Exercise 1

a) No, Let consider R^3 $V_1 = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$, $V_2 = \begin{pmatrix} 4 \\ -3 \\ -11 \end{pmatrix}$, $V_3 = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix}$

and assume that

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0$$

$$\alpha_1 \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix} + \alpha_2 \begin{pmatrix} 4 \\ -3 \\ -11 \end{pmatrix} + \alpha_3 \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -3\alpha_1 \\ +2\alpha_1 \\ 5\alpha_1 \end{pmatrix} + \begin{pmatrix} 4\alpha_2 \\ -3\alpha_2 \\ -11\alpha_2 \end{pmatrix} + \begin{pmatrix} 2\alpha_3 \\ -3\alpha_3 \\ 7\alpha_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

we can write it like

$$-3\alpha_1 + 4\alpha_2 + 2\alpha_3 = 0$$
$$2\alpha_1 - 3\alpha_2 - 3\alpha_3 = 0$$
$$5\alpha_1 - 11\alpha_2 + 7\alpha_3 = 0$$

It is

$$\begin{pmatrix} -3 & 4 & 2 \\ 2 & -3 & -3 \\ 5 & -11 & 7 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-3\alpha_1 + 4\alpha_2 + 2\alpha_3 = 0$$
$$2\alpha_1 - 3\alpha_2 - 3\alpha_3 = 0$$
$$5\alpha_1 - 11\alpha_2 + 7\alpha_3 = 0$$

Multiply equation 2 by $-5/2$ and add in equation 3

$$-3\alpha_1 + 4\alpha_2 + 2\alpha_3 = 0$$
$$2\alpha_1 - 3\alpha_2 - 3\alpha_3 = 0$$
$$-7\alpha_2 + \frac{29}{2}\alpha_3 = 0$$

Multiply equation one by $\frac{2}{3}$ and add in equation 2

$$-3\alpha_1 + 4\alpha_2 + 2\alpha_3 = 0$$
$$-\frac{1}{3}\alpha_2 - \frac{5}{3}\alpha_3 = 0$$
$$-7\alpha_2 + \frac{29}{2}\alpha_3 = 0$$

Multiply equation 2 by -21 and add in equation 3

$$-3\alpha_1 + 4\alpha_2 + 2\alpha_3 = 0$$
$$-\frac{1}{3}\alpha_2 - \frac{5}{3}\alpha_3 = 0$$
$$\frac{99}{2}\alpha_3 = 0$$

$\alpha_3 = 0$, $\alpha_2 = 0$, $\alpha_1 = 0$

and we have

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0$$
$$0 + 0 + 0 = 0$$
$$0 = 0$$

It is linear Independent.
That mean it is not dependent

b) Yes, Let assume R^3 and

$$V_1 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, V_2 = \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}, V_3 = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}, V_4 = \begin{pmatrix} 5 \\ 9 \\ -4 \end{pmatrix}$$

and assume that

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 + \alpha_4 V_4 = 0$$

thenwe have

$$\begin{pmatrix} 2\alpha_1 \\ 3\alpha_1 \\ 4\alpha_1 \end{pmatrix} + \begin{pmatrix} -3\alpha_2 \\ 5\alpha_2 \\ -1\alpha_2 \end{pmatrix} + \begin{pmatrix} 4\alpha_3 \\ 7\alpha_3 \\ -3\alpha_3 \end{pmatrix} + \begin{pmatrix} 5\alpha_4 \\ 9\alpha_4 \\ -4\alpha_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$2\alpha_1 - 3\alpha_2 + 4\alpha_3 + 5\alpha_4 = 0$$
$$3\alpha_1 + 5\alpha_2 + 7\alpha_3 + 9\alpha_4 = 0$$
$$4\alpha_1 - \alpha_2 - 3\alpha_3 - 4\alpha_4 = 0$$

Multiply equation 1 by -2 and add equation 3

$$2\alpha_1 - 3\alpha_2 + 4\alpha_3 + 5\alpha_4 = 0$$
$$3\alpha_1 + 5\alpha_2 + 7\alpha_3 + 9\alpha_4 = 0$$
$$5\alpha_2 - 11\alpha_3 - 14\alpha_4 = 0$$

Multiply equation 1 by $-\frac{3}{2}$ and add equation 2.

