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Exercise 2

a) Suppose

$$a = \begin{pmatrix} 2 & -1 & 5 & -2 \\ 3 & 6 & -9 & 2 \\ -4 & 3 & 7 & -11 \end{pmatrix}$$

Multiply row 1 by 2 and add in row 3

$$= \begin{pmatrix} 2 & -1 & 5 & -2 \\ 3 & 6 & -9 & 2 \\ 0 & 1 & 17 & -15 \end{pmatrix}$$

Multiply row 1 by $-\frac{3}{2}$ and add row 2

$$= \begin{pmatrix} 2 & -1 & 5 & -2 \\ 0 & 15/2 & -23/2 & 5 \\ 0 & 1 & 17 & -15 \end{pmatrix}$$

Multiply ~~equation~~ row 2 by $-\frac{2}{15}$ and add row 3

$$= \begin{pmatrix} 2 & -1 & 5 & -2 \\ 0 & 15/2 & -\frac{33}{2} & 5 \\ 0 & 0 & \frac{96}{5} & -\frac{47}{3} \end{pmatrix}$$

Now we have 3 independent row according to defination we know coloum rank is equal to row rank than rank(a) = 3 Ans

b) det

$$a = \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 5 & 4 & 3 & 7 & 6 \\ 6 & 4 & 2 & 10 & 8 \\ -10 & -8 & -6 & -14 & -12 \\ 4 & 5 & -2 & -7 & 13 \end{pmatrix}$$

Multiply row 1 by -2 and add row 3

$$= \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 5 & 4 & 3 & 7 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ -10 & -8 & -6 & -14 & -12 \\ 4 & 5 & -2 & -7 & 13 \end{pmatrix}$$

Multiply row 2 by 2 and add row 4

$$= \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 5 & 4 & 3 & 7 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 5 & -2 & -7 & 13 \end{pmatrix}$$

Multiply row 1 by $-\frac{4}{3}$ and add row 5

$$= \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 5 & 4 & 3 & 7 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 7/3 & -\frac{10}{3} & -\frac{41}{3} & 23/3 \end{pmatrix}$$

Multiply row 1 by $-5/3$ and add row 2

$$= \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 0 & 2/3 & 4/3 & -4/3 & -2/3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 7/3 & -\frac{10}{3} & -\frac{44}{3} & 23/3 \end{pmatrix}$$

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Swap row 5 and row 3

$$= \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 0 & 2/3 & 4/3 & -4/3 & -2/3 \\ 0 & 7/3 & \frac{-10}{3} & \frac{-41}{3} & \frac{23}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Multiply row 2 by $\frac{-7}{2}$ and add row 3

$$= \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 0 & 2/3 & 4/3 & -4/3 & -\frac{2}{3} \\ 0 & 0 & -8 & -9 & 10 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Now we have 3 independent row than the rank (a) = 3 <u>Ans</u>

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Exercise 3

a) Let consider R^5

$$V_1 = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}, V_2 = \begin{pmatrix} -1 \\ 2 \\ 0 \\ -2 \\ 1 \end{pmatrix}, V_3 = \begin{pmatrix} 8 \\ 7 \\ 6 \\ 5 \\ 4 \end{pmatrix}, V_4 = \begin{pmatrix} 0 \\ 3 \\ 1 \\ -1 \\ 2 \end{pmatrix}, V_5 = \begin{pmatrix} 10 \\ 8 \\ 6 \\ 4 \\ 2 \end{pmatrix}$$

and we know vector space $V = \text{span}\{v_1, v_2, \dots, v_5\}$

let assume that

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 + \alpha_4 V_4 + \alpha_5 V_5 = 0$$

$$\alpha_1 \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} -1 \\ 2 \\ 0 \\ -2 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 8 \\ 7 \\ 6 \\ 5 \\ 4 \end{pmatrix} + \alpha_4 \begin{pmatrix} 0 \\ 3 \\ 1 \\ -1 \\ 2 \end{pmatrix} + \alpha_5 \begin{pmatrix} 10 \\ 8 \\ 6 \\ 4 \\ 2 \end{pmatrix} = 0$$

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$4\alpha_1 + 2\alpha_2 + 7\alpha_3 + 3\alpha_4 + 8\alpha_5 = 0$$
$$3\alpha_1 + 0 + 6\alpha_3 + \alpha_4 + 6\alpha_5 = 0$$
$$2\alpha_1 - 2\alpha_2 + 5\alpha_3 - \alpha_4 + 4\alpha_5 = 0$$
$$\alpha_1 + \alpha_2 + 4\alpha_3 + 2\alpha_4 + 2\alpha_5 = 0$$

