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Exercise 2

Prove - $(AB)^T = B^T A^T \neq A^T B^T$

$$(AB)^T = \left(\begin{pmatrix} 4 & 2 \\ 3 & 7 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ -2 & 3 \end{pmatrix} \right)^T$$

$$= \begin{pmatrix} 16 & 2 \\ 1 & 8 \end{pmatrix}^T$$

$$(AB)^T = \begin{pmatrix} 16 & 1 \\ 2 & 8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 5 & -1 \\ -2 & 3 \end{pmatrix}^T \begin{pmatrix} 4 & 2 \\ 3 & 7 \end{pmatrix}^T$$

$$= \begin{pmatrix} 5 & -2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & 7 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 16 & 1 \\ 2 & 8 \end{pmatrix}$$

$$(AB)^T = B^T A^T$$

$$A^T B^T = \begin{pmatrix} 4 & 2 \\ 3 & 7 \end{pmatrix}^T \begin{pmatrix} 5 & -1 \\ -2 & 3 \end{pmatrix}^T$$

$$= \begin{pmatrix} 4 & 3 \\ 2 & 7 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -1 & 3 \end{pmatrix}$$

$$A^T B^T = \begin{pmatrix} 17 & 1 \\ 3 & 17 \end{pmatrix}$$

That's proved.

$$(AB)^T = B^T A^T \neq A^T B^T$$

Exercise 3

- $AA + BB \Rightarrow (AA + BB)^T \Rightarrow (AA)^T + (BB)^T$
  $= A^T A^T + B^T B^T = AA + BB$ It is symmetric
- $(B-A)(A+B) \Rightarrow ((B-A)(A+B))^T$
  $= (A+B)^T (B-A)^T \Rightarrow (A^T + B^T)(B^T - A^T)$
  $= (A+B)(B-A)$ It is not symmetric

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- $BAB \Rightarrow (BAB)^T \Rightarrow \cancel{(BA)} (B)^T (BA)^T$

$= (B)^T (A)^T (B)^T \Rightarrow BAB$. It is symmetric

- $BABA \Rightarrow (BABA)^T \Rightarrow (BA)^T (BA)^T$

$= (A^T B^T)(A^T B^T) = ABAB$. It is not symmetric

Exercise 4

(a) Let $V = R^2$

Suppose that $v = \begin{pmatrix} x \\ y \end{pmatrix}$ and $w = \begin{pmatrix} w \\ z \end{pmatrix} \in R$

than as we know sum of real number is also real number

$$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} w \\ z \end{pmatrix} = \begin{pmatrix} x+w-3 \\ y+z+1 \end{pmatrix} \begin{matrix} \in R \\ \in R \end{matrix} \in R^2$$

as we know $R^2 = V$.

then.

$$\begin{pmatrix} x+w-3 \\ y+z+1 \end{pmatrix} \in V$$

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- $\forall \alpha \in R, \forall v \in V \rightarrow \alpha \odot v \in V$

Suppose $\forall v = \begin{pmatrix} x \\ y \end{pmatrix} \in R$ and $\forall \alpha \in R$

then we know multiplication of two real number is also real number.

$$\alpha \odot v = \begin{pmatrix} \alpha x - 3\alpha + 3 \\ \alpha y + \alpha - 1 \end{pmatrix}\begin{matrix} \in R \\ \in R \end{matrix} \in R^2$$

as we know $R^2 = V$ then.

$$\begin{pmatrix} \alpha x - 3\alpha + 3 \\ \alpha y + \alpha - 1 \end{pmatrix} \in V$$

(b) Let $u, v, w \in R$ ~~than~~ and $V = R^2$

$$u = \begin{pmatrix} x \\ y \end{pmatrix}, \quad v = \begin{pmatrix} w \\ z \end{pmatrix}, \quad w = \begin{pmatrix} p \\ q \end{pmatrix}$$

(A1) $(u \oplus v) \oplus w = u \oplus (v \oplus w)$

$$\left(\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} w \\ z \end{pmatrix}\right) \oplus \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \oplus \left(\begin{pmatrix} w \\ z \end{pmatrix} \oplus \begin{pmatrix} p \\ q \end{pmatrix}\right)$$

$$\begin{pmatrix} x + w - 3 \\ y + z + 1 \end{pmatrix} \oplus \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} w + p - 3 \\ z + q + 1 \end{pmatrix}$$

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$$\begin{pmatrix} x+w+p-6 \\ y+z+q+2 \end{pmatrix} = \begin{pmatrix} x+w+p-6 \\ y+z+q+2 \end{pmatrix}$$

It is true.

(A2) $U \oplus V = V \oplus U$

$$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} w \\ z \end{pmatrix} = \begin{pmatrix} w \\ z \end{pmatrix} \oplus \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x+w-3 \\ y+z+1 \end{pmatrix} = \begin{pmatrix} w+x-3 \\ z+y+1 \end{pmatrix}$$

If w, x is real number than $w+x$ we can write $x+w$

$$\begin{pmatrix} x+w-3 \\ y+z+1 \end{pmatrix} = \begin{pmatrix} x+w-3 \\ y+z+1 \end{pmatrix}$$

It is true.

(A3) $U \oplus O = U$ and let zero vector $= \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \oplus O_v = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

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$$\begin{pmatrix} x+3-3 \\ y-3+1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

It is true.

