SATISH KUMAR

Bonus Question

Exercise 1

$$\underline{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \quad \underline{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \quad \underline{w} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

1) $\underline{u} + \underline{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0+(-2) \\ 2+3 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

2) $\underline{v} - \underline{w} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} -2-6 \\ 3-(-2) \end{pmatrix} = \begin{pmatrix} -8 \\ 5 \end{pmatrix}$

3) $5\underline{v} = 5 \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \cdot (-2) \\ 5 \cdot 3 \end{pmatrix} = \begin{pmatrix} -10 \\ 15 \end{pmatrix}$

4) $-2\underline{u} + \underline{v} = -2\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

5) $\underline{v} + \frac{1}{2}\underline{w} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \cdot 6 \\ \frac{1}{2} \cdot (-2) \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} =$

$\begin{pmatrix} -2+3 \\ 3+(-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

6) $-\underline{u} + \underline{w} - 2\underline{u} = -1\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

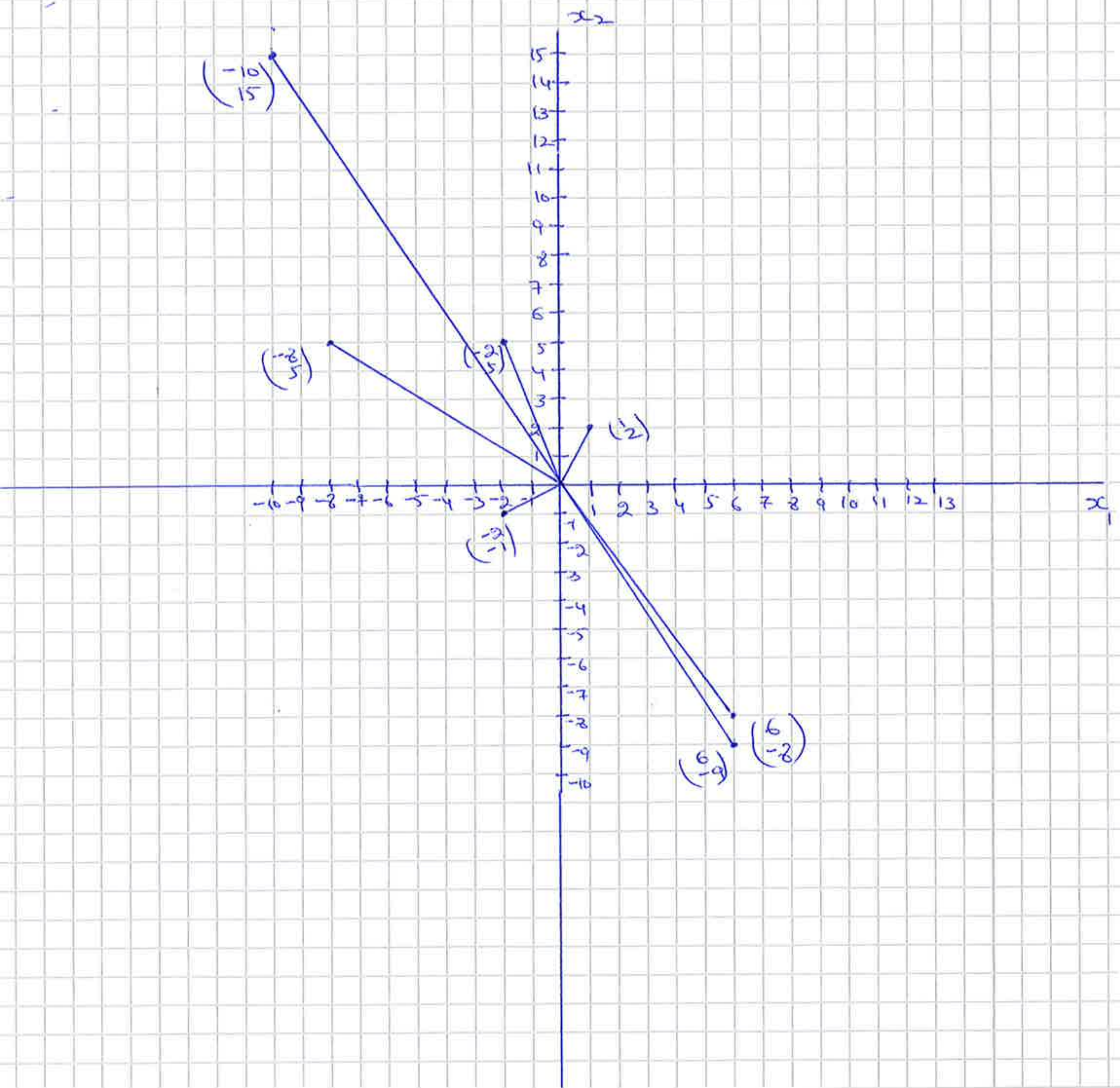
$\begin{pmatrix} -1 \cdot 0 \\ -1 \cdot 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \cdot 0 \\ 2 \cdot 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 0+6-0 \\ -2+(-2)-4 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 6 \\ -8 \end{pmatrix}$

