Introduction to SFG

Alternative method to block diagram representation, developed by Samuel Jefferson Mason.

Advantage: the availability of a flow graph gain formula, also called Mason’s gain formula.

A signal-flow graph consists of a network in which nodes are connected by directed branches.

It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Comparison of BD and SFG

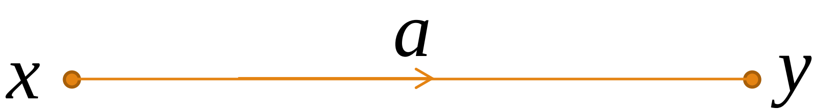
Block Diagram	Signal Flow Graph
	
In this case at each step, block diagram is to be redrawn. That's why it is tedious method. So there is a wastage of time and space.	Only one time SFG is to be drawn and then Mason's gain formula is to be valuated. So time and space is saved.

Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

- The signal flow graph of the equation is shown below;

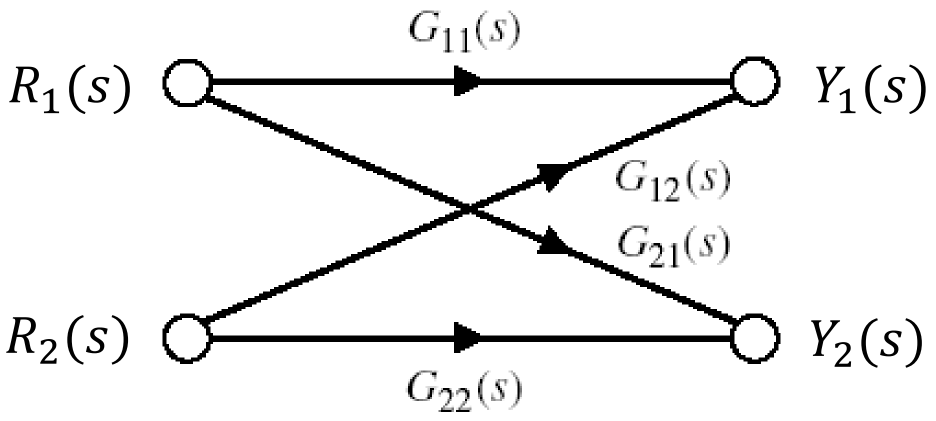


- Every variable in a signal flow graph is represented by a **Node**.
- Every transmission function in a signal flow graph is represented by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal flow.

Signal-Flow Graph Models

$$Y_1(s) = G_{11}(s) \cdot R_1(s) + G_{12}(s) \cdot R_2(s)$$

$$Y_2(s) = G_{21}(s) \cdot R_1(s) + G_{22}(s) \cdot R_2(s)$$



Signal-Flow Graph Models

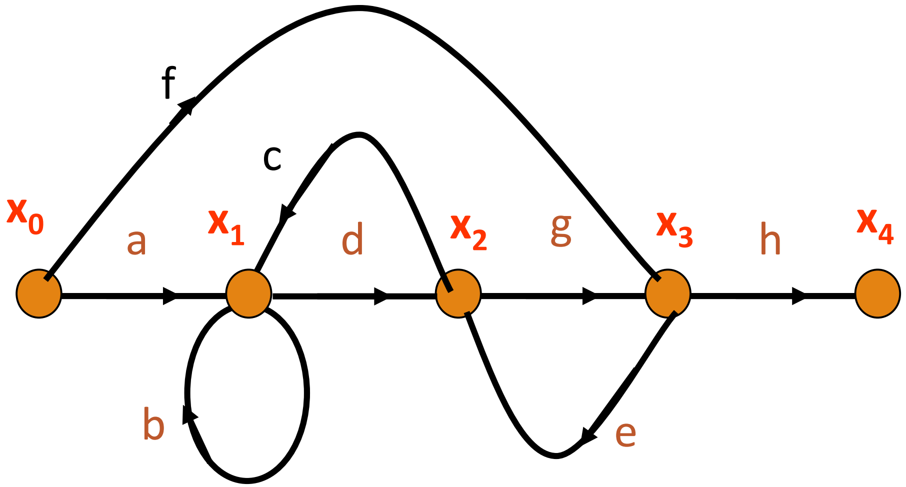
x_o is input and x_4 is output

$$x_1 = ax_0 + bx_1 + cx_2$$

$$x_2 = dx_1 + ex_3$$

$$x_3 = fx_0 + gx_2$$

$$x_4 = hx_3$$



Terminologies

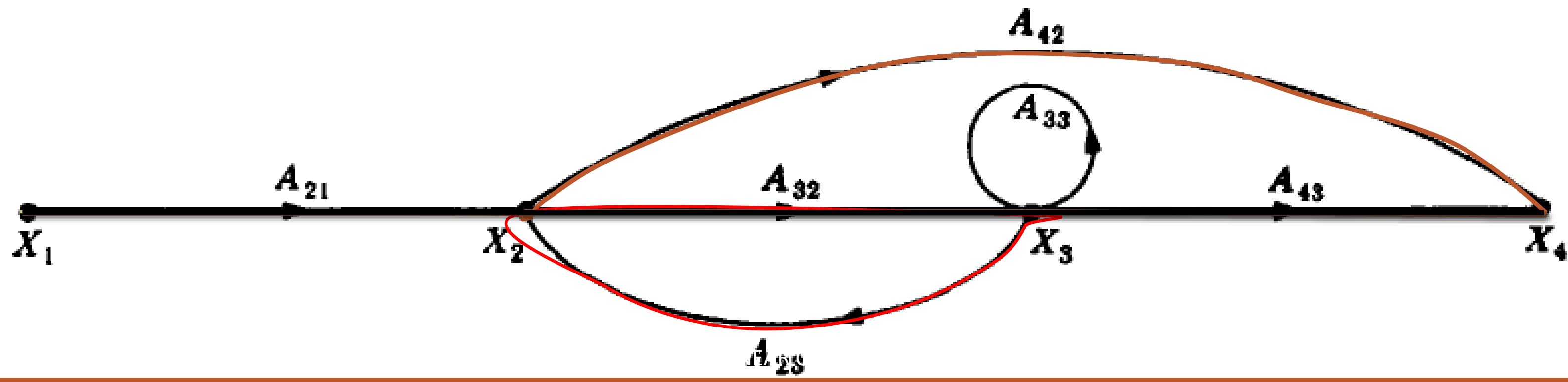
- An **input node** or source contain only the outgoing branches. i.e., X_1
- An **output node** or sink contain only the incoming branches. i.e., X_4
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e.,

  X_1 *to* X_2 *to* X_3 *to* X_4 X_1 *to* X_2 *to* X_4 X_2 *to* X_3 *to* X_4

- A **forward path** is a path from the input node to the output node. i.e.,

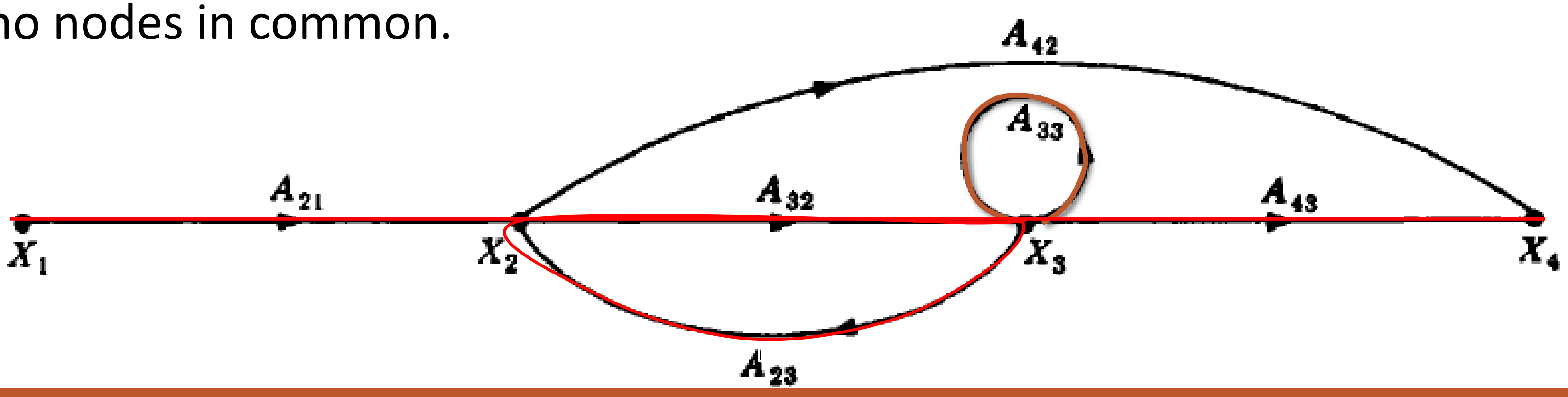
  X_1 *to* X_2 *to* X_3 *to* X_4, and X_1 *to* X_2 *to* X_4, are forward paths.

- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; X_2 *to* X_3 **and back to** X_2 is a feedback path.

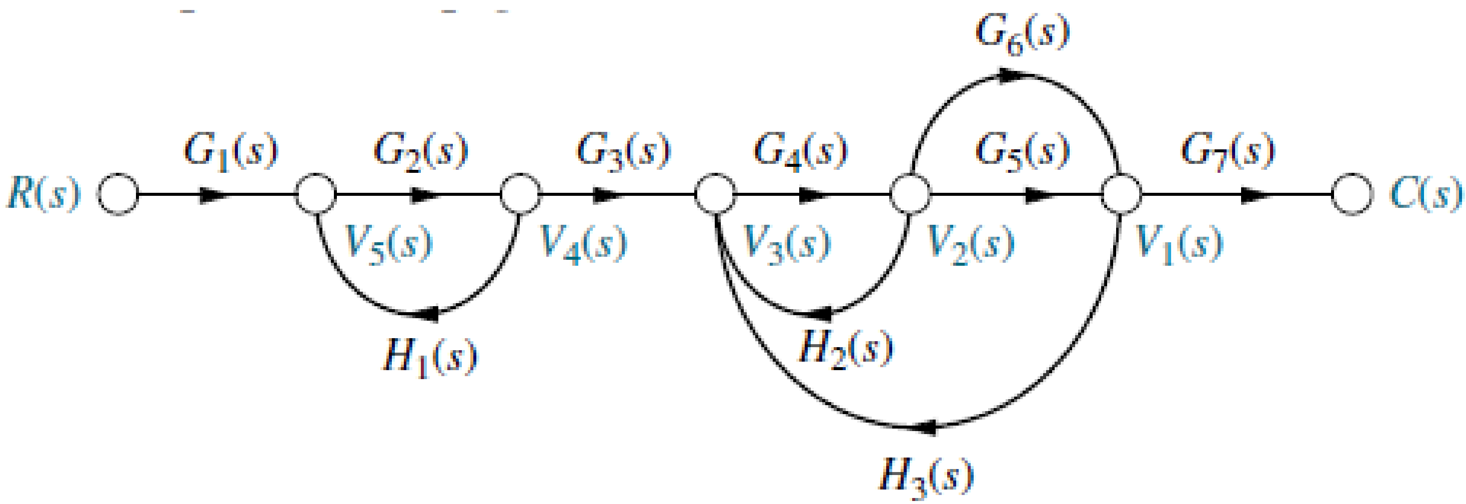


Terminologies

- A **self-loop** is a feedback loop consisting of a single branch. i.e.; A_{33} is a self loop.
- The **gain** of a branch is the transmission function of that branch.
- The **path gain** is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X_1 *to* X_2 *to* X_3 *to* X_4 is $A_{21}A_{32}A_{43}$
- The **loop gain** is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.

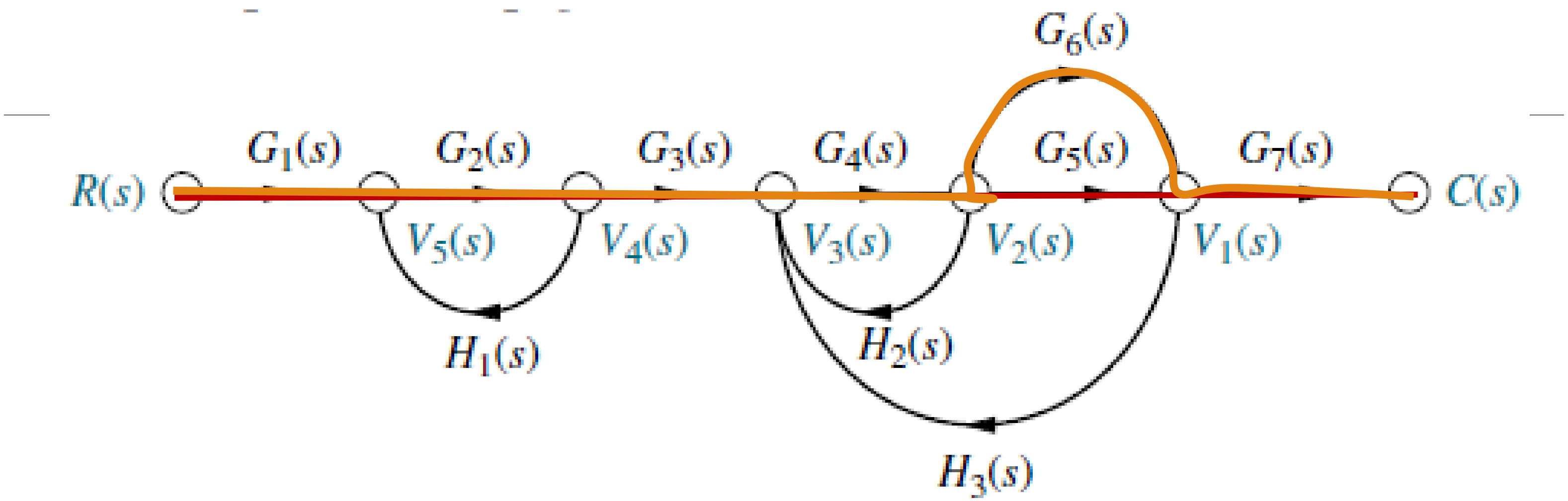


Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths (loops).
e) Determine the loop gains of the feedback loops.
f) Determine the path gains of the forward paths.
g) Non-touching loops

Consider the signal flow graph below and identify the following

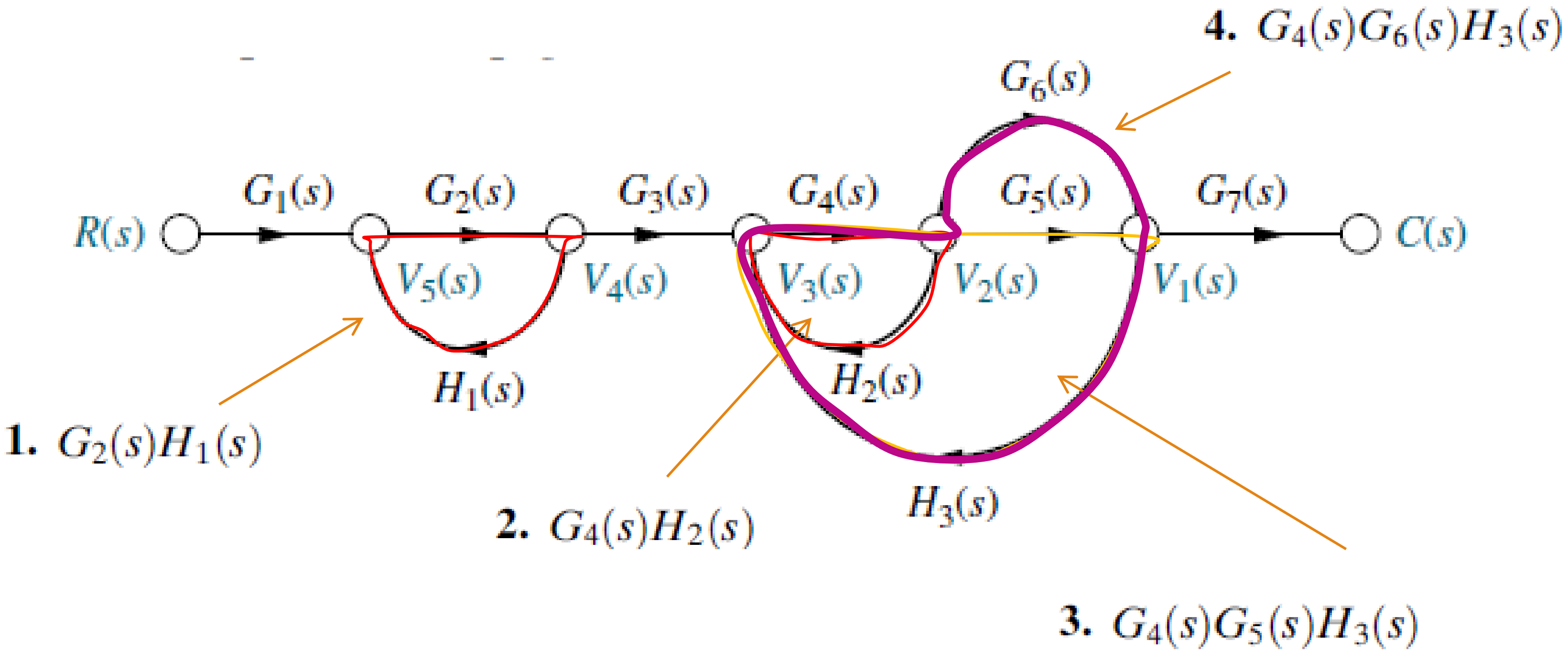


- There are two forward path gains;