(A4) $U \oplus (-U) = 0_v$ and $0_v = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \left(-\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$= \begin{pmatrix} x+(-x)+3 \\ y+(-y)-1 \end{pmatrix}$$

$$= \begin{pmatrix} +3 \\ -1 \end{pmatrix} = 0_v$$

It is true.

(A5) let $\forall \alpha \in R$

$$\alpha(U \oplus V) = \alpha U \oplus \alpha V$$

$$\alpha \odot \left(\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} w \\ z \end{pmatrix}\right) = \alpha \odot \begin{pmatrix} x \\ y \end{pmatrix} \oplus \alpha \odot \begin{pmatrix} w \\ z \end{pmatrix}$$

$$\alpha \odot \begin{pmatrix} x+w-3 \\ y+z+1 \end{pmatrix} = \begin{pmatrix} \alpha x - 3\alpha + 3 \\ \alpha y + \alpha - 1 \end{pmatrix} \oplus \begin{pmatrix} \alpha w - 3\alpha + 3 \\ \alpha z + \alpha - 1 \end{pmatrix}$$

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$$\begin{pmatrix} \alpha(x+w-3)-3\alpha+3 \\ \alpha(y+z+1)+\alpha-1 \end{pmatrix} = \begin{pmatrix} \alpha x-3\alpha+3+\alpha w-3\alpha+3-3 \\ \alpha y+\alpha-1+\alpha z+\alpha-1+1 \end{pmatrix}$$

$$\begin{pmatrix} \alpha(x+w-3)-3\alpha+3 \\ \alpha(y+z+1)+\alpha-1 \end{pmatrix} = \begin{pmatrix} \alpha x+\alpha w-3\alpha-3x+3 \\ \alpha y+\alpha z+\alpha+\alpha-1 \end{pmatrix}$$

$$\begin{pmatrix} \alpha(x+w-3)-3\alpha+3 \\ \alpha(y+z+1)+\alpha-1 \end{pmatrix} = \begin{pmatrix} \alpha(x+w-3)-3\alpha+3 \\ \alpha(y+z+1)+\alpha-1 \end{pmatrix}$$

It is true

(A6) $\forall u \in V, \forall \alpha, \beta \in R$

$$(\alpha+\beta)\odot u = (\alpha \odot u) \oplus (\beta \odot u)$$

$$\left((\alpha+\beta) \odot \begin{pmatrix} x \\ y \end{pmatrix} \right) = \left(\alpha \odot \begin{pmatrix} x \\ y \end{pmatrix} \right) \oplus \left(\beta \odot \begin{pmatrix} x \\ y \end{pmatrix} \right)$$

L.H.S R.H.S

R.H.S

$$= \begin{pmatrix} \alpha x-3\alpha+3 \\ \alpha y+\alpha-1 \end{pmatrix} \oplus \begin{pmatrix} \beta x-3\beta+3 \\ \beta y+\beta-1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha x-3\alpha+3+\beta x-3\beta+3-3 \\ \alpha y+\alpha-1+\beta y+\beta-1+1 \end{pmatrix}$$

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$$= \begin{pmatrix} (\alpha+\beta)x - 3(\alpha+\beta)+3 \\ (\alpha+\beta)y + (\alpha+\beta) - 1 \end{pmatrix}$$

$$= (\alpha+B) \odot \begin{pmatrix} x \\ y \end{pmatrix}$$

It is true

(A7) $\forall v \in V, \forall \alpha, \beta \in R$

$$(\alpha\beta)\odot v = \alpha(\beta \odot v)$$

$$(\alpha\beta)\odot\begin{pmatrix} x \\ y \end{pmatrix} = \alpha\odot\left(\beta\odot\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\begin{pmatrix} (\alpha\beta)x - 3(\alpha\beta)+3 \\ (\alpha\beta)y + (\alpha\beta) - 1 \end{pmatrix} = \alpha\odot\begin{pmatrix} \beta x - 3\beta + 3 \\ \beta y + \beta - 1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha\beta x - 3\alpha\beta + 3\alpha - 3\alpha + 3 \\ \alpha\beta y - \alpha\beta - \alpha + \alpha - 1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha\beta x - 3(\alpha\beta) + 3 \\ \alpha\beta y + (\alpha\beta) - 1 \end{pmatrix}$$

It is true.

(A8) $\forall v \in V$, 1 is scalar here

$$1 \odot v \Rightarrow 1 \odot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} x - 3 + 3 \\ y + 1 - 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$$

It is true. Proved

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Exercise 5

$$A = \begin{pmatrix} 2 & -2 \\ 10 & 5 \end{pmatrix}$$

Zero vector in space $= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\frac{1}{3}A$ vector $= \frac{1}{3} \cdot \begin{pmatrix} 2 & -2 \\ 10 & 5 \end{pmatrix} = \begin{pmatrix} 2/3 & -2/3 \\ 10/3 & 5/3 \end{pmatrix}$

$-A$ vector $= -\begin{pmatrix} 2 & -2 \\ 10 & 5 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ -10 & -5 \end{pmatrix}$

The smallest subspace containing A is αA, $\forall \alpha \in R$

Exercise 6

(a) The subspace of M that contains A ~~by~~ but doesnot contain B is αA, $\forall \alpha \in R$

(b)

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(B) $A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 0 \\ 0 & -3 \end{pmatrix}$

Multiply A with $1/2$ and B with $\frac{1}{3}$ and than $A - B = I$

(C) M that contains no nonzero diagonal matrics.

$A = \begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix} \ \forall x, y \in R$

that have all main diagonal are zero.