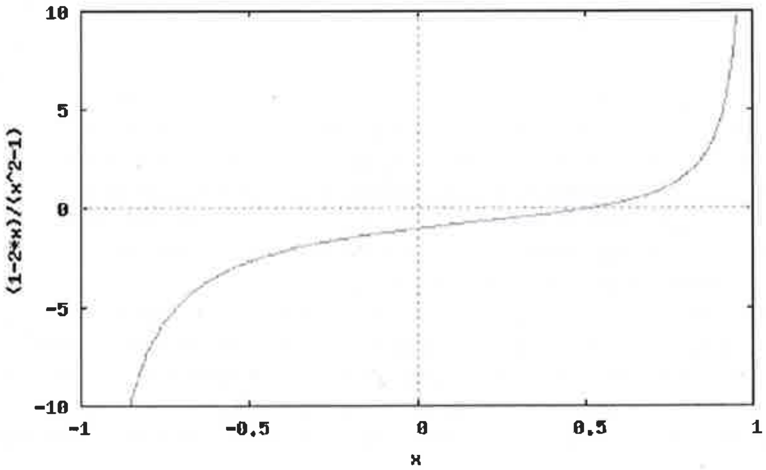
SATISH KUMAR

Exercise ①

```
(%i65) f(x):=(1-(2*x))/((x^2)-1
       ),
       wxplot2d( f(x),[x,-1,1],[y,-10,10]);
plot2d: expression evaluates to non-numeric value somewhere in plotting range.
plot2d: some values were clipped.
```

(%t65)




```
ev : improper argument:
 -- an error. To debug this try: debugmode(true);
```

-->

Exercise 3 (a)

```
(%i66) c : (-0.5) + (0.2*%i);
       z[0] : c;
       z[n] := ((z[n - 1])^2) + c;
       z1 : makelist(realpart(z[n]),n,0,20);
       z2 : makelist((imagpart(z[n])),n,0,20);
       wxplot2d([discrete,z1,z2],[style,[points,1,1]]);
```

(%o66) 0.2 %i -0.5

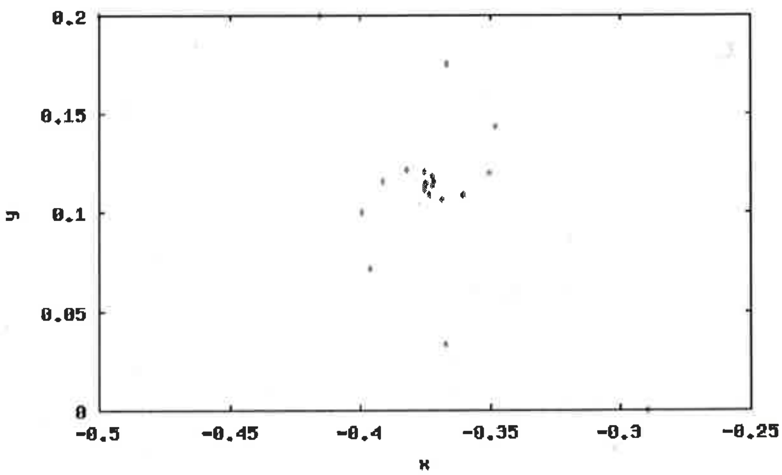
(%o67) 0.2 %i -0.5

(%o68) $z_n := z_{n-1}^2 + c$

(%o69) [-0.5 , -0.29 , -0.4159 , -0.36702719 , -.3664226914007039 , -.3964667488440351 , -.3479303012292202 , -.3994746508653273 , -.3504790294026833 , -.3915331763473662 , -.3601523023029966 , -.3822111955690897 , -.3686416746077245 , -.3756023189862402 , -.3735490418981834 , -.3723749156975639 , -.3753682653353813 , -.3715937524185706 , -0.375393021158023 , -.3720145026972935 , -.3747414412517399]

(%o70) [0.2 , 0.0 , 0.2 , 0.03364 , 0.1753064106568 , .07152750637467661 , 0.143283444189422 , .1002946962040285 , .1198696225005033 , .1159764221023151 , .1091827661457553 , .1213551508015931 , .1072334054473119 , .1209385956840277 , .1091503660122798 , .1184539707065537 , .1117814252882107 , .1160816005856956 , .1137296049032153 , .1146134000405475 , .1147243059629396]

(%t71)



(%o71)

3) c)

```
(%i72) c : (-0.3) + (0.8*%i);
       z[0] : c;
       z[n] := ((z[n - 1])^2) + c;
       z1 : makelist(realpart(z[n]),n,0,30);
       z2 : makelist((imagpart(z[n])),n,0,30);
       wxplot2d([discrete,z1,z2],[style,[points,1,1]]);
```

