Project: Kinematics Pick & Place

Rubric Points

Here I will consider the rubric points individually and describe how I addressed each point in my implementation.

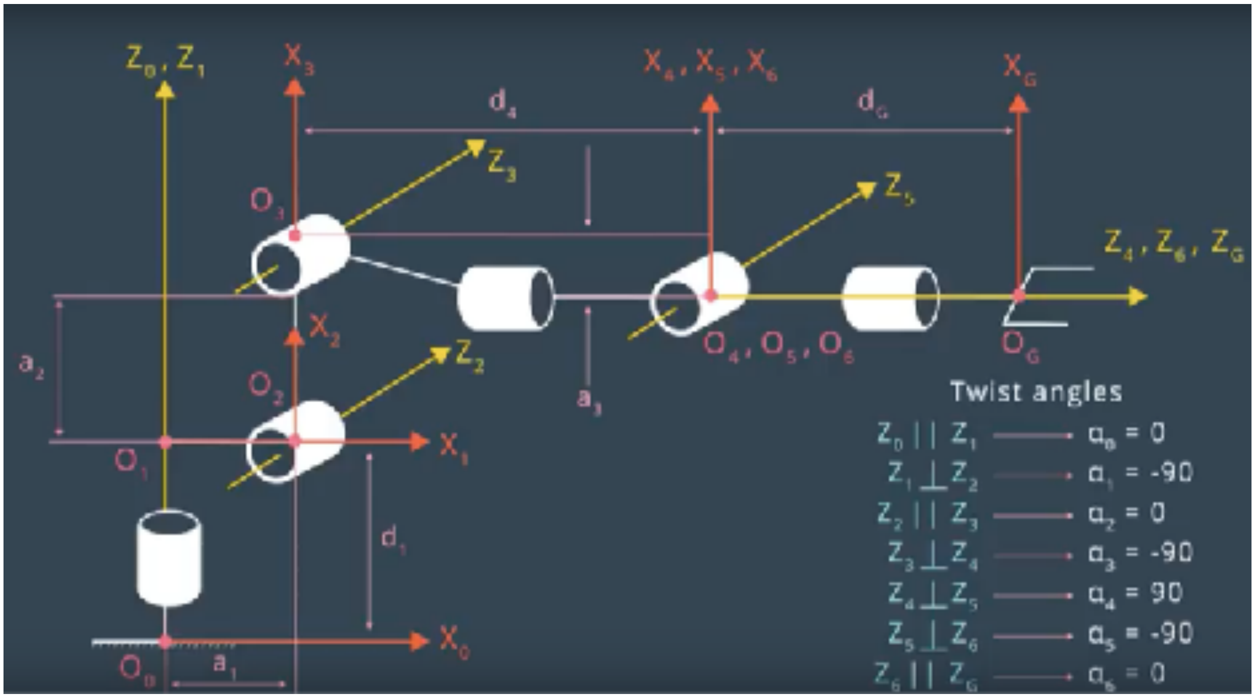
Writeup / README

1. Provide a Writeup / README that includes all the rubric points and how you addressed each one. You can submit your writeup as markdown or pdf.
You're reading it!

