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Exercise 1

$$f_n = \begin{cases} n=0 & f_0 = 0 \\ n=1 & f_1 = 1 \\ n>1 & f_n = f_{n-1} + f_{n-2} \end{cases} \quad \text{for } n \in N$$

Suppose f_{3n} is even.

Base case- $n=0$

$f_{3n} = 0$

$f_{3(0)} = 0$

$f_0 = 0$ and

Induction ~~ho~~ step-

Suppose f_{3n} is even for all $n \in N$

and $f_{3n} = f_{3n-1} + f_{3n-2}$

$f_{3(n+1)} = f_{3(n+1)-1} + f_{3(n+1)-2}$

$f_{3n+3} = f_{3n+2} + f_{3n+1}$

$= f_{3n+1} + f_{3n} + f_{3n+1}$

$= 2 \cdot f_{3n+1} + f_{3n}.$

Than f_{3n+3} is even. And we know that the ~~this~~ if we multiply any number with 2 is even. and we can see the every 3^{rd} number is even in this series.

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Exercise 2

a) Ratio test define $\sum_{n=0}^{\infty} a_n$

$$L = \lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)$$

If $L<1$ the series is convergent
if $L>1$ the serie is divergent
If $L=1$ the serie may be divergent or Convergent

Let we have $\sum_{k=0}^{\infty} \frac{4}{3^k}$

$$L = \lim_{k\to\infty}\left|\frac{\frac{4}{3^{k+1}}}{\frac{4}{3^k}}\right|$$

$$= \lim_{k\to\infty}\left|\frac{4}{3^{k+1}}\cdot\frac{3^k}{4}\right|$$

$$= \lim_{k\to\infty}\left|\frac{1}{3}\right|$$

$$= \frac{1}{3} < 1$$

It means that series is convergent

Sum of series.

$$\sum_{k=0}^{\infty}\frac{4}{3^k} = 4\sum_{k=0}^{\infty}\frac{1}{3^k}$$

$\cancel{\frac{1}{3^k}}$ $r = \frac{1}{3^k}$

$a = 4$

$$a.\frac{\text{First term}}{1-r} \Rightarrow 4.\frac{\left(\frac{1}{3^0}\right)}{1-\frac{1}{3^k}} \Rightarrow \frac{4}{\frac{2}{3}} \Rightarrow 2\cdot 3 \Rightarrow 6$$

The sum of series is 6 Ans

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b) Let $\sum_{n=1}^{\infty} \frac{n^4}{2^n}$

$$L = \lim_{n\to\infty} \left| \frac{\frac{(n+1)^4}{2^{n+1}}}{\frac{n^4}{2^n}} \right|$$

If $L < 1$ the series is ~~di~~ convergent.
If $L > 1$ the series is divergent
If $L = 1$ the series maybe divergent or convergent

$$L = \lim_{n\to\infty} \left| \frac{\frac{(n+1)^4}{2^{n+1}}}{\frac{n^4}{2^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^4}{2^{(n+1)}} \cdot \frac{2^n}{n^4} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^4}{2 \cdot n^4} \right|$$

$$= \frac{1}{2} \lim_{n\to\infty} \left| \frac{n^4 + 4n^3 + 6n^2 + 4n + 1}{n^4} \right|$$

$$= \frac{1}{2} \lim_{n\to\infty} \left| \frac{n^4}{n^4} + \frac{4n^3}{n^4} + \frac{6n^2}{n^4} + \frac{4n}{n^4} + \frac{1}{n^4} \right|$$

$$= \frac{1}{2} \left| \lim_{n\to\infty}(1) + 4\lim_{n\to\infty}\left(\frac{1}{n}\right) + 6\lim_{n\to\infty}\left(\frac{1}{n^2}\right) + 4\lim_{n\to\infty}\left(\frac{1}{n^3}\right) + \lim_{n\to\infty}\left(\frac{1}{n^4}\right) \right|$$

$$= \frac{1}{2} \left| 1 + 0 + 0 + 0 + 0 \right|$$

$$= \frac{1}{2} < 1$$

This series is convergent.
According to maxima sum is 150.

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c) Let $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n+2}$

If $L<1$ the series is convergent
If $L>1$ the series is divergent
If $L=1$ The series may be convergent or divergent

Let

$$L = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}(n+1)}{(n+1)+2}}{\frac{(-1)^n n}{n+2}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)^n (n+1)}{(n+3)} \cdot \frac{(n+2)}{(-1)^n n} \right|$$

$$= \lim_{n\to\infty} \frac{(n+1)(n+2)}{(n+3)(n)}$$

$$= \lim_{n\to\infty} \frac{n^2+4n+3}{n^2+3n}$$

$$= \lim_{n\to\infty} \frac{\frac{n^2}{n^2}+\frac{4n}{n^2}+\frac{3}{n^2}}{\frac{n^2}{n^2}+\frac{3n}{n^2}}$$

$$= \frac{\lim_{n\to\infty}(1)+4\lim_{n\to\infty}\left(\frac{1}{n}\right)+3\lim_{n\to\infty}\left(\frac{1}{n^2}\right)}{\lim_{n\to\infty}(1)+3\lim_{n\to\infty}\left(\frac{1}{n}\right)}$$

$$= \frac{1+0+0}{1+0} \Rightarrow 1$$

The ratio test is inconclusive.

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Exercise 3

a)

```
X(n) := sum(1/k!, k, 0, n);
euler: bfloat(%e), fpprec:4;
t1: 1;
X_n: bfloat(x(1)), fpprec:4;
For i:2 while not(is(equal(euler, x_n))) do (
    X_n: bfloat(x(i));
    fpprec:4,
    t1 = t1+1
    Print("Current:" x_n)
    Print("Times:" t1)
```

$x_{n_7} = 2.71860$

It takes 7 cycles to equal to 2.71860.

b)

```
y(n) := ~~Sum~~((1+1/n)^n;
euler: bfloat(%e), fpprec:4;
t1: 1;
y_n: bfloat(y(1)), fpprec: 4;
For i:2 while not(is(equal(euler, y_n))) do (
    y_n: bfloat(y(i)),
    fpprect: 4;
    t1=t1+1
    Print(y_n)
    Print(ti)
```

$x_{n_{4823}} = 2.71860$

It takes 4823 cycles than equal to 2.71860

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c)

```
z(n) := sum(n/(n!)^1/n);
euler : bfloat(%e), fpprec:4;
t1 : 1;

z_n: bfloat(z(1)), fpprec:4;
For i:2 while not(is(equal(euler, z_n))) do(
        z_n: bfloat(z(i)),
        fpprec:4,
        t1: t1+1
        Print(z_n)
        Print(t1)
```

It take >4823 than equal to 2.71860