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Exercise 2

(a) $2x+2y=4$
$\quad 11y=0$

$$A=\begin{pmatrix}2 & 2\\ 0 & 11\end{pmatrix}\in R^{2,2}, \quad x=\begin{pmatrix}x\\ y\end{pmatrix}$$

$$B=\begin{pmatrix}4\\ 0\end{pmatrix}$$

A is an upper trangular. we can use backward substitution to find the solution.

$$11y=0 \Rightarrow y=\frac{0}{11} \Rightarrow y=0$$

$$2x+2y=4 \Rightarrow 2x+2(0)=4 \Rightarrow 2x+0=4 \Rightarrow 2x=4$$
$$\Rightarrow x=\frac{4}{2} \Rightarrow x=2$$

(B) $y=3$
$\quad x-y=8$

$$A=\begin{pmatrix}0 & 1\\ 1 & -1\end{pmatrix}\in R^{2,2} \quad x=\begin{pmatrix}x\\ y\end{pmatrix}, B=\begin{pmatrix}3\\ 8\end{pmatrix}$$

we can use forward substitution.

$$y=3$$
$$x-y=8 \Rightarrow x-3=8 \Rightarrow x=8+3 \Rightarrow x=\cancel{8}\,11$$

c) $2x-4y+2z=3$
$\quad y+z=-2$
$\quad 4z=8$

$$A=\begin{pmatrix}2 & -4 & 2\\ 0 & 1 & 1\\ 0 & 0 & 4\end{pmatrix} \quad x=\begin{pmatrix}x\\ y\\ z\end{pmatrix} \quad B=\begin{pmatrix}3\\ 2\\ 8\end{pmatrix}$$

It is upper trangular and use backward substitution

$$4z=8 \Rightarrow z=\frac{8}{4} \Rightarrow z=2$$

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$y+z=-2 \Rightarrow y+2=-2 \Rightarrow y=-2-2 \Rightarrow$

$\Rightarrow y=-4$

$2x-4y+2z=3 \Rightarrow 2x-4(-4)+2(2)=3$

$\Rightarrow 2x+16+4=3 \Rightarrow 2x=3-20 \Rightarrow 2x=-17$

$x=\frac{-17}{2} \Rightarrow x=-8.5$

(d) $-x-y=-8$
$4y=10$ $A=\begin{pmatrix}-1 & -1\\ 0 & 4\end{pmatrix}, x=\begin{pmatrix}x\\ y\end{pmatrix}, B=\begin{pmatrix}-8\\ 10\end{pmatrix}$

It is upper trangular and use the backward substitution.

$4y=10 \Rightarrow y=\frac{10}{4} \Rightarrow y=2.5$

$-x-y=-8 \Rightarrow -x-(2.5)=-8$

$-x=-8+2.5 \Rightarrow -x=-5.5 \Rightarrow x=5.5$

e) $7x+7y-7z=0$
$y+z=7$
$-z=7$
$A=\begin{pmatrix}7 & 7 & -7\\ 0 & 1 & 1\\ 0 & 0 & -1\end{pmatrix}, x=\begin{pmatrix}x\\ y\\ z\end{pmatrix}, B=\begin{pmatrix}0\\ 7\\ 7\end{pmatrix}$

It is an upper trangular and use backward substitution.

$-z=7 \Rightarrow z=-7$

$y+z=7 \Rightarrow y+(-7)=7 \Rightarrow y=7+7 \Rightarrow y=14$

$7x+7y-7z=0 \Rightarrow 7x+7(14)-7(-7)=0$

$\Rightarrow 7x+98+49=0 \Rightarrow 7x=147 \Rightarrow x=\frac{147}{7} \Rightarrow x=21$

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(f) $-w=5$

$z-w=1$

$3y-z-w=0$

$3x+y+2z+w=12$

$$A=\begin{pmatrix}0&0&0&-1\\0&0&1&-1\\0&3&-1&-1\\3&1&2&1\end{pmatrix},\quad x=\begin{pmatrix}x\\y\\z\\w\end{pmatrix},\quad B=\begin{pmatrix}5\\1\\0\\12\end{pmatrix}$$

we use forward substitution.

$-w=5 \Rightarrow w=-5$

$z-w=1 \Rightarrow z-(-5)=1 \Rightarrow z+5=1 \Rightarrow z=1-5$

$\Rightarrow z=-4$

$3y-z-w=0 \Rightarrow 3y-(-4)-(-5)=0$

$3y+4+5=0 \Rightarrow 3y+9=0 \Rightarrow 3y=-9 \Rightarrow y=\frac{-9}{3}$

$y=-3$

$3x+y+2z+w=12 \Rightarrow 3x+(-3)+2(-4)+(-5)=12$

$\cancel{\Rightarrow 3x+(-3)-8-5=12 \Rightarrow 3x-10=12}$

$\cancel{\Rightarrow 3x=12+10 \Rightarrow x=\frac{22}{3} \Rightarrow x\doteq 7.3}$

