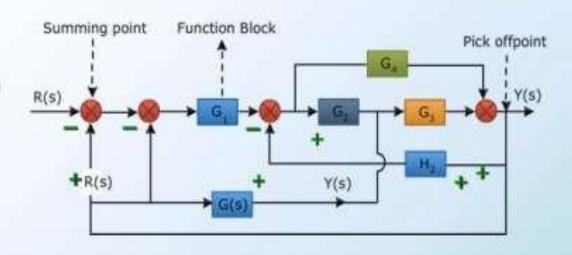
Block Diagram

- Block diagram (BD) is a diagram of a system in which the principal parts are represented by blocks connected by lines, that show the relation ships of blocks
- A block diagram comprises of following components
 - · Summing point
 - Pick off point
 - Function block
- A block diagram has:
 - Rules for BD Reduction
 - Advantages
 - Limitations



Block Diagram Representation

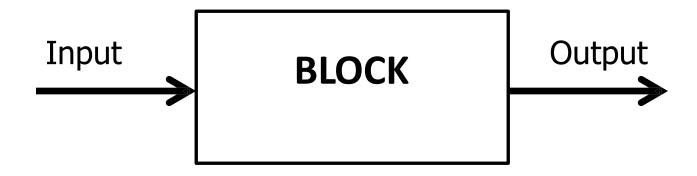
Need of Block Diagram Algebra

> If the system is simple & has limited parameters then it is easy to analyze such systems using the methods discussed earlier i.e. transfer function, if the system is complicated and also have number of parameters then it is very difficult to analyze it.

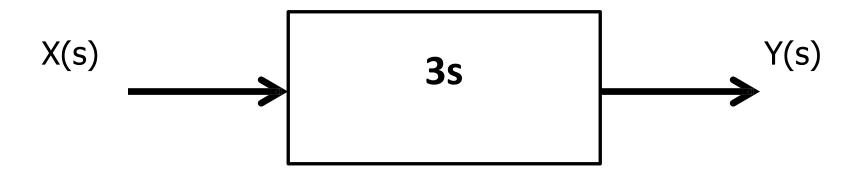
Need of Block Diagram Algebra

- ➤ To overcome this problem block diagram representation method is used.
- It is a simple way to represent any practically complicated system. In this each component of the system is represented by a separate block known as functional block.
- > These blocks are interconnected in a proper sequence.

➤ <u>Block Diagram:</u> It is shorthand, pictorial representation of the cause and effect relationship between input and output of a physical system.

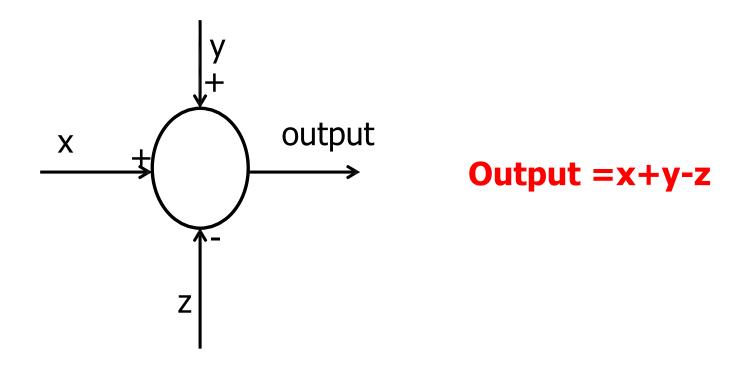


➤ <u>Output:</u> The value of the input is multiplied to the value of block gain to get the output.

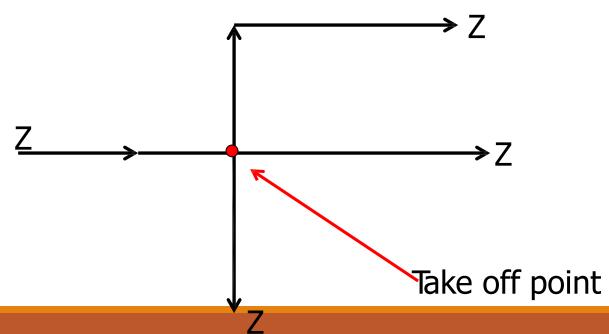


Output Y(s) = 3s. X(s)

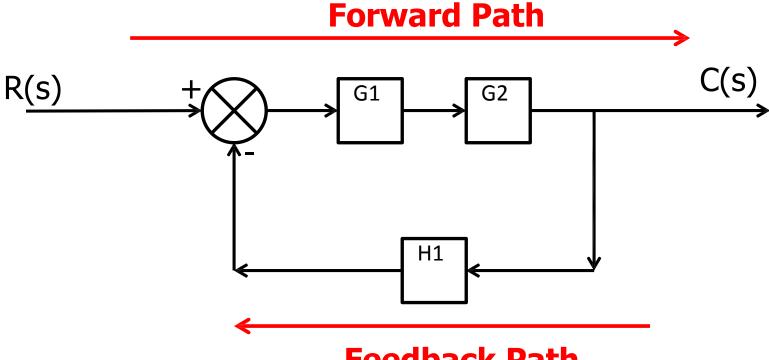
Summing Point: Two or more signals can be added/ substracted at summing point.



Take off Point: The output signal can be applied to two or more points from a take off point.



Forward Path: The direction of flow of signal is from input to output

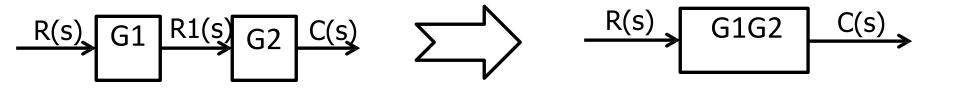


Feedback Path

Feedback Path: The direction of flow of signal is from output to input

Rule 1: For blocks in cascade

Gain of blocks connected in cascade gets multiplied with each other.



$$R1(s)=G1R(s)$$

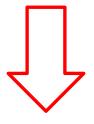
$$C(s) = G2R1(s)$$

= $G1G2R(s)$

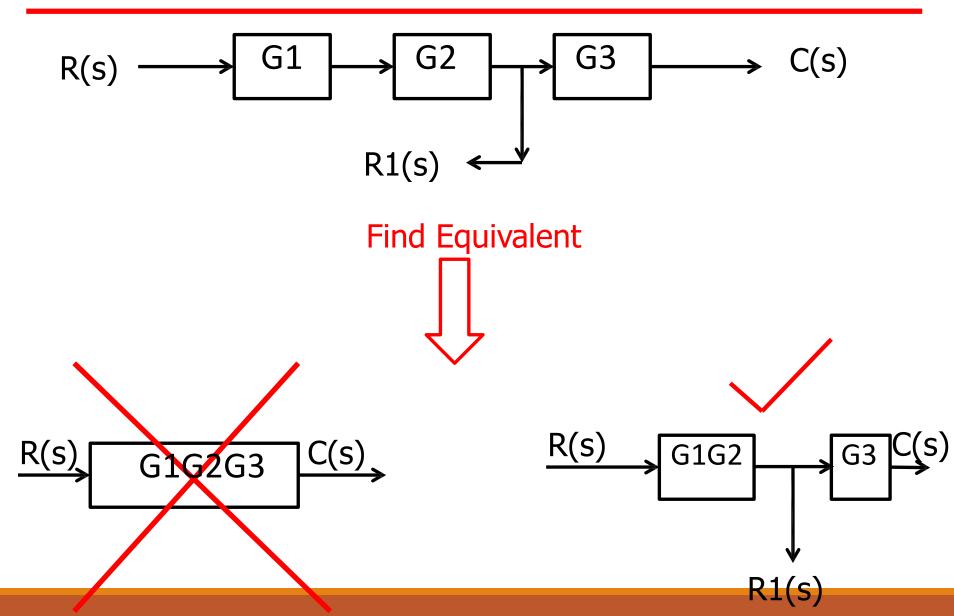
$$C(s) = G1G2R(s)$$



Find Equivalent

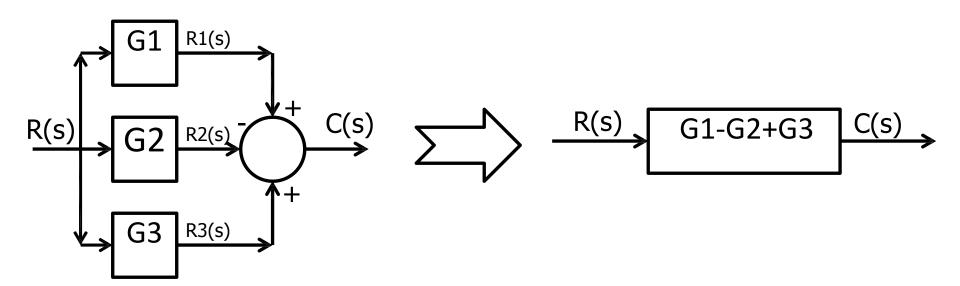


$$R(s) \longrightarrow G1G2G3 \longrightarrow C(s)$$



Rule 2: For blocks in Parallel

Gain of blocks connected in parallel gets added algebraically.

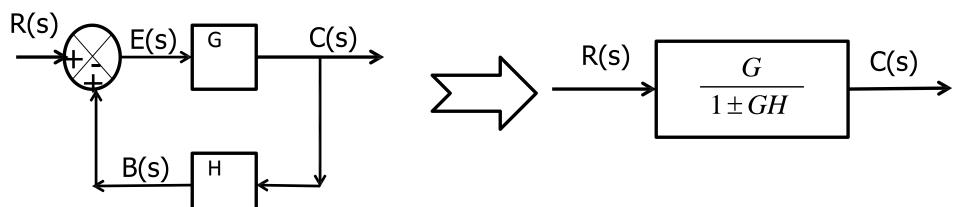


$$C(s) = R1(s)-R2(s)+R3(s)$$

= $G1R(s)-G2R(s)+G3R(s)$
 $C(s) = (G1-G2+G3) R(s)$

$$C(s) = (G1-G2+G3) R(s)$$

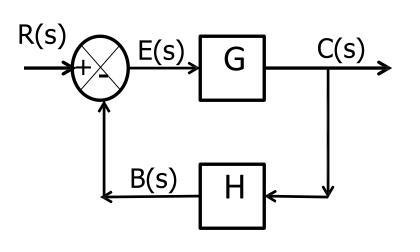
Rule 3: Eliminate Feedback Loop



$$\frac{C(s)}{R(s)} = \frac{G}{1 \pm GH}$$

In General

From Shown Figure,



$$E(s) = R(s) - B(s)$$

and

$$C(s) = G.E(s)$$

$$= G[R(s) - B(s)]$$

$$= GR(s) - GB(s)$$

But

$$B(s) = H.C(s)$$

$$\therefore C(s) = G.R(s) - G.H.C(s)$$

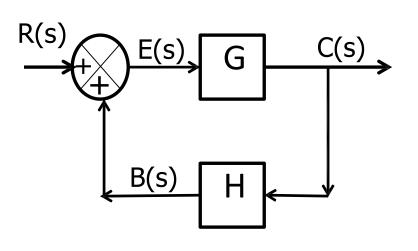
$$C(s) + G.H = GR(s)$$

$$\therefore C(s)\{1+G.H\} = G.R(s)$$

For Negative Feedback

$$\therefore \frac{C(s)}{R(s)} = \frac{G}{1 + GH}$$

From Shown Figure,



$$E(s) = R(s) + B(s)$$

and

$$C(s) = G.E(s)$$

$$= G[R(s) + B(s)]$$

$$= GR(s) + GB(s)$$

But

$$B(s) = H.C(s)$$

$$\therefore C(s) = G.R(s) + G.H.C(s)$$

$$C(s) - G.H = GR(s)$$

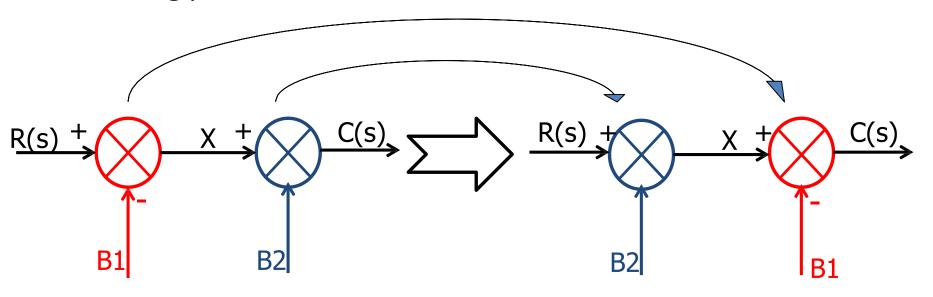
$$\therefore C(s)\{1-G.H\} = G.R(s)$$

For Positive Feedback

$$\therefore \frac{C(s)}{R(s)} = \frac{G}{1 - GH}$$

Rule 4: Associative Law for Summing Points

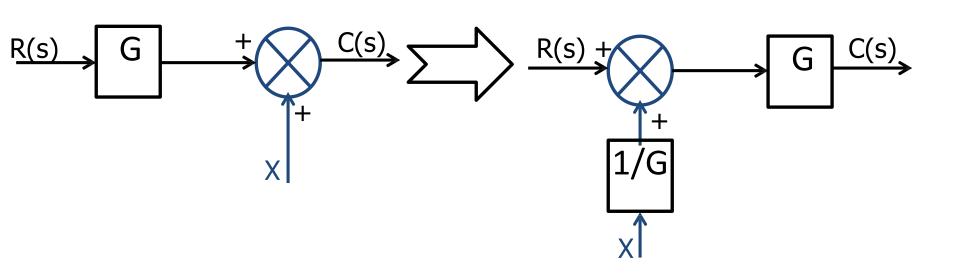
The order of summing points can be changed if two or more summing points are in series



$$X=R(s)-B1$$

 $C(s)=X-B2$
 $C(s)=R(s)-B1-B2$

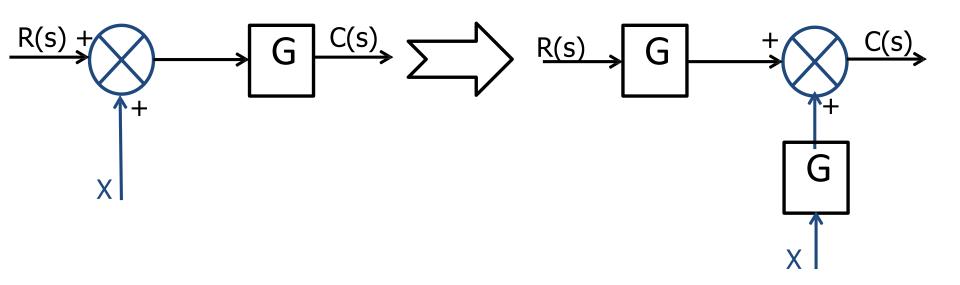
Rule 5: Shift summing point before block



$$C(s)=R(s)G+X$$

$$C(s)=G\{R(s)+X/G\}$$
$$=GR(s)+X$$

Rule 6: Shift summing point after block

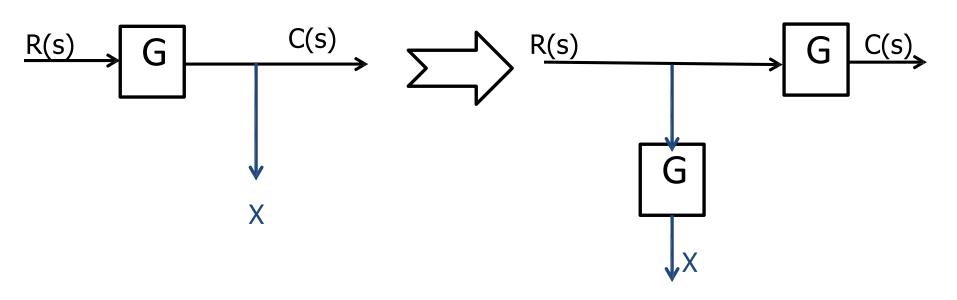


$$C(s)=G\{R(s)+X\}$$
$$=GR(s)+GX$$

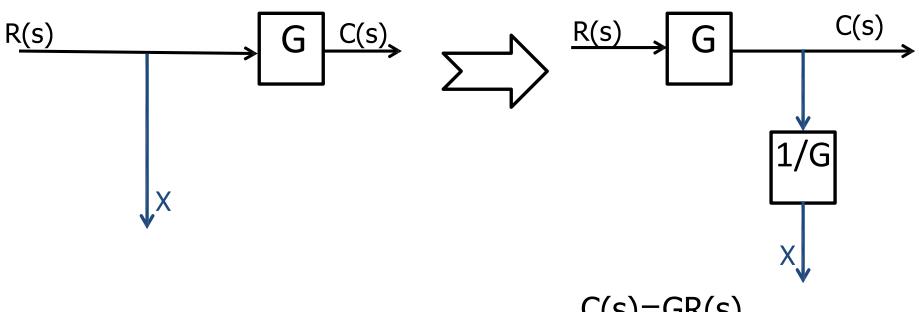
$$C(s)=GR(s)+XG$$

= $GR(s)+XG$

Rule 7: Shift a take off point before block



Rule 8: Shift a take off point after block



While solving block diagram for getting single block equivalent, the said rules need to be applied. After each simplification a decision needs to be taken. For each decision we suggest preferences as

First Choice

First Preference: Rule 1 (For series)

Second Preference: Rule 2 (For parallel)

Third Preference: Rule 3 (For FB loop)

Second Choice

(Equal Preference)

Rule 4 Adjusting summing order

Rule 5/6 Shifting summing point before/after block

Rule7/8 Shifting take off point before/after block

Feed Forward Transfer function

error ratio

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

closed loop transfer function

 $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

characteristic equation

$$1 + G(s)H(s) = 0$$

closed loop poles and zeros if K=10.

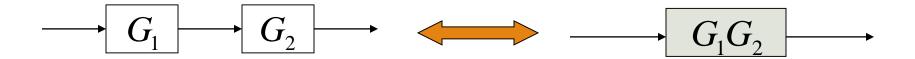
G(s)

(1+K)s+1

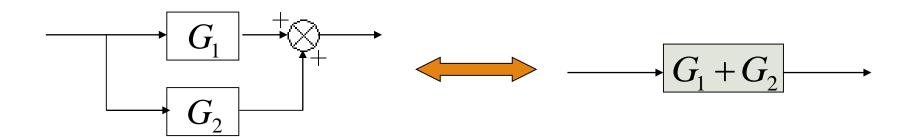
H(s)

Reduction techniques

1. Combining blocks in cascade

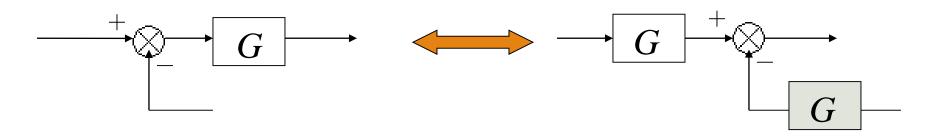


2. Combining blocks in parallel

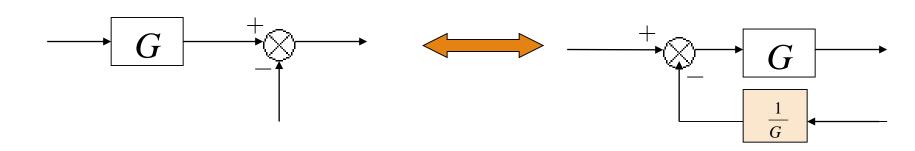


Reduction techniques

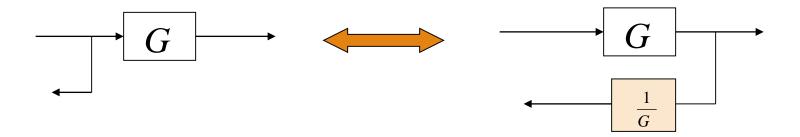
3. Moving a summing point behind a block



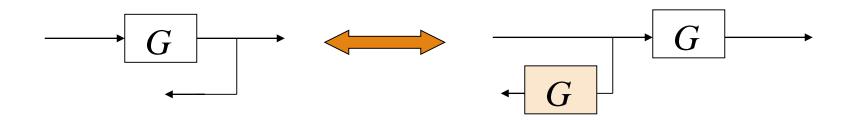
4. Moving a summing point ahead of a block



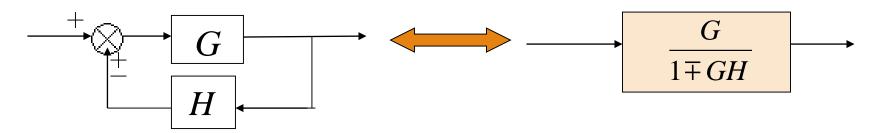
5. Moving a pickoff point behind a block

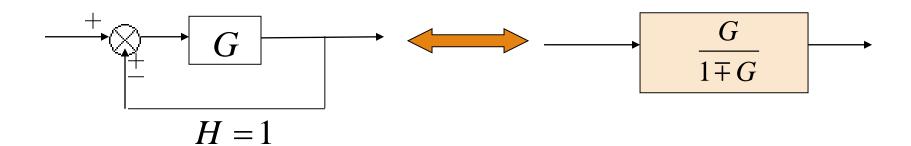


6. Moving a pickoff point ahead of a block

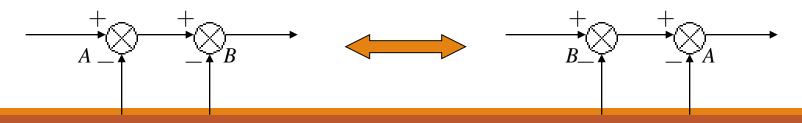


7. Eliminating a feedback loop

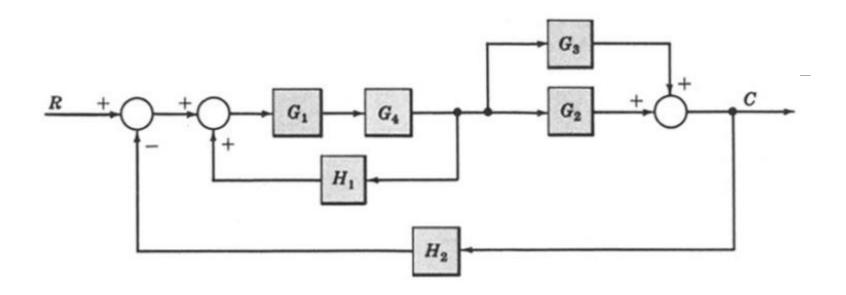




8. Swap with two neighboring summing points



Example: Reduce the Block Diagram to Canonical Form.



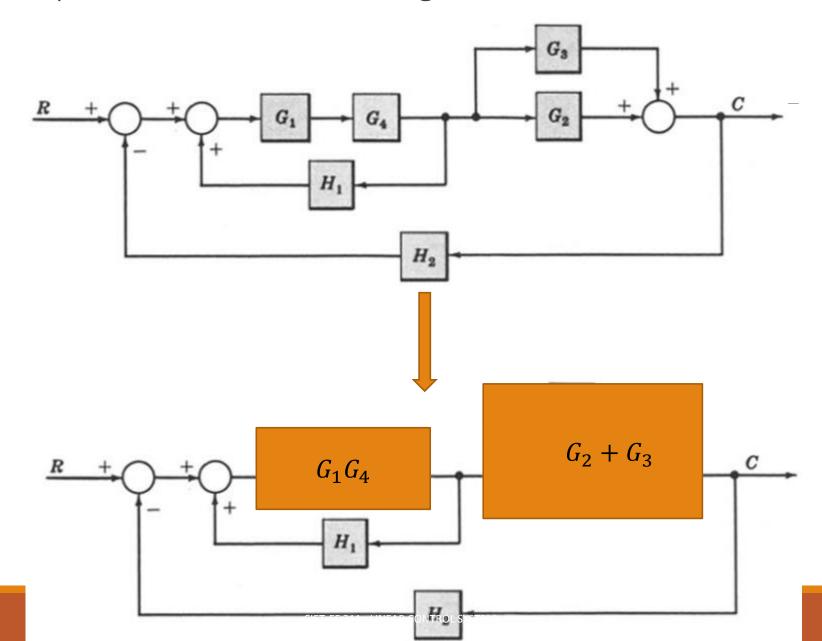
Combine all cascade block using rule-1



Combine all parallel block using rule-2



Example: Reduce the Block Diagram to Canonical Form.

