Fixed-Priority Tasks with Arbitrary Response Times (Section 6.6 of Liu)

- TDA scheduling condition valid only if each job of every task completes before the next job of that task is released.
- We now consider schedulability check for systems in which tasks may have relative deadlines larger than their periods.
  - **Note:** In this model, a task may have multiple ready jobs. We assume they are scheduled on a FIFO basis.

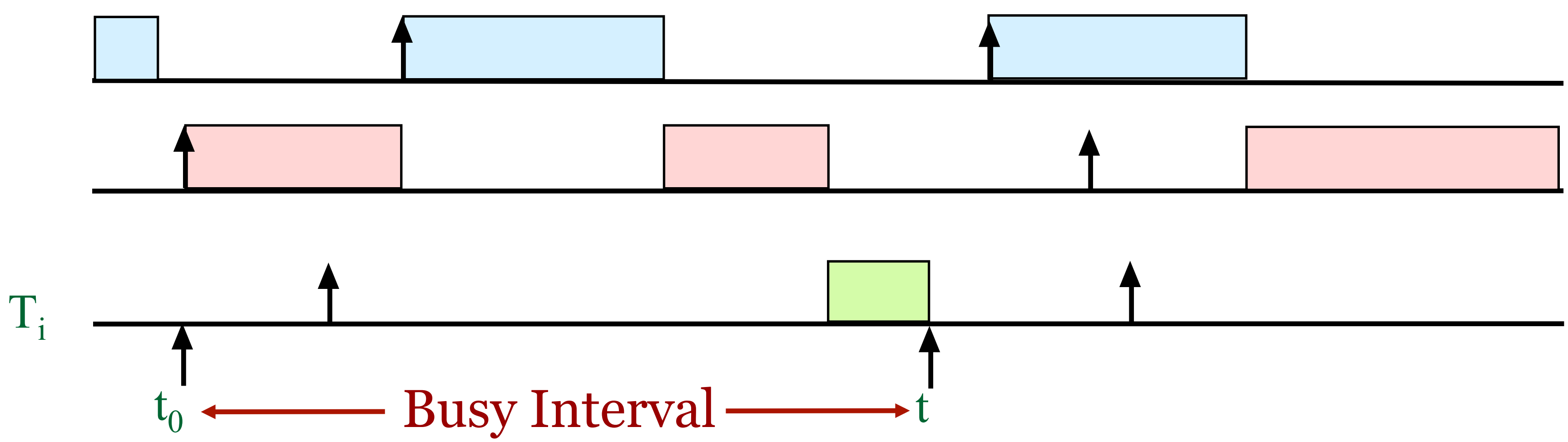
It Ain't So Easy …

- If relative deadline of every task is less than its period
  - "Critical instant" scenario can be used
  - i.e., consider only first job of each task in an in-phase system
- If relative deadlines of tasks can be larger than their periods
  - Response time of first job in in-phase system no longer guaranteed to be longest
  - Intuitive explanation
    - First job has interference only from higher-priority jobs
    - Second & subsequent jobs may also have interference from previous job of same task & hence longer response times

Busy Intervals

Definition: A **level-π_i busy interval** $(t_o, t]$ begins at an instant t_o when
(1) all jobs in $\mathbf{T_i}$ released before this instant have completed, and
(2) a job in $\mathbf{T_i}$ is released.
The interval ends at the first instant t after t_o when all jobs in $\mathbf{T_i}$ released since t_o are complete.

Example:



Busy Intervals (Continued)

Definition: We say that a level-π_i busy interval is **in phase** if the first jobs of all tasks that execute in the interval are released at the same time.

General TDA Method

Test tasks from highest-priority task (T_1) to lowest. When considering T_i, assume that all the tasks are in phase and the first level-π_i busy interval begins at time zero.

Consider the subset $\mathbf{T_i}$ of tasks with priorities π_i or higher.
(i) If first job of every task in $\mathbf{T_i}$ completes by the end of the first period of that task

- Check whether the first job $J_{i,1}$ of T_i meets its deadline
- T_i is schedulable if $J_{i,1}$ completes in time
- Otherwise, T_i is not schedulable.

(ii) If (i) doesn't apply, then do the following.

(a) Compute length of in-phase level-π_i busy interval

- Solve equation $t = \sum_{k=1,\ldots,i} \lceil t/p_k \rceil e_k$ iteratively
- Start from $t^{(1)} = \sum_{k=1,\ldots,i} e_k$
- Continue until $t^{(l+1)} = t^{(l)}$ for some $l \geq 1$
- Solution $t^{(l)}$ is length of level-π_i busy interval

(b) Check schedulability

- Compute response times of all $\lceil t^{(l)}/p_i \rceil$ jobs of T_i in in-phase level-π_i busy interval
- T_i is schedulable if all of these jobs complete in time
- Otherwise T_i is not schedulable.