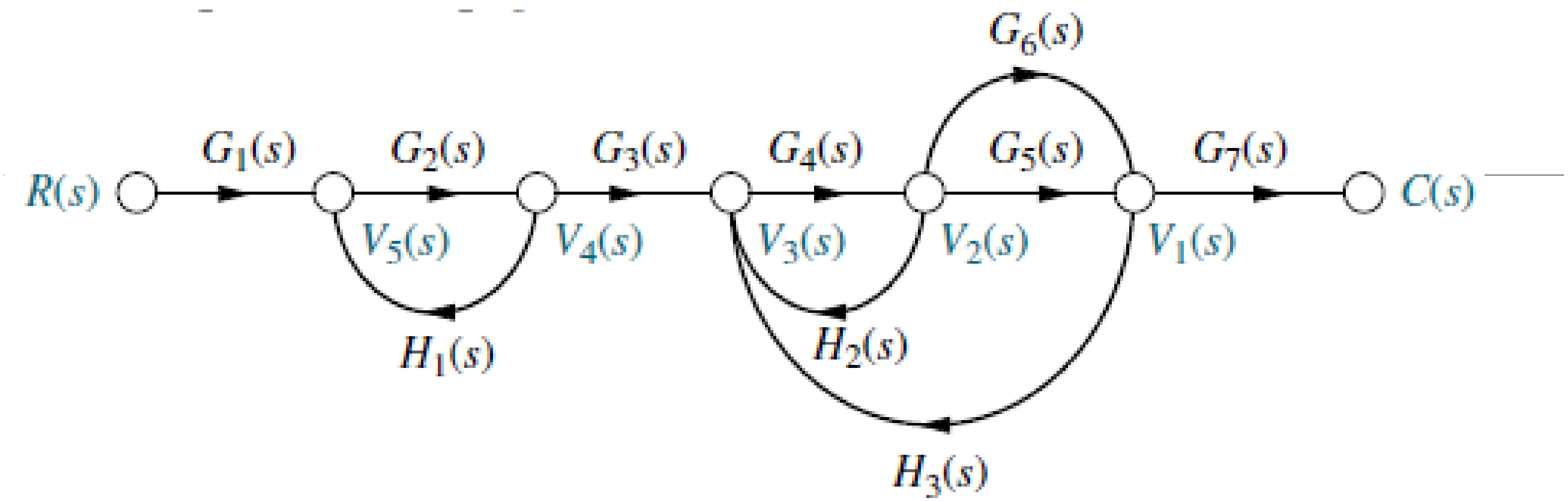
1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$ **2.** $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

Consider the signal flow graph below and identify the following

- There are four loops



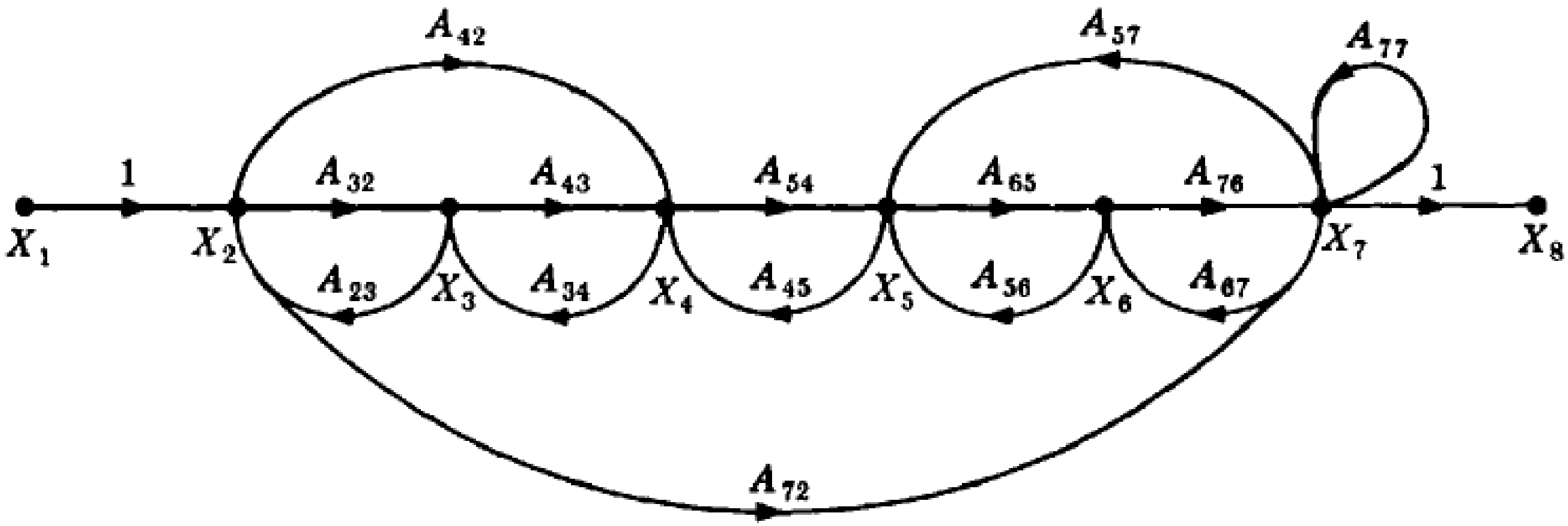
Consider the signal flow graph below and identify the following



- Nontouching loop gains;

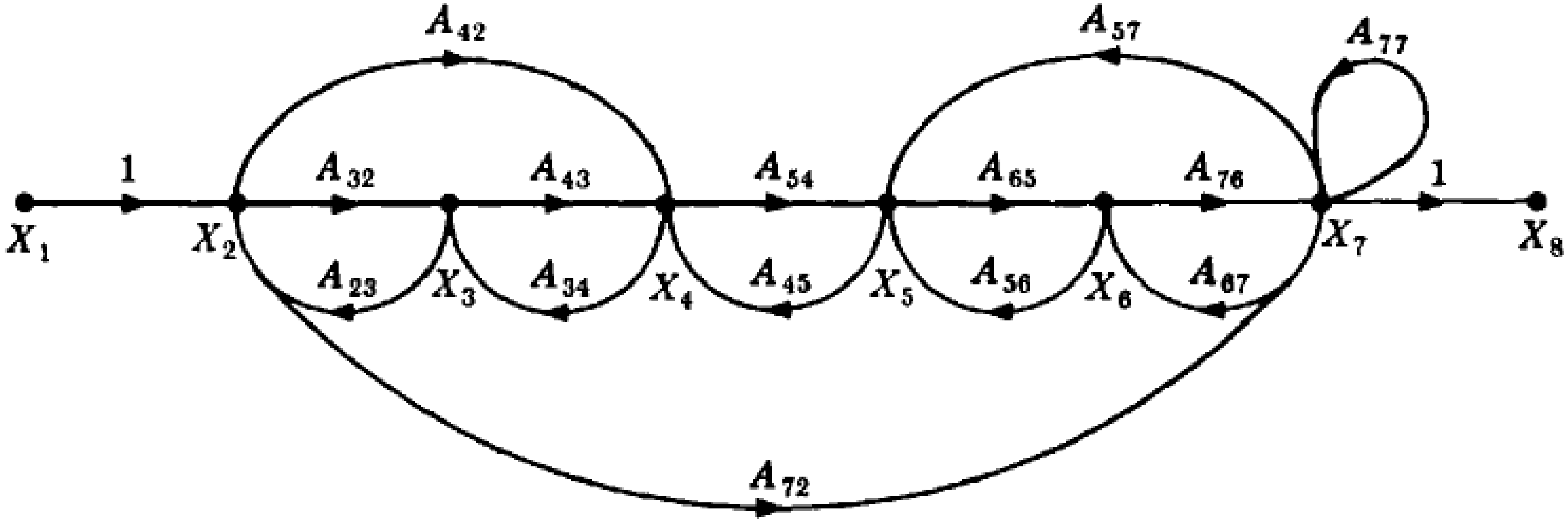
1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths.
e) Self loop.
f) Determine the loop gains of the feedback loops.
g) Determine the path gains of the forward paths.

