Exercise 5

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix}$$

And

$$A - \lambda I = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 0 \\ 2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 2 & 0 \\ 2 & 3-\lambda & -2 \\ 0 & -2 & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I)$$

$$= \det \begin{bmatrix} 4-\lambda & 2 & 0 \\ 2 & 3-\lambda & -2 \\ 0 & -2 & 2-\lambda \end{bmatrix}$$

$$= (4-\lambda) \begin{vmatrix} 3-\lambda & -2 \\ -2 & 2-\lambda \end{vmatrix} - 2 \begin{vmatrix} 2 & -2 \\ 0 & 2-\lambda \end{vmatrix} + 0 \begin{vmatrix} 2 & 3-\lambda \\ 0 & -2 \end{vmatrix}$$

$$= (4-\lambda)\left((3-\lambda)(2-\lambda) - (-2)(-2)\right) - 2\left((2)(2-\lambda) - 0(-2)\right) + 0$$

$$= (4-\lambda)\left(6 - 3\lambda - 2\lambda + \lambda^2 - 4\right) - 2(4 - 2\lambda)$$

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$$= (4-\lambda)(\lambda^2-5\lambda+2) - 8 + 4\lambda$$

$$= 4\lambda^2 - 20\lambda + 8 - \lambda^3 + 5\lambda^2 - 2\lambda - 8 + 4\lambda$$

$$= -\lambda^3 + 9\lambda^2 - 18\lambda + 0$$

$$= \lambda(-\lambda^2 + 9\lambda - 18)$$

$$= -\lambda(\lambda^2 - 9\lambda + 18)$$

$$= -\lambda(\lambda-3)(\lambda-6)$$

than

$$\lambda = 0$$
$$\lambda = 3$$
$$\lambda = 6$$

than

$$AX = 0$$

$$\begin{bmatrix} 4-0 & 2 & 0 \\ 2 & 3-0 & -2 \\ 0 & -2 & 2-0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix}$$

than

$$\begin{bmatrix} 4 & 2 & 0 \\ 2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 4 & 2 & 0 & 0 \\ 2 & 3 & -2 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

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Multiply row 1 by $-\frac{1}{2}$ and add row 2

$$= \left[\begin{array}{ccc|c} 4 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

~~Multiply~~ Add row 2 and 3

$$= \left[\begin{array}{ccc|c} 4 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$4x_1 + 2x_2 = 0$$

$$2x_2 - 2x_3 = 0$$

$x_3 = 1$

then

$$2x_2 - 2(1) = 0$$

$$x_2 = \frac{2}{2}$$

$$x_2 = 1$$

$$4x_1 + 2(1) = 0$$

$$x_1 = \frac{-2}{4} = -\frac{1}{2}$$

~~he~~

Eigen vector is $\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$

$\lambda = 3$

$$= \begin{bmatrix} 4-3 & 2 & 0 \\ 2 & 3-3 & -2 \\ 0 & -2 & 2-3 \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & -2 & -1 \end{bmatrix}$$

than

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 2 & 0 & -2 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right]$$

Multiply row 1 by -2 and add row 2

$\Leftrightarrow$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right]$$

Multiply row 2 by $-\frac{1}{2}$ and add row 3

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

than

$$x_1 + 2x_2 = 0$$
$$-4x_2 - 2x_3 = 0$$

$x_3 = 1$

than

$$-4x_2 - 2(1) = 0$$
$$x_2 = -\frac{2}{4} = -\frac{1}{2}$$

$$x_1 + 2\left(-\frac{1}{2}\right) = 0$$
$$x_1 = 1$$

than $\begin{bmatrix} 1 \\ -\frac{1}{2} \\ 1 \end{bmatrix}$

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$\lambda = 6$

than

$$= \begin{bmatrix} 4-6 & 2 & 0 \\ 2 & 3-6 & -2 \\ 0 & -2 & 2-6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & 0 \\ 2 & -3 & -2 \\ 0 & -2 & -4 \end{bmatrix}$$

than

$$\begin{bmatrix} -2 & 2 & 0 \\ 2 & -3 & -2 \\ 0 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} -2 & 2 & 0 & 0 \\ 2 & -3 & -2 & 0 \\ 0 & -2 & -4 & 0 \end{array}\right]$$

Multiply row Add row 1 and 2

$$= \left[\begin{array}{ccc|c} -2 & 2 & 0 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -2 & -4 & 0 \end{array}\right]$$

Multiply row 2 by -2 and add row 3

$$= \left[\begin{array}{ccc|c} -2 & 2 & 0 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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$$-2x_1 + 2x_2 = 0$$
$$-x_2 - 2x_3 = 0$$

then $x_3 = 1$

$$-x_2 - 2(1) = 0$$
$$-x_2 = 2$$
$$x_2 = -2$$

and

$$-2x_1 + 2x_2 = 0$$
$$-2x_1 + 2(-2) = 0$$
$$-2x_1 = 4$$
$$x_1 = -2$$

the Eigen vector is $\begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$