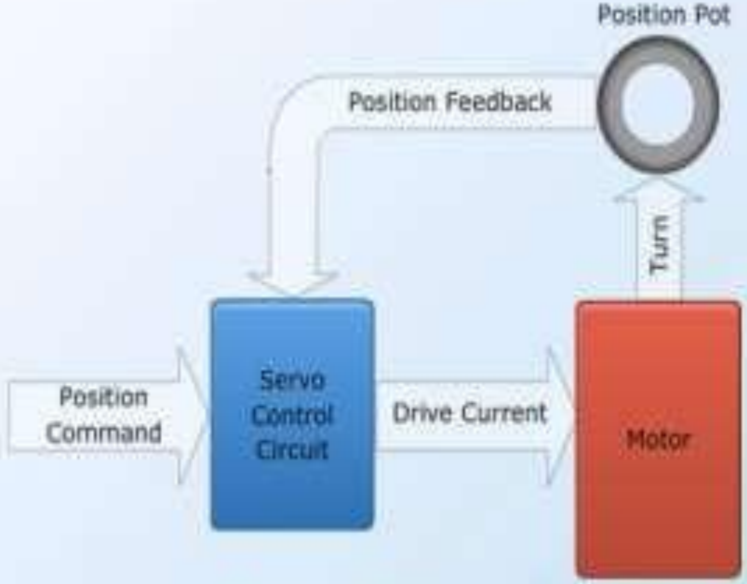
BLOCK DIAGRAM METHOD	SFG METHOD
1. It is a pictorial representation of function performed by each component and its flow of signal.	1. It is a graphical representation of a relationship between variable of a set of linear algebraic equations written in the form of cause and effect relationship.
2. It can be used to represent linear as well as non-linear systems.	2. It can be used to represent only linear system.
3. No direct formula is available to find the overall transfer function of the system.	3. Mason gain formula is used to find overall transfer function.
4. Step by step procedure is followed to find the transfer function.	4. Transfer function can be obtained in one step.
5. It is not a systematic method.	5. It is a systematic method.
6. It indicated more realistically the signal flows of the system than the original system itself	6. It is constrained by more rigid mathematical rules than a block diagram.
7. For a system block diagram is not unique.	7. SFG is not unique.

Servo Motors

- A servo motor is a d.c. or a.c. or any other motor combined with a position sensing device
- The servo motor has some control circuits and a potentiometer that is connected to the output shaft
- Requirements for Good Servo Motor
- Servo motors are of three types
  - D.C. Servo Motor
  - A.C. Servo Motor
  - Special Servo Motor



Servo Motors

When the objective of the system is to control the position of a object then the system is called SERVOMECHANISM.

Features of Servomotor:

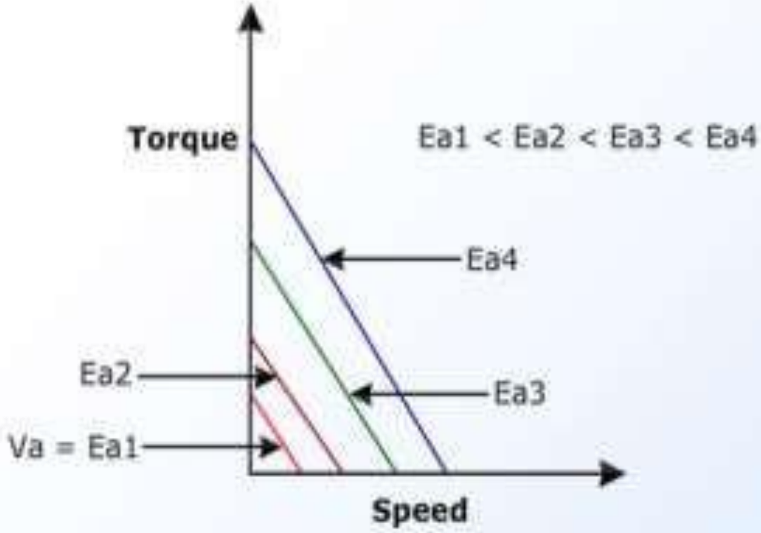
1. Linear relationship between the speed and electric control signal.
2. Steady State Stability.
3. Wide range of speed control.
4. Linearity of mechanical characteristics throughout the entire speed range.
5. Low mechanical and electrical inertia.
6. Fast response

DC SERVOMOTOR	AC SERVOMOTOR
Advantages:	Advantages:
1. Higher output than from 50 Hz motor of same size.	1. Lower Cost
2. Linearity of characteristics.	2. Higher Efficiency
3. Easier Speed control form zero speed to full speed in both directions.	3. Less maintenance since due to absence of commutator and brushes.
4. High Torque to inertia ratio which give quick response to control signals.	4. Delivers 2 to 2.5 times their rated torque.
5. Low weight, Low Inertia and Low Armature inductance.	
6.Low electrical and mechanical time constants.	
7. Delivers 3 times their rated torque for Short time.	

DC SERVOMOTOR	AC SERVOMOTOR
Disadvantages:	Disadvantages:
1. They are very expensive.	1. Characteristics are non-linear.
	2. Difficult to control
Applications:	Applications:
Used for Large Power Applications Eg: Machine Tools and Robotics	Used for Low power Applications Eg: Servo Instruments (X-Y Recorder) Computer related Applications (Tape drives, Printers etc)