7)

$$-\underline{u} - 3\underline{v} + \underline{u} = -1\begin{pmatrix}0\\2\end{pmatrix} - 3\begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}0\\2\end{pmatrix} \Rightarrow \begin{pmatrix}-1\cdot 0\\-1\cdot 2\end{pmatrix} - \begin{pmatrix}3\cdot(-2)\\3\cdot 3\end{pmatrix} + \begin{pmatrix}0\\2\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}0\\-2\end{pmatrix} - \begin{pmatrix}-6\\9\end{pmatrix} + \begin{pmatrix}0\\2\end{pmatrix} \Rightarrow \begin{pmatrix}0-(-6)+0\\-2-9+2\end{pmatrix} \Rightarrow \begin{pmatrix}6\\-9\end{pmatrix}$$



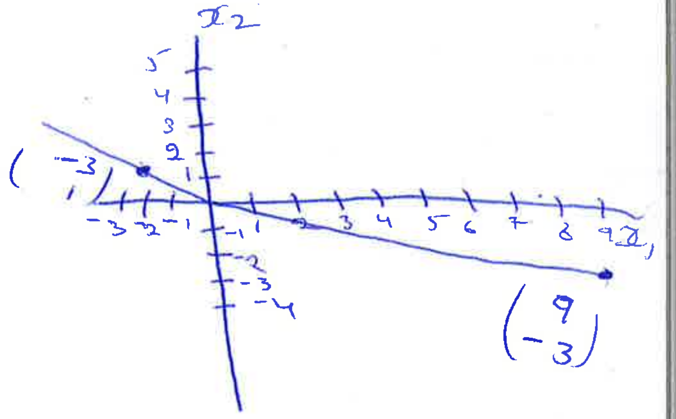
(b) $\underline{V} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \\ -1 \end{pmatrix}$ is Not defined because ~~it~~ vector ~~that~~ has different dimension. Mean the different number of componant.

$$\underline{w} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 100 \\ -59 \\ 6 \end{pmatrix} \Rightarrow \begin{pmatrix} -1+100 \\ 3+(-59) \\ 4+6 \end{pmatrix} \Rightarrow \begin{pmatrix} 99 \\ -56 \\ 10 \end{pmatrix}$$

