Assignment - 6

1. (a) $x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$

	Assumpⁿ
$P(x_2) \Rightarrow P(x_2)$	
$P(x_3/x_2) \Rightarrow P(x_3)$	$x_2 \perp x_3$
$P(x_4/x_2,x_3) \Rightarrow P(x_4)$	$x_2, x_3 \perp x_4$
$P(x_5/x_2,x_3,x_4) \Rightarrow P(x_5/x_2,x_3)$	$x_5 \perp x_4 / x_2, x_3$
$P(x_6/x_2,x_3,x_4,x_5)$	$x_6 \perp x_2, x_5 / x_3, x_4$
$P(x_7/x_2,x_3,x_4,x_5,x_6)$	$x_7 \perp x_2, x_3, x_4, x_6 / x_5$
$P(x_8/x_2,x_3,x_4,x_5,x_6,x_7)$	$x_8 \perp x_2, x_4, x_7 / x_3, x_5, x_6$
$P(x_9/x_2,x_3,x_4,x_5,x_6,x_7,x_8)$	$x_9 \perp x_2, x_3, x_4, x_6 / x_5, x_7, x_8$

According Bayesian Network

$P(x_2)$
$P(x_3)$
$P(x_4)$
$P(x_5/x_2,x_3)$
$P(x_6/x_3,x_4)$
$P(x_7/x_5)$
$P(x_8/x_3,x_5,x_6)$
$P(x_9/x_5,x_7,x_8)$

(b) i) Formula $\rightarrow n! - 1$ ∴ Solⁿ $\rightarrow 9! - 1$

ii) $x_2 \rightarrow 1$ independent.
$x_3 \rightarrow 2$ independent.
$x_4 \rightarrow 3$ independent.
$x_5 \rightarrow (3 \times 2)(4) \leftarrow$ for x_5.
$\rightarrow 24$ independent
$x_6 \rightarrow (4 \times 3)(5) \rightarrow 60$ independent.
$x_7 \rightarrow 5 \times 6 \rightarrow 30$ independent.
$x_8 \rightarrow 6 \times 5 \times 3 \times 7 \rightarrow 630$ independent.
$x_9 \rightarrow 8 \times 7 \times 5 \times 9 \rightarrow 2240$ independent

So finally 2990 independent

c) ① T
② F
③ T
④ F
⑤ T

Q.2] a) P(B) A, C, D

$P(A)\,P(B/A)\,P(C/B)\,P(D/C)$

$$\sum_A P(A)\,P(B/A)\sum_C P(C/B)\sum_D P(D/C)$$

c	$f_1(c)$
t	$0.82 + 0.18 = 1$
f	$0.37 + 0.63 = 1$

$$\sum_A P(A)\,P(B/A)\underbrace{\sum_C P(C/B)\,f_1(c)}_{f_2(B)}$$

B	f(B)
t	$0.7 \times 1 + 0.3 \times 1 = 1$
f	$0.4 \times 1 + 0.6 \times 1 = 1$

$$\sum_A P(A)\, P(B/A)\, f(B)$$

T

$f_3(B)$

B	$f_3(B)$
t	$0.4 * 0.1 + 0.6 \times 0.3 = 0.52$
f	$0.4 \times 0.9 + 0.6 \times 0.2 = 0.48$

b) $P(C/A=T)$

Eliminating B & D

$$\therefore P(C/A=T) = P(A=T) \sum_B P(B/A=T)\, P(C/B) \sum_D P(D/C)$$

$$f_1(C) = \sum_D P(D/C)$$

C	$f_1(C)$
T	$0.7 + 0.3 = 1$
f	$0.4 + 0.6 = 1$

$$P(C/A=T) = P(A=T) \sum_B P(B/A=T)\, P(C/B) \cdot 1$$

$$f_2(A,C) = \sum_B P(B/A=T)\, P(C/B)$$

C	$P(B/A=T)P(C/B)$
T	$0.1 \times 0.7 + 0.9 \times 0.4 = 0.43$
F	$0.1 \times 0.3 + 0.9 \times 0.6 = 0.57$

(4)

$P(C/A=T) = P(A=T) \cdot f_2(A,C)$

C	$P(A=T) \cdot f_2(A,C)$
t	$0.4 \times 0.43 = 0.172$
f	$0.4 \times 0.57 = 0.228$

Now, $0.228 + 0.172 = 0.40$

After Normalizing we get the following

$P(C/A=T)$	C	$P(C/A=T)$
	T	$0.172/0.40 = 0.43$
	F	$0.228/0.40 = 0.57$

~~Q3~~ ~~Given~~ ~~Decision~~

c) $P(A, B/C=T; D=F)$

∴ Any variable cannot be eliminated.