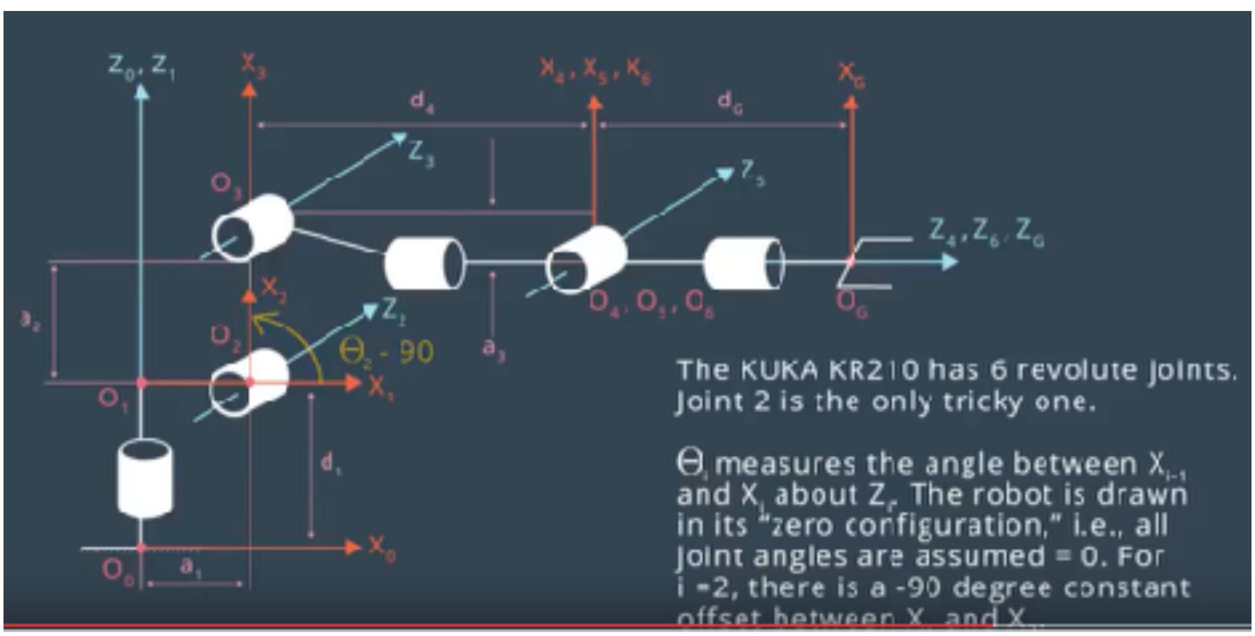
Kinematic Analysis

1. Run the forward_kinematics demo and evaluate the kr210.urdf.xacro file to perform kinematic analysis of Kuka KR210 robot and derive its DH parameters.

DH Parameters



1/ The twist angles alpha is angle between z_i-1 and z_i measured about x_i-1 in (+ in anti-clockwise direction)
2/ The link length a_i-1 is the distance from z_i-1 to z_i
3/ The link offset d_i is the signed distance between x_i-1 and x_i measured along z_i axis.
4/ The joint variable Qi is the angle between x_i-1 and x_i about zi



In zero configuration all angles are Qi angles are zero only X1 and X2 are are perpendicular and have and angle of -90 degree.