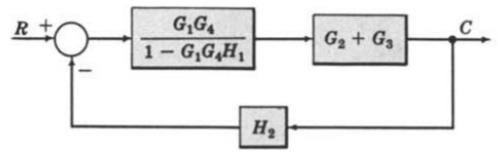


Example Continue.

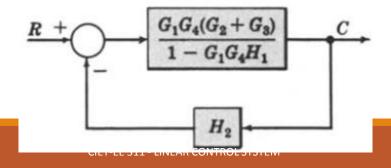
Eliminate all minor feedback loops using rule-7

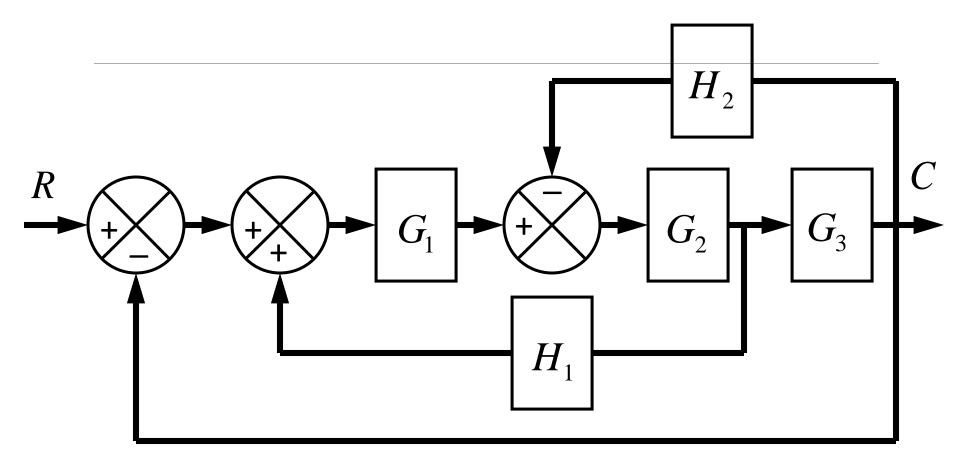


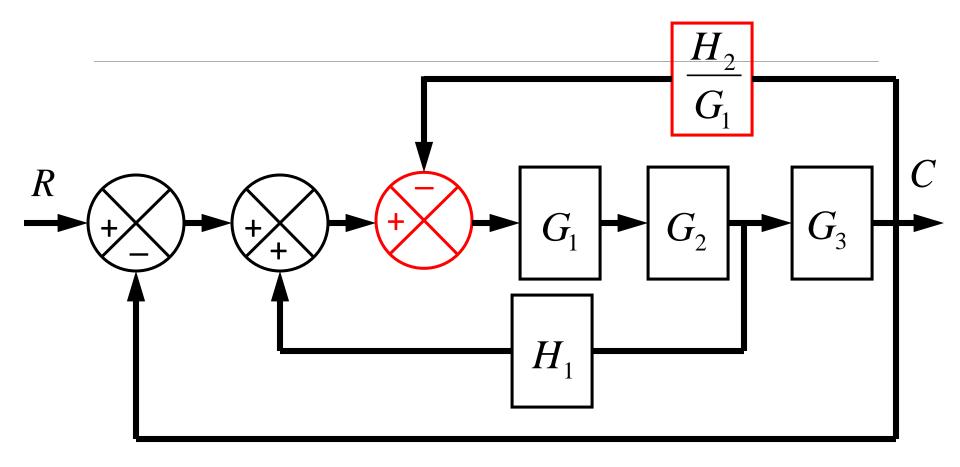
After the elimination of minor feedback loop the block diagram is reduced to as shown below

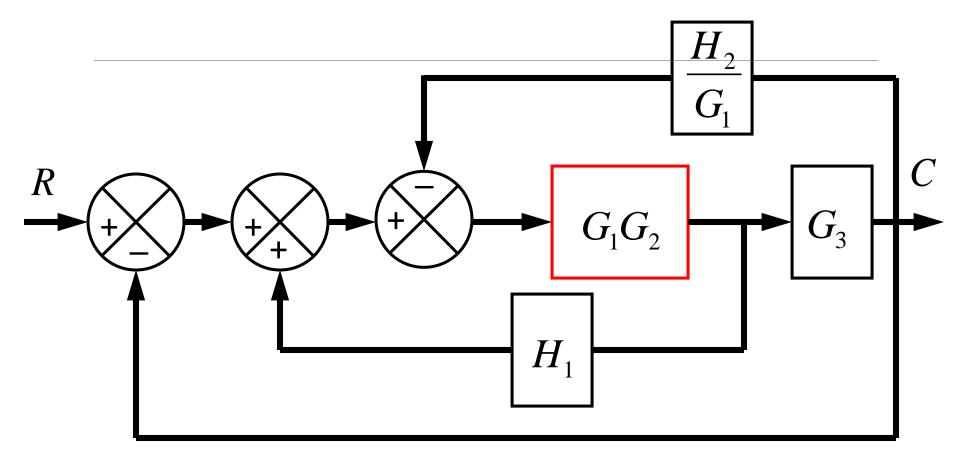


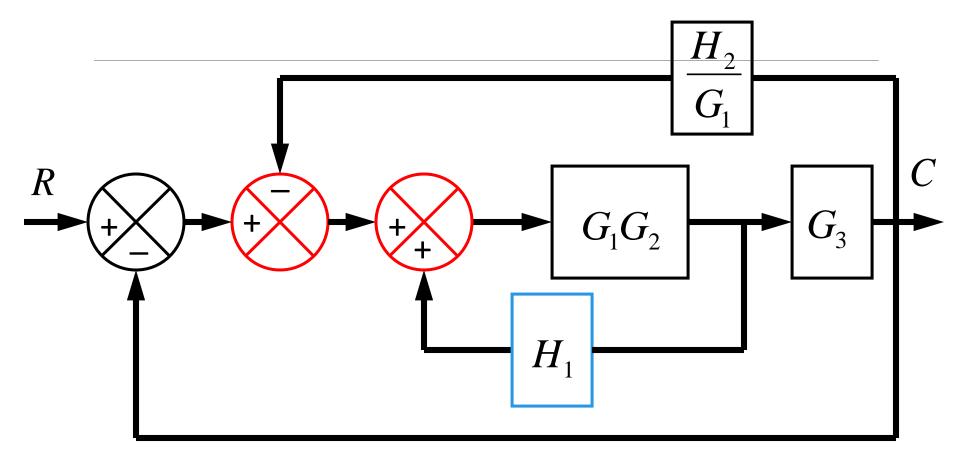
Again blocks are in cascade are removed using rule-1

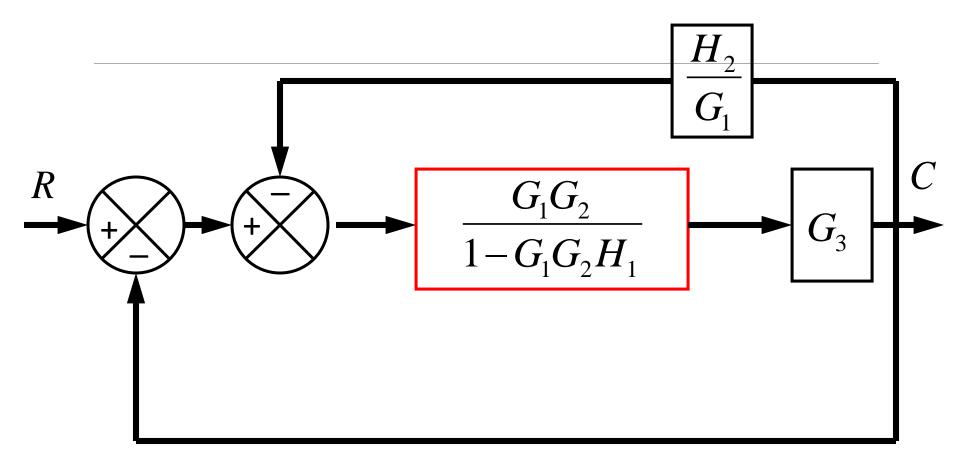


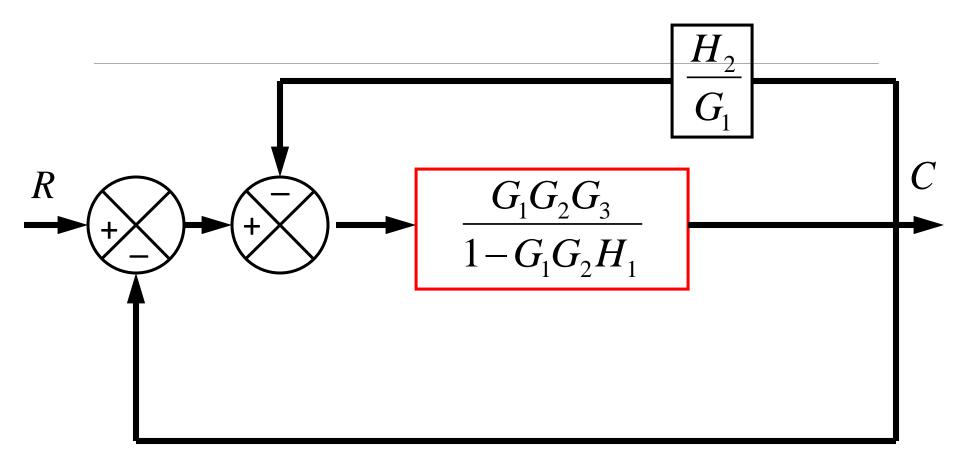


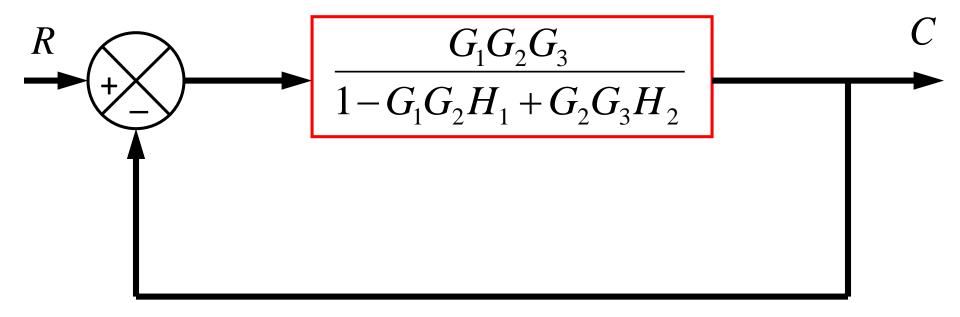




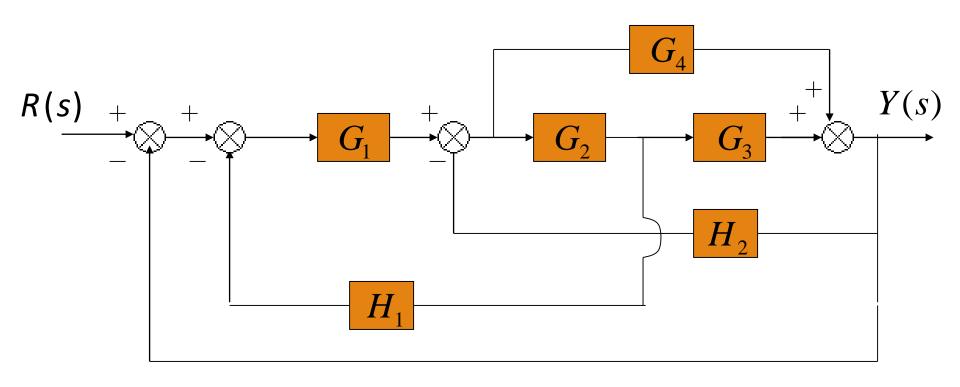


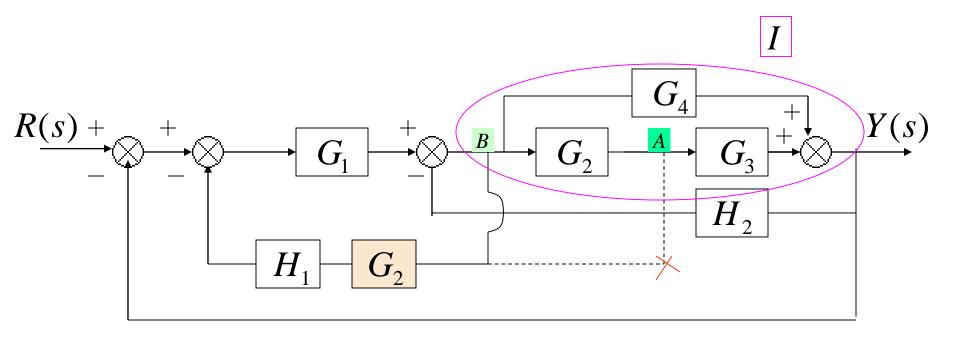






Find the transfer function of the following block diagram

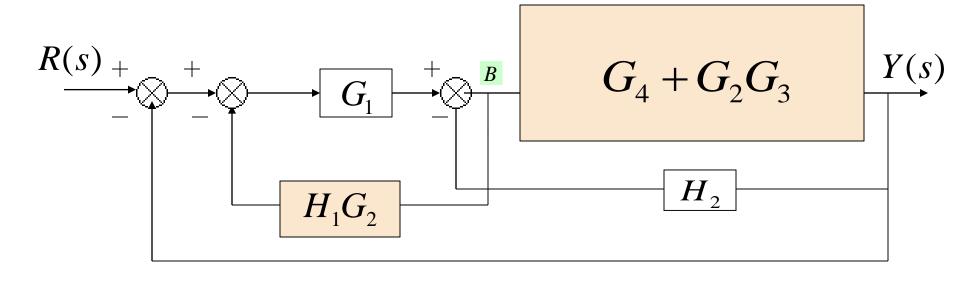


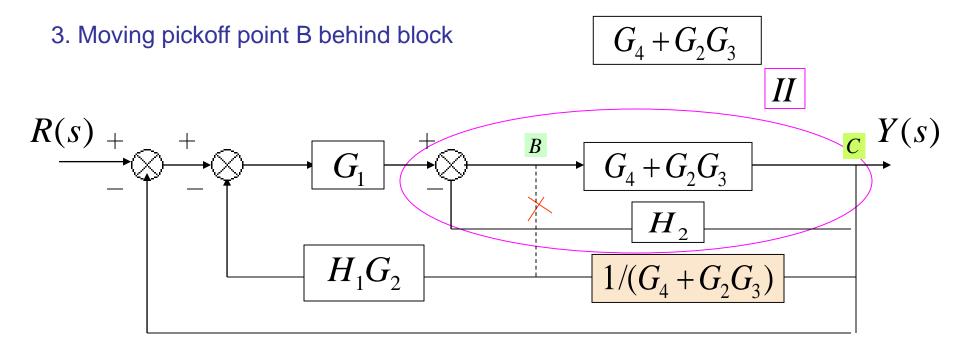


Solution:

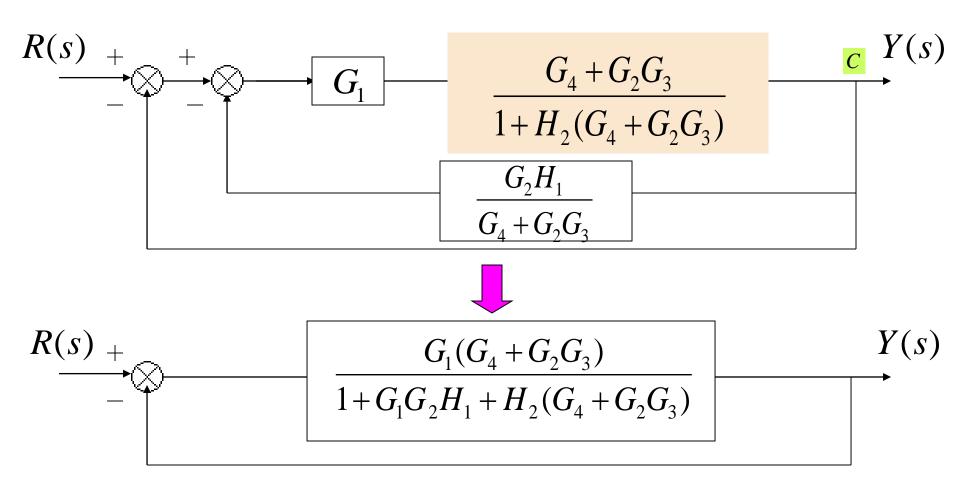
- 1. Moving pickoff point A ahead of block $oxedown_2$
- 2. Eliminate loop I & simplify

$$G_4 + G_2G_3$$



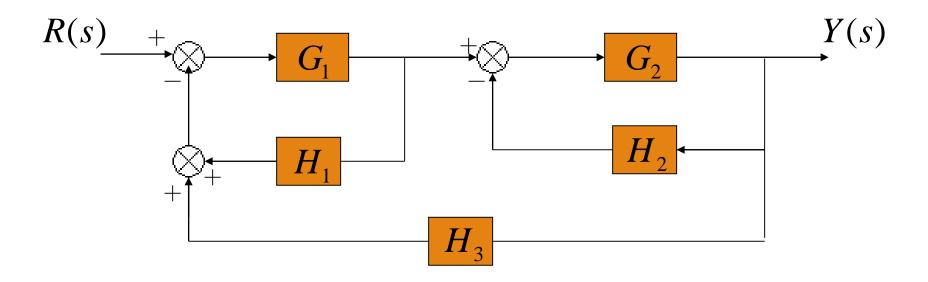


4. Eliminate loop III



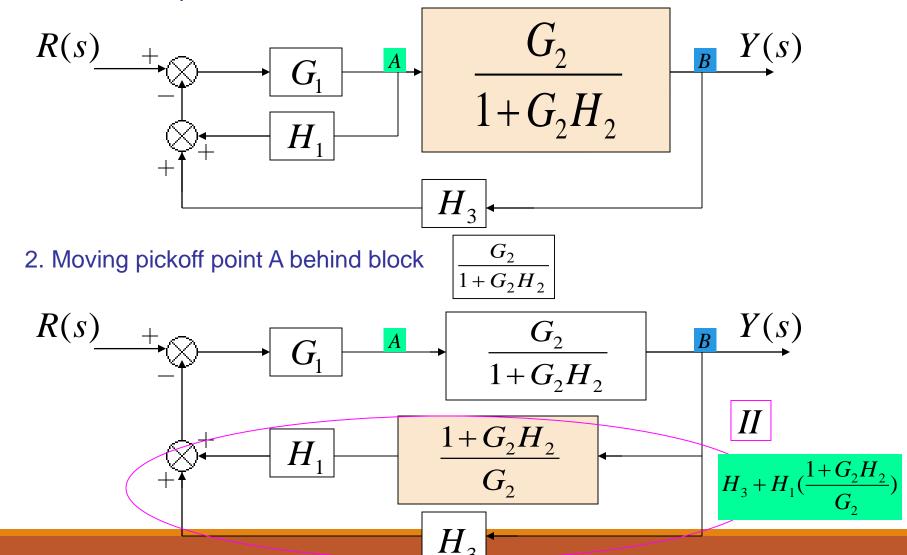
$$\frac{Y(s)}{R(s)} = \frac{G_1(G_4 + G_2G_3)}{1 + G_1G_2H_1 + H_2(G_4 + G_2G_3) + G_1(G_4 + G_2G_3)}$$

Find the transfer function of the following block diagrams

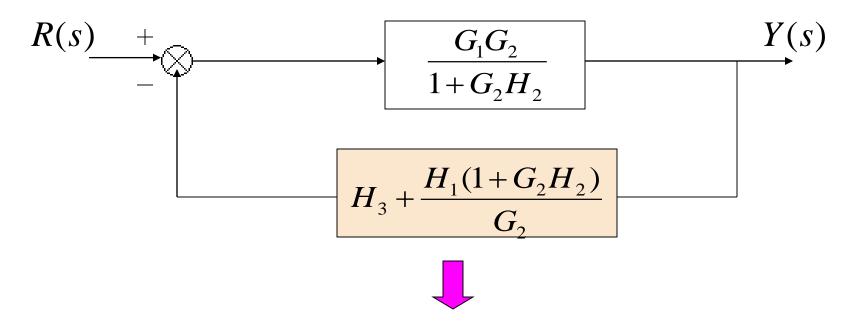


Solution:

1. Eliminate loop I

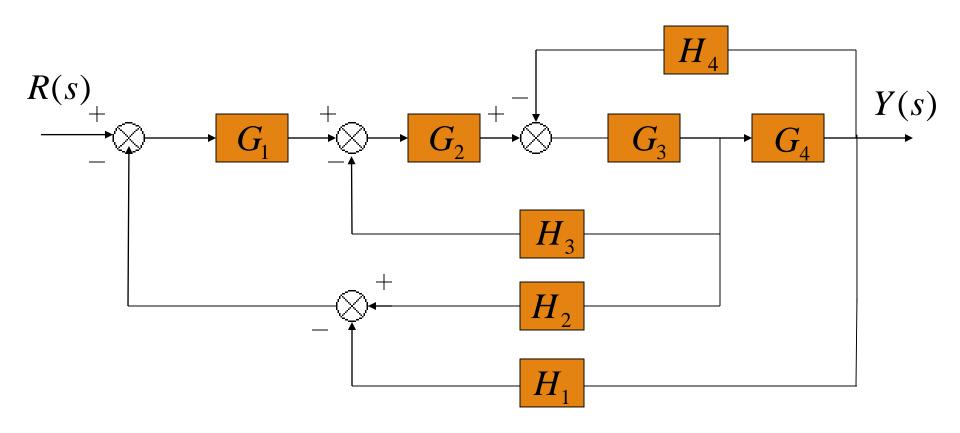


3. Eliminate loop II

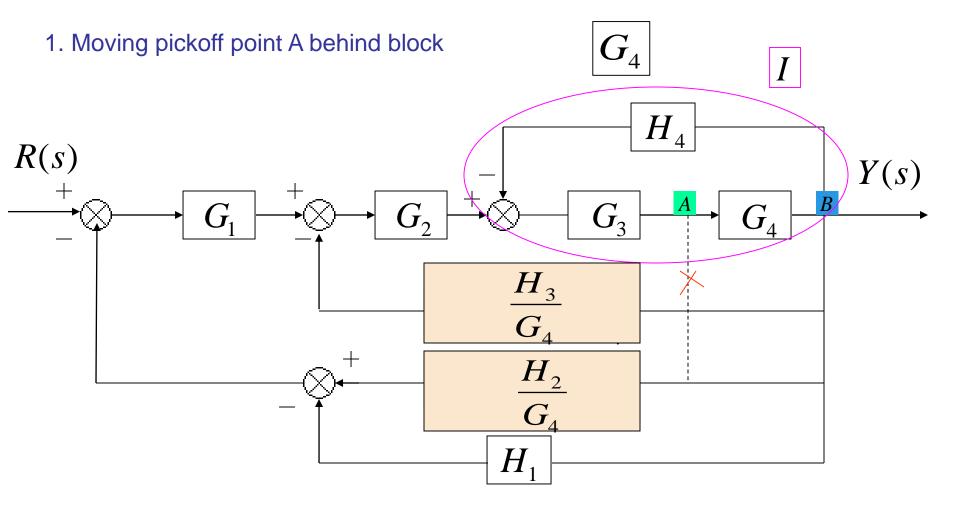


$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_3 + G_1 H_1 + G_1 G_2 H_1 H_2}$$