D.C. Servomotor

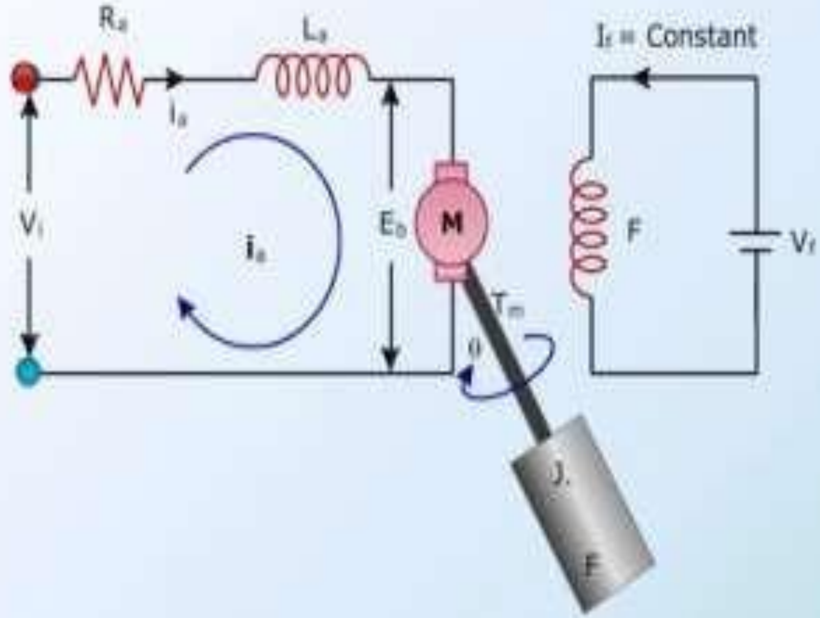
Characteristics of D.C. Servomotor



Characteristics

Transfer Function of an Armature Controlled D.C. Motor

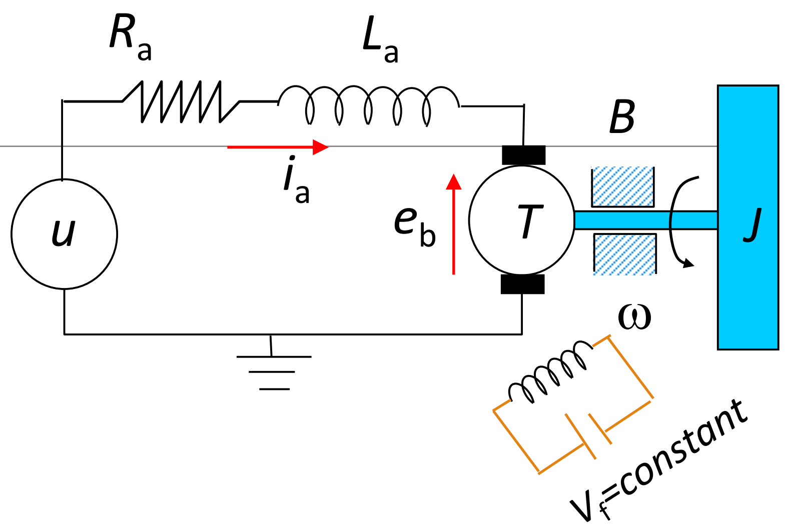
- R_a : Resistance of armature in ohms
- L_a : Inductance of armature in Henries
- i_a : Armature current. and 'If' Field current
- V_i : Applied armature voltage
- E_b : Back e.m.f. in volts
- T_m : Torque developed by the motor
- θ : Angular displacement
- J : Equivalent moment of inertia
- F : Equivalent Viscous friction



Armature Controlled D.C Motor

Armature Controlled D.C Motor

Input: voltage *u*
Output: Angular velocity ω



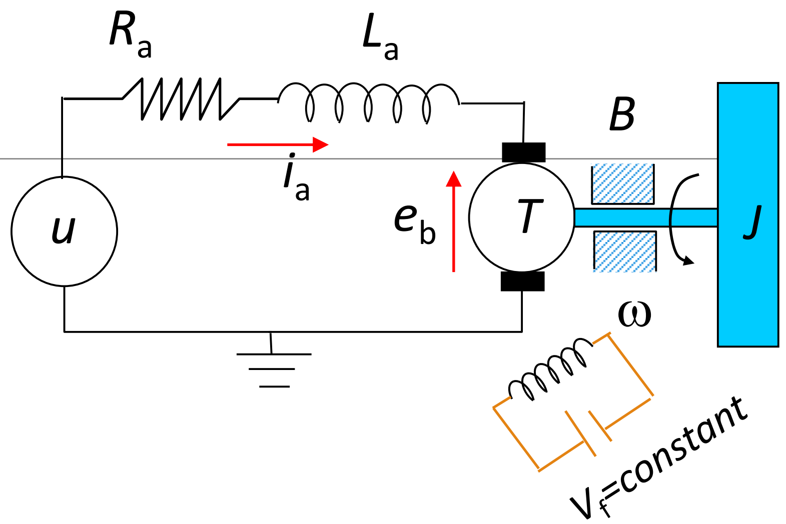
Electrical Subsystem (loop method):

$$u = R_a i_a + L_a \frac{di_a}{dt} + e_b, \qquad where \quad e_b = back\text{-}emf\ voltage$$

Mechanical Subsystem

$$T_{motor} = J\dot{\omega} + B\omega$$

Armature Controlled D.C Motor



<u>Power Transformation</u>:

Torque-Current: $T_{motor} = K_t i_a$

Voltage-Speed: $e_b = K_b \omega$

- Combing previous equations results in the following mathematical model:

$$\begin{cases} L_a \dfrac{di_a}{dt} + R_a i_a + K_b \omega = u \\ J\dot{\omega} + B\omega - K_t i_a = 0 \end{cases}$$

Armature Controlled D.C Motor

Taking Laplace transform of the system's differential equations with zero initial conditions gives:

$$\begin{cases} \left(L_a s + R_a\right) I_a(s) + K_b \Omega(s) = U(s) \\ \left(Js + B\right)\Omega(s) - K_t I_a(s) = 0 \end{cases}$$

Eliminating I_a yields the input-output transfer function

$$\frac{\Omega(s)}{U(s)} = \frac{K_t}{L_a J s^2 + \left(JR_a + BL_a\right)s + BR_a + K_t K_b}$$

Armature Controlled D.C Motor

Reduced Order Model

Assuming small inductance, $L_a \approx 0$

$$\frac{\Omega(s)}{U(s)} = \frac{K_t}{JR_a s + (BR_a + K_t K_b)}$$

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + e_b$$

$$T = K_t i_a$$

$$T = J \frac{d\omega}{dt} + B\omega$$

$$e_b = K_b \omega$$

$$\omega = \frac{d\theta}{dt}$$

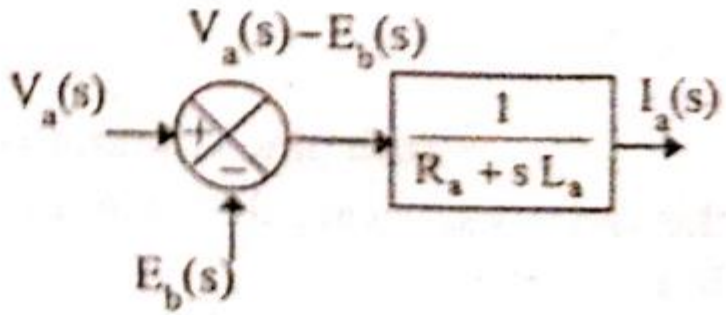
BLOCK DIAGRAM REPRESENTATION

On taking Laplace transform

$$V_a(s) = I_a(s)\,R_a + L_a\,s\,I_a(s) + E_b(s)$$

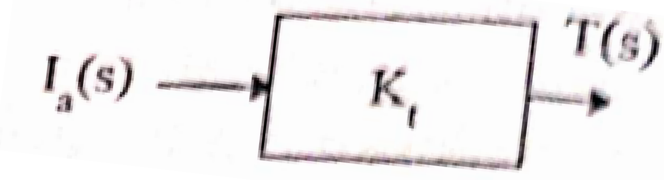
$$V_a(s) - E_b(s) = I_a(s)\left[R_a + s\,L_a\right]$$

