter $0 < \hat{\beta} < 1$ that gives more information of splited sampling intervals. Also, the external disturbances of the proposed system are attenuated by using the H_∞ performance. Furthermore, the stabilization conditions expressed as linear matrix inequalities (LMIs), the proposed closed-loop system's asymptotic stability is ensured under the designed controller. Finally, comparison and simulation results demonstrate the effectiveness and feasibility of the proposed method and the designed control scheme.**

***Index Terms*—H_∞, linear matrix inequalities (LMIs), memory sampled-data control (SDC), permanent magnet synchronous generator (PMSG), Takagi–Sugeno (T–S) fuzzy system.**

I. Introduction

THE development and utilization of wind energy systems to satisfy the electrical demand has been received much attention in recent years [1], [2], [3], [4], [5], [6], [7]. This wind energy balances the overall production and consumption without impacting the environment. A wind turbine system (WTS) converts the natural energy available at the system location into electrical energy. Therefore, the WTS has been received more focus among researchers to improve the maximum power from the environment [2], [3]. Generally, WTS is divided into two categories: 1) variable-speed WTS (VSWTS) and 2) fixed-speed WTS [1]. The VSWTS is far better than the fixed-speed WTS due to its higher reliability, optimal power production, lower mechanical stress, and stable performance under grid disturbance. In this regard, VSWTS becomes, the most popular form of modern WTS. Moreover, the VSWTS generator is essential in converting mechanical energy into electrical energy.

Nowadays, there are two types of generators primarily used in the WTS, namely, doubly-fed induction generator (DFIG) [4] and permanent magnet synchronous generator (PMSG) [5]. Compared to DFIG, PMSG allows the generator to run at low speeds without using a gearbox. In addition, it minimizes the weight and size of nacelle equipment, as well as mechanical losses and maintenance requirements. Therefore, many researchers have investigated the dynamical analysis for PMSG-based WTS via the state-space model in [5], [6], and [7]. In particular, the wind energy generation system is a nonlinear time-varying system that suffers from various wind loads, vibration loads, and all kinds of variable loads. Further, due to their nonlinearity presence in the PMSG-based WTS, its security and stability control research has become more popular among the research community. In this regard, the stability of a nonlinear PMSG model has been established in [6] under impulsive observer-based output control by using the Lyapunov stability. Also, the modeling and simulation of multiscale transients for PMSG-based WTS has been examined in [7]. Moreover, Deng et al. [8] have analyzed the stabilization problem of PMSG-based WTS.

Furthermore, due to the nonlinear properties of PMSG-based WTS, it becomes a huge challenge to examine the model's dynamical behavior. In this context, the Takagi–Sugeno (T–S) fuzzy technique is regarded as the most helpful tool for expressing the model nonlinearities in the linear form together with the membership functions [9], [10], [11], [12], [14], [15], [16], [17], [18], [19]. Moreover, Bououden et al. [11] have derived linear matrix inequality (LMI)-based stabilization conditions for the VSWTS under the predictive control scheme based on the T–S fuzzy technique. Meanwhile, the stabilization of the T–S fuzzy system with time-delay has been analyzed in [12]. On the other hand, the sampled-data control (SDC) scheme has been received much attention among the control community due to its advantages, such as maximum performance, reliability, lower cost of SDC equipment, and easy implementation [20], [21], [22], [23], [24]. Besides that, the SDC system has both discrete-time and continuous-time signals, with sampling information employed in control at sampling instants during the sampling intervals. In this aspect, the SDC has been examined

Manuscript received 18 April 2022; revised 14 October 2022; accepted 16 February 2023. Date of publication 14 March 2023; date of current version 16 June 2023. This work was supported in part by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education under Grant NRF2016R1A6A1A03013567, Grant NRF2021R1A2B5B01001484, and Grant NRF-2022K2A9A2A06045121. This article was recommended by Associate Editor D. Yue. *(Corresponding author: Young Hoon Joo.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TSMC.2023.3250653.

Digital Object Identifier 10.1109/TSMC.2023.3250653

for the nonlinear dynamical model of wind turbine generators (see [23], [24]). In addition, the memory-based SDC (MBSDC) is more successful than the traditional SDC because it integrates the unavoidable fixed transmission delay in the transfer of signal from sampler to the controller as well as zero-order hold (ZOH) at sampling instant t_k [25], [26], [27]. Therefore, in recent years, an MBSDC for T–S fuzzy systems has been widely designed. For instance, Ge et al. [25] and Liu et al. [26] designed an MBSDC for the nonlinear model and then presented their stabilization requirements based on the Lyapunov stability and LMI technique. Moreover, they examined the stabilization of a WTS based on PMSG by using a fuzzy-based memory SDC (FBMSDC) technique [27].

The larger sampling interval will reduce the number of samples in this SDC system. Hence, the sampling interval is significant with lower and upper bounds. In addition, the larger upper bound makes the stability region more efficient and conservative [24]. For this purpose, the stability of the SDC system has been studied using a time-dependent Lyapunov function (LF)-based approach [28]. In addition, Saberi and Zamani [31], Liu and Fridman [30], and Lee and Park [29] have analyzed the stability results through a discontinuous LF approach. Especially, the stability analysis for the SDC system has been examined via the free-matrix-based time-dependent discontinuous LF approach in [29]. The stabilization of switched time-varying delay systems has been examined in [31] based on the improved discontinuous LF approach. Nowadays, the looped functional has been gained more focus in the SDC design of fuzzy systems because it does not need to be a positive definite between sampling times, and also it uses all information from $x(t_k)$ to $x(t)$ and $x(t)$ to $x(t_{k+1})$. With this approach, Zeng et al. [32], Shanmugam and Joo [33], [34], and Hua et al. [35] have provided efficient results for obtaining the largest sampling interval through the looped functional.

Motivated by the aforementioned analysis, the T–S FBMSDC is formulated for a nonlinear model of the WTS based on PMSG via an improved looped functional (ILF) approach. This study's main contribution is summarized as follows.

1) The T–S fuzzy system represents the nonlinear model of PMSG-based WTS into the linear submodels. Then, to investigate the stabilization problem of the WTS based on PMSG, the FBMSDC is proposed, which contains the constant signal transmission delay.
2) A fractional parameter $0 < \hat{\beta} < 1$ is introduced in the ILF to produce the information of the sample intervals $[t_k, t_k + \hat{\beta}\eta_1(t)]$, $[t_k + \hat{\beta}\eta_1(t), t]$, $[t, t + \hat{\beta}\eta_2(t)]$ and $[t + \hat{\beta}\eta_2(t), t_{k+1}]$. Further, the information regarding the states $x(t), x(t_k), x(t_{k+1}), x(t-\tau), x(t_k-\tau)$ and the internal relationship between them are taken into account in an augmented type LKF in $V(t)$.
3) The adequate derived conditions are expressed in the form of LMIs, and the control gain matrices (CGMs) are calculated by solving the set of LMIs, which ensure the steady-state stability performance with external disturbances.
4) Finally, the proposed control design approach for PMSG-based WTS is validated through the numerical simulation, and the comparison example of the Lorenz system is more effective compared to previous studies, which demonstrates the superiority of the proposed method.

This study is structured as follows. Section II describes the T–S fuzzy model of nonlinear systems. Section III formulates the PMSG modeling, aerodynamic torque, and T–S fuzzy system. The overall principle findings of this study are described in Section IV. Section V describes numerical validation and comparison result. Finally, Section VI describes the conclusion of this study.

Notations: $\mathbb{R}^n$ denotes the Euclidean n-dimensional space. $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. diag$\{\cdots\}$ represents a block-diagonal matrix. I_n stands for the $n \times n$ identity matrix. In a matrix, the notation $*$ represents the symmetric term of a symmetric matrix. Sym$\{X\} = X + X^T$. The matrix $X > 0(< 0)$ is a positive definite (negative definite).

II. Preliminaries

Consider the following the T–S fuzzy system of the nonlinear model with disturbance is considered as follows.

Plant Rule i: IF $\Psi_1(t)$ is δ_{i1} and $\Psi_2(t)$ is $\delta_{i2}, \ldots,$ and $\Psi_p(t)$ is δ_{ip}, THEN

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A_i\mathbf{x}(t) + B_iu(t) + F_i\omega(t), i = 1, 2, \ldots, p\\ y(t) &= C_i\mathbf{x}(t)\end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ indicates the state vector, $u(t) \in \mathbb{R}^m$ is a control input, the external disturbance is denoted by $\omega(t) \in \mathbb{R}^p$. $y(t) \in \mathbb{R}^k$ is a system output. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $F_i \in \mathbb{R}^{n\times p}$, and $C_i \in \mathbb{R}^{k\times n}$ are the known constant matrices. The premise variable is represented by $\Psi(t) = [\Psi_1(t); \Psi_2(t); \cdots ; \Psi_p(t)]$ and δ_{ij},$(j = 1, 2, \ldots, p)$ is the T–S fuzzy set. p denotes the number of fuzzy rules. Then, system (1) can be described as follows:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{p} \vartheta_i(\Psi(t))(A_i\mathbf{x}(t) + B_iu(t) + F_i\omega(t))\\ y(t) &= \sum_{i=1}^{p} \vartheta_i(\Psi(t))C_i\mathbf{x}(t)\end{aligned} \tag{2}$$

where $\vartheta_i(\Psi(t))$ represents the normalized membership function that satisfies

$$\vartheta_i(\Psi(t)) = \frac{\varphi_i(t)}{\sum_{i=1}^{p}\varphi_i(t)} \geq 0, \sum_{i=1}^{p}\vartheta_i(\Psi(t)) = 1$$

with $\varphi_i(t) = \prod_{j=1}^{p}\delta_{ij}(\Psi_j(t))$ is the grade membership of $\Psi_j(t)$ in δ_{ij}.

Control Rule j: IF $\Psi_1(t_k)$ is δ_{j1} and $\Psi_2(t_k)$ is $\delta_{j2}, \ldots,$ and $\Psi_p(t_k)$ is δ_{jp}, THEN

$$u(t) = K_j\mathbf{x}(t_k) + L_j\mathbf{x}(t_k - \tau), t_k \leq t < t_{k+1}, k \in \mathbb{N} \tag{3}$$

where K_j and L_j are the CGM of fuzzy SDC. Furthermore, the τ is a constant time delay

$$0 < t_{k+1} - t_k = h_k \in [h_l, h_u]. \tag{4}$$

In addition, (4) satisfies the sampling interval. The known positive scalars are h_l and h_u. As a result, the proposed control is described as follows:

$$u(t) = \sum_{j=1}^{p} \vartheta_j(\Psi(t_k))\Big(K_j\mathbf{x}(t_k) + L_j\mathbf{x}(t_k-\tau)\Big). \tag{5}$$

Finally, by substituting (5) in (2), the closed-loop system is obtained as follows:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{p}\sum_{j=1}^{p} \vartheta_i(\Psi(t))\vartheta_j(\Psi(t_k))\\ &\quad\times \big(A_i\mathbf{x}(t) + B_iK_j\mathbf{x}(t_k) + B_iL_j\mathbf{x}(t_k-\tau) + F_i\omega(t)\big)\\ y(t) &= \sum_{i=1}^{p} \vartheta_i(\Psi(t))C_i\mathbf{x}(t).\end{aligned} \tag{6}$$

III. Problem Formulation

In this section, we discuss the nonlinear model of PMSG-based WTS. Further, the T–S fuzzy system is derived using IF-THEN membership functions.

