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Exercise 2

let. $Q \in \mathbb{R}^{n,n}$ is orthogonal and we know that

$$Q Q^T = I$$

and $\det(I) = 1$

$$\det(Q Q^T) = \det(I)$$

$$\det(Q) \cdot \det(Q^T) = 1$$

and we know that $\det(Q^T) = \det(Q)$

$$\det(Q) \cdot \det(Q) = 1$$

$$(\det Q)^2 = 1$$

square root both sides.

$$\det Q = \pm 1$$

Proved

Exercise 3

Prove $\det(B) = (\det(B_1))(\det(B_2))$

Let $\det(B) = \det\begin{pmatrix} B_1 & A \\ 0 & B_2 \end{pmatrix}$

$$= \det\left(B_1 \begin{pmatrix} 1 & A/B_1 \\ 0 & B_2 \end{pmatrix}\right) \quad \text{Properties (3)}$$

$$= \det\left(B_1 \cdot B_2 \begin{pmatrix} 1 & A/B_1 \\ 0 & 1 \end{pmatrix}\right) \quad \text{Properties (3)}$$

$$= \det(B_1) \cdot \det(B_2) \cdot \det\begin{pmatrix} 1 & A/B_1 \\ 0 & 1 \end{pmatrix}$$

$$= \det(B_1) \cdot \det(B_2) \cdot 1$$

$$= \det(B_1) \cdot \det(B_2) \quad \underline{\text{Proved}}$$

Exercise 4

$$P_A(\lambda) = \det(A - \lambda I)$$

(a) $$P_B(\lambda) = \det(B - \lambda I)$$

$$= \det\left(\begin{pmatrix} 3/2 & -3 \\ 1 & -6 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$= \det\left(\begin{pmatrix} 3/2 & -3 \\ 1 & -6 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right)$$

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$$= \det\begin{pmatrix} 3/2 - \lambda & -3 \\ 1 & -6-\lambda \end{pmatrix}$$

$$= \left(\frac{3}{2} - \lambda\right)(-6-\lambda) - (-3)(1)$$

$$= -9 - \frac{3}{2}\lambda + 6\lambda + \lambda^2 + 3$$

$$= \lambda^2 + \frac{9}{2}\lambda - 6 \quad \underline{\text{Ans}}$$

b)

$$C = \begin{pmatrix} 0 & 1 & 0 \\ 3 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix}$$

$$P_C(\lambda) = \det(C - \lambda I)$$

$$= \det\left(\begin{pmatrix} 0 & 1 & 0 \\ 3 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} -\lambda & 1 & 0 \\ 3 & 1-\lambda & 3 \\ 1 & 0 & 1-\lambda \end{pmatrix}$$

$$= -\lambda\begin{vmatrix} 1-\lambda & 3 \\ 0 & 1-\lambda \end{vmatrix} - 1\begin{vmatrix} 3 & 3 \\ 1 & 1-\lambda \end{vmatrix} + 0\begin{vmatrix} 3 & 1-\lambda \\ 1 & 0 \end{vmatrix}$$

$$= -\lambda(1-\lambda)(1-\lambda) - 1\big(3(1-\lambda) - 3.1\big) + 0$$

$$= -\lambda(1-\lambda-\lambda+\lambda^2) + 3 - 3 + 3\lambda$$

$$= -\lambda + 2\lambda^2 - \lambda^3 + 3\lambda$$

$$= -\lambda^3 + 2\lambda^2 + 2\lambda$$

$$= \lambda^3 - 2\lambda^2 - 2\lambda$$

c)

$$D = \begin{pmatrix} -\cos(a) & \sin(a) \\ \sin(a) & \cos(a \end{pmatrix}$$

$$P_D(\lambda) = \det\left(\begin{pmatrix} -\cos(a) & \sin(a) \\ \sin(a) & \cos(a) \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} -\cos(a) - \lambda & \sin(a) \\ \sin(a) & \cos(a) - \lambda \end{pmatrix}$$

$$= (-\cos(a) - \lambda)(\cos(a) - \lambda) - \sin(a)\sin(a)$$

$$= -(\cos(a))^2 + \cos(a)\lambda - \cos(a)\lambda + \lambda^2 - (\sin(a))^2$$

$$= -\cos^2(a) + \lambda^2 - (\sin(a))^2$$

$$= \lambda^2 - (\cos(a))^2 - (\sin(a))^2$$

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Exercise 5

$$det(A) = \frac{1}{3}$$

(a) $det(3A) = 3^4 \cdot det(A) = 54 \times \frac{1}{3} = 27$

$det(-A) = (-1)^4 det(A) = 1 \cdot \frac{1}{3} = \frac{1}{3}$

$det(A)^2 = det(A) \cdot det(A) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$det(A^{-1}) = \frac{1}{det(A)} = \frac{1}{1/3} = 3$

(b) $det(A) = \frac{1}{8}$

$det(2A) = 2^3 det(A) = 8 \cdot \frac{1}{8} = 1$

$det(-A) = (-1)^3 det(A) = -1 \cdot \frac{1}{8} = -\frac{1}{8}$

$det(A)^2 = det\, A \cdot det\, A = \frac{1}{8} \cdot \frac{1}{8} = \frac{1}{64}$

$det(A^{-1}) = \frac{1}{det(A)} = \frac{1}{1/8} = 8$

Ans