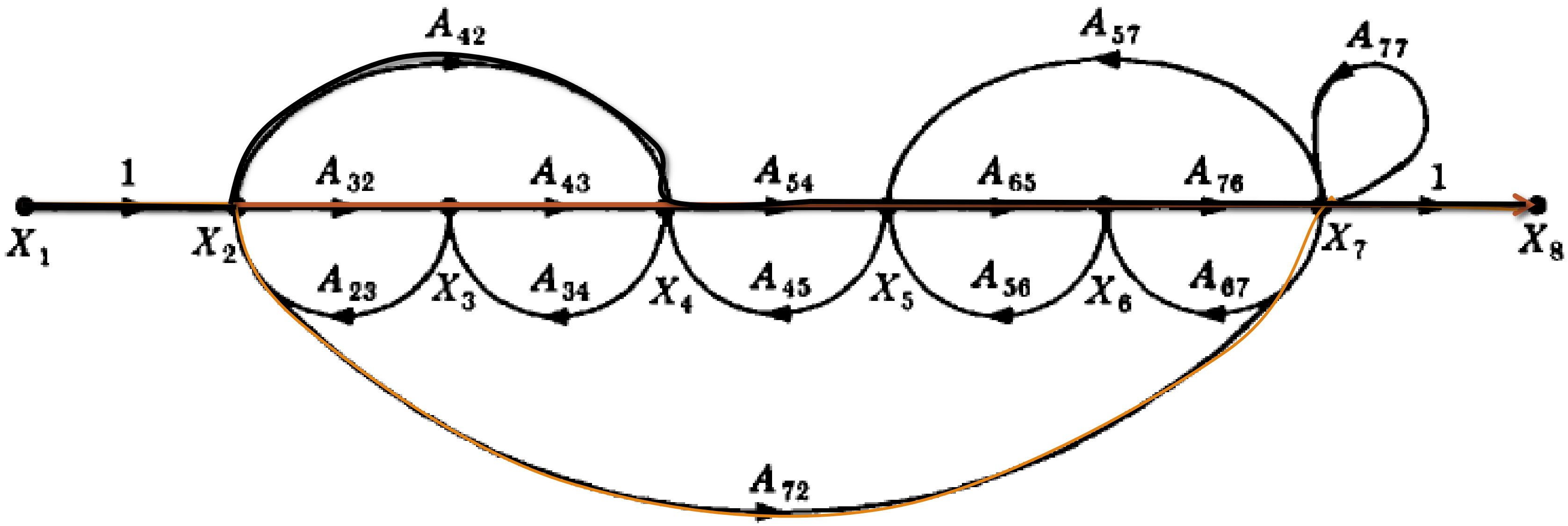
Input and output Nodes



a) Input node X_1

b) Output node X_8

(c) Forward Paths

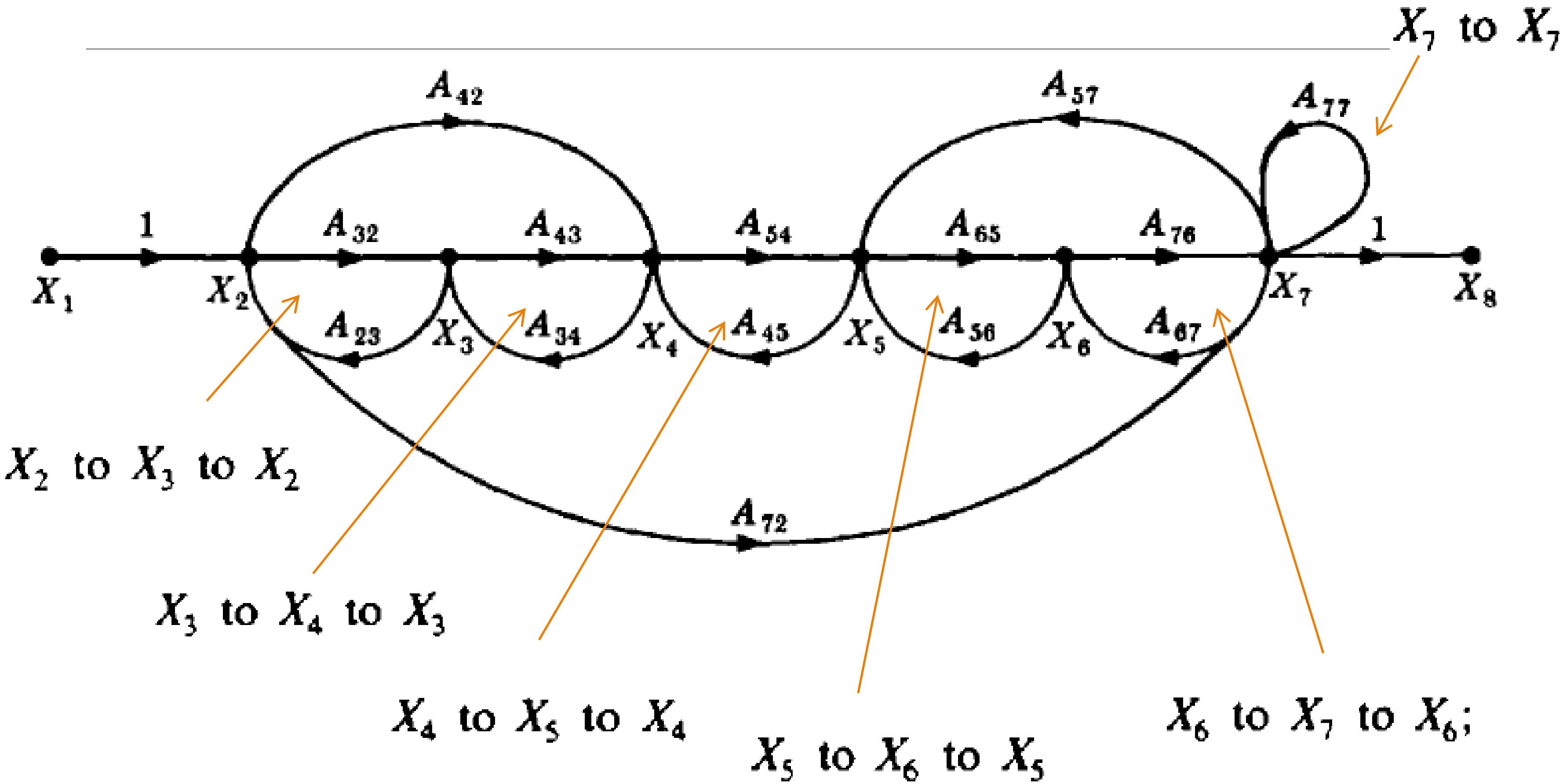


X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8

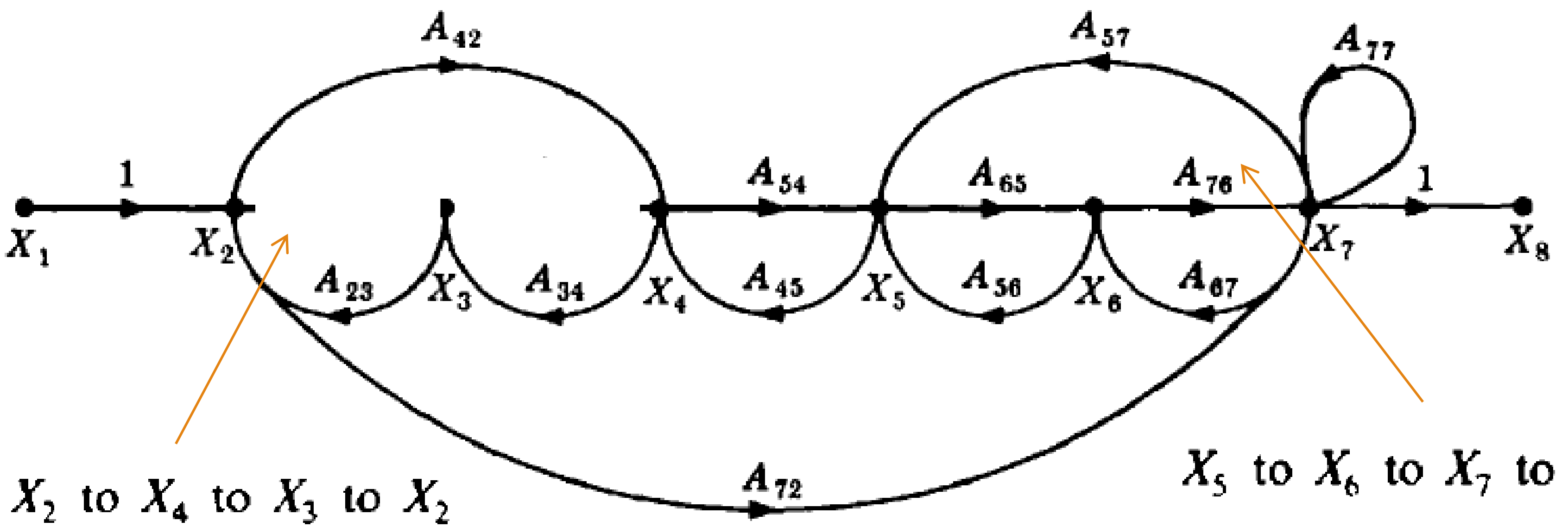
X_1 to X_2 to X_7 to X_8

X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

(d) Feedback Paths or Loops



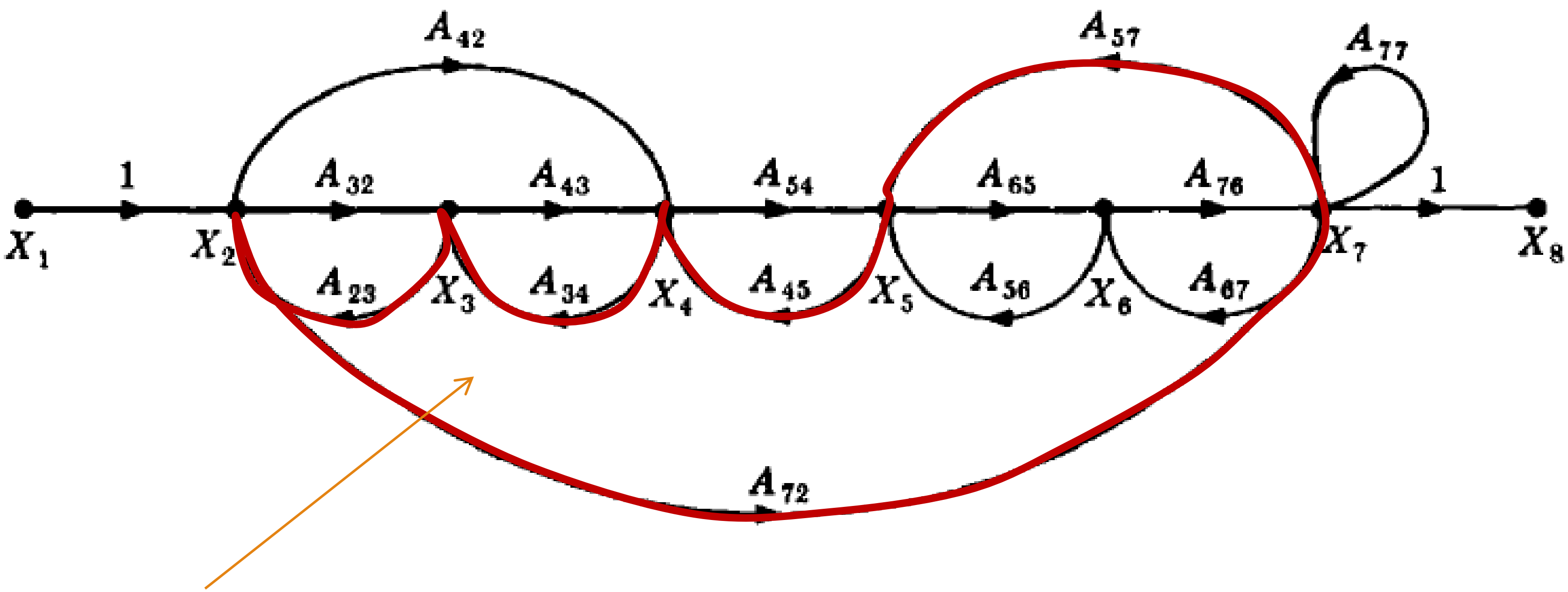
(d) Feedback Paths or Loops



X_2 to X_4 to X_3 to X_2

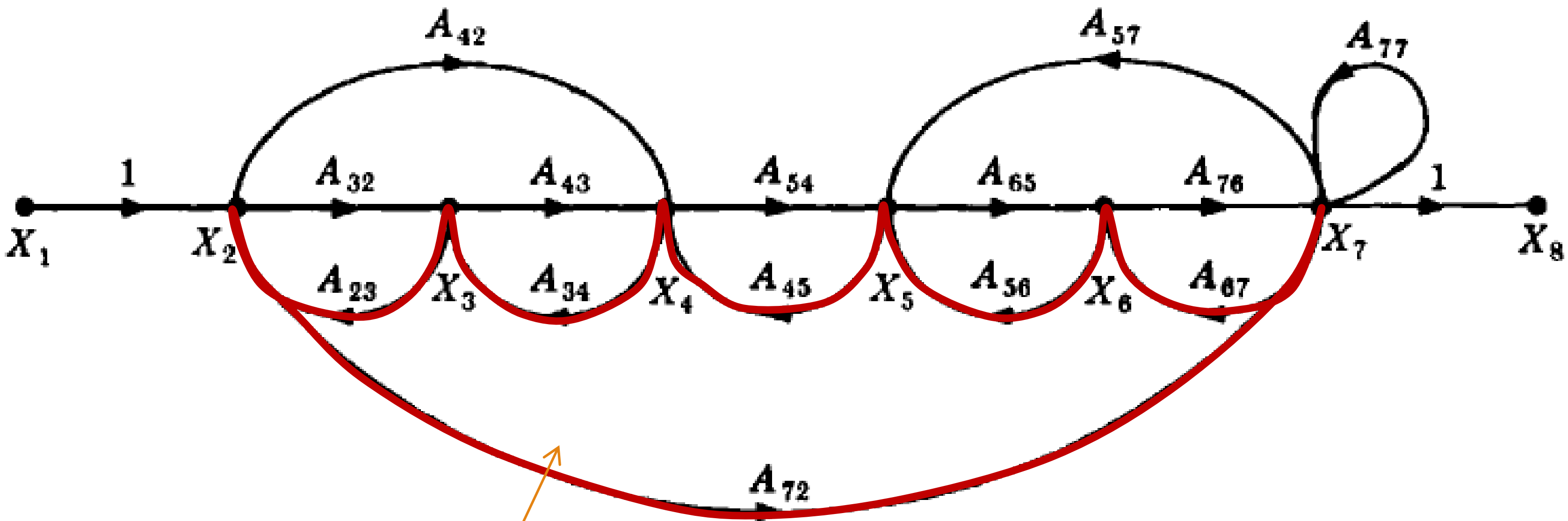
X_5 to X_6 to X_7 to X_5

(d) Feedback Paths or Loops



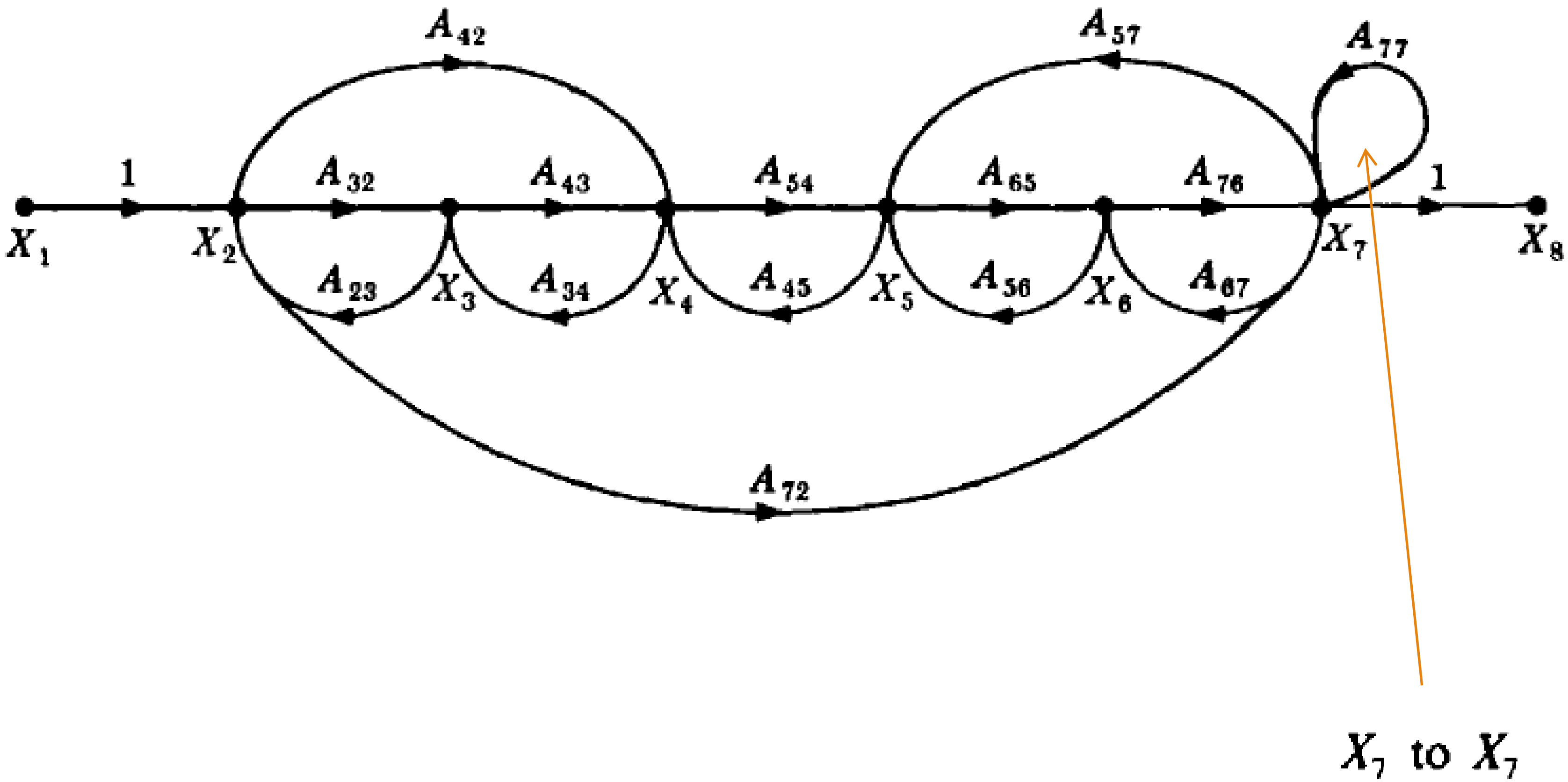
X_2 to X_7 to X_5 to X_4 to X_3 to X_2

(d) Feedback Paths or Loops



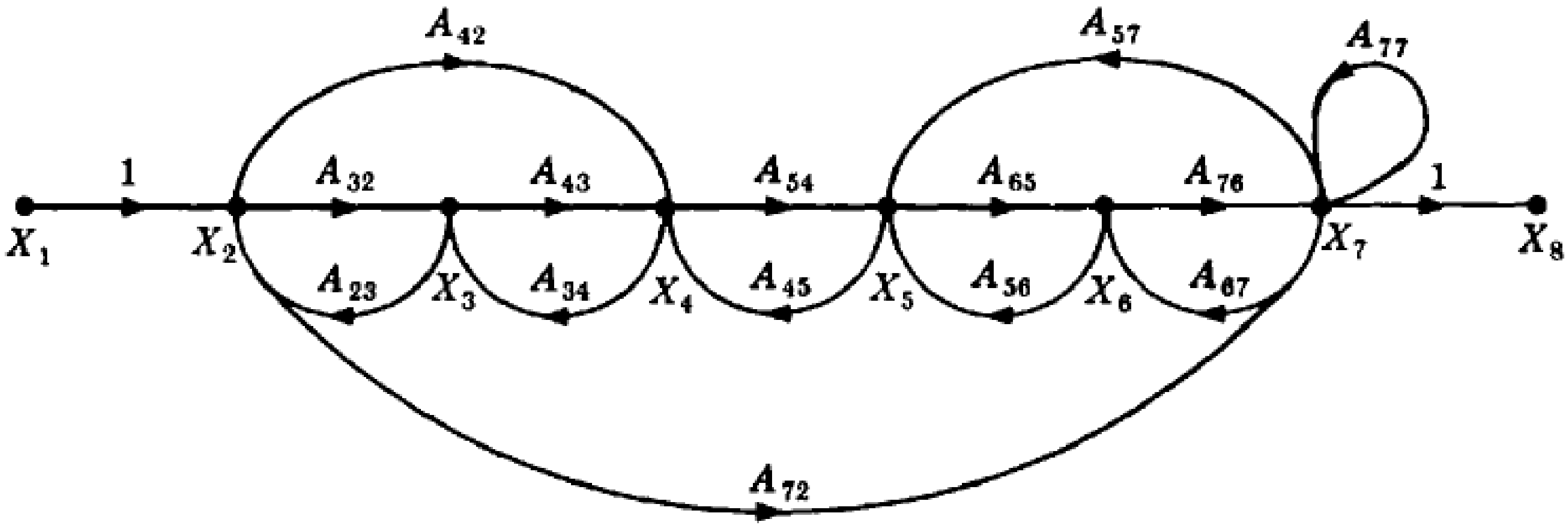
X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X_2

(e) Self Loop(s)



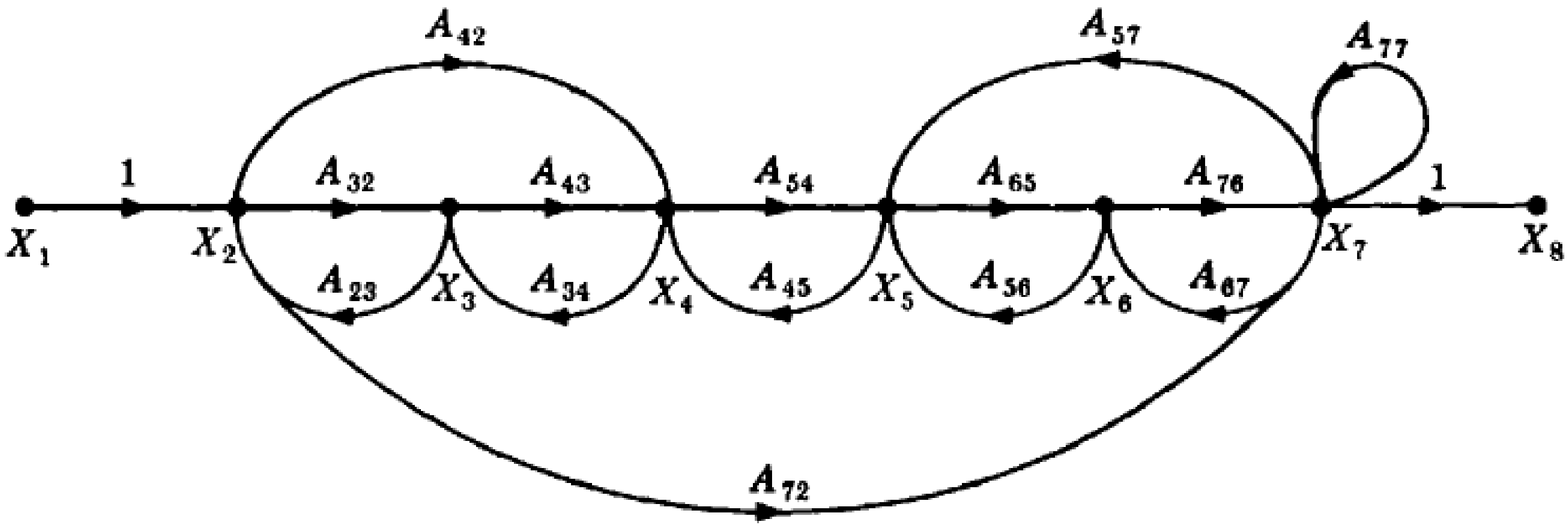
1
A_{42}
A_{32}
A_{43}
A_{54}
A_{57}
A_{77}
A_{65}
A_{76}
1
X_1
X_2
X_3
X_4
X_5
X_6
X_7
X_8
A_{23}
A_{34}
A_{45}
A_{56}
A_{67}
A_{72}
X_7 to X_7

(f) Loop Gains of the Feedback Loops



$A_{32}A_{23}$	$A_{76}A_{67}$	$A_{72}A_{57}A_{45}A_{34}A_{23}$
$A_{43}A_{34}$	$A_{65}A_{76}A_{57}$	$A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$
$A_{54}A_{45}$	A_{77}	
$A_{65}A_{56}$	$A_{42}A_{34}A_{23}$	

(g) Path Gains of the Forward Paths



$A_{32} A_{43} A_{54} A_{65} A_{76}$

A_{72}

$A_{42} A_{54} A_{65} A_{76}$

Mason’s Rule (Mason, 1953)

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- On the other hand, Mason’s rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

Mason's Rule:

- The transfer function, ***C(s)/R(s),*** of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

Where

n = number of forward paths.
P_i = the i^{th} forward-path gain.
Δ = Determinant of the system
Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function. Since $\Delta=0$ is the system characteristic equation.

Mason's Rule:

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

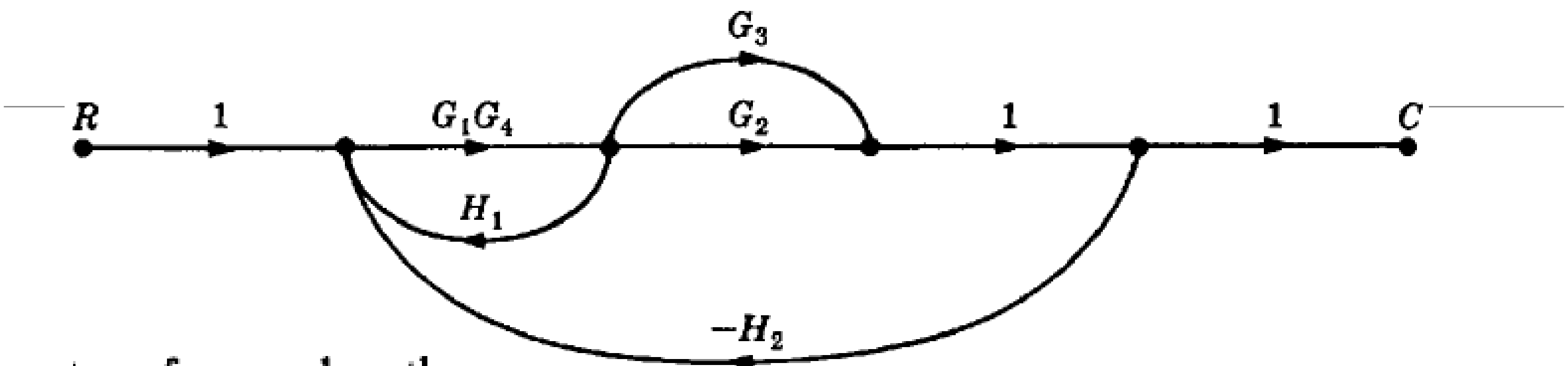
Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

Δ_i = value of Δ for the part of the block diagram that does not touch the i-th forward path (Δ_i = 1 if there are no non-touching loops to the i-th path.)