Computing Response Times

Lemma 6-6: The maximum response time $W_{i,j}$ of the j-th job of T_i in an in-phase level-π_i busy period is equal to the smallest value of t that satisfies the equation

$$t = w_{i,j}(t + (j-1)p_i) - (j-1)p_i$$

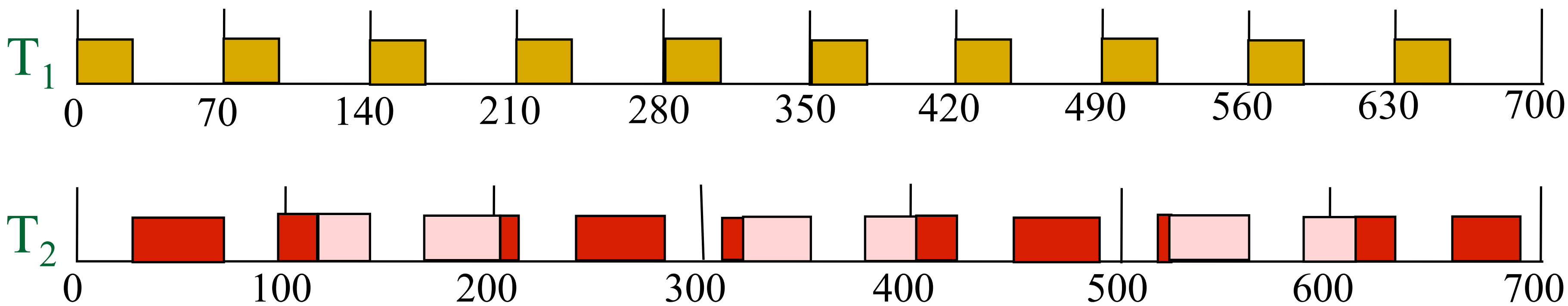
where

$$w_{i,j}(t) = je_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{p_k} \right\rceil \times e_k \qquad \text{for } (j-1)p_i < t \le w_{i,j}(t)$$

The recurrence given in the lemma can be solved iteratively, as described before.

Example

Consider: $T_1 = (70, 26)$, $T_2 = (100, 62, 120)$
Here's a schedule:



T_2's seven jobs have the following response times, respectively: 114, 102, 116, 104, 118, 106, 94.

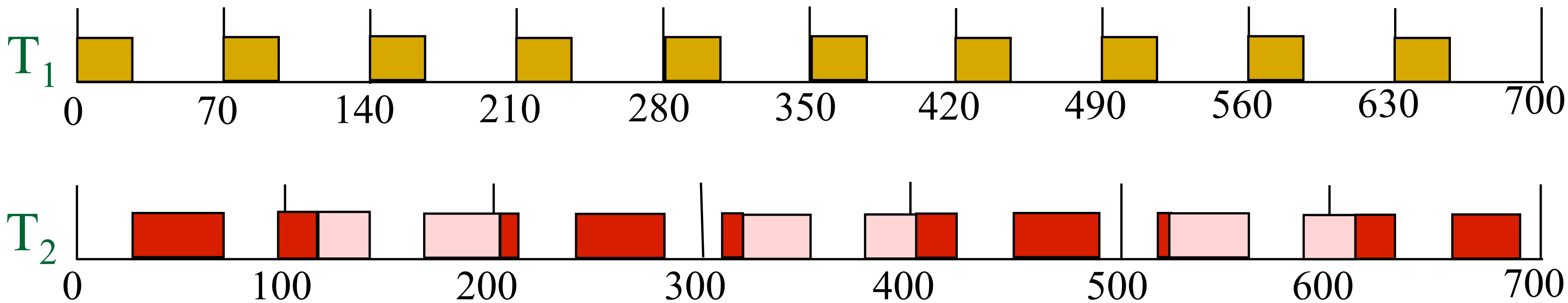
Note that the first job's response time is not the longest.

Bottom Line: We have to consider **all** jobs in an in-phase busy interval.