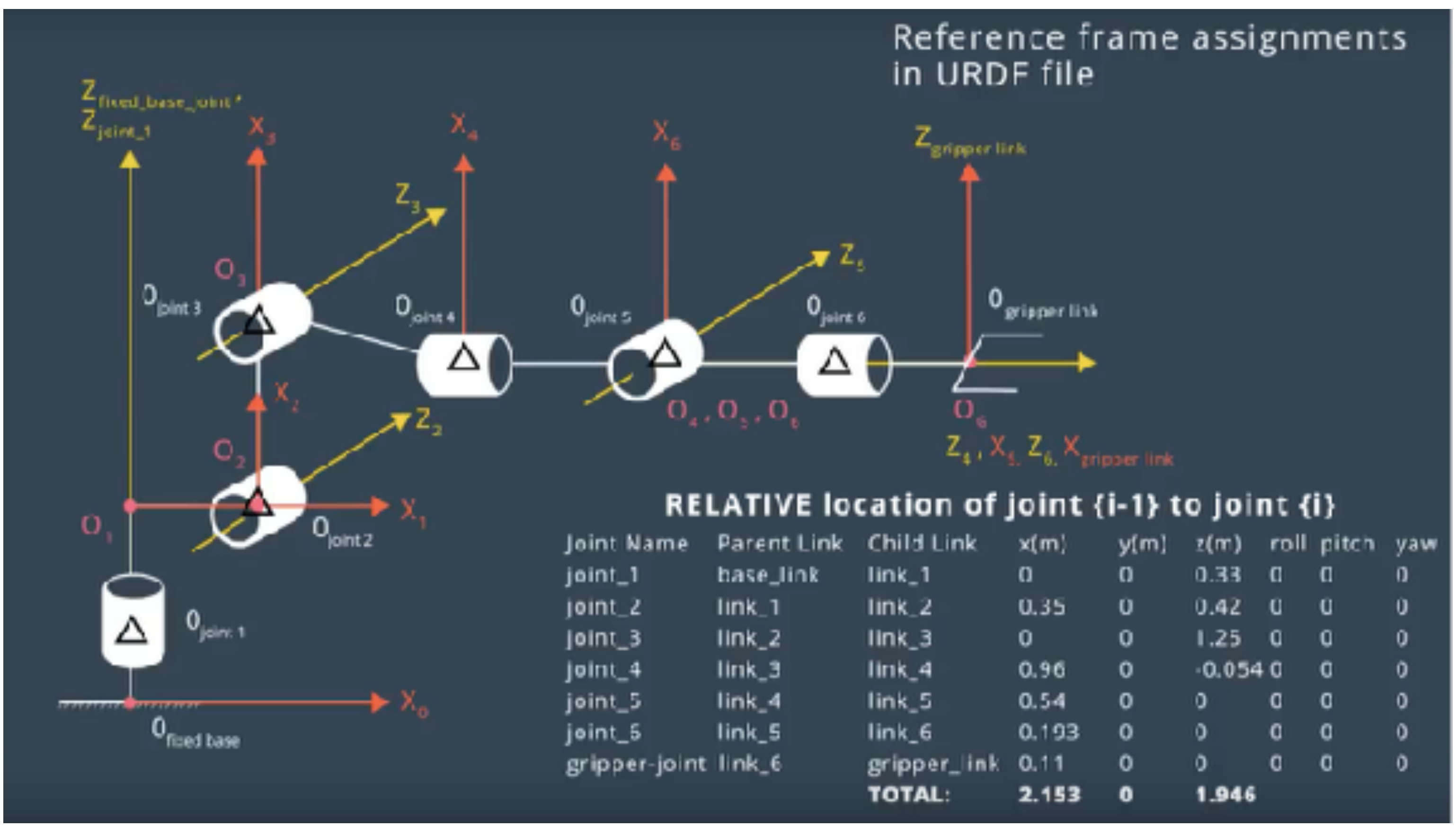
The URDF file was used to come up with numerical values of ‘a’ and ‘d’

From following lines we can see X, Y, Z of every joint w.r.t its parent link

```
<joint name="joint_1" type="revolute">
    <origin xyz="0 0 0.33" rpy="0 0 0"/>

```

The following snapshot shows how x,y and z for every joint can be collected from UDRDF file. Since these x,y,z are relative we need to add them as per our DH picture to come up with DH parameters



Joint Name	Parent Link	Child Link	x(m)	y(m)	z(m)	roll	pitch	yaw
joint_1	base_link	link_1	0	0	0.33	0	0	0
joint_2	link_1	link_2	0.35	0	0.42	0	0	0
joint_3	link_2	link_3	0	0	1.25	0	0	0
joint_4	link_3	link_4	0.96	0	-0.054	0	0	0
joint_5	link_4	link_5	0.54	0	0	0	0	0
joint_5	link_5	link_6	0.193	0	0	0	0	0
gripper-joint	link_6	gripper_link	0.11	0	0	0	0	0
		TOTAL:	2.153	0	1.946			

For example the d1 = distance from base_link to link_1 (z-axis)+ distance from link_1 to link_2(z-axis)

D1 = 0.33 + 0.42 = 0.75

A1 = distance of z2 to z1 along x-axis
A1 = 0.35

Joint	alp(i-1)	a(i-1)	Di	Qi
1	0	0	0.75	Q1
2	-pi/2	0.35	0	Q2-pi/2
3	0	1.25	0	Q3
4	-pi/2	-0.054	1.5	Q4
5	pi/2	0	0	Q5
6	-pi/2	0	0	Q6
G	0	0	0.303	Q7

2. Using the DH parameter table you derived earlier, create individual transformation matrices about each joint. In addition, also generate a generalized homogeneous transform between base_link and gripper_link using only end-effector(gripper) pose.

