SATISH KUMAR

(a) $x - 3y = -2$
$2x - 3y = 5$

$A = \begin{pmatrix} 1 & -3 \\ 2 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

we can multiply equation 1 by $(-2) = \frac{-2}{2} = -\frac{a_{21}}{a_{11}}$ and add to equation 2. And this is $\mathcal{E}_{21}\left(\frac{-a_{21}}{a_{11}}\right)$

$$\left(\begin{array}{cc|c} 1 & -3 & -2 \\ 2 & -3 & 5 \end{array}\right) \xRightarrow{\mathcal{E}_{21}(-2)} \left(\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 3 & 9 \end{array}\right)$$

then $x - 3y = -2$
$0 \cdot x + 3y = 9$

It is upper triangular and use backward substitution.

$0 \cdot x + 3y = 9 \Rightarrow 3y = 9 \Rightarrow y = \frac{9}{3} \Rightarrow y = 3$

$x - 3y = -2 \Rightarrow x - 3(3) = -2$

$\Rightarrow x = -2 + 9 \Rightarrow x = 7$

$y = 3$
$x = 7$ Proved

(b) $-x + 3y = 1$

$5x + y = 3$

$$\left(\begin{array}{cc|c} -1 & 3 & 1 \\ 5 & 1 & 3 \end{array}\right) \xRightarrow{E_{21}(5)} \left(\begin{array}{cc|c} -1 & 3 & 1 \\ 0 & 16 & 8 \end{array}\right)$$

we ~~can~~ multiply first equation by 5 and add in 2nd equation. that is

$= \frac{5}{5} = 1 = -\frac{a_{21}}{a_{11}}$ .and we use

$E_{21}\left(-\frac{a_{21}}{a_{11}}\right)$

then we can

$$-x + 3y = 1$$
$$0 \cdot x + 16y = 8$$

It is upper triangular and we Can use backward substitution

$0 \cdot x + 16y = 8 \Rightarrow 16y = 8 \Rightarrow y = \frac{8}{16} = y = \frac{1}{2}$

$-x + 3y = 1 \Rightarrow -x + 3\left(\frac{1}{2}\right) = 1$

$\Rightarrow -x = 1 - \frac{3}{2} \Rightarrow -x = \frac{2-3}{2}$

$= -x = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$

Proved.