RATISH KUMAR

Exercise 1

Given $\exists$ $x = \frac{b}{a} = ba^{-1}$

Let $a, b, x \in \mathbb{Q}$ and $a \neq 0$

$x = ba^{-1}$

than multiply both sides by a

$a \cdot x = (ba^{-1}) \cdot a$

$a \cdot x = b \cdot (a^{-1}a)$ (F1) associativity

$a \cdot x = b \cdot (1)$ (Multiplicative inverse)

$a \cdot x = b$ (F3) Identity elements.

Proved

Exercise 2

Let $x, y \in \mathbb{Q}$ and $0 < x < y$

$\underset{LHS}{x < y} = \underset{R.H.S}{x^2 < y^2}$

Let LHS

$x < y$

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Multiply both sides with $(x+y)$

$$x\cdot(x+y) < y\cdot(x+y)$$

$$x\cdot x + x\cdot y < y\cdot x + y\cdot y \quad \text{(F5) distributivity}$$

$$x\cdot x + x\cdot y < x\cdot y + y\cdot y \quad \text{(F2) Commutativity}$$

$$x\cdot x + x\cdot y < y\cdot y + x\cdot y \quad \text{(F2) commutativity}$$

Add both sides $(-xy)$

$$(x\cdot x + x\cdot y) + (-x\cdot y) < (y\cdot y + x\cdot y) + (-x\cdot y)$$

$$(x\cdot x + x\cdot y) + (-x\cdot y) < (y\cdot y + x\cdot y) + (-x\cdot y)$$

$$x\cdot x + (x\cdot y - x\cdot y) < y\cdot y + (x\cdot y - x\cdot y)$$

(F1) associativity

$$x\cdot x + (0) < y\cdot y + (0) \quad \text{(F4) inverse element}$$

$$x\cdot x < y\cdot y \quad \text{(F3) Identity}$$

$$x^2 < y^2$$

Proved

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Exercise 3

$$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|$$

$n \in N$ and $x_1, \ldots, x_n \in Q$

Base case $n = 1$

than $|x_1| \leq |x_1|$ is true.

Induction hypothesis-

Suppose

$$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|$$

for $n \in N$ and $x_1 \cdots x_n \in Q$

Induction step-

$$|x_1 + x_2 + \cdots + x_{n+1}| \leq |x_1| + |x_2| + \cdots + |x_{n+1}|$$

$= |x_1 + x_2 + \cdots + x_{n+1}|$

$\cancel{=} |x_1 + x_2 + \cdots + x_n + x_{n+1}|$

$\leq |x_1 + x_2 + \cdots + x_n| + |x_{n+1}|$ (triangle inequality)

$\leq |x_1| + |x_2| + \cdots + |x_n| + |x_{n+1}|$ (Induction hypothesis)

$\leq |x_1| + |x_2| + \cdots + |x_{n+1}|$

Proved.

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Bonus Exercise

Suppose $a, b, c \in R$

$$a+b=c$$

we prove

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$$

$$\frac{b+a}{ab}=\frac{1}{c} \Rightarrow (b+a)c = ab$$

we assume that $a \neq 0$, $b \neq 0$ and $c \neq 0$ because we ~~w~~ have $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ and then Value $1/0$ is not defined.

$$cb+ac=ab$$

$$b(a+b)+a(a+b)=ab$$

$$ba+b^2+a^2+ab=ab$$

$$a^2+b^2+2ab=ab$$

$$a^2+b^2+2ab-ab=0$$

$$a^2+b^2+ab=0$$

The only possible value for a and b in following equation is 0

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5/5

$$ba + a^2 + b^2 = 0$$
$$0 \cdot 0 + 0^2 + 0^2 = 0$$
$$0 = 0$$

But this contradicts the fact that a, b, c must be different from 0.