(%o72) 0.8 %i -0.3

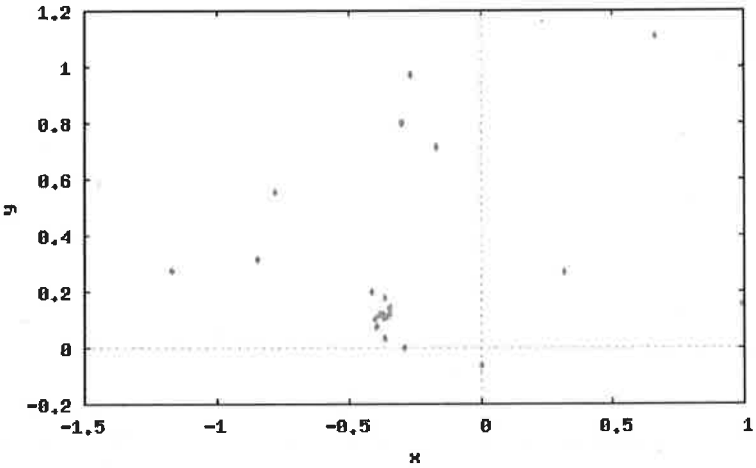
(%o73) 0.8 %i -0.3

(%o74) $z_n := z_{n-1}^2 + c$

(%o75) [-0.3 , -0.29 , -0.4159 , -0.36702719 , -.3664226914007039 , -.3964667488440351 , -.3479303012292202 , -.3994746508653273 , -.3504790294026833 , -.3915331763473662 , -.3601523023029966 , -.3822111955690897 , -.3686416746077245 , -.3756023189862402 , -.3735490418981834 , -.3723749156975639 , -.3753682653353813 , -.3715937524185706 , -0.375393021158023 , -.3720145026972935 , -.3747414412517399 , -.1727305185872469 , -.7799831539720992 , .002193811901105291 , -.3039874506203618 , -.8471481399938057 , .3191966684057583 , -.2701247241404034 , -1.170480168491423 , .9942617690897773 , 0.664328664414678]

(%o76) [0.8 , 0.0 , 0.2 , 0.03364 , 0.1753064106568 , .07152750637467661 , 0.143283444189422 , .1002946962040285 , .1198696225005033 , .1159764221023151 , .1091827661457553 , .1213551508015931 , .1072334054473119 , .1209385956840277 , .1091503660122798 , .1184539707065537 , .1117814252882107 , .1160816005856956 , .1137296049032153 , .1146134000405475 , .1147243059629396 , .7140160964736849 , .5533352587529174 , -.06318436065213628 , .7997227707952753 , .3137886274057847 , .2683490958839605 , .9713122747517152 , .2752490794570105 , .1556528222000989 , 1.109519300728974]

(%t77)



(%o77)

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Exercise 1

$$f(x) = \frac{1-2x}{x^2-1}$$

$$f'(x) = \frac{\frac{d}{dx}(1-2x)(x^2-1) - (1-2x)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{-2(x^2-1) - (1-2x)(2x)}{(x^2-1)^2}$$

$$= \frac{-2x^2+2 -2x+4x^2}{(x^2-1)^2}$$

$$= \frac{2x^2-2x+2}{(x^2-1)^2}$$

$$= \frac{x^2+x^2-2x+1+1}{(x^2-1)^2}$$

$$= \frac{x^2+1+(x-1)^2}{(x^2-1)^2}$$

According to defination of monotonically increasing if $f'(x) \geq 0$ for all $x \in (a,b)$. Than In our equation we put $x \in (-1,1)$ than we got the positive number because the we put value in numerator than square of positive number is positive and square of negative number is also positive. And Denominator is also we got the positive number and if positive number divide by positive that

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is also positive. And if we got the positive number than we know that positive number is always bigger or equal than Zero that mean $f'(x) \geq 0$

And if $f'(x) \geq 0$ than $f(x)$ is monotonically increasing on the open interval $(-1, 1)$

Exercise 2

a)

$$\frac{1}{1-i}$$

Algebraic form is $a+ib$

than

$$= \frac{1}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{1+i}{(1)^2-(i)^2} \qquad \text{we know } i^2 = -1$$

$$= \frac{1+i}{1-(-1)}$$

$$= \frac{1+i}{1+1}$$

$$= \frac{1+i}{2}$$

$$= \frac{1}{2} + \frac{1}{2} i$$

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Polar form

$a = \frac{1}{2}, \quad b = \frac{1}{2}$

$r = \sqrt{a^2+b^2} = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{\frac{1+1}{4}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$

$\theta = \arctan\frac{b}{a} = \arctan\frac{\frac{1}{2}}{\frac{1}{2}} = \arctan\frac{1}{2}\times 2 = \arctan 1$

