

- a. (2, 1)(3, 1)(8, 6)(8, 1)(6, 1)
- b. The array $\langle n, n-1, \dots, 2, 1 \rangle$ has the most inversions. It has $n(n-1)/2$ of them.
- c. The running time of insertion-sort is $\Theta(n+i)$, where i is the number of inversions.
- d. We need to create two procedures:
 1. MERGE-COUNT, which accepts the same arguments as MERGE.
 2. MERGE-SORT-COUNT, which accepts the same arguments as MERGE-SORT.

The procedures are defined as follows.

MERGE-COUNT(A, p, q, r)

1. $n_L = q - p + 1$
2. $n_R = r - q$
3. let $L[0 : n_L - 1]$ and $R[0 : n_R - 1]$ be new arrays
4. **for** $i = 0$ **to** $n_L - 1$
5. $L[i] = A[p + i]$
6. **for** $j = 0$ **to** $n_R - 1$
7. $R[j] = A[q + j + 1]$
8. $i = 0$
9. $j = 0$
10. $k = p$
11. $inversions = 0$ // Added part
12. **while** $i < n_L$ and $j < n_R$
13. **if** $L[i] \leq R[j]$
14. $A[k] = L[i]$
15. $i = i + 1$
16. **else**
17. $A[k] = R[j]$
18. $j = j + 1$
19. $inversions = inversions + n_L - i$ // Added part
20. $k = k + 1$

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21. while  $i < n_L$ 
22.    $i = i + 1$ 
23.    $k = k + 1$ 
24. while  $j < n_R$ 
25.    $j = j + 1$ 
26.    $k = k + 1$ 
27. return  $inversions$  // Added part
    MERGE-SORT-COUNT( $A, p, r$ )
1. if  $p \geq r$ 
2.   return 0
3.  $q = \lfloor (p + r)/2 \rfloor$ 
4.  $inversions = 0$ 
5.  $inversions = inversions + \text{MERGE-SORT-COUNT}(A, p, q)$ 
6.  $inversions = inversions + \text{MERGE-SORT-COUNT}(A, q+1, r)$  // Count-
   ing inversions within the two parts.
7.  $inversions = inversions + \text{MERGE-COUNT}(A, p, q, r)$  // Counting in-
   versions between them.
8. return  $inversions$ 

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We then simply make a copy C of the array we want to sort, and run $\text{MERGE-SORT-COUNT}(C, 1, n)$ to get our answer.