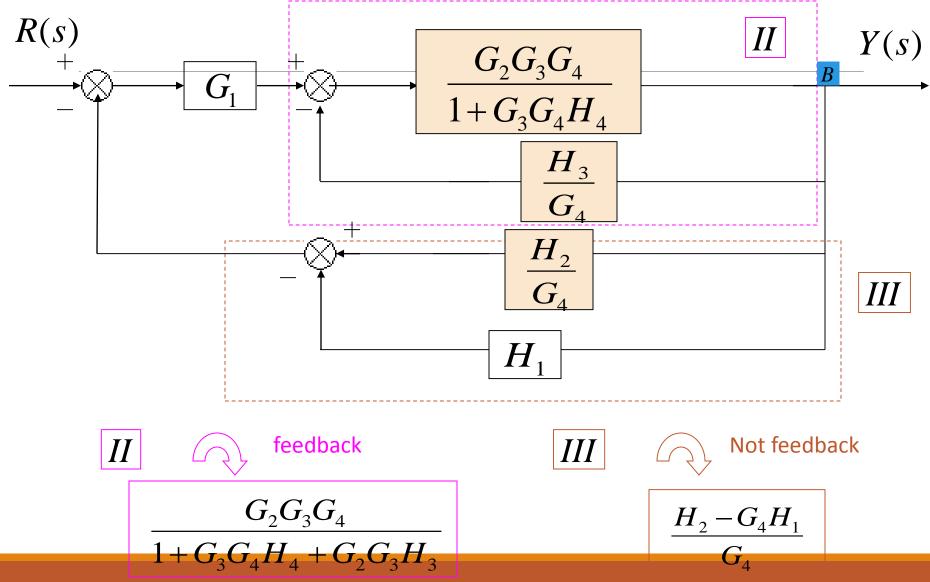
Find the transfer function of the following block diagrams



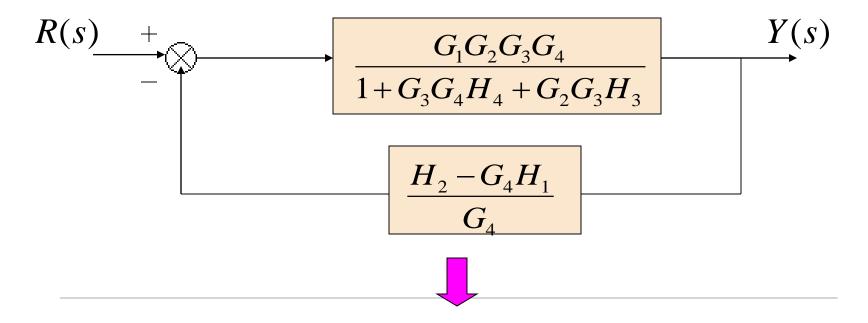
Solution:



2. Eliminate loop I and Simplify

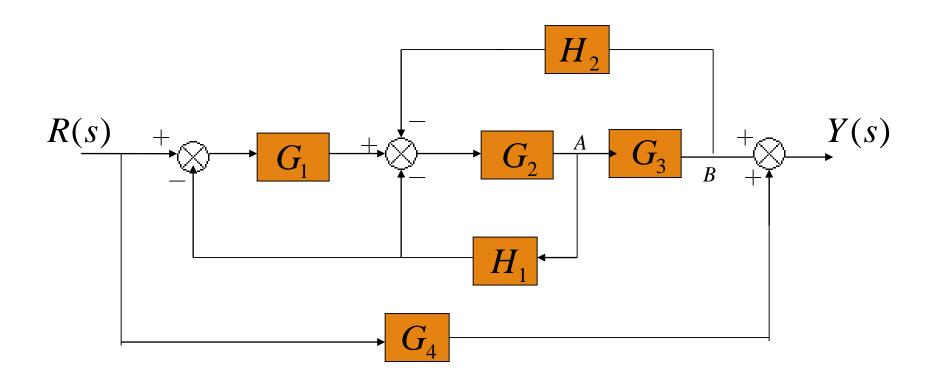


3. Eliminate loop II & IIII

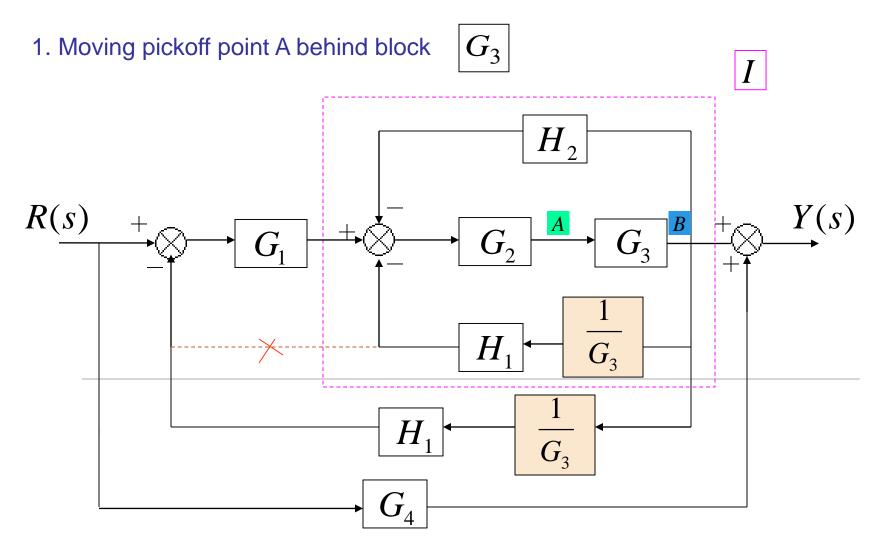


$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_2 G_3 H_3 + G_3 G_4 H_4 + G_1 G_2 G_3 H_2 - G_1 G_2 G_3 G_4 H_1}$$

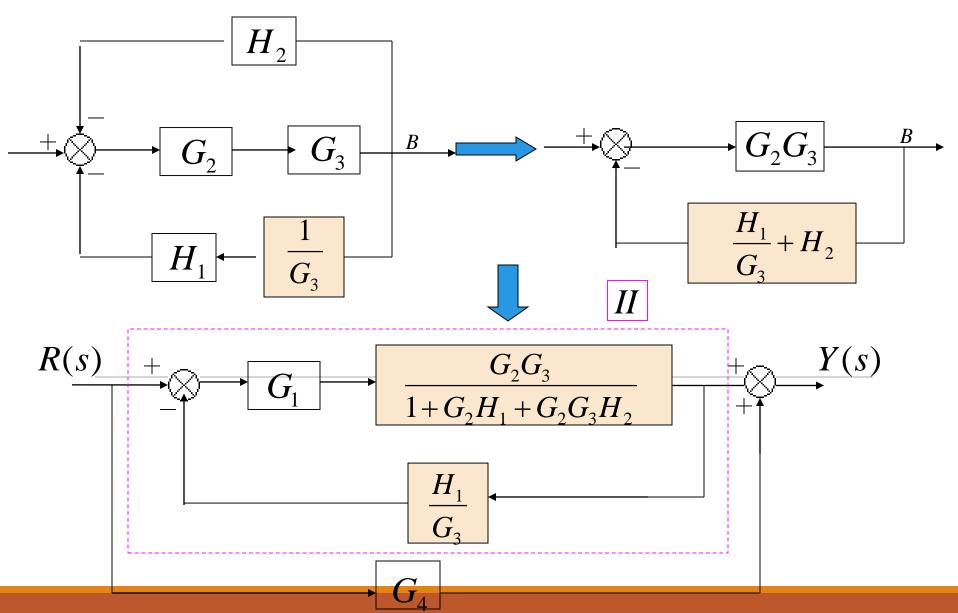
Find the transfer function of the following block diagrams



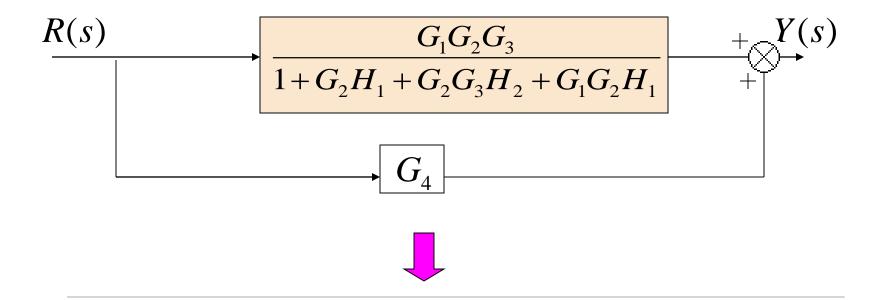
Solution:



2. Eliminate loop I & Simplify

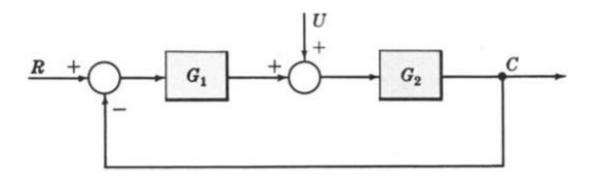


3. Eliminate loop II

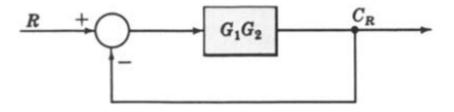


$$T(s) = \frac{Y(s)}{R(s)} = G_4 + \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 H_1}$$

Example: Multiple Input System. Determine the output C due to inputs R and U using the Superposition Method.

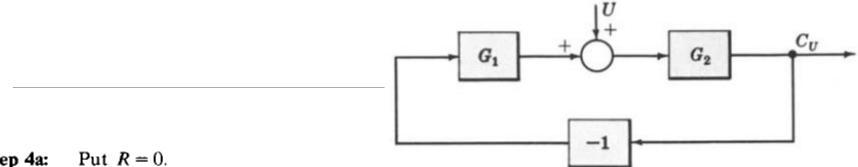


- Step 1: Put $U \equiv 0$.
- **Step 2:** The system reduces to



Step 3: the output C_R due to input R is $C_R = [G_1G_2/(1 + G_1G_2)]R$.

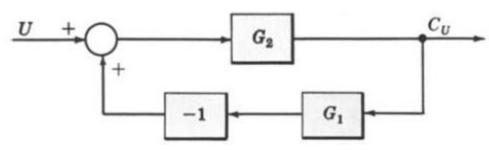
Example: Continue.



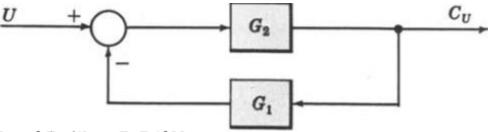
Step 4a:

Step 4b: Put -1 into a block, representing the negative feedback effect:

Rearrange the block diagram:



Let the -1 block be absorbed into the summing point:



the output C_U due to input U is $C_U = [G_2/(1 + G_1G_2)]U$. Step 4c:

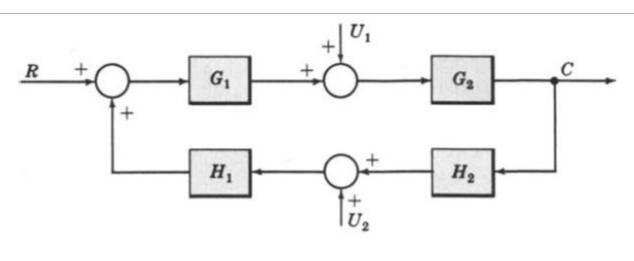
Example: Continue.

Step 5: The total output is $C = C_R + C_U$

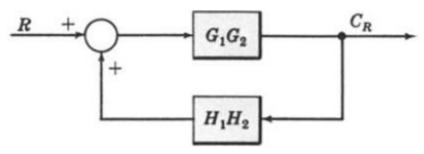
$$= \left[\frac{G_1 G_2}{1 + G_1 G_2} \right] R + \left[\frac{G_2}{1 + G_1 G_2} \right] U$$

$$= \left[\frac{G_2}{1 + G_1 G_2}\right] \left[G_1 R + U\right]$$

Example: Multiple-Input System. Determine the output C due to inputs R, U1 and U2 using the Superposition Method.



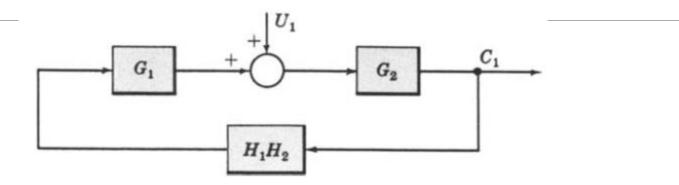
Let $U_1 = U_2 = 0$.



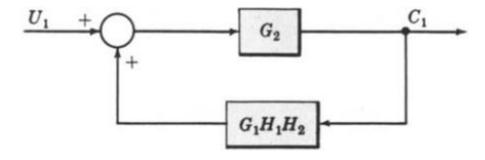
$$C_R = [G_1G_2/(1 - G_1G_2H_1H_2)]R$$

Example: Continue.

Now let $R = U_2 = 0$.



Rearranging the blocks, we get

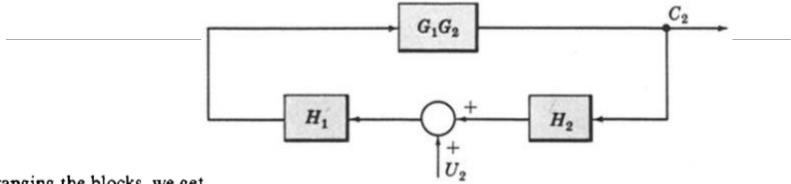


$$C_1 = [G_2/(1 - G_1G_2H_1H_2)]U_1$$

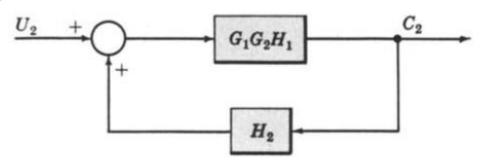
where C_1 is the response due to U_1 acting alone.

Example: Continue.

Finally, let $R = U_1 = 0$.



Rearranging the blocks, we get

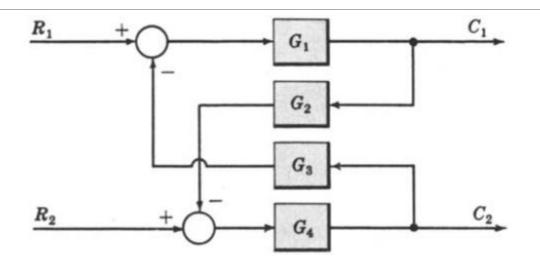


$$C_2 = [G_1G_2H_1/(1 - G_1G_2H_1H_2)]U_2$$

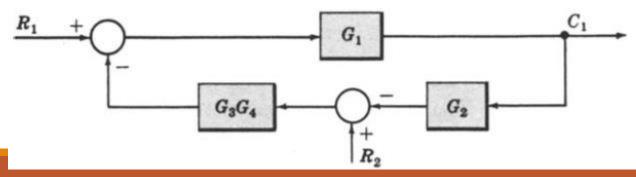
where C_2 is the response due to U_2 acting alone.

By superposition, the total output is
$$C = C_R + C_1 + C_2 = \frac{G_1 G_2 R + G_2 U_1 + G_1 G_2 H_1 U_2}{1 - G_1 G_2 H_1 H_2}$$

Example: Multi-Input Multi-Output System. Determine C1 and C2 due to R1 and R2.

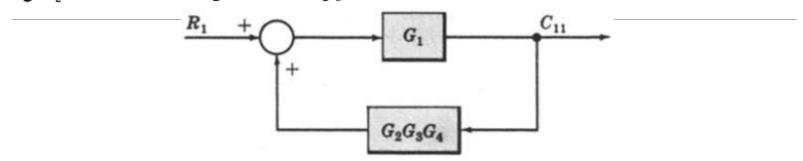


First ignoring the output C_2 .



Example: Continue.

Letting $R_2 = 0$ and combining the summing points,



Hence C_{11} , the output at C_1 due to R_1 alone, is $C_{11} = G_1 R_1 / (1 - G_1 G_2 G_3 G_4)$.

For
$$R_1 = 0$$
,
$$R_2 + G_1G_3G_4$$

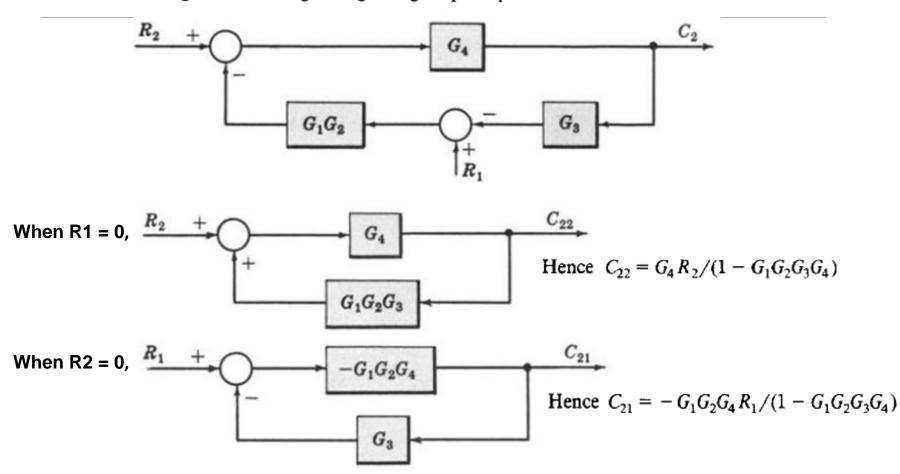
$$G_2$$

Hence $C_{12} = -G_1G_3G_4R_2/(1 - G_1G_2G_3G_4)$ is the output at C_1 due to R_2 alone.

Thus
$$C_1 = C_{11} + C_{12} = (G_1 R_1 - G_1 G_3 G_4 R_2)/(1 - G_1 G_2 G_3 G_4)$$

Example: Continue.

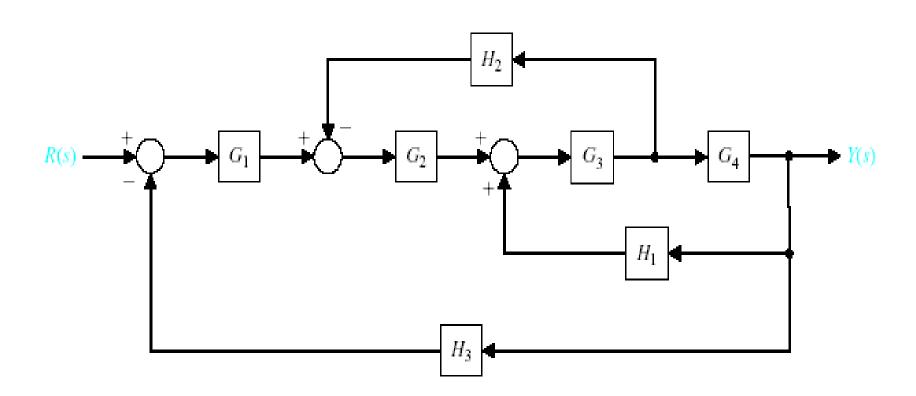
Now we reduce the original block diagram, ignoring output C_1 .



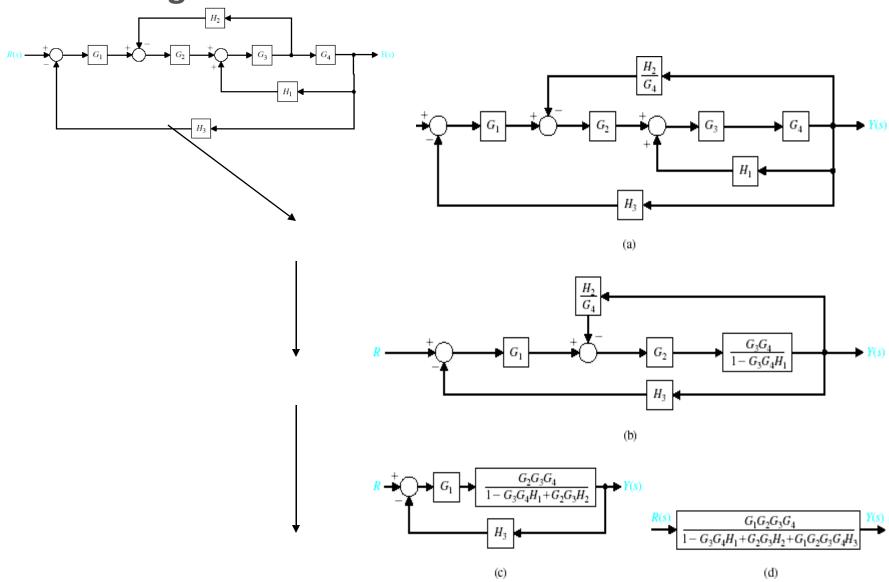
Finally,
$$C_2 = C_{22} + C_{21} = (G_4 R_2 - G_1 G_2 G_4 R_1)/(1 - G_1 G_2 G_3 G_4)$$

Block Diagram Models

Example

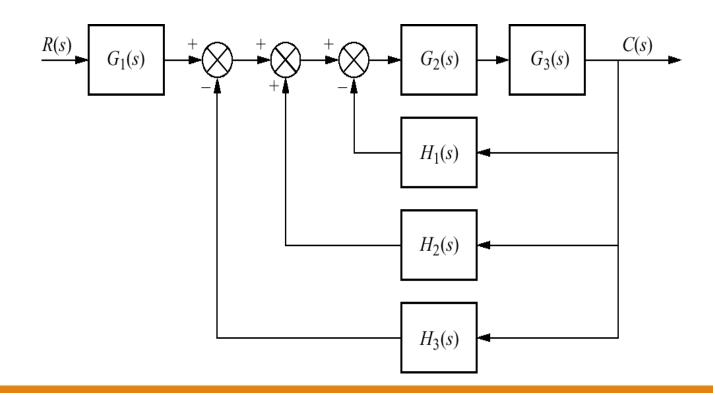


Block Diagram Models



Block diagram reduction via familiar forms for Example

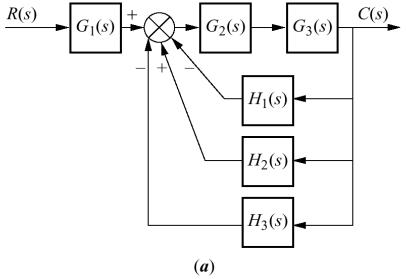
Problem: Reduce the block diagram shown in figure to a single transfer function

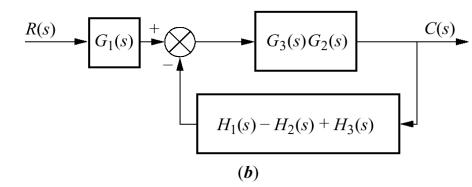


Block diagram reduction via familiar forms for Example Cont.

