SATISH KUMAR

Exercise 1

$$-x+y+2z=a$$
$$-16x+2y+3z+w=2b$$
$$7x-y+4w=-9c$$

we can write.

$$\begin{pmatrix} -1 & 1 & 2 & 0 \\ -16 & 2 & 3 & 1 \\ 7 & -1 & 0 & 4 \end{pmatrix} = \begin{pmatrix} a \\ 2b \\ -9c \end{pmatrix}$$

Multiply row 1 by -16 and row 2

$$\begin{pmatrix} -1 & 1 & 2 & 0 \\ 0 & -14 & -29 & 1 \\ 7 & -1 & 0 & 4 \end{pmatrix} = \begin{pmatrix} a \\ -16a+2b \\ -9c \end{pmatrix}$$

Multiply row 1 by 7 and add row 3

$$\begin{pmatrix} -1 & 1 & 2 & 0 \\ 0 & -14 & -29 & 1 \\ 0 & 6 & 14 & 4 \end{pmatrix} = \begin{pmatrix} a \\ 2b-16a \\ 7a-9c \end{pmatrix}$$

Multiply row 2 by $\frac{3}{7}$ and add in row 3

$$\begin{pmatrix} -1 & 1 & 2 & 0 \\ 0 & -14 & -29 & 1 \\ 0 & 0 & 11/7 & \frac{31}{7} \end{pmatrix} = \begin{pmatrix} a \\ 2b-16a \\ \frac{6b+a-63c}{7} \end{pmatrix}$$

SATISH KUMAR

In this w is free variable.
and we use backward substitution.

$$\frac{11}{7}z + \frac{31}{7}w = \frac{6b+a-63c}{7}$$

$$\frac{11z+31w}{7} = \frac{6b+a-63c}{7}$$

$$11z+31w = 6b+a-63c$$

$$11z = 6b+a-63c-31w$$

$$z = \frac{6b+a-63c-31w}{11}$$

$$-14y-29z+w = 2b-16a$$

$$-14y = 2b-16a+29z-w$$

$$-14y = 2b-16a+29\left(\frac{6b+a-63c-31w}{11}\right)-w$$

$$-14y = 2b-16a+\frac{174b+29a-1827c-899w}{11}-w$$

$$-14y = \frac{22b-176a+174b+29a-1827c-899w-11w}{11}$$

$$-14y = \frac{196b-147a-1827c-910w}{11}$$

$$y = \frac{-196b+147a+1827c+910w}{154}$$

$$y = \frac{-28b+21a+261c+130w}{22}$$

JATISH KUMAR

$$-x+y+2z=a$$

$$-x=a-y-2z$$

$$-x=-(y+2z-a)$$

$$x=y+2z-a$$

$$x=\frac{-28b+21a+261c+130w}{22}+2\left(\frac{6b+a-63c-31w}{11}\right)-a$$

$$=\frac{-28b+21a+261c+130w+24b+4a-252c-124w-22a}{22}$$

$$=\frac{-4b+3a+9c+6w}{22}$$

(b) Now we have

$a=12,\ 2b=2 \Rightarrow b=1,\ -9c=-18 \Rightarrow c=2$

$$\begin{pmatrix}\frac{-4b+3a+9c}{22}\\ \frac{-28b+21a+261c}{22}\\ \frac{6b+a-63c}{11}\end{pmatrix}+\begin{pmatrix}6/22\,w\\ 130/22\,w\\ 6/22\,w\end{pmatrix}$$

$$\begin{pmatrix}25/11\\ 378/11\\ -\frac{108}{11}\end{pmatrix}+\begin{pmatrix}6/22\\ 130/22\\ 6/22\end{pmatrix}w$$