A. Nonlinear PMSG-Based WTS Model

The mechanical torque $T_m(t)$ and $\lambda(t)$ denotes the tip speed ratio, which is determined as follows:

$$T_m(t) = \frac{\xi\pi R^3\vartheta^2(t)C_P\big(\lambda,\hat{\alpha}\big)}{2\lambda(t)},\ \lambda(t) = \frac{R\omega_g(t)}{\vartheta(t)} \tag{7}$$

where

$$\begin{aligned}C_P\big(\lambda,\hat{\alpha}\big) &= 0.5109\left(\frac{116}{\lambda_j} - 0.4\hat{\alpha} - 5\right)e^{\frac{-21}{\lambda_j}}\\ \lambda_j &= \frac{1}{\lambda + 0.08\hat{\alpha}} - \frac{0.035}{\hat{\alpha}^3+1}.\end{aligned}$$

The wind turbine power coefficient is denoted by $C_P(\lambda,\hat{\alpha})$, where λ and $\hat{\alpha}$ are tip speed ratio and pitch angle with $\vartheta(t)$ denoting the wind speed. From (7), the extraction of maximum power is ensured by the power coefficient C_P to a maximum possible value ($C_{P\max}$) and this can be achieved only with the optimum tip speed ratio value (λ_{opt}) and $\hat{\alpha} = 0^0$. In addition, the design of the PMSG model addressed in [36], [37], and [38] has been utilized for this study. Let us take the following state space system represented in the d–q axis from [39]:

$$\begin{aligned}V_{sd}(t) &= r_s i_{sd}(t) - \omega_e(t)L_{sq}i_{sq}(t) + L_{sd}\frac{di_{sd(t)}}{dt}\\ V_{sq}(t) &= r_s i_{sq}(t) - \omega_e(t)L_{sd}i_{sd}(t) + L_{sq}\frac{di_{sq(t)}}{dt}\\ &\quad + \omega_e(t)\vartheta_f\\ T_e(t) &= 1.5n_p i_{sq}(t)\vartheta_f\end{aligned} \tag{8}$$

where $V_{sd}(t)$,$V_{sq}(t)$ and $i_{sd}(t)$, $i_{sq}(t)$ represent the stator voltages and stator currents in the d–q axis reference frame, respectively. In addition, the voltages are regarded to be the control inputs. Further, the electrical rotor speed, denoted by $\omega_e(t)$, is defined as $\omega_e(t) = n_p\omega_g(t)$. Here, $\omega_g(t)$, $T_e(t)$, and ρ represent the generator rotational speed, generator torque, and friction coefficient, respectively. In addition, the mechanical performance of PMSG-based WTS is expressed as follows:

$$\mathcal{J}\frac{d\omega_g(t)}{dt} = T_e(t) - T_m(t) - \rho\omega_g(t). \tag{9}$$

Finally, based on (7)–(9), the state-space model for the nonlinear system of PMSG-based WTS can be written as follows:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + Bu(t) + F\omega(t)\\ y(t) &= C\mathbf{x}(t)\end{aligned} \tag{10}$$

where A is system matrix, B is the input matrix, and F is the disturbance matrix, respectively.

$$\mathbf{x}(t) = [\omega_g(t)\ i_{sq}(t)\ i_{sd}(t)]^T,\ u(t) = [V_{sq}(t)\ V_{sd}(t)]^T$$

$$A = \begin{bmatrix} -\frac{\rho}{\mathcal{J}} & \frac{1.5n_p\vartheta_f}{\mathcal{J}} & 0\\ -\frac{n_p\vartheta_f}{L_{sq}} & -\frac{r_s}{L_{sq}} & -\frac{n_pL_{sd}\omega_g(t)}{L_{sq}}\\ 0 & \frac{n_pL_{sq}\omega_g(t)}{L_{sd}} & -\frac{r_s}{L_{sd}}\end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0\\ \frac{1}{L_{sq}} & 0\\ 0 & \frac{1}{L_{sd}}\end{bmatrix},\ C = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

$$F = \begin{bmatrix}-\frac{T_0\omega_g(t)}{\mathcal{J}} & 0 & 0\end{bmatrix}^T.$$

B. T–S Fuzzy Approach

The T–S fuzzy model of the system (10) can be expressed as follows using the T–S fuzzy technique stated in (1) and (2):

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{2}\vartheta_i(\Psi(t))(A_i\mathbf{x}(t) + B_iu(t) + F_i\omega(t))\\ y(t) &= \sum_{i=1}^{2}\vartheta_i(\Psi(t))C_i\mathbf{x}(t)\end{aligned} \tag{11}$$

where $\mathbf{x}(t) = [\omega_g(t)\ i_{sq}(t)\ i_{sd}(t)]^T, u(t) = [V_{sq}(t)\ V_{sd}(t)]^T$, $\vartheta_1(\omega_g(t)) = ([\mathcal{M}_2 - \omega_g(t)]/[\mathcal{M}_2 - \mathcal{M}_1])$ and $\vartheta_2(\omega_g(t)) = 1 - \vartheta_1(\omega_g(t)), \omega_g(t) \in (\omega_{g\min}, \omega_{g\max}) = [\mathcal{M}_1, \mathcal{M}_2], T_0 = (\xi\pi R^5 C_{P\max}/2\lambda_{\text{opt}}^3)$. Also

$$A_1 = \begin{bmatrix} -\frac{\rho}{\mathcal{J}} & \frac{1.5n_p\vartheta_f}{\mathcal{J}} & 0\\ -\frac{n_p\vartheta_f}{L_{sq}} & -\frac{r_s}{L_{sq}} & -\frac{n_pL_{sd}\mathcal{M}_1}{L_{sq}}\\ 0 & \frac{n_pL_{sq}\mathcal{M}_1}{L_{sd}} & -\frac{r_s}{L_{sd}}\end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{\rho}{\mathcal{J}} & \frac{1.5n_p\vartheta_f}{\mathcal{J}} & 0\\ -\frac{n_p\vartheta_f}{L_{sq}} & -\frac{r_s}{L_{sq}} & -\frac{n_pL_{sd}\mathcal{M}_2}{L_{sq}}\\ 0 & \frac{n_pL_{sq}\mathcal{M}_2}{L_{sd}} & -\frac{r_s}{L_{sd}}\end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 & 0\\ \frac{1}{L_{sq}} & 0\\ 0 & \frac{1}{L_{sd}}\end{bmatrix}$$

$$F_1 = \begin{bmatrix}-\frac{T_0\mathcal{M}_1}{\mathcal{J}} & 0 & 0\end{bmatrix}^T$$

$$F_2 = \begin{bmatrix}-\frac{T_0\mathcal{M}_2}{\mathcal{J}} & 0 & 0\end{bmatrix}^T$$

$$C_1 = C_2 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

$$\mathcal{M}_1 = -105, \text{and } \mathcal{M}_2 = 105.$$

For the above T–S fuzzy model with the SDC technique, we define the problem as follows.

Problem 1: We design an FBMSDC (5) with the following objectives for the given system (11).

1) When $\omega(t) = 0$, then equilibrium point of (11) is asymptotically stable.
2) The following inequality holds with a zero initial condition and $\gamma > 0$:

$$\int_0^\infty y^T(t)y(t)dt \le \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt. \tag{12}$$

IV. Main Results

In this section, by utilizing the improved Looped-Lyapunov functional and LMI approach under the FBMSDC scheme, we derive T–S fuzzy models of PMSG-based WTS.

Throughout the work, we use the following notations:

$$\begin{aligned}
&\eta_1(t) = t - t_k, \quad \eta_2(t) = t_{k+1} - t\\
&\xi(t) = \Big[\mathbf{x}(t);\ \mathbf{x}(t_k);\ \mathbf{x}(t_{k+1});\ \mathbf{x}\Big(t_k + \hat{\beta}\eta_1(t)\Big);\\
&\quad \mathbf{x}\Big(t + \hat{\beta}\eta_2(t)\Big);\ \dot{\mathbf{x}}(t);\ \int_{t_k+\hat{\beta}\eta_1(t)}^{t} \mathbf{x}(\varsigma)d\varsigma;\\
&\quad \int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \mathbf{x}(\varsigma)d\varsigma;\ \int_{t}^{t+\hat{\beta}\eta_2(t)} \mathbf{x}(\varsigma)d\varsigma;\\
&\quad \int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \mathbf{x}(\varsigma)d\varsigma;\ \mathbf{x}(t-\tau);\ \mathbf{x}(t_k-\tau);\ \omega(t)\Big].
\end{aligned}$$

Now, consider the following solution to Problem 1.

Theorem 1: A set of known scalars is given $k, h_l, h_u\tau, \gamma$, and $0 < \hat{\beta} < 1$. The symmetric matrices are $O > 0$, $P > 0$, $\mathbb{R}_c > 0$, $\mathbb{Q}_c > 0$, $\mathbb{S}_c > 0$ $(c = 1, 2)$, T_d $(d = 1, \ldots, 4)$, J_1, J_5, J_8, and J_{10} and the any matrices are J_2, J_3, J_4, J_6, J_7, J_9, N_c, M_c, U_c, H_c, W_c, X_c, Y_c, and Z_c. The following conditions are satisfied with the appropriate dimensional matrices:

$$\begin{bmatrix} \psi_{ij}^1 + h_k\psi_{ij}^2 & \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4\\ * & -\mathbb{S}_1 & 0 & 0 & 0\\ * & * & -\mathbb{S}_1 & 0 & 0\\ * & * & * & -\mathbb{Q}_1 & 0\\ * & * & * & * & -\mathbb{Q}_2 \end{bmatrix} < 0 \tag{13}$$

$$\begin{aligned}
&\Pi_1 = \sqrt{h_k\Big(1-\hat{\beta}\Big)}N \ \ \Pi_2 = \sqrt{h_k\hat{\beta}}M\\
&\Pi_3 = \sqrt{h_k\Big(1-\hat{\beta}\Big)}U \ \ \Pi_4 = \sqrt{h_k\hat{\beta}}H
\end{aligned}$$

$$\begin{bmatrix} \psi_{ij}^1 + h_k\psi_{ij}^3 & \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4\\ * & -\mathbb{S}_2 & 0 & 0 & 0\\ * & * & -\mathbb{S}_2 & 0 & 0\\ * & * & * & -\mathbb{R}_2 & 0\\ * & * & * & * & -\mathbb{R}_1 \end{bmatrix} < 0$$

$$\begin{aligned}
&\Pi_5 = \sqrt{h_k\Big(1-\hat{\beta}\Big)}Y \ \ \Pi_6 = \sqrt{h_k\hat{\beta}}X\\
&\Pi_7 = \sqrt{h_k\Big(1-\hat{\beta}\Big)}Z \ \ \Pi_8 = \sqrt{h_k\hat{\beta}}W
\end{aligned} \tag{14}$$

for all $0 < h_k \le [h_l, h_u]$, where

$$\begin{aligned}
\psi_{ij}^1 = {}& \mathrm{He}\Big(\upsilon_1^T P\upsilon_6 + \big[\upsilon_1^T U_1 + \upsilon_7^T U_2\big]\upsilon_7\\
&+ \big[\upsilon_1^T H_1 + \upsilon_8^T H_2\big]\upsilon_8 + \big[\upsilon_1^T W_1 + \upsilon_9^T W_2\big]\upsilon_9\\
&+ \big[\upsilon_1^T Z_1 + \upsilon_{10}^T Z_2\big]\upsilon_{10} + \big[\upsilon_1^T N_1 + \upsilon_4^T N_2\big][\upsilon_1 - \upsilon_4]\\
&+ \big[\upsilon_2^T M_1 + \upsilon_4^T M_2\big][\upsilon_4 - \upsilon_2] + \big[\upsilon_3^T Y_1 + \upsilon_5^T Y_2\big][\upsilon_3 - \upsilon_5]\\
&+ \big[\upsilon_1^T X_1 + \upsilon_5^T X_2\big][\upsilon_5 - \upsilon_1] + \Big[\upsilon_1^T\tau^2 + \upsilon_{11}^T\tau^2\Big]\mathbb{R}[\upsilon_{11}]\\
&+ \Big[\frac{\pi^2}{4}\upsilon_{11}^T O\upsilon_{12}\Big]\Big) - \Big(\tau^2\upsilon_1^T\mathbb{R}\upsilon_1 - \tau^2\upsilon_6^T\mathbb{R}\upsilon_6 + \upsilon_7^T T_1\upsilon_7\\
&+ \upsilon_8^T T_2\upsilon_8 + \upsilon_9^T T_3\upsilon_9 + \upsilon_{10}^T T_4\upsilon_{10} + \frac{\pi^2}{4}\upsilon_{11}^T O\upsilon_{11}\\
&+ \frac{\pi^2}{4}\upsilon_{12}^T O\upsilon_{12}\Big) + \mathrm{He}\Big(\big[\upsilon_1^T + k\upsilon_6^T\big]G\\
&\times \big[\big(A_i\upsilon_1 + B_iK_j\upsilon_2 + B_iL_j\upsilon_{12} + F_i\upsilon_{13}\big) - \upsilon_6\big]\Big)\\
&+ \upsilon_1^T C_i^T C_i\upsilon_1 - \gamma^2\upsilon_{13}^T\upsilon_{13}
\end{aligned}$$