Steps in solving Example

- a. collapse summing junctions; form
 - b. equivalent cascaded system in the forward path
- c. form equivalent parallel system in the feedback path;
- d. form equivalent feedback system and multiply by cascadedG₁(s)

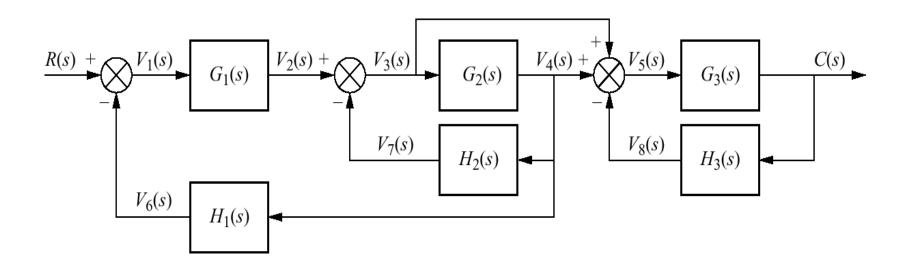




$$\begin{array}{c|c}
R(s) & \hline
G_3(s)G_2(s)G_1(s) & C(s) \\
\hline
1 + G_3(s)G_2(s)[H_1(s) - H_2(s) + H_3(s)] & C(s)
\end{array}$$
(c)

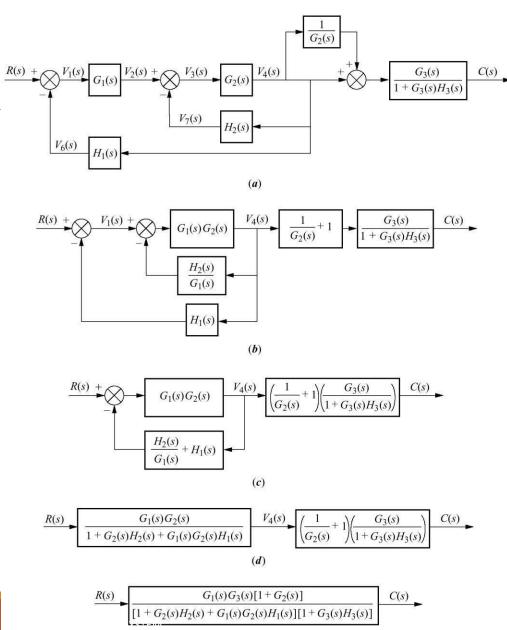
Block diagram reduction by moving blocks Example

Problem: Reduce the block diagram shown in figure to a single transfer function



Steps in the block diagram reduction for Example

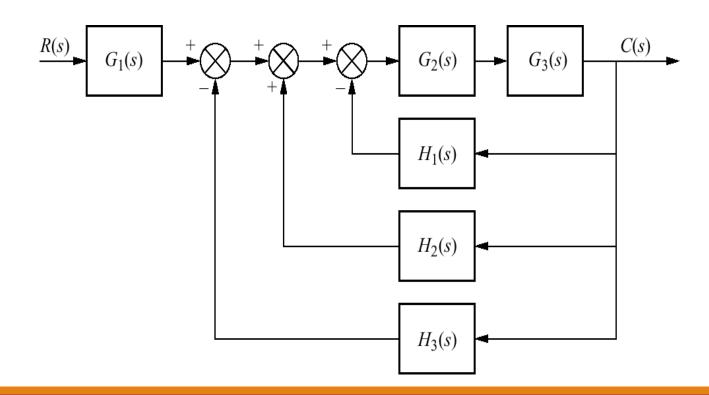
- a)Move G₂(s) to the left past of pickoff point to create parallel subsystems, and reduce the feedback system of G₃(s) and H₃(s)
- b)Reduce parallel pair of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past summing junction
- c)Collapse the summing junctions, add the 2 feedback elements, and combine the last 2 cascade blocks
- d)Reduce the feedback system to the left
- e)finally, Multiple the 2 cascade blocks and obtain final result.



(e)

Block diagram reduction via familiar forms for Example

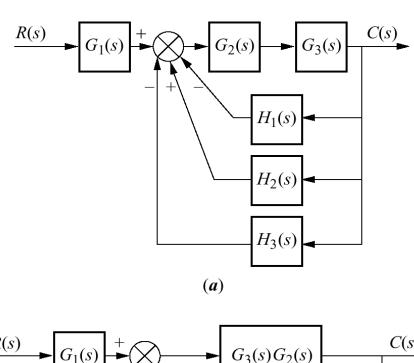
Problem: Reduce the block diagram shown in figure to a single transfer function

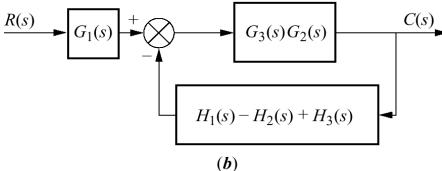


Block diagram reduction via familiar forms for Example Cont.

Steps in solving Example

- a. collapse summing junctions;
- b. form equivalent cascaded system in the forward path
- c. form equivalent parallel system in the feedback path;
- d. form equivalent feedback system and multiply by cascadedG₁(s)



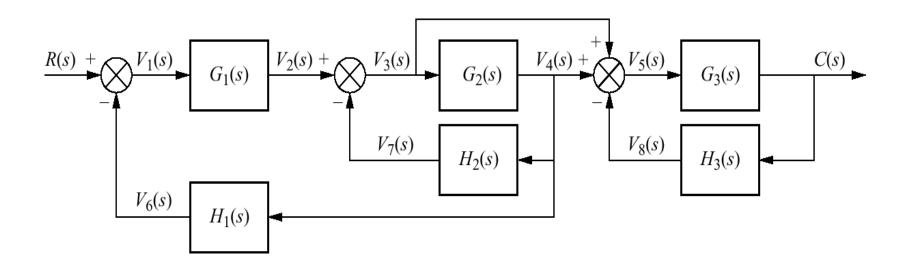


$$\begin{array}{c|c}
R(s) & G_3(s)G_2(s)G_1(s) & C(s) \\
\hline
1 + G_3(s)G_2(s)[H_1(s) - H_2(s) + H_3(s)] & C(s)
\end{array}$$

(c)

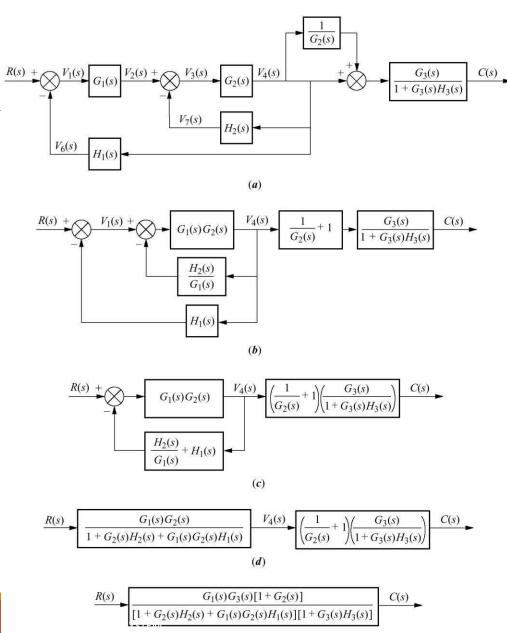
Block diagram reduction by moving blocks Example

Problem: Reduce the block diagram shown in figure to a single transfer function



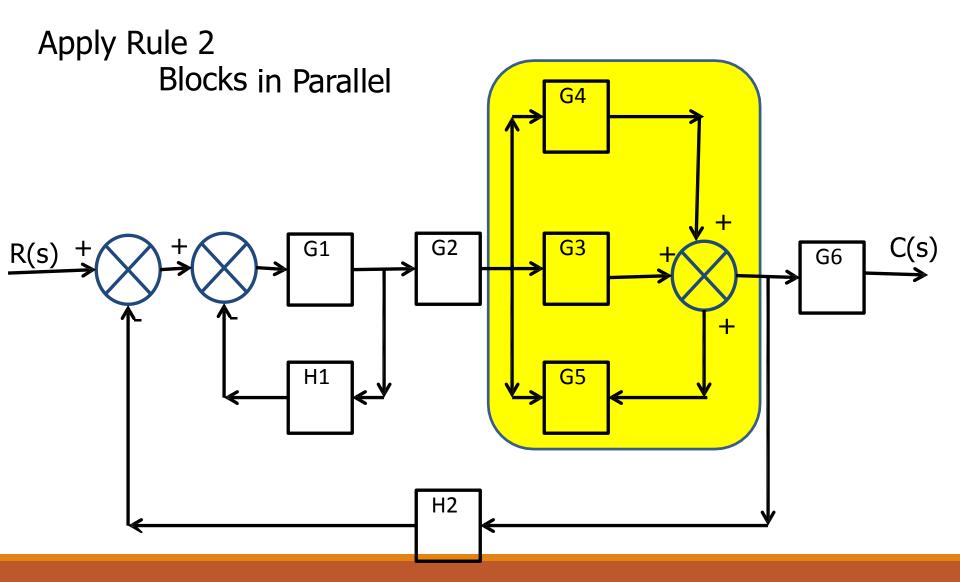
Steps in the block diagram reduction for Example

- a)Move G₂(s) to the left past of pickoff point to create parallel subsystems, and reduce the feedback system of G₃(s) and H₃(s)
- b)Reduce parallel pair of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past summing junction
- c)Collapse the summing junctions, add the 2 feedback elements, and combine the last 2 cascade blocks
- d)Reduce the feedback system to the left
- e)finally, Multiple the 2 cascade blocks and obtain final result.

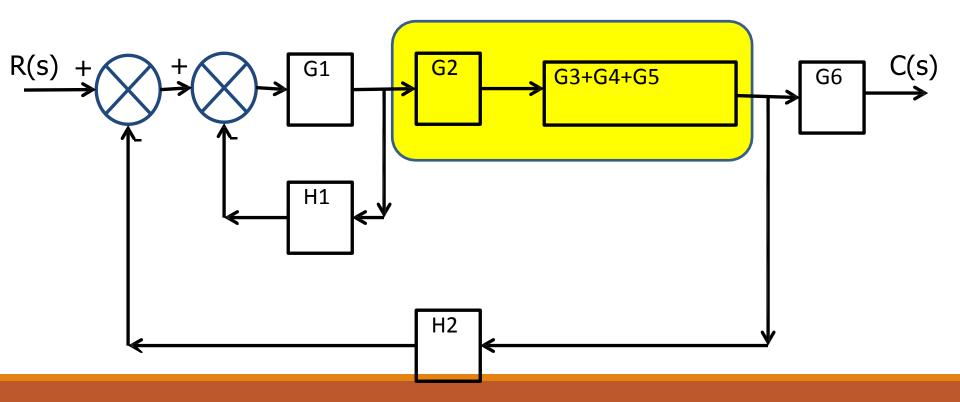


(e)

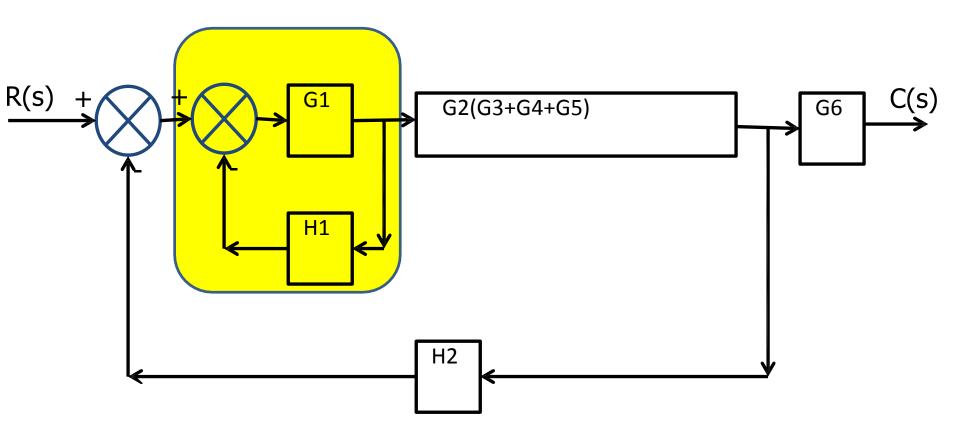
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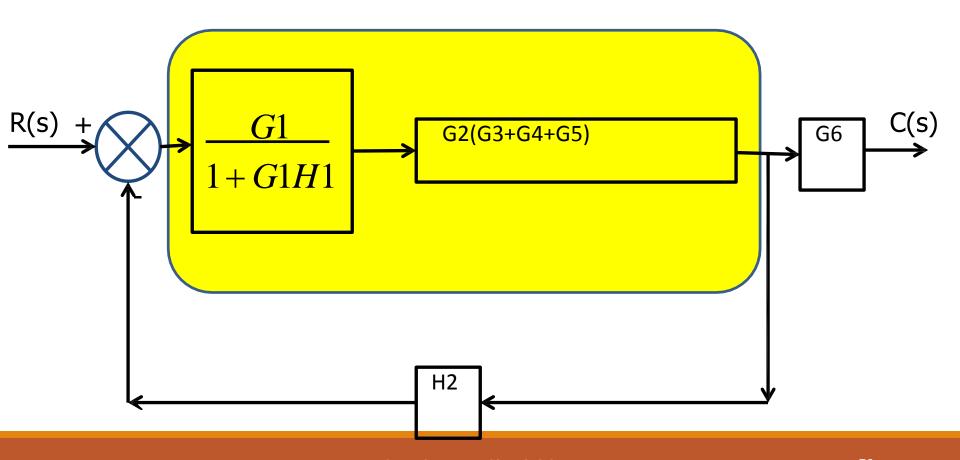
Apply Rule 1 Blocks in series



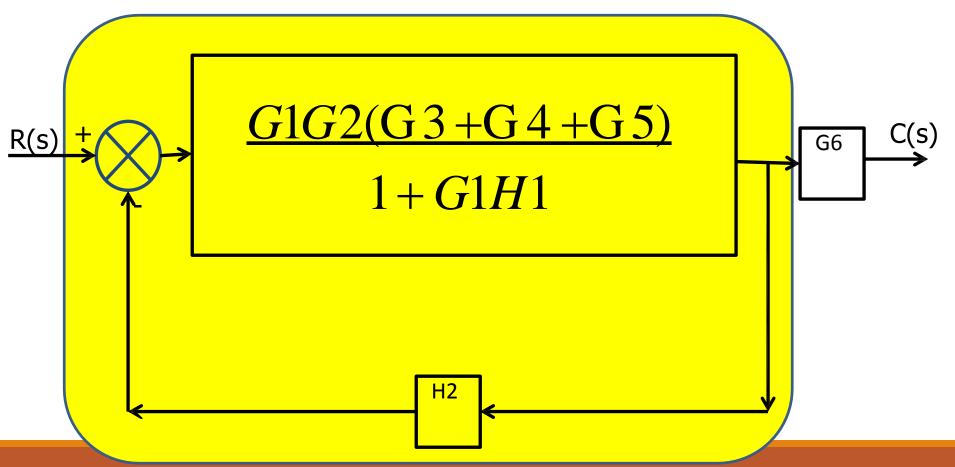
Apply Rule 3 Elimination of feedback loop



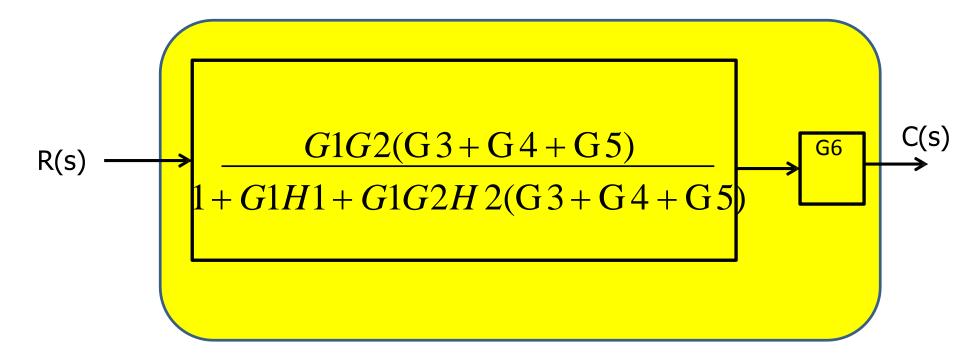
Apply Rule 1 Blocks in series



Apply Rule 3 Elimination of feedback loop

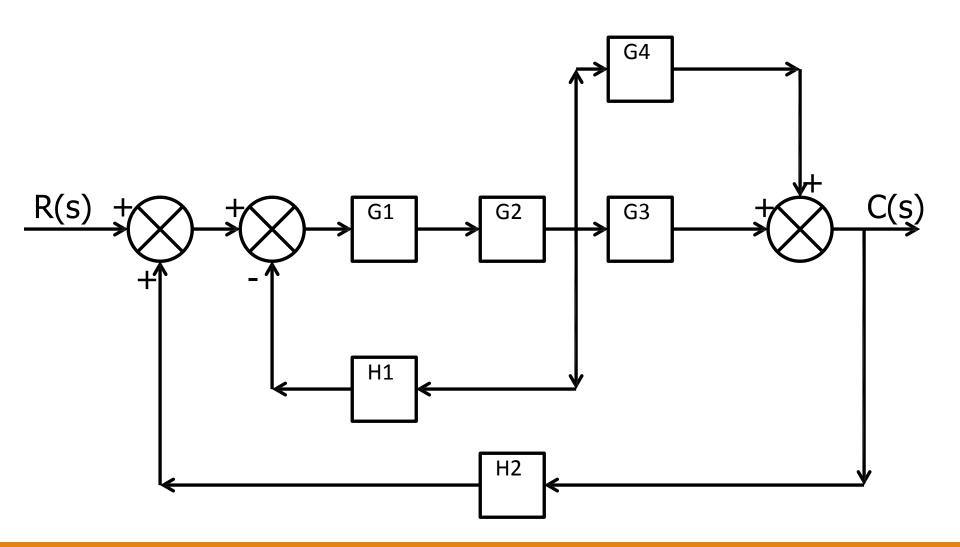


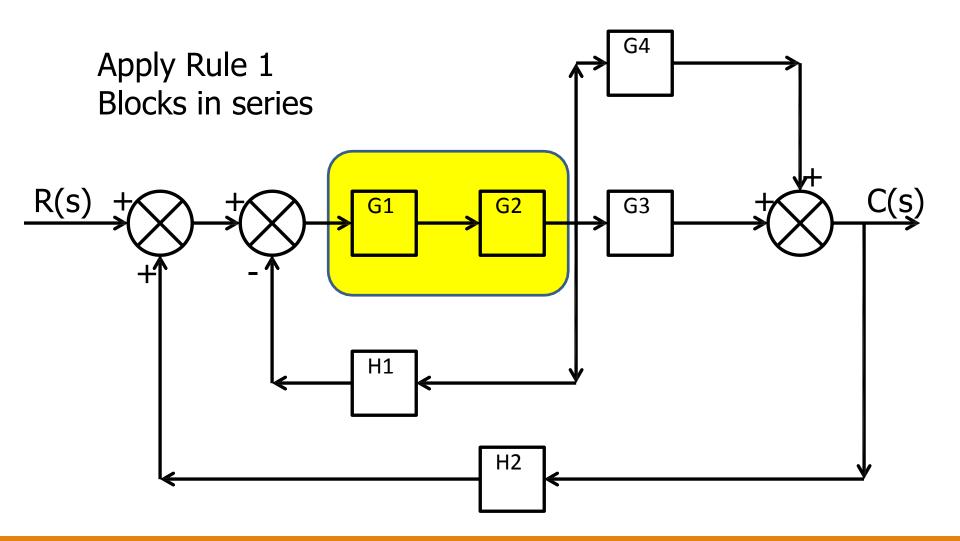
Apply Rule 1 Blocks in series

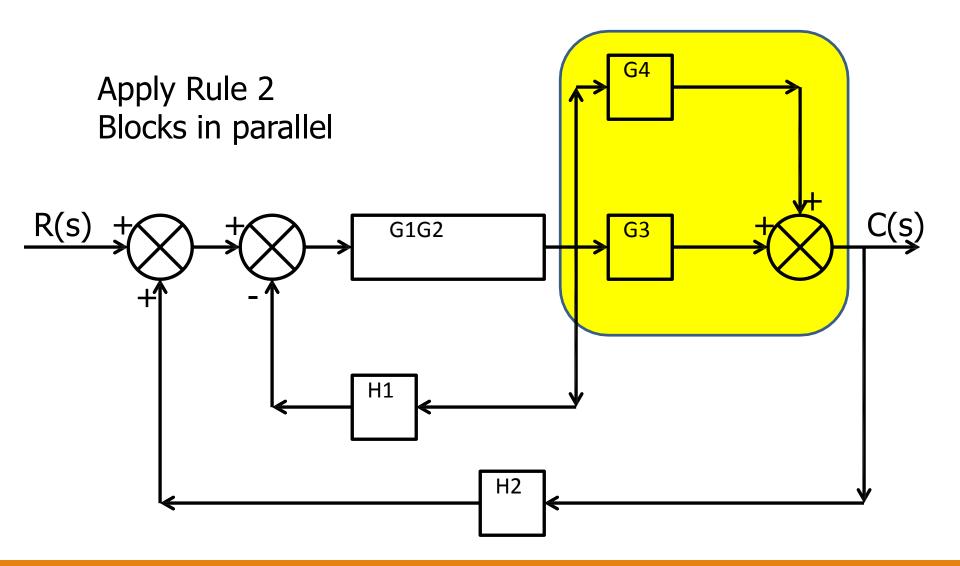


R(s)
$$G1G2G6(G3+G4+G5)$$
 C(s) $1+G1H1+G1G2H2(G3+G4+G5)$

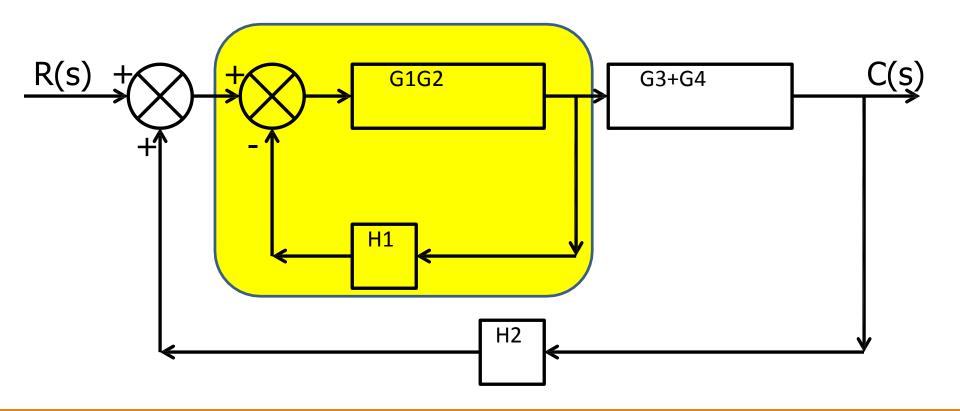
$$\frac{C(s)}{R(s)} = \frac{G1G2G6(G3+G4+G5)}{1+G1H1+G1G2H2(G3+G4+G5)}$$



