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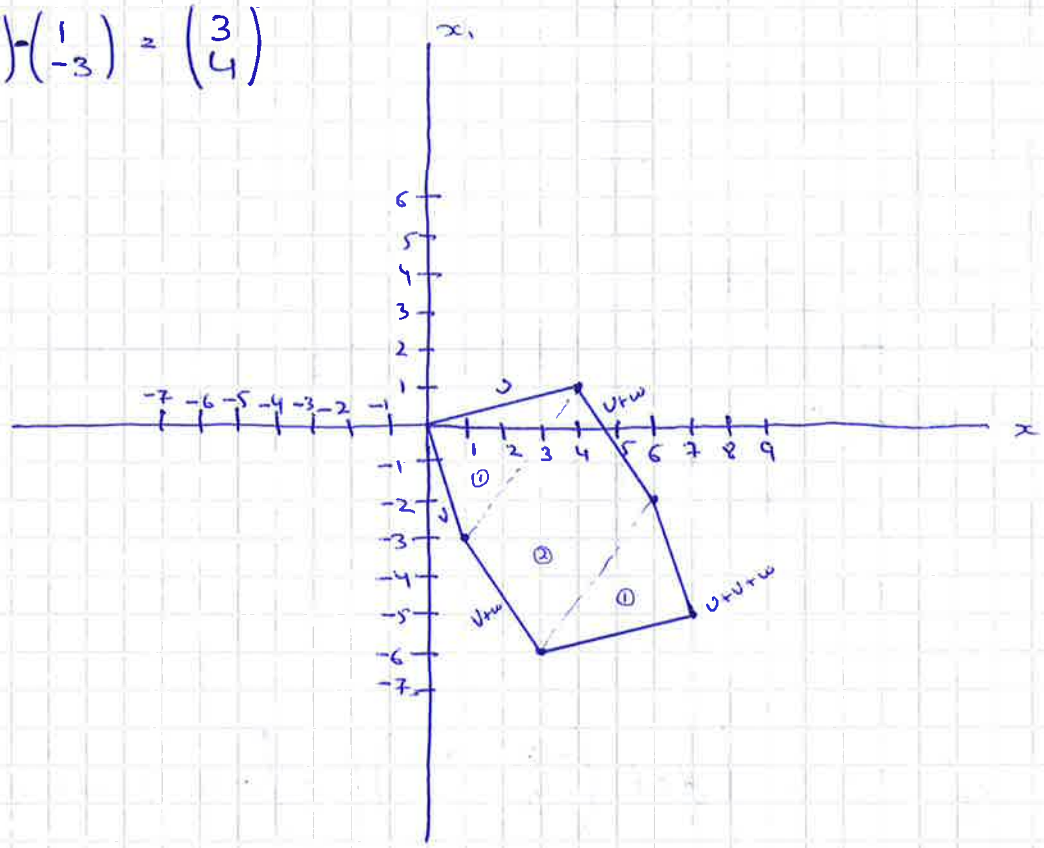
Exercise 1

$$U+W = \begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}6\\-2\end{pmatrix}$$

$$V+W = \begin{pmatrix}1\\-3\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}3\\-6\end{pmatrix}$$

$$U+V+W = \begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}7\\-5\end{pmatrix}$$

$$U-V = \begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}$$



$$\text{Area of } \cancel{p} \text{ ①} = \left|\det\begin{pmatrix}4 & 1\\1 & -3\end{pmatrix}\right| = |-12-1| = |-13| = 13$$

$$\text{Area of ②} = \left|\det\begin{pmatrix}U-V & W\\U-V & W\end{pmatrix}\right| = \left|\det\begin{pmatrix}3 & 2\\4 & -3\end{pmatrix}\right| = |-9-8| = |-17| = 17$$