Systematic approach

1. Calculate forward path gain P_i for each forward path *i.*
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. Calculate Δ from steps 2,3,4 and 5
6. Calculate Δ_i as portion of Δ not touching forward path *i*

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



There are two forward paths:

$P_1 = G_1G_2G_4 \qquad P_2 = G_1G_3G_4$

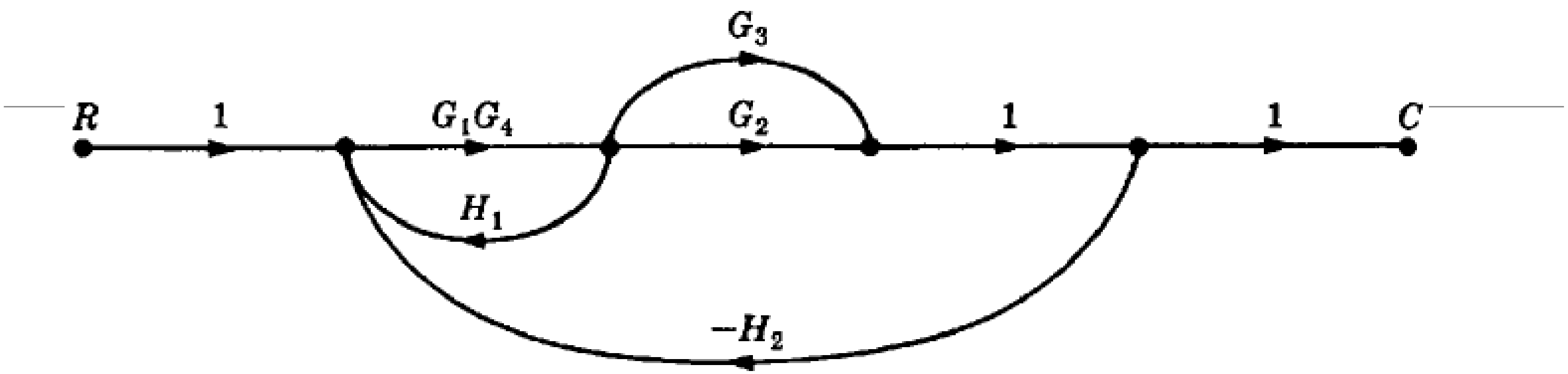
Therefore,

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



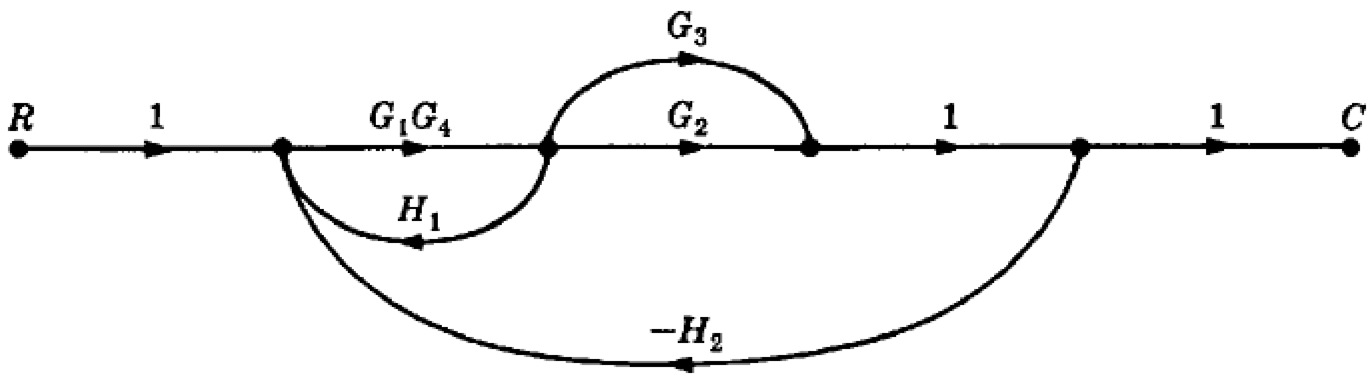
There are no non-touching loops, therefore

Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - \left(L_1 + L_2 + L_3\right)$$

$$\Delta = 1 - \left(G_1 G_4 H_1 - G_1 G_2 G_4 H_2 - G_1 G_3 G_4 H_2\right)$$

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+...

$\Delta_1 = 1$

Eliminate forward path-2

Δ_2 = 1- (sum of all individual loop gains)+...

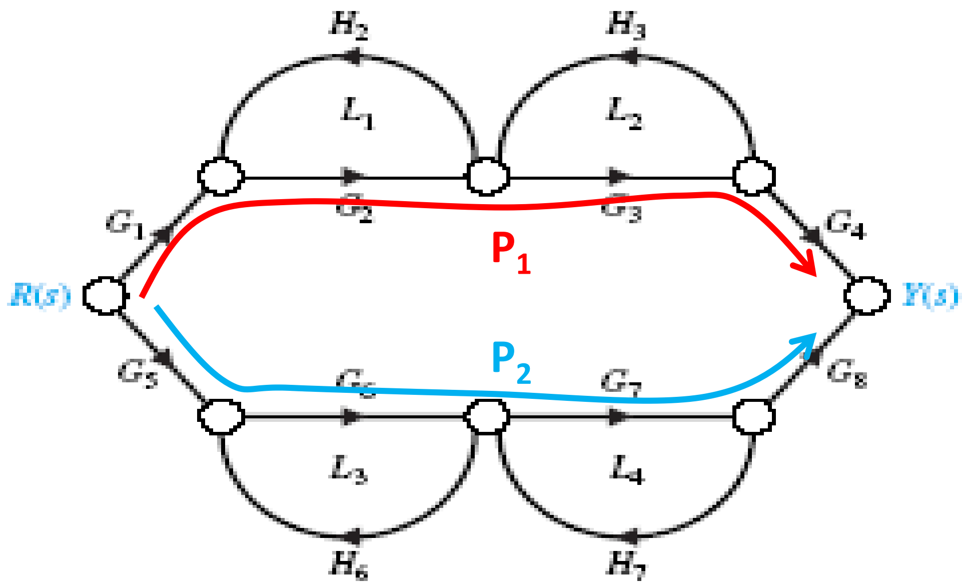
$\Delta_2 = 1$

Example#1: Continue

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Example#2: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



1. Calculate forward path gains for each forward path.

$$P_1 = G_1 G_2 G_3 G_4 \text{ (path 1)} \quad \text{and} \quad P_2 = G_5 G_6 G_7 G_8 \text{ (path 2)}$$

2. Calculate all loop gains.

$$L_1 = G_2 H_2, \quad L_2 = H_3 G_3, \quad L_3 = G_6 H_6, \quad L_4 = G_7 H_7$$

3. Consider two non-touching loops.

$L_1 L_3$ $\quad$ $L_1 L_4$

$L_2 L_4$ $\quad$ $L_2 L_3$

Example#2: continue

4. Consider three non-touching loops.

   None.

5. Calculate Δ from steps 2,3,4.

$$\Delta = 1 - \left(L_1 + L_2 + L_3 + L_4\right) + \left(L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4\right)$$

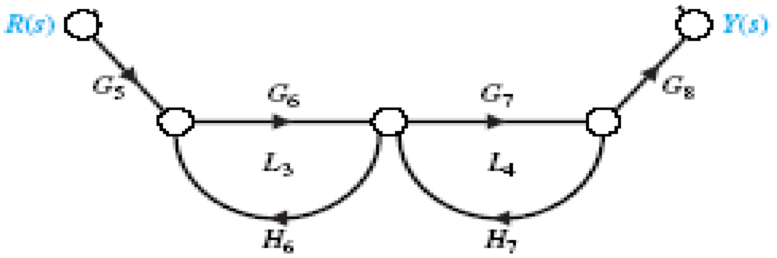
$$\Delta = 1 - \left(G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7\right) + \left(G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7\right)$$

Example#2: continue

Eliminate forward path-1

$$\Delta_1 = 1 - \left(L_3 + L_4\right)$$

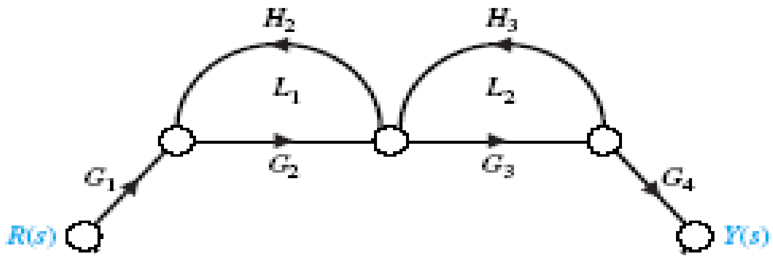
$$\Delta_1 = 1 - \left(G_6 H_6 + G_7 H_7\right)$$



Eliminate forward path-2

$$\Delta_2 = 1 - \left(L_1 + L_2\right)$$

$$\Delta_2 = 1 - \left(G_2 H_2 + G_3 H_3\right)$$



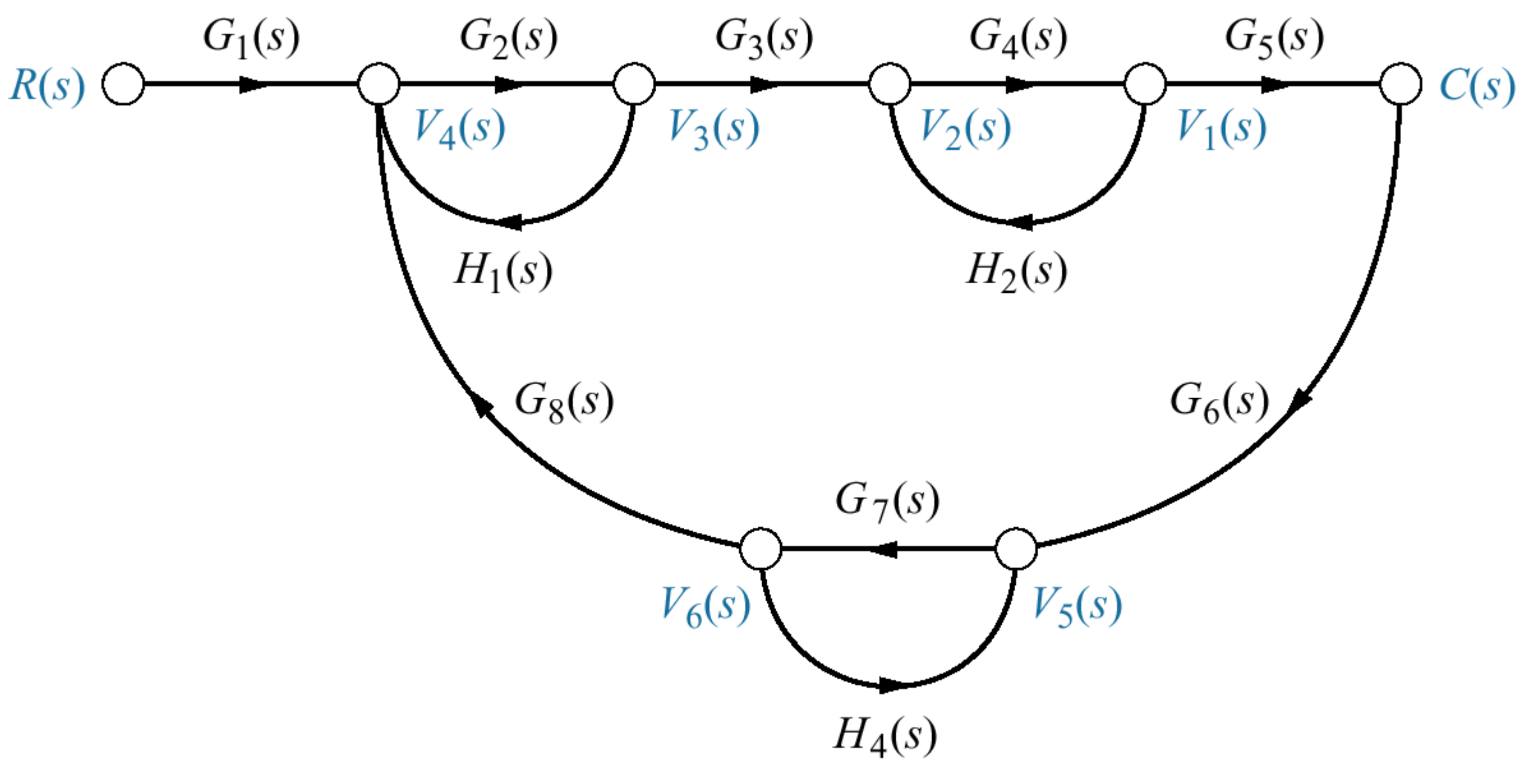
Example#2: continue

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4\left[1-\left(G_6H_6 + G_7H_7\right)\right] + G_5G_6G_7G_8\left[1-\left(G_2H_2 + G_3H_3\right)\right]}{1-\left(G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7\right) + \left(G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7\right)}$$

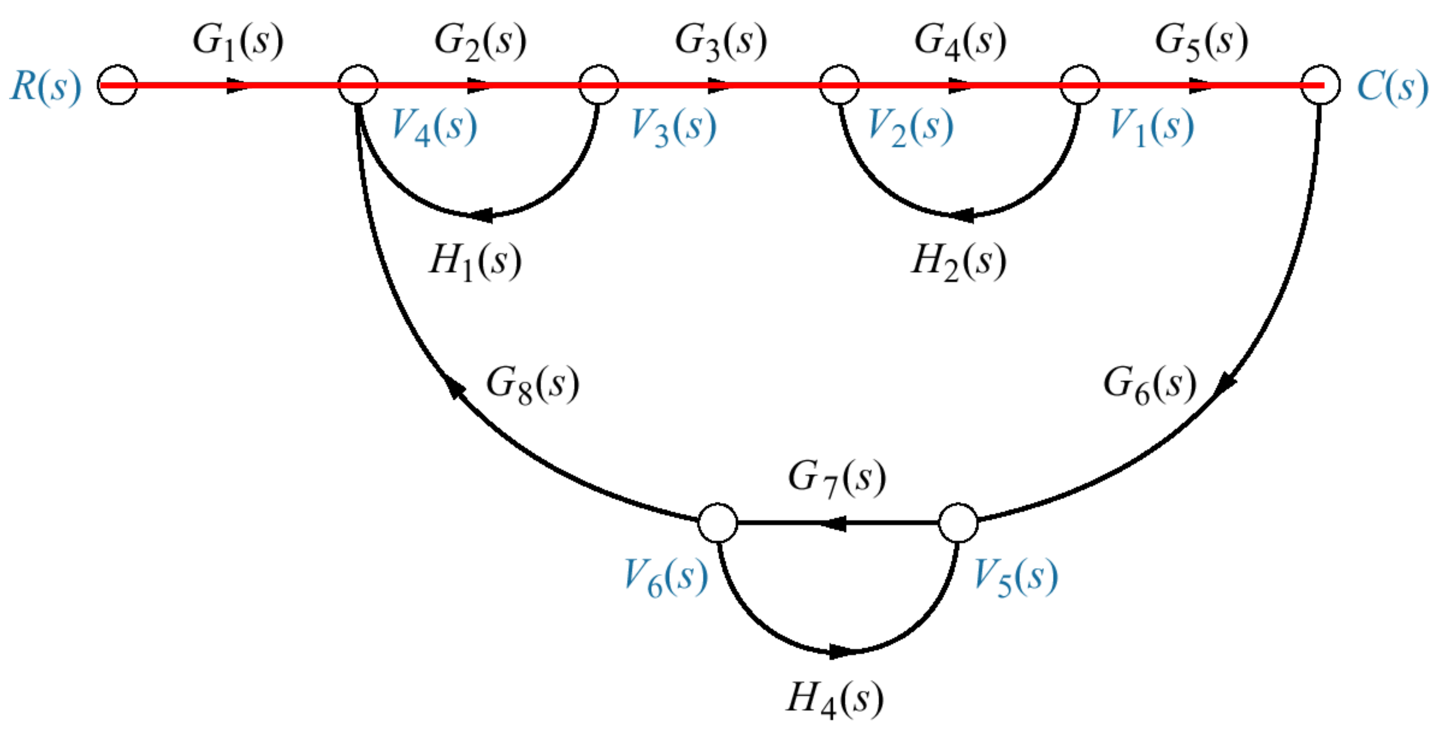
Example#3

Find the transfer function, C(s)/R(s), for the signal-flow graph in figure below.



Example#3

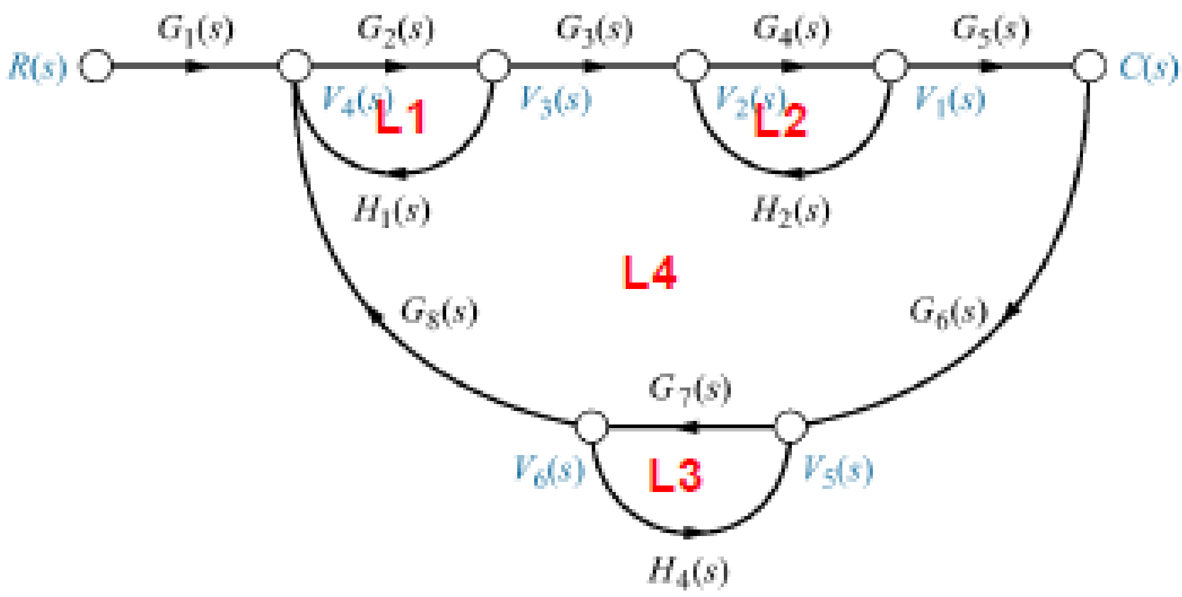
There is only one forward Path.



$$P_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$$

Example#3

There are four feedback loops.

