SATISH KUMAR

Exercise 1

$S \to \mathbb{R}$, $f(x) = \sqrt{9+x}$ for $S = [-9, +\infty)$

$g: T \to \mathbb{R}$, $g(x) = x^2$ for $T = \mathbb{R}$

$\text{dom}(f) = [-9, +\infty)$ and $\text{dom}(g) = \cancel{(+\infty, +\infty)}\ \mathbb{R}$

(a) $\text{dom}(f+g) = \text{dom}(\sqrt{9+x} + x^2)$

$= [-9, +\infty)$

$\text{dom}(f \circ g) = \text{dom}(\sqrt{9+x} \cdot x^2)$

$= [-9, +\infty)$

b) $(f \circ g)(x)$

$= (f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{9+x^2}$

$x = 0$

$(f \circ g)(0) = f(g(0)) = f(0^2) = \sqrt{9+(0)^2} = \sqrt{9} = 3$

$x = -4$

$(f \circ g)(-4) = f(g(-4)) = f((-4)^2) = \sqrt{9+(-4)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$

$x = 4$

$(f \circ g)(4) = f(g(4)) = f((4)^2) = \sqrt{9+(4)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$

c) No,

The function $f \circ g \neq g \circ f$

$$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{9+x^2}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{9+x}) = (\sqrt{9+x})^2 = 9+x$$

Exercise 2

There are two fixed point of f.

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```
(%i1) f(x) := exp(x - 2);
```

(%o1) $f(x):=\exp(x-2)$

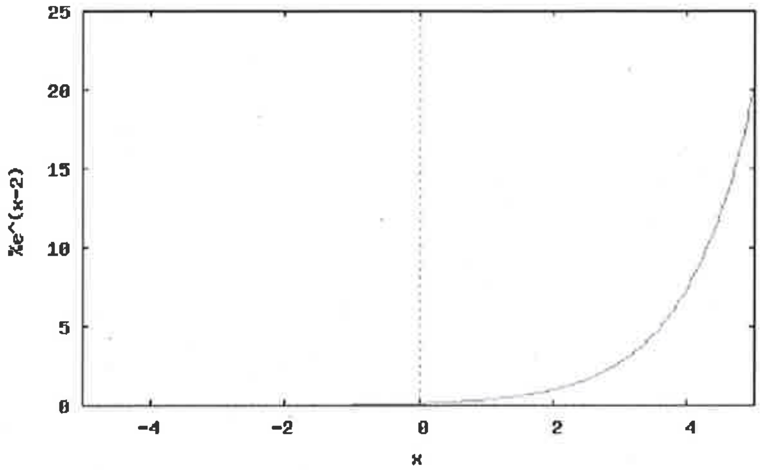
```
(%i2) g(x) := x;
```

(%o2) $g(x):=x$

```
(%i3) /* Plotting f(x) */

      wxplot2d(f(x), [x, -5, 5]);
```

(%t3)

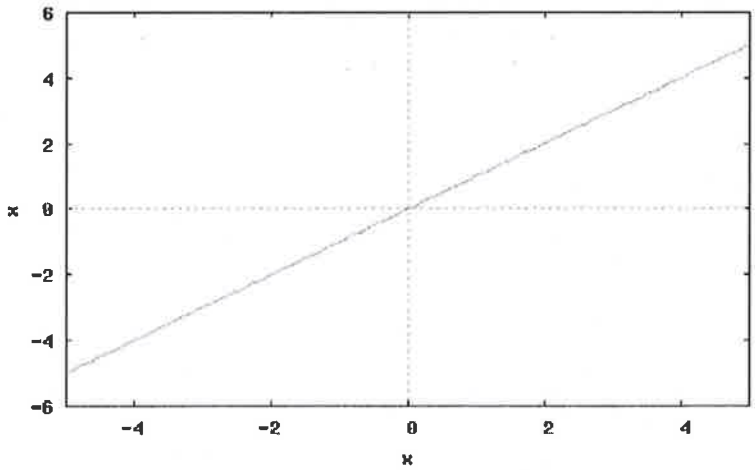


(%o3)