$$\therefore I_a(s) = \frac{1}{R_a + s\,L_a}\left[V_a(s) - E_b(s)\right]$$



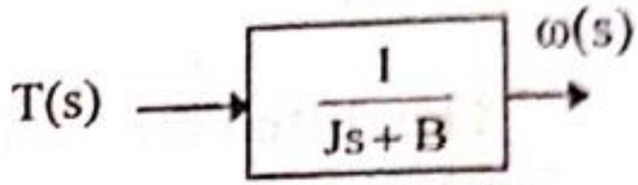
Block Diagram Contd..

$$T(s) = K_t \, I_a(s)$$

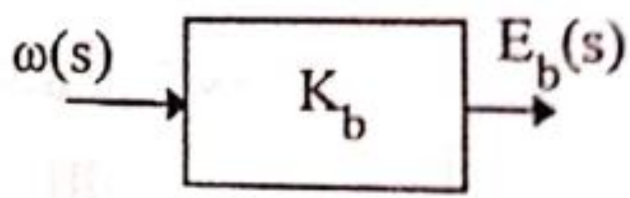


$$T(s) = (Js + B)\,\omega(s)$$

$$\therefore \omega(s) = \frac{1}{Js + B}\,T(s)$$

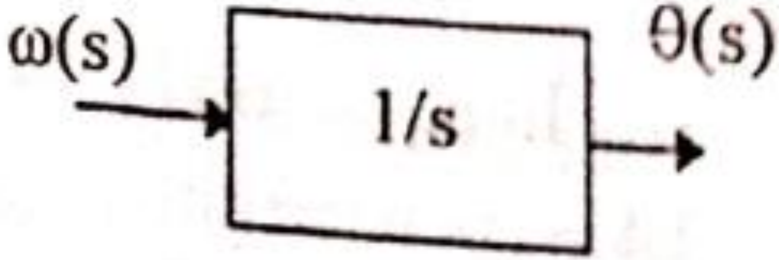


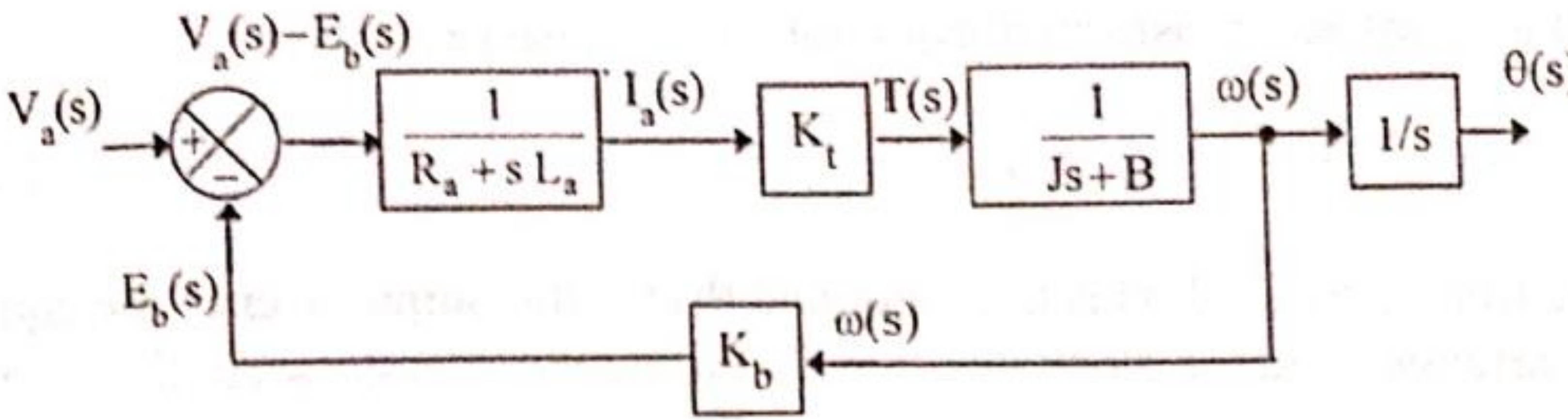
$$E_b(s) = K_b \, \omega(s)$$



$$\omega(s) = s\,\theta(s)$$

$$\theta(s) = \frac{1}{s}\,\omega(s)$$

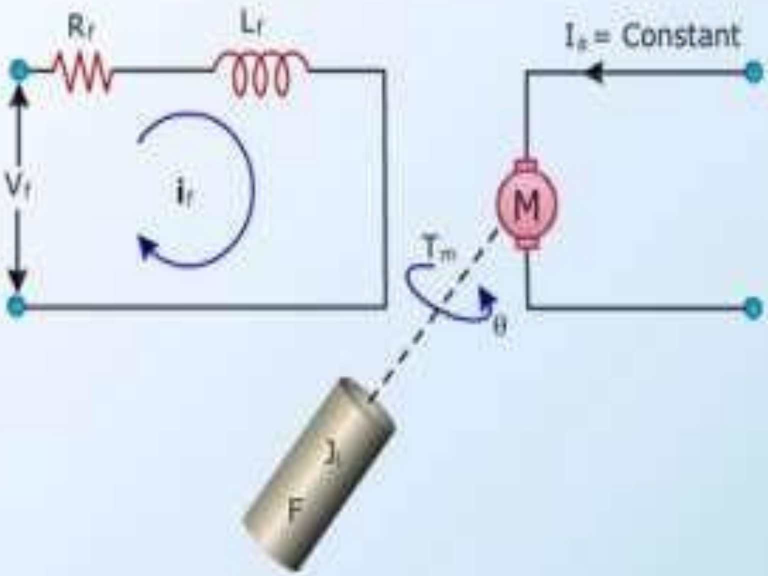




Block diagram of armature controlled dc motor

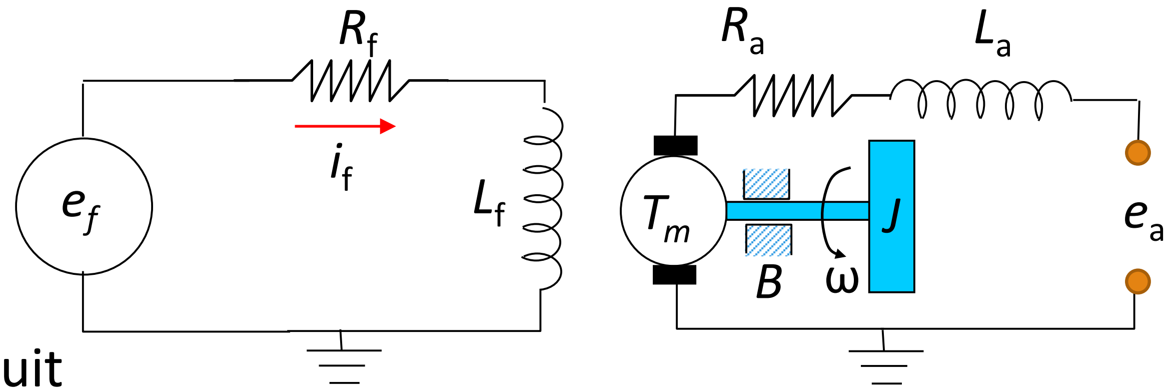
Transfer Function of an Field Controlled D.C. Motor

- From the block diagram of field controlled d.c. motor,
  - R_f: Field winding resistance
  - L_f: Field winding inductance
  - V_f: Field control voltage
  - I_f: Field current
  - T_m: Torque developed by motor
  - J: Equivalent moment of inertia
  - F: Equivalent Viscous friction
  - θ: Angular displacement



Field Controlled D.C. Motor

Field Controlled D.C Motor



Applying KVL at field circuit

$$e_f = i_f R_f + L_f \frac{di_f}{dt}$$

Mechanical Subsystem