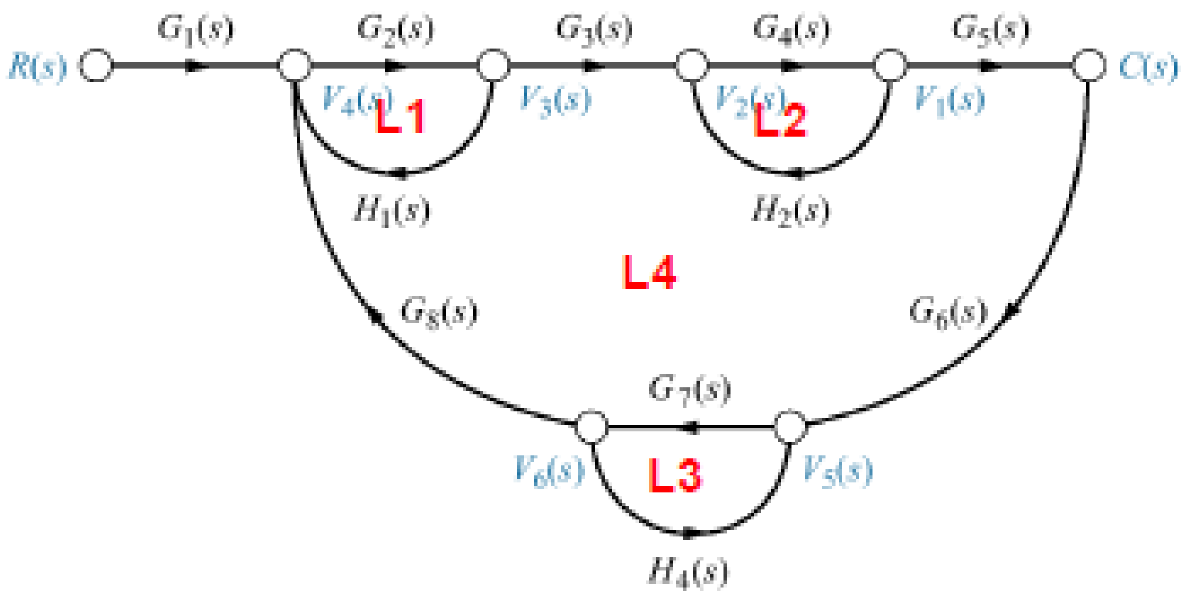


L1. $G_2(s)H_1(s)$ L3. $G_7(s)H_4(s)$

L2. $G_4(s)H_2(s)$ L4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$

Example#3

Non-touching loops taken two at a time.

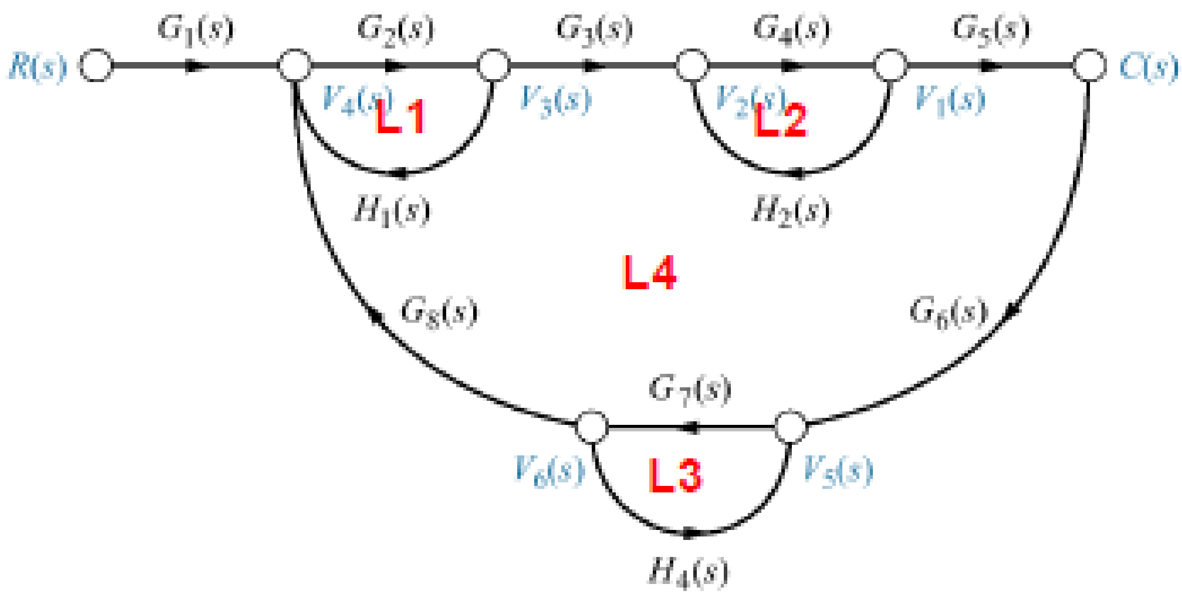


L1 and L2: $G_2(s)H_1(s)G_4(s)H_2(s)$ L2 and L3: $G_4(s)H_2(s)G_7(s)H_4(s)$

L1 and L3: $G_2(s)H_1(s)G_7(s)H_4(s)$

Example#3

Non-touching loops taken three at a time.



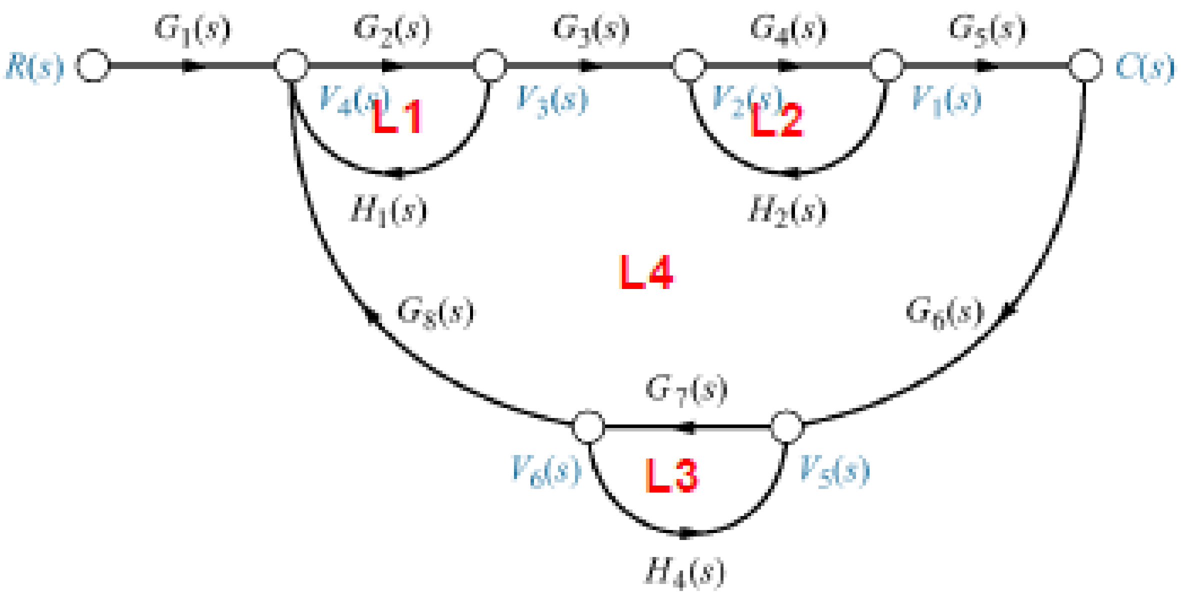
L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Example#3

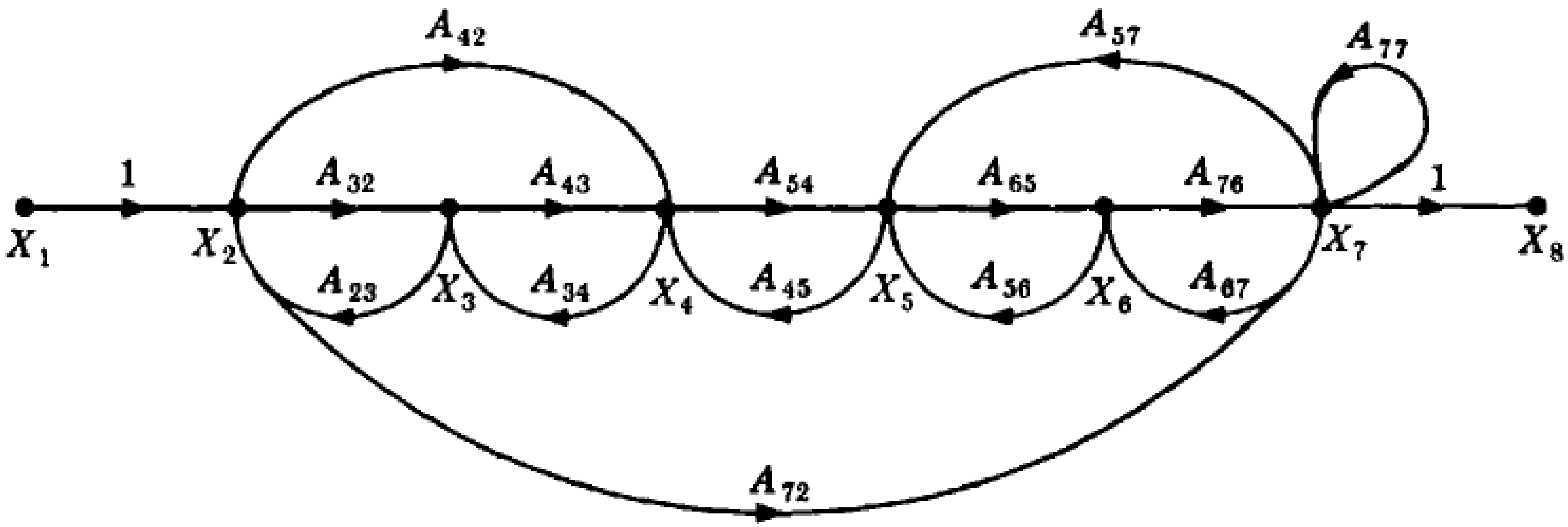
$$\Delta = 1 - [G_2(s)H_1(s) + G_4(s)H_2(s)$$
$$+ G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)]$$
$$+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s)$$
$$+ G_4(s)H_2(s)G_7(s)H_4(s)]$$
$$- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

Eliminate forward path-1

$$\Delta_1 = 1 - G_7(s)H_4(s)$$



Example#4: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph

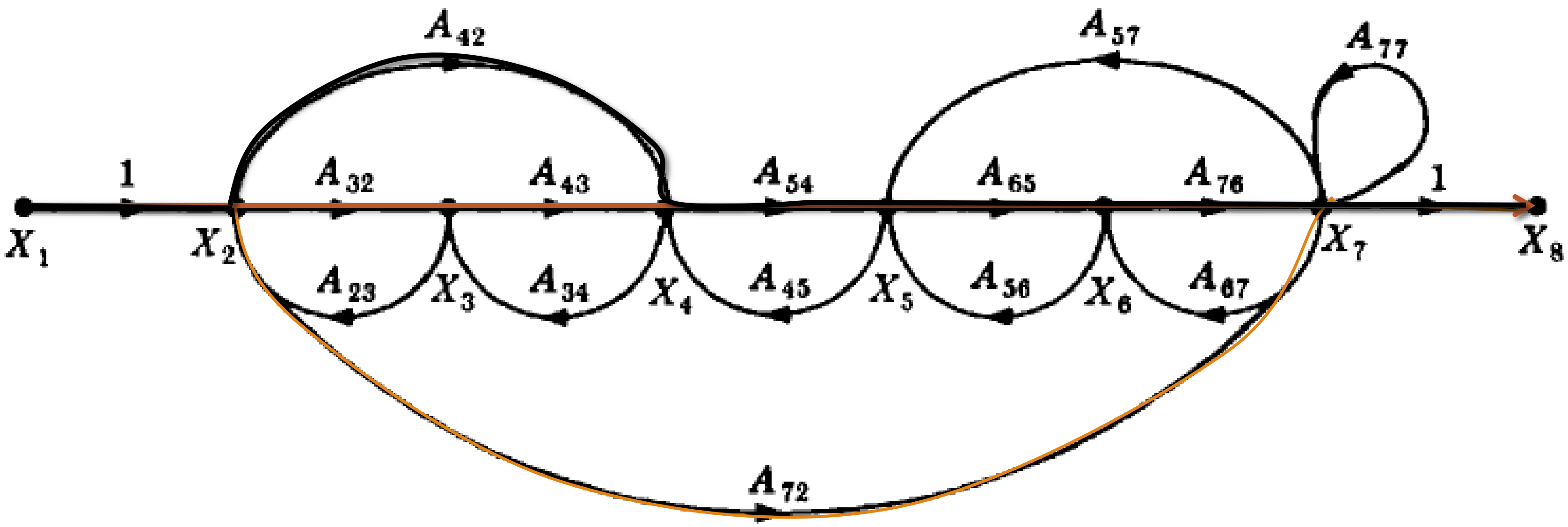


There are three forward paths, therefore n=3.

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{3} P_i \Delta_i}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

Example#4: Forward Paths

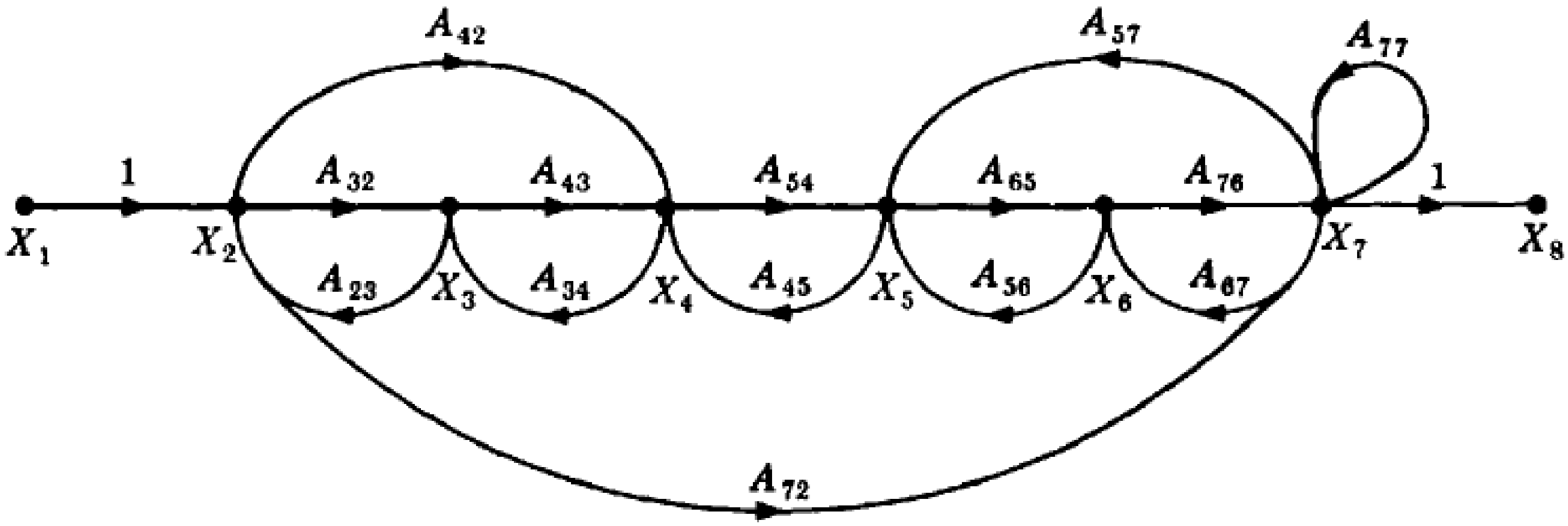
$$P_3 = A_{42} A_{54} A_{65} A_{76}$$



$$P_1 = A_{32} A_{43} A_{54} A_{65} A_{76}$$

$$P_2 = A_{72}$$

Example#4: Loop Gains of the Feedback Loops

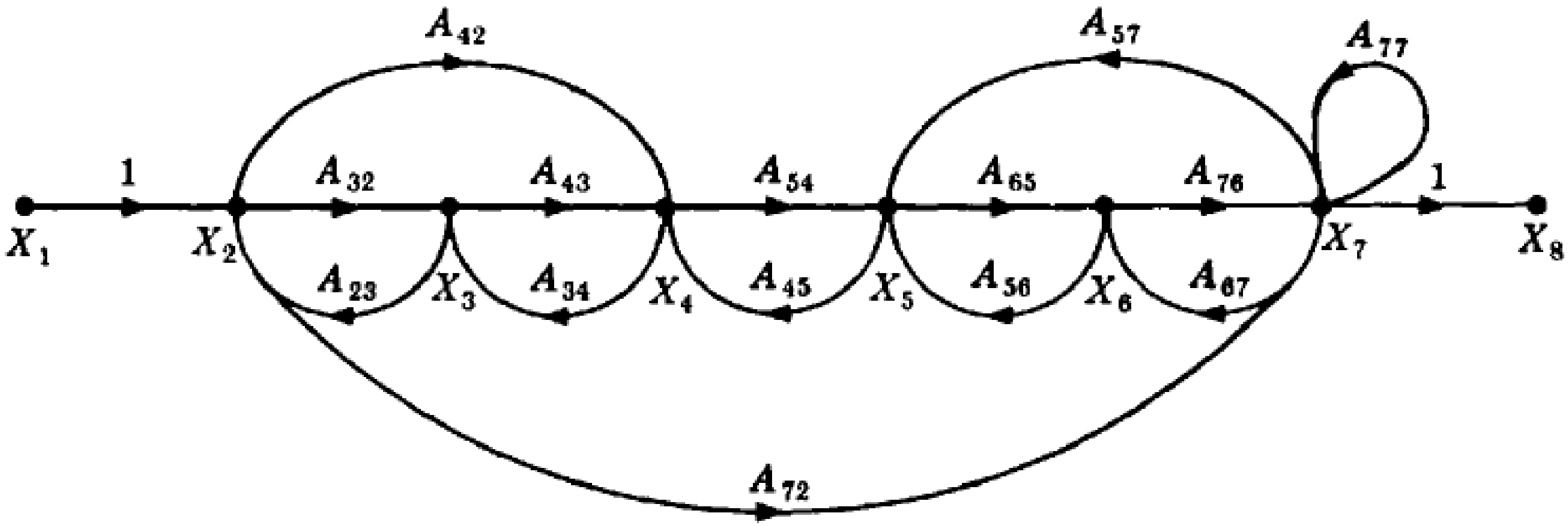


$$L_1 = A_{32} A_{23}$$
$$L_2 = A_{43} A_{34}$$
$$L_3 = A_{54} A_{45}$$
$$L_4 = A_{65} A_{56}$$

$$L_5 = A_{76} A_{67}$$
$$L_6 = A_{77}$$
$$L_7 = A_{42} A_{34} A_{23}$$
$$L_8 = A_{65} A_{76} A_{67}$$

$$L_9 = A_{72} A_{57} A_{45} A_{34} A_{23}$$
$$L_{10} = A_{72} A_{67} A_{56} A_{45} A_{34} A_{23}$$

Example#4: two non-touching loops



L_1L_3 $\quad$ L_2L_4 $\quad$ L_3L_5 $\quad$ L_4L_6 $\quad$ L_5L_7 $\quad$ L_7L_8