$$\begin{aligned}
\psi_{ij}^2 = {}& -\Big[\Big(1-\hat{\beta}\Big)\upsilon_5^T\mathbb{R}_1\upsilon_5 - \upsilon_1^T\mathbb{R}_1\upsilon_1 - \Big(1-\hat{\beta}\Big)\upsilon_5^T\mathbb{R}_2\upsilon_5\Big]\\
&+ \upsilon_6\mathbb{S}_2\upsilon_6 - \mathrm{He}\Big(\upsilon_9^T T_3\Big(\Big(1-\hat{\beta}\Big)\upsilon_5 - \upsilon_1\Big)\\
&- \upsilon_{10}^T T_4\Big(1-\hat{\beta}\Big)\upsilon_5\Big)\\
&- \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}^T \begin{bmatrix} J_1 & J_2 & J_3 & J_4\\ * & J_5 & J_6 & J_7\\ * & * & J_8 & J_9\\ * & * & * & J_{10}\end{bmatrix} \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\psi_{ij}^3 = {}& \Big[\upsilon_1^T\mathbb{Q}_1\upsilon_1 - \hat{\beta}\upsilon_4^T\mathbb{Q}_1\upsilon_4 + \hat{\beta}\upsilon_4^T\mathbb{Q}_2\upsilon_4\Big]\\
&+ \upsilon_6^T\mathbb{S}_1\upsilon_6 + \mathrm{He}\Big([\upsilon_7]^T T_1\Big(\upsilon_1 - \hat{\beta}\upsilon_4\Big) + [\upsilon_8]^T T_2\hat{\beta}\upsilon_4\Big)\\
&+ \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}^T \begin{bmatrix} J_1 & J_2 & J_3 & J_4\\ * & J_5 & J_6 & J_7\\ * & * & J_8 & J_9\\ * & * & * & J_{10}\end{bmatrix} \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
N &= \Big[N_1^T\ 0_n\ 0_n\ N_2^T\ \underbrace{0_n \ldots 0_n}_{9\text{ times}}\Big]^T\\
M &= \Big[0_n\ M_1^T\ 0_n\ M_2^T\ \underbrace{0_n \ldots 0_n}_{9\text{ times}}^{T}\Big]\\
U &= \Big[U_1^T\ \underbrace{0_n \ldots 0_n}_{5\text{ times}}\ U_2^T\ \underbrace{0_n \ldots 0_n}_{6\text{ times}}\Big]^T\\
H &= \Big[H_1^T\ \underbrace{0_n \ldots 0_n}_{6\text{ times}}\ H_2^T\ \underbrace{0_n \ldots 0_n}_{5\text{ times}}\Big]^T\\
Y &= \Big[0_n\ 0_n\ Y_1^T\ 0_n\ Y_2^T\ \underbrace{0_n \ldots 0_n}_{8\text{ times}}\Big]^T\\
X &= \Big[X_1^T\ 0_n\ 0_n\ X_2^T\ \underbrace{0_n \ldots 0_n}_{9\text{ times}}\Big]^T
\end{aligned}$$

$$Z = \Big[Z_1^T \underbrace{0_n \dots 0_n}_{8\text{ times}} Z_2^T \underbrace{0_n \dots 0_n}_{3\text{ times}}\Big]^T$$
$$W = \Big[W_1^T \underbrace{0_n \dots 0_n}_{7\text{ times}} W_2^T \underbrace{0_n \dots 0_n}_{4\text{ times}}\Big]^T$$
$$\upsilon_\varrho = \big[0_{n,(\varrho-1)n}\ I_n\ 0_{n,(13-\varrho)n}\big],\ \varrho = 1, 2, 3, \dots, 13.$$

Finally, the H_∞ disturbance reduction level γ and the system (6) is asymptotically stable.

Proof: Let us assume the following LKF for the T–S fuzzy system (6):

$$V(t) = \sum_{i=1}^{3} V_i(t) + \sum_{i=4}^{8} \chi_i(t) \tag{15}$$

where

$$V_1(t) = \mathbf{x}^T(t)P\mathbf{x}(t)$$
$$V_2(t) = \tau \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{\mathbf{x}}^T(\varsigma)\mathbb{R}\dot{\mathbf{x}}(\varsigma)d\varsigma\ d\beta$$
$$V_3(t) = h^2 \int_{t_k-\tau}^{t} \dot{\mathbf{x}}^T(\varsigma)O\dot{\mathbf{x}}(\varsigma)d\varsigma - \frac{\pi^2}{4}\int_{t_k-\tau}^{t-\tau}(\mathbf{x}(\varsigma)-\mathbf{x}(t_k-\tau))^T O(\mathbf{x}(\varsigma)-\mathbf{x}(t_k-\tau))d\varsigma$$
$$\chi_4(t) = \eta_2(t)\bigg(\int_{t_k+\hat{\beta}\eta_1(t)}^{t}\mathbf{x}^T(\varsigma)\mathbb{Q}_1\mathbf{x}(\varsigma)d\varsigma + \int_{t_k}^{t_k+\hat{\beta}\eta_1(t)}\mathbf{x}^T(\varsigma)\mathbb{Q}_2\mathbf{x}(\varsigma)d\varsigma\bigg)$$
$$\chi_5(t) = -\eta_1(t)\bigg(\int_{t}^{t+\hat{\beta}\eta_2(t)}\mathbf{x}^T(\varsigma)\mathbb{R}_1\mathbf{x}(\varsigma)d\varsigma + \int_{t+\hat{\beta}\eta_2(t)}^{t_k+1}\mathbf{x}^T(\varsigma)\mathbb{R}_2\mathbf{x}(\varsigma)d\varsigma\bigg)$$
$$\chi_6(t) = \eta_2(t)\bigg(\int_{t_k}^{t}\dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_1\dot{\mathbf{x}}(\varsigma)d\varsigma - \eta_1(t)\int_{t}^{t_k+1}\dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_2\dot{\mathbf{x}}(\varsigma)d\varsigma\bigg)$$
$$\begin{aligned}\chi_7(t) = \eta_2(t)\Bigg(&\bigg[\int_{t_k+\hat{\beta}\eta_1(t)}^{t}\mathbf{x}(\varsigma)d\varsigma\bigg]^T T_1\bigg[\int_{t_k+\hat{\beta}\eta_1(t)}^{t}\mathbf{x}(\varsigma)d\varsigma\bigg] + \bigg[\int_{t_k}^{t_k+\hat{\beta}\eta_1(t)}\mathbf{x}(\varsigma)d\varsigma\bigg]^T T_2\bigg[\int_{t_k}^{t_k+\hat{\beta}\eta_1(t)}\mathbf{x}(\varsigma)d\varsigma\bigg]\Bigg)\\ &- \eta_1(t)\Bigg(\bigg[\int_{t}^{t+\hat{\beta}\eta_2(t)}\mathbf{x}(\varsigma)d\varsigma\bigg]^T \times T_3\bigg[\int_{t}^{t+\hat{\beta}\eta_2(t)}\mathbf{x}(\varsigma)d\varsigma\bigg] + \bigg[\int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}}\mathbf{x}(\varsigma)d\varsigma\bigg]^T T_4\bigg[\int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}}\mathbf{x}(\varsigma)d\varsigma\bigg]\Bigg)\end{aligned}$$
$$\chi_8(t) = \eta_1(t)\eta_2(t)\begin{bmatrix}\mathbf{x}(t_k)\\ \mathbf{x}(t_{k+1})\\ \mathbf{x}(t-\tau)\\ \mathbf{x}(t_k-\tau)\end{bmatrix}^T \begin{bmatrix}J_1 & J_2 & J_3 & J_4\\ * & J_5 & J_6 & J_7\\ * & * & J_8 & J_9\\ * & * & * & J_{10}\end{bmatrix} \times \begin{bmatrix}\mathbf{x}(t_k)\\ \mathbf{x}(t_{k+1})\\ \mathbf{x}(t-\tau)\\ \mathbf{x}(t_k-\tau)\end{bmatrix}.$$

The positiveness of $V_1(t)$ and $V_2(t)$ is guaranteed by $P > 0$, $\mathbb{R} > 0$ and the proof process of $V_3(t) > 0$ can be found in [12] and [13]. Therefore, the positiveness of $V_i(t)$ can be easily ensured and Looped functional $\chi_i(t)$ satisfies the condition $\chi_i(t_k) = \chi_i(t_{k+1}) = 0$ as in [32], where $i = 4, 5, 6, 7, 8$. In other words, $\chi_i(t)$ is not required to be positive definite. We have taken a time derivative of (15) using the states of the system (6)

$$\dot{V}(t) = \sum_{i=1}^{3} \dot{V}_i(t) + \sum_{i=4}^{8} \dot{\chi}_i(t) \tag{16}$$

where

$$\dot{V}_1(t) = 2\mathbf{x}^T(t)P\dot{\mathbf{x}}(t)$$
$$\dot{V}_2(t) \le \tau^2\dot{\mathbf{x}}^T(t)\mathbb{R}\dot{\mathbf{x}}(t) - [\mathbf{x}(t)-\mathbf{x}(t-\tau)]^T\mathbb{R}[\mathbf{x}(t)-\mathbf{x}(t-\tau)]$$
$$\dot{V}_3(t) = h^2\dot{\mathbf{x}}^T(t)O\dot{\mathbf{x}}(t) - \frac{\pi^2}{4}\big[(\mathbf{x}(t-\tau)-\mathbf{x}(t_k-\tau))^T O \times (\mathbf{x}(t-\tau)-\mathbf{x}(t_k-\tau))\big]$$
$$\begin{aligned}\dot{\chi}_4(t) = &-\bigg(\int_{t_k+\hat{\beta}\eta_1(t)}^{t}\mathbf{x}^T(\varsigma)\mathbb{Q}_1\mathbf{x}(\varsigma)d\varsigma + \int_{t_k}^{t_k+\hat{\beta}\eta_1(t)}\mathbf{x}^T(\varsigma)\mathbb{Q}_2\mathbf{x}(\varsigma)d\varsigma\bigg)\\ &+ \eta_2(t)\Big[\mathbf{x}^T(t)\mathbb{Q}_1\mathbf{x}(t) - \hat{\beta}\mathbf{x}^T\big(t_k+\hat{\beta}\eta_1(t)\big) \times \mathbb{Q}_1\mathbf{x}\big(t_k+\hat{\beta}\eta_1(t)\big) + \hat{\beta}\mathbf{x}^T\big(t_k+\hat{\beta}\eta_1(t)\big) \times \mathbb{Q}_2\mathbf{x}\big(t_k+\hat{\beta}\eta_1(t)\big)\Big]\end{aligned}$$
$$\begin{aligned}\dot{\chi}_5(t) = &-\bigg(\int_{t}^{t+\hat{\beta}\eta_2(t)}\mathbf{x}^T(\varsigma)\mathbb{R}_1\mathbf{x}(\varsigma)d\varsigma + \int_{t+\hat{\beta}\eta_2(t)}^{t_k+1}\mathbf{x}^T(\varsigma)\mathbb{R}_2\mathbf{x}(\varsigma)d\varsigma\bigg) - \eta_1(t)\\ &\times \Big[\big(1-\hat{\beta}\big)\mathbf{x}^T\big(t+\hat{\beta}\eta_2(t)\big)\mathbb{R}_1\mathbf{x}\big(t+\hat{\beta}\eta_2(t)\big) - \mathbf{x}^T(t)\mathbb{R}_1\mathbf{x}(t) - \big(1-\hat{\beta}\big)\mathbf{x}^T\big(t+\hat{\beta}\eta_2(t)\big) \times \mathbb{R}_2\mathbf{x}\big(t+\hat{\beta}\eta_2(t)\big)\Big]\end{aligned}$$
$$\dot{\chi}_6(t) = -\int_{t_k}^{t}\dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_1\dot{\mathbf{x}}(\varsigma)d\varsigma + \eta_2(t)\dot{\mathbf{x}}^T(t)\mathbb{S}_1\dot{\mathbf{x}}(t)$$