$$T_m = J\dot{\omega} + B\omega$$

Field Controlled D.C Motor

Torque-Current: $T_m = K_f i_f$

where K_f: torque constant

Combing previous equations and taking Laplace transform (considering initial conditions to zero) results in the following mathematical model:

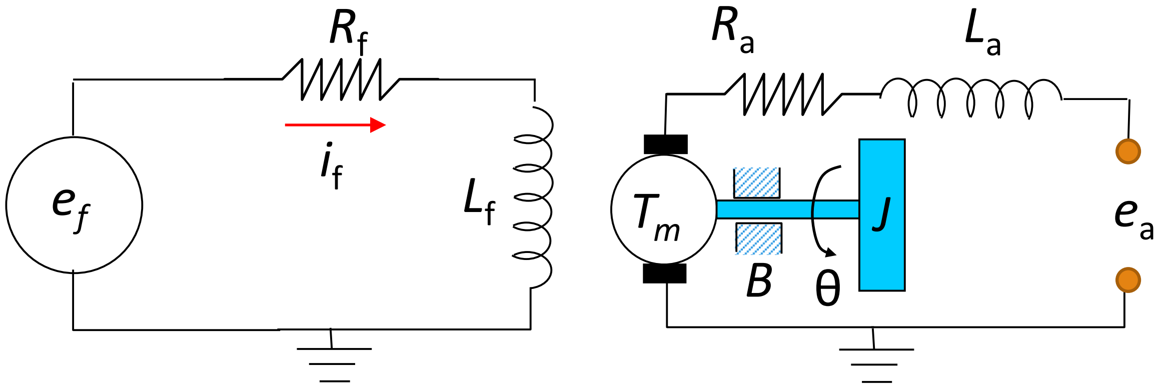
$$\begin{cases} E_f(s) = R_f I_f(s) + sL_f I_f(s) \\ Js\Omega(s) + B\Omega(s) = K_f I_f(s) \end{cases}$$

Field Controlled D.C Motor

Eliminating $I_f(S)$ yields

$$\frac{\Omega(s)}{E_f(s)} = \frac{K_f}{(Js + B)(L_f s + R_f)}$$

If angular position θ is output of the motor



$$\frac{\theta(s)}{E_f(s)} = \frac{K_f}{s(Js + B)(L_f s + R_f)}$$

Block Diagram Representation

$$v_f = R_f\, i_f + L_f \frac{di_f}{dt}$$

$$T = K_{tf}\, i_f$$

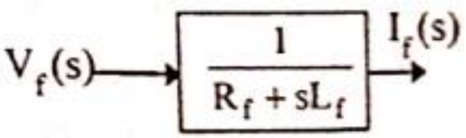
$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt}$$

On taking Laplace transform of equation

$$V_f(s) = R_f\, I_f(s) + L_f\, s\, I_f\,(s)$$

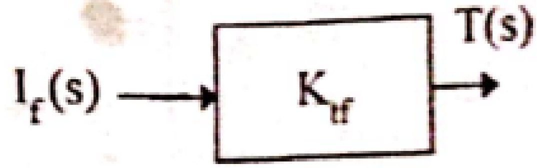
$$V_f(s) = I_f(s)\left[R_f + sL_f\right]$$

$$\therefore I_f(s) = \frac{1}{R_f + s\,L_f} V_f(s)$$



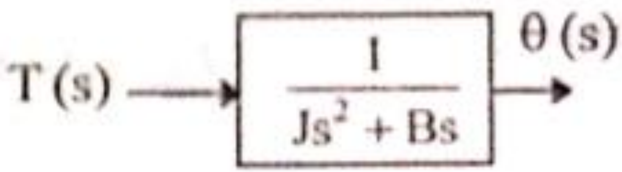
Block Diagram Contd..

