ZATISH KUMAR

Exercise 1 :-)

a) $I_{rs}A$ is effect is that it takes row s of A, put it into row r and replace everything with zero.

b) $(I_{rs}+I_{sr})A$ effect that it takes row s of A put it into row r and takes row r of A put it into row s and replace other with ~~z~~ zero.

c) $P_{rs}A$ effect that it takes row s of A put it into row r and takes row r of A put it into row s and ~~other~~ ~~Value~~ - + keep other row same.

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Bonus

Exercise 2

(a)

$$x - 3y = -2$$
$$2x - 3y = 5$$

$$A = \begin{pmatrix} 1 & -3 \\ 2 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

we can multiply equation 1 by $(-2) = \frac{-2}{2} = -\frac{a_{21}}{a_{11}}$ and add to equation 2 And this is $E_{21}\left(\frac{-a_{21}}{a_{11}}\right)$

$$\left(\begin{array}{cc|c} 1 & -3 & -2 \\ 2 & -3 & 5 \end{array}\right) \xRightarrow{E_{21}(-2)} \left(\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 3 & 9 \end{array}\right)$$

then

$$x - 3y = -2$$
$$0 \cdot x + 3y = 9$$

It is upper triangular and use backward substitution.

$$0 \cdot x + 3y = 9 \Rightarrow 3y = 9 \Rightarrow y = \frac{9}{3} \Rightarrow y = 3$$

$$x - 3y = -2 \Rightarrow x - 3(3) = -2$$
$$\Rightarrow x = -2 + 9 \Rightarrow x = 7$$

$$y = 3$$
$$x = 7$$

Proved

(b) $-x + 3y = 1$
$5x + y = 3$

$$\left(\begin{array}{cc|c} -1 & 3 & 1 \\ 5 & 1 & 3 \end{array}\right) \overset{\mathcal{E}_{21}(5)}{\Longrightarrow} \left(\begin{array}{cc|c} -1 & 3 & 1 \\ 0 & 16 & 8 \end{array}\right)$$

we ~~can~~ multiply first equation by 5 and add in 2nd equation. that is $= \frac{5}{5} = -1 = -\frac{a_{21}}{a_{11}}$ and we use $\mathcal{E}_{21}\left(-\frac{a_{21}}{a_{11}}\right)$

then we can

$$-x + 3y = 1$$
$$0 \cdot x + 16y = 8$$

It is upper triangular and we can use backward substitution

$$0 \cdot x + 16y = 8 \Rightarrow 16y = 8 \Rightarrow y = \frac{8}{16} = y = \frac{1}{2}$$

$$-x + 3y = 1 \Rightarrow -x + 3\left(\frac{1}{2}\right) = 1$$

$$\Rightarrow -x = 1 - \frac{3}{2} \Rightarrow -x = \frac{2-3}{2}$$

$$= -x = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

Proved.

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Exercise 3 :-

$$x+2y+z=1$$
$$2x+y+5z=2$$
$$2x+2y-3z=4$$

Multiply 1st equation by -2 and add in 2nd equation. that is $\frac{-2}{2} = -1 = -\frac{a_{21}}{a_{11}}$

That equation multiplying $Ax=b$ by $E_{21}\left(\frac{-a_{21}}{a_{11}}\right)$

$$x+2y+z=1$$
$$-5y+3z=0$$
$$2x+2y-3z=4$$

Multiply first equation by -2 and add in equation 3. and we use

$$E_{31}\left(\frac{-a_{31}}{a_{11}}\right) = \frac{-2}{2} = -\frac{a_{31}}{a_{11}}$$

$$x+2y+z=1$$
$$-5y+3z=0$$
$$-2y-5z=2$$

Multiple second equation by $-\frac{2}{5}$ and add in equation 3. and we use

$$\frac{-\frac{2\times 5}{5}}{2} = -\frac{a_{31}}{a_{21}} \Rightarrow E_{31}\left(\frac{-a_{31}}{a_{21}}\right)$$