$$
\begin{aligned}
&- \int_{t}^{t_k+1} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_2\dot{\mathbf{x}}(\varsigma)d\varsigma + \eta_1(t)\dot{\mathbf{x}}^T(t)\mathbb{S}_2\dot{\mathbf{x}}(t)\\
\dot{\chi}_7(t) = &\Bigg(\left[\int_{t_k+\hat{\beta}\eta_1(t)}^{t} \mathbf{x}(\varsigma)d\varsigma\right]^T T_1 \left[\int_{t_k+\hat{\beta}\eta_1(t)}^{t} \mathbf{x}(\varsigma)d\varsigma\right]\\
&+ \left[\int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \mathbf{x}(\varsigma)d\varsigma\right]^T T_2 \left[\int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \mathbf{x}(\varsigma)d\varsigma\right]\Bigg)\\
&+ 2\eta_2(t)\Bigg(\left[\int_{t_k+\hat{\beta}\eta_1(t)}^{t} \mathbf{x}(\varsigma)d\varsigma\right]^T T_1\\
&\times \left(\mathbf{x}(t) - \hat{\beta}\mathbf{x}\left(t_k + \hat{\beta}\eta_1(t)\right)\right)\\
&+ \left[\int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \mathbf{x}(\varsigma)d\varsigma\right]^T T_2\left(\hat{\beta}\mathbf{x}\left(t_k + \hat{\beta}\eta_1(t)\right)\right)\Bigg)\\
&- \Bigg(\left[\int_{t}^{t+\hat{\beta}\eta_2(t)} \mathbf{x}(\varsigma)d\varsigma\right]^T T_3 \left[\int_{t}^{t+\hat{\beta}\eta_2(t)} \mathbf{x}(\varsigma)d\varsigma\right]\\
&+ \left[\int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \mathbf{x}(\varsigma)d\varsigma\right]^T T_4 \left[\int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \mathbf{x}(\varsigma)d\varsigma\right]\Bigg)\\
&- 2\eta_1(t)\Bigg(\left[\int_{t}^{t+\hat{\beta}\eta_2(t)} \mathbf{x}(\varsigma)d\varsigma\right]^T T_3\\
&\times \left(\left(1-\hat{\beta}\right)\mathbf{x}\left(t + \hat{\beta}\eta_2(t) - \mathbf{x}(t)\right)\right)\\
&- \left[\int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \mathbf{x}(\varsigma)d\varsigma\right]^T T_4\left(\left(1-\hat{\beta}\right)\mathbf{x}\left(t + \hat{\beta}\eta_2(t)\right)\right)\Bigg)\\
\dot{\chi}_8(t) = &-\eta_1(t)\begin{bmatrix}\mathbf{x}(t_k)\\ \mathbf{x}(t_{k+1})\\ \mathbf{x}(t-\tau)\\ \mathbf{x}(t_k-\tau)\end{bmatrix}^T \begin{bmatrix} J_1 & J_2 & J_3 & J_4\\ * & J_5 & J_6 & J_7\\ * & * & J_8 & J_9\\ * & * & * & J_{10}\end{bmatrix}\\
&\times \begin{bmatrix}\mathbf{x}(t_k)\\ \mathbf{x}(t_{k+1})\\ \mathbf{x}(t-\tau)\\ \mathbf{x}(t_k-\tau)\end{bmatrix} + \eta_2(t)\begin{bmatrix}\mathbf{x}(t_k)\\ \mathbf{x}(t_{k+1})\\ \mathbf{x}(t-\tau)\\ \mathbf{x}(t_k-\tau)\end{bmatrix}^T\\
&\times \begin{bmatrix} J_1 & J_2 & J_3 & J_4\\ * & J_5 & J_6 & J_7\\ * & * & J_8 & J_9\\ * & * & * & J_{10}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t_k)\\ \mathbf{x}(t_{k+1})\\ \mathbf{x}(t-\tau)\\ \mathbf{x}(t_k-\tau)\end{bmatrix}.
\end{aligned}
$$

Now

$$
\begin{aligned}
-\int_{t_k}^{t} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_1\dot{\mathbf{x}}(\varsigma)d\varsigma = &-\int_{t_k+\hat{\beta}\eta_1(t)}^{t} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_1\dot{\mathbf{x}}(\varsigma)d\varsigma\\
&- \int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_1\dot{\mathbf{x}}(\varsigma)d\varsigma
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
-\int_{t}^{t_{k+1}} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_2\dot{\mathbf{x}}(\varsigma)d\varsigma = &-\int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_2\dot{\mathbf{x}}(\varsigma)d\varsigma\\
&- \int_{t}^{t+\hat{\beta}\eta_2(t)} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_2\dot{\mathbf{x}}(\varsigma)d\varsigma.
\end{aligned}
\tag{18}
$$

The integral terms are considered as follows using [34, Lemma 1]:

$$
\begin{aligned}
&- \int_{t_k+\hat{\beta}\eta_1(t)}^{t} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_1\dot{\mathbf{x}}(\varsigma)d\varsigma\\
&\le \left(1-\hat{\beta}\right)\eta_1(t)\xi^T(t)N\mathbb{S}_1^{-1}N^T\xi(t)\\
&+ 2\xi^T(t)N\left[\mathbf{x}(t) - \mathbf{x}\left(t_k + \hat{\beta}\eta_1(t)\right)\right]
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&- \int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_1\dot{\mathbf{x}}(\varsigma)d\varsigma\\
&\le \hat{\beta}\eta_1(t)\xi^T(t)M\mathbb{S}_1^{-1}M^T\xi(t)\\
&+ 2\xi^T(t)M\left[\mathbf{x}\left(t_k + \hat{\beta}\eta_1(t)\right) - \mathbf{x}(t_k)\right]
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&- \int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_2\dot{\mathbf{x}}(\varsigma)d\varsigma\\
&\le \left(1-\hat{\beta}\right)\eta_2(t)\xi^T(t)Y\mathbb{S}_2^{-1}Y^T\xi(t)\\
&+ 2\xi^T(t)Y\left[\mathbf{x}(t_{k+1}) - \mathbf{x}\left(t + \hat{\beta}\eta_2(t)\right)\right]
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
&- \int_{t}^{t+\hat{\beta}\eta_2(t)} \dot{\mathbf{x}}^T(\varsigma)\mathbb{S}_2\dot{\mathbf{x}}(\varsigma)d\varsigma\\
&\le \hat{\beta}\eta_2(t)\xi^T(t)X\mathbb{S}_2^{-1}X^T\xi(t)\\
&+ 2\xi^T(t)X\left[\mathbf{x}\left(t + \hat{\beta}\eta_2(t)\right) - \mathbf{x}(t)\right].
\end{aligned}
\tag{22}
$$

Moreover

$$
\begin{aligned}
&- \int_{t_k+\hat{\beta}\eta_1(t)}^{t} \mathbf{x}^T(\varsigma)\mathbb{Q}_1\mathbf{x}(\varsigma)d\varsigma\\
&\le \left(1-\hat{\beta}\right)\eta_1(t)\xi^T(t)U\mathbb{Q}_1^{-1}U^T\xi(t)\\
&+ 2\xi^T(t)U\int_{t_k+\hat{\beta}\eta_1(t)}^{t} \mathbf{x}(\varsigma)d\varsigma
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&- \int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \mathbf{x}^T(\varsigma)\mathbb{Q}_2\mathbf{x}(\varsigma)d\varsigma\\
&\le \hat{\beta}\eta_1(t)\xi^T(t)H\mathbb{Q}_2^{-1}H^T\xi(t)\\
&+ 2\xi^T(t)H\int_{t_k}^{t_k+\hat{\beta}\eta_1(t)} \mathbf{x}(\varsigma)d\varsigma
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&- \int_{t}^{t+\hat{\beta}\eta_2(t)} \mathbf{x}^T(\varsigma)\mathbb{R}_1\mathbf{x}(\varsigma)d\varsigma\\
&\le \hat{\beta}\eta_2(t)\xi^T(t)W\mathbb{R}_1^{-1}W^T\xi(t)\\
&+ 2\xi^T(t)W\int_{t}^{t+\hat{\beta}\eta_2(t)} \mathbf{x}(\varsigma)d\varsigma
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&- \int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \mathbf{x}^T(\varsigma)\mathbb{R}_2\mathbf{x}(\varsigma)d\varsigma\\
&\le \left(1-\hat{\beta}\right)\eta_2(t)\xi^T(t)Z\mathbb{R}_2^{-1}Z^T\xi(t)\\
&+ 2\xi^T(t)Z\int_{t+\hat{\beta}\eta_2(t)}^{t_{k+1}} \mathbf{x}(\varsigma)d\varsigma.
\end{aligned}
\tag{26}
$$

The resulting equation holds for any nonsingular matrix G

$$2\big[\mathbf{x}^T(t)+k\dot{\mathbf{x}}^T(t)\big]G\Bigg[\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i(\Psi(t))\vartheta_j(\Psi(t_k))\times\Big(A_i\mathbf{x}(t)+B_iK_j\mathbf{x}(t_k)+B_iL_j\mathbf{x}(t_k-\tau)+F_i\omega(t)\Big)-\dot{\mathbf{x}}(t)\Bigg]=0. \tag{27}$$

Finally, we can get the following equation by substituting (19)–(26) in (16) and adding (27) in (16):

$$\dot{V}(t)+y^T(t)y(t)-\gamma^2\omega^T(t)\omega(t)\leq\sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i(\Psi(t))\vartheta_j(\Psi(t_k))\xi^T(t)\Big(\Omega_{ij}\Big)\xi(t)<0$$

where

$$\Omega_{ij}=\psi_{ij}^1+\eta_1(t)\Phi_{ij}^1+\eta_2(t)\Phi_{ij}^2$$

with

$$\begin{aligned}\Phi_{ij}^1&=\psi_{ij}^2+\Big(1-\hat{\beta}\Big)N\mathbb{S}_1^{-1}N^T+\hat{\beta}M\mathbb{S}_1^{-1}M^T\\&\quad+\Big(1-\hat{\beta}\Big)U\mathbb{Q}_1^{-1}U^T+\hat{\beta}H\mathbb{Q}_2^{-1}H^T\\\Phi_{ij}^2&=\psi_{ij}^3+\Big(1-\hat{\beta}\Big)Y\mathbb{S}_2^{-1}Y^T+\hat{\beta}X\mathbb{S}_2^{-1}X^T\\&\quad+\Big(1-\hat{\beta}\Big)Z\mathbb{R}_2^{-1}Z^T+\hat{\beta}W\mathbb{R}_1^{-1}W^T.\end{aligned}$$

In terms of the convex combination method, if $\Omega_{ij}<0$ holds, then the following inequalities also hold:

$$\psi_{ij}^1+h_k\Phi_{ij}^1<0 \tag{28}$$

$$\psi_{ij}^1+h_k\Phi_{ij}^2<0 \tag{29}$$

for all $h_k\in(0,[h_l,h_u]]$. When the Schur complement is applied, (28) and (29) are equal to (13) and (14). Hence, $\dot{V}(t)<0$ indicates that model (6) is asymptotically stable under the given H_∞ disturbance reduction level γ. Finally, the proof is completed. ∎

The obtained conditions (13) and (14) become non-LMIs when K_j and L_j are assumed to be unknown in Theorem 1 stability condition. To find the control gains K_j and L_j, the stabilization condition is obtained as follows.

