SATISH KUMAR



```
(%i1) x: 256;
      e: 10^(-15);
      k: 1;
      a[0]: 1;
      b[0]: x/a[0];

      av1(a,b) := (a+b)/2;
      av2(a,b) := 2*(a*b)/(a+b);

      a[n] := av1(a[n-1],b[n-1]);
      b[n] := x/a[n];

      while abs(a[k]-b[k]) >= e do
      (k:k+1);
      done;
      display(k,float(a[k]));
```

(%o1) 256

(%o2) $\frac{1}{1000000000000000}$

(%o3) 1

(%o4) 1

(%o5) 256

(%o6) $\mathrm{av1}(a,b):=\frac{a+b}{2}$

(%o7) $\mathrm{av2}(a,b):=\frac{2\,(a\,b)}{a+b}$

(%o8) $a_n:=\mathrm{av1}(a_{n-1},b_{n-1})$

(%o9) $b_n:=\frac{x}{a_n}$

(%o10) *done*

(%o11) *done*

$k=9$

$\mathrm{float}\left(\frac{488447341304082595702063324879[570\ \mathrm{digits}]235568570066354256638774280193}{305279588315051622313789577959[569\ \mathrm{digits}]782519602746476827920035086848}\right)=16.0$

(%o12) *done*

JATISH KUMAR



```
--> x: 256;
    e: 10^(-15);
    k: 1;
    a[0]: 5;
    b[0]: x/a[0];

    av1(a,b) := (a+b)/2;
    av2(a,b) := 2*(a*b)/(a+b);

    a[n] := av1(a[n-1],b[n-1]);
    b[n] := x/a[n];

    while abs(a[k]-b[k]) >= e do
    (k:k+1);
    done;
    display(k,float(a[k]));
```

(%o1) 256

(%o2) $\frac{1}{1000000000000000}$

(%o3) 1

(%o4) 5

(%o5) $\frac{256}{5}$

(%o6) $\mathrm{av1}(a, b) := \frac{a+b}{2}$

(%o7) $\mathrm{av2}(a, b) := \frac{2\,(a\,b)}{a+b}$

(%o8) $a_n := \mathrm{av1}(a_{n-1}, b_{n-1})$

(%o9) $b_n := \frac{x}{a_n}$

(%o10) *done*

(%o11) *done*

$k = 6$

$\mathrm{float}\left(\frac{2094135925513637135563968214139439414392642928834622526499100158087627462852239752961}{130883495344602320694128283100846063657695674716548811688058670193506296708644057920}\right) = 16.0$

(%o12) *done*

```
--> x: 256;
    e: 10^(-15);
    k: 1;
    a[0]: 16;
    b[0]: x/a[0];

    av1(a,b) := (a+b)/2;
    av2(a,b) := 2*(a*b)/(a+b);

    a[n] := av1(a[n-1],b[n-1]);
    b[n] := x/a[n];

    while abs(a[k]-b[k]) >= e do
    (k:k+1);
    done;
    display(k,float(a[k]));
```

(%o1) 256

(%o2) $\frac{1}{1000000000000000}$

(%o3) 1

(%o4) 16

(%o5) 16

(%o6) $\mathrm{av1}(a,b):=\frac{a+b}{2}$

(%o7) $\mathrm{av2}(a,b):=\frac{2\,(a\,b)}{a+b}$

(%o8) $a_n:=\mathrm{av1}(a_{n-1},b_{n-1})$

(%o9) $b_n:=\frac{x}{a_n}$

(%o10) *done*

(%o11) *done*

$k=1$

float(16)=16.0

(%o12) *done*

RATISH KUMAR 

## Exercise 1

Yes, It makes impact on the result. If $a_0$ value is small than it takes more steps. And if $a_0$ is big value than it takes less steps.

## Exercise 2

We have to need 31 books to build a stack where the top book extends the table by twice its full length.

The position of ~~top~~ books is

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\cdots\cdots+\frac{1}{62} \geq 2$$

$$\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{31}\right) \geq 2$$

$$= \frac{1}{2}\cdot\sum_{k=1}^{31}\frac{1}{k} \cancel{\geq} 2$$

SATISH KUMAR 

Exercise 3

Show that $\sum_{k=1}^{\infty} \frac{k}{2^k} = 2$

1) Formula for the partial sum is

$$S_n = \frac{2^{n+1} - n - 2}{2^n}$$

2) Induction

$$S_n = \frac{2^{n+1} - n - 2}{2^n} \quad \text{for all } n \in N$$

The base case $n = 1$

$$S_1 = \frac{2^{1+1} - 1 - 2}{2^1} = \frac{2^2 - 3}{2} \Rightarrow \frac{1}{2} \checkmark$$

Induction step.

$$S_{n+1} = \frac{2^{n+1+1} - (n+1) - 2}{2^{n+1}}$$

we have

$$S_{n+1} \Rightarrow S_n + \frac{n+1}{2^{n+1}}$$

I.H

$$= \frac{2^{n+1} - n - 2}{2^n} + \frac{n+1}{2^{n+1}}$$

# ZATISH KUMAR



$$= \frac{2\cdot 2^{n+1} - 2n - 4 + n + 1}{2^{n+1}}$$

$$= \frac{2^{n+2} - n - 3}{2^{n+1}}$$ Proved

3) Limit

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty}\left(\frac{2^{n+1} - n - 2}{2^n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2\cdot 2^n}{2^n} - n\cdot\frac{1}{2^n} - 2\cdot\frac{1}{2^n}\right)$$

$$= 2\lim_{n\to\infty}(1) - \lim_{n\to\infty}(n)\cdot\lim_{n\to\infty}\left(\frac{1}{2^n}\right) - 2\lim_{n\to\infty}\left(\frac{1}{2^n}\right)$$

$$= 2\cdot 1 - \infty\cdot 0 - 2\cdot 0$$

$$= 2$$ Ans