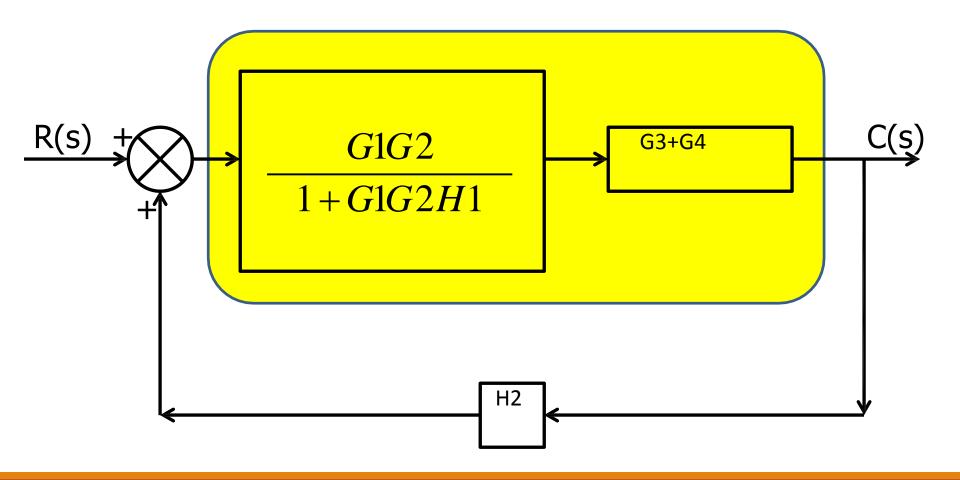




Apply Rule 3 Elimination of feedback loop

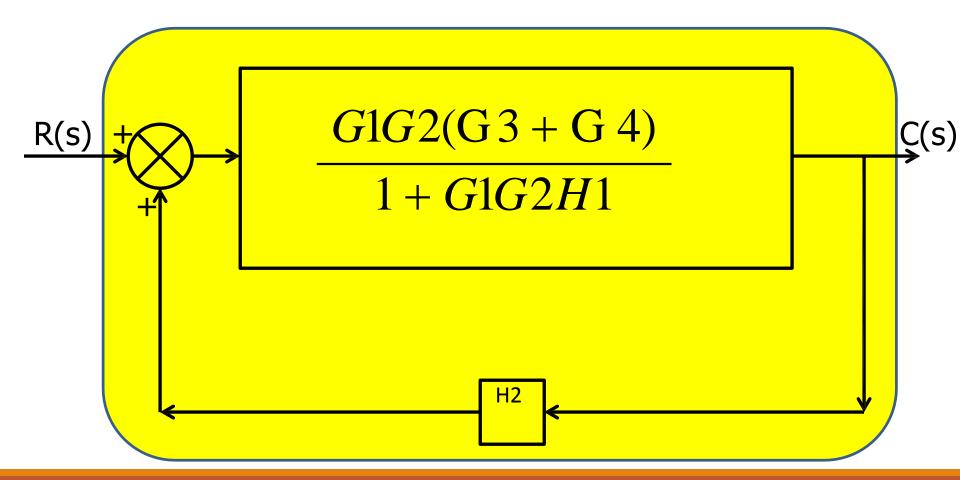


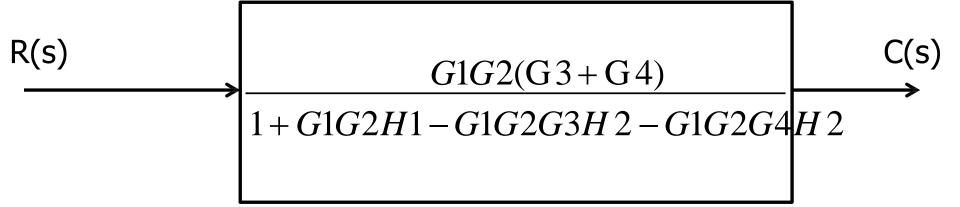
Apply Rule 2 Blocks in series



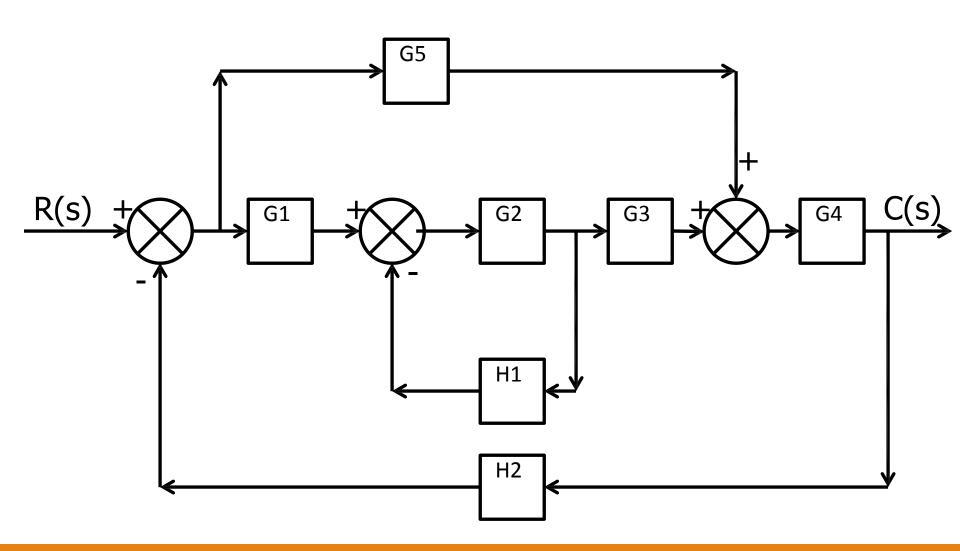
Apply Rule 3

Elimination of feedback loop

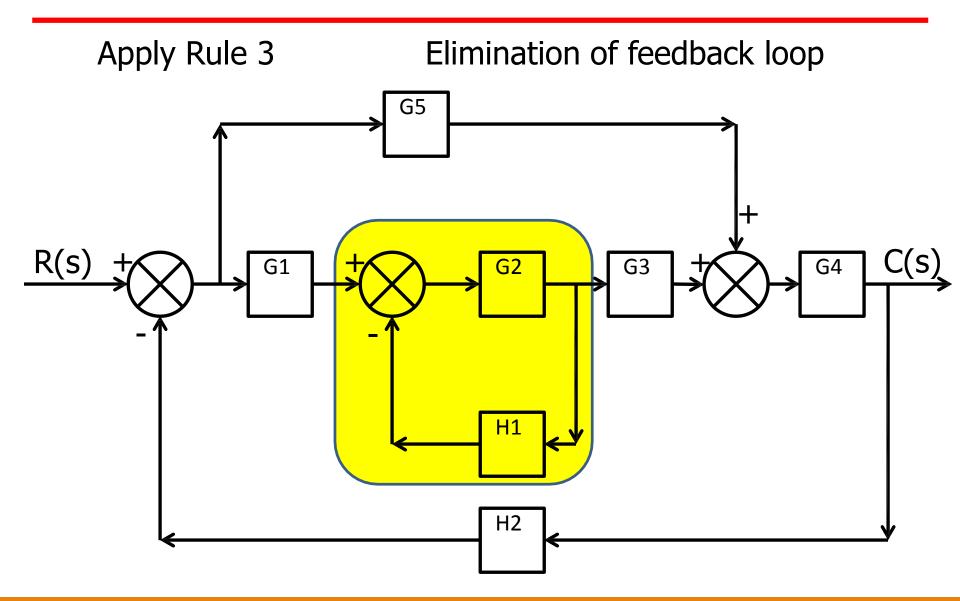




$$\frac{C(s)}{R(s)} = \frac{G1G2(G3+G4)}{1+G1G2H1-G1G2G3H2-G1G2G4H2}$$

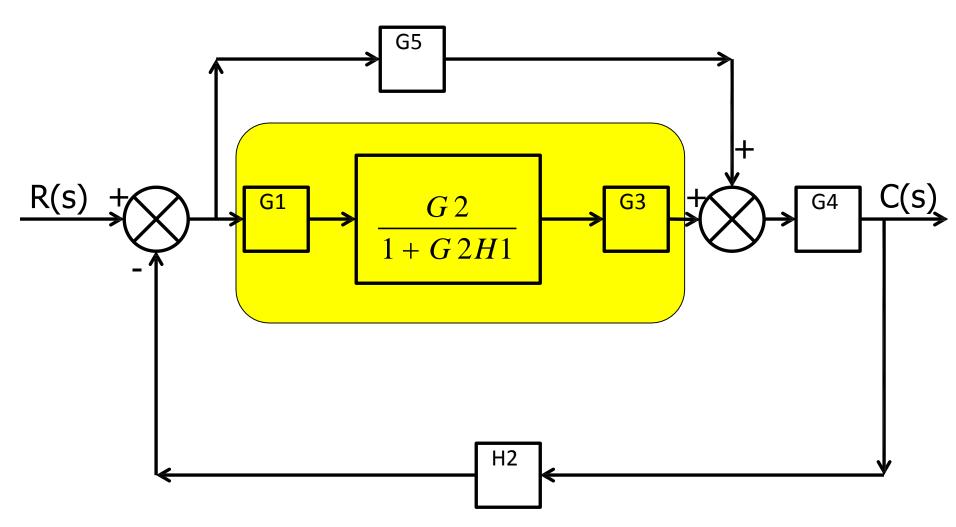


cont....



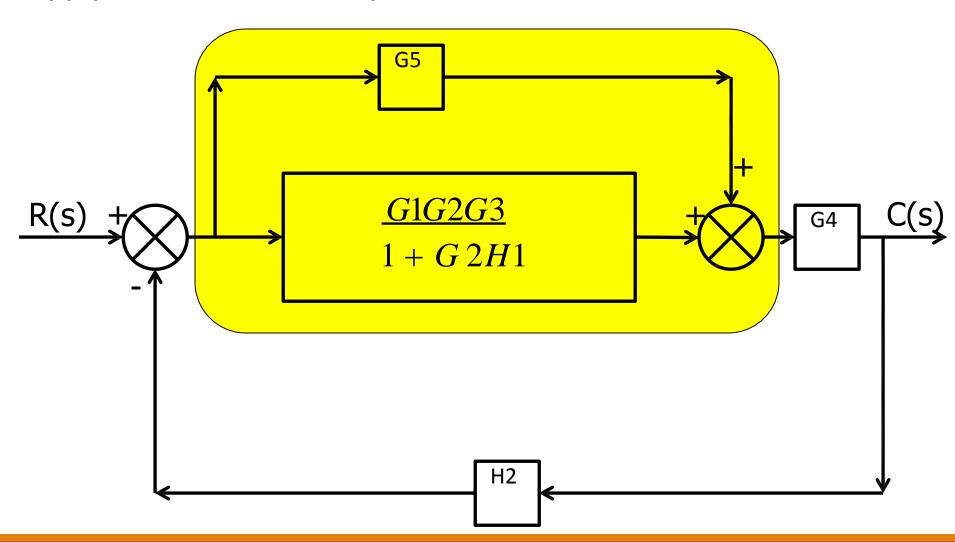
cont....

Apply Rule 1 Blocks in series

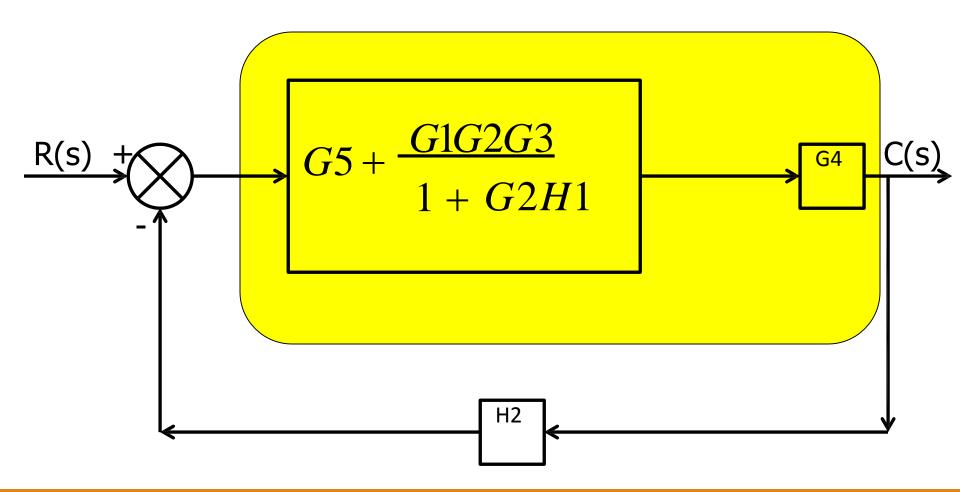


cont....

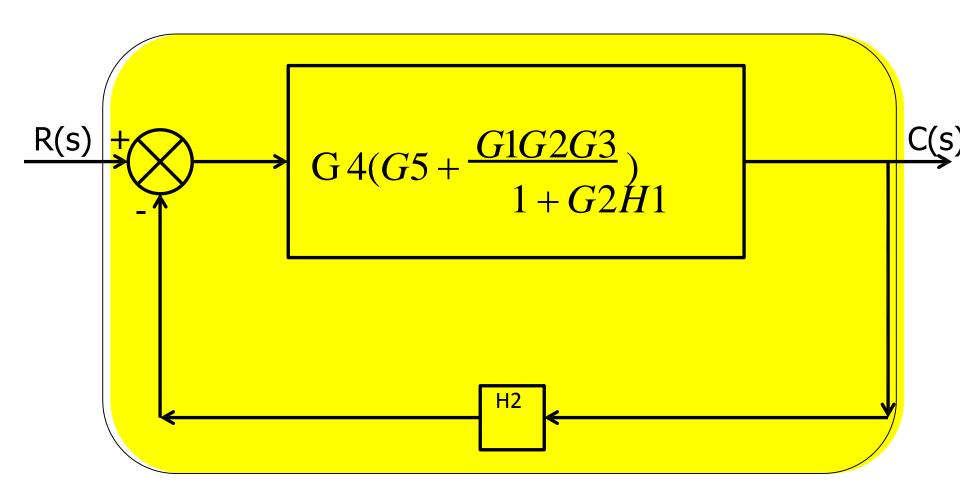
Apply Rule 2 Blocks in parallel



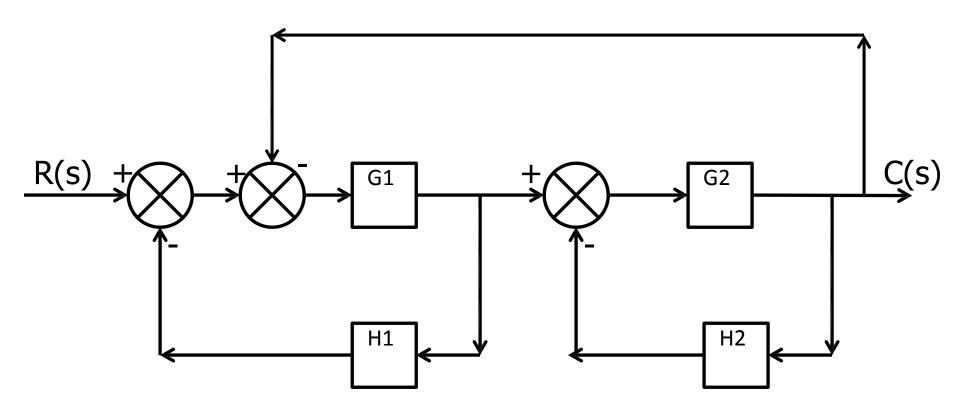
Apply Rule 1 Blocks in series



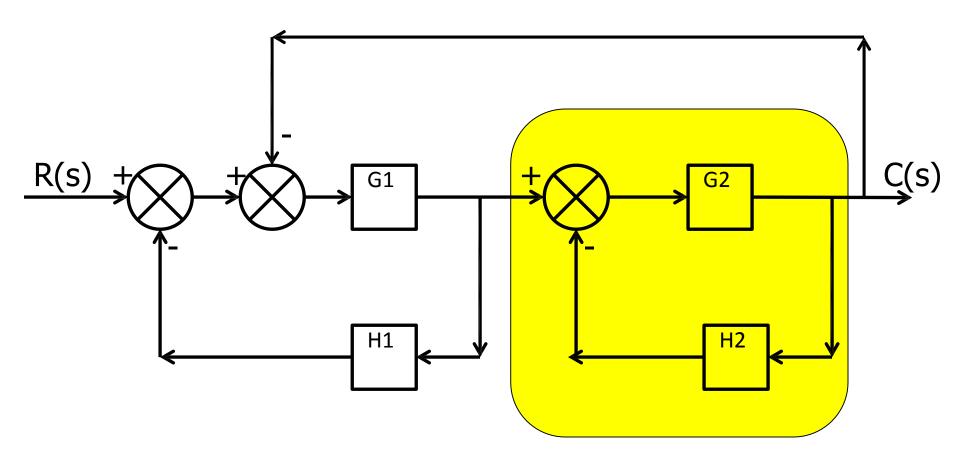
Apply Rule 3 Elimination of feedback loop

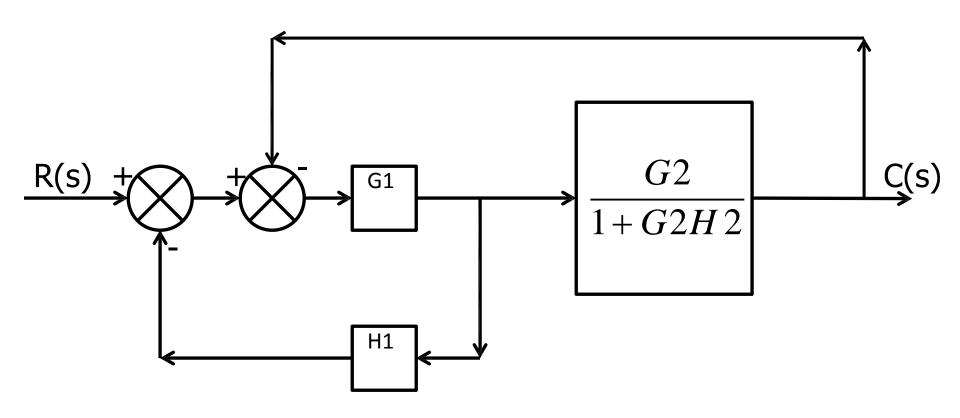


R(s)
$$\frac{G4G5 + G2G4G5H1 + G1G2G3G4}{1 + G2H1 + G4G5H2 + G2G4G5H1H2 + G1G2G3G4} C(s)$$



Apply Rule 3 Elimination of feedback loop





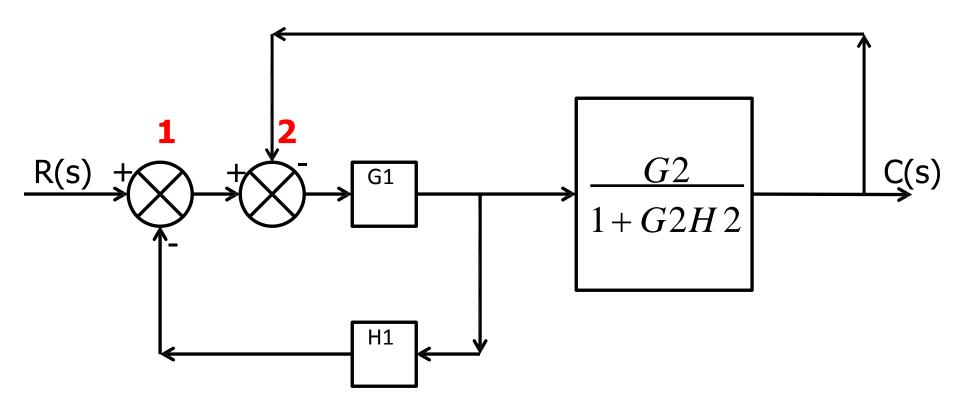
- ➤ Now Rule 1, 2 or 3 cannot be used directly.
- There are possible ways of going ahead.
 - a. Use Rule 4 & interchange order of summing so that Rule 3 can be used on G.H1 loop.
 - b. Shift take off point after $\frac{G^2}{1+G^2H^2}$ block reduce by Rule 1, followed by Rule 3.

Which option we have to use????

cont....

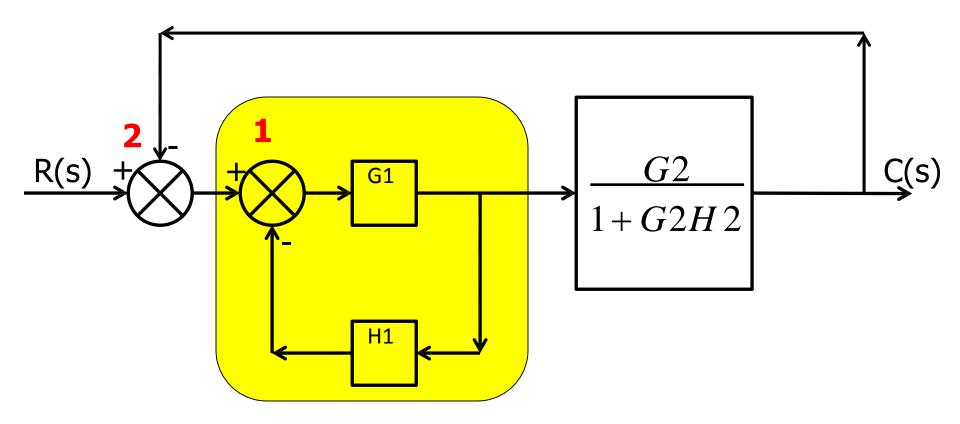
Apply Rule 4

Exchange summing order



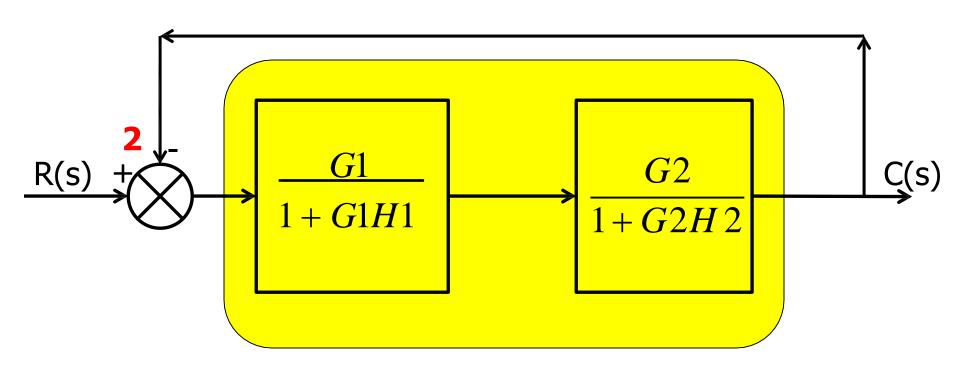
cont....

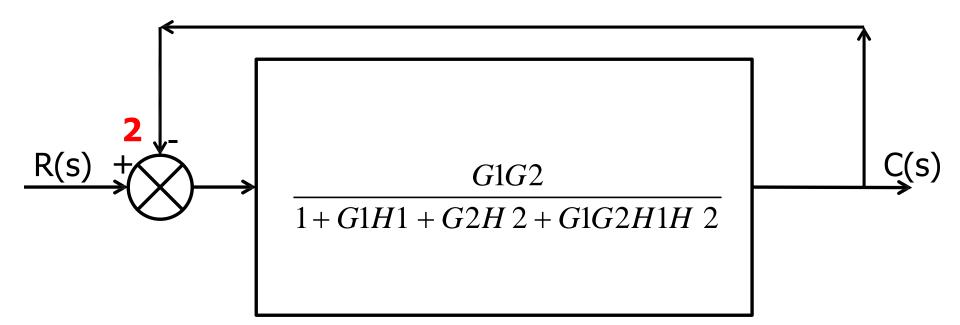
Apply Rule 3 Elimination feedback loop



cont....