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$$x + 2y + z = 1$$
$$-5y + 3z = 0$$
$$-\frac{31}{5} z = 2$$

Now we have uppar triangular and we can use backward substitution.

$$-\frac{31}{5} z = 2 \Rightarrow z = 2 \times \frac{5}{-31} \Rightarrow z = -\frac{10}{31}$$

$$-5y + 3z = 0 \Rightarrow -5y + 3\left(-\frac{10}{31}\right) = 0$$

$$\Rightarrow -5y = \frac{30}{31} \Rightarrow y = \frac{30}{(31)(-5)} \Rightarrow \frac{30}{-155}$$

$$\Rightarrow y = \frac{-30}{155}$$

$$x + 2y + z = 1 \Rightarrow x + 2\left(\frac{-30}{155}\right) + \left(\frac{-10}{31}\right) = 1$$

$$\Rightarrow x + \frac{(-60) + (-50)}{155} = 1 \Rightarrow x - \frac{110}{155} = 1$$

$$\Rightarrow x = \cancel{\frac{1 + 110}{45}} \; 1 + \frac{110}{155} \Rightarrow x = \frac{155 + 110}{155}$$

$$\Rightarrow x = \frac{265}{155} \Rightarrow x = \frac{53}{31}$$

Ans

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Exercise 4:

$$4\sin(\alpha) + 3\tan(\beta) + 2\cos(\gamma) = 1$$
$$-\sin(\alpha) + 2\tan(\beta) - \cos(\gamma) = 2$$
$$\sin(\alpha) + 2\tan(\beta) - 3\cos(\gamma) = -2$$

Let $x = \sin(\alpha)$, $y = \tan(\beta)$, $z = \cos(\gamma)$
then we have

$$4x + 3y + 2z = 1$$
$$-x + 2y - z = 2$$
$$x + 2y - 3z = -2$$

Multiply 2nd equation by 4 and add with 1st equation $\frac{-4}{+4} = \frac{-a_{21}}{a_{11}} = \mathcal{E}_{21}\left(\frac{-a_{21}}{a_{11}}\right)$

$$4x + 3y + 2z = 1$$
$$11y - 2z = 9$$
$$x + 2y - 3z = -2$$

Multiply 3rd equation by -4 and add with first equation. $\frac{-4}{4} = \frac{-a_{31}}{a_{11}} = \mathcal{E}_{31}\left(\frac{-a_{31}}{a_{11}}\right)$

$$4x + 3y + 2z = 1$$
$$11y - 2z = 9$$
$$-5y + 14z = 9$$

Multiply 3rd equation by $\frac{11}{5}$ and add in equation 2nd $= \frac{\frac{11}{5} \times 5}{-11} = -\frac{a_{31}}{a_{21}} = \varepsilon_{31}\left(\frac{-a_{31}}{a_{21}}\right)$

$$4x + 3y + 2z = 1$$
$$11y - 2z = 9$$
$$\frac{144}{5} z = \frac{144}{5}$$

Now we have upper triangular and we can use backward substitution.

$\frac{144}{5} z = \frac{144}{5} \Rightarrow z = \frac{144}{5} \times \frac{5}{144} \Rightarrow z = 1$

$11y - 2z = 9 \Rightarrow 11y - 2(1) = 9 \Rightarrow 11y = 9 + 2$

$\Rightarrow y = \frac{11}{11} \Rightarrow y = 1$

$4x + 3y + 2z = 1 \Rightarrow 4x + 3(1) + 2(1) = 1$

$\Rightarrow 4x = 1 - 5 \Rightarrow 4x = -4 \Rightarrow x = \frac{-4}{4}$

$\Rightarrow x = -1$

$z = 1,\ y = 1,\ x = -1$ than

no(d) satisfied with this solution.