The transformation matrix is given as follows

```
TF =  [          cos(q),          -sin(q),         0,            a]
      [sin(q)*cos(alp), cos(q)*cos(alp), -sin(alp), -sin(alp)*d]
      [sin(q)*sin(alp), cos(q)*sin(alp),  cos(alp),  cos(alp)*d]
      [               0,               0,         0,            1]

T0_1:        [cos(q1), -sin(q1), 0,    0]
             [sin(q1),  cos(q1), 0,    0]
             [      0,        0, 1, 0.75]
             [      0,        0, 0,    1]

T1_2:        [sin(q2),  cos(q2), 0, 0.35]
             [      0,        0, 1,    0]
             [cos(q2), -sin(q2), 0,    0]
             [      0,        0, 0,    1]
```

```
T2_3:        [cos(q3), -sin(q3), 0, 1.25]
             [sin(q3),  cos(q3), 0,    0]
             [      0,        0, 1,    0]
             [      0,        0, 0,    1]
T3_4:        [ cos(q4),  -sin(q4), 0, -0.054]
             [       0,         0, 1,    1.5]
             [-sin(q4),  -cos(q4), 0,      0]
             [       0,         0, 0,      1]

T4_5:        [cos(q5), -sin(q5),  0, 0]
             [      0,        0, -1, 0]
             [sin(q5),  cos(q5),  0, 0]
             [      0,        0,  0, 1]

T5_6:        [ cos(q6), -sin(q6), 0, 0]
             [       0,        0, 1, 0]
             [-sin(q6), -cos(q6), 0, 0]
             [       0,        0, 0, 1]

T6_G:        [1, 0, 0,     0]
             [0, 1, 0,     0]
             [0, 0, 1, 0.303]
             [0, 0, 0,     1]
```

```
T0_G = Matrix([[((sin(q1)*sin(q4) + sin(q2 + q3)*cos(q1)*cos(q4))*cos(q5) +
sin(q5)*cos(q1)*cos(q2 + q3))*cos(q6) - (-sin(q1)*cos(q4) + sin(q4)*sin(q2 +
q3)*cos(q1))*sin(q6), -((sin(q1)*sin(q4) + sin(q2 +
q3)*cos(q1)*cos(q4))*cos(q5) + sin(q5)*cos(q1)*cos(q2 + q3))*sin(q6) +
(sin(q1)*cos(q4) - sin(q4)*sin(q2 + q3)*cos(q1))*cos(q6), -(sin(q1)*sin(q4) +
sin(q2 + q3)*cos(q1)*cos(q4))*sin(q5) + cos(q1)*cos(q5)*cos(q2 + q3),
-0.303*sin(q1)*sin(q4)*sin(q5) + 1.25*sin(q2)*cos(q1) - 0.303*sin(q5)*sin(q2
+ q3)*cos(q1)*cos(q4) - 0.054*sin(q2 + q3)*cos(q1) +
0.303*cos(q1)*cos(q5)*cos(q2 + q3) + 1.5*cos(q1)*cos(q2 + q3) +
0.35*cos(q1)], [((sin(q1)*sin(q2 + q3)*cos(q4) - sin(q4)*cos(q1))*cos(q5) +
sin(q1)*sin(q5)*cos(q2 + q3))*cos(q6) - (sin(q1)*sin(q4)*sin(q2 + q3) +
cos(q1)*cos(q4))*sin(q6), -((sin(q1)*sin(q2 + q3)*cos(q4) -
sin(q4)*cos(q1))*cos(q5) + sin(q1)*sin(q5)*cos(q2 + q3))*sin(q6) -
(sin(q1)*sin(q4)*sin(q2 + q3) + cos(q1)*cos(q4))*cos(q6), -(sin(q1)*sin(q2 +
q3)*cos(q4) - sin(q4)*cos(q1))*sin(q5) + sin(q1)*cos(q5)*cos(q2 + q3),
1.25*sin(q1)*sin(q2) - 0.303*sin(q1)*sin(q5)*sin(q2 + q3)*cos(q4) -
0.054*sin(q1)*sin(q2 + q3) + 0.303*sin(q1)*cos(q5)*cos(q2 + q3) +
1.5*sin(q1)*cos(q2 + q3) + 0.35*sin(q1) + 0.303*sin(q4)*sin(q5)*cos(q1)], [-
(sin(q5)*sin(q2 + q3) - cos(q4)*cos(q5)*cos(q2 + q3))*cos(q6) -
sin(q4)*sin(q6)*cos(q2 + q3), (sin(q5)*sin(q2 + q3) - cos(q4)*cos(q5)*cos(q2
+ q3))*sin(q6) - sin(q4)*cos(q6)*cos(q2 + q3), -sin(q5)*cos(q4)*cos(q2 + q3)
- sin(q2 + q3)*cos(q5), -0.303*sin(q5)*cos(q4)*cos(q2 + q3) - 0.303*sin(q2 +
q3)*cos(q5) - 1.5*sin(q2 + q3) + 1.25*cos(q2) - 0.054*cos(q2 + q3) + 0.75],
[0, 0, 0, 1]])
```