$= 45^\circ$

Polar form is $= \cancel{\frac{1}{\sqrt{2}}\left(\cos(45^\circ) + i\sin(45^\circ)\right)}$

$= \cancel{\frac{1}{\sqrt{2}} e^{i45^\circ}} = \frac{1}{\sqrt{2}} e^{i\cdot 0.7854}$

b) $\frac{1-i}{1+i}$

Algebraic form

$= \frac{1-i}{1+i} \times \frac{1-i}{1-i} \Rightarrow \frac{1-i-i+i^2}{(1)^2-(i)^2} = \frac{1-2i+(-1)}{1-(-1)} = \frac{-2i}{2}$

$= -i$

Polar form is $a=0, \ b=-1$

$r = \sqrt{(0)^2+(-1)^2} = \sqrt{1} = 1$

$\theta = \arctan\left(\frac{-1}{0}\right) = -90^\circ$

$= 1 \cdot e^{i\cdot(-90^\circ)}$

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c) $\dfrac{(1+2i)^2}{2+3i}$

Algebraic form

$$= \frac{(1)^2+(2i)^2+4i}{2+3i} = \frac{1+4i^2+4i}{2+3i} = \frac{-3+4i}{2+3i}$$

$$= \frac{(-3+4i)}{2+3i} \times \frac{2-3i}{2-3i} \Rightarrow \frac{-6+8i+9i-12i^2}{(2)^2-(3i)^2}$$

$$= \frac{6+17i}{4-9(-1)} = \frac{6+17i}{13} = \frac{6}{13} + \frac{17}{13}i$$

$$= \frac{6}{13} + \frac{17}{13}i$$

Polar form $a = \frac{6}{13}$, $b = \frac{17}{13}$

$$r = \sqrt{\left(\frac{6}{13}\right)^2 + \left(\frac{17}{13}\right)^2} = \sqrt{\frac{36+289}{169}} = \sqrt{\frac{325}{169}}$$

$$= 1.38$$

$$\theta = \arctan \frac{\frac{17}{13}}{\frac{6}{13}} = \arctan \frac{17}{13} \cdot \frac{13}{6} = \arctan \frac{17}{6} = \arctan \frac{17}{6}$$

$$Z = 1.38\, e^{i(1.2315)}$$

d) $\overline{(-1+2i)^2}$

~~Polar form~~

$$= \overline{(-1)^2 + (2i)^2 + 2(-1)(2i)} = \overline{1+4i^2-4i} = \overline{-3-4i}$$

$$= -3+4i$$

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~~A~~ Polar form $a = -3,\ b = 4$

$$r = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\theta = \arctan \frac{4}{-3} = -0.927$$

$$z = 5e^{i(-0.927)}$$

e) $\left(\frac{4-i}{2+i}\right)^2$

algebraic form

$$= \frac{(4-i)^2}{(2+i)^2} = \frac{16+i^2-8i}{4+i^2+4i} = \frac{15-8i}{3+4i}$$

$$= \frac{15-8i}{3+4i} \times \frac{3-4i}{3-4i} = \frac{45-24i-60i+32i^2}{(3)^2-(4i)^2} = \frac{13-84i}{9+16}$$

$$= \frac{13-84i}{25} = \frac{13}{25} + \left(\frac{-84}{25}\right)i$$

Polar form $a = \frac{13}{25},\ b = -\frac{84}{25}$

$$r = \sqrt{\left(\frac{13}{25}\right)^2 + \left(\frac{-84}{25}\right)^2} = \sqrt{\frac{169+7056}{625}} = \sqrt{\frac{7225}{625}}$$

$$= \sqrt{\frac{289}{25}} = \frac{17}{5} = 3.4$$

$$\theta = \arctan \frac{-84}{25} \Big/ \frac{13}{25} = \arctan \frac{-84}{25} \times \frac{25}{13} = \arctan \frac{-84}{13}$$

$$= -1.417$$

$$z = 3.4\, e^{i(-1.417)}$$

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Exercise 3

B) The limit with maxima is ~~-3.73+0.114i~~

-3.73 + 0.114i

C) No, with the other value of c. It is not converges. Then it gets the randomalley. And it is not limit. The value of z[n] is ~~not~~ randomaly. And the ~~point~~ value is some where. is increasing and some where is decreasing.

Bonus Exercise

~~let suppose the maximum value f(M) = f(m) minimum value. So all value of f on [a, b] are equal.~~ ~~and f is constant on [a, b].~~

Let f is constant on [a, b] than $f'(x)=0$ for all $x \in [a, b]$. Suppose there exist $c \in [a, b]$ that $f(c) > f(a)$. A similar argument given if $f(c) < f(a)$. Then there exist $x \in (a, b)$ such that $f(x)$ is maximum than $f'(x) = 0$. Example is $x^5 + x^4 - 1$