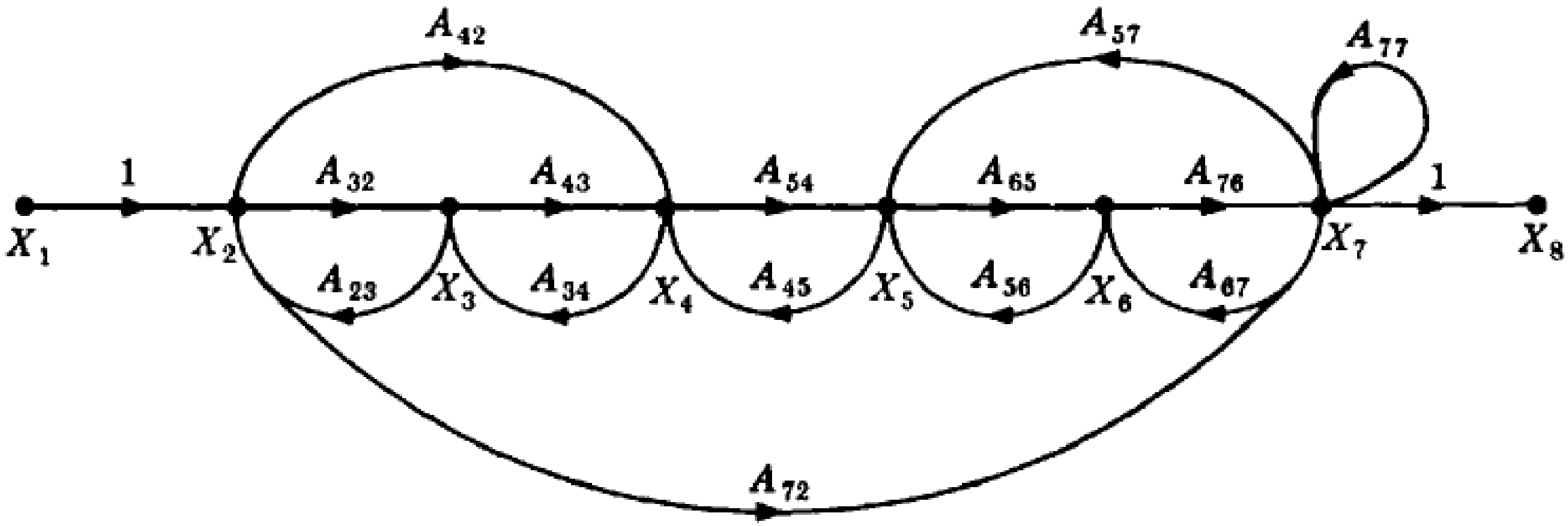
L_1L_4 $\quad$ L_2L_5 $\quad$ L_3L_6 $\quad$ L_4L_7

L_1L_5 $\quad$ L_2L_6

L_1L_6 $\quad$ L_2L_8

L_1L_8

Example#4: Three non-touching loops



L_1L_3 L_2L_4 L_3L_5 L_4L_6 L_5L_7 L_7L_8

L_1L_4 L_2L_5 L_3L_6 L_4L_7

L_1L_5 L_2L_6

L_1L_6 L_2L_8

L_1L_8

Rules for Drawing of SFG from Block Diagram

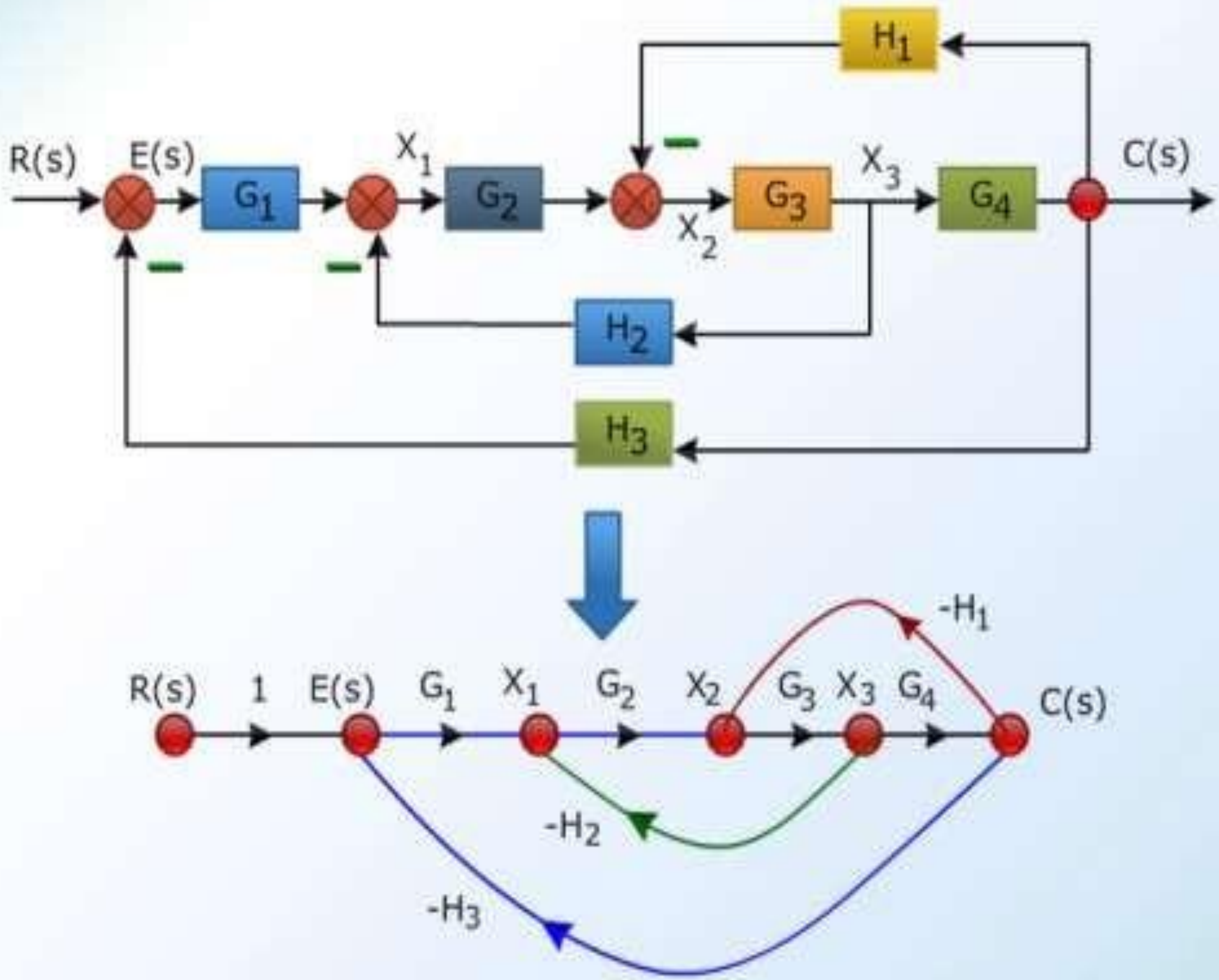
- All variables, summing points and take off points are represented by nodes
- If a summing point is placed before a take off point then the summing point and take off point shall be represented by a single node
- If a summing point is placed after a take off point then the summing point and take off point shall be represented by separate nodes connected by a branch having transmittance unity

Converting Block Diagram to Signal-Flow Graph

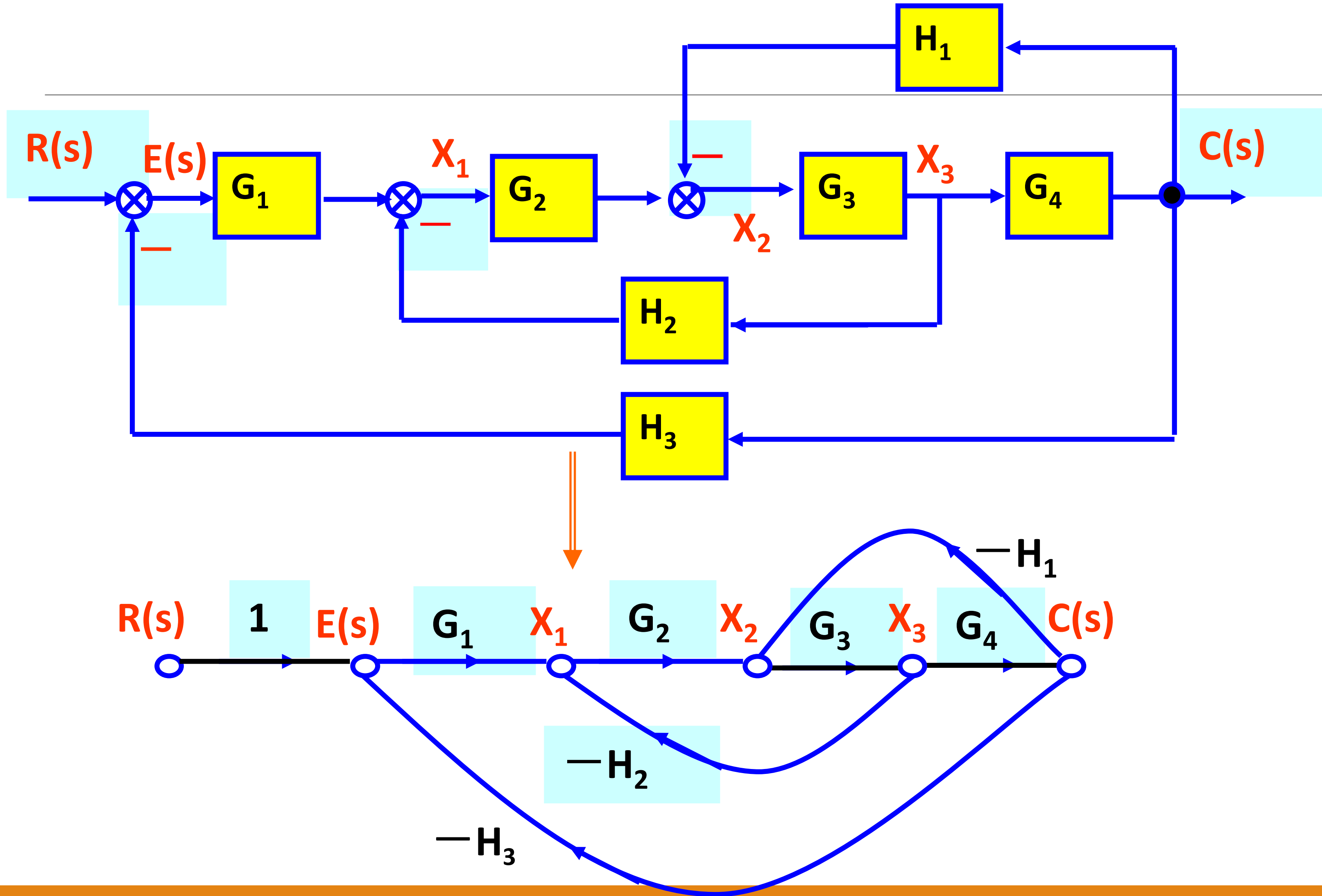
Procedure

- Assume nodes at input, output, at every summing point, at every branch point and in between cascaded blocks
- Draw the nodes separately as small circles and number the circles in the order 1, 2, 3…. or name them in terms of variables names
- From the block diagram find the gain between each node in the main forward path
- Connect all the corresponding circles by straight line and mark gain between the nodes
- Draw the feed forward paths between various nodes and mark the gain of feed forward path along with sign
- Draw the feedback paths between various nodes and mark the gain of feedback paths along with sign

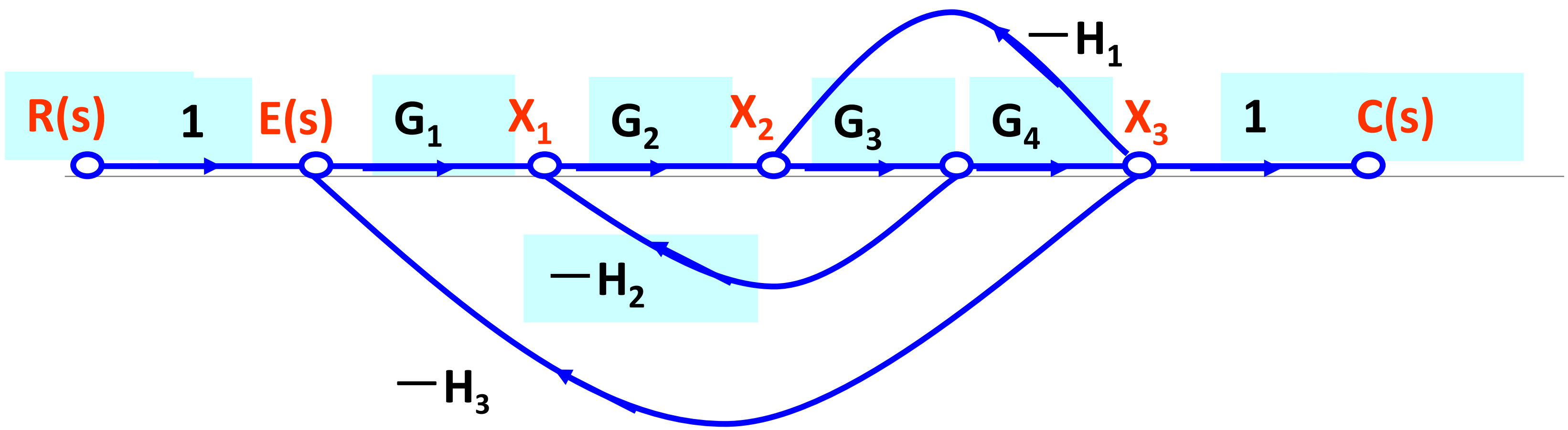
From Block Diagram to Signal-flow Graph Models