Area of given vector = ① + ②

$= 13 + 17 = 30$

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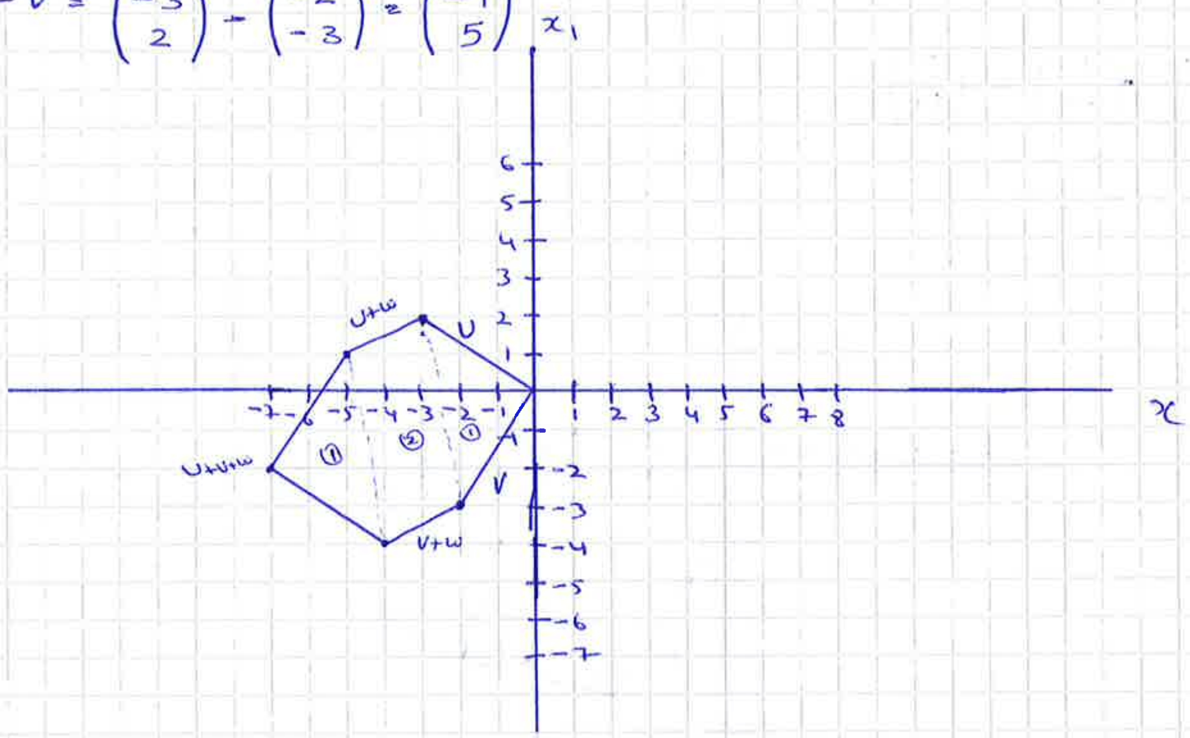
b) $U+W = \begin{pmatrix}-3\\2\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}-5\\1\end{pmatrix}$

$V+W = \begin{pmatrix}-2\\-3\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}-4\\-4\end{pmatrix}$

$U+V+W = \begin{pmatrix}-3\\2\end{pmatrix} + \begin{pmatrix}-2\\-3\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}-7\\-2\end{pmatrix}$

$U-V = \begin{pmatrix}-3\\2\end{pmatrix} - \begin{pmatrix}-2\\-3\end{pmatrix} = \begin{pmatrix}-1\\5\end{pmatrix}$



Area of ① $= \left| \det\begin{pmatrix}-3 & -2\\ 2 & -3\end{pmatrix} \right| = \left|(-3)(-3)-(-2)(2)\right| =$

$= |9+4| = |13| = 13$

Area of ② $= \left| \det\begin{pmatrix}u-v & w\\ u-v & w\end{pmatrix} \right| = \left| \det\begin{pmatrix}-1 & -2\\ 5 & -1\end{pmatrix} \right|$

$= \left|(-1)(-1)-(-2)(5)\right| = |1+10| = |11| = 11$

Area of vectors $= 13 + 11 = 24$ Ans

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Exercise 2

we have

$$\det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij} \quad \text{for } i \in \{1 \ldots n\}$$

Suppose that we have matrix upper triangular than

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

$$\det(A) = \det \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

$$= a_{11}(a_{22} \cdot a_{33} - 0 \cdot a_{23}) - a_{12}(0 \cdot a_{33} - 0 \cdot a_{23}) + a_{13}(0 \cdot 0 - 0 \cdot a_{22})$$

$$= a_{11} a_{22} a_{33} - 0 - 0 + 0$$

$$= a_{11} a_{22} a_{33}$$

That is proved determinant of triangular matrix ~~is~~ can be computed as product of main diagonal.