Example

Let's apply Lemma 6-6 to our previous example:

$T_1 = (70, 26), T_2 = (100, 62, 120)$



$$W_{2,1} = \text{minimum } t \text{ s.t.}$$

$$t = w_{2,1}(t)$$

$$= e_2 + \sum_{k=1}^{i-1} \left\lceil \frac{t}{p_k} \right\rceil \times e_k$$

$$= 62 + \left\lceil \frac{t}{70} \right\rceil \times 26$$

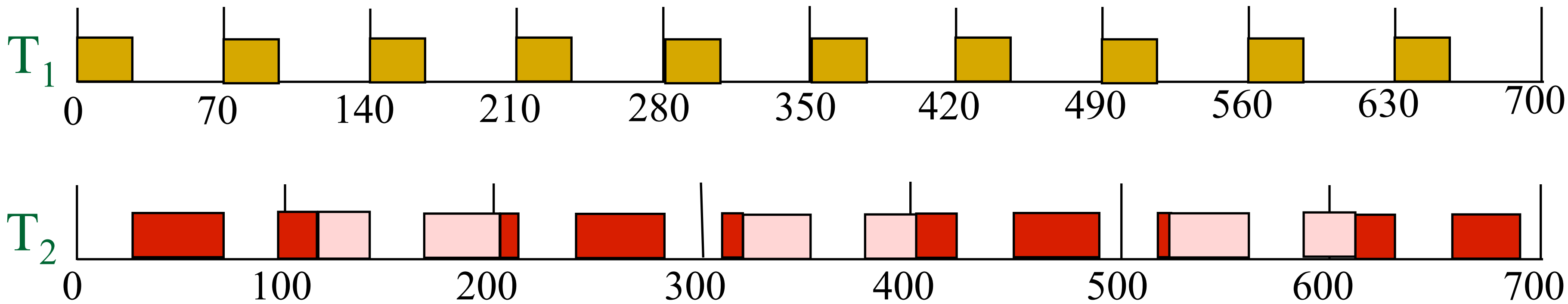
$$??114 = 62 + \left\lceil \frac{114}{70} \right\rceil \times 26$$

$$= 62 + 2 \times 26$$

$$= 114 \quad \text{Yes!}$$

Example (Continued)

$T_1 = (70, 26), T_2 = (100, 62)$



$$W_{2,2} = \text{minimum } t \text{ s.t.}$$

$$t = w_{2,2}(t + p_2) - p_2$$

$$= 2 \times e_2 + \sum_{k=1}^{i-1} \left\lceil \frac{t+100}{p_k} \right\rceil \times e_k - 100$$

$$= 124 + \left\lceil \frac{t+100}{70} \right\rceil \times 26 - 100$$

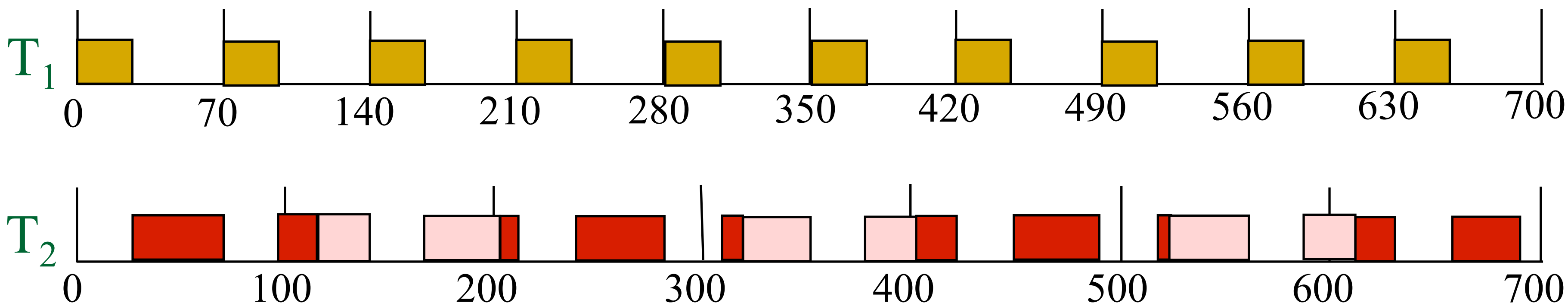
$$?? \; 102 = 124 + \left\lceil \frac{202}{70} \right\rceil \times 26 - 100$$

$$= 124 + 3 \times 26 - 100$$

$$= 102 \quad \text{Yes!}$$

Example (Continued)

$T_1 = (70, 26), T_2 = (100, 62)$



$$W_{2,3} = \text{minimum } t \text{ s.t.}$$

$$t = w_{2,3}(t + 2 \times p_2) - 2 \times p_2$$

$$= 3 \times e_2 + \sum_{k=1}^{i-1} \left\lceil \frac{t+200}{p_k} \right\rceil \times e_k - 200$$

$$= 186 + \left\lceil \frac{t+200}{70} \right\rceil \times 26 - 200$$

$$??116 = 186 + \left\lceil \frac{316}{70} \right\rceil \times 26 - 200$$

$$= 186 + 5 \times 26 - 200$$

$$= 116 \quad \text{Yes!}$$