$\Rightarrow 3x-3-8-5=12 \Rightarrow 3x-16=12$

$\Rightarrow 3x=12+16 \Rightarrow x=\frac{28}{3}$

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Exercise 3

$$\left.\begin{array}{l} ax+by=c \\ ax+dy=e \end{array}\right\} \quad a\neq 0$$

~~Suppose~~ $ax+by=c \Rightarrow ax=c-by$

$\Rightarrow x=\dfrac{c-by}{a}$

Suppose $y=0$ than we have

$x=\dfrac{c}{a}-\dfrac{b(0)}{a} \Rightarrow x=\dfrac{c}{a}$

than we know

$ax+dy=e \Rightarrow \cancel{a}\left(\dfrac{c}{\cancel{a}}\right)+d(0)=e$

$\Rightarrow \quad c+0=e \Rightarrow c=e$

then we assume $y=1$ then

$x=\dfrac{c-b(1)}{a} \Rightarrow x=\dfrac{c-b}{a}$

$a\left(\dfrac{c-b}{\cancel{a}}\right)+d(1)=e \Rightarrow c-b+d=e$

as we know $c=e$ than

$c-b+d=c \Rightarrow -b+d=c-c \Rightarrow -b+d=0$

$\Rightarrow d=b$

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If $a=0$ than

$$0(x)+by=c \Rightarrow y=\frac{c}{b}$$
$$0(x)+dy=e \Rightarrow y=\frac{e}{d}$$

and than we donot have any value for x.

B) Suppose that $c=0$ and $a\neq 0$ than $d\neq 0$

$$ax+by=j$$
$$0\cdot x+dy=h \Rightarrow 0+dy=h \Rightarrow y=\frac{h}{d}$$
$$ax+by=j \Rightarrow ax+b\left(\frac{h}{d}\right)=j$$
$$\Rightarrow ax=j-\frac{hb}{d} \Rightarrow ax=\frac{dh-hb}{d}$$
$$\Rightarrow x=\frac{dh-hb}{d}$$ than $ad-bc\neq 0$

Suppose $a=0$ and $b=0$ than

$$0\cdot x+by=j \Rightarrow y=\frac{j}{b}$$
$$0\cdot x+dy=h \Rightarrow y=\frac{h}{d}$$

and than $ad-bc=0$ or both $ad=bc$ than it is equal to zero.

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Exercise 4

$$A = \begin{pmatrix} a_{1,1} & & & 0 \\ a_{2,1} & a_{2,2} & & \\ \vdots & \vdots & \ddots & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{pmatrix}, \quad B = \begin{pmatrix} b_{1,1} & & & 0 \\ b_{2,1} & b_{2,2} & & \\ \vdots & \vdots & \ddots & \\ b_{n,1} & b_{n,2} & \cdots & b_{n,n} \end{pmatrix}$$

$$C = AB$$

$$C = \begin{pmatrix} a_{1,1} b_{1,1} & & 0 \\ a_{2,1} \cdot b_{1,1} + a_{2,2} \cdot b_{2,1} & a_{2,2} \cdot b_{2,2} & \\ \vdots & & \ddots \\ a_{n,1} \cdot b_{1,1} + \cdots + a_{n,n} \cdot b_{n,1} & \cdots & a_{n,n} \cdot b_{n,n} \end{pmatrix}$$

And it is prove C is a lower trangular when A is lower trangular and B is also lower trangular than the multiplication is also lower trangular.

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Exercise 5

a) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 5 \end{pmatrix}$ $\quad I_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$B = I_{12} A$

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$

b) The effect of the multiplication on the matrix A is the first coloum of A matrix is same as second coloum of B matrix after multiplicat and all other is zero.

Let $E_{12}(3) = I + 3(I_{12})$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad 3(I_{12}) = 3\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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$$\mathcal{E}_{12}(3) = \begin{pmatrix} 1+0 & 0+3 & 0+0 \\ 0+0 & 1+0 & 0+0 \\ 0+0 & 0+0 & 1+0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

c) $S = \mathcal{E}_{12}(3)\, A$

$$S = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 7 & 10 \\ 1 & 2 & 3 \\ 2 & -1 & 5 \end{pmatrix}$$

d) The effect of the multiplication on matrix A is second and 3 Third row of S is same as 2nd and 3rd row of A matrix.

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Let $\mathcal{E}_{12}(-3) = I - 3I_{12}$

$$\mathcal{E}_{12}(-3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1-0 & 0-3 & 0-0 \\ 0-0 & 1-0 & 0-0 \\ 0-0 & 0-0 & 1-0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

e)

$$M = \mathcal{E}_{12}(-3)\, S$$

$$= \begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 7 & 10 \\ 1 & 2 & 3 \\ 2 & -1 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 5 \end{pmatrix}$$

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b) we obtain the value same as value ~~m~~ matrix A. Because $\mathcal{E}_{12}(-3)$ affect the matrix in the opposite way. In last step

$$\mathcal{E}_{12}(3)S \Rightarrow \mathcal{E}_{12}(3) \cdot \mathcal{E}_{12}(-3) \cdot A$$

$$= 1 \cdot A \Rightarrow A$$