Apply Rule 1 Bocks in series

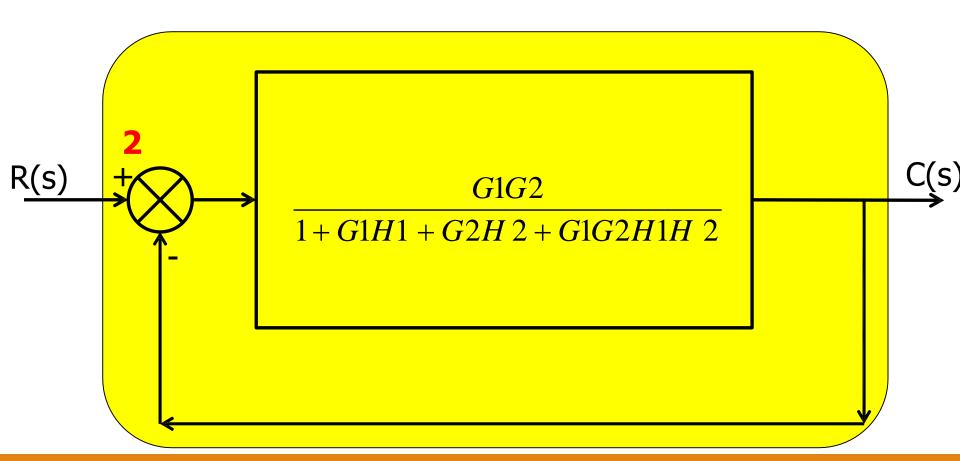


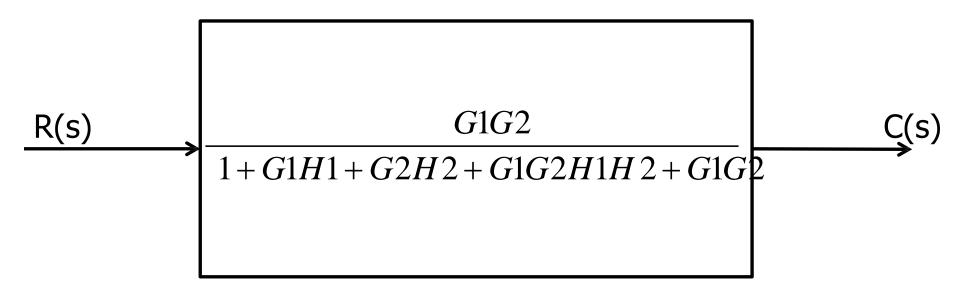


Now which Rule will be applied
-----It is blocks in parallel
-----It is feed back loop

cont....

Let us rearrange the block diagram to understand Apply Rule 3 Elimination of feed back loop

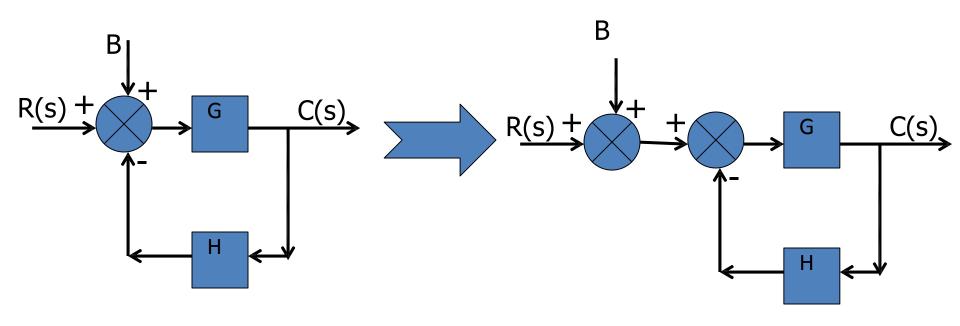


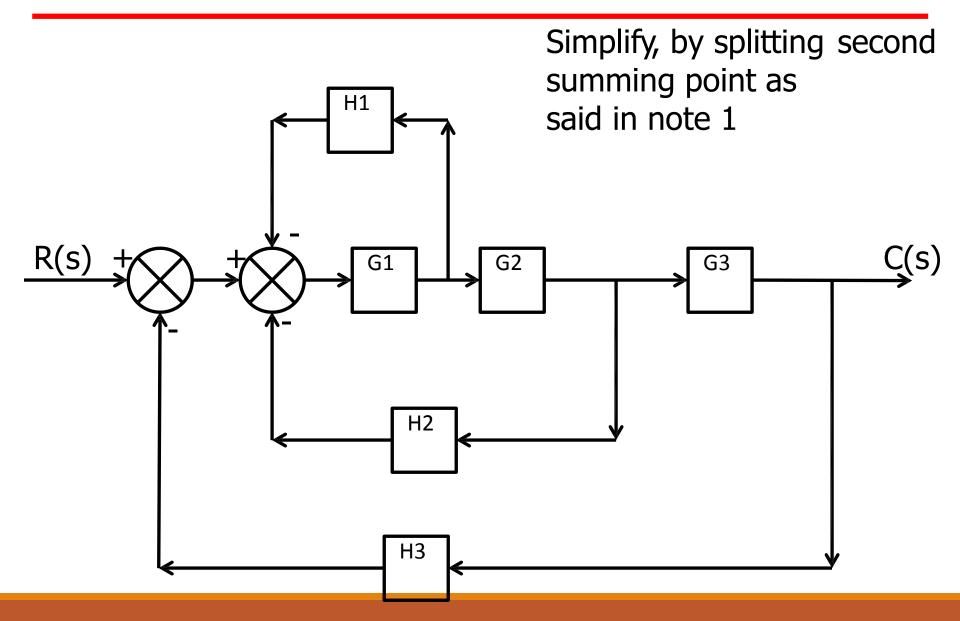


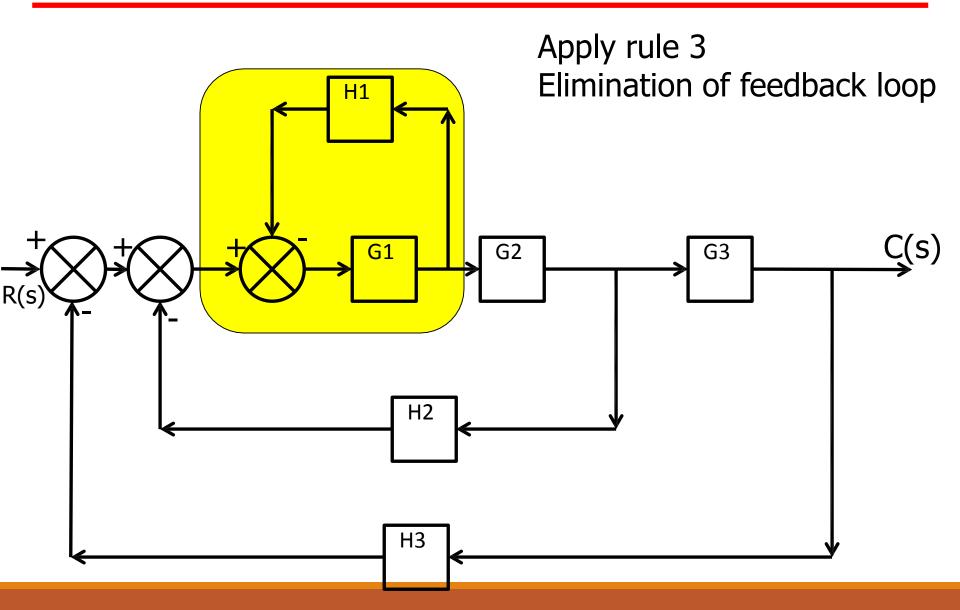
$$\frac{C(s)}{R(s)} = \frac{G1G2}{R(s) 1 + G1H1 + G2H2 + G1G2H1H2 + G1G2}$$

Note 1: According to Rule 4

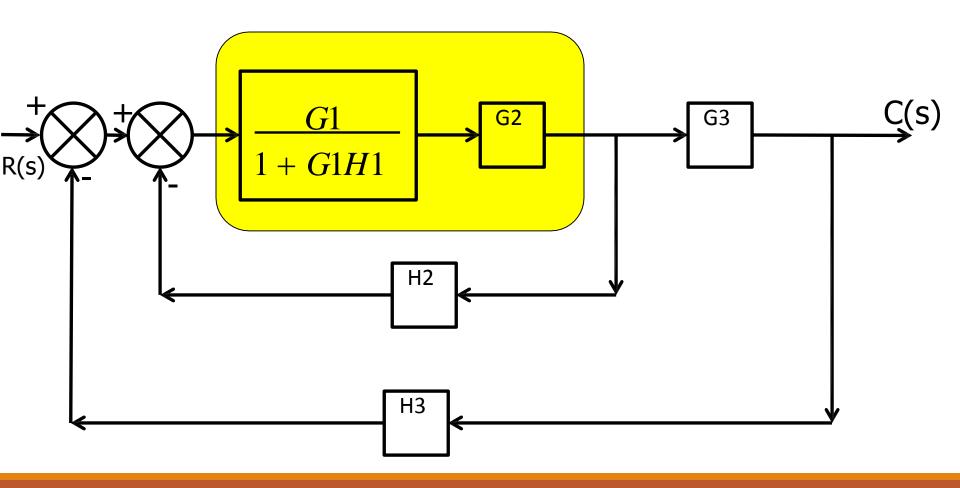
➤ By corollary, one can split a summing point to two summing point and sum in any order



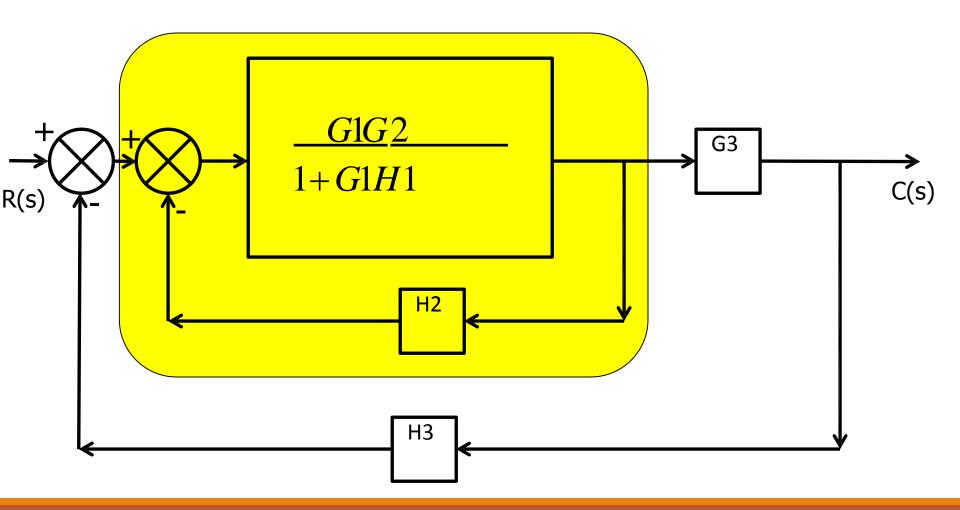




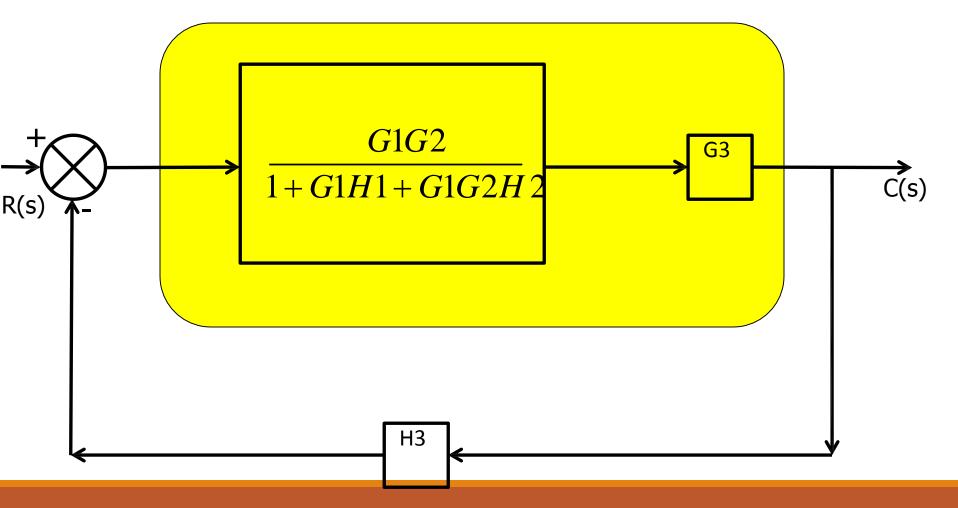
Apply rule 1 Blocks in series



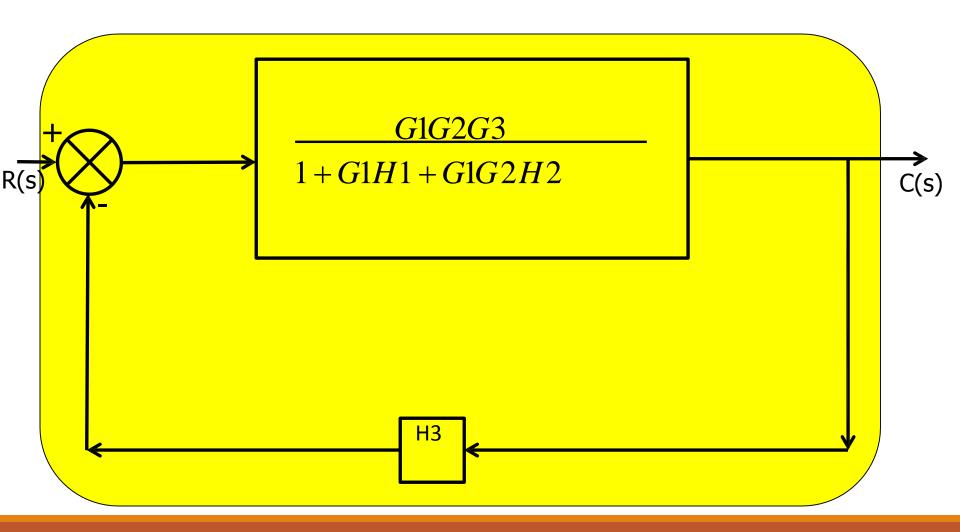
Elimination of feedback loop

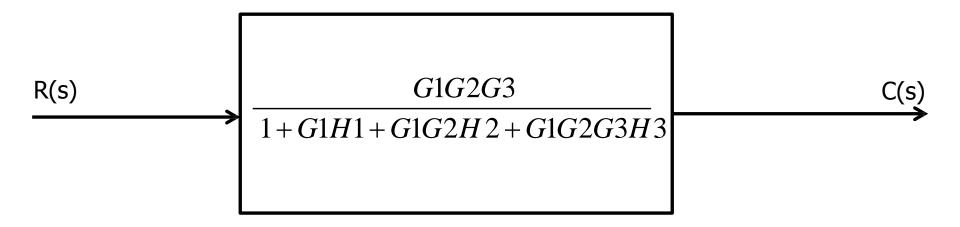


Blocks in series

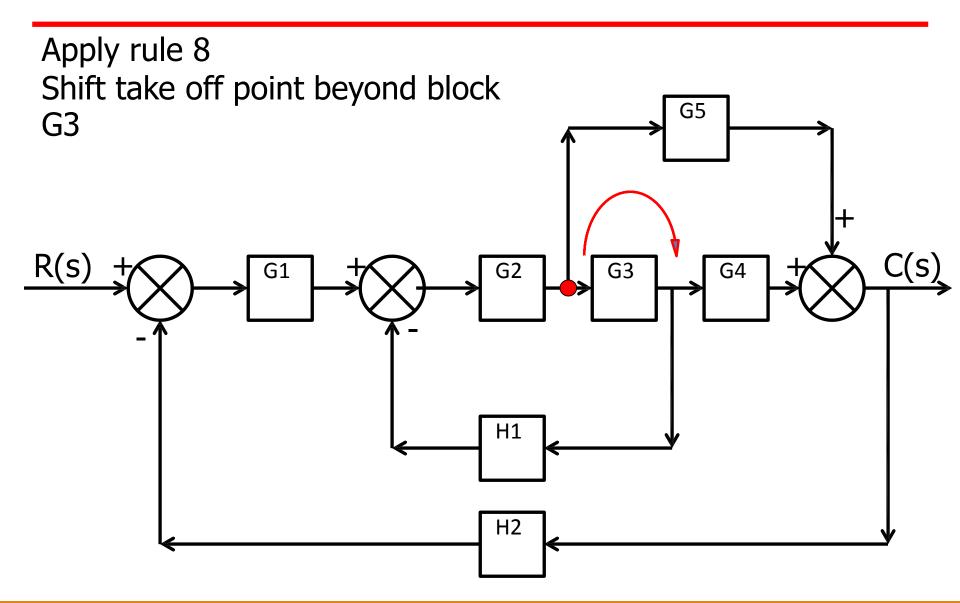


Elimination of feedback loop

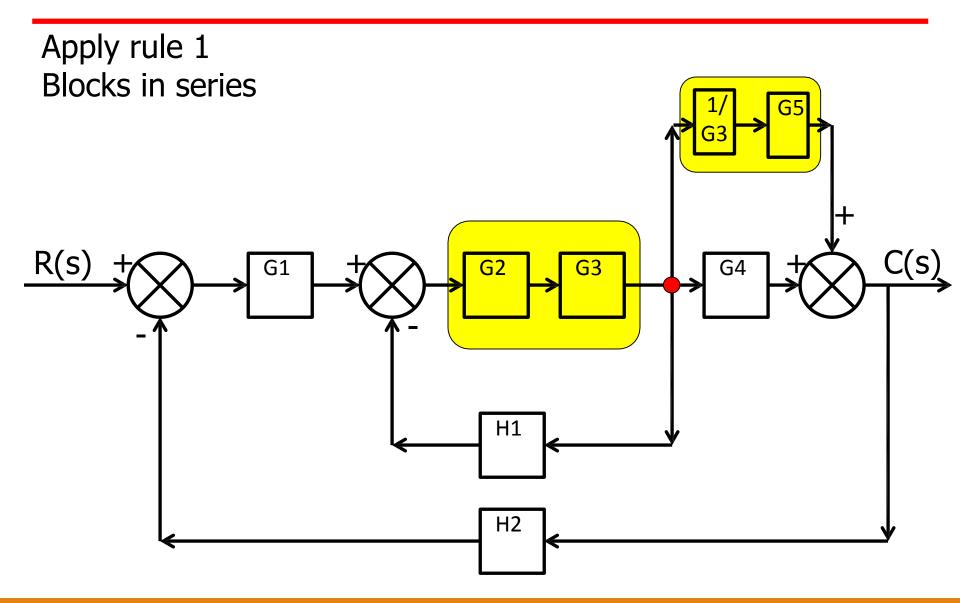




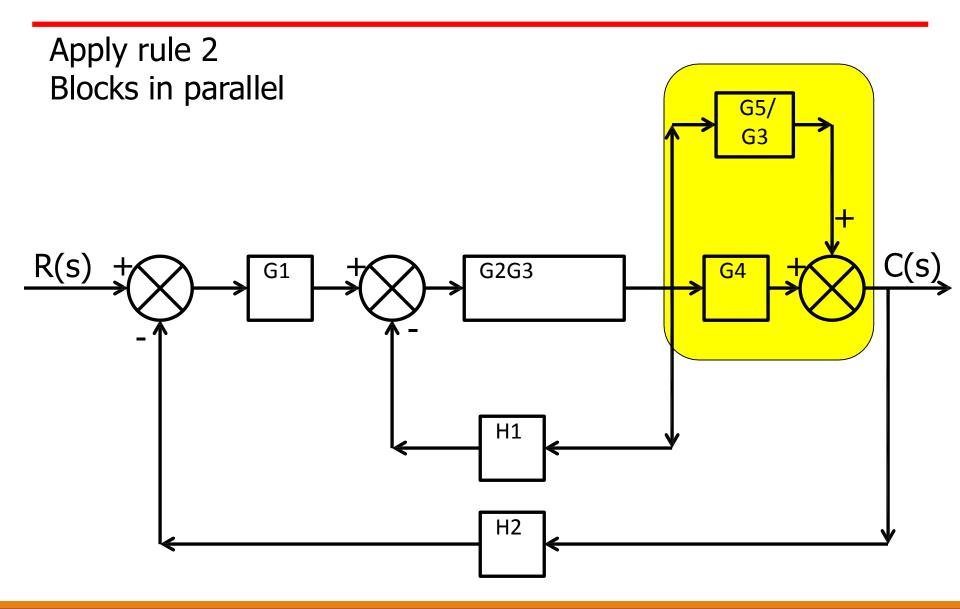
$$\frac{C(s)}{R(s)} = \frac{G1G2G3}{1 + G1H1 + G1G2H 2 + G1G2G3H 3}$$



cont....

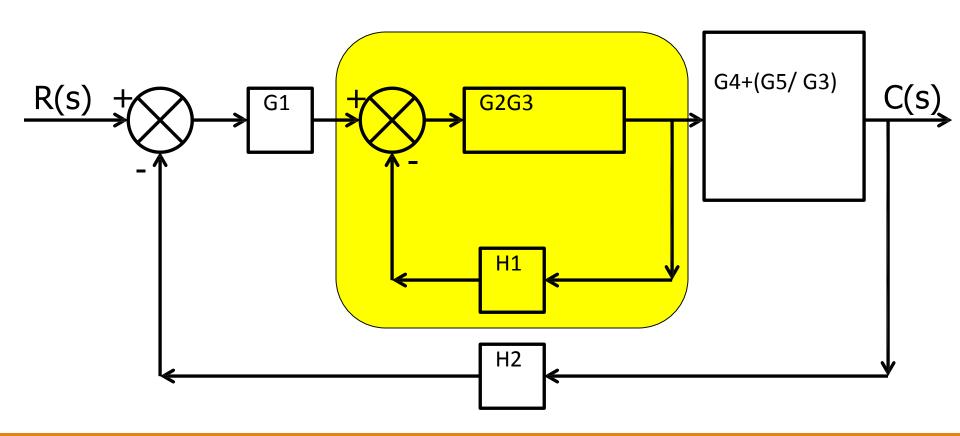






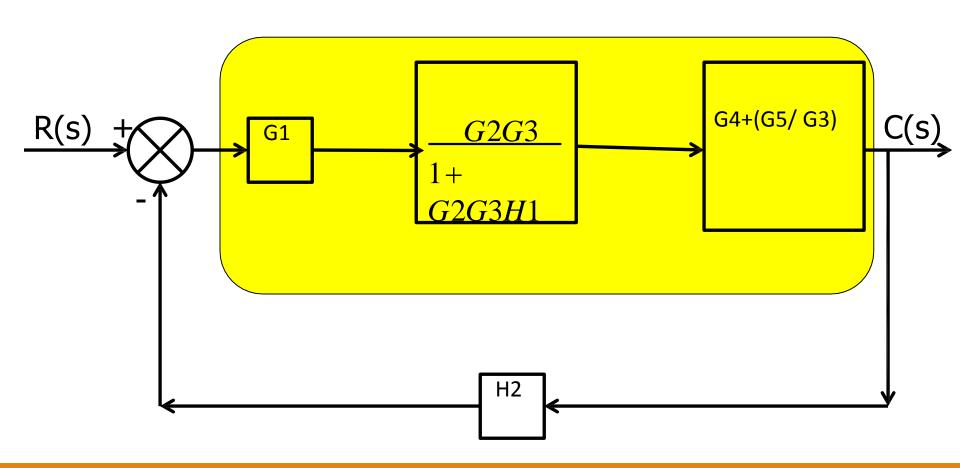
cont....