Exercise-2

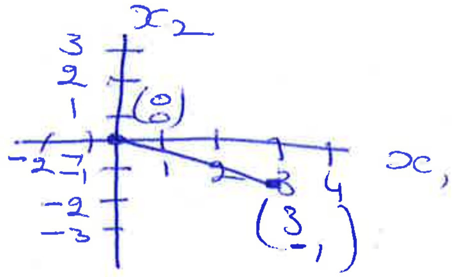
(a) $\begin{pmatrix} 9 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

is a line



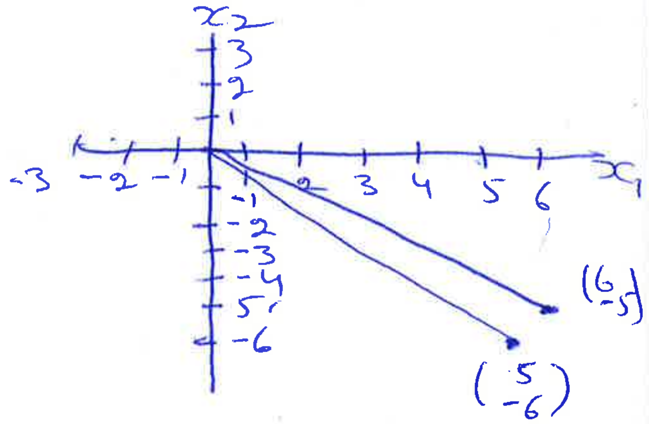
(B) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

is a line.



(C) $\begin{pmatrix} 6 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

is a plain.



(d) $\begin{pmatrix} -15 \\ 100 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -20 \end{pmatrix}$

is a line

Exercise - 3

(a) $\underline{V} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$, $\underline{w} = \begin{pmatrix} -3 \\ 3 \\ -1 \end{pmatrix}$

Given = $c\underline{V} - d\underline{w} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$

$$c\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} - d\begin{pmatrix} -3 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2c \\ -2c \\ -c \end{pmatrix} - \begin{pmatrix} -3d \\ 3d \\ -d \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2c+3d \\ -2c-3d \\ -c+d \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

we know. $2c+3d = -1$

$2c = -1-3d \Rightarrow c = \frac{-1-3d}{2}$

$-c+d = -2$

$$-\left(\frac{-1-3d}{2}\right) + d = -2$$

$$\frac{1+3d+2d}{2} = -2$$

$1 + 5d = -2 \times 2$

$\cancel{1}\ 5d = -4-1$

$d = \frac{-\cancel{5}1}{\cancel{5}} \Rightarrow d = -1$

$-c+d = -2$

$-c+(-1) = -2 \Rightarrow -c = -2+1$

$\Rightarrow$ [illegible]

(B) $\cos\theta = \dfrac{\underline{U} \cdot \underline{V}}{\|\underline{U}\| \cdot \|\underline{V}\|}$

(1) $\begin{pmatrix} \sqrt{3} \\ \sqrt{2} \end{pmatrix}$ and $\begin{pmatrix} -\sqrt{2} \\ \sqrt{3} \end{pmatrix}$

~~Cos θ =~~

Suppose = $\underline{U} = \begin{pmatrix} \sqrt{3} \\ \sqrt{2} \end{pmatrix}$ and $\underline{V} = \begin{pmatrix} -\sqrt{2} \\ \sqrt{3} \end{pmatrix}$

$\|\underline{U}\| = \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2} \Rightarrow \sqrt{3+2} = \sqrt{5}$

$\|\underline{V}\| = \sqrt{(-\sqrt{2})^2 + \sqrt{3}} \Rightarrow \sqrt{2+3} \Rightarrow \sqrt{5}$

$\cos\theta = \dfrac{\sqrt{3} \cdot (-\sqrt{2}) + \sqrt{2} \cdot \sqrt{3}}{\sqrt{5} \cdot \sqrt{5}}$

$\Rightarrow \dfrac{-\cancel{\sqrt{6}} + \cancel{\sqrt{6}}}{5} \Rightarrow \dfrac{0}{5} \Rightarrow 0$

$\cos\theta = 0 \Rightarrow \theta = 90^\circ$ Ans

(2) $\underline{U} = \begin{pmatrix} \sqrt{3} \\ \sqrt{3} \end{pmatrix}$ and $\underline{V} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$\|\underline{U}\| = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{3+3} = \sqrt{6}$

