

On average $(n + 1)/2$ elements need to be checked. Let A be a random variable which takes on values from 1 to n , and is equal to the number of comparisons needed to find the element. The probability distribution function $P(A)$ is uniform, thus we have :

$$P(A = i) = \frac{1}{n}$$

Thus, the expected value of A is :

$$\begin{aligned} E(A) &= \sum_{i=1}^n \frac{i}{n} \\ &= \frac{n(n+1)}{2n} \\ &= \frac{n+1}{2} \end{aligned}$$

For the worst case, ie. if the element is not present or is the last element in the array, we will obviously have n elements that need to be checked.

The time taken by each line in the linear search algorithm is as follows:

1. c_1
2. c_2k
3. c_3k
4. c_4 (since it is executed only once)
5. c_5

Where k is the number of checked performed by the algorithm. Thus the running time of the algorithm is $\Theta(k) = \Theta(n)$ both in the average and worst case.