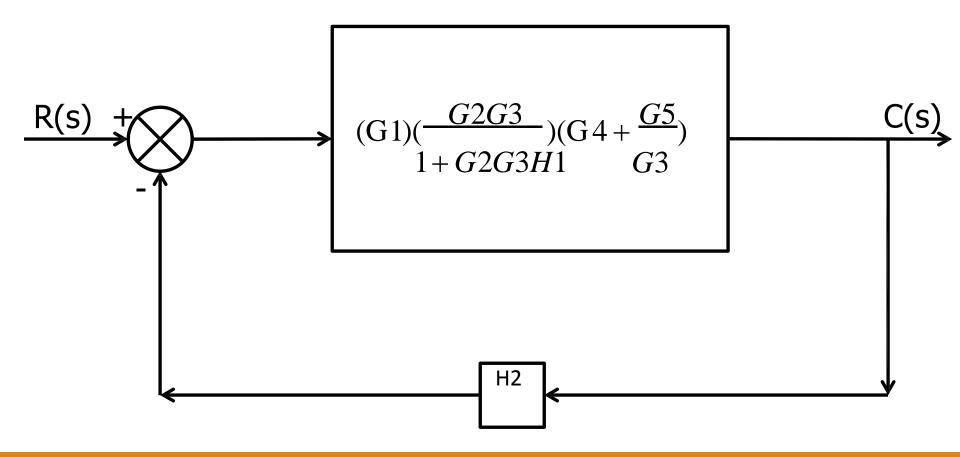
Apply rule 3 Feedback loop



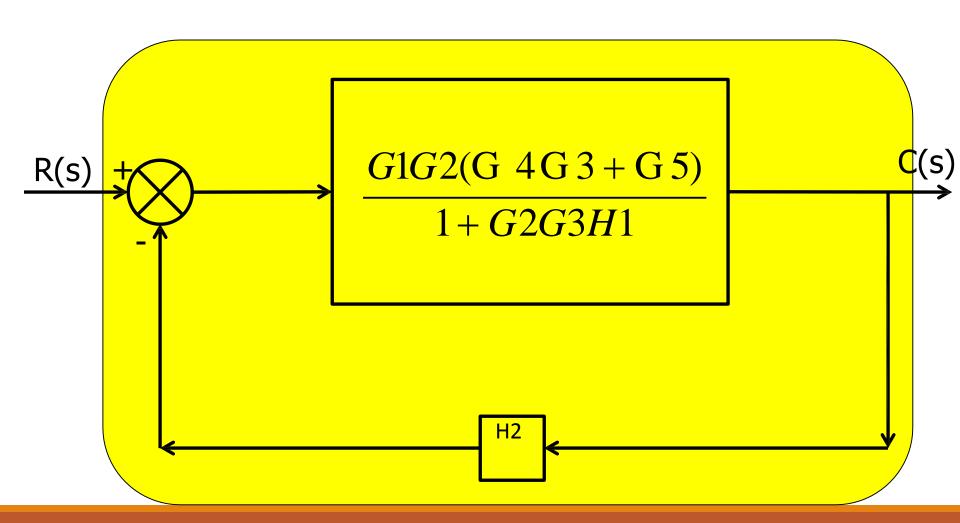
cont....

Apply rule 1 Blocks in series





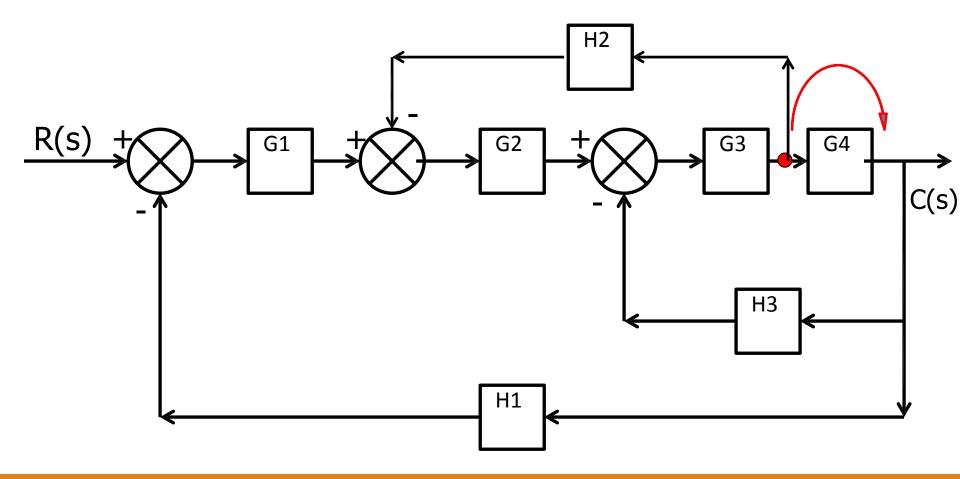
Feedback loop



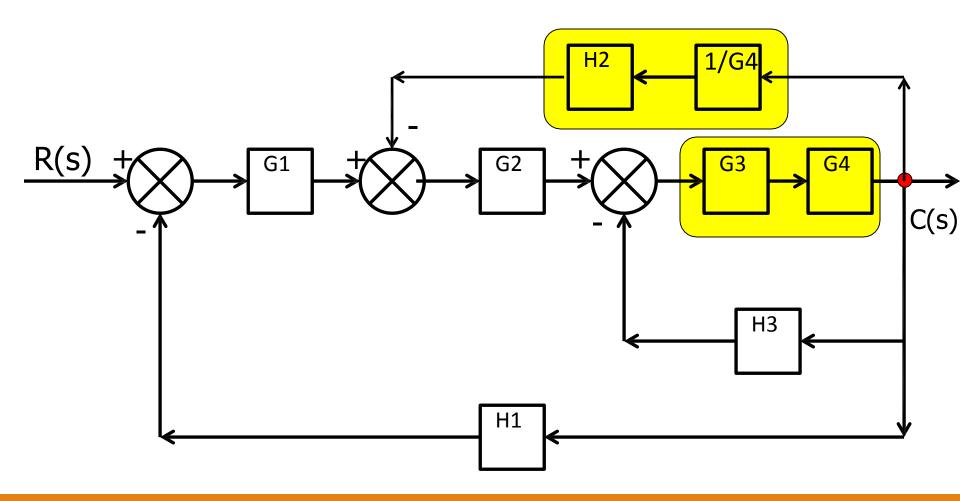
R(s)
$$G1G2(G 4G 3 + G 5)$$
 $C(s)$ $1 + G2G3H1 + G1G2H 2(G 3G 4 + G 5)$

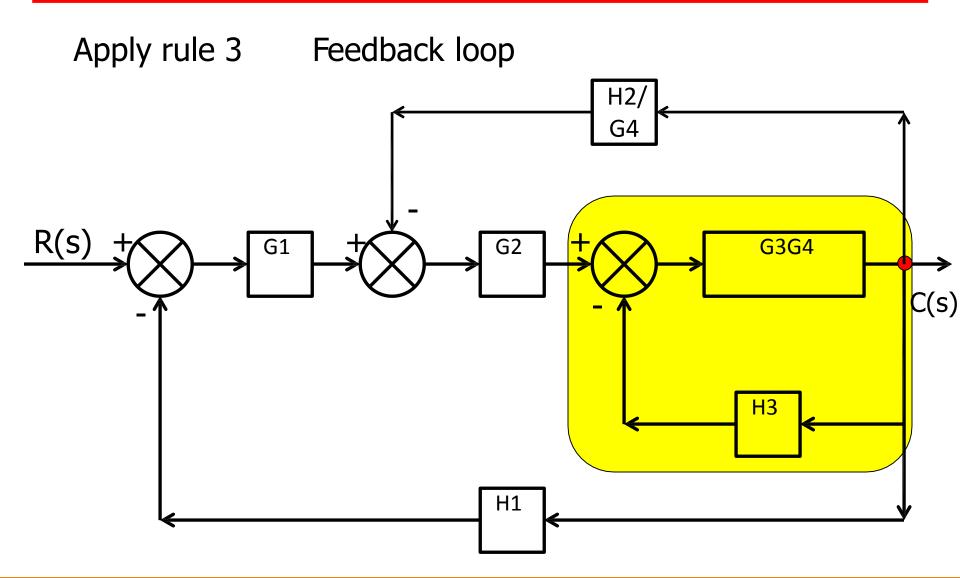
$$\frac{C(S)}{R(S)} = \frac{G1G2(G4G3+G5)}{1+G2G3H1+G1G2H2(G3G4+G5)}$$

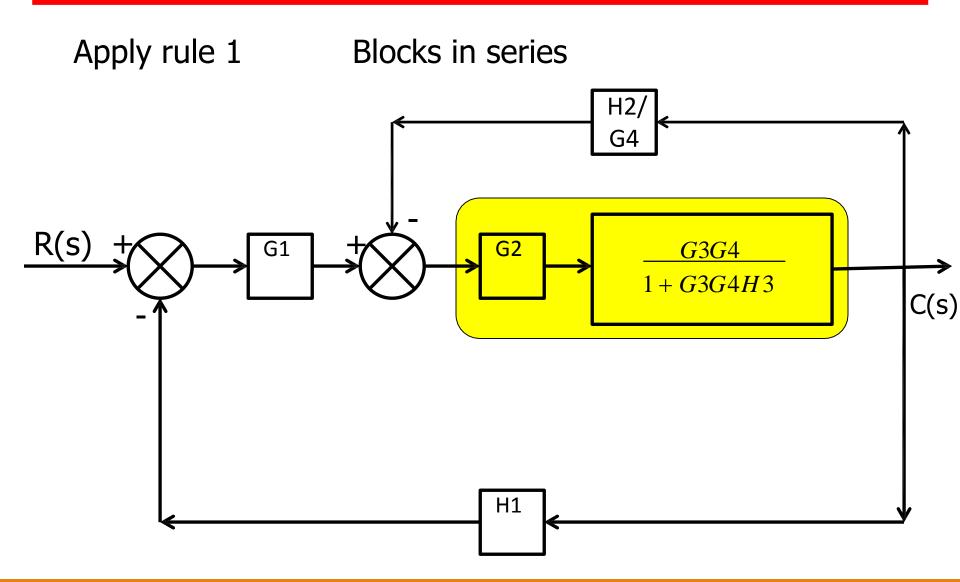
Apply rule 8 Shift take off point after block G4



Apply rule 1 Blocks in series

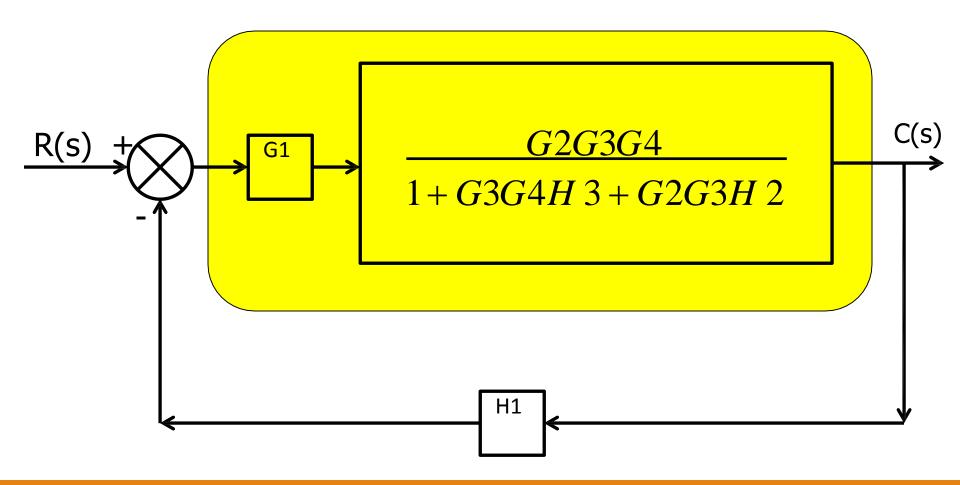




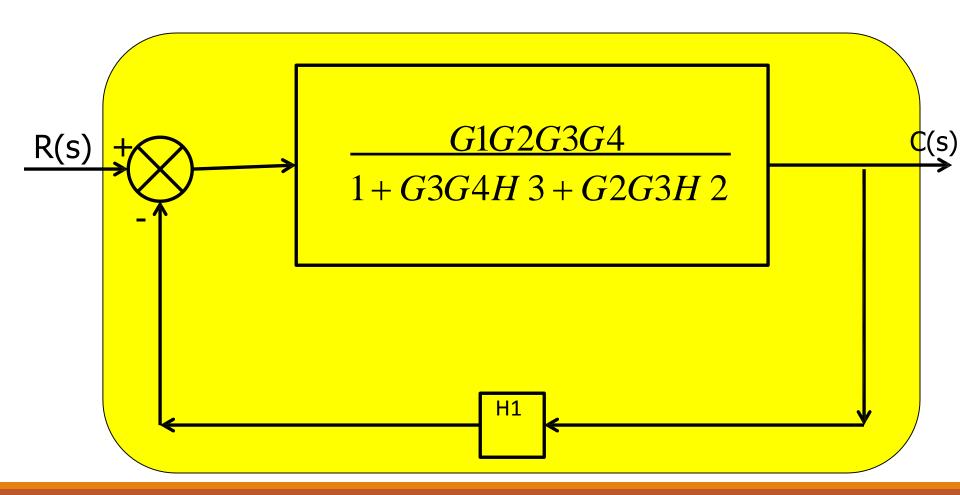


Feedback loop Apply rule 3 G1 *G2G3G4* 1 + G3G4H3**C**(s) H1

Blocks in series



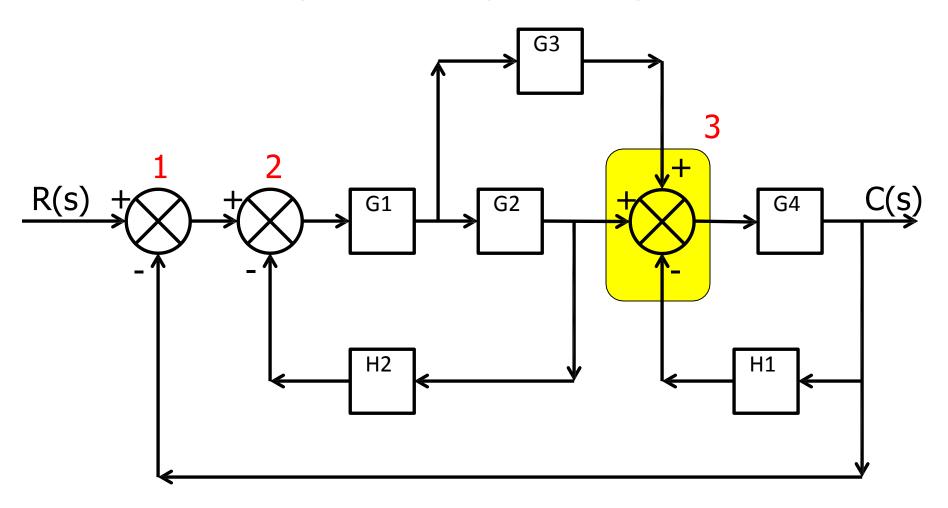
Feedback loop



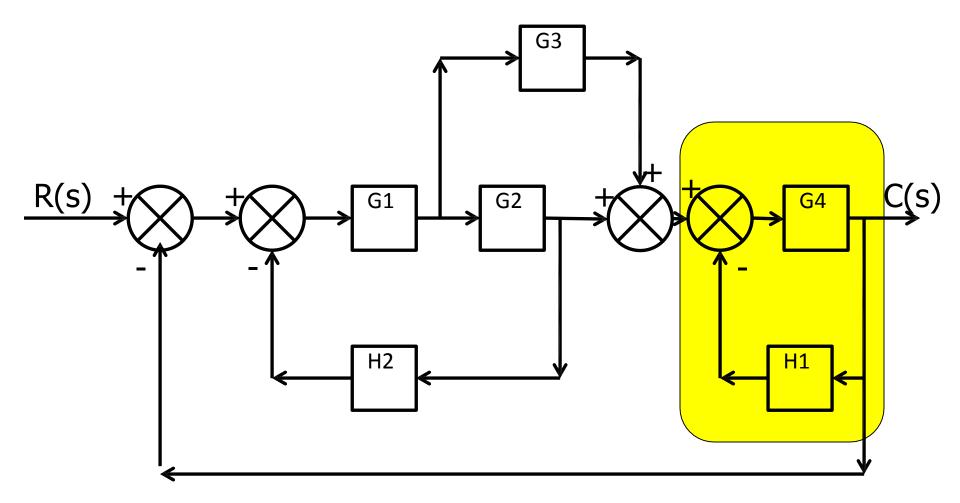
R(s)
$$G1G2G3G4$$
 C(s) $1+G3G4H3+G2G3H2+G1G2G3G4H1$

$$\frac{C(S)}{R(S)} = \frac{G1G2G3G4}{1 + G3G4H3 + G2G3H2 + G1G2G3G4H1}$$

Simplify, by splitting 3rd summing point as given in Note 1



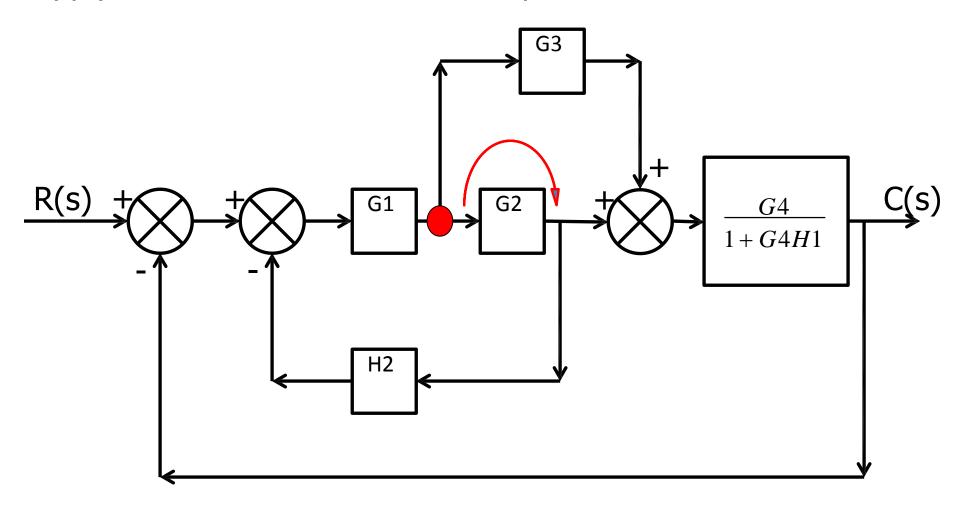
Elimination of Feedback loop



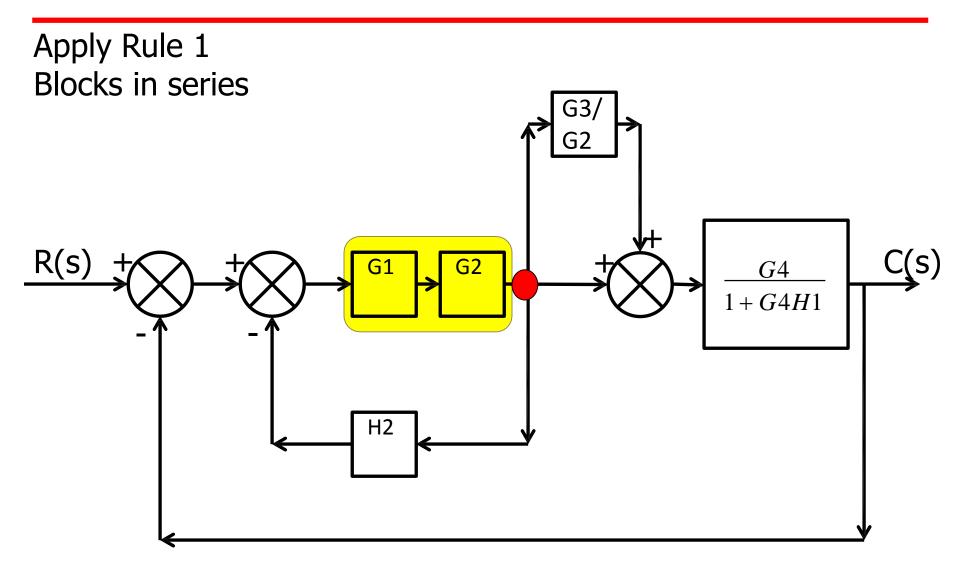
cont....

Apply Rule 8

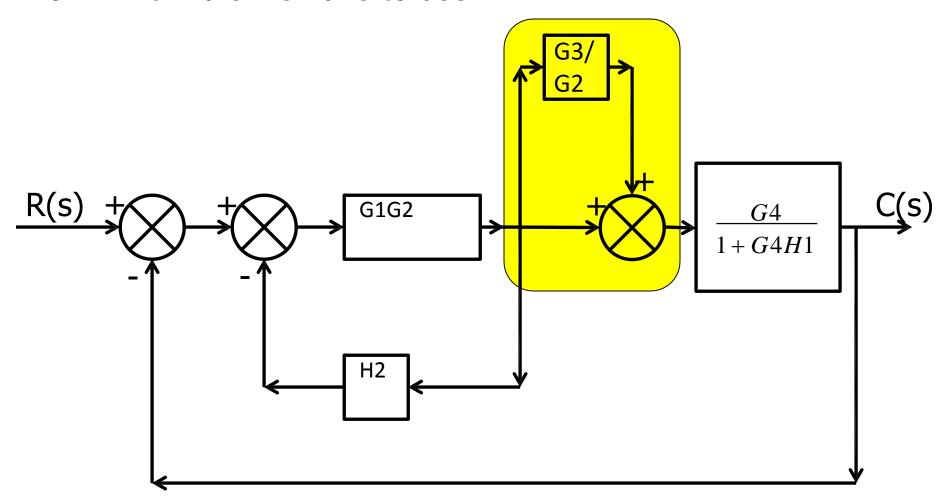
Shift take off point after block



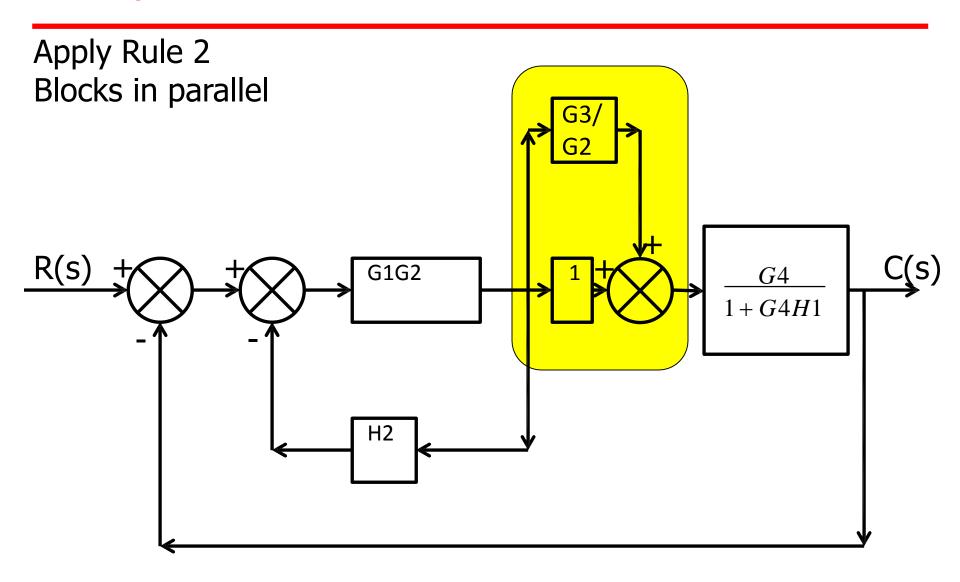




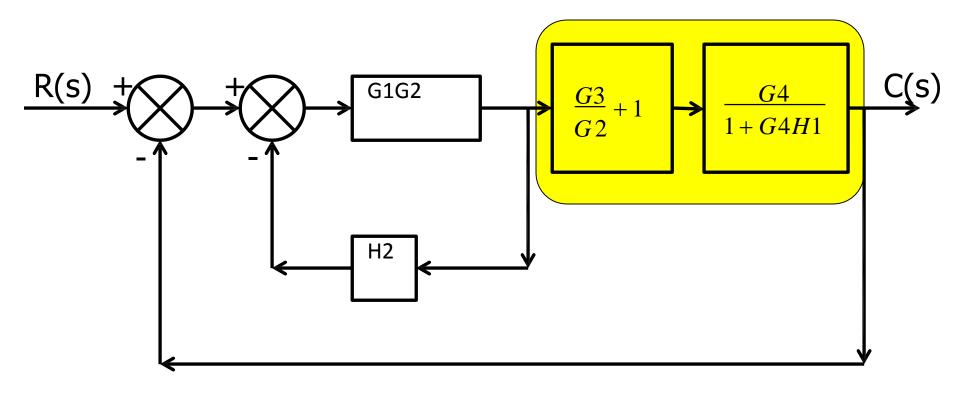
Now which rule we have to use?