$\|\underline{V}\| = \sqrt{(1/\sqrt{2})^2 + (\frac{1}{\sqrt{2}})^2} \Rightarrow \sqrt{\frac{1}{2} + \frac{1}{2}} \Rightarrow \sqrt{1} = 1$

$\cos\theta = \dfrac{\sqrt{3} \cdot \frac{1}{\sqrt{2}} + \sqrt{3} \cdot \frac{1}{\sqrt{2}}}{\sqrt{6} \cdot 1}$

Exercise - 3

(a) $\underline{v} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$, $\underline{w} = \begin{pmatrix} -3 \\ 3 \\ -1 \end{pmatrix}$

Given $= c\underline{v} - d\underline{w} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$

$$c\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} - d\begin{pmatrix} -3 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2c \\ -2c \\ -c \end{pmatrix} - \begin{pmatrix} -3d \\ 3d \\ -d \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2c+3d \\ -2c-3d \\ -c+d \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

we know. $2c+3d = -1$

$2c = -1-3d \Rightarrow c = \dfrac{-1-3d}{2}$

$-c+d = -2$

$-\left(\dfrac{-1-3d}{2}\right) + d = -2$

$\dfrac{1+3d+2d}{2} = -2$

$1+5d = -2 \times 2$

$\cancel{1+}\, 5d = -4-1$

$d = \dfrac{-\cancel{5}}{\cancel{5}}1 \Rightarrow d = -1$

$-c+d = -2$

$-c+(-1) = -2 \Rightarrow -c = -2+1$

$$\Rightarrow \frac{\frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}}}{\sqrt{6}} \Rightarrow \frac{\sqrt{3}+\sqrt{3}}{\sqrt{2}\cdot\sqrt{6}}$$

$$\Rightarrow \frac{2\sqrt{3}}{\sqrt{12}} \Rightarrow \frac{\cancel{2\sqrt{3}}}{\cancel{2\sqrt{3}}} = 1$$

$\cos\theta = 1 \Rightarrow \theta = 0°$ Ans

(3) $\underline{U} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\underline{V} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

$\|U\| = \sqrt{(1)^2+(0)^2} = \sqrt{1} \Rightarrow 1$

$\|V\| = \sqrt{(1/\sqrt{2})^2+(-\frac{1}{\sqrt{2}})^2} \Rightarrow \sqrt{\frac{1}{2}+\frac{1}{2}} \Rightarrow \sqrt{1} \Rightarrow 1$

$$\cos\theta = \frac{1\cdot 1/\sqrt{2} + 0\cdot(-\frac{1}{\sqrt{2}})}{1\cdot 1}$$

$\cos\theta \Rightarrow \frac{1}{\sqrt{2}}$ ~~$\Rightarrow \theta =$~~

Multiply and divide by $\sqrt{2}$

$$= \frac{\sqrt{2}\cdot 1}{\sqrt{2}\cdot\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}$$

$\theta = 315°$

Ans

Exercise-4

(a) Prove

$$\langle \lambda \underline{v} + \mu \underline{w}, \underline{x} \rangle = \lambda \langle \underline{v}, x \rangle + \mu \langle \underline{w}, \underline{x} \rangle$$

$\forall \underline{v}, \underline{w} \in R^n, \lambda, \mu \in R$

$\Rightarrow (\lambda \underline{v} + \mu \underline{w})_1 x_1 + \text{---} + (\lambda \underline{v} + \mu \underline{w})_n x_n$

$\Rightarrow (\lambda \underline{v} \cdot x_1) + (\mu \underline{w} \cdot x_1) + \text{---} + (\lambda \underline{v} \cdot x_n) + (\mu \underline{w} \cdot x_n)$

$\Rightarrow \lambda(\underline{v} \cdot x_1) + \mu(\underline{w} \cdot x_1) + \text{---} + \lambda(\underline{v} \cdot x_n) + \mu(\underline{w} \cdot x_n)$