Theorem 2: A set of known scalars is given k,h_l,h_u,τ,γ and $0<\hat{\beta}<1$. The symmetric matrices are $\hat{O}>0$, $\hat{P}>0$, $\hat{\mathbb{R}}_c>0$, $\hat{\mathbb{Q}}_c>0$, $\hat{\mathbb{S}}_c>0$ $(c=1,2)$, $\hat{T}_d$ $(d=1,\ldots,4)$, $\hat{J}_1$, $\hat{J}_5$, $\hat{J}_8$, and $\hat{J}_{10}$ and the any matrices are $\hat{J}_2$, $\hat{J}_3$, $\hat{J}_4$, $\hat{J}_6$, $\hat{J}_7$, $\hat{J}_9$, $\hat{N}_c$, $\hat{M}_c$, $\hat{U}_c$, $\hat{H}_c$, $\hat{W}_c$, $\hat{X}_c$, $\hat{Y}_c$, and $\hat{Z}_c$. The following conditions are satisfied with the appropriate dimensional matrices:

$$\begin{bmatrix}\hat{\psi}_{ij}^1+h_k\hat{\psi}_{ij}^2 & \hat{\Pi}_1 & \hat{\Pi}_2 & \hat{\Pi}_3 & \hat{\Pi}_4 & \hat{G}\\ * & -\hat{\mathbb{S}}_1 & 0 & 0 & 0 & 0\\ * & * & -\hat{\mathbb{S}}_1 & 0 & 0 & 0\\ * & * & * & -\hat{\mathbb{Q}}_1 & 0 & 0\\ * & * & * & * & -\hat{\mathbb{Q}}_2 & 0\\ * & * & * & * & * & -1\end{bmatrix}<0 \tag{30}$$

$$\hat{\Pi}_1=\sqrt{h_k\Big(1-\hat{\beta}\Big)}\hat{N}\quad\hat{\Pi}_2=\sqrt{h_k\hat{\beta}}\hat{M}$$

$$\hat{\Pi}_3=\sqrt{h_k\Big(1-\hat{\beta}\Big)}\hat{U}\quad\hat{\Pi}_4=\sqrt{h_k\hat{\beta}}\hat{H}$$

$$\begin{bmatrix}\hat{\psi}_{ij}^1+h_k\hat{\psi}_{ij}^3 & \hat{\Pi}_5 & \hat{\Pi}_6 & \hat{\Pi}_7 & \hat{\Pi}_8 & \hat{G}\\ * & -\hat{\mathbb{S}}_2 & 0 & 0 & 0 & 0\\ * & * & -\hat{\mathbb{S}}_2 & 0 & 0 & 0\\ * & * & * & -\hat{\mathbb{R}}_2 & 0 & 0\\ * & * & * & * & -\hat{\mathbb{R}}_1 & 0\\ * & * & * & * & * & -1\end{bmatrix}<0$$

$$\hat{\Pi}_5=\sqrt{h_k\Big(1-\hat{\beta}\Big)}\hat{Y}\quad\hat{\Pi}_6=\sqrt{h_k\hat{\beta}}\hat{X}$$

$$\hat{\Pi}_7=\sqrt{h_k\Big(1-\hat{\beta}\Big)}\hat{Z}\quad\hat{\Pi}_8=\sqrt{h_k\hat{\beta}}\hat{W} \tag{31}$$

for all $0<h_k\leq[h_l,h_u]$, where

$$\begin{aligned}\psi_{ij}^1&=\mathrm{He}\Big(\upsilon_1^T\hat{P}\upsilon_6+\Big[\upsilon_1^T\hat{U}_1+\upsilon_7^T\hat{U}_2\Big]\upsilon_7\\&\quad+\Big[\upsilon_1^T\hat{H}_1+\upsilon_8^T\hat{H}_2\Big]\upsilon_8+\Big[\upsilon_1^T\hat{W}_1+\upsilon_9^T\hat{W}_2\Big]\upsilon_9\\&\quad+\Big[\upsilon_1^T\hat{Z}_1+\upsilon_{10}^T\hat{Z}_2\Big]\upsilon_{10}+\Big[\upsilon_1^T\hat{N}_1+\upsilon_4^T\hat{N}_2\Big][\upsilon_1-\upsilon_4]\\&\quad+\Big[\upsilon_2^T\hat{M}_1+\upsilon_4^T\hat{M}_2\Big][\upsilon_4-\upsilon_2]+\Big[\upsilon_3^T\hat{Y}_1+\upsilon_5^T\hat{Y}_2\Big][\upsilon_3-\upsilon_5]\\&\quad+\Big[\upsilon_1^T\hat{X}_1+\upsilon_5^T\hat{X}_2\Big][\upsilon_5-\upsilon_1]+\Big[\upsilon_1^T\tau^2+\upsilon_{11}^T\tau^2\Big]\hat{\mathbb{R}}[\upsilon_{11}]\\&\quad+\Big[\frac{\pi^2}{4}\upsilon_{11}^T\hat{O}\upsilon_{12}\Big]\Big)-\Big(\tau^2\upsilon_1^T\hat{\mathbb{R}}\upsilon_1-\tau^2\upsilon_6^T\hat{\mathbb{R}}\upsilon_6+\upsilon_7^T\hat{T}_1\upsilon_7\\&\quad+\upsilon_8^T\hat{T}_2\upsilon_8+\upsilon_9^T\hat{T}_3\upsilon_9+\upsilon_{10}^T\hat{T}_4\upsilon_{10}+\frac{\pi^2}{4}\upsilon_{11}^T\hat{O}\upsilon_{11}\\&\quad+\frac{\pi^2}{4}\upsilon_{12}^T\hat{O}\upsilon_{12}\Big)+He\big(\big[\upsilon_1^T+k\upsilon_6^T\big]\\&\quad\times\big[\big(A_i\mathcal{G}\upsilon_1+B_i\mathcal{E}_j\upsilon_2+B_i\mathcal{V}_j\upsilon_{12}+F_i\upsilon_{13}\big)-\mathcal{G}\upsilon_6\big]\big)\\&\quad-\gamma^2\upsilon_{13}^T\upsilon_{13}.\\\psi_{ij}^2&=-\Big[\Big(1-\hat{\beta}\Big)\upsilon_5^T\hat{\mathbb{R}}_1\upsilon_5-\upsilon_1^T\hat{\mathbb{R}}_1\upsilon_1-\Big(1-\hat{\beta}\Big)\upsilon_5^T\hat{\mathbb{R}}_2\upsilon_5\Big]\\&\quad+\upsilon_6\hat{\mathbb{S}}_2\upsilon_6-He\Big(\upsilon_9^T\hat{T}_3\Big(\Big(1-\hat{\beta}\Big)\upsilon_5-\upsilon_1\Big)\\&\quad-\upsilon_{10}^T\hat{T}_4\Big(1-\hat{\beta}\Big)\upsilon_5\Big)\\&\quad-\begin{bmatrix}\upsilon_2\\\upsilon_3\\\upsilon_{11}\\\upsilon_{12}\end{bmatrix}^T\begin{bmatrix}\hat{J}_1 & \hat{J}_2 & \hat{J}_3 & \hat{J}_4\\ * & \hat{J}_5 & \hat{J}_6 & \hat{J}_7\\ * & * & \hat{J}_8 & \hat{J}_9\\ * & * & * & \hat{J}_{10}\end{bmatrix}\begin{bmatrix}\upsilon_2\\\upsilon_3\\\upsilon_{11}\\\upsilon_{12}\end{bmatrix}\\\psi_{ij}^3&=\Big[\upsilon_1^T\hat{\mathbb{Q}}_1\upsilon_1-\hat{\beta}\upsilon_4^T\hat{\mathbb{Q}}_1\upsilon_4+\hat{\beta}\upsilon_4^T\hat{\mathbb{Q}}_2\upsilon_4\Big]\\&\quad+\upsilon_6^T\hat{\mathbb{S}}_1\upsilon_6+He\Big([\upsilon_7]^T\hat{T}_1\Big(\upsilon_1-\hat{\beta}\upsilon_4\Big)+[\upsilon_8]^T\hat{T}_2\hat{\beta}\upsilon_4\Big)\\&\quad+\begin{bmatrix}\upsilon_2\\\upsilon_3\\\upsilon_{11}\\\upsilon_{12}\end{bmatrix}^T\begin{bmatrix}\hat{J}_1 & \hat{J}_2 & \hat{J}_3 & \hat{J}_4\\ * & \hat{J}_5 & \hat{J}_6 & \hat{J}_7\\ * & * & \hat{J}_8 & \hat{J}_9\\ * & * & * & \hat{J}_{10}\end{bmatrix}\begin{bmatrix}\upsilon_2\\\upsilon_3\\\upsilon_{11}\\\upsilon_{12}\end{bmatrix}\end{aligned}$$

$$\hat{N}=\Big[\hat{N}_1^T\ 0_n\ 0_n\ \hat{N}_2^T\ \underbrace{0_n\ldots0_n}_{9\text{ times}}\Big]^T$$

$$\hat{M}=\Big[0_n\ \hat{M}_1^T\ 0_n\ \hat{M}_2^T\ \underbrace{0_n\ldots0_n}_{9\text{ times}}\Big]^T$$

$$\hat{U} = \Big[\hat{U}_1^T \ \underbrace{0_n \dots 0_n}_{5 \text{ times}} \ \hat{U}_2^T \ \underbrace{0_n \dots 0_n}_{6 \text{ times}}\Big]^T$$
$$\hat{Y} = \Big[0_n \ 0_n \ \hat{Y}_1^T \ 0_n \ \hat{Y}_2^T \ \underbrace{0_n \dots 0_n}_{8 \text{ times}}\Big]^T$$
$$\hat{X} = \Big[\hat{X}_1^T \ 0_n \ 0_n \ \hat{X}_2^T \ \underbrace{0_n \dots 0_n}_{9 \text{ times}}\Big]^T$$
$$\hat{Z} = \Big[\hat{Z}_1^T \ \underbrace{0_n \dots 0_n}_{8 \text{ times}} \ \hat{Z}_2^T \ \underbrace{0_n \dots 0_n}_{3 \text{ times}}\Big]^T$$
$$\hat{W} = \Big[\hat{W}_1^T \ \underbrace{0_n \dots 0_n}_{7 \text{ times}} \ \hat{W}_2^T \ \underbrace{0_n \dots 0_n}_{4 \text{ times}}\Big]^T$$
$$\hat{G}_1 = \Big[CI\mathcal{G}^T \ \underbrace{0_n \dots 0_n}_{10 \text{ times}}\Big]^T.$$

Furthermore, the CGM are obtained from $K_j = \mathcal{E}_j\mathcal{G}^T$, $L_j = \mathcal{V}_j\mathcal{G}^T$.