Multipey equation 5 by -2 and add equation 4

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$4\alpha_1 + 2\alpha_2 + 7\alpha_3 + 3\alpha_4 + 8\alpha_5 = 0$$
$$3\alpha_1 + 0 + 6\alpha_3 + \alpha_4 + 6\alpha_5 = 0$$
$$0 - 4\alpha_2 - 3\alpha_3 - 5\alpha_4 + 0 = 0$$
$$\alpha_1 + \alpha_2 + 4\alpha_3 + 2\alpha_4 + 2\alpha_5 = 0$$

Multiply equation 2 by $-1/4$ and add eq 5

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 0\alpha_5 = 0$$
$$4\alpha_1 + 2\alpha_2 + 7\alpha_3 + 3\alpha_4 + 8\alpha_5 = 0$$
$$3\alpha_1 + 0 + 6\alpha_3 + \alpha_4 + 6\alpha_5 = 0$$
$$0 - 4\alpha_2 - 3\alpha_3 - 5\alpha_4 + 0 = 0$$
$$0 + \frac{1}{2}\alpha_2 + \frac{9}{4}\alpha_3 + \frac{5}{4}\alpha_4 + \cancel{\tfrac{3}{5}}\alpha_5 = 0$$

Multiply equation 1 by $-\frac{3}{5}$ and add equation 3

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$4\alpha_1 + 2\alpha_2 + 7\alpha_3 + 3\alpha_4 + 8\alpha_5 = 0$$
$$0 + \frac{3}{5}\alpha_2 + \frac{6}{5}\alpha_3 + \alpha_4 + 0 = 0$$
$$0 - 4\alpha_2 - 3\alpha_3 - 5\alpha_4 + 0 = 0$$
$$0 + \frac{1}{2}\alpha_2 + \frac{9}{4}\alpha_3 + \frac{5}{4}\alpha_4 + \cancel{\tfrac{8}{5}}\alpha_5 = 0$$

Multiply equation 1 by $-\frac{4}{5}$ and add equation 2

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$0 + \frac{14}{5}\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + \frac{3}{5}\alpha_2 + \frac{6}{5}\alpha_3 + \alpha_4 + 0 = 0$$
$$0 - 4\alpha_2 - 3\alpha_3 - 5\alpha_4 + 0 = 0$$
$$0 + \frac{1}{2}\alpha_2 + \frac{9}{4}\alpha_3 + \frac{5}{4}\alpha_4 + \cancel{\tfrac{8}{5}}\alpha_5 = 0$$

Multiply equation 4 by $\frac{1}{8}$ and add equation 5

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$0 + \frac{14}{5}\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + \frac{3}{5}\alpha_2 + \frac{6}{5}\alpha_3 + \alpha_4 + 0 = 0$$
$$0 - 4\alpha_2 - 3\alpha_3 - 5\alpha_4 + 0 = 0$$
$$0 + \cancel{\tfrac{1}{2}} * \frac{15}{8}\alpha_3 + \frac{5}{8}\alpha_4 + 0 = 0$$

Multiply equation 3 by $\frac{20}{3}$ and add eq. 4

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$0 + \frac{14}{5}\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + \frac{3}{5}\alpha_2 + \frac{6}{5}\alpha_3 + \alpha_4 + 0 = 0$$
$$0 + 0 + 5\alpha_3 + \frac{5}{3}\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{8}\alpha_3 + \frac{5}{8}\alpha_4 + 0 = 0$$

Multiply equation 2 by $-\frac{3}{14}$ and add equation 3

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$0 + \frac{14}{5}\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{14}\alpha_3 + \frac{5}{14}\alpha_4 + 0 = 0$$
$$0 + 0 + 5\alpha_3 + \frac{5}{3}\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{8}\alpha_3 + \frac{5}{8}\alpha_4 + 0 = 0$$

Multiply equation 4 by $\frac{-3}{8}$ and add eq. 5

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$0 + \frac{14}{5}\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{14}\alpha_3 + \frac{5}{14}\alpha_4 + 0 = 0$$
$$0 + 0 + 5\alpha_3 + \frac{5}{3}\alpha_4 + 0 = 0$$
$$0 + 0 + 0 + 0 + 0 = 0$$

Multiply equation 3 by $-\frac{3}{14}$ and add eq. 4

$$5\alpha_1 - \alpha_2 + 8\alpha_3 + 0 + 10\alpha_5 = 0$$
$$0 + 14\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{14}\alpha_3 + \frac{5}{14}\alpha_4 + 0 = 0$$
$$0 = 0$$
$$0 = 0$$

