Assignment 1

Exercises

Q1 Let x be the percentage of the points earned by doing the homework assignments.

$$\begin{aligned}
&50\% \cdot 30\% + 40\% \cdot 40\% + x \cdot 30\% = 60\% \\
&0.5 \cdot 0.3 + 0.4 \cdot 0.4 + x \cdot 30\% = 0.6 \\
&0.15 + 0.16 + 0.3 \cdot x = 0.6 \\
&0.3 \cdot x = 0.6 - 0.31 \\
&x = \frac{0.29}{0.3} = 0.97
\end{aligned}$$

In percentage it will be $0.97 \cdot 100 = 97\%$ of the points in the homework assignments.

Q2 Prove that for all $n \in \mathbb{N}, \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$$P(n) = \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Base case $n = 0$

$$\sum_{k=0}^{0} k^2 = 0 = \frac{0(0+1)(2*0+1)}{6}$$

Let n be an arbitrary natural number, assuming $P(n)$ we want to prove $P(n+1)$, that is:

$$\sum_{k=0}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

We start from

$$\sum_{k=0}^{n+1} k^2 = \sum_{k=0}^{n} k^2 + (n+1)^2$$

Now we can use the inductive step $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$$\begin{aligned}\sum_{k=0}^{n+1} k^2 &= \sum_{k=0}^{n} k^2 + (n+1)^2 \\ &= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \\ &= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)(n(2n+1) + 6(n+1))}{6} = \\ &= \frac{(n+1)(2n^2 + n + 6n + 6)}{6} = \frac{(n+1)(2n^2 + 7n + 6)}{6} = \\ &= \frac{(n+1)(2n^2 + 4n + 3n + 2 * 3)}{6} = \frac{(n+1)(2n(n+2) + 3(n+2))}{6} \\ &= \frac{(n+1)(2n+3)(n+2)}{6}\end{aligned}$$

We have just proved by induction that for all $n \in \mathbb{N}, \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

Q3 Prove that $(x+y)(x-y) = (xx - yy)$

$(x+y)(x-y) = (x+y)(x+(-y))$ def of -
$= (x+(-y))(x+y)$ (by F2)
$= [(x+(-y))x] + [(x+(-y))y]$ (by F5)
$= [x(x+(-y))] + [y(x+(-y))]$ (by F2)
$= [(xx) + x(-y)] + [(yx) + (y(-y))]$ (by F5)
$= [(xx) + x(-y)] + [(xy) + (y(-y))]$ (by F2)
$= [(xx) + x(-y)] + (xy) + (y(-y))$ (remove parentheses by F1)
$= \{[(xx) + x(-y)] + xy\} + (y(-y))$ (by F1)
$= \{xx + [x(-y) + (xy)]\} + (y(-y))$ (by F1)
$= \{xx + [x((-y) + y)]\} + (y(-y))$ (by F5)
$= \{xx + [x(y + (-y))]\} + (y(-y))$ (by F2)
$= \{xx + [x(0)]\} + (y(-y))$ (by F4)
$= x(x+0) + y(-y)$ (by F5)
$= xx + y(-y)$ (by F3)
Now we just have to show that $xx + y(-y)$ is equal to $xx - yy$.

We will prove that $y(-y) = -(yy)$.
If $y(-y)$ is the additive inverse of yy then $yy + y(-y) = 0$.

Let's consider $yy + y(-y)$.
$= y(y + (-y))$ (by F5)
$= y0$ (by F4)
Therefore:
$yy + y(-y) = y0$

and $y0 = y(0+0)$ (by F3)
$= y0 + y0$ (by F5)
Then:
$y0 = y0 + y0$

By F4 we know that $y0 + (-(y0)) = 0$
Since $y0 = y0 + y0$;
$y0 + (-(y0)) = (y0 + y0) + (-y0)) = 0$
$= y0 + (y0 + (-y0)) = 0$ (by F1)

$= y0 + 0 = 0$ (by F4)
$y0 = 0$ (by F3)

Since $yy + y(-y) = y0$
and $y0 = 0$;
$yy + y(-y) = 0$
Therefore $y(-y)$ is the additive inverse of yy.
We also know that $yy + -(yy) = 0$
therefore
$y(-y) = -(yy)$

Since $xx + y(-y)$ and $y(-y) = -(yy)$ we conclude
$xx + (-(yy)) = xx - yy$ (by the definition of subtraction).

Q4 Bonus Exercise
If $a = 1, b = 2$ and $c = 3$
then we can write $1 + 2 = 3$ as $a + b = c$.
In the first step we multiply both sides with $(a - b)$ which is $1 - 2 = -1$.
That is $(a + b)(a - b) = c(a - b)$.
Expanding we obtain $a^2 - b^2 = ac - bc$ which is $1 - 4 = 3 - 6 = -3$.
Then we add $b^2 - ac$ to both sides obtaining:
$a^2 - b^2 + b^2 - ac = ac - bc + b^2 - ac$
$a^2 - ac = b^2 - bc$
that is $1 - 3 = 4 - 6 = -2$
Now we add ab to both sides resulting in:
$a^2 - ac + ab = b^2 - bc + ab$
which is $1 - 3 + 1 \cdot 2 = 4 - 6 + 1 \cdot 2 = 0$
Finally, both sides of the equality can be written as:

$$a(a + b - c) = b(b + a - c)$$

They are being divided by the common factor $(a + b - c)$ which in this case is equal to zero.
Since you can't divide a number by zero, you cannot conclude that $a = b$.