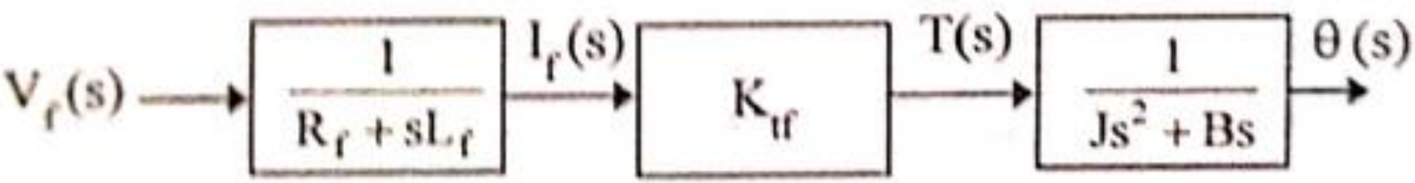
$$T(s) = K_{tf} \; I_f(s)$$



$$T(s) = (J s^2 + Bs)\, \theta(s)$$

$$\therefore \theta(s) = \frac{1}{Js^2 + Bs} T(s)$$

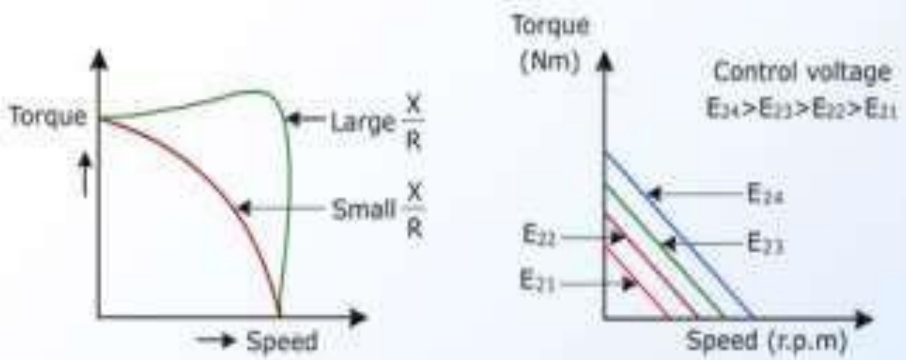




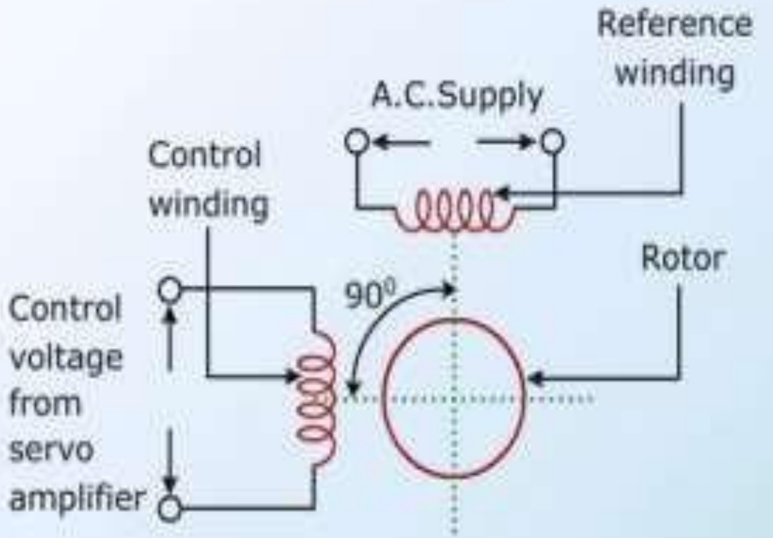
Block diagram of field controlled dc motor

A.C Servo Motor

- The a.c. servomotor is basically a "two-phase induction motor"
- The output power of a.c. servomotor varies from fraction of a watt to a few hundred watts
- The operating frequency is 50 Hz to 400 Hz



Torque Speed Characteristics



A.C Motor

CONSTRUCTION OF AC SERVOMOTOR

STATOR

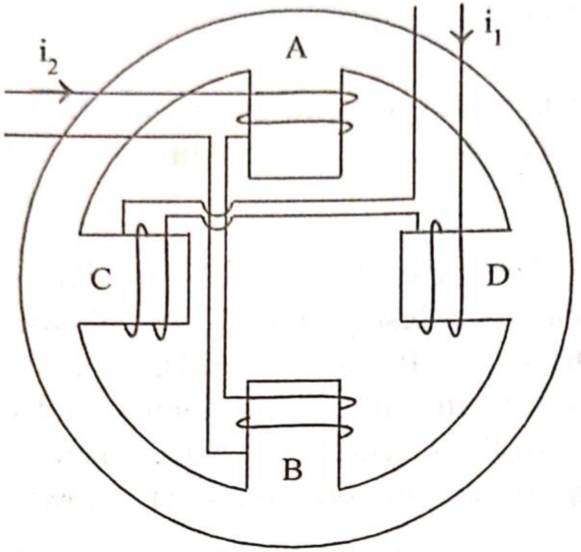
- It has two pole-pair mounted on the inner periphery of the stator such that axes are spaced by 90.
- Each pole-pair carries a winding. 1) Reference winding 2) Control Winding.

ROTOR

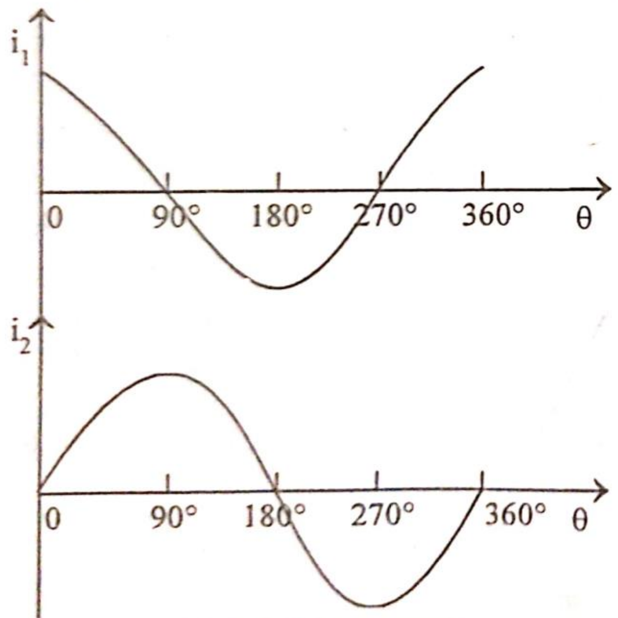
Squirrel-Cage or Drag-Cup Type construction.

- **Squirrel-Cage** is made of Laminations.
- The rotor bars are placed on the slots and short circuited at the both ends by end rings
- Diameter is kept small in order to reduce inertia and to obtain good accelerating characteristics.