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Exercise 5

a)

$$x - 2y = b_1$$
$$2x - 9y = b_2$$
$$7x - 3y = b_3$$
$$2x - y = b_4$$

Multiply 1st equation by -2 and add in second equation. $\frac{-2}{\cancel{2}} = \frac{-a_{21}}{a_{11}} = \mathcal{E}_{21}\left(\frac{-a_{21}}{a_{11}}\right)$

$$x - 2y = b_1$$
$$-5y = -2b_1 + b_2$$
$$7x - 3y = b_3$$
$$2x - y = b_4$$

than

$$-5y = -2b_1 + b_2 \Rightarrow y = \frac{2b_1 - b_2}{5}$$

$$x - 2y = b_1 \Rightarrow x - 2\left(\frac{2b_1 - b_2}{5}\right) = b_1$$

$$\Rightarrow x = b_1 + \frac{4b_1 - 2b_2}{5} = \frac{5b_1 + 4b_1 - 2b_2}{5}$$

$$\Rightarrow \frac{9b_1 - 2b_2}{5}$$

$$7x - 3y = b_3$$

$$7\left(\frac{9b_1 - 2b_2}{5}\right) - 3\left(\frac{2b_1 - b_2}{5}\right) = b_3$$

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$$\frac{63b_1 - 14b_2}{5} - \frac{6b_1 + 3b_2}{5} = b_3$$

$$\frac{63b_1 - 14b_2 - 6b_1 + 3b_2}{5} = b_3$$

$$\frac{57b_1 - 11b_2}{5} = b_3$$

$$0 = 5b_3 - 57b_1 + 11b_2$$

$$2x - y = b_4$$

$$2\left(\frac{9b_1 - 2b_2}{5}\right) - \left(\frac{2b_1 - b_2}{5}\right) = b_4$$

$$\frac{18b_1 - 4b_2}{5} - \frac{2b_1 + b_2}{5} = b_4$$

$$\frac{18b_1 - 4b_2 - 2b_1 + b_2}{5} = b_4$$

$$16b_1 - 3b_2 = 5b_4$$

$$\Rightarrow \quad 0 = 5b_4 - 16b_1 + 3b_2$$

This system is if b_3 and b_4 is true.

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b)
$$x_1 - 3x_2 + 5x_3 = b_1$$
$$x_1 + 3x_2 - x_3 = b_2$$
$$-x_1 + 2x_2 + 3x_3 = b_3$$

Multiply first equation with -1 and add in 2^{nd} equation $\frac{-1}{1} = -\frac{a_{21}}{a_{11}} = E_{21}\left(\frac{-a_{21}}{a_{11}}\right)$

$$x_1 - 3x_2 + 5x_3 = b_1$$
$$6x_2 - 8x_3 = b_2 - b_1$$
$$-x_1 + 2x_2 + 3x_3 = b_3$$

Multiply first equation by $+1$ and add in 1^{st} equation. $\frac{-1}{1} = -\frac{a_{31}}{a_{11}} = E_{31}\left(\frac{-a_{31}}{a_{11}}\right)$

$$x_1 - 3x_2 + 5x_3 = b_1$$
$$6x_2 - 8x_3 = b_2 - b_1$$
$$-x_2 + 8x_3 \quad b_1 + b_3$$

~~Add~~ equation Multiply 3^{rd} equation by 6 and add with 2^{nd} equation: $\frac{6}{6} = -\frac{a_{31}}{a_{21}}$

$$x_1 - 3x_2 + 5x_3 = b_1$$
$$6x_2 - 8x_3 = b_2 - b_1$$
$$40x_3 = 5b_1 + b_2 + 6b_3$$