Generalized homogenous vector using end-effector pose

$$T = \left[\begin{array}{ccc|c} & & & P_x \\ & R_T & & P_y \\ & & & P_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

3. Decouple Inverse Kinematics problem into Inverse Position Kinematics and inverse Orientation Kinematics; doing so derive the equations to calculate all individual joint angles.

The last three joints of Kuka_arm are revolute joints, such a design is called a spherical wrist and the common point of intersection is called the wrist center. The advantage of such a design is that it kinematically decouples the position and orientation of the end effector. Mathematically, this means that instead of solving *twelve* nonlinear equations simultaneously (one equation for each term in the first three rows of the overall homogeneous transform matrix), it is now possible to independently solve two simpler problems: first, the Cartesian coordinates of the wrist center, and then the composition of rotations to orient the end effector. Physically speaking, a six degree of freedom serial manipulator with a spherical wrist would use the first three joints to control the *position* of the wrist center while the last three joints would orient the end effector as needed.

The end effector matrix looks like follows

$$\begin{bmatrix} l_x & m_x & n_x & p_x \\ l_y & m_y & n_y & p_y \\ l_z & m_z & n_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step1: We can calculate wrist centre position using following equation

$$w_x = p_x - (d_6 + l) \cdot n_x$$
$$w_y = p_y - (d_6 + l) \cdot n_y$$
$$w_z = p_z - (d_6 + l) \cdot n_z$$

Where,
Px, Py, Pz = end-effector positions
Wx, Wy, Wz = wrist positions
d6 = from DH table

l = end-effector length

We have to calculate nx, ny, and nz to substitute in above equation to get wrist centre.