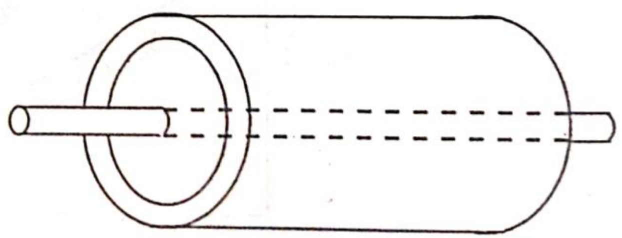
- **Drag-Cup Type** construction is employed for very low inertia applications.
- It is in form of hollow cylinder made of aluminium as the cylinder acts itself as short circuited rotor conductors.



a. stator



b. Exciting currents



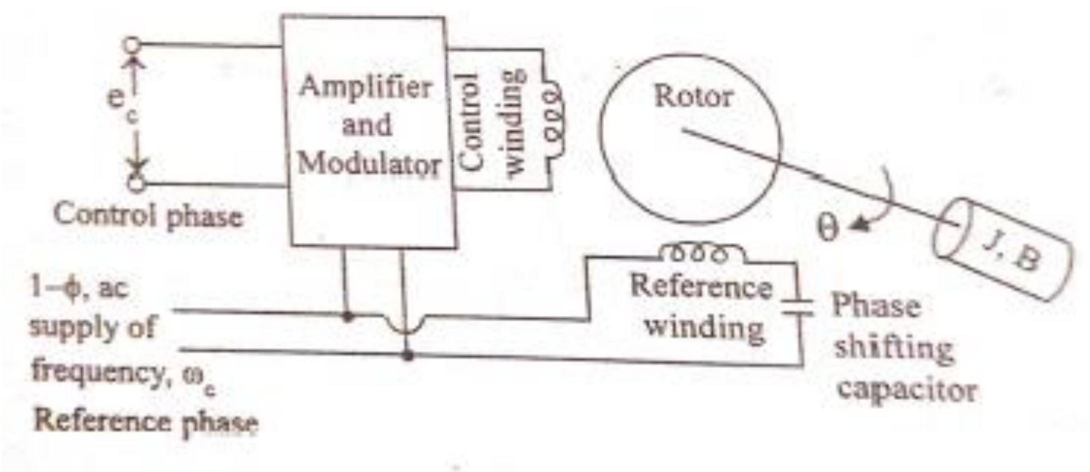
c. Rotor

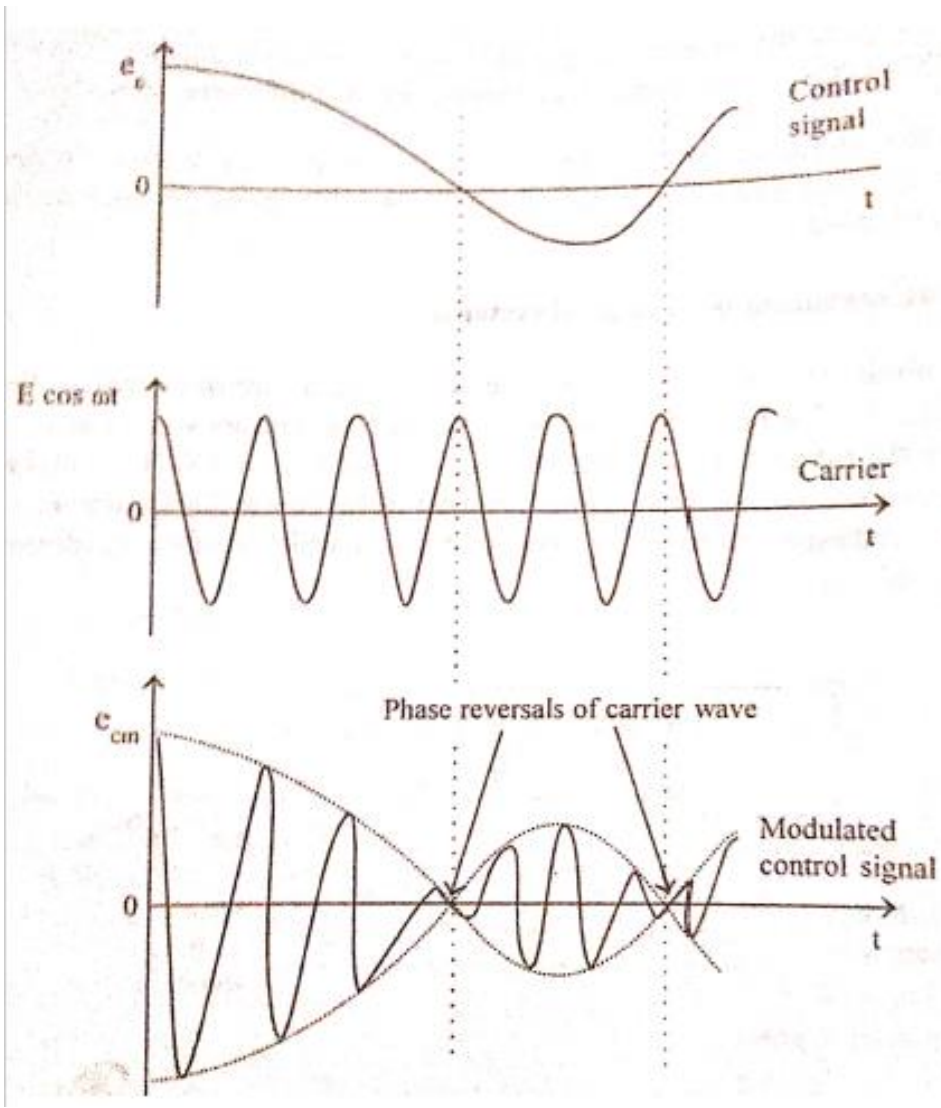
WORKING PRINCIPLE OF AC SERVOMOTOR AS INDUCTION MOTOR

- The stator winding are excited by voltages of equal rms magnitude and 90 phase difference.
- The currents give rise to a rotating magnetic field of constant magnitude.
- For the excitation currents shown there will be clockwise rotation. For Anti-Clockwise rotation, Current **i1** must be phase shifted by 180.
- The rotating magnetic field sweeps over the rotor conductors.
- The rotor conductors experience a change in flux and so voltages are induced in rotor conductors.
- This voltage circulated current in the short circuited rotor conductors and create rotor flux.
- Due to the interaction of stator and rotor flux, a mechanical force is developed on the rotor and rotor starts moving or rotating.