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Exercise 3

a) $V = \left| \det \begin{pmatrix} 3 & 1 & 2 \\ 2 & 3 & 5 \\ 2 & 5 & 6 \end{pmatrix} \right|$

$= \left| 3 \cdot \begin{vmatrix} 3 & 5 \\ 5 & 6 \end{vmatrix} - 1 \begin{vmatrix} 2 & 5 \\ 2 & 6 \end{vmatrix} + 2 \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} \right|$

$= \left| 3(3 \cdot 6 - 5 \cdot 5) - 1(2 \cdot 6 - 2 \cdot 5) + 2(2 \cdot 5 - 2 \cdot 3) \right|$

$= \left| 3 \cdot (-7) - 1 \cdot 2 + 2 \cdot 4 \right|$

$= \left| -21 - 2 + 8 \right|$

$= \left| -15 \right|$

$= 15$

b) $V = \left| \det \begin{pmatrix} 1 & -3 & 2 \\ -3 & 4 & 1 \\ -1 & 3 & -2 \end{pmatrix} \right|$

$= \left| 1 \begin{vmatrix} 4 & 1 \\ 3 & -2 \end{vmatrix} - (-3) \begin{vmatrix} -3 & 1 \\ -1 & -2 \end{vmatrix} + 2 \begin{vmatrix} -3 & 4 \\ -1 & 3 \end{vmatrix} \right|$

$= \left| 1(4 \cdot (-2) - 3 \cdot 1) + 3((-3)(-2) - (-1)(1) + 2((-3)(3) - 4(-1)) \right|$

$= \left| -11 + 21 - 10 \right|$

$= \left| -21 + 21 \right|$

$= |0|$

$= 0$

c) $$V = \left| \det \begin{pmatrix} 5 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & -6 \end{pmatrix} \right|$$

$$= \left| 5 \begin{vmatrix} -1 & 3 \\ 1 & -6 \end{vmatrix} - 2 \begin{vmatrix} 2 & 3 \\ 3 & -6 \end{vmatrix} + (-1) \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} \right|$$

$$= \left| 5\big((-1)(-6) - 3\cdot 1\big) - 2\big((2)(-6) - (3)(3)\big) + (-1)\big(2\cdot 1 - (-1)\cdot 3\big) \right|$$

$$= |15 + 42 - 5|$$

$$= |52|$$

$$= 52$$

d) $$V = \left| \det \begin{pmatrix} -2 & 3 & 5 \\ -4 & 5 & 1 \\ -5 & 2 & -2 \end{pmatrix} \right|$$

$$= \left| (-2) \begin{vmatrix} 5 & 1 \\ 2 & -2 \end{vmatrix} - 3 \begin{vmatrix} -4 & 1 \\ -5 & -2 \end{vmatrix} + 5 \begin{vmatrix} -4 & 5 \\ -5 & 2 \end{vmatrix} \right|$$

$$= \left| (-2)\big(5(-2) - (1)(2)\big) - 3\big((-4)(-2) - 1(-5)\big) + 5\big((-4)(2) - 5(-5)\big) \right|$$

$$= |24 - 39 + 85|$$

$$= |70|$$

$$= 70$$

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Exercise 4

(a)

$$\begin{pmatrix} 4 & -1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\cancel{x_{11} = \frac{\det B_{11}}{\det(A)}}$$

$$\det(A) = 4\cdot(-2) - (-1)(2) = -6$$

$$x_{11} = \frac{\det B_{11}}{\det A} = \frac{\begin{pmatrix} 1 & -1 \\ 0 & -2 \end{pmatrix}}{-6} = \frac{1(-2)-(-1)(0)}{-6} = \frac{-2}{-6} = \frac{1}{3}$$

$$x_{21} = \frac{\det B_{12}}{\det(A)} = \frac{\begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}}{-6} = \frac{4(0)-(1)(2)}{-6} = \frac{-2}{-6} = \frac{1}{3}$$