We know roll, pitch and yaw of end effector from ROS by converting quaternions

Those can be substituted in following equation to get Rrpy

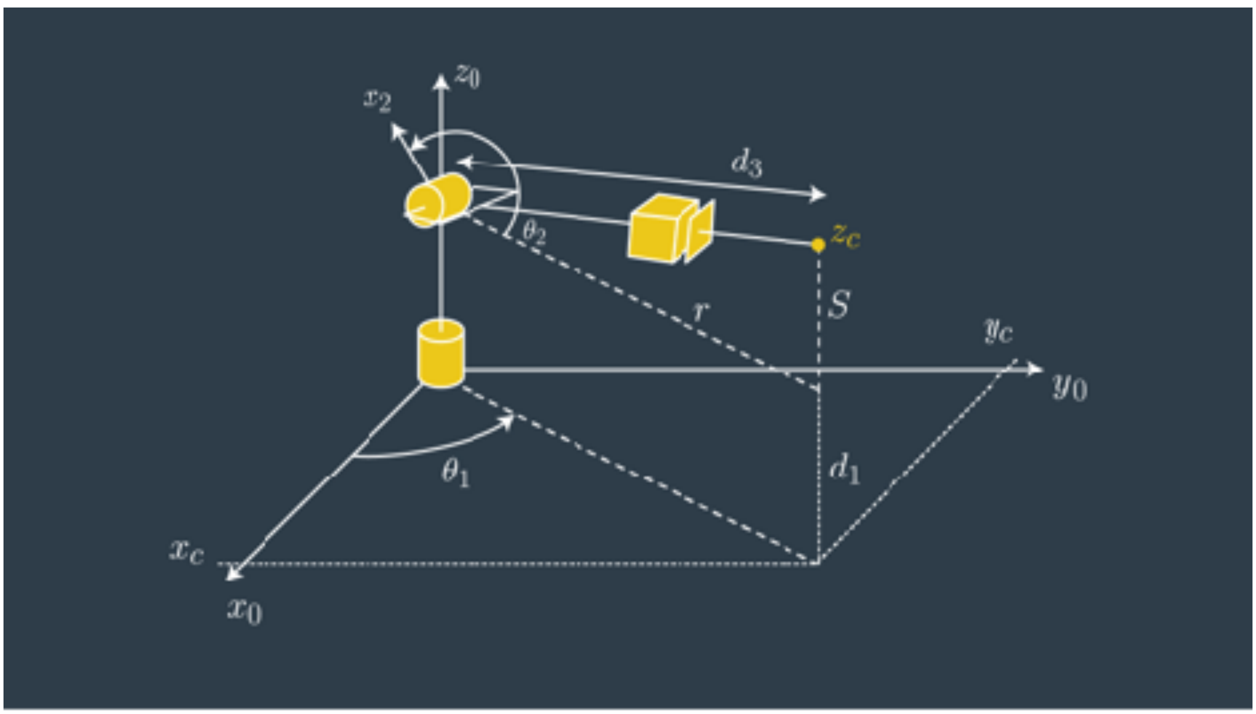
```
Rrpy = Rot(Z, yaw) * Rot(Y, pitch) * Rot(X, roll) * R_corr
```

We can extract nx, ny, nz from Rrpy matrix

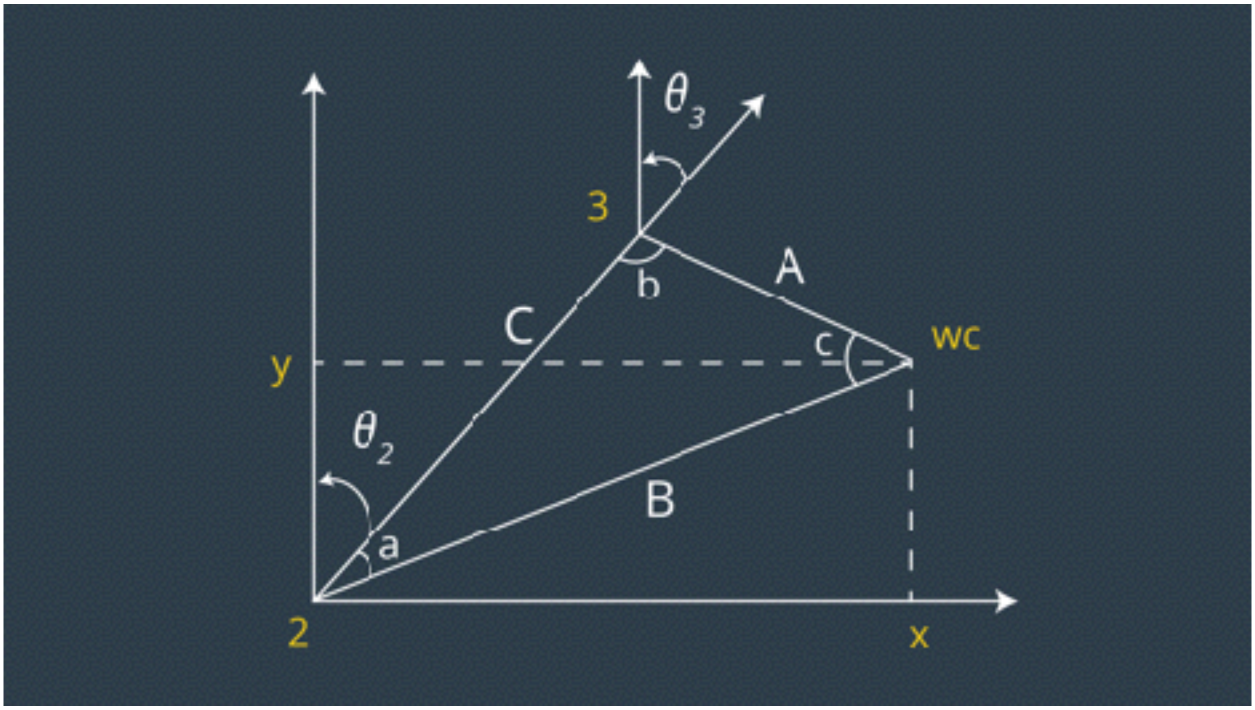
This will give us wx, wy, wz which represents wrist centre

