Exercise 7

a)

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & -3 & 3 \\ 3 & 5 & 3 \end{pmatrix}$$

Multiply Row 1 by -3 and add Row 3

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & -3 & 3 \\ 0 & -1 & 0 \end{pmatrix}$$

Multiply Row 1 by -2 and add Row 2

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -7 & -3 \\ 0 & -1 & 0 \end{pmatrix}$$

Multiply Row 2 by $-\frac{1}{7}$ and add Row 3

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -7 & -3 \\ 0 & 0 & \frac{3}{7} \end{pmatrix}$$

The rank of this matrix $= 3$

(b)

$$\begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{pmatrix}$$

Multiply Row 1 by -1 and add Row 3

$$\begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

Multiply Row 1 by -1 and add Row 2

$$\begin{pmatrix} 4 & 4 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The Rank of this matrix = 1

9)

$$\begin{pmatrix} -6 & 2 & -4 \\ 6 & -2 & 4 \\ -12 & 4 & -8 \end{pmatrix}$$

Multiply Row 1 by -2 and add Row 3

$$\begin{pmatrix} -6 & 2 & -4 \\ 6 & -2 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

Add Row 1 and Row 2

$$\begin{pmatrix} -6 & 2 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The Rank of matrix = 1

d)

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 4 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

Multiply row 1 by -1 and add row 3

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 4 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Multiply row 1 by -1 and add row 2

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The rank of matrix = 2

e)

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The rank of matrix = 0