Multiply equation 2 by $\frac{1}{14}$ and add eq. 1

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$$5\alpha_1 + 0 + \frac{563}{70}\alpha_3 + \frac{3}{14}\alpha_4 + 10\alpha_5 = 0$$
$$0 + 14\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{14}\alpha_3 + \frac{5}{14}\alpha_4 + 0 = 0$$
$$0 = 0$$
$$0 = 0$$

Multiply equation 3 by $-\frac{563}{75}$ and add eq 1

$$5\alpha_1 + 0 + 0 + \frac{518}{210}\alpha_4 + 10\alpha_5 = 0$$
$$0 + 14\alpha_2 + \frac{3}{5}\alpha_3 + 3\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{14}\alpha_3 + \frac{15}{14}\alpha_4 + 0 = 0$$
$$0 = 0$$
$$0 = 0$$

Multiply equation 3 by $-\frac{15}{14}$ and eq. 2

$$5\alpha_1 + 0 + 0 + \frac{518}{210}\alpha_4 + 10\alpha_5 = 0$$
$$0 - 15\alpha_2 + 0 - \frac{30}{14}\alpha_4 + 0 = 0$$
$$0 + 0 + \frac{15}{14}\alpha_3 + \frac{15}{14}\alpha_4 + 0 = 0$$
$$0 = 0$$
$$0 = 0$$

As we can α_4 and α_5 are arbitrary and V_1, V_2, V_3 are linearly independent than basis $b = \{V_1, V_2, V_3\}$

Ans

b) Let consider that vector space V_1, V_2, V_3 are linearly independent
let assume that $C = \{ C_1 V_1 + C_2 V_2 + C_3 V_3$
and $D = D_1 V_1 + D_2 V_2 + D_3 V_3$ than $C_i = D_i$ for $i = 1 \dots n$
Now we have

$$C_1 V_1 + C_2 V_2 + C_3 V_3 = D_1 V_1 + D_2 V_2 + D_3 V_3$$
$$C_1 V_1 - D_1 V_1 + C_2 V_2 - D_2 V_2 + C_3 V_3 - D_3 V_3 = 0$$
$$(C_1 - D_1) V_1 + (C_2 - D_2) V_2 + (C_3 - D_3) V_3 = 0$$

since V_1, V_2, V_3 are linearly independent and $(C_i = D_i)$ it mean $(C_i - D_i = 0)$ for all $i = 1 \dots n$.

Exercise 4 -

Yes, we have. $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Let consider. R^3

$$V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad V_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad V_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad V_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Let assume that

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 + \alpha_4 V_4 = 0$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \alpha_4 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$\alpha_1 + 0 + 0 + \alpha_4 = 0$$
$$0 + \alpha_2 + 0 + \alpha_4 = 0$$
$$0 + 0 + \alpha_3 + \alpha_4 = 0$$

than we find

$\alpha_1 = -\alpha_4$ and $\alpha_2 = -\alpha_4$, $\alpha_3 = -\alpha_4$

~~than~~ it mean α_4 is arbitrary.

If we take ~~first~~ 3 vector. R^3

$$V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad V_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Let assume

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\alpha_1 + 0 + 0 = 0$$
$$0 + \alpha_2 + 0 = 0$$
$$0 + 0 + \alpha_3 = 0$$

Than $\alpha_1 = 0$, $\alpha_2 = 0$, $\alpha_3 = 0$ it mean

~~α_1~~ V_1, V_2, V_3 are linearly independent.

Ans

Exercise 5

The dimension of vector spake M of 3×4 matrices is 12

Let assume $M_{m,n}$ the set of $m \times n$ matrics. The standard basis for $M_{m,n}$ is the set B_{ij} for $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots n$. where B_{ij} is the $m \times n$ matrices whose entries are all zeros. except for the ij entry which is 1

The set has mn element, therefore

$$\dim(M_{m,n}) = mn$$

Exercise 6

Let

$$a = \begin{pmatrix} 2 & 4 & -1 & 3 \\ 6 & 12 & -3 & 9 \\ 3 & 5 & 0 & 4 \\ -2 & -4 & 1 & -3 \end{pmatrix}$$

Multiply row 1 by -3 and add row 2

$$= \begin{pmatrix} 2 & 4 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 3 & 5 & 0 & 4 \\ -2 & -4 & 1 & -3 \end{pmatrix}$$

Add row 1 and row 4

$$= \begin{pmatrix} 2 & 4 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 3 & 5 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Multiply row 1 by $\frac{-3}{2}$ and add row 3

$$= \begin{pmatrix} 2 & 4 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 3/2 & -1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Swap row 2 and row 3

$$= \begin{pmatrix} 2 & 4 & -1 & 3 \\ 0 & -1 & 3/2 & -1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The basis for vector space generated by the rows of Matrix is

$$\{(2,4,-1,3), (0,-1,3/2,-1/2)\}$$

Ans