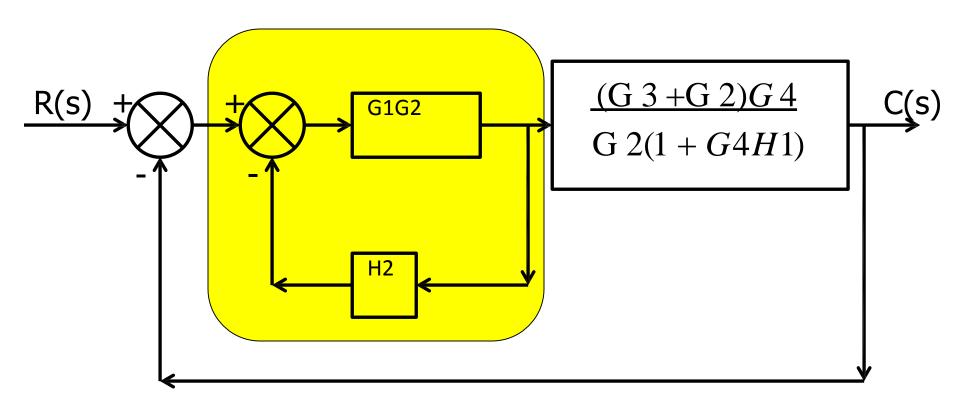




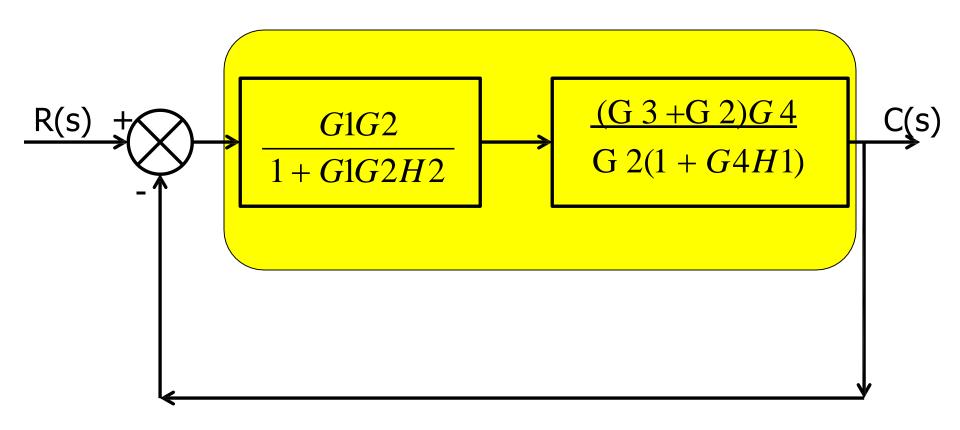
Blocks in series



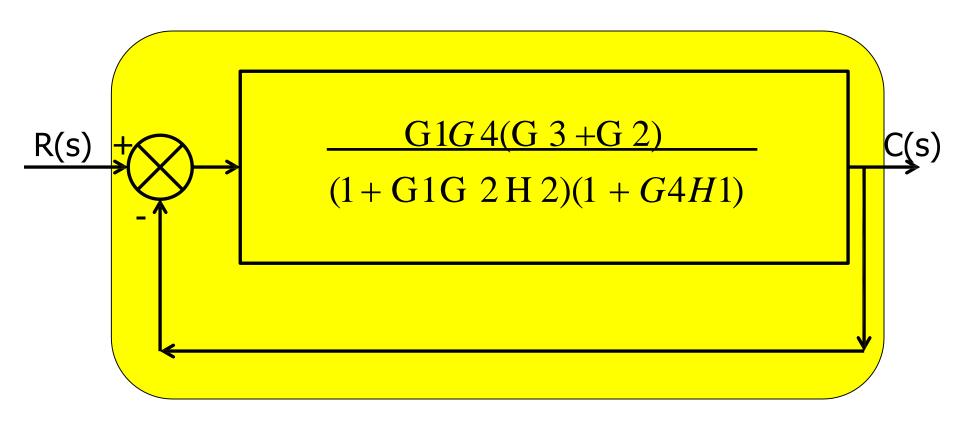
Elimination of Feedback Loop



Blocks in series



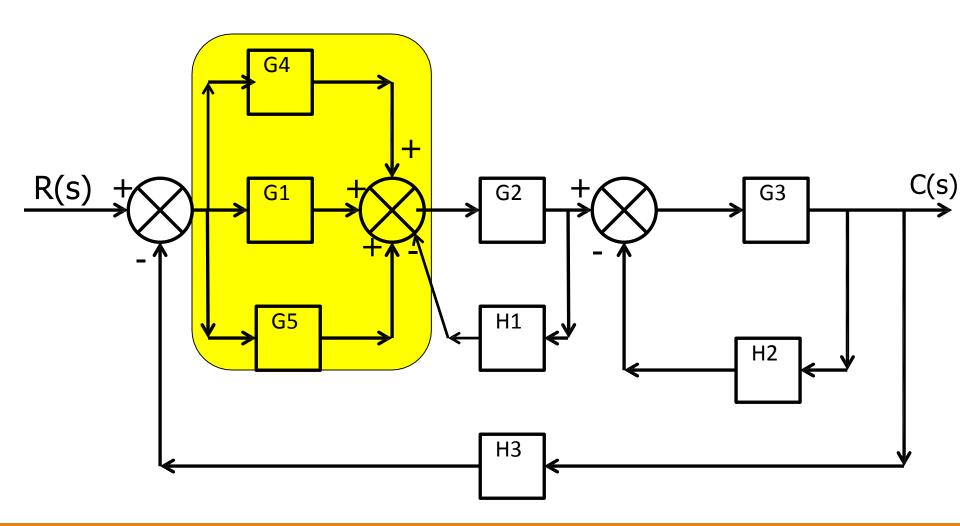
Elimination of Feedback loop



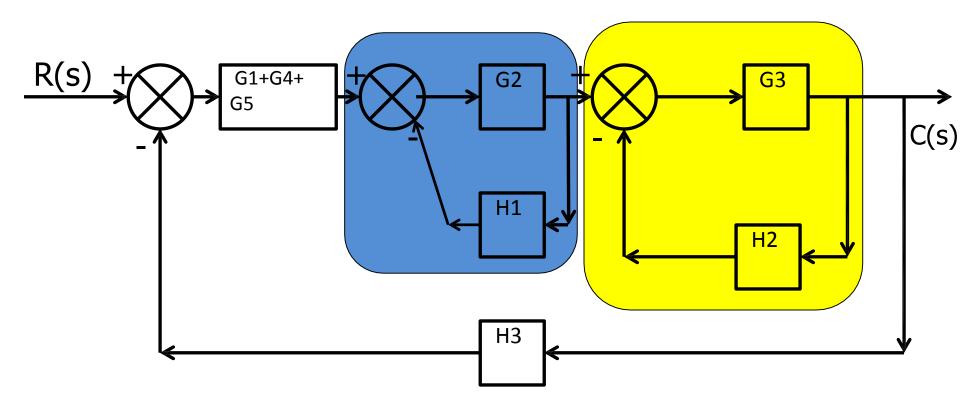
R(s)
$$G1G4(G3+G2)$$
 C(s) $+G4H1+G1G2H2+G1G2G4H1H2+G1G4(G2+G3)$

Example 9

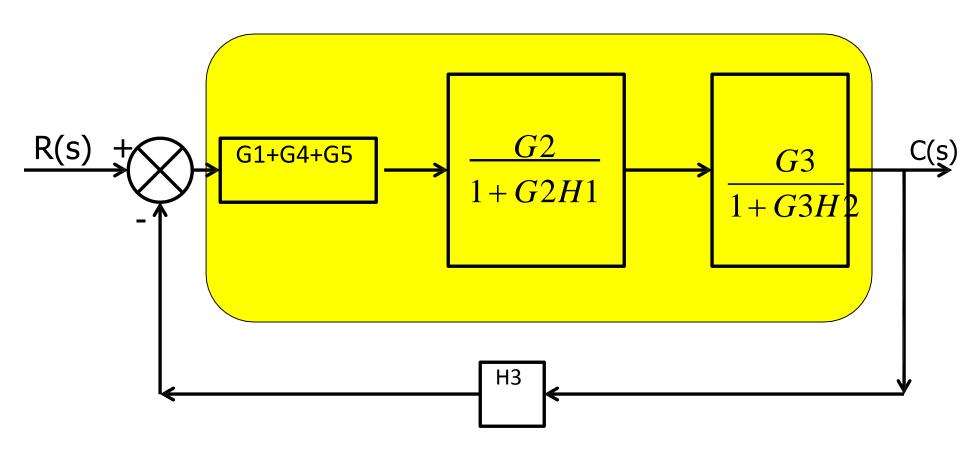
Apply rule 2 Blocks in Parallel



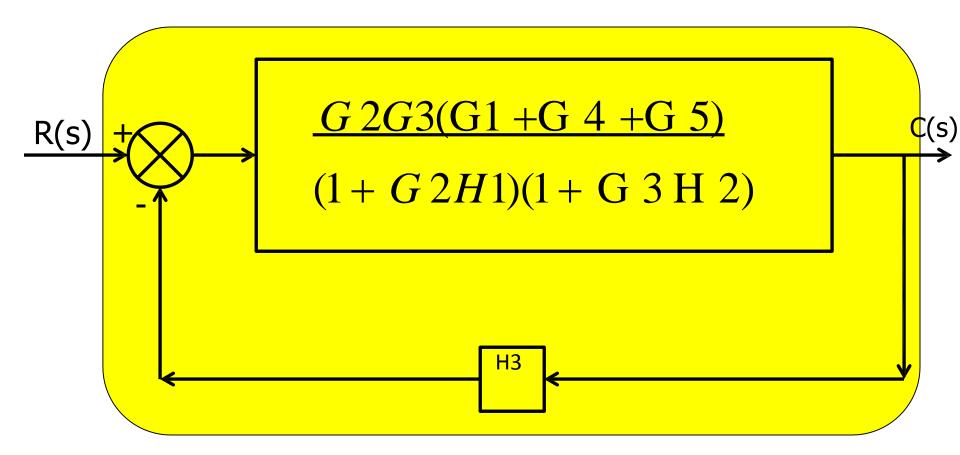
Apply rule 3 Elimination of Feedback Loop



Apply rule 1 Blocks in Series



Apply rule 3 Elimination of Feedback loop

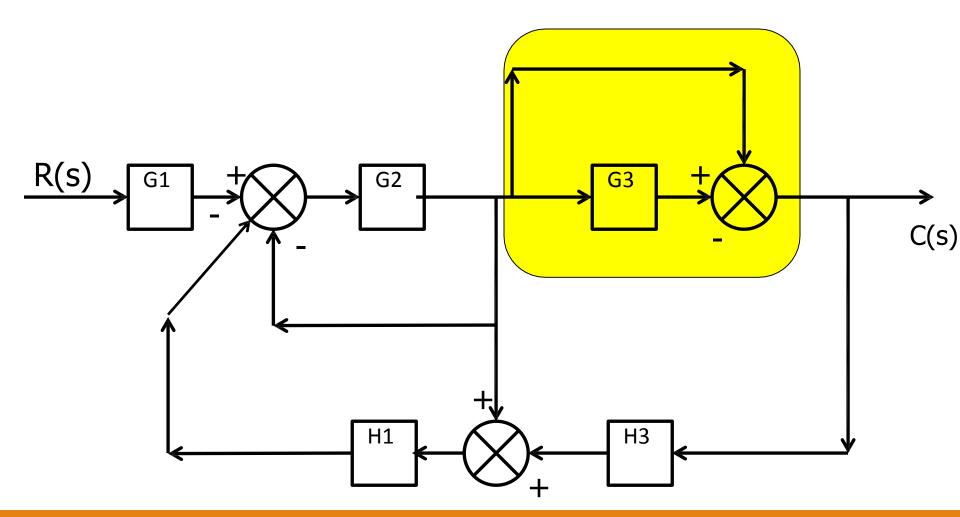


R(s)
$$G2G3(G1+G4+G5) \qquad C(s)$$

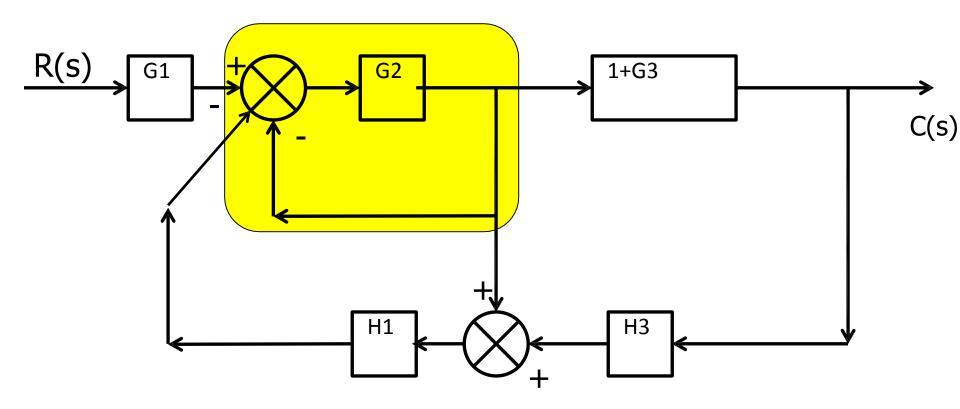
$$1+G2H1+G3H2+G2G3H1H2+G2G3H3(G1+G4+G5)$$

Example 10

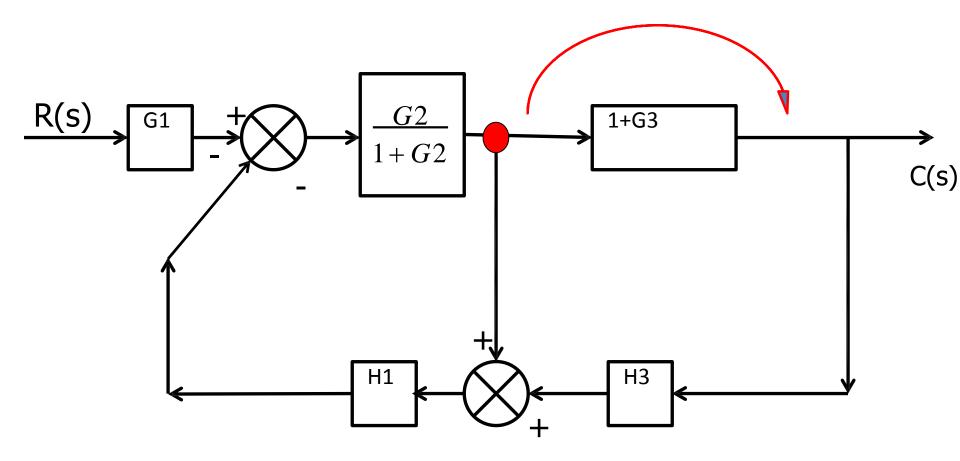
Apply rule 2 Blocks in Parallel



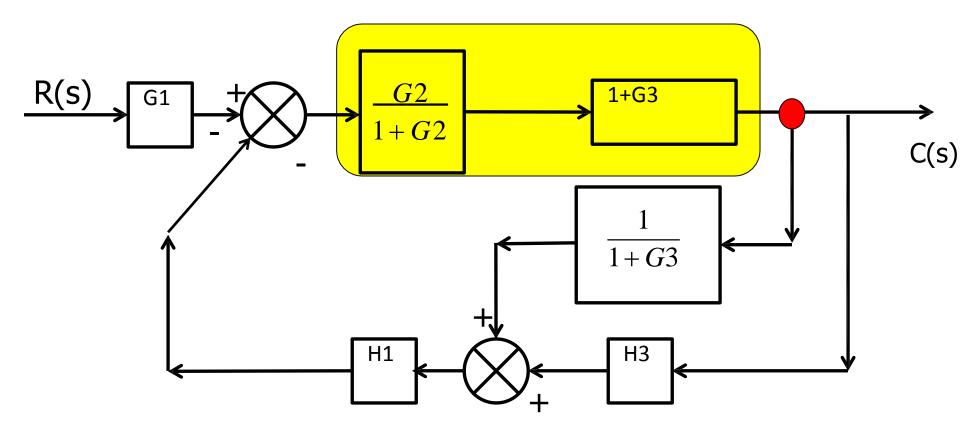
Elimination of Feedback Loop



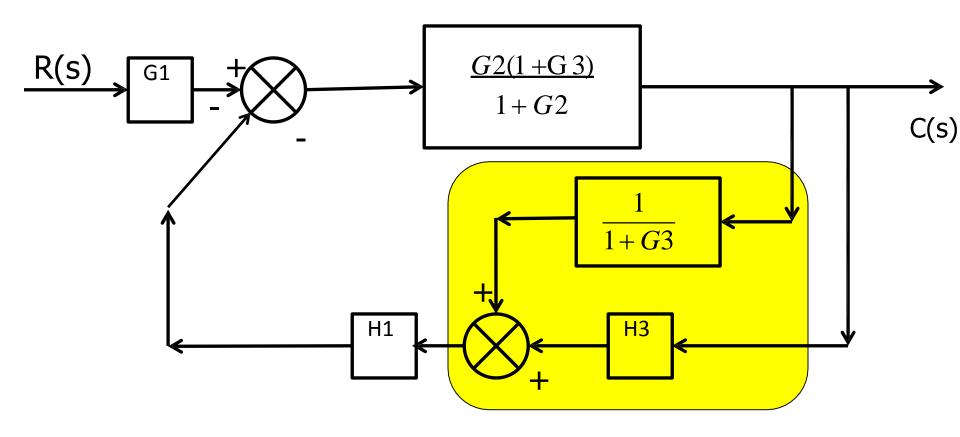
Shift take off point after block



Blocks in series



Blocks in Parallel

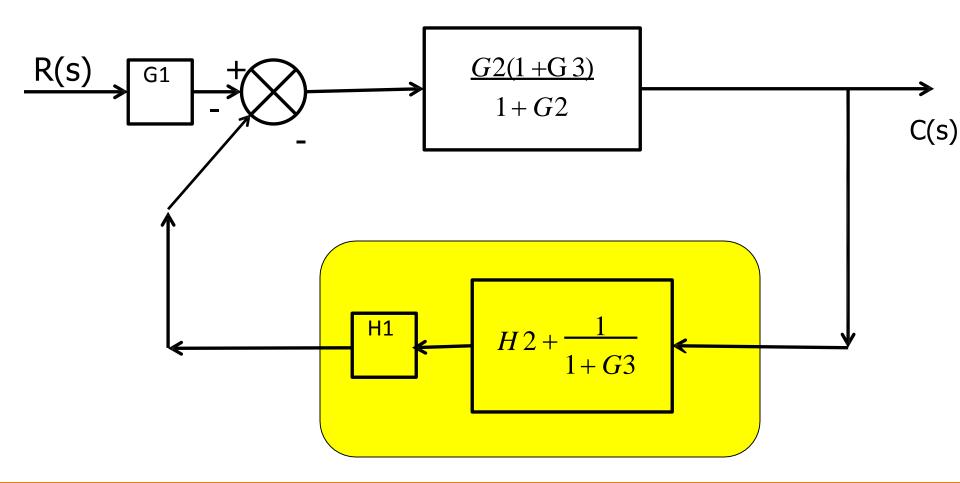


Example 10

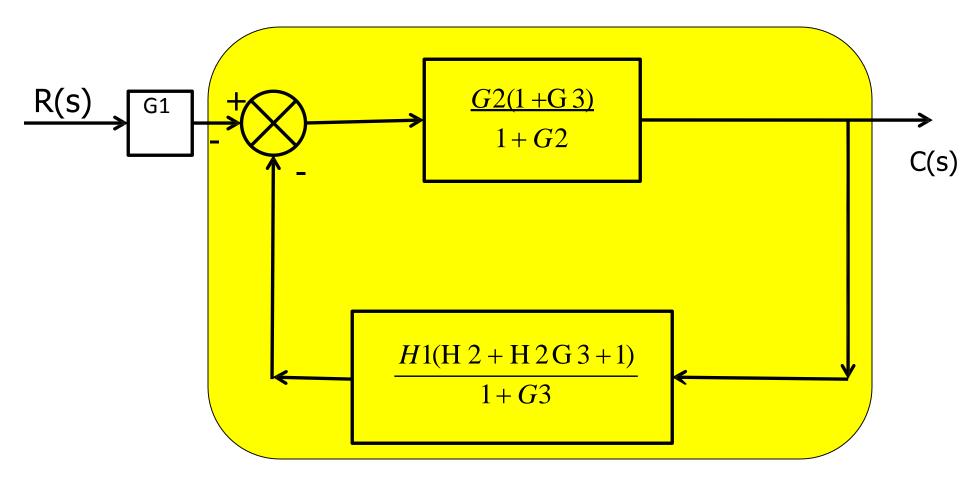
cont....

Apply rule 1

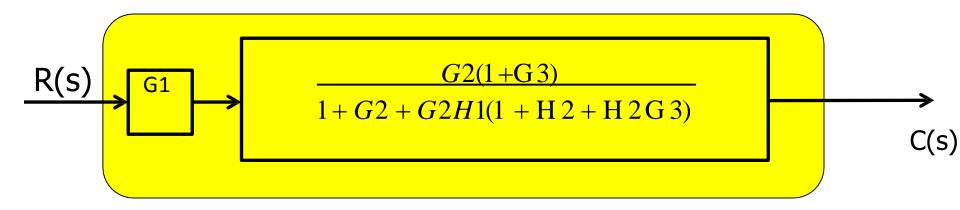
Blocks in Series



Elimination of Feedback loop



Blocks in series



R(s)
$$\frac{G1G2(1+G3)}{1+G2+G2H1(1+H2+H2G3)}$$
 C(s)

$$\frac{C(s)}{R(s)} = \frac{G1G2(1+G3)}{1+G2+G2H1(1+H2+H2G3)}$$