01.

GENERALISED-MATRIX-MULTIPLY-RECURSIVE(A, B, C, n)

if $\lg n \in \mathbb{W}$

    MATRIX-MULTIPLY-RECURSIVE(A, B, C, n)

else

    GENERALISED-MATRIX-MULTIPLY-RECURSIVE$(A, B, C, n-1)$

    for $i = 1$ **to** n

        for $j = 1$ **to** n

            $c_{in} = c_{in} + a_{ij} b_{jn}$

    for $i = 1$ **to** $n - 1$

        for $j = 1$ **to** n

            $c_{ni} = c_{in} + a_{nj} b_{bji}$

The running time of this procedure can be written as follows

$$T(n) = \begin{cases} c_1 n^3 & \lg n \in \mathbb{W} \\ T(n-1) + c_3(n^2 + n(n-1)) & \lg n \notin \mathbb{W} \end{cases}$$

To show that this is still $\Theta(n^3)$, let us assume that we have

$$\begin{aligned} n &= 2^k - 1, k \in \mathbb{N} \\ &= 2^{k-1} + (2^{k-1} - 1) \\ &= (\frac{n+1}{2}) + (\frac{n-1}{2}) \end{aligned}$$

Thus, we will have to keep expanding the second case of the recursion until we get

$$\begin{aligned} T(n) &= T(\frac{n+1}{2}) + c_3 \sum_{k=(n-1/2)}^{n} 2k^2 - k \\ &= (\frac{n+1}{2})^3 + c_3 \sum_{k=(n-1/2)}^{n} 2k^2 - k \end{aligned}$$

The highest order term in this is still n^3, thus the the running time can still be characterised as $\Theta(n^3)$.

02.

For the first case, the answer is $\Theta(k^2 n^3)$, because the resultant matrix has $kn \times kn$ elements, which means you need to perform k^2 matrix multiplications. For the latter case however, the answer just $\Theta(kn^3)$ because the resultant matrix has only $n \times n$ elements, we just need to multiply each matrix of the row of k matrices with the respective element of the column of k matrices, which results in k multiplications.

Thus, the second case is asymptotically faster by a factor of k.

03.

It now takes $2n^2$ steps to partition the matrices instead of just 1. Thus the new recursion equation becomes

$$T(n) = 8T(n/2) + c_2 n^2$$

By repeatedly expanding this, we will get

$$\begin{aligned} T(n) &= 2^{3 \lg n} T(n/2^{\lg n}) + c_2(1 + 2 + 2^1 \ldots 2^{\lg(n)-1}) n^2 \\ &= c_1 n^3 + c_2(2^{lgn} - 1) n^2 \\ &= c_1 n^3 + c_2 n^3 - c_2 n^2 \\ &= \Theta(n^3) \end{aligned}$$

04.

MATRIX-ADD-RECURSIVE(A, B, C, n)

1. **if** $n = 1$
2. $\quad c_{11} = a_{11} + b_{11}$
3. $\quad$ **return**
4. partition A, B, C into $A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{12}, B_{21}, B_{22}, C_{11}, C_{12}, C_{21}, C_{22}$
5. MATRIX-ADD-RECURSIVE$(A_{11}, B_{11}, C_{11}, n/2)$
6. MATRIX-ADD-RECURSIVE$(A_{12}, B_{12}, C_{12}, n/2)$
7. MATRIX-ADD-RECURSIVE$(A_{21}, B_{21}, C_{21}, n/2)$
8. MATRIX-ADD-RECURSIVE$(A_{22}, B_{22}, C_{22}, n/2)$

The recurrence is:

$$T(n) = 4T(n/2) + c_2$$

The solution to this recurrence is $\Theta(n^2)$

In the other case, the recurrence is:

$$T(n) = 4T(n/2) + c_2 n^2$$

This recurrence expands to

$$T(n) = c_1 n^2 + c_2 n^2 \lg n$$

Thus the solution to this recurrence is $\Theta(n^2 \lg n)$