

LINEAR-SEARCH(A, x, n)

1. $index = \text{NIL}$
2. **for** $i = 1$ **to** n
3. **if** $A[i] = x$
4. $index = i$
5. **return** $index$

Loop Invariant: After the i^{th} iteration of the for loop in lines 2-4, if the sub-array $A[1 : i]$ contains x , then $A[index] = x$, otherwise $index = \text{NIL}$.

Initialisation: Before the 1st iteration, x obviously cannot be in the sub-array $A[1 : 0]$, since it has no elements. Accordingly, $x = \text{NIL}$.

Maintenance: Before the i^{th} iteration, let us say $index = k \ni A[k] = x$ if x is present in the sub-array $A[1 : i - 1]$ and NIL if not. If $A[i] = x$, then after the i^{th} iteration, $index$ holds i . This satisfies our loop invariant, since it means $A[1 : i]$ contains x with $A[index] = A[i] = x$.

Now let us take the alternate case, if $A[i] \neq x$, then $index$ equals to k or NIL depending on if $A[1 : i - 1]$ contains x . This also satisfies our loop invariant, since if both $A[1 : i - 1]$ and $A[i]$ don't contain x , then $A[1 : i]$ doesn't contain x , and accordingly $index = \text{NIL}$. Otherwise, if $A[1 : i - 1]$ does contain x , then so does $A[1 : i]$, with $A[index] = A[k] = x$.

Termination: The for loop has i go from 1 to n , which means there are n iterations of the loop. Thus after the n^{th} iteration, if $A[1 : n]$ doesn't contain x $index = \text{NIL}$, otherwise $A[index] = x$.