$\Rightarrow \lambda \langle \underline{v}, x \rangle + \mu \langle \underline{w}, x \rangle$

Ans

(b) $\langle \underline{v}, \underline{w} \rangle = \langle \underline{w}, \underline{v} \rangle \quad \forall \underline{v}, \underline{w} \in R^n$

$v_1 w_1 + v_2 w_2 + \text{----} + v_n w_n$

$w_1 v_1 + w_2 v_2 + \text{----} + w_n v_n$

$\langle \underline{w}, \underline{v} \rangle = \langle \underline{w}, \underline{v} \rangle$ Ans

$\langle \underline{v}, \underline{v} \rangle > 0 \quad \forall \underline{v} \in R^n, \underline{v} \neq 0$

~~&~~

$\underline{v}_1 \cdot \underline{v}_1 + \underline{v}_2 \cdot \underline{v}_2 + \cdots + \underline{v}_n \cdot \underline{v}_n$

And it is proved in positive we multiply and ~~as~~ number it is bigger than zero.

<u>Negative</u> :- If $\underline{v}$ is negative ~~w.~~ Negative multiply with with negative is positive.

$\langle \underline{v}, \underline{v} \rangle = 0 \Leftrightarrow \underline{v} = 0$

$\underline{v}_1 \cdot v_1 + \underline{v}_2 \cdot \underline{v}_2 + \cdots + \underline{v}_n \underline{v}_n$

$0 \cdot 0 + 0 \cdot 0 + \cdots + 0 \cdot 0$

$= 0$

<u>Ans</u>

(4)
(B)

$$\|\lambda \underline{V}\| = |\lambda| \cdot \|\underline{V}\|$$

$$\|\lambda \underline{V}\| \Rightarrow \sqrt{\lambda^2 V_1^2 + \lambda^2 V_2^2 + \cdots + \lambda^2 V_n^2}$$

$$\Rightarrow \sqrt{\lambda^2 (V_1^2 + V_2^2 + \cdots + V_n^2)}$$

$$\Rightarrow \cancel{|\lambda| \sqrt{V_1 \cdot V_1 + V_2 \cdot V_2 + \cdots + V_n V_n}}$$

$$\Rightarrow |\lambda| \cdot \sqrt{V_1^2 + V_2^2 + \cdots + V_n^2}$$

$$= |\lambda| \cdot \|\underline{V}\| \quad \text{Ans-}$$

$$\|\underline{V}\| \geq 0 \quad \forall \underline{V} \in R^n$$

$$\sqrt{V_1^2 + V_2^2 + \cdots + V_n^2} \geq 0$$

It is proved because if it is positive or 0 than it is equal or bigger than zero.
And if $\underline{V}$ is ~~equal~~ negative than square of negative is positive.

$\|\underline{V}\| = 0 \Longleftrightarrow \underline{V} = \underline{0}$

$\Rightarrow \sqrt{V_1^2 + V_2^2 + + --- + V_n^2}$

$\Rightarrow \sqrt{0^2 + 0^2 + --- + 0^2}$

$\Rightarrow \sqrt{0} \Rightarrow 0$ Ans

c) $\|\underline{U} + \underline{V}\|^2 = \|\underline{U}\|^2 + \|\underline{V}\|^2$

$\cancel{\langle \underline{U}+\underline{V}, \underline{U}+\underline{V} \rangle} (\underline{U} + \underline{V})^2$

$\Rightarrow (\underline{U}+\underline{V}) \cdot (\underline{U}+\underline{V}) \Rightarrow \underline{U}^2 + 2\underline{U}\underline{V} + \underline{V}^2$

$\Rightarrow \underline{U}^2 + \underline{V}^2 + 2\underline{U}\,\underline{V}$

And it is given $\cos \measuredangle(U, V) = 0$

$\Rightarrow \underline{U}^2 + V^2 + 2(0) \Rightarrow \underline{U}^2 + V^2$

$\Rightarrow \|\underline{U}\|^2 + \|V\|^2$

Ans