01.
First, matrices S_1 through S_{10} are made.

$$S_1 = B_{12} - B_{22} = 8 - 2 = (6)$$
$$S_2 = A_{11} + A_{12} = 1 + 3 = (4)$$
$$S_3 = A_{21} + A_{22} = 7 + 5 = (12)$$
$$S_4 = B_{21} - B_{11} = 4 - 6 = (-2)$$
$$S_5 = A_{11} + A_{22} = 1 + 5 = (6)$$
$$S_6 = B_{11} + B_{22} = 6 + 2 = (8)$$
$$S_7 = A_{12} - A_{22} = 3 - 5 = (-2)$$
$$S_8 = B_{21} + B_{22} = 4 + 2 = (6)$$
$$S_9 = A_{11} - A_{21} = 1 - 7 = (-6)$$
$$S_{10} = B_{11} + B_{12} = 6 + 8 = (14)$$

Then, matrices P_1 through P_7 are made.

$$P_1 = A_{11}S_1 = (1)(6) = (6)$$
$$P_2 = S_2B_{22} = (4)(2) = (8)$$
$$P_3 = S_3B_{11} = (12)(6) = (72)$$
$$P_4 = A_{22}S_4 = (5)(-2) = (-10)$$
$$P_5 = S_5S_6 = (6)(8) = (48)$$
$$P_6 = S_7S_8 = (-2)(6) = (-12)$$
$$P_7 = S_9S_{10} = (-6)(14) = (-84)$$

Finally, we perform the required additions

$$C_{11} = C_{11} + P_5 + P_4 - P_2 + P_6 = (0) + (48) + (-10) - (8) + (-12) = (18)$$
$$C_{12} = C_{12} + P_1 + P_2 = (0) + (6) + (8) = (14)$$
$$C_{21} = C_{21} + P_3 + P_4 = (0) + (72) + (-10) = (62)$$
$$C_{22} = C_{22} + P_5 + P_1 - P_3 - P_7 = (0) + (48) + (6) - (72) - (-84) = (66)$$

Thus the resultant is

$$\begin{pmatrix} 18 & 14 \\ 62 & 66 \end{pmatrix}$$

02.
STRASSEN-MULTIPLY(A, B, C, n)

1. **if** $n = 1$

2. $c_{11} = c_1 1 + a_{11} b_{11}$

3. **return**

4. partition A, B, C into $A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{12}, B_{21}, B_{22}, C_{11}, C_{12}, C_{21}, C_{22}$ each of size $n/2$

5. let $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}$ be matrices of size $n/2$ with all elements 0

6. MATRIX-SUBTRACT($B_{12}, B_{22}, S_1, n/2$)

7. MATRIX-ADD($A_{11}, A_{12}, S_2, n/2$)

8. MATRIX-ADD($A_{21}, B_{22}, S_3, n/2$)

9. MATRIX-SUBTRACT($B_{21}, B_{11}, S_4, n/2$)

10. MATRIX-ADD($A_{11}, A_{22}, S_5, n/2$)

11. MATRIX-ADD($B_{11}, B_{22}, S_6, n/2$)

12. MATRIX-SUBTRACT($A_{12}, A_{22}, S_8, n/2$)

13. MATRIX-ADD($B_{21}, B_{22}, S_8, n/2$)

14. MATRIX-SUBTRACT($A_{11}, A_{21}, S_9, n/2$)

15. MATRIX-ADD($B_{11}, B_{12}, S_{10}, n/2$)

16. let $P_1, P_2, P_3, P_4, P_5, P_6, P_7$ be matrices of size $n/2$ with all elements 0

17. STRASSEN-MULTIPLY($A_{11}, S_1, P_1, n/2$)

18. STRASSEN-MULTIPLY($S_2, B_{22}, P_2, n/2$)

19. STRASSEN-MULTIPLY($S_3, B_{11}, P_3, n/2$)

20. STRASSEN-MULTIPLY($A_{22}, S_4, P_4, n/2$)

21. STRASSEN-MULTIPLY($S_5, S_6, P_5, n/2$)

22. STRASSEN-MULTIPLY($S_7, S_8, P_6, n/2$)

23. STRASSEN-MULTIPLY($S_9, S_{10}, P_7, n/2$)

24. MATRIX-ADD($P_5, P_6, C_{11}, n/2$)

25. MATRIX-SUBTRACT($P_4, P_2, C_{11}, n/2$)

26. MATRIX-ADD($P_1, P_2, C_{12}, n/2$)

27. MATRIX-ADD($P_3, P_4, C_{21}, n/2$)

28. MATRIX-SUBTRACT($P_1, P_3, C_{11}, n/2$)

29. MATRIX-SUBTRACT($P_5, P_7, C_{11}, n/2$)

Try to write MATRIX-ADD and MATRIX-SUBTRACT yourself!

03.

With k multiplications, we can write a modified recursive equation for Strassen's algorithm.

$T(n) = kT(n/3) + \Theta(n^2)$

By expanding, you will find the solution to this recursion equation to be

$T(n) = n^{\log_3 k}$

For the running time of this algorithm to be $o(n^{\lg 7})$, we need

$$\log_3 k < \lg 7$$

$$\implies k < 3^{\lg 7}$$

$$\implies k < 21.85$$

Thus, the maximum value of k is 21.

04.

We have

$$\log_{70} 143640 < \log_{68} 132464 < \log_{72} 155424 < \lg 7$$

Thus, the method with the best asymptotic running time is the one which multiples 70×70 matrices, and it is even faster than Strassen's algorithm by a factor of $n^{0.0122322323093}$

05.

COMPLEX-MULTIPLY(a, b, c, d)

1. $P_1 = ac$
2. $P_2 = bd$
3. $P_3 = (a + b)(c + d)$
4. $R = P_1 - P_2$
5. $I = P3 - P_1 - P_2$
6. **return** $\langle R, I \rangle$

06 .

The algorithm can square a $2n \times 2n$ matrix in $\Theta(n^2)$ time as well, thus we can construct a $2n \times 2n$ matrix like this

$$\begin{pmatrix} A & B \\ 0 & 0 \end{pmatrix}$$

Squaring it gives

$$\begin{pmatrix} A^2 & AB \\ 0 & 0 \end{pmatrix}$$

Then, simply partitioning the matrix to gain its topright quarter gives our result.