Ans

SATISH KUMAR

Exercise 2

a) Let $U = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $V = \begin{pmatrix} 4 \\ 10 \end{pmatrix}$

$$\cos\theta = \frac{U \cdot V}{\|U\| \cdot \|V\|}$$

$$\|U\| = \sqrt{(3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$\|V\| = \sqrt{(4)^2 + (10)^2} = \sqrt{16+100} = \sqrt{116}$$

$$\cos\theta = \frac{4 \cdot 3 + 2 \cdot 10}{\sqrt{13} \cdot \sqrt{116}} \Rightarrow \frac{32}{2\sqrt{377}} \Rightarrow 34.5^\circ$$

b) Let $U = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$, $V = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

$$\|U\| = \sqrt{(3)^2 + (-2)^2 + (0)^2} = \sqrt{9+4} = \sqrt{13}$$

$$\|V\| = \sqrt{(1)^2 + (7)^2 + (2)^2} = \sqrt{1+49+4} = \sqrt{54}$$

$$\cos\theta = \frac{3 \cdot 1 + 7 \cdot (-2) + 0 \cdot 2}{\sqrt{13} \cdot \sqrt{54}} \Rightarrow \frac{-11}{3\sqrt{78}} \Rightarrow 114.5^\circ$$

c) Let $U = \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix}$, $V = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$

Not defined.

SATISH KUMAR

Exercise 3

a) Yes, let $x, y, z \in R$

$$V_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + V_2 \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + V_3 \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

has solution $V_1 = x$, $V_2 = y/2$, $V_3 = -z/3$

b) Yes, let $x, y, z \in R$

$$V_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + V_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + V_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$V_1 = x$
$V_2 = y$
$V_3 = z - x - y$

c) No, let $x, y, z \in R$

$$V_1 \begin{pmatrix} 0 \\ -3 \\ -8 \end{pmatrix} + V_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$V_1 = -y/3,\ \frac{-z}{8}$, $V_2 = x$ and the third component is zero.

d) Yes, let $x, y, z \in R$

$$V_1 \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + V_2 \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} + V_3 \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + V_4 \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

SATISH KUMAR

It is

$$\begin{pmatrix} 3 & 5 & 1 & 3 \\ 0 & 3 & -1 & 2 \\ 1 & 0 & 4 & 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Multiply row 3 by -3 and add row 1

$$\begin{pmatrix} 0 & 5 & -11 & -6 \\ 0 & 3 & -1 & 2 \\ 1 & 0 & 4 & 3 \end{pmatrix} = \begin{pmatrix} -3z+x \\ y \\ z \end{pmatrix}$$

Multiply row 2 by $-\frac{3}{5}$ and add row 1

$$\begin{pmatrix} 0 & 0 & 28/5 & 28/5 \\ 0 & 3 & -1 & 2 \\ 1 & 0 & 4 & 3 \end{pmatrix} = \begin{pmatrix} \frac{9z-3x+5y}{5} \\ y \\ z \end{pmatrix}$$

V_4 is free variable. and it has many solution.

e) ~~Yet~~ Yes, let $x, y, z \in R$

$$V_1 \begin{pmatrix} 11 \\ 22 \\ 1 \end{pmatrix} + V_2 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + V_3 \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} + V_4 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

It is

$$\begin{pmatrix} 11 & 1 & 2 & 1 \\ 22 & -2 & 4 & 0 \\ 1 & 1 & 4 & 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Multiply row 1 by -2 and add row 2

$$\begin{pmatrix} 11 & 1 & 2 & 1 \\ 0 & -4 & 0 & -2 \\ 1 & 1 & 4 & 1 \end{pmatrix} = \begin{pmatrix} x \\ y-2x \\ z \end{pmatrix}$$

Multiply row 1 by $-\frac{1}{11}$ and add row 3

$$\begin{pmatrix} 11 & 1 & 2 & 1 \\ 0 & -4 & 0 & -2 \\ 0 & \frac{10}{11} & \frac{42}{11} & \frac{10}{11} \end{pmatrix} = \begin{pmatrix} x \\ y-2x \\ z-\frac{1}{11}x \end{pmatrix}$$

Multiply row 2 by $\frac{10}{44}$ and add row 3

$$\begin{pmatrix} 11 & 1 & 2 & 1 \\ 0 & -4 & 0 & -2 \\ 0 & 0 & \frac{42}{11} & \frac{10}{22} \end{pmatrix} = \begin{pmatrix} x \\ y-2x \\ \frac{10y-6x+44z}{44} \end{pmatrix}$$

v_4 is free variable and has many solution.

