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Exercise 1

a)

$$\int_0^1 xe^x\,dx$$

$$\int xe^x\,dx \quad , f(x) = x\ , g'(x) = e^x$$
$$g(x) = e^x$$

$$= xe^x - \int 1\cdot e^x\,dx$$

$$= xe^x - e^x$$

$$= e^x(x-1)$$

$$\int_{0=a}^{1=b} xe^x = F(b) - F(a) \quad \text{for } F(x) = \int xe^x$$

$$= e^1(1-1) - e^0(0-1)$$

$$= 0 - (-1)$$

$$= 1$$

b)

$$\int x\sin(x)\,dx$$

$$f(x) = x \qquad g'(x) = \sin x$$
$$g(x) = -\cos(x)$$

$$= x(-\cos x) - \int 1\cdot(-\cos(x))$$

$$= -x\cos(x) + \sin x$$

$$= \sin(x) - x\cos(x)$$

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c)

$$\int e^x \sin(x)\, dx$$

$$f(x) = \sin(x) \quad , \; g'(x) = e^x$$
$$g(x) = e^x$$

$$= e^x \sin(x) - \int e^x \cos(x)\, dx$$

$$f(x) = \cos(x) \quad , \; g'(x) = e^x$$
$$g(x) = e^x$$

$$= e^x \sin(x) - \left(e^x \cos(x) - \int e^x (-\sin(x))\, dx \right)$$

$$= e^x \sin(x) - e^x \cos(x) + \int e^x (-\sin(x)\, dx$$

$$\int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)\, dx$$

Add $\int e^x \sin(x)\, dx$ both sides

$$\int e^x \sin(x) dx + \int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x)$$

$$2\int e^x \sin(x)\, dx = e^x (\sin(x) - \cos(x))$$

Divide by 2 both sides

$$\int e^x \sin(x)\, dx = \frac{e^x (\sin(x) - \cos(x))}{2}$$

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Exercise 2

a) $$\int \frac{1}{1+x}\, dx$$

Let $y = 1+x$

$dy = dx$

$$= \int \frac{1}{y}\, dy$$

$$= \log(y) + c$$

$$= \log(1+x) + c$$

b) $$\int_0^{\sqrt{\pi}} x \sin(x^2)\, dx$$

Let $y = x^2$

$dy = 2x\, dx$

$x = \frac{1}{2} dy$

then $$\int x \sin(x^2)\, dx$$

$$= \int \sin(y) \cdot x\, dx$$

$$= \int \sin(y) \cdot \frac{1}{2}\, dy$$

$$= \frac{1}{2} \int \sin(y)\, dy$$

$$= \frac{1}{2}(-\cos(y))$$

$$= -\frac{1}{2}\cos(\cancel{y}\, x^2)$$

then

$$\int_0^{\sqrt{\pi}} x \sin(x^2)\, dx = -\frac{1}{2}\left(\cos((\sqrt{\pi})^2) - \cos(0^2)\right)$$

$$= -\frac{1}{2}\left(\cos(\pi) - \cos(0)\right)$$

$$= -\frac{1}{2}(-1-1)$$

$$= \frac{\cancel{2}}{\cancel{2}} = 1 \quad \underline{\text{Ans}}$$

c)

$$\int_0^1 e^{\sqrt{x}}\, dx$$

Let $y = \sqrt{x}$

$$dy = \frac{1}{2\sqrt{x}}\, dx$$

$$2\sqrt{x}\, dy = dx$$

$$\int e^{\sqrt{x}}\, dx$$

$$= \int e^{y} \cdot 2\sqrt{x}\, dy$$

$$= 2\int e^y \cdot y \, dy$$

$f(y) = y$, $g'(y) = e^y$; $g(y) = e^y$

$$= 2\left(y \cdot e^y - \int 1 \cdot e^y \, dy\right)$$

$$= 2\left(y \cdot e^y - e^y\right)$$

$$= 2e^y(y-1)$$

$$= 2e^{\sqrt{x}}(\sqrt{x}-1)$$

than

$$\int_0^1 e^{\sqrt{x}} \, dx = 2e^{\sqrt{1}}(\sqrt{1}-1) - 2e^{\sqrt{0}}(\sqrt{0}-1)$$

$$= 0 - 2(-1)$$

$$= 2$$

Exercise 3

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$

Suppose $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$

$$\frac{y^2}{b^2} \leq 1 - \frac{x^2}{a^2}$$

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$$y = \pm\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}$$

$$= \frac{b}{a}\sqrt{a^2-x^2}$$

Since ellipse extends from $x=-a$ to $x=a$ and area of top half of ecllipse is

$$= \int_{-a}^{a} \frac{b}{a}\sqrt{a^2-x^2}$$

And area of whole ecllipse is

$$= 2\int_{-a}^{a} \frac{b}{a}\sqrt{a^2-x^2}$$

And use substitution $x = a\sin\theta$ then $dx = a\cos\theta\, d\theta$ and $\sin\theta = \frac{x}{a}$ and then $\sin(\theta) = \frac{-a}{a} = -1$ when $x=-a$ mean $\theta = -\pi/2$ and $x=a$ than $\theta = \pi/2$

than

$$= 2\int_{-\pi/2}^{\pi/2} \frac{b}{a}\sqrt{a^2-a^2\sin^2\theta}\, dx$$

$$= 2\frac{b}{a}\int_{-\pi/2}^{\pi/2} \sqrt{a^2-a^2\sin^2\theta}\, dx$$

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~~Multiply~~

$$\leq 2\frac{b}{a}\int_{-\pi/2}^{\pi/2}\sqrt{a^2-a^2\sin^2\theta}\cdot a\cos\theta\, d\theta$$

$$\leq 2\frac{b}{a}\int_{-\pi/2}^{\pi/2}\sqrt{a^2\cos^2\theta}\cdot a\cos\theta\, d\theta$$

$$\leq 2\frac{b}{a}\int_{-\pi/2}^{\pi/2}a^2\cos^2\theta\, d\theta$$

$$\leq 2ab\int_{-\pi/2}^{\pi/2}\frac{1+\cos(2\theta)}{2}\, d\theta$$

$$\leq ab\int_{-\pi/2}^{\pi/2}1+\cos(2\theta)\, d\theta$$

$$\leq ab\left(\theta+\frac{\sin(2\theta)}{2}\right)\Bigg|_{-\pi/2}^{\pi/2}$$

$$\leq ab\left[\left(\frac{\pi}{2}+0\right)-\left(-\frac{\pi}{2}+0\right)\right]$$

$$\leq \pi ab$$

The area of ellipse is πab

Ans