From Block Diagram to Signal-Flow Graph Models Example#5



From Block Diagram to Signal-Flow Graph Models
Example#5

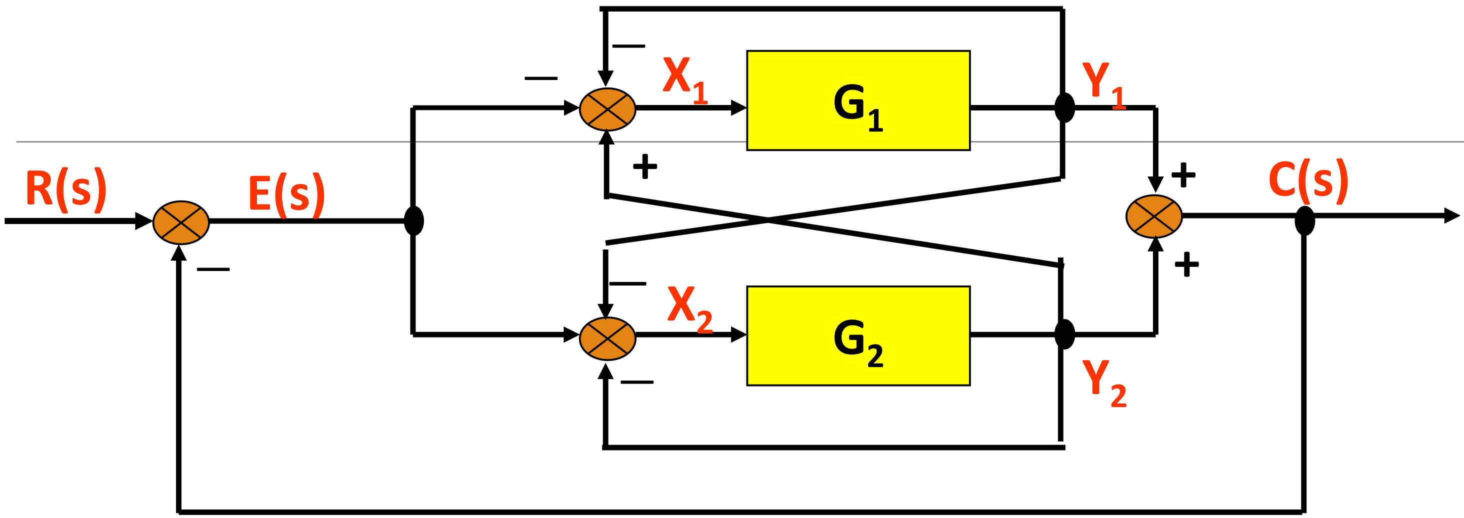


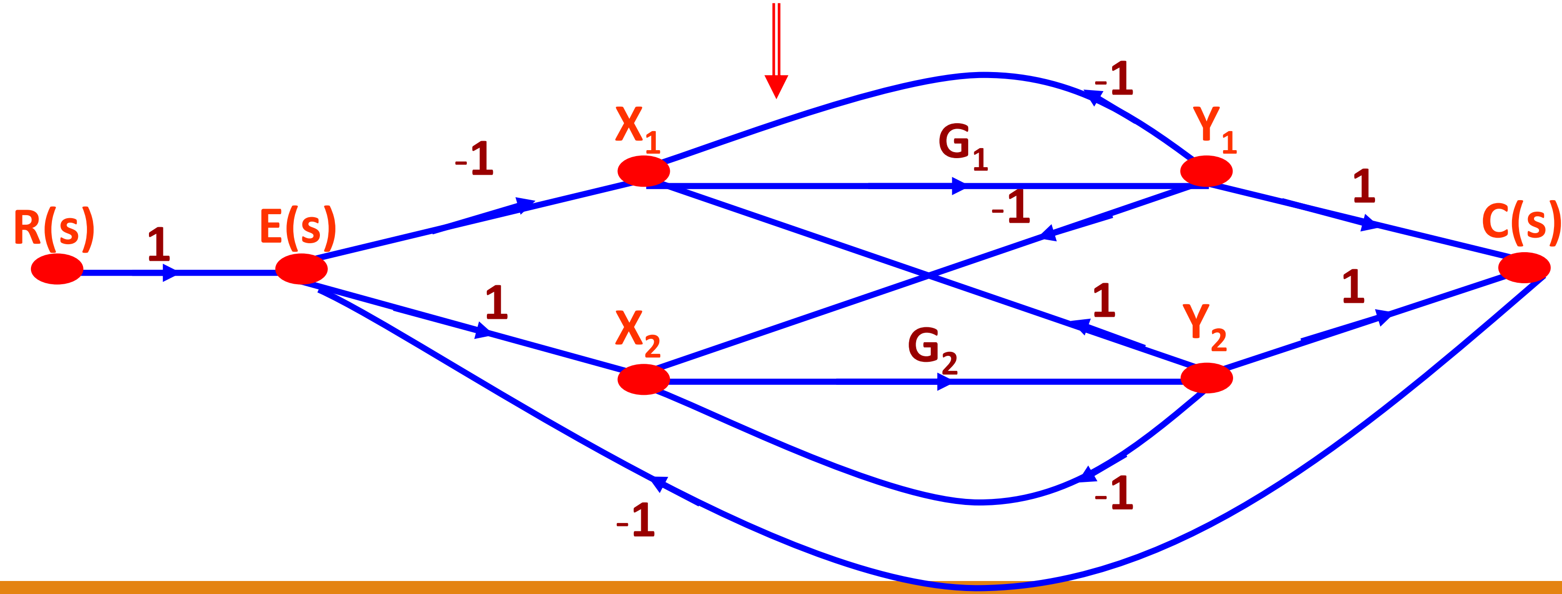
$$\Delta = 1 + (G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1)$$

$$P_1 = G_1G_2G_3G_4; \qquad \Delta_1 = 1$$

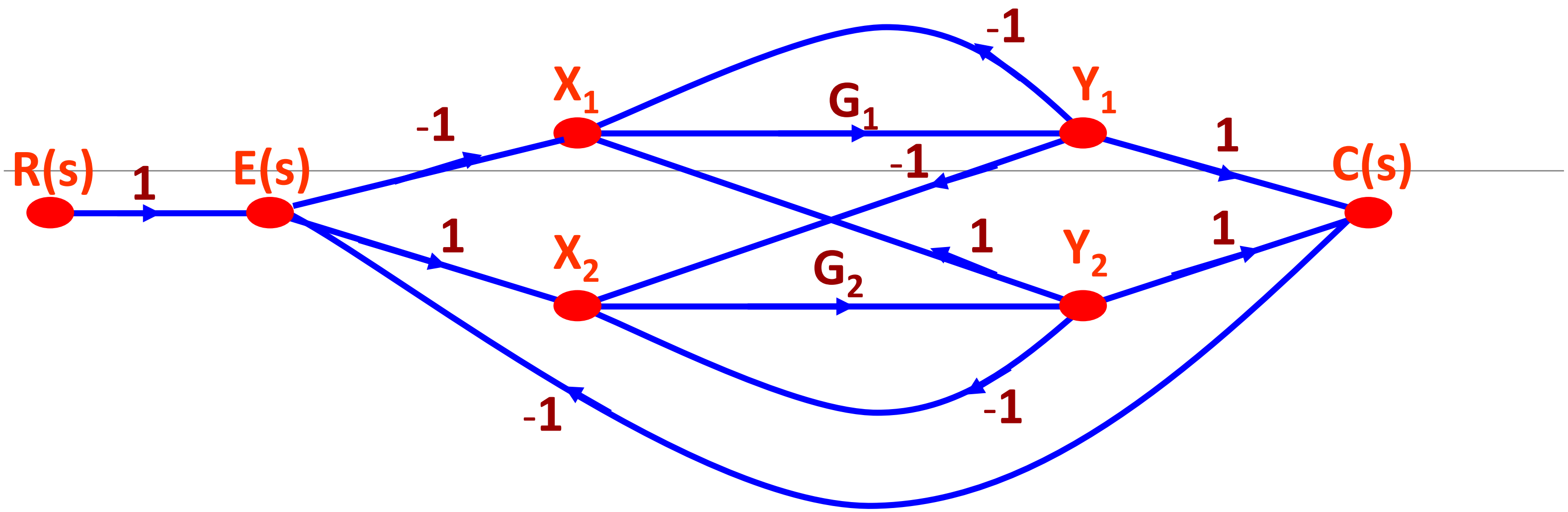
$$G = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1}$$

Example#6





Example#6



7 loops:

$$[G_1 \cdot (-1)]; \quad [G_2 \cdot (-1)]; \quad [G_1 \cdot (-1) \cdot G_2 \cdot 1]; \quad [(-1) \cdot G_1 \cdot 1 \cdot (-1)];$$

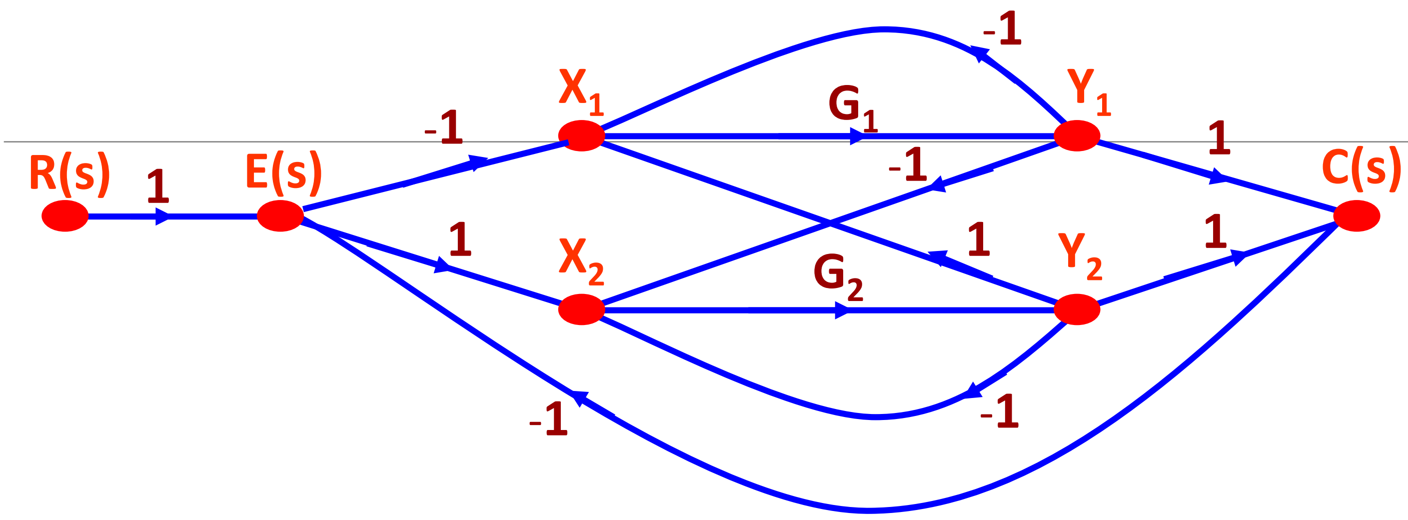
$$[(-1) \cdot G_1 \cdot (-1) \cdot G_2 \cdot 1 \cdot (-1)]; \quad [1 \cdot G_2 \cdot 1 \cdot (-1)]; \quad [1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1 \cdot (-1)].$$

3 '2 non-touching loops' :

$$[G_1 \cdot (-1)] \cdot [G_2 \cdot (-1)]; \quad [(-1) \cdot G_1 \cdot 1 \cdot (-1)] \cdot [G_2 \cdot (-1)];$$

$$[1 \cdot G_2 \cdot 1 \cdot (-1)] \cdot [G_1 \cdot (-1)].$$

Example#6



Then: $$\Delta = 1 + 2G_2 + 4G_1G_2$$

4 forward paths:

$$p_1 = (-1)\cdot G_1 \cdot 1 \qquad \Delta_1 = 1 + G_2$$

$$p_2 = (-1)\cdot G_1 \cdot (-1)\cdot G_2 \cdot 1 \qquad \Delta_2 = 1$$

$$p_3 = 1 \cdot G_2 \cdot 1 \qquad \Delta_3 = 1 + G_1$$

$$p_4 = 1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1 \qquad \Delta_4 = 1$$

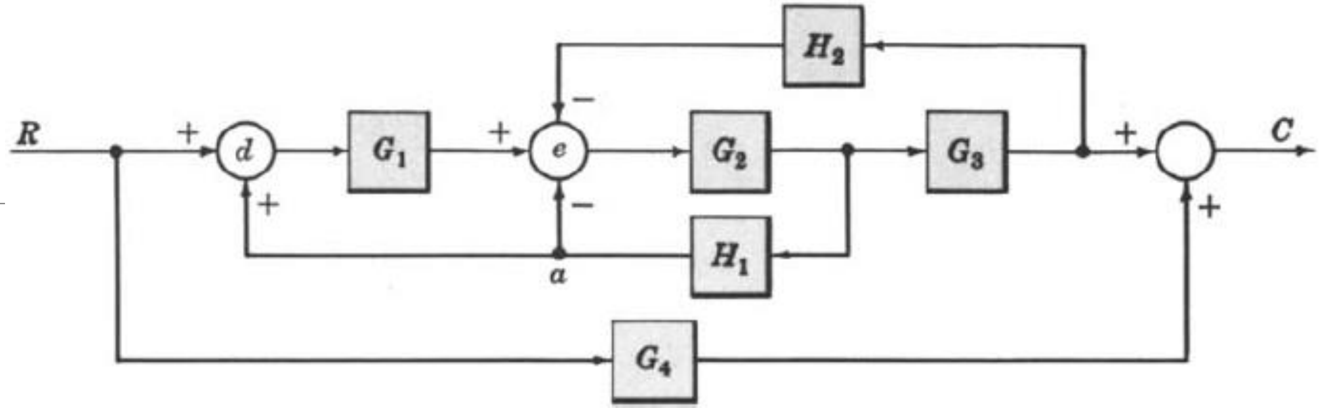
Example#6

We have

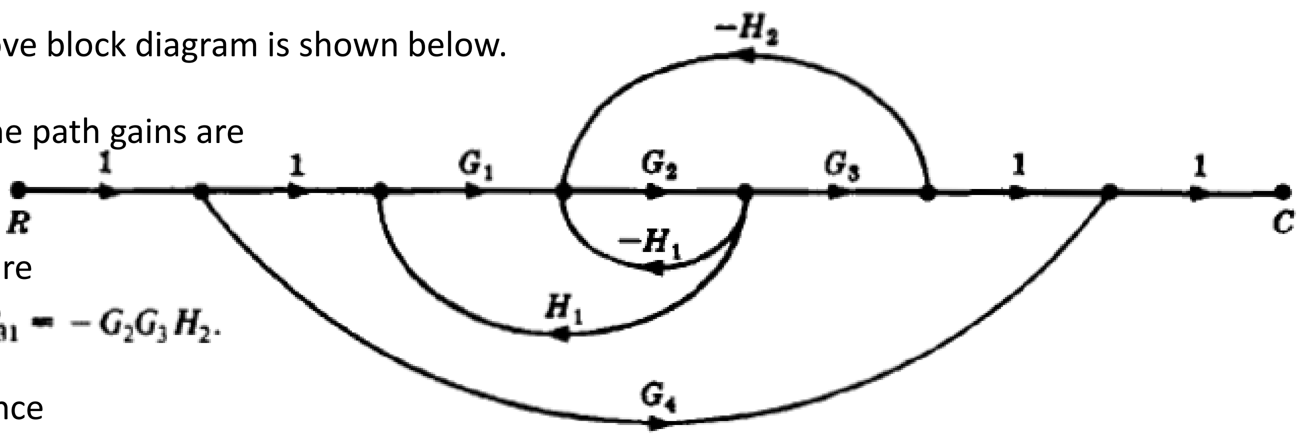
$$\frac{C(s)}{R(s)} = \frac{\sum p_k \Delta_k}{\Delta}$$

$$= \frac{G_2 - G_1 + 2G_1G_2}{1 + 2G_2 + 4G_1G_2}$$

Example-7: Determine the transfer function C/R for the block diagram below by signal flow graph techniques.



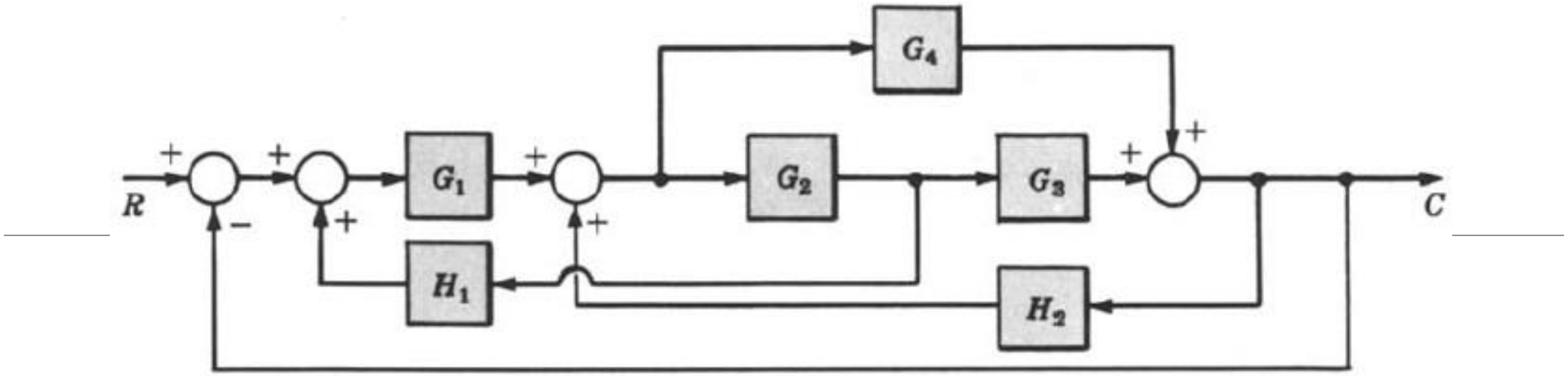
- The signal flow graph of the above block diagram is shown below.

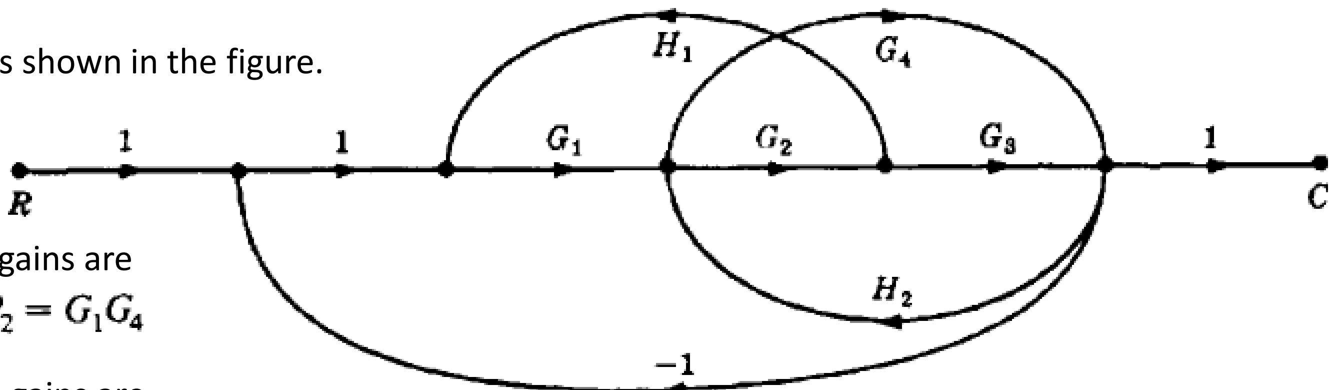


- There are two forward paths. The path gains are
  $P_1 = G_1G_2G_3$ and $P_2 = G_4$
- The three feedback loop gains are
  $P_{11} = -G_2H_1,\ P_{21} = G_1G_2H_1,\ P_{31} = -G_2G_3H_2.$
- No loops are non-touching, hence
  $\Delta = 1 - (P_{11} + P_{21} + P_{31})$
- Because the loops touch the nodes of P1, hence $\Delta_1 = 1$
- Since no loops touch the nodes of P2, therefore $\Delta_2 = \Delta.$
- Hence the control ratio T = C/R is

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_4 + G_2G_4H_1 - G_1G_2G_4H_1 + G_2G_3G_4H_2}{1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2}$$

Example-6: Find the control ratio C/R for the system given below.