WORKING PRINCIPLE OF AC SERVOMOTOR IN CONTROL SYSTEM

- The reference winding is excited by constant voltage source with a frequency in the range of 50 to 1000 Hz.
- By using frequencies of 400 Hz or higher, the system can be made less susceptible to low frequency noise.
  (So used in AIRCRAFT and MISSILE CONTROL SYSTEM)
- Control Signal used is of low frequency (0 to 20 Hz)



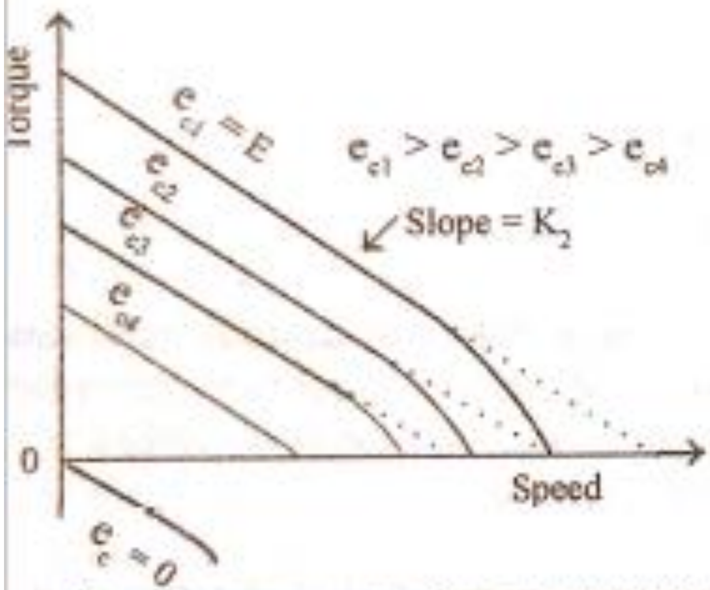


Let e_c = Control signal

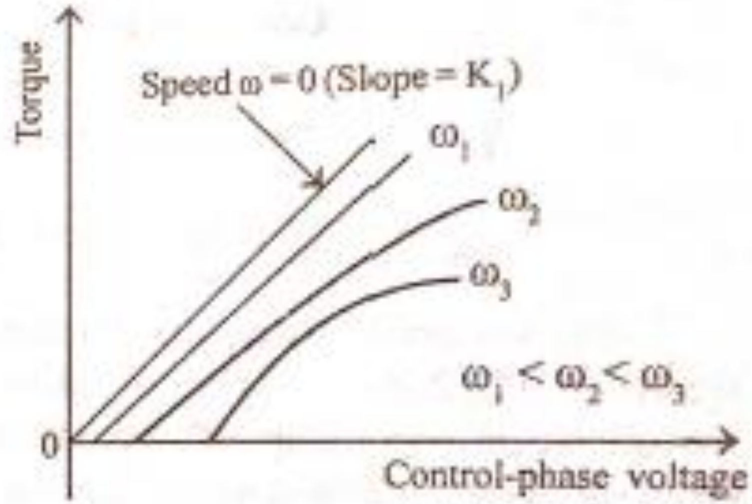
e_{car} = $E \cos \omega_c t$ = Carrier signal

e_{cm} = Modulated control signal.

$$e_{cm} = |E + e_c| \cos \omega_c t \quad \text{for } e_c > 0$$
$$= |E + e_c| \cos(\omega_c t + \pi) \quad \text{for } e_c < 0$$



a. Speed-torque curves of an ac servomotor



b. Control voltage Vs Torque curves of an ac servomotor

Let, T_m = Torque developed by servomotor

q = Angular displacement of rotor

$w = \frac{d\theta}{dt}$ = Angular speed

T_l = Torque required by the load

J = Moment of inertia of load and the rotor

B = Viscous-frictional coefficient of load and the rotor.

K_1 = Slope of control-phase voltage Vs Torque characteristic

K_2 = Slope of speed-torque characteristic.

Torque developed by motor, $T_m = K_1 e_c - K_2 \frac{d\theta}{dt}$

Load torque, $T_l = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt}$

At equilibruim the motor torque is equal to load torque.

$$\therefore J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = K_1 e_c - K_2 \frac{d\theta}{dt}$$

$$J s^2\theta(s) + B s\theta(s) = K_1 E_c(s) - K_2 s\theta(s)$$

$$\left[Js^2 + Bs + K_2 s\right]\theta(s) = K_1 E_c(s)$$

$$\therefore \frac{\theta(s)}{E_c(s)} = \frac{K_1}{s(Js + B + K_2)} = \frac{K_1/(B + K_2)}{s\left(\frac{J}{B + K_2}s + 1\right)}$$

$$= \frac{K_m}{s(\tau_m s + 1)}$$

Where, $K_m = \frac{K_1}{B + K_2}$ = Motor gain constant

$\tau_m = \frac{J}{B + K_2}$ = Motor time constant

Synchros

- Here, a rotating electromagnetic device transmits angular information
- It forms a variable-coupling transformer
- Types of Synchros
  - Transmitters and receivers
  - Differentials
  - Control transformers and linear transformers
  - Resolvers and differential resolvers
  - Transolvers
- Most widely used Synchros are:
  - 3-phase transmitter/receiver pair
  - Synchro receiver is electrically identical to the transmitter