Proof: Define $\mathcal{G} = G^{-1}$, $\mathcal{G}P\mathcal{G} = \hat{P}$, $\mathcal{G}O\mathcal{G} = \hat{O}$, $\mathcal{G}\mathbb{R}_f\mathcal{G} = \hat{\mathbb{R}}_f$, $(f = 1, 2)$, $\mathcal{G}\mathbb{Q}_f\mathcal{G} = \hat{\mathbb{Q}}_f$, $\mathcal{G}\mathbb{S}_f\mathcal{G} = \hat{\mathbb{S}}_f$, $\mathcal{G}T_o\mathcal{G} = \hat{T}_o$, $(o = 1, \dots, 4)$, $\mathcal{G}J_p\mathcal{G} = \hat{J}_p$, $(p = 1, \dots, 10)$, $\mathcal{G}N\mathcal{G} = \hat{N}$, $\mathcal{G}M_f\mathcal{G} = \hat{M}_f$, $\mathcal{G}U_f\mathcal{G} = \hat{U}_f$, $\mathcal{G}W_f\mathcal{G} = \hat{W}_f$, $\mathcal{G}Y\mathcal{G} = \hat{Y}$, $\mathcal{G}Z\mathcal{G} = \hat{Z}$, $\Theta = \mathrm{diag}\{\underbrace{\mathcal{G}, \dots, \mathcal{G}I}_{12 \text{ times}}\}$, $\mathcal{E}_j = K_j\mathcal{G}$, and $\mathcal{V}_j = L_j\mathcal{G}$. Now, multiply (13) and (14) by Θ on the both side, the LMIs (30) and (31) can be obtained using the Schur complement. Then, with a H_∞ disturbance attenuation level γ, the system (6) is asymptotically stable. Finally, the proof is completed. ■

Remark 1: The stability and stabilization criteria for the T–S fuzzy system (6) with H_∞ attenuation level are provided by Theorems 1 and 2. The T–S fuzzy systems are considered to determine the conservatism of the proposed LKF

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p}\vartheta_i(\Psi(t))\vartheta_j(\Psi(t_k)) \times \big(A_i\mathbf{x}(t) + B_iK_j\mathbf{x}(t_k) + B_iL_j\mathbf{x}(t_k - \tau)\big). \quad (32)$$

The stabilization condition of the above T–S fuzzy systems is summarized in the following corollary.

Corollary 1: A set of known scalars is given $k, h_l, h_u, \tau, \gamma$ and $0 < \hat{\beta} < 1$. The symmetric matrices are $\hat{O} > 0$, $\hat{P} > 0$, $\hat{\mathbb{R}}_c > 0$, $\hat{\mathbb{Q}}_c > 0$, $\hat{\mathbb{S}}_c > 0$ $(c = 1, 2)$, $\hat{T}_d$ $(d = 1, \dots, 4)\hat{J}_1$, $\hat{J}_5$, $\hat{J}_8$, and $\hat{J}_{10}$ and the any matrices are $\hat{J}_2$, $\hat{J}_3$, $\hat{J}_4$, $\hat{J}_6$, $\hat{J}_7$, $\hat{J}_9$, $\hat{N}_c$, $\hat{M}_c$, $\hat{U}_c$, $\hat{H}_c$, $\hat{W}_c$, $\hat{X}_c$, $\hat{Y}_c$, and $\hat{Z}_c$. The following conditions are satisfied with the appropriate dimensional matrices:

$$\begin{bmatrix} \hat{\Gamma}_{ij}^1 + h_k\hat{\Gamma}_{ij}^2 & \hat{\Pi}_1 & \hat{\Pi}_2 & \hat{\Pi}_3 & \hat{\Pi}_4 \\ * & -\hat{\mathbb{S}}_1 & 0 & 0 & 0 \\ * & * & -\hat{\mathbb{S}}_1 & 0 & 0 \\ * & * & * & -\hat{\mathbb{Q}}_1 & 0 \\ * & * & * & * & -\hat{\mathbb{Q}}_2 \end{bmatrix} < 0 \quad (33)$$
$$\hat{\Pi}_1 = \sqrt{h_k\big(1-\hat{\beta}\big)}\hat{N} \quad \hat{\Pi}_2 = \sqrt{h_k\hat{\beta}}\hat{M}$$
$$\hat{\Pi}_3 = \sqrt{h_k\big(1-\hat{\beta}\big)}\hat{U} \quad \hat{\Pi}_4 = \sqrt{h_k\hat{\beta}}\hat{H}$$
$$\begin{bmatrix} \hat{\Gamma}_{ij}^1 + h_k\hat{\Gamma}_{ij}^3 & \hat{\Pi}_5 & \hat{\Pi}_6 & \hat{\Pi}_7 & \hat{\Pi}_8 \\ * & -\hat{\mathbb{S}}_2 & 0 & 0 & 0 \\ * & * & -\hat{\mathbb{S}}_2 & 0 & 0 \\ * & * & * & -\hat{\mathbb{R}}_2 & 0 \\ * & * & * & * & -\hat{\mathbb{R}}_1 \end{bmatrix} < 0$$
$$\hat{\Pi}_5 = \sqrt{h_k\big(1-\hat{\beta}\big)}\hat{Y} \quad \hat{\Pi}_6 = \sqrt{h_k\hat{\beta}}\hat{X}$$
$$\hat{\Pi}_7 = \sqrt{h_k\big(1-\hat{\beta}\big)}\hat{Z} \quad \hat{\Pi}_8 = \sqrt{h_k\hat{\beta}}\hat{W} \quad (34)$$

for all $0 < h_k \le [h_l, h_u]$, where

$$\begin{aligned}\Gamma_{ij}^1 = \mathrm{He}\Big(&\upsilon_1^T\hat{P}\upsilon_6 + \big[\upsilon_1^T\hat{U}_1 + \upsilon_7^T\hat{U}_2\big]\upsilon_7 \\ &+ \big[\upsilon_1^T\hat{H}_1 + \upsilon_8^T\hat{H}_2\big]\upsilon_8 + \big[\upsilon_1^T\hat{W}_1 + \upsilon_9^T\hat{W}_2\big]\upsilon_9 \\ &+ \big[\upsilon_1^T\hat{Z}_1 + \upsilon_{10}^T\hat{Z}_2\big]\upsilon_{10} + \big[\upsilon_1^T\hat{N}_1 + \upsilon_4^T\hat{N}_2\big][\upsilon_1 - \upsilon_4] \\ &+ \big[\upsilon_2^T\hat{M}_1 + \upsilon_4^T\hat{M}_2\big][\upsilon_4 - \upsilon_2] + \big[\upsilon_3^T\hat{Y}_1 + \upsilon_5^T\hat{Y}_2\big][\upsilon_3 - \upsilon_5] \\ &+ \big[\upsilon_1^T\hat{X}_1 + \upsilon_5^T\hat{X}_2\big][\upsilon_5 - \upsilon_1] + \big[\upsilon_1^T\tau^2 + \upsilon_{11}^T\tau^2\big]\hat{\mathbb{R}}[\upsilon_{11}] \\ &+ \Big[\frac{\pi^2}{4}\upsilon_{11}^T\hat{O}\upsilon_{12}\Big]\Big) - \Big(\tau^2\upsilon_1^T\hat{\mathbb{R}}\upsilon_1 - \tau^2\upsilon_6^T\hat{\mathbb{R}}\upsilon_6 + \upsilon_7^T\hat{T}_1\upsilon_7 \\ &+ \upsilon_8^T\hat{T}_2\upsilon_8 + \upsilon_9^T\hat{T}_3\upsilon_9 + \upsilon_{10}^T\hat{T}_4\upsilon_{10} + \frac{\pi^2}{4}\upsilon_{11}^T\hat{O}\upsilon_{11} \\ &+ \frac{\pi^2}{4}\upsilon_{12}^T\hat{O}\upsilon_{12}\Big) + \mathrm{He}\big(\big[\upsilon_1^T + k\upsilon_6^T\big] \\ &\times \big[\big(A_i\mathcal{G}\upsilon_1 + B_i\mathcal{E}_j\upsilon_2 + B_i\mathcal{V}_j\upsilon_{12}\big) - \mathcal{G}\upsilon_6\big]\big)\end{aligned}$$

$$\begin{aligned}\Gamma_{ij}^2 = &-\Big[\big(1-\hat{\beta}\big)\upsilon_5^T\hat{\mathbb{R}}_1\upsilon_5 - \upsilon_1^T\hat{\mathbb{R}}_1\upsilon_1 - \big(1-\hat{\beta}\big)\upsilon_5^T\hat{\mathbb{R}}_2\upsilon_5\Big] \\ &+ \upsilon_6\hat{\mathbb{S}}_2\upsilon_6 - \mathrm{He}\Big(\upsilon_9^T\hat{T}_3\Big(\big(1-\hat{\beta}\big)\upsilon_5 - \upsilon_1\Big) \\ &- \upsilon_{10}^T\hat{T}_4\big(1-\hat{\beta}\big)\upsilon_5\Big) \\ &- \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}^T \begin{bmatrix}\hat{J}_1 & \hat{J}_2 & \hat{J}_3 & \hat{J}_4\\ * & \hat{J}_5 & \hat{J}_6 & \hat{J}_7\\ * & * & \hat{J}_8 & \hat{J}_9\\ * & * & * & \hat{J}_{10}\end{bmatrix} \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\Gamma_{ij}^3 = &\Big[\upsilon_1^T\hat{\mathbb{Q}}_1\upsilon_1 - \hat{\beta}\upsilon_4^T\hat{\mathbb{Q}}_1\upsilon_4 + \hat{\beta}\upsilon_4^T\hat{\mathbb{Q}}_2\upsilon_4\Big] \\ &+ \upsilon_6^T\hat{\mathbb{S}}_1\upsilon_6 + \mathrm{He}\Big([\upsilon_7]^T\hat{T}_1\big(\upsilon_1 - \hat{\beta}\upsilon_4\big) + [\upsilon_8]^T\hat{T}_2\hat{\beta}\upsilon_4\Big) \\ &+ \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}^T \begin{bmatrix}\hat{J}_1 & \hat{J}_2 & \hat{J}_3 & \hat{J}_4\\ * & \hat{J}_5 & \hat{J}_6 & \hat{J}_7\\ * & * & \hat{J}_8 & \hat{J}_9\\ * & * & * & \hat{J}_{10}\end{bmatrix} \begin{bmatrix}\upsilon_2\\ \upsilon_3\\ \upsilon_{11}\\ \upsilon_{12}\end{bmatrix}\end{aligned}$$

$$\hat{N} = \Big[\hat{N}_1^T \ 0_n \ 0_n \ \hat{N}_2^T \ \underbrace{0_n \dots 0_n}_{9 \text{ times}}\Big]^T$$
$$\hat{M} = \Big[0_n \ \hat{M}_1^T \ 0_n \ \hat{M}_2^T \ \underbrace{0_n \dots 0_n}_{9 \text{ times}}\Big]^T$$
$$\hat{U} = \Big[\hat{U}_1^T \ \underbrace{0_n \dots 0_n}_{5 \text{ times}} \ \hat{U}_2^T \ \underbrace{0_n \dots 0_n}_{6 \text{ times}}\Big]^T$$

$$\hat{H} = \begin{bmatrix} \hat{H}_1^T \ \underbrace{0_n \ldots 0_n}_{6 \text{ times}} \ \hat{H}_2^T \ \underbrace{0_n \ldots 0_n}_{5 \text{ times}} \end{bmatrix}^T$$

$$\hat{Y} = \begin{bmatrix} 0_n \ 0_n \ \hat{Y}_1^T \ 0_n \ \hat{Y}_2^T \ \underbrace{0_n \ldots 0_n}_{8 \text{ times}} \end{bmatrix}^T$$

$$\hat{X} = \begin{bmatrix} \hat{X}_1^T \ 0_n \ 0_n \ \hat{X}_2^T \ \underbrace{0_n \ldots 0_n}_{9 \text{ times}} \end{bmatrix}^T$$

$$\hat{Z} = \begin{bmatrix} \hat{Z}_1^T \ \underbrace{0_n \ldots 0_n}_{8 \text{ times}} \ \hat{Z}_2^T \ \underbrace{0_n \ldots 0_n}_{3 \text{ times}} \end{bmatrix}^T$$

$$\hat{W} = \begin{bmatrix} \hat{W}_1^T \ \underbrace{0_n \ldots 0_n}_{7 \text{ times}} \ \hat{W}_2^T \ \underbrace{0_n \ldots 0_n}_{4 \text{ times}} \end{bmatrix}^T.$$

The CGM are calculated from $K_j = \mathcal{E}\mathcal{G}^{-1}$ and $L_j = \mathcal{V}\mathcal{G}^{-1}$.