```
(%i5) /* Plotting g(x) */

      wxplot2d(g(x), [x, -5, 5]);
```

(%t5)

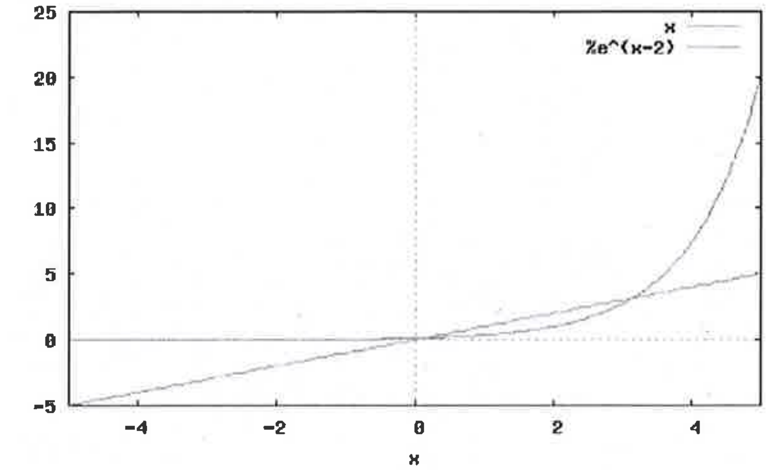


(%o5)

(%i4) /* This shows up intersection between f(x) and g(x) */

```
wxplot2d([g(x), f(x)], [x, -5, 5]);
```

(%t4)



(%o4)

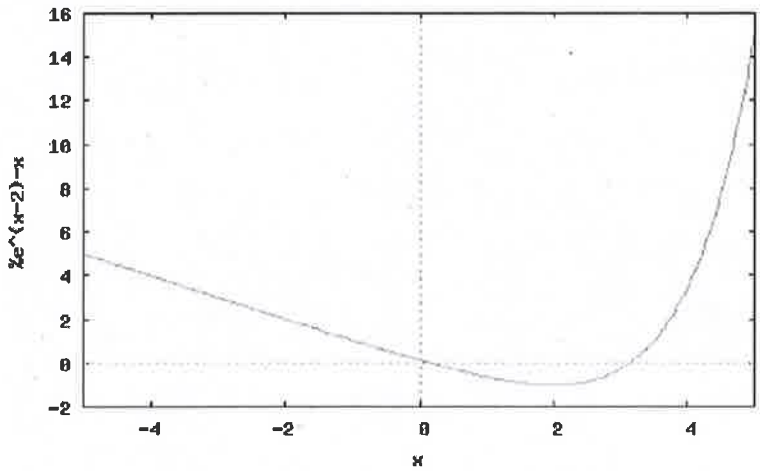
(%i6) h(x) := f(x) - x;

(%o6) $h(x):=f(x)-x$

(%i7) /* This is the function we need to study */

```
wxplot2d(h(x), [x, -5, 5]);
```

(%t7)



(%o7)

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Exercise 2

Since we are more familiar with Python (and because we have had some problems with Maxima), we decided to implement the *bisection algorithm* directly with Python. I think you have no problems understanding Python (even if you don't know it), because it's like pseudo code. The definition of a function starts with the keyword "def". Under the signature (heading) of each function, you have a description of the function and of the parameters (variables) the function requires. What ever starts with # and that is between triple quotes is a comment. If you need any help, just contact the authors.

The following is the code we have used:

```
import math as m

def h(x):
    """Function of which we want to find the root"""
    return m.exp(x - 2) - x


def same_signs(a, b):
    """Returns True if a and b have the same sign"""
    return a * b > 0


def bisect(f, a, b, tol=0.00001):
    """
    func: reference to the function
    a: first x of the domain for f (f(a) should have different sign of f(b))
    b: second x of the domain for f (f(b) should have different sign of f(a))
    tol: maximum acceptable error or tolerance (epsilon)
    this parameter is useful because not always we manage
    to obtain an root easily exactly, but we need an approximation
    """

    # print(f(a), "\n", f(b))

    e = (a + b) / 2 # point between a and b (estimation)

