a. It is $\Theta(n)$ because the for loop runs n times and performs a constant time operation each time. **b.** NAIVE-HORNER(A, n, x)

1. $p = 0$
2. **for** $i = 0$ **to** n
3. $\quad term = A[i]$
4. $\quad$ **for** $j = 1$ **to** i
5. $\quad\quad term = term \times x$
6. $\quad p = p + term$
7. **return** p

The running time of this algorithm is $\Theta(n^2)$. It is slower than HORNER.

c. Initialisation: Before the 1^{st} iteration, we have

$$p = 0 = \sum_{k=0}^{-1} A[n+1]$$

Maintenance: Let us suppose that the loop invariant is true before the iteration with $i = a + 1$. We have:

$$p = \sum_{k=0}^{n-a} A[k+a] \cdot x^k$$

Then, after the iteration, we have:

$$p = A[a+1] + x \cdot \sum_{k=0}^{n-a-2} A[k+a+2] \cdot x^k$$

$$= A[a+1] + \sum_{k=0}^{n-a-2} A[k+a+2] \cdot x^{k+1}$$

$$= A[a+1] + \sum_{k=1}^{n-a-1} A[k+a+1] \cdot x^k$$

$$= A[0+a+1] \cdot x^0 + \sum_{k=1}^{n-a-1} A[k+a+1] \cdot x^k$$

$$= \sum_{k=0}^{n-a-1} A[k+a+1] \cdot x^k$$

Combined with the fact that i is updated so that $i = a$ before the next iteration, we can see that the loop invariant is maintained with each loop.

Termination: The for loop goes on until i is updated such that $i = -1$, at which point, the loop stops before it can be executed. At which point, by the loop invariant, we have:

$$p = \sum_{k=0}^{n} A[n] \cdot x^k$$

Thus the correctness of the HORNER procedure is proven.