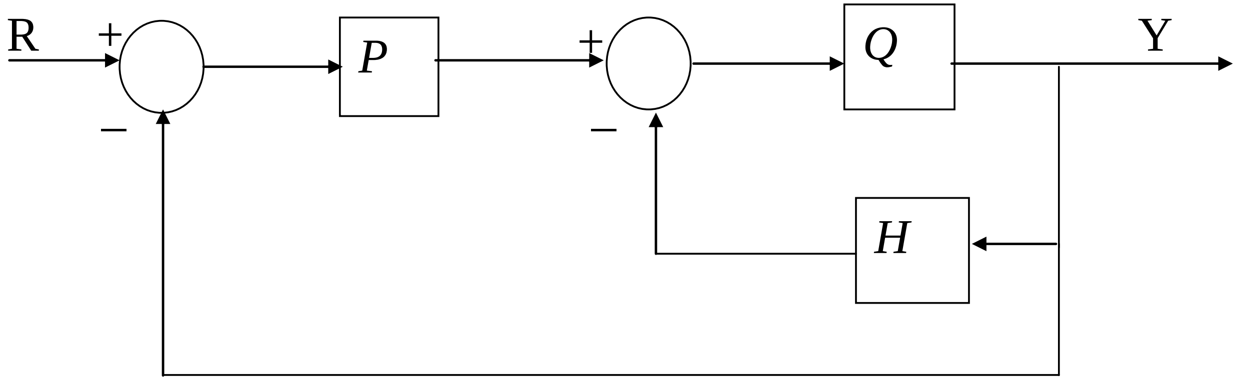
- The signal flow graph is shown in the figure.
- The two forward path gains are $P_1 = G_1G_2G_3$ and $P_2 = G_1G_4$
- The five feedback loop gains are $P_{11} = G_1G_2H_1,\ P_{21} = G_2G_3H_2,\ P_{31} = -G_1G_2G_3,$ $P_{41} = G_4H_2,$ and $P_{51} = -G_1G_4.$
- All feedback loops touches the two forward paths, hence $\Delta_1 = \Delta_2 = 1$
- There are no non-touching loops, hence

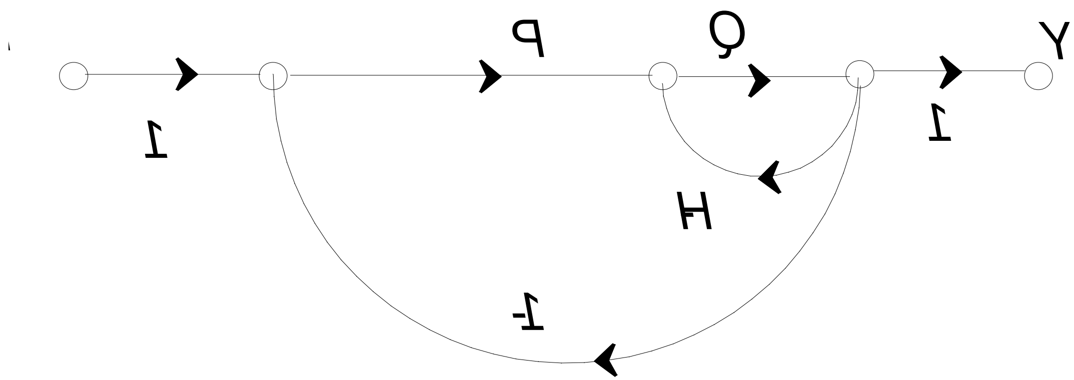
$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51})$$
$$= 1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4$$

- Hence the control ratio T = $\dfrac{C}{R} = \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \dfrac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4}$

Example:

Determine the transfer function of $Y(s)/R(s)$ the following block diagram.





$L_1 = -Q.H$ and $L_2 = -P.Q.1$.

$\sum L_i = L_1 + L_2 = -Q.H - P.Q$.

$\sum L_i L_j = 0$ etc.

$$\Delta = 1 - \sum L_i + \sum L_i L_j + \sum L_i L_j L_k - ..... = 1 + Q.H + P.Q$$

$P_1 = 1.P.Q.1$,

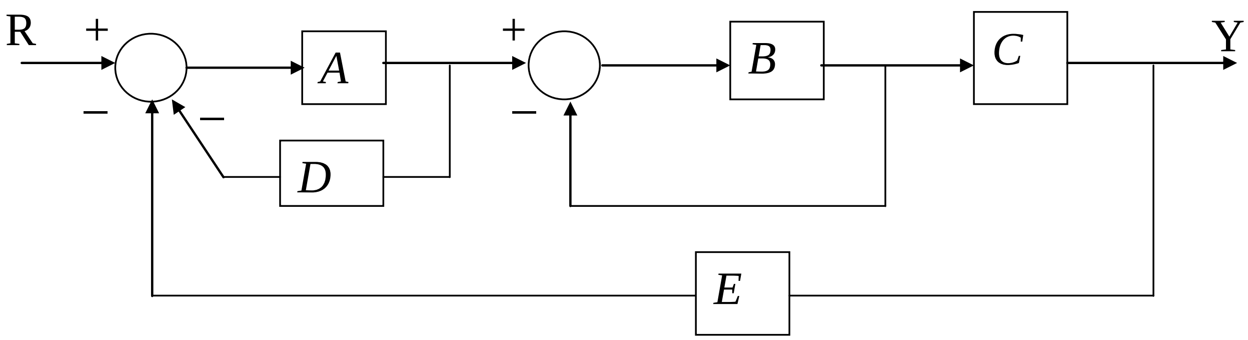
$\Delta_1 = 1$.

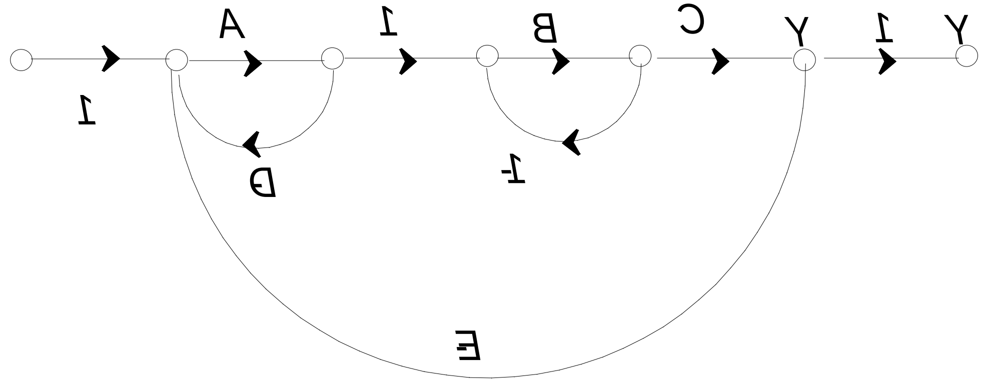
Transfer function

$$\frac{Y}{R} = \frac{P.Q}{1 + Q.H + P.Q}$$

Example:

Determine $Y(s)/R(s)$.





$L_1 = -A.D$, $L_2 = -1.B$ and $L_3 = -A.1.B.C.E$

$\sum L_i = L_1 + L_2 + L_3 = -A.D - B - A.B.C.E$

$\sum L_i L_j = L_1.L_2 = -A.D. - B = A.D.B$

$\sum L_i L_j L_k = 0$

$$\Delta = 1 - \sum L_i + \sum L_i L_j + \sum L_i L_j L_k - .....$$

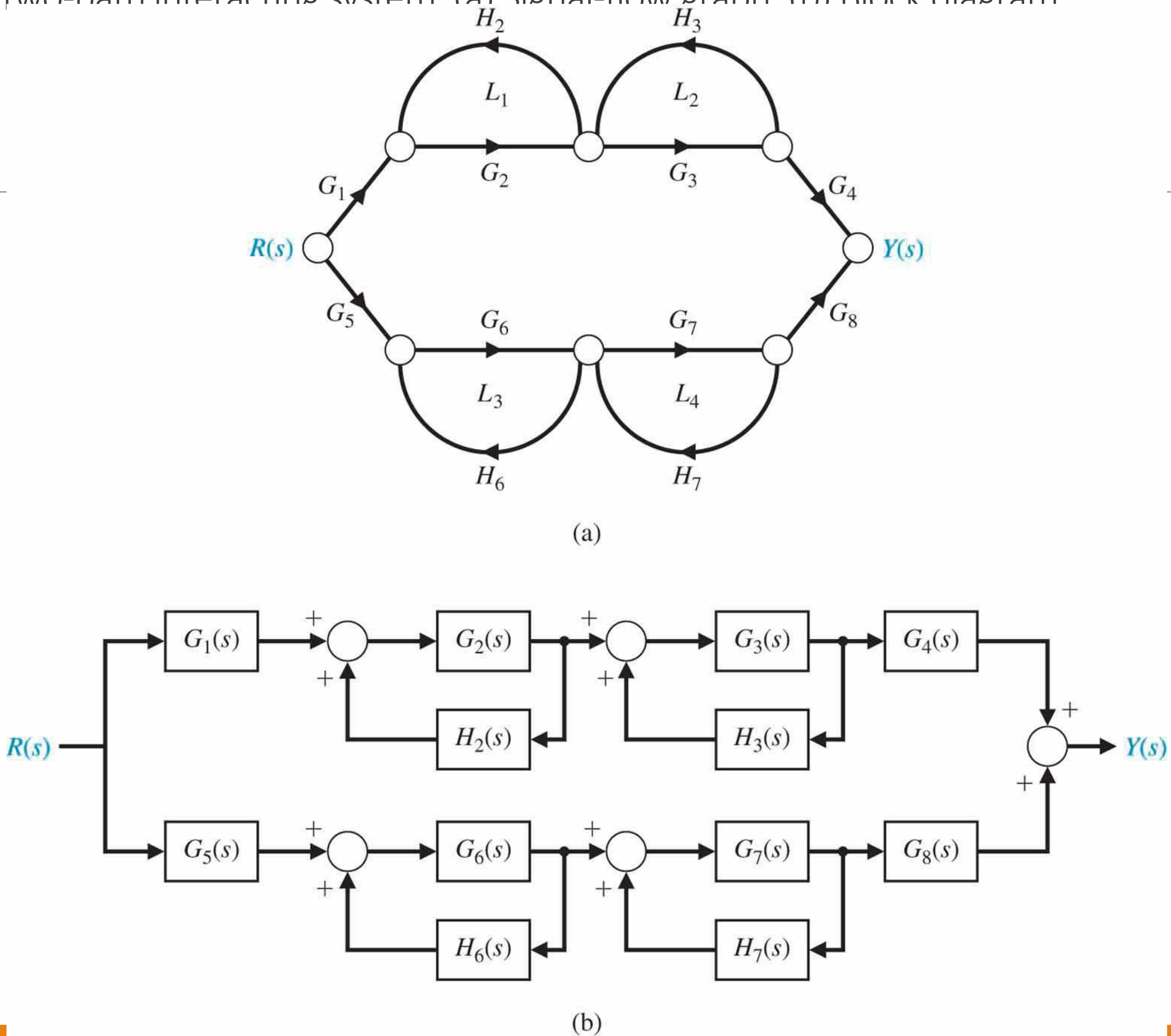
$$= 1 - [-AD - B - ABCE] + ADB$$

$$= 1 + AD + B + ABCE + ADB$$

$$\Delta_1 = 1$$

$$\frac{Y(S)}{R(s)} = \frac{ABC}{1 + AD + B + ABCE + ADB}$$

Figure 2.31 Two-path interacting system. (a) Signal-flow graph. (b) Block diagram.



$P_1 = G_1G_2G_3G_4$ (path 1)

$P_2 = G_5G_6G_7G_8$ (path 2)

There are four self loops:

$$L_1 = G_2H_2\,,\ L_2 = H_3G_3,\ \ L_3 = G_6H_6,\ and\ \ L_4 = G_7H_7$$

Loops L_1 and L_2 do not touch L_3 and L_4,

$$\Delta = 1-(L_1+L_2+L_3+L_4)+(L_1L_3+L_1L_4+L_2L_3+L_2L_4)$$

Removing loops that touch path 1,

$$L_1 = L_2 = 0$$

$$\Delta_1 = 1-(L_3+L_4)$$

Removing loops that touch path 2,

$$L_3 = L_4 = 0$$

$$\Delta_2 = 1-(L_1+L_2)$$

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4(1-L_3-L_4)+G_5G_6G_7G_8(1-L_1-L_2)}{1-L_1-L_2-L_3-L_4+L_1L_3+L_1L_4+L_2L_3+L_2L_4}$$

Using the block diagram,

$$Y_1(s) = G_1(s)\left[\frac{G_2(s)}{1-G_2(s)H_2(s)}\right]\left[\frac{G_3(s)}{1-G_3(s)H_3(s)}\right]G_4(s)R(s)$$

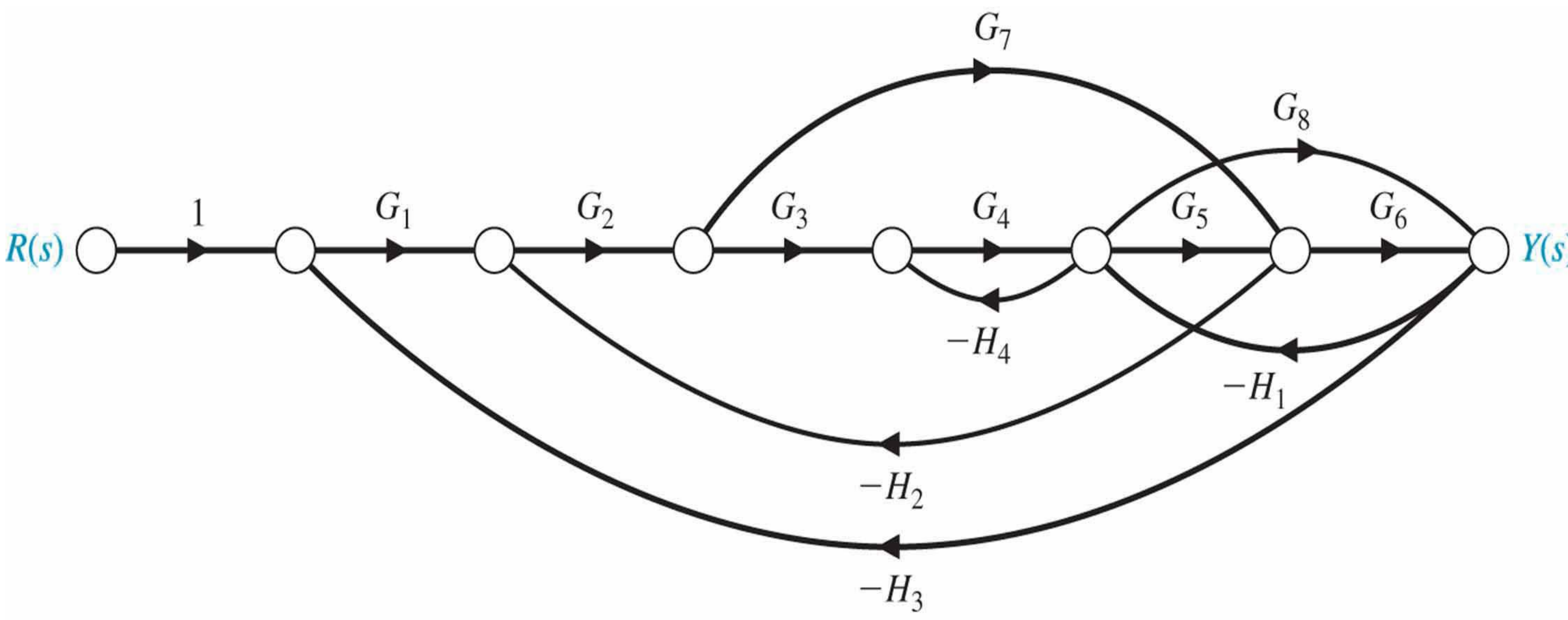
$$= \left[\frac{G_1(s)G_2(s)G_3(s)G_4(s)}{(1-G_2(s)H_2(s))(1-G_3(s)H_3(s))}\right]R(s)$$

$$Y_2(s) = G_5(s)\left[\frac{G_6(s)}{1-G_6(s)H_6(s)}\right]\left[\frac{G_7(s)}{1-G_7(s)H_7(s)}\right]G_8(s)R(s)$$

$$= \left[\frac{G_5(s)G_6(s)G_7(s)G_8(s)}{(1-G_6(s)H_6(s))(1-G_7(s)H_7(s))}\right]R(s)$$

$$Y(s) = Y_1(s) + Y_2(s) = \left[\frac{G_1(s)G_2(s)G_3(s)G_4(s)}{\left(1 - G_2(s)H_2(s)\right)\left(1 - G_3(s)H_3(s)\right)} + \frac{G_5(s)G_6(s)G_7(s)G_8(s)}{\left(1 - G_6(s)H_6(s)\right)\left(1 - G_7(s)H_7(s)\right)} \right] R(s)$$

Figure Multiple-loop system.



Find the paths,

$$P_1 = G_1G_2G_3G_4G_5G_6,$$

$$P_2 = G_1G_2G_7G_6,$$

$$P_3 = G_1G_2G_3G_4G_8$$

Find the feedback loops:

$$L_1 = -G_2G_3G_4G_5H_2,\ L_2 = -G_5G_6H_1,\ L_3 = -G_8H_1,\ L_4 = -G_7H_2G_2,$$

$$L_5 = -G_4H_4,\ L_6 = -G_1G_2G_3G_4G_5G_6H_3,\ L_7 = -G_1G_2G_7G_6H_3,$$

$$L_8 = -G_1G_2G_3G_4G_8H_3$$

Loop L_5 does not touch L_4 or L_7, and loop L_3 does not touch L_4

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4)$$

$$\Delta_1 = \Delta_3 = 1 \quad \text{and} \quad \Delta_2 = 1 - L_5 = 1 + G_4H_4$$

$$\frac{Y(s)}{R(s)} = \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}$$