Step2: Calculate first three angles from wrist centre

```
theta1 = atan2(wy, wx) as shown in following picture
```



We will use below picture to get theta2 and theta3




```
side_a = 1.50
r = sqrt(wx*wx+wy*wy) - 0.35 # a1: 0.35
side_b = sqrt((r*r) + pow(wz - 0.75, 2))
side_c = 1.25

angle_a = acos((side_b * side_b + side_c * side_c - side_a * side_a) /
(2 * side_b * side_c))

angle_b = acos((side_a * side_a + side_c * side_c - side_b * side_b) /
(2 * side_a * side_c))

angle_c = acos((side_a * side_a + side_b * side_b - side_c * side_c) /
(2 * side_a * side_b))


theta2 = pi/2 - angle_a - atan2(wz - 0.75, r)
theta3 = pi/2 - (angle_b + 0.036) # 0.036 is for sag in link4
```

Step3: Calculate last three angles which is Inverse orientation

The resultant rotation is of the following form f

```
R0_6 = R0_1*R1_2*R2_3*R3_4*R4_5*R5_6
```

Since the overall RPY (**R**oll **P**itch **Y**aw) rotation between base_link and gripper_link must be equal to the product of individual rotations between respective links, following holds true:

```
R0_6 = Rrpy
```

where,
Rrpy = Homogeneous RPY rotation between base_link and gripper_link as calculated above.
We can substitute the values we calculated for joints 1 to 3 in their respective individual rotation matrices and pre-multiply both sides of the above equation by **inv(R0_3)** which leads to:

```
R3_6 = inv(R0_3) * Rrpy
```

```
theta4 = atan2(R3_6[2,2], -R3_6[0,2])
theta5 = atan2(sqrt(R3_6[0,2]*R3_6[0,2] + R3_6[2,2]*R3_6[2,2]),
R3_6[1,2])
theta6 = atan2(-R3_6[1,1], R3_6[1,0])
```

Project Implementation

1. Fill in the IK_server.py file with properly commented python code for calculating Inverse Kinematics based on previously performed Kinematic Analysis. Your code must guide the robot to successfully complete 8/10 pick and place

cycles. Briefly discuss the code you implemented and your results.

