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Exercise 1

(a) $f(x) = \frac{2(x+4)}{x+2}$

quotient rules.

$$= \frac{\left(\frac{d}{dx}(2(x+4))\right)(x+2) - 2(x+4)\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$= \frac{2\left(\frac{d}{dx}(x+4)\right)(x+2) - 2(x+4)\cdot 1}{(x+2)^2}$$

$$= \frac{2\cdot 1(x+2) - (2x+8)}{(x+2)^2}$$

$$= \frac{2x+4-2x-8}{(x+2)^2}$$

$$= \frac{-4}{(x+2)^2}$$ Ans

b) $g(x) = \sin(2x+3)$

Chain rules

$$\frac{d}{dx}\sin(2x+3)\,\frac{d}{dx}(2x+3)$$

$$= \cos(2x+3)\cdot 2$$

$$= 2\cos(2x+3)$$ Ans

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c) $h(z) = \sqrt[z]{(3z^2-2)^2}$

$= (3z^2-2)^{\frac{2}{z}}$

$= (3z^2-2)^{\frac{2}{z}} \cdot \frac{d}{dz}\left(\log(3z^2-2)\cdot\frac{2}{z}\right)$

$= 2(3z^2-2)^{\frac{2}{z}} \cdot \frac{d}{dz}\left(\frac{\log(3z^2-2)}{z}\right)$

$= 2(3z^2-2)^{\frac{2}{z}} \cdot \frac{\left(z\left(\frac{d}{dz}(\log(3z^2-2)\right) - \left(\frac{d}{dz}z\right)\cdot\log(3z^2-2)\right)}{z^2}$

$= 2(3z^2-2)^{\frac{2}{z}} \cdot \frac{\left(\frac{z}{3z^2-2}\cdot\frac{d}{dz}(3z^2-2) - 1\cdot\log(3z^2-2)\right)}{z^2}$

$= 2(3z^2-2)^{\frac{2}{z}} \frac{\left(\frac{6z^2}{(3z^2-2)} - \log(3z^2-2)\right)}{z^2}$

$= 2(3z^2-2)^{\frac{2}{z}}\left(\frac{6}{(3z^2-2)} - \frac{\log(3z^2-2)}{z^2}\right)$ Ans

d) $P(x) = \exp(\cos(x)) + (\sin(x)+1)^2$

$= \frac{d}{dx}(\exp(\cos(x))) + \frac{d}{dy}(\sin(x)+1)^2$

$= (\exp(\cos(x)))\frac{d}{dx}\cos(x) + 2\cdot(\sin(x)+1)\cdot\frac{d}{dx}(\sin(x)+1)$

$= -\sin(x)\exp(\cos(x)) + 2(\sin(x)+1)\cdot\cos(x)$

$= -\sin(x)\exp(\cos(x)) + 2\cos(x)(\sin(x)+1)$ Ans

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e) $q(y) = \arcsin(y)$

$$= \frac{d}{dy} \sin^{-1}(y).$$

$$= \frac{1}{\frac{d}{dy} \sin(\sin^{-1}(y))}$$

Suppose $\sin^{-1}(y) = x$

$$= \frac{1}{\frac{d}{dy} \sin(x)}$$

$$= \frac{1}{\cos(x)}$$

And we know that

$$\sin^2(x) + \cos^2(x) = 1$$

$$\cos(x) = \mp\sqrt{1 - \sin^2 x} \quad \text{for } x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$= \frac{1}{\sqrt{1 - \sin^2 x}}$$

And $\sin^2 x$ is smaller than 1 than it is always positive.

Exercise 2

$$f(x) = x - 5\arctan(2x+1) \quad \text{for } [-5,5]$$

$$= \frac{d}{dx} x - 5\frac{d}{dx} \tan^{-1}(2x+1)$$

$$= 1 - 5 \cdot \frac{1}{(2x+1)^2+1} \cdot \frac{d}{dx}(2x+1)$$

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$$= 1 - \frac{5 \cdot 2}{(2x+1)^2+1}$$

$$0 = 1 - \frac{10}{4x^2+1+4x+1}$$

$$0 = \frac{4x^2+4x+2-10}{4x^2+2+4x}$$

$$0 = 4x^2+4x-8$$

$$0 = x^2+x-4$$

$$= (x+2)(x-1)$$

than

$$x = -2 \text{ and } x = 1$$

for ~~initial~~ $f : [-5, -2]$,

the minimum is

$$f(x) = x - 5\arctan(2x+1)$$

$$f(-5) = -5 - 5\arctan(-9)$$

$$= -5 + \cancel{5.03}\ 7.3$$

$$= 2.3$$

$$f(-2) = -5 - 5\arctan(2(-2)+1)$$

$$= -5 + 6.2$$

$$= 1.2$$

If $f''(x)$ is bigger than 0 that is minimum and if $f''(x)$ is smaller than 0 that is maximum.

