

To prove by induction, we only need to show that the inductive hypothesis  $T(n) = n \lg n$  holds true for the base case, and that it holds true recursively.

**Base Case:** Since  $n$  is given to be a power of 2, the base case is when  $n = 2$ . For this case, we have  $T(2) = 2 \lg 2 = 2 \times 1 = 2$

**Recursion Equation:** We know that for  $n > 2$

$$T(n) = 2T(n/2) + n$$

Since  $n$  is a power of 2, so is  $n/2$ . Now, assuming our inductive hypothesis is true for  $T(n/2)$ , we have

$$T(n) = 2T(n/2) + n$$

$$T(n) = 2(n/2)(\lg(n/2)) + n$$

$$T(n) = n(\lg(n) - 1) + n$$

$$T(n) = n(\lg(n) - n + n)$$

$$T(n) = n \lg(n)$$

Thus the proof is complete.