Proof: We can obtain the conditions (33) and (34) by using the Lyapunov functional $V(t)$ in (15) and the corresponding proof of Theorems 1 and 2. Then, with an H_∞ disturbance reduction level γ, the system (6) is asymptotically stable. Finally, the proof is completed. ■

Remark 2: As a result of the studies on the SDC system's stability, more attention has been given to the Looped-Lyapunov functionals. For example, Zeng et al. [32] and Shanmugam and Joo [33], [34] introduced looped functionals, which use all interval sampling information and lead to less conservative outcomes. However, this study introduced a fractional parameter, which is $0 < \hat{\beta} < 1$ and interval of sampling $[t_k, t_{k+1})$ has been splitted into $[t_k, t_k + \hat{\beta}\eta_1(t)]$, $[t_k + \hat{\beta}\eta_1(t), t]$, $[t, t+\hat{\beta}\eta_2(t)]$ and $[t+\hat{\beta}\eta_2(t), t_{k+1}]$. We have introduced ILF on the basis of these splitted intervals in $\sum_{i=4}^{8} \chi_i(t)$. which contains all sampling information of the interval.

Remark 3: The two-sided looped function $\chi(t)$ used in this study has more sampling information, which helps to improve the relationship between the terms of $\xi(t)$. Because of the time delay, $V_3(t)$ includes the delayed state information from $(t_k - \tau)$ to $(t - \tau)$. To summarize, states $\mathbf{x}(t)$, $\mathbf{x}(t_k)$, $\mathbf{x}(t_{k+1})$, $\mathbf{x}(t-\tau)$, and $\mathbf{x}(t_k - \tau)$ and to handle the closed-loop system's stability problem (6), the internal relationship between them is taken into account, which helps to improve the derived theoretical results.

V. Numerical Validation

This section validates the derived stabilization criteria with the PMSG-based WTS model and Lorenz system, which shows the merit of the proposed control technique and derived sufficient conditions.

A. PMSG-Based WTS

Table I shows parameter values for PMSG-based WTS and the scalars $\hat{\beta} = 0.5$, $k = 0.05$, $\gamma = 0.1$, $h_l = 0.01$, and $h_u = 0.05$, the LMIs are solved in Theorem 2. Then, the CGM has calculated as follows:

$$K_1 = \begin{bmatrix} -13.3637 & -37.4659 & 0.0000 \\ -0.0000 & -0.0000 & -58.2461 \end{bmatrix}$$

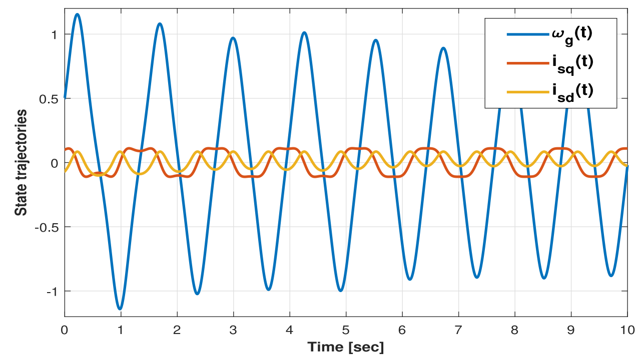

Fig. 1. Dynamical behavior of system (11) without control inputs and the state $\omega_g(t)$, $i_{sq}(t)$, and $i_{sd}(t)$.

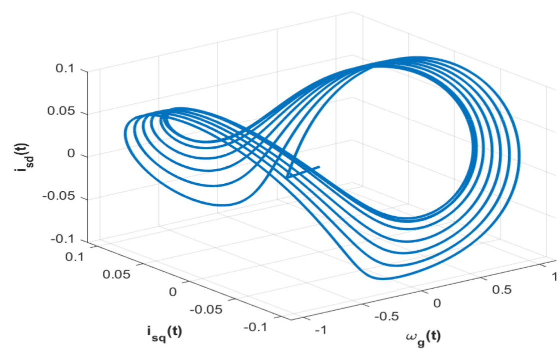

Fig. 2. Phase portraits of $\omega_g(t)$, $i_{sq}(t)$, and $i_{sd}(t)$ of system (11) with $u(t) = 0$.

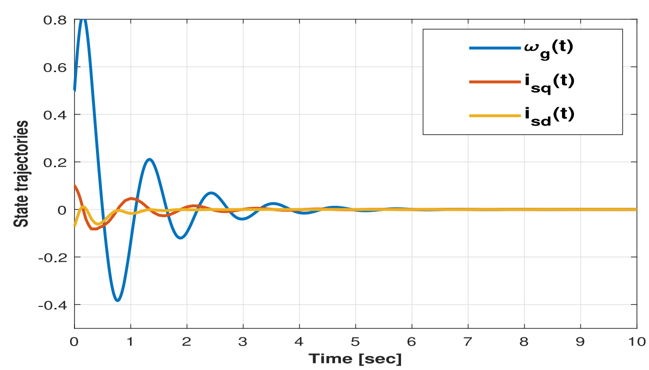

Fig. 3. Dynamic behavior of system (11) with control inputs $V_{sq}(t)$ and $V_{sd}(t)$ and the state $\omega_g(t)$, $i_{sq}(t)$, and $i_{sd}(t)$.

$$K_2 = \begin{bmatrix} -13.3637 & -37.4659 & 0.0000 \\ -0.0000 & -0.0000 & -58.2461 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.0003 & -0.0020 & -0.0000 \\ -0.0000 & -0.0000 & -0.0011 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0.0003 & -0.0020 & -0.0000 \\ -0.0000 & -0.0000 & -0.0011 \end{bmatrix}.$$

Based on the above CGM and $\omega(t) = 0.1 sint$, the simulation results of WTS based on PMSG (11) are presented in Figs. 1–4. From Figs. 1 and 2, we can notice that the state trajectories exhibit the chaotic behavior with the initial condition $\mathbf{x}(0) = [0.5 \ 0.1 \ -0.07]^T$ and without control input $u(t) = 0$. Then, the proposed MBSDC technique (5) is applied to the PMSG model (11), where the system states $\omega_g(t)$, $i_{sq}(t)$, and

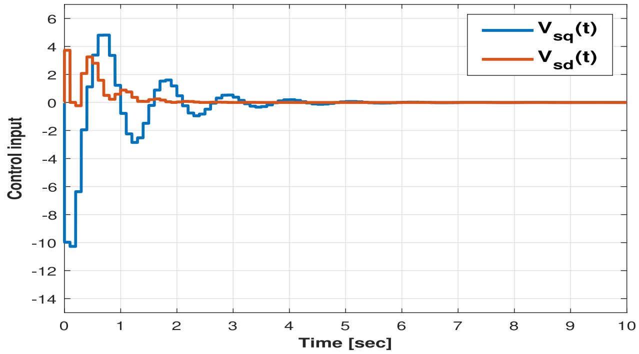

Fig. 4. Dynamic behavior of system (11) and the control trajectories $V_{sq}(t)$ and $V_{sd}(t)$.

TABLE I
PMSG PARAMETERS [39]

Parameter	Description	Numerical Value
r_s	Stator Resistance	0.4 Ω
L_{sd}, L_{sq}	Stator reactance on the $d-q$ frame	17.5 mH
n_p	Number of poles	10
ξ	Air density	1.25 kg/m^3
R	Blade Radius	1.94 m
$\vartheta(t)$	Wind speed	12m/s
$\mathcal{J}$	Moment of Inertia	0.188 $kg.m^2$
ϑ_f	Magnetic flux linkage	0.445 Wb
ρ	Friction coefficient	0.009 $N.m.sec$
λ_{opt}	Tip speed ratio	8.1
C_{Pmax}	Maximum power coefficient	0.42

$i_{sd}(t)$ become stable, as illustrated in Fig. 3. Moreover, Fig. 4 shows the time response of the control $V_{sq}(t)$, $V_{sd}(t)$ under the proposed FBMSDC. Therefore, from the simulation results (Figs. 1–4), we can demonstrate that the proposed FBMSDC technique transforms the chaotic behavior of state response into stable using estimated gain matrices.

B. Comparison Example

Consider the Lorenz system as follows:

$$\begin{cases} \dot{\mathbf{x}}_1(t) = -a\mathbf{x}_1(t) + a\mathbf{x}_2(t) + u_1(t) \\ \dot{\mathbf{x}}_2(t) = c\mathbf{x}_1(t) - \mathbf{x}_2(t) - \mathbf{x}_1(t)\mathbf{x}_3(t) \\ \dot{\mathbf{x}}_3(t) = \mathbf{x}_1(t)\mathbf{x}_2(t)b\mathbf{x}_3(t) \end{cases} \tag{35}$$

where $\mathbf{x}_1(t), \mathbf{x}_2(t)$, and $\mathbf{x}_3(t)$ are the state variables, $u(t)$ denotes the control input and also a, b, and c are the constants. Moreover, the Lorenz system with $\mathbf{x}_1(t) \in [-d, d]$ can be represented as the T–S fuzzy model with the following parameters:

$$\mathcal{A}_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & -d & -b \end{bmatrix} \text{ and}$$
$$B_1 = B_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$$

with $a = 10, b = 8/3, c = 28$, and $d = 25$ and the membership functions are

$$\vartheta_1(\Psi(t)) = 1/2\left(1 + \frac{\mathbf{x}_1(t)}{d}\right), \text{ and } \vartheta_2(\Psi(t)) = 1 - \vartheta_1(\Psi(t)).$$

TABLE II
CALCULATING THE MAXIMUM SAMPLING INTERVAL h_u WHEN $h_l = h_u$

Methods	h_u
[40]	0.0158
[41]	0.0299
[42]	0.0347
[10]	0.0412
[43]	0.0423
[27]	0.125 ($\tau = 0.01$)
Corollary 1	0.1632 ($\tau = 0.09$)

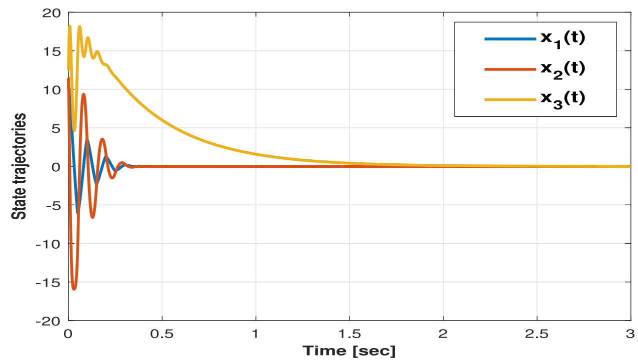

Fig. 5. Dynamic behavior of state trajectories $\mathbf{x}_1(t), \mathbf{x}_2(t)$, and $\mathbf{x}_3(t)$.

By choosing $\hat{\beta} = 0.5$, $k = 1.5$, and solving the adequate condition in Corollary 1, we can calculate the largest sample interval $h_u = 0.1632$ when $\tau = 0.09$ and a comparison results on the obtained largest sampling interval with the existing approach are illustrated in Table II. From Table II, we can observe that the adequate conditions obtained less conservative results than existing methods [10], [27], [40], [41], [42], [43].