$$x_{21} = \frac{\det B_{21}}{\det(A)} = \frac{\begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}}{-6} = \frac{0\cdot(-2)-(-1)(1)}{-6} = \frac{1}{-6} = -\frac{1}{6}$$

$$x_{22} = \frac{\det B_{22}}{\det(A)} = \frac{\begin{pmatrix} 4 & 0 \\ 2 & 1 \end{pmatrix}}{-6} = \frac{4(1)-0\cdot 2}{-6} = \frac{4}{-6} = -\frac{2}{3}$$

$$A^{-1} = \begin{pmatrix} 1/3 & -1/6 \\ 1/3 & -2/3 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & -3 & 2 \\ 2 & 2 & 3 \\ -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det(A) = 1\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix}$$

$$= (4-3) + 3(4+3) + 2(2+2)$$

$$= 1 + 21 + 8 \Rightarrow 30$$

$$x_{11} = \frac{\det B_{11}}{\det A} = \frac{\begin{vmatrix} 1 & -3 & 2 \\ 0 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix}}{30} = \frac{1(4-3)+0+0}{30} = \frac{1}{30}$$

$$x_{21} = \frac{\det B_{12}}{\det A} = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \\ -1 & 0 & 2 \end{vmatrix}}{30} = \frac{0-1(4-(-3)+0}{30} = -\frac{7}{30}$$

$$x_{31} = \frac{\det B_{13}}{\det A} = \frac{\begin{vmatrix} 1 & -3 & 1 \\ 2 & 2 & 0 \\ -1 & 1 & 0 \end{vmatrix}}{30} = \frac{0+0+1(2-(-2)}{30} = \frac{4}{30} = \frac{2}{15}$$

$$x_{12} = \frac{\det B_{21}}{\det A} = \frac{\begin{vmatrix} 0 & -3 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix}}{30} = \frac{0-1((-3)(2)-2)}{30} = \frac{8}{30} = \frac{4}{15}$$

$$x_{22} = \frac{\det B_{22}}{\det A} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ -1 & 0 & 2 \end{vmatrix}}{30} = \frac{0+1(2-(-2)+0}{30} = \frac{4}{30} = \frac{2}{15}$$

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~~$a_{32} =$~~

$$x_{32} = \frac{\det B_{23}}{\det A} = \frac{\begin{pmatrix} 1 & -3 & 0 \\ 2 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix}}{30} = \frac{0 - 1(1-(-3)(-1)) + 0}{30} = \frac{2}{30} = \frac{1}{15}$$

$$x_{13} = \frac{\det B_{31}}{\det(A)} = \frac{\begin{pmatrix} 0 & -3 & 2 \\ 0 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}}{30} = \frac{0 - 0 + 1\left((-3)(3) - (2)(2)\right)}{30} = \frac{-13}{30}$$

$$x_{23} = \frac{\det B_{32}}{\det A} = \frac{\begin{pmatrix} 1 & 0 & 2 \\ 2 & 0 & 3 \\ -1 & 1 & 2 \end{pmatrix}}{30} = \frac{-0 + 0 - 1(3 - (2)(2))}{30} = \frac{1}{30}$$

$$x_{33} = \frac{\det B_{33}}{\det A} = \frac{\begin{pmatrix} 1 & -3 & 0 \\ 2 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}}{30} = \frac{0 - 0 + 1(2 - (-3)(2))}{30} = \frac{8}{30} = \frac{4}{15}$$

$$A^{-1} = \begin{pmatrix} 1/30 & 4/15 & -13/30 \\ -7/30 & 2/15 & 1/30 \\ 2/15 & 1/15 & 4/15 \end{pmatrix}$$

c)
$$\begin{pmatrix} 4 & 7 & -1 \\ 1 & 4 & -4 \\ 2 & 5 & -3 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det A = 4\begin{vmatrix} 4 & -4 \\ 5 & -3 \end{vmatrix} - 7\begin{vmatrix} 1 & -4 \\ 2 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix}$$

$$= 32 - 35 + 3$$

$$= 0$$

$\det(A) = 0$ it means inverse doesnot exist