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Exercise 1

a) $A|I = \left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 5 & 6 & 0 & 1 \end{array}\right)$ Multiply equation 1 by 2 and add in equation 2.

$= \left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 9 & 0 & 2 & 1 \end{array}\right)$ Multiply equation 2 by $-\frac{2}{9}$ and add equation 1

$= \left(\begin{array}{cc|cc} 0 & -3 & 5/9 & -2/9 \\ 9 & 0 & 2 & 1 \end{array}\right)$ Multiply equation 2 by $\frac{1}{9}$

$= \left(\begin{array}{cc|cc} 0 & -3 & 5/9 & -2/9 \\ 1 & 0 & 2/9 & 1/9 \end{array}\right)$ Multiply equation 1 by $-\frac{1}{3}$

$= \left(\begin{array}{cc|cc} 0 & 1 & -5/27 & 2/27 \\ 1 & 0 & 2/9 & 1/9 \end{array}\right)$ exchange the row

$I|A^{-1} = \left(\begin{array}{cc|cc} 1 & 0 & 2/9 & 1/9 \\ 0 & 1 & -5/27 & 2/27 \end{array}\right)$

$A^{-1} = \begin{pmatrix} 2/9 & 1/9 \\ -5/27 & 2/27 \end{pmatrix} \cancel{\Rightarrow (A^{-1})^T}$

$(A^{-1})^T = \begin{pmatrix} 2/9 & -5/27 \\ 1/9 & 2/27 \end{pmatrix}$

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~~A^{T}~~ $A = \begin{pmatrix} 2 & -3 \\ 5 & 6 \end{pmatrix}$

$A^T = \begin{pmatrix} 2 & 5 \\ -3 & 6 \end{pmatrix}$

$A^T | I = \left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -3 & 6 & 0 & 1 \end{array}\right)$ Multiply equation 2 by +1 and add in equation 1

$= \left(\begin{array}{cc|cc} -1 & 11 & 1 & 1 \\ -3 & 6 & 0 & 1 \end{array}\right)$ Multiply equation 1 by -3 and add in equation 2

$= \left(\begin{array}{cc|cc} -1 & 11 & 1 & 1 \\ 0 & -27 & -3 & -2 \end{array}\right)$ Multiply equation 2 by $-\frac{1}{27}$

$= \left(\begin{array}{cc|cc} -1 & 11 & 1 & 1 \\ 0 & 1 & 3/27 & 2/27 \end{array}\right)$ Multiply equation 2 by -11 and add in equation 1

$= \left(\begin{array}{cc|cc} -1 & 0 & -6/27 & 5/27 \\ 0 & 1 & 3/27 & 2/27 \end{array}\right)$ Multiply equation 1 by -1

$I | (A^T)^{-1} = \left(\begin{array}{cc|cc} 1 & 0 & 2/9 & -5/27 \\ 0 & 1 & 1/9 & 2/27 \end{array}\right)$

$(A^T)^{-1} = \begin{pmatrix} 2/9 & -5/27 \\ 1/9 & 2/27 \end{pmatrix}$

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b)

$$A = \begin{pmatrix} 0 & 3 \\ 0 & 6 \end{pmatrix}$$

$$A|I = \left(\begin{array}{cc|cc} 0 & 3 & 1 & 0 \\ 0 & 6 & 0 & 1 \end{array}\right)$$ multiply equation 1 by -2 and add in equation 2

$$= \left(\begin{array}{cc|cc} 0 & 3 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right)$$

The inverse matrix doesnot exist.

$$A^T = \begin{pmatrix} 0 & 0 \\ 3 & 6 \end{pmatrix}$$

$$A^T/I = \left(\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 3 & 6 & 0 & 1 \end{array}\right)$$

$(A^T)^{-1}$ doesnot exist.

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c) $A = \begin{pmatrix} 0 & c \\ c & 2 \end{pmatrix}$, $c \neq 0$

$A|I = \left(\begin{array}{cc|cc} 0 & c & 1 & 0 \\ c & 2 & 0 & 1 \end{array}\right)$ Multiply equation 1 by $\frac{1}{c}$

$= \left(\begin{array}{cc|cc} 0 & 1 & 1/c & 0 \\ c & 2 & 0 & 1 \end{array}\right)$ Multiply equation 1 by -2 and add equation 1

$= \left(\begin{array}{cc|cc} 0 & 1 & 1/c & 0 \\ c & 0 & -2/c & 1 \end{array}\right)$ Multiply equation 2 by $\frac{1}{c}$

$= \left(\begin{array}{cc|cc} 0 & 1 & 1/c & 0 \\ 1 & 0 & -2/c^2 & 1/c \end{array}\right)$ Exchange the row.

$= \left(\begin{array}{cc|cc} 1 & 0 & -2/c^2 & 1/c \\ 0 & 1 & 1/c & 0 \end{array}\right)$

$A^{-1} = \begin{pmatrix} -2/c^2 & 1/c \\ 1/c & 0 \end{pmatrix}$

$(A^{-1})^T = \begin{pmatrix} -2/c^2 & 1/c \\ 1/c & 0 \end{pmatrix}$

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$$A = \begin{pmatrix} 0 & c \\ c & 2 \end{pmatrix}, \quad c \neq 0$$

$$A^T = \begin{pmatrix} 0 & c \\ c & 2 \end{pmatrix}$$

In this equation the matrix $A = A^T$ it mean that it is symmatric. Then the $A^{-1} = (A^T)^{-1}$ is also same

$$(A^T)^{-1} = \begin{pmatrix} -2/c^2 & 1/c \\ 1/c & 0 \end{pmatrix}$$

$$(A^T)^{-1} = A^{-1}$$

~~But $(A^T)^{-1} \neq (A^{-1})^T$~~

Then ~~this~~ e~~quation~~ Matrix

$$\Rightarrow A^{-1} = (A^{-1})^T = (A^T)^{-1}$$

Proved