$$40x_3 = 5b_1 + b_2 + 6b_3$$
$$x_3 = \frac{5b_1 + b_2 + 6b_3}{40}$$

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$$6x_2 - 8x_3 = b_2 - b_1$$

$$6x_2 - 8\left(\frac{5b_1 + b_2 + 6b_3}{5}\right) = b_2 - b_1$$

$$6x_2 = b_2 - b_1 + \left(\frac{40b_1 + 8b_2 + 48b_3}{5}\right)$$

$$6x_2 = \frac{5b_2 - 5b_1 + 40b_1 + 8b_2 + 48b_3}{5}$$

$$x_2 = \frac{35b_1 + 13b_2 + 48b_3}{30}$$

$$x_1 - 3x_2 + 5x_3 = b_1$$

$$x_1 - 3\left(\frac{35b_1 + 13b_2 + 48b_3}{30}\right) + 5\left(\frac{5b_1 + b_2 + 6b_3}{5}\right) = b_1$$

$$x_1 - \frac{35b_1 - 13b_2 - 48b_3 + 50b_1 + 10b_2 + 60b_3}{10} = b_1$$

$$x_1 + \frac{15b_1 - 3b_2 + 12b_3}{10} = b_1$$

$$x_1 = b_1 - \left(\frac{15b_1 - 3b_2 + 12b_3}{10}\right)$$

$$x_1 = \frac{10b_1 - 15b_1 + 3b_2 - 12b_3}{10}$$

$$x_1 = \frac{-5b_1 + 3b_2 - 12b_3}{10}$$

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Exercise 6

$$A|I = \left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 3 & 1 & 4 & 0 & 0 & 1 \end{array}\right)$$

Multiply row 2 ~~and~~ by -1 and add row 1

$$E_{21} = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 3 & 1 & 4 & 0 & 0 & 1 \end{array}\right)$$

Multiply row 2 by -3 and add row 3

$$E_{31} = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & -8 & -2 & 0 & -3 & 1 \end{array}\right)$$

Multiply row 1 by -1 and add row 2

$$E_{21} = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 3 & 3 & -1 & 2 & 0 \\ 0 & -8 & -2 & 0 & -3 & 1 \end{array}\right)$$

Add row 2 and row 3

$$E = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & -5 & 1 & -1 & -1 & 1 \\ 0 & -8 & -2 & 0 & -3 & 1 \end{array}\right)$$

Multiply row 2 by $-\frac{8}{5}$ and add in row 3

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$$E_2 = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & -5 & 1 & -1 & -1 & 1 \\ 0 & 0 & -\frac{18}{5} & 8/5 & -7/5 & -3/5 \end{array}\right)$$

Multiply row 3 by $-5/18$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & -5 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & -4/9 & 7/18 & 3/6 \end{array}\right)$$

Add row 1 and 3

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/9 & -11/18 & 1/6 \\ 0 & -5 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & -4/9 & 7/18 & 1/6 \end{array}\right)$$

Multiply row 3 by -1 and add row 2

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/9 & -11/18 & 1/6 \\ 0 & -5 & 0 & -5/9 & -25/18 & 5/6 \\ 0 & 0 & 1 & -4/9 & 7/18 & 1/6 \end{array}\right)$$

Multiply row 2 by $-\frac{1}{5}$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/9 & -11/18 & 1/6 \\ 0 & 1 & 0 & 1/9 & 5/18 & -1/6 \\ 0 & 0 & 1 & -4/9 & 7/18 & 1/6 \end{array}\right)$$

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$$I | A^{-1} = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/9 & -11/18 & 1/6 \\ 0 & 1 & 0 & 1/9 & 5/18 & -1/6 \\ 0 & 0 & 1 & -4/9 & 7/18 & 1/6 \end{array}\right)$$

The matrix B is equal to A^{-1}

And we will calculate

$$AB = A.A^{-1} = I$$

$$A.B = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 5/9 & -11/18 & 1/6 \\ 1/9 & 5/18 & -1/6 \\ -4/9 & 7/18 & 1/6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B.A = A^{-1}.A = I$$

$$B.A = \begin{pmatrix} 5/9 & -11/18 & 1/6 \\ 1/9 & 5/18 & -1/6 \\ -4/9 & 7/18 & 1/6 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

~~AB~~ Yes the name of matrix B is A^{-1}

Proved