Exercise 5

a) $\quad -x + 2y = -3$
$\quad\quad 2x + 3y = 8$

$$A = \begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$$

$x = \dfrac{\det B_1}{\det A} = \dfrac{\begin{vmatrix} -3 & 2 \\ 8 & 3 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix}} = \dfrac{-3 \cdot 3 - 8 \cdot 2}{-1 \cdot 3 - 2 \cdot 2} =$

$\quad\quad = \dfrac{-9-16}{-3-4} = \dfrac{-25}{-7} = \dfrac{25}{7}$

$y = \dfrac{\det B_2}{\det A} = \dfrac{\begin{vmatrix} -1 & -3 \\ 2 & 8 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix}} = \dfrac{-1 \cdot 8 - 2 \cdot -3}{-1 \cdot 3 - 2 \cdot 2}$

$\quad\quad = \dfrac{-2}{-7} = \dfrac{2}{7}$

$x = \dfrac{25}{7} \quad$ and $\quad y = \dfrac{2}{7}$

b)

$$2x - y = 5$$
$$-x - 4y = -3$$

$$A = \begin{pmatrix} 2 & -1 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$x = \frac{\det B_1}{\det A} = \frac{\begin{vmatrix} 5 & -1 \\ -3 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & -4 \end{vmatrix}} = \frac{5 \cdot (-4) - (-1)(-3)}{2 \cdot (-4) - (-1)(-1)}$$

$$= \frac{-20 - 3}{-8 - 1} = \frac{-23}{-9} = \frac{23}{9}$$

$$y = \frac{\det B_2}{\det A} = \frac{\begin{vmatrix} 2 & 5 \\ -1 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & -4 \end{vmatrix}} = \frac{2 \cdot (-3) - 5(-1)}{2 \cdot (-4) - (-1)(-1)}$$

$$= \frac{-6 + 5}{-8 - 1} = \frac{-1}{-9} = \frac{1}{9}$$

$$x = \frac{23}{9} \quad \text{and} \quad y = \frac{1}{9}$$

c)

$$x - 2y + z = -2$$
$$3x + y + 2z = 1$$
$$-x + y - 3z = 4$$

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 3 & 1 & 2 \\ -1 & 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$x = \frac{\det B_1}{\det A} = \frac{\begin{vmatrix} -2 & -2 & 1 \\ 1 & 1 & 2 \\ 4 & 1 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 2 \\ -1 & 1 & -3 \end{vmatrix}}$$

$$= \frac{(-2)(1)(3) + (-2)(2)(4) + (1)(1)(1) - (2)(1)(2) - (-2)(1)(-3) - (4)(1)(1)}{(1)(1)(-3) + (-2)(-1)(2) + (1)(3)(1) - (1)(1)(2) - (-2)(3)(-3) - (1)(-1)(1)}$$

$$= \frac{6 - 16 + 1 + 4 - 6 - 4}{-3 + 4 + 3 - 2 - 18 + 1} = \frac{-15}{-15} = 1$$

$$y = \frac{\det B_2}{\det A} = \frac{\begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 2 \\ -1 & 4 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 2 \\ -1 & 1 & -3 \end{vmatrix}}$$

$$= \frac{(1)(1)(-3) + (-2)(2)(-1) + (1)(3)(4) - (-1)(1)(1) - (4)(2)(1) - (3)(-3)(-2)}{(1)(1)(-3) + (-2)(-1)(2) + (1)(3)(1) - (1)(1)(2) - (-2)(3)(-3) - (1)(-1)(1)}$$

$$= \frac{-3 + 4 + 12 + 1 - 8 - 18}{-3 + 4 + 3 - 2 - 18 + 1} = \frac{-12}{-15} = \frac{4}{5}$$

$$Z = \frac{\det B_3}{\det A} = \frac{\begin{vmatrix} 1 & -2 & -2 \\ 3 & 1 & 1 \\ -1 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 2 \\ -1 & 1 & -3 \end{vmatrix}}$$

$$= \frac{(1)(1)(4) + (-2)(1)(-1) + (-2)(3)(1) - (-1)(1)(-2) - (1)(1)(1) - (4)(3)(-2)}{(1)(1)(-3) + (-2)(-1)(2) + (1)(3)(1) - (1)(1)(2) - (-2)(3)(-3) - (1)(-1)(1)}$$

$$= \frac{4 + 2 - 6 - 2 - 1 + 24}{-3 + 4 + 3 - 2 - 18 + 1} = \frac{21}{-15} = \frac{-7}{5}$$