Moreover, we can calculate the CGM $K_j = \mathcal{E}_j\mathcal{G}^{-1}$ and $L_j = \mathcal{V}_j\mathcal{G}^{-1}$, for maximum sampling interval $h_u = 0.1632$. The control gains are attained as follows:

$$K_1 = \begin{bmatrix} -15.8766 & -3.3482 & 0.0000 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -15.8766 & -3.3482 & 0.0000 \end{bmatrix}$$
$$L_1 = \begin{bmatrix} -0.0091 & 0.0026 & -0.0000 \end{bmatrix}$$
$$L_2 = \begin{bmatrix} -0.0091 & 0.0026 & -0.0000 \end{bmatrix}.$$

Using the aforementioned CGM and initial condition of $\mathbf{x}(0) = [10.5\ \ 11.5\ \ 12.5]^T$, the state responses are shown in Fig. 5, revealing that the states are asymptotically stable under the proposed model FBMSDC. Moreover, the control response of the FBMSDC is also shown in Fig. 6. As a result, we can conclude that the proposed method achieves the stabilized performance for the Lorenz system and also, a longer sample interval value than previous methods can be obtained as well.

VI. CONCLUSION

In this study, we have investigated the design of FBMSDC to achieve the stable performance for nonlinear PMSG-based WTS against external disturbance. A memory-based fuzzy SDC has been designed, which contains both discrete and continuous-time control signals designed for the T–S fuzzy PMSG model. In addition, new sufficient conditions have

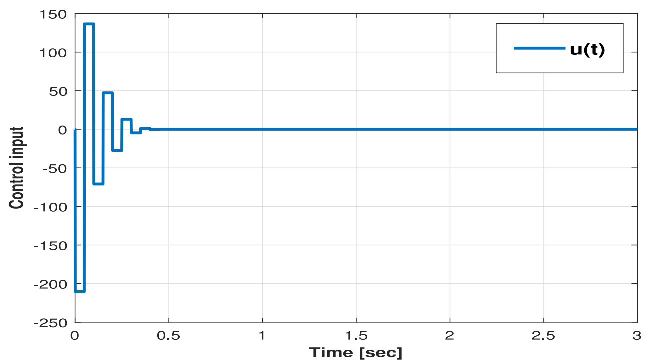

Fig. 6. Dynamic behavior of control response $u(t)$.

been derived using an improved looped Lyapunov functional with sampling intervals $[t_k, t_{k+1})$ splited into four nonuniform intervals. The obtained sufficient condition with an H_∞ disturbance attenuation level has been capable of achieving the asymptotically stable performance of the T–S fuzzy system in a closed-loop form. Finally, we have demonstrated that the proposed technique outperforms the existing works based on the obtained largest sampling interval and less conservatism through nonlinear PMSG-based WTS and Lorenz system.

REFERENCES

[1] J. L. Rodríguez-Amenedo, S. Arnaltes, and M. A. Rodríguez, "Operation and coordinated control of fixed and variable speed wind farms," *Renew. Energy*, vol. 33, no. 3, pp. 406–414, Mar. 2008.

[2] L. Shanmugam, and Y. H. Joo, "Stability and stabilization for T–S fuzzy large-scale interconnected power system with wind farm via sampled data control," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2134–2144, Apr. 2021.

[3] X.-X. Yin, Y.-G. Lin, W. Li, H. -W. Liu, and Y.-J. Gu, "Fuzzylogic sliding-mode control strategy for extracting maximum wind power," *IEEE Trans. Energy Convers.*, vol. 30, no. 4, pp. 1267–1278, Dec. 2015.

[4] R. Venkateswaran and Y. H. Joo, "Retarded sampled-data control design for interconnected power system with DFIG-based wind farm: LMI approach," *IEEE Trans. Cybern.*, vol. 52, no. 7, pp. 5767–5777, Jul. 2022.

[5] C. N. Bhende, S. Mishra, and S. G. Malla, "Permanent magnet synchronous generator-based standalone wind energy supply system," *IEEE Trans. Sustain. Energy*, vol. 2, no. 4, pp. 361–373, Oct. 2011.

[6] A. Vinodkumar, M. Prakash, and Y. H. Joo, "Impulsive observer-based output control for PMSG-based wind energy conversion system," *IET Control Theory Appl.*, vol. 13, no. 13, pp. 2056–2064, Sep. 2019.

[7] H. Ye, B. Yue, X. Li, and K. Strunz, "Modeling and simulation of multiscale transients for PMSG-based wind power systems," *Wind Energy*, vol. 20, no. 8, pp. 1349–1364, Aug. 2017.

[8] Y. Deng, Y. He, D. Tian, W. He, and P. Ding, "Analysis of the stability control of wind turbines generator system based on the Lure Lyapunov function," in *Proc. Int. Conf. Renew. Power Gener.*, Oct. 2015, pp. 1–9, doi: 10.1049/cp.2015.0429.

[9] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 15, no. 1, pp. 116–132, Jan./Feb. 1985.

[10] Z.-P. Wang and H.-N. Wu, "On fuzzy sampled-data control of chaotic systems via a time-dependent Lyapunov functional approach," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 819–829, Apr. 2015.

[11] S. Bououden, M. Chadli, S. Filali, and A. E. Hajjaji, "Fuzzy model based multivariable predictive control of a variable speed wind turbine: LMI approach," *Renew. Energy*, vol. 37, no. 1, pp. 434–439, Jan. 2012.

[12] C. Hua, S. Wu, and X. Guan, "Stabilization of T–S fuzzy system with time delay under sampled-data control using a new looped-functional," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 2, pp. 400–407, Feb. 2020.

[13] K. Liu, and E. Fridman, "Wirtinger's inequality and Lyapunov-based sampled-data stabilization," *Automatica*, vol. 48, no. 1, pp. 102–108, Jan. 2012.

[14] M. Wang, J. Qiu, M. Chadli, and M. Wang, "A switched system approach to exponential stabilization of sampled-data T–S fuzzy systems with packet dropouts," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 3145–3156, Dec. 2016.

[15] X. -H. Chang and G. -H. Yang, "Nonfragile H_∞ filter design for T–S fuzzy systems in standard form," *IEEE Trans. Ind. Electron.*, vol. 61, no. 7, pp. 3448–3458, Jul. 2014.

[16] Q. X. Chen and X. H. Chang, "Resilient filter of nonlinear network systems with dynamic event-triggered mechanism and hybrid cyber attack," *Appl. Math. Comput.*, vol. 434, Dec. 2022, Art. no. 127419.

[17] J. Wang, Y. Zhang, L. Su, J. H. Park, and H. Shen, "Model-based fuzzy filtering for discrete-time semi-markov jump nonlinear systems using semi-Markov kernel," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 7, pp. 2289–2299, Jul. 2022.

[18] J. Wang, J. Xia, H. Shen, M. Xing, and J. H. Park, "$\mathcal{H}_\infty$ Synchronization for fuzzy markov jump chaotic systems with piecewise-constant transition probabilities subject to PDT switching rule," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 10, pp. 3082–3092, Oct. 2021.

[19] X. Liu, J. Xia, J. Wang, and H. Shen, "Interval type-2 fuzzy passive filtering for nonlinear singularly perturbed PDT-switched systems and its application," *J. Syst. Sci. Complexity*, vol. 34, no. 6, pp. 2195–2218, Jan. 2021.

[20] S. Kuppusamy and Y. H. Joo, "Stabilization criteria for T–S Fuzzy systems with multiplicative sampled-data control gain uncertainties," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 10, pp. 4082–4092, Oct. 2022.

[21] Y. Liu and S. M. Lee, "Stability and stabilization of Takagi–Sugeno fuzzy systems via sampled-data and state quantized controller," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 3, pp. 635–644, Jun. 2016.

[22] N. Gunasekaran and Y. H. Joo, "Stochastic sampled-data controller for T–S fuzzy chaotic systems and its applications," *IET Control Theory Appl.*, vol. 13, no. 12, pp. 1834–1843, Jun. 2019.

[23] V. Sharmila, R. Rakkiyappan, and Y. H. Joo, "Fuzzy sampled-data control for DFIG-based wind turbine with stochastic actuator failures," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2199–2211, Apr. 2021.

[24] L. Shanmugam and Y. H. Joo, "Design of interval type-2 fuzzy-based sampled-data controller for nonlinear systems using novel fuzzy Lyapunov functional and its application to PMSM," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 1, pp. 542–551, Jan. 2021.

[25] C. Ge, J. H. Park, C. Hua, and X. Guan, "Nonfragile consensus of multiagent systems based on memory sampled-data control," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 1, pp. 391–399, Jan. 2021.

[26] Y. Liu, J. H. Park, B.-Z. Guo, and Y. Shu, "Further results on stabilization of chaotic systems based on fuzzy memory sampled-data control," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 1040–1045, Apr. 2018.

[27] L. Shanmugam and Y. H. Joo, "Stabilization of permanent magnet synchronous generator-based wind turbine system via fuzzy-based sampled-data control approach," *Inf. Sci.*, vol. 559, pp. 270–285, Jun. 2021.

[28] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421–427, Feb. 2010.

[29] T. H. Lee and J. H. Park, "Stability analysis of sampled-data systems via free-matrix-based time-dependent discontinuous Lyapunov approach," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3653–3657, Jul. 2017.

[30] K. Liu and E. Fridman, "Networked-based stabilization via discontinuous Lyapunov functionals," *Int. J. Robust Nonlinear Control*, vol. 22, no. 4, pp. 420–436, Mar. 2012.

[31] M. M. Saberi and I. Zamani, "Stability and stabilisation of switched time-varying delay systems: A multiple discontinuous Lyapunov function approach," *Int. J. Syst. Sci.*, vol. 51, no. 13, pp. 2378–2409, Jul. 2020.

[32] H. B. Zeng, K. L. Teo, and Y. He, "A new looped-functional for stability analysis of sampled-data systems," *Automatica*, vol. 82, pp. 328–331, Aug. 2017.

[33] L. Shanmugam and Y. H. Joo, "Further stability and stabilization condition for sampled-data control systems via looped-functional method," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 68, no. 1, pp. 301–305, Jan. 2021.

[34] L. Shanmugam and Y. H. Joo, "Stability criteria for fuzzy-based sampled-data control systems via a fractional parameter-based refined looped Lyapunov functional," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 7, pp. 2538–2549, Jul. 2022.

[35] C. Hua, S. Wu, and X. Guan, "Stabilization of T–S fuzzy system with time delay under sampled-data control using a new looped-functional," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 2, pp. 400–407, Feb. 2020.

[36] C. Chatri and M. Ouassaid, "Design of fuzzy control T–S for wind energy conversion system based PMSG using LMI approach," in *Proc. IEEE 5th Int. Congr. Inf. Sci. Technol.*, Dec. 2018, pp. 466–471, doi: 10.1109/CIST.2018.8596515.

[37] P. Mani, R. Rajan, and Y. H. Joo, "Design of observer-based eventtriggered fuzzy ISMC for TS fuzzy model and its application to PMSG," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2221–2231, Apr. 2021.

[38] S. Hwang, J. B. Park, and Y. H. Joo, "Disturbance observer-based integral fuzzy sliding-mode control and its application to wind turbine system," *IET Control Theory Appl.*, vol. 13, no. 12, pp. 1891–1900, Aug. 2019.

[39] V. Gandhi and Y. H. Joo, "T–S fuzzy sampled-data control for nonlinear systems with actuator faults and its application to wind energy system," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 2, pp. 462–474, Feb. 2022.

[40] H. K. Lam and F. H. F. Leung, "Stabilization of chaotic systems using linear sampled-data controller," *Int. J. Bifurcation Chaos*, vol. 17 no. 6, pp. 2021–2031, Jun. 2007.

[41] X. -L. Zhu, B. Chen, D. Yue, and Y. Wang, "An improved input delay approach to stabilization of fuzzy systems under variable sampling," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 330–341, Apr. 2012.

[42] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Sampled-data fuzzy control of chaotic systems based on T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 153–163, Feb. 2014.

[43] T. H. Lee and J. H. Park, "New methods of fuzzy sampled-data control for stabilization of chaotic systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 12, pp. 2026–2034, Dec. 2018.

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