    # Distance between e and our root is at most (b - a) / 2
    max_error = (b - a) / 2

    while max_error > tol:
        # repeat loop until (b - a) / 2 <= tol

        if f(e) == 0:
            break

        if not same_signs(f(a), f(e)):
            b = e

        elif not same_signs(f(b), f(e)):
            a = e

        # Estimating a new point between a and b
        e = (a + b) / 2

        # Distance between e and our root is at most (b - a) / 2
        max_error = (b - a) / 2
    return e
Print (bisect(h,1,5))
```

The output when a=1 and b=5 is 3.14612.... If for example a=0 and b=1 we obtain 0.1585922.....

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Exercise 3

As we can see in graph the minimum is -0.5 and maximum is 1.5.

$$f(x) = \frac{3x}{x^2-2x+4}$$

$$\frac{d}{dx}f(x) = f'(x) = \frac{d}{dx}\left(\frac{3x}{x^2-2x+4}\right)$$

$$= 3\left(\frac{d}{dx}\left(\frac{x}{x^2-2x+4}\right)\right)$$

$$= 3\left(\frac{(x^2-2x+4)\frac{d}{dx}(x) - \frac{d}{dx}(x^2-2x+4)\cdot x}{(x^2-2x+4)^2}\right)$$

$$= 3\left(\frac{(x^2-2x+4)\cdot 1 - (2x-2)\cdot x}{(x^2-2x+4)^2}\right)$$

$$= 3\left(\frac{x^2-2x+4-2x^2+2x}{(x^2-2x+4)^2}\right)$$

$$= 3\left(\frac{-x^2+12}{(x^2-2x+4)^2}\right) = \left(\frac{-3x^2+12}{(x^2-2x+4)^2}\right)$$

$$\frac{-3x^2+12}{(x^2-2x+4)^2} = 0$$

$$-3x^2+12 = 0$$

$$-3x^2 = -12$$

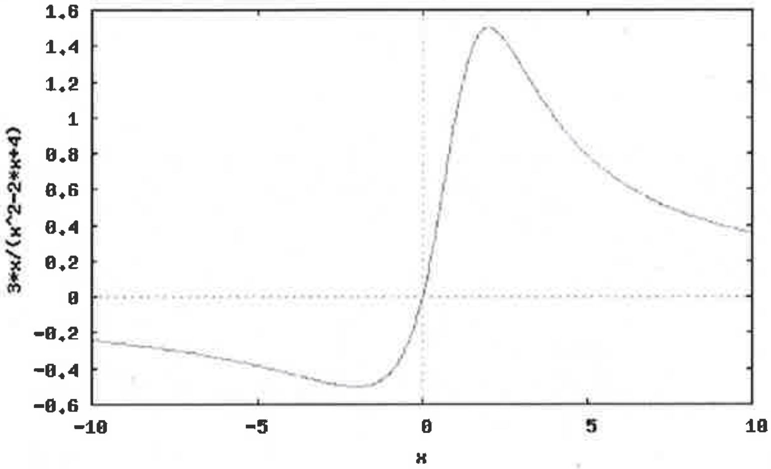
$$x^2 = 4$$

$$x = \pm 2$$

(%i1) /*Plotting and finding the minimum and the maximum of f(x) := (3x)/(x^2 - 2*x + 4)*/ f(x) := (3*x)/(x^2 - 2*x + 4)

(%o1) $f(x):=\frac{3\,x}{x^2-2\,x+4}$

(%i2) wxplot2d(f(x), [x, -10, 10]);

(%t2) 

(%o2)

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$$f(2) = \frac{3(2)}{(2)^2 - 2(2) + 4} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$f(-2) = \frac{3(-2)}{(-2)^2 - 2(-2) + 4} = \frac{-6}{12} = \frac{-1}{2} = -0.5$$

Bonus Exercise →

$$P(x) = \sum_{k=0}^{n} a_k x^k$$

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$$

Let $p(x)$ be a polynomical of odd degree. Let $l = \lim_{x \to -\infty} (p(x))$ and $R = \lim_{x \to +\infty} (p(x))$ And l is negative infinity and R is positive infinity for degree of ($p(x)$ is odd). Thus p has a positive and negative value $p(a) < 0 < p(b)$ for some a and b. p is also continuous. So by intermediate value theorm there exist a real number $c \in R$ that $a < c < b$ such that $p(c) = 0$.

⇒ If the degree of polynomical is even. The function behaves the same way as x approaches both positive and negatives infinity. If the conficient of the term with the greatest exponent is positive. $p(x)$ approach positive infinity at the both ends. If the coefficient is negative, $p(x)$ approach negatives infinity both sides.

=> If the degrees of the polynomial function is odd. The function exhibits opposite behaviour as x approach positive and negative infinity. If cofficient is positive the function increase as x increase and decrease x decrease. And if cofficient is negative the function decrease as x increase and increase as x decrease.