Number Systems

Place value of numbers:

123 -> 1 hundred + 2 tens + 3 ones

In general a number ab is represented in terms of its place value as -> 10a+b

Classification of numbers:

Numbers are broadly classified as real numbers and complex numbers. We will be looking into only real numbers.

Real numbers can be classified as follows

- Rational Numbers- Numbers, which can be represented in the form p/q where q is not equal to zero
  eg: 2/3, 8/1

- Irrational- Numbers which cannot be represented in the form p/q where q not equal to zero
  eg: $\sqrt{3}$, $\sqrt{2}$

- Natural Numbers- > {1, 2, 3, …..∞} -> +ve counting numbers

- Whole numbers -> {0, 1, 2, 3 …..∞} – natural no: including zero

- Integers -> set of whole number along with -ve numbers {-∞.........-3, -2, ….0, 1, 2, ….∞}

- Even Numbers-> divisible by 2 {2, 4, 6…}
- odd Numbers-> not divisible by 2 {1, 3, 5, …}
- Prime Numbers-> no factors other than 1 & itself {2, 3, 5, 7, 11….}
- Composite Numbers -> numbers having other factors besides 1& itself {4, 6, 8, 9, 10, 12….}
- Coprimes - numbers which are primes with respect to each other. They have no common factors other than 1

  Eg:- 22,9. Co primes necessarily need not be prime numbers

Factors: To factor a number means to break it up into integers that can be multiplied together to get the original number

eg: 12 -> 2 x 6, 4 x 3 where 1, 2, 6, 4, 3, 12 are factors of 12

Note: For every number 1 and itself are always factors.

Finding factors of a number:

If a number is expressed in terms of its prime factors as $\mathrm{A}^{\mathrm{x}} \times \mathrm{B}^{\mathrm{y}} \times \mathrm{C}^{\mathrm{z}}$ then the number of factors of the given number is $(x+1) \times (y+1) \times (z+1)$

The number of prime factors of this number is given by (x+y+z) and the number of unique prime factors of this number is no. of base values (a, b, c) (in above example it is 3 i.e. the three unique prime factors here are a, b and c)

Division Method: Dividend = (Divisor × Quotient) + Remainder

Divisibility by 2 - all even numbers

Divisibility by 3 - sum of digits of numbers is divisible by 3

Ex: 729 -> 7 + 2 + 9 = 18 is divisible by 3 so the number is divisible by 3

Divisibility by 4 – last 2 digits of numbers is divisible by 4

Ex: 724 - last 2 digits is 24 which is divisible by 4. Therefore, the number is divisible by 4

Divisibility by 5 – last digit 5/0

Divisibility by 6 – number should be divisible by 3 & 2 for it to be divisible by 6

Divisibility by 7 – multiply unit digit by 2, subtract that value from remaining digits. Keep continuing this process

Ex: 11347

Last digit of 11347 is 7 -> 7 x 2 = 14

1134 - 14 = 1120

last digit of 1120 is 0 -> 0 x 2 = 0

112 - 0 = 112

last digit of 112 is 2 -> 2 x 2 = 4

11 - 4 = 7 => div by 7

Divisibility by 8 - last 3 digits divisible by 8

2512 -> 512 divisible by 8 so the number is divisible by 8

Divisibility by 9 - sum of digits is divisible by 9

Ex: 729 -> 7 + 2 + 9 = 18 is divisible by 9. So the number is divisible by 9

Divisibility by 10 – last digit '0'

Divisibility by 11 -> find sum of alternatives digits & subtracts thus & equal to zero or multiple of 11.

Ex: 1 2 3 2 1

1 + 3 + 1 = 5

2 + 2 = 4

5 - 4 = 1 so not divisible by 11

Ex: 1 4 6 4 1

1 + 6 + 1 = 8

4 + 4 = 8

8 - 8 = 0 so divisible by 11

Divisibility by 12 - > Number should be divisible by 4 & 3

Divisibility by 13 -> Multiply last digit by 4. Add it to remaining digit. Keep continuing this process

Ex: 50661 -> 1 x 4 = 4

5066 + 4 = 5070 -> 0 x 4 = 0

507 + 0 = 507 -> 7 x 4 = 28

50 + 28 = 78.

78 is divisible by 13. So the number is divisible by 13

Divisibility by 17 -> multiply last digit with 5. Subtract the answer from the remaining digits. Keep continuing the process

Ex: 3978 -> 8 x 5 = 40

397 - 40 = 357 -> 7 x 5 = 35

35- 35 = 0

So divisible by 17.

Finding the unit digit of the given number with certain power value.

Pattern method:

1. If the unit digit of the given number is 0, 1, 5 or 6, then the same number will be the unit digit of the given number (i.e., 0, 1, 5 or 6).
2. If the unit digit of the given number is 4, then we have to check the power value ie., whether the given power is odd or even power. If it is odd power then, the unit digit is 4, if the power is even number, then the unit will be 6.
3. The same condition is applicable for 9. If it odd power then, the unit digit is 9, if it is even power, then the unit digit is 1.
4. If the unit digit of the given number is 2, 3, 7, or 8, then

Unit digit 2	$2^1 = 2$
	$2^2 = 4$
	$2^3 = 8$
	$2^4 = 6$
Unit digit 3	$3^1 = 3$
	$3^2 = 9$
	$3^3 = 7$
	$3^4 = 1$
Unit digit 7	$7^1 = 7$
	$7^2 = 9$
	$7^3 = 3$

	$7^4 = 1$
Unit digit 8	$8^1 = 8$
	$8^2 = 4$
	$8^3 = 2$
	$8^4 = 6$

Shortcut formulae:

1. A number being divided by d_1 and d_2 successively leaves remainders r_1 and r_2 respectively. Then remainder when the same number is divided by $d_1 \times d_2$

   Remainder = $d_1 \times r_{2} + r_1$