Hence,

$$P(A, B/C=T, D=F) = \frac{P(A, B, C, D)}{P(C=T, D=F)}$$

i) $P(A, B, C=T, D=F)$

A	B	$P(A, B, C=T, D=F)$
T	T	$0.4 \times 0.1 \times 0.7 \times 0.18 = 0.00504$
T	F	$0.4 \times 0.9 \times 0.4 \times 0.18 = 0.02592$
F	T	$0.6 \times 0.8 \times 0.7 \times 0.18 = 0.06048$
F	F	$0.6 \times 0.2 \times 0.4 \times 0.18 = 0.00864$

So these are final values for each

2) $P(C=T, D=F)$

By eliminating the A & B

$P(A) = P(B/A) \cdot P(C/B) \cdot P(D/C)$

$P(C=T, D=F) = P(D/C) \sum_A P(A) \sum_B P(B/A) P(C/B)$

$= P(D=F/C=T) \sum_A P(A) \sum_B P(B/A) P(C=T/B)$

$F_1(A, C)$.

A	$P(B/A)\,P(C=T/B)$
T	$0.1 \times 0.7 + 0.9 \times 0.4 = 0.07 + 0.36 = 0.43$
F	$0.8 \times 0.7 + 0.2 \times 0.4 = 0.69$.

$P(C=T, D=F) = P(D=F/C=T) \sum P(A) \cdot f_1(A, C)$

$f_2(C)$.

$f_2(C) = 0.43 \times 0.4 + 0.64 \times 0.6 \Rightarrow 0.172 + 0.384$

$= 0.172 + 0.384$

$= 0.556$.

$P(C=T, D=F) = P(D=F/C=T) \cdot f_2(C)$

$P(C=T, D=F) = 0.556 \times 0.18$

$= 0.10008$

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∴, By all computations,

$P(A, B/C=T, D=F)$

A	B	$P(A, B/C=T, D=F)$
T	T	$0.00504/0.10008 = 0.05036$
T	F	$0.07592/0.10008 = 0.25899$
F	T	$0.06048/0.10008 = 0.60432$
F	F	$0.00864/0.10008 = 0.08633$

This is final value of $P(A, B/C=T, D=F$

9.3] a) Given Decision Network:-

we get following,

~~Action~~

Y	Action	U(Y, Action)
T	a	800
T	~a	400
F	a	200
F	~a	1000

a) what action should be taken.

is it P(y)=?

$P(x, y, z) = P(x)\, P(y|x)\, P(z|y)$

$\sum_x P(x)\, P(y|x) \sum_z P(z|y)$.

So,

Y	$f_1(y)$
T	0.9 + 0.1 = 1
F	0.2 + 0.8 = 1

$\Rightarrow \sum_x P(x)\, P(y|x)\, f_1(y)$.

3)

x	y	f2(y)	
T	T	0.4×0.7 = 0.28	> F
T	F	0.6×0.7 = 0.42	
F	T	0.4×0.3 = 0.12	> F
F	F	0.6×0.3 = 0.18	

Y	P(Y)
T	0.5
F	0.5

$u(a) = 0.5 \times 500 + 0.5 \times 200 = 500$

$u(\neg a) = 0.5 \times 400 + 0.5 \times 1000 = 700$

Expected Utility:-

Action a is 500

$\neg a$ is 700.

Since $u(\neg a) > u(a)$, the action that is taken is expected as '$\neg a$' with

MEU = 700

Q.3.

b) Value of info Z?

For z, we need P(z).

$$\sum_x P(x) \sum_y P(y/x) P(z/y)$$

X	Z	$f_1(x,z)$
T	T	0.2×0.9 + 0.8×0.2 = 0.34
T	F	0.2×0.1 + 0.8×0.8 = 0.66
F	T	0.7×0.9 + 0.3×0.2 = 0.69
F	F	0.7×0.1 + 0.3×0.8 = 0.31.

⇒ implies $\sum_x P(x) f_1(x,z)$

X	Z	$f_2(Z)$	
T	T	0.34×0.4 = 0.136	T
T	F	0.66×0.4 = 0.264	
F	T	0.69×0.6 = 0.414	F
F	F	0.31×0.6 = 0.186.	

Z	P(Z)
T	0.55
F	0.45

P(Y/Z = T)

Y		P(Y/Z = T)
T	0.5 x 0.9 = 0.45/.55	0.82
F	0.5 x 0.2 = 0.10/0.55	0.18.

u(a) = 0.82 x 800 + 0.18 x 200 = 692
u(~a) = 0.82 x 400 + 0.18 x 1000 = 508.

∴ action is a.