$$2\alpha_1 - 3\alpha_2 + 4\alpha_3 + 5\alpha_4 = 0$$
$$\frac{19}{2}\alpha_2 + \alpha_3 + \frac{3}{2}\alpha_4 = 0$$
$$5\alpha_2 - 11\alpha_3 - 14\alpha_4 = 0$$

Multiply equation 2 by $-\frac{10}{19}$ and add in equation 3

$$2\alpha_1 - 3\alpha_2 + 4\alpha_3 + 5\alpha_4 = 0$$
$$\frac{19}{2}\alpha_2 + \alpha_3 + \frac{3}{2}\alpha_4 = 0$$
$$-\frac{219}{19}\alpha_3 + \frac{266}{19}\alpha_4 = 0$$

It mean $\alpha_4 = \frac{219}{19}\alpha_3 \cdot \frac{19}{266}$

$$\alpha_4 = \frac{219}{266}$$

That mean it is not linear independent. Then it is dependent.

c) No, let assume R^3

$$V_1 = \begin{pmatrix} -4 \\ 5 \\ 3 \end{pmatrix}, \quad V_2 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \quad V_3 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

and assume that

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0$$

than we have.

$$\begin{pmatrix} -4\alpha_1 \\ 5\alpha_1 \\ 3\alpha_1 \end{pmatrix} + \begin{pmatrix} 2\alpha_2 \\ 3\alpha_2 \\ -1\alpha_2 \end{pmatrix} + \begin{pmatrix} 3\alpha_3 \\ 2\alpha_3 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

It is

$$-4\alpha_1 + 2\alpha_2 + 3\alpha_3 = 0$$
$$5\alpha_1 + 3\alpha_2 + 2\alpha_3 = 0$$
$$3\alpha_1 + \alpha_2 + \alpha_3 = 0$$

Multiply equation 1 by $\frac{3}{4}$ and add in equation 3

$$-4\alpha_1 + 2\alpha_2 + 3\alpha_3 = 0$$
$$5\alpha_1 + 3\alpha_2 + 2\alpha_3 = 0$$
$$\frac{1}{2}\alpha_2 + \frac{13}{4}\alpha_3 = 0$$

Multiply equation 1 by $\frac{5}{4}$ and add in equation 2

$$-4\alpha_1 + 2\alpha_2 + 3\alpha_3 = 0$$
$$\frac{11}{2}\alpha_2 + \frac{23}{4}\alpha_3 = 0$$
$$\frac{1}{2}\alpha_2 + \frac{13}{4}\alpha_3 = 0$$

Multiply equation 2 by $-\frac{1}{11}$ and add in equation 3

$$-4\alpha_1 + 2\alpha_2 + 3\alpha_3 = 0$$
$$\frac{11}{2}\alpha_2 + \frac{23}{4}\alpha_3 = 0$$
$$\frac{30}{11}\alpha_3 = 0$$

That mean $\alpha_3 = 0$ and $\alpha_2 = 0$ and $\alpha_1 = 0$ that mean it is linear independend. Therefore it isnot linear dependent.

d) Yes, Let consider R^3

$$V_1 = \begin{pmatrix} \cancel{\sin}\cos(\frac{\pi}{2}) \\ \sin(0) \\ \tan(\pi) \end{pmatrix}, V_2 = \begin{pmatrix} \cos(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) \\ \tan(2\pi) \end{pmatrix}, V_3 = \begin{pmatrix} \cos(0) \\ \sin(\frac{\pi}{2}) \\ \tan(0) \end{pmatrix}$$

and we ~~kno~~ put value ~~of~~

$$V_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, V_2 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}, V_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

let assume

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0$$

than we have.

$$\frac{1}{\sqrt{2}}\alpha_2 + \alpha_3 = 0$$

$$\frac{1}{\sqrt{2}}\alpha_2 + \alpha_3 = 0$$

$$0 = 0$$

Multipley equation 1 by -1 and add in equation 2

$$\frac{1}{\sqrt{2}}\alpha_2 + \alpha_3 = 0$$

$$0 = 0$$

$$0 = 0$$

It mean $\alpha_3 = -\frac{1}{\sqrt{2}}\alpha_2$ it is not linear independent than it is linear dependent.