$$f''(x) = \frac{d}{dx}\, 1 - \frac{10}{(2x+1)^2+1}$$

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$$= 10 \frac{d}{dx}\left(\frac{1}{(2x+1)^2+1}\right)$$

$$= \frac{10 \frac{d}{dx}(2x+1)^2}{((2x+1)^2+1)^2}$$

$$= \frac{10 \cdot 2 \cdot (2x+1) \cdot \frac{d}{dx}(2x+1)}{((2x+1)^2+1)^2}$$

$$= \frac{40(2x+1)}{((2x+1)^2+1)^2}$$

$$f''(-2) = \frac{40(2(-2)+1)}{((2(-2)+1)^2+1)^2}$$

$$= \frac{-120}{100} = -1.2$$

$$f''(1) = \frac{40(2(1)+1)}{((2(1)+1)^2+1)^2}$$

$$= \frac{120}{100} = 1.2$$

It mean local maximum is -2 and local minimum is 1 for $[-5, 5]$

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Exercise 3

a) $$\lim_{x \to 0} \frac{\sin x}{x}$$

$$= \lim_{x \to 0} \frac{\frac{d}{dx}\sin(x)}{\frac{d}{dx}x}$$

$$= \lim_{x \to 0} \frac{\cos(x)}{1}$$

$$= \frac{\cos(0)}{1} = \frac{1}{1} = 1$$ Ans

b) $$\lim_{x \to 0} \frac{a^x - 1}{x}$$

$$= \lim_{x \to 0} \frac{\frac{d}{dx}a^x - 1}{\frac{d}{dx}x}$$

$$= \lim_{x \to 0} \frac{a^x \log(a)}{1}$$

$$= \frac{a^0 \log(a)}{1} = 1 \cdot \log(a) = \log(a)$$ Ans

c) $$\lim_{x \to 0} \frac{\log(1+x)}{x}$$

$$= \lim_{x \to 0} \frac{\frac{d}{dx}\log(1+x)}{\frac{d}{dx}x}$$

$$= \lim_{x \to 0} \frac{\frac{1}{x+1}}{1}$$

$$= \frac{\frac{1}{0+1}}{1} = \frac{1}{1} = 1$$ Ans

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d) $$\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$$

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos(x))}{\frac{d}{dx}x^2}$$

$$= \lim_{x\to 0} \frac{\sin(x)}{2x}$$

$$= \lim_{x\to 0} \frac{\frac{d}{dx}\sin(x)}{2\frac{d}{dx}x}$$

$$= \lim_{x\to 0} \frac{\cos(x)}{2}$$

$$= \frac{\cos(0)}{2} = \frac{1}{2} = \frac{1}{2}$$ Ans

e) $$\lim_{x\to\infty} \frac{x^n}{e^x}$$

$$= \lim_{x\to\infty} \frac{\frac{d}{dx}x^n}{\frac{d}{dx}e^x}$$

$$= \lim_{x\to\infty} \frac{n.x^{n-1}}{e^x}$$

Second derivate $$= \lim_{x\to\infty} \frac{n\frac{d}{dx}(x^{n-1})}{\frac{d}{dx}e^x}$$

$$= \lim_{x\to\infty} \frac{n(n-1)x^{n-2}}{e^x}$$

and apply hopital rule n times than we get.

$$= \lim_{x\to\infty} \frac{n!\,x^{n-n}}{e^x}$$

$$= n!\lim_{x\to\infty} \frac{1}{e^x}$$

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And we know that $e^x > x$ than

$\frac{1}{e^x} < \frac{1}{x}$ and $\lim_{x \to \infty} \frac{1}{x} = 0$ therefore

$\lim_{x \to \infty} \frac{1}{e^x} = 0$ Ans

Bonus exercise

$f : [0,5]$ $\quad f(x) = x^4 + x^2$

It is continous and strictly increasing and the dirivative of fuction is

$$= \frac{d}{dx} x^4 + \frac{d}{dx} x^2$$

$$= 4x^3 + 2x$$

and $f'(0) = 4(0)^3 + 2(0) = 0$

According to theorm we know that f is strictly increasing, if $f'(x) > 0$ for all $x \in [a, b)$ but our f is equal to zero when x is zero.