P(Y/Z = F) -

Y		P(Y/Z = F)
T	0.5 x 0.1 = 0.05 /0.45	0.11
F	0.5 x 0.8 = 0.40/0.45	0.89.

u(a) = 0.11 x 800 + 0.89 x 200 = 266
u(~a) = 0.11 x 400 + 0.89 x 1000 = 934.

MEU before Z = 700 — (i)
Z = T = 692. — (ii)
Z = F = 934. — (iii)

$\therefore$ VOI(z) = 0.55 × (i) + 0.45 × (ii) } — (i)

$= (0.55 \times 692) + (0.45 \times 934) - 700$

VOI(z) = 1009.

c) Value of Info of X. (VOI)

VOI (x) = P(X = T) . P(MEU / X = T) + P(X = F)

P(MEU / X = F) − P(MEU)

BN, P(X) · P(Y/X) · P(Z/Y).

Eliminate X/Z.

$\sum_x$ P(X = T) P(Y/X = T) $\sum_z$ P(Z/Y).

$\sum_x$ P(X = T) P(Y/X = T)

$f_1(y)$

Y	P(X = T) P(Y/X = T).
T	0.4 × 0.2 = 0.08
F	0.4 × 0.8 = 0.32.

Normalize

Y	f_1(y)
T	0.08 / 0.4 = 0.2
F	0.32 / 0.4 = 0.8.

1) If a.

∴ a = 0.2 × 400 + 0.8 × 1000 = 880.

2) If we consider a,
a = 0.2 × 800 + 0.8 × 200 = 320.

So P(MEU / X=T) = 880.
∴ a.

X = F

$$\sum_{x} P(x=F)\, P(Y/x=F)$$

Y	P(x=F)P(Y/x=F)
T	0.6 × 0.7 = 0.42.
F	0.6 × 0.3 = 0.18.

∴ Normalize

Y	P(X=F) P(Y/X=F)
T	0.42/0.6 = 0.7
F	0.18/0.6 = 0.3.

∴ a = 0.7 × 400 + 0.3 × 1000
= 580.

for a = 0.7 × 800 + 0.3 × 700.
= 560 + 60
= 620.

P(MEU / x = F) = 620.
∴ a.

VOI(X) = 880 × 0.4 + 620 × 0.6 = 24.

Q-3] d) Given Z = T.

MEU = 692. as per 3)b.

VOI (X / Z = T) → ?.

∴ i) For action a.

P(Y / Z = T, X = T, action = a).

P(Y / Z = T, X = T).

$$P(Y/Z=T, X=T) = \frac{\sum_{x} \sum_{z} P(x=T, Y, Z=T)}{}$$

= P(X = T) P(Y / X = T) P(Z = T / Y)

Y	$P(Y/Z=T, X=T) = P(X=T)P(Y/X=T)P(Z=T/Y)$	
T	$0.9\times0.2\times0.4 = 0.072$	0.529
T	$0.9\times0.8\times0.1 = 0.064$	0.471

After Normalization.

EU =>

Y	$P(Y/Z=T, X=T)\,U(Y, Action=a)$
T	$0.529\times800 = 423.2$
F	$0.471\times200 = 94.2$

} 517.4

ii) For ¬a.

EU =>

Y	$P(Y/Z=T, X=T)\,U(Y, Action=\neg a)$
T	$0.529\times400 = 211.6$
F	$0.471\times1000 = 471$

} 682.6

MEU when X = F

i) For Action = a.

$P(Y/Z=T, X=F)\,a) = P(Y/Z=T, X=F)$

$P(Y/Z=T, X=F) = \sum_x \sum_z P(X=F, Y/Z=T)$

$= P(X=F)\,P(Y/X=F)\,P(Z=T/Y)$

Y	$P(Y/Z=T, X=F) = P(X=F)\,P(Y/X=F)\,P(Z=T/Y)$	
T	$0.6 \times 0.7 \times 0.9 = 0.378$	0.913
F	$0.6 \times 0.3 \times 0.2 = 0.036$	0.087

N

EU =>

Y	$P(Y/Z=T, X=F)\,U(Y, \text{No})$	
T	$0.913 \times 800 = 730.4$	747.8
F	$0.087 \times 200 = 17.4$	

MEU when X=F.

X	$F_1(x)$
T	$0.2 \times 0.9 + 0.8 \times 0.2 = 0.34$
F	$0.7 \times 0.9 + 0.3 \times 0.2 = 0.69$.

X	$P(x/2=T) = P(x) \cdot F_2(x)$
T	$0.4 \times 0.34 = 0.136$
F	$0.6 \times 0.69 = 0.414$

$\{0.25, 0.75\}$

$\downarrow$

N^m_2

$VOI(X_2(2=T) = 0.25 \times 682.57 + 0.75 \times 7 \times 7.8 - 68$

$= 39.5$