Step1: DH parameters

```
        # Create Modified DH parameters
        s = {alp0:     0, a0:       0, d1:   0.75, q1: q1,
             alp1: -pi/2, a1:    0.35, d2:      0, q2: q2-pi/2,
             alp2:     0, a2:    1.25, d3:      0, q3: q3,
            }
            ##Not needed## alp3: -pi/2, a3:  -0.054, d4:   1.50, q4: q4,
            ##Not needed## alp4:  pi/2, a4:       0, d5:      0, q5: q5,
            ##Not needed## alp5: -pi/2, a5:       0, d6:      0, q6: q6,
            ##Not needed## alp6:     0, a6:       0, d7:  0.303, q7: 0}

```

Step2: Individual Transform matrix

```
        # Define Modified DH Transformation matrix
        def TF_Matrix(alp, a, d, q):
             TF = Matrix([[         cos(q),         -sin(q),         0,          a],
                          [sin(q)*cos(alp), cos(q)*cos(alp), -sin(alp), -sin(alp)*d],
                          [sin(q)*sin(alp), cos(q)*sin(alp),  cos(alp),  cos(alp)*d],
                          [              0,               0,         0,          1]])
              return TF


         # Create individual transformation matrices
         T0_1 = TF_Matrix(alp0, a0, d1, q1).subs(s)
         T1_2 = TF_Matrix(alp1, a1, d2, q2).subs(s)
         T2_3 = TF_Matrix(alp2, a2, d3, q3).subs(s)
         ##Not needed ##T3_4 = TF_Matrix(alp3, a3, d4, q4).subs(s)
         ##Not needed ##T4_5 = TF_Matrix(alp4, a4, d5, q5).subs(s)
         ##Not needed ##T5_6 = TF_Matrix(alp5, a5, d6, q6).subs(s)
```

```
        ##Not needed ##T6_G = TF_Matrix(alp6, a6, d7, q7).subs(s)

```

Step3: Rotation matrix for end effector with correction between DH and URDF

```
        r, p, y = symbols('r p y')
        ROT_x = Matrix([[1,       0,        0],
                        [0, cos(r), -sin(r)],
                        [0, sin(r),  cos(r)]])

        ROT_y = Matrix([[cos(p),  0,  sin(p)],
                        [0,       1,       0],
                        [-sin(p), 0,  cos(p)]])

        ROT_z = Matrix([[cos(y), -sin(y),  0],
                        [sin(y),  cos(y),  0],
                        [     0,       0,  1]])


        R0_3 = simplify(T0_1[0:3,0:3] * T1_2[0:3,0:3] * T2_3[0:3,0:3])
        ROT_G = simplify(ROT_z * ROT_y * ROT_x)
        ROT_error = ROT_z.subs(y, radians(180)) * ROT_y.subs(p, radians(-90))
        ROT_G = simplify(ROT_G * ROT_error)
```

Step4: Wrist centre from end effector pose

```
            wx = px - 0.303 * ROT_G_eval[0,2]
            wy = py - 0.303 * ROT_G_eval[1,2]
            wz = pz - 0.303 * ROT_G_eval[2,2]
```

Step5: Calculate first three angles

```
            # Calculate joint angles using Geometric IK method
            theta1 = atan2(wy, wx)
            side_a = 1.50
            r = sqrt(wx*wx+wy*wy) - 0.35 # a1: 0.35
            side_b = sqrt((r*r) + pow(wz - 0.75, 2))
            side_c = 1.25

            angle_a = acos((side_b * side_b + side_c * side_c - side_a * side_a) / (2 * side_b * side_c))
```

```
        angle_b = acos((side_a * side_a + side_c * side_c - side_b * side_b) / (2 * side_a * side_c))
        angle_c = acos((side_a * side_a + side_b * side_b - side_c * side_c) / (2 * side_a * side_b))


        theta2 = pi/2 - angle_a - atan2(wz - 0.75, r)
        theta3 = pi/2 - (angle_b + 0.036) # 0.036 is for sag in link4

```

Step6: Calculate last three angles

```
        R0_3_eval = R0_3.evalf(subs={'q1':theta1, 'q2':theta2, 'q3':theta3})

        R3_6 = R0_3_eval.inv("LU") * ROT_G_eval

        theta4 = atan2(R3_6[2,2], -R3_6[0,2])
        theta5 = atan2(sqrt(R3_6[0,2]*R3_6[0,2] + R3_6[2,2]*R3_6[2,2]), R3_6[1,2])
        theta6 = atan2(-R3_6[1,1], R3_6[1,0])

```

Pick and Place in action