SATISH KUMAR

Exercise 1

1) Matrix transforms

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

is $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

2) Matrix transforms

$$\begin{bmatrix} \cancel{a} & \cancel{b} \\ \cancel{c} & \cancel{d} \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 2a+6b & 4a+8b \\ 2c+6d & 4c+8d \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

then we have

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

row 1 multiply by -3 and add row 2

$$\begin{bmatrix} 2 & 4 \\ 0 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

first row divide by 2

SATISH KUMAR

$$\begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1/2 & 0 \\ -3 & 1 \end{bmatrix}$$

row 2 divide by -4

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1/2 & 0 \\ 3/4 & -1/4 \end{bmatrix}$$

row 2 multiply by -2 and add row 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1/2 \\ 3/4 & -1/4 \end{bmatrix}$$

Now we have

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1/2 \\ 3/4 & -1/4 \end{bmatrix}$$

$$\begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1/2 \\ 3/4 & -1/4 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1/2 \\ 3/4 & -1/4 \end{bmatrix}$$

is $\begin{bmatrix} -1 & 1/2 \\ 3/4 & -1/4 \end{bmatrix}$ is inverse of matrix

(3) Because $(2,6) = 2(1,3)$

If $A = (1,3) = \cancel{(0,0)}\ \cancel{(1,0)}\ \cancel{(0,1)}\ (1,0)$

than $(2,6) = \cancel{(0,2)}\ (2,0)$

It is not $(0,1)$

That's why does not matrix transforms.

4) Matrix transforms

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} r & s \\ t & u \end{bmatrix}$$

$$\begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix} \to \begin{bmatrix} r & s \\ t & u \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \to \begin{bmatrix} r & s \\ t & u \end{bmatrix}$$

is

$$\begin{bmatrix} r & s \\ t & u \end{bmatrix}$$

5) Matrix transforms

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiply row 1 by $-\frac{c}{a}$ and add row 2

$$\begin{bmatrix} a & b \\ 0 & d-\frac{bc}{a} \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ -c/a & 1 \end{bmatrix}$$

~~Multip~~ divide row 1 by a

$$\begin{bmatrix} 1 & b/a \\ 0 & \frac{da-bc}{a} \end{bmatrix} \to \begin{bmatrix} 1/a & 0 \\ -c/a & 1 \end{bmatrix}$$

divide row 2 by $\frac{da-bc}{a}$.

$$\begin{bmatrix} 1 & b/a \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1/a & 0 \\ \frac{-c}{da-bc} & \frac{a}{da-bc} \end{bmatrix}$$

row 2 multiply by $-\frac{b}{a}$ and add row 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{d}{da-bc} & \frac{-b}{da-bc} \\ \frac{-c}{da-bc} & \frac{a}{da-bc} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \frac{1}{da-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \frac{1}{da-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \frac{1}{da-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

is $\frac{1}{da-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

5) Impossible condition is

$$da-bc = 0$$
$$da = bc$$

Exercise 2

1) change of basis matrix is $B^{-1}D$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2) Change of basis matrix is $B^{-1}D$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$$

3) change of basis matrix is $B^{-1}D$

$$= \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 & 1/2 \\ 3/2 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 & 1/2 \\ 3/2 & -1/2 \end{pmatrix}$$

(4) change of basis ~~of~~ matrix is $B^{-1}D$

$$= \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 3/2 \\ 1 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 1/2 \\ 1 & 1/2 \end{pmatrix}$$

JATISH KUMAR

Exercise 3

1) In this liner transformation

$$\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix} \to \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$$

Multiply row 1 by -3 and add row 2

$$\begin{bmatrix} 1 & -1 \\ 0 & 6 \end{bmatrix} \to \begin{bmatrix} 3 & 3 \\ -8 & -7 \end{bmatrix}$$

divide row 2 by 6

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 3 & 3 \\ -8/6 & -7/6 \end{bmatrix}$$

add row 2 and 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 5/3 & 11/6 \\ -8/6 & -7/6 \end{bmatrix}$$

is $\begin{bmatrix} 5/3 & 11/6 \\ -8/6 & -7/6 \end{bmatrix}$

2) transformation send this vector is

$$= \begin{pmatrix} 5/3 & 11/6 \\ -4/3 & -7/6 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{6} \\ -\frac{1}{6} \end{pmatrix}$$

3) The transformation matrix is $A = \begin{pmatrix} 5/3 & 11/6 \\ -4/3 & -7/6 \end{pmatrix}$

and $S = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

and representation with respect of two basis is $(S^T)^{-1} A S^T$

than $\left(\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}^T\right)^{-1} \begin{pmatrix} 5/3 & 11/6 \\ -4/3 & -7/6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}^T$

$$= \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 5/3 & 11/6 \\ -4/3 & -7/6 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3/2 & -2 \\ -1/2 & 1 \end{pmatrix} \begin{pmatrix} 5/3 & 11/6 \\ -4/3 & -7/6 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 31/6 & 61/12 \\ -\frac{13}{6} & -\frac{25}{12} \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{185}{12} & \frac{431}{12} \\ \frac{-77}{12} & -\frac{179}{12} \end{pmatrix}$$

Exercise 4

a) $A_{EE} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$\cancel{f(v_1) =}$

$$f = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$f(v_1) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+2+1 \\ -1+0+1 \\ -1+2+0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

$$f(v_2) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0-3+2 \\ -2+0+2 \\ -2-3+0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -5 \end{pmatrix}$$

$$f(v_3) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+2+1 \\ 1+0+1 \\ 1+2+0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$$

$$\det A_{\varepsilon\varepsilon} = \det \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$= 0 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= -1(0-1) + 1(1-0)$$

$$= 1 + 1$$

$$= 2$$

b) $P(v_1, v_2, v_3) = \left| \det \begin{pmatrix} -1 & -2 & 1 \\ 2 & -3 & 2 \\ 1 & 2 & 1 \end{pmatrix} \right|$

$= -1 \begin{vmatrix} -3 & 2 \\ 2 & 1 \end{vmatrix} - (-2) \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$

$= -1(-3-4) + 2(2-2) + 1(4+3)$

$= 7 + 0 + 7 \Rightarrow 14$

$P(f(v_1), f(v_2), f(v_3) = \left| \det \begin{pmatrix} 3 & -1 & 3 \\ 0 & 0 & 2 \\ 1 & -5 & 3 \end{pmatrix} \right|$

$= 3 \begin{vmatrix} 0 & 2 \\ -5 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} + 3 \begin{vmatrix} 0 & 0 \\ 1 & -5 \end{vmatrix}$

$= 3(0-(-10)) + 1(0-2) + 0$

$= 30 - 2 \Rightarrow 28$

c) Yes, it is connection between the volume of P, ~~Pf~~ $f(P)$ and $\det(A_{EE})$

Because.

$\det(f(P)) = \det(P) \cdot \det A_{EE}$

$28 = 2 \cdot 14$

$28 = 28$

Ans

SATISH KUMAR

Exercise 5

Let V, W be two vector spaces and $f : V \to W$ a linear map. Let $w_1, \dots w_n$ be element of W that are linearly independent and let $v_1 \dots v_n \in V$ such that

$f(v_i) = w_i$ for $i = 1 \dots n$

Suppose that v_i is not liner independent and $v_i = \alpha v_i$ for α is scalar. than

$f(v_i) = w_i$ and than $f(\alpha v_1) = \alpha w_i$ and $\alpha f(v_i) = \alpha w_i$. Since we know that w_i is linearly independent and it is contradiction. Then $v_1 \dots v_n$ is also linearly independent.