Exercise 4

⑥ Column space of matrix is span the coloumn of matrix

$$C(A) = \{ y \in R^m : y = \alpha_1 y_1 + \alpha_2 y_2 + \cdots + \alpha_n y_n \mid \alpha_i \in R \}$$

SATISH KUMAR

let we $x = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$

and we have $Ax = y$

$Ax = y$ we can write like.

$= (c_1, c_2, c_3 \cdots c_n) \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \mid \alpha_i \in R^m$

$= \alpha_1 c_1 + \alpha_2 c_2 + \cdots + \alpha_n c_n \Rightarrow y$

that is equal to y because it is span$\{c_1, c_2, \cdots c_n\}$ and we can say

$C(A) = \{ y \in R^n : y = Ax, x \in R^m \}$

Exercise 5

$$f(x) = \begin{pmatrix} 2x + 1x_2 + 2x_3 - x_4 \\ x_1 + 2x_3 - 2x_4 \\ 3x_1 + 5x_2 + x_3 - 3x_4 \end{pmatrix}$$

we can write

$$\begin{pmatrix} 2 & 1 & 2 & -1 \\ 1 & 0 & 2 & -2 \\ 3 & 5 & 1 & -3 \end{pmatrix}$$

Multiply row 2 by -3 and add row 3

$$\begin{pmatrix} 2 & 1 & 2 & -1 \\ 1 & 0 & 2 & -2 \\ 0 & 5 & -5 & 3 \end{pmatrix}$$

Multiply row 1 by $\frac{-1}{2}$ and add row 2

$$\begin{pmatrix} 2 & 1 & 2 & -1 \\ 0 & -\frac{1}{2} & 1 & -3/2 \\ 0 & 5 & -5 & 3 \end{pmatrix}$$

Multiply row 2 by 10 and add row 3

$$\begin{pmatrix} 2 & 1 & 2 & -1 \\ 0 & 1/2 & 1 & -3/2 \\ 0 & 0 & 5 & -12 \end{pmatrix}$$

In this function x_4 is free variable.

a) Basis of column space is

$\cancel{C(A)}$ $C(A) = \left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

because this column are linearly independent.

Dimension of column space is $\underline{\underline{3}}$

because we have 3 linearly independent coloumn.

SATISH KUMAR

b) Basis of null space is

$$5x_3 - 12x_4 = 0$$

$$x_3 = \frac{12}{5}x_4$$

$$-\frac{1}{2}x_2 + x_3 - \frac{3}{2}x_4 = 0$$

$$-\frac{1}{2}x_2 = \frac{3}{2}x_4 - x_3$$

$$-\frac{1}{2}x_2 = \frac{3}{2}x_4 - \frac{12}{5}x_4 = \frac{15x_4 - 24x_4}{10} = -\frac{9}{10}x_4$$

$$x_2 = \frac{9}{5}x_4$$

$$2x_1 + x_2 + 2x_3 - x_4 = 0$$

$$2x_1 = x_4 - 2x_3 - x_2$$

$$2x_1 = x_4 - \frac{24}{5}x_4 - \frac{9}{5}x_4 = \frac{5x_4 - 24x_4 - 9x_4}{5} = \frac{-28}{5}x_4$$

$$x_1 = -\frac{14}{5}x_4$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} -14/5\, x_4 \\ 9/5\, x_4 \\ 12/5\, x_4 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + x_4 \begin{bmatrix} -14/5 \\ 9/5 \\ 12/5 \\ 1 \end{bmatrix} = 0$$

The basis of null space is $\left\{ \begin{bmatrix} -14/5 \\ 9/5 \\ 12/5 \\ 1 \end{bmatrix} \right\}$

Dimension of null space is $\underline{\underline{1}}$ Ans

SATISH KUMAR

Exercise 6

a) ~~Yes~~ No

$$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Multiply row 1 by 2 and add row 2

$$\begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

It has no solution. than it is not in the column space of matrix

b) Yes,

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & -2 \\ 0 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Multiply row 1 by -2 and add row 2

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -4 \\ 0 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Multiply row 2 by $\frac{3}{2}$ and add row 3

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -4 \\ 0 & 0 & -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 5/2 \end{pmatrix}$$

It ~~has~~ is in the Column space of matrix.

SATISH KUMAR

c) No,

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

It is not in the column space because in 6th row is zero ~~its~~ ~~not~~ equal ~~to 1~~ and it has no solution.