Synchro Transmitter and Receiver

Transmitters

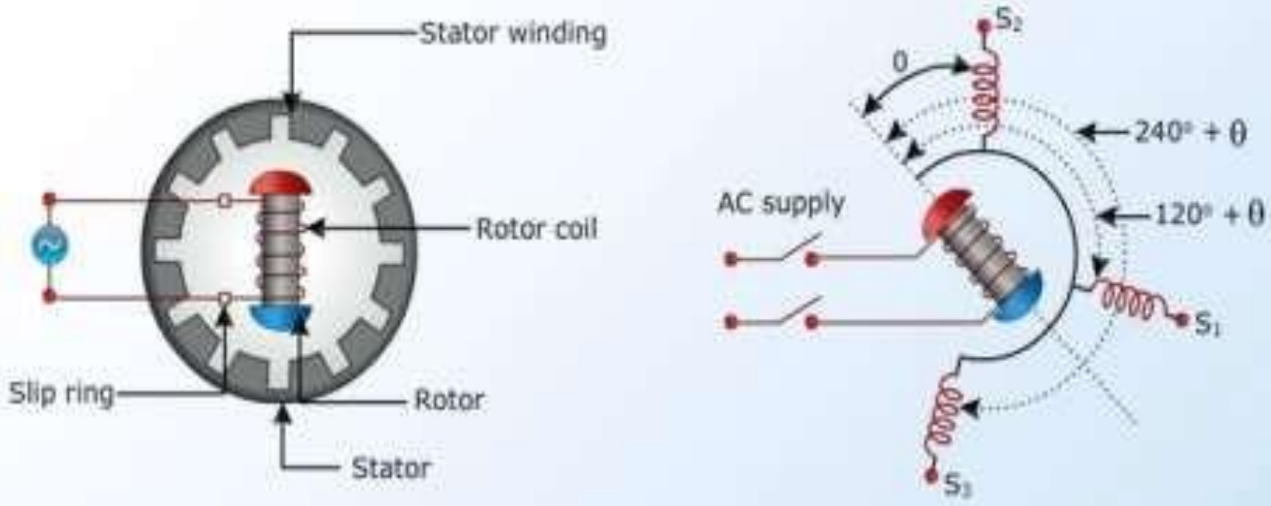
- The transmitter or Synchro generator consists of a rotor with a single winding and a stator with three windings placed 120 degrees apart
- When the mechanical device moves, the mechanically attached rotor also moves
- The rotor induces a voltage in each of the stator windings based on the rotor's angular position

Transmitter Stator

Transmitter Rotor

Schematic Diagram of Synchro Transmitter

- The synchro transmitter consists of a rotor with a single winding and a stator with three windings, placed 120 degrees apart



Synchro Transmitter

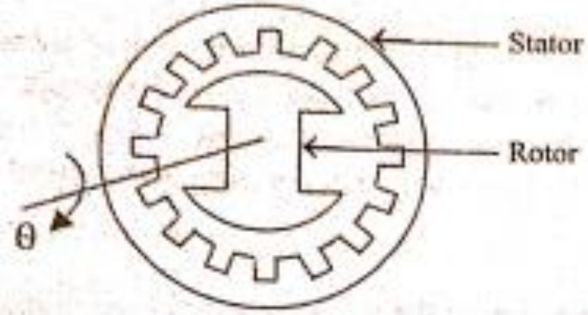


Fig a : Constructional features

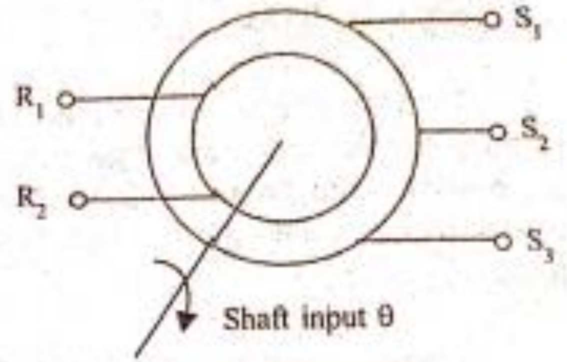


Fig b : Schematic symbol of a synchro transmitter

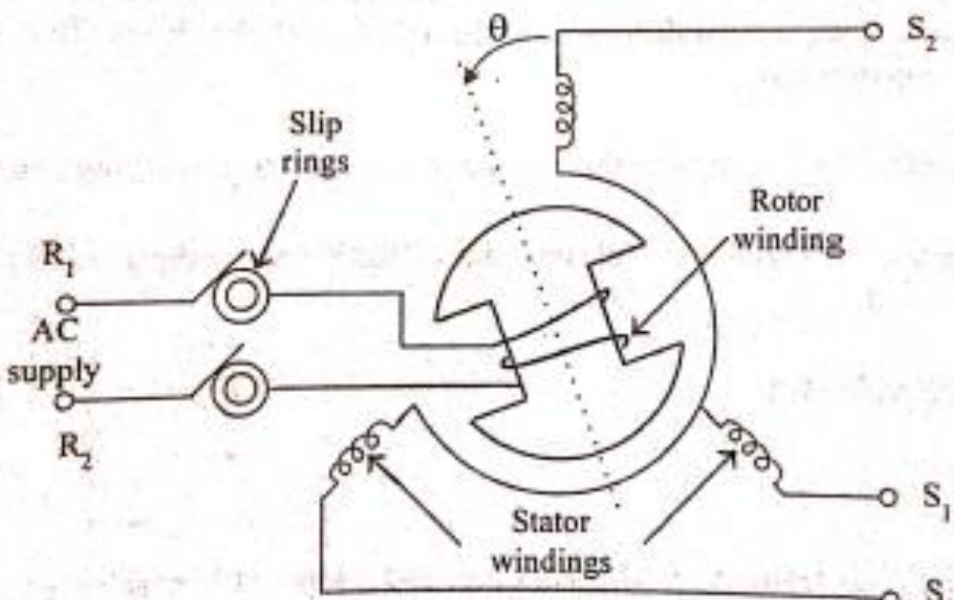


Fig c : Electrical circuit

Let e_r = Instantaneous value of ac voltage applied to rotor

e_{S1}, e_{S2}, e_{S3} = Instantaneous value of emf induced in stator coils s_1, s_2, s_3 with respect to neutral respectively.

E_r = Maximum value of rotor excitation voltage

ω = Angular frequency of rotor excitation voltage.

K_t = Turns ratio of stator and rotor windings.

K_c = Coupling coefficient

θ = Angular displacement of rotor with respect to reference

$\therefore$ The instantaneous value of excitation voltage, $e_r = E_r \sin \omega t$

The magnitude of induced emf are proportional to the turns ratio and coupling coefficient. The turns ratio Kt is a constant , but coupling coefficient , Kc is a function of rotor angular position.

Induced EMF in stator coil = Kt*Kc*Er*sin(wt)

Let S2 be the reference vector.
When ɵ = 0, Flux linkage is max. with S2 and
When ɵ =90, Flux linkage is min. with S2

$\therefore$ Coupling coefficient, K_c for coil-s_2 = $K_1 \cos\theta$
Coupling coefficient, K_c for coil-s_3 = $K_1 \cos(\theta - 120^o)$
Coupling coefficient, K_c for coil-s_1 = $K_1 \cos(\theta - 240^o)$

Hence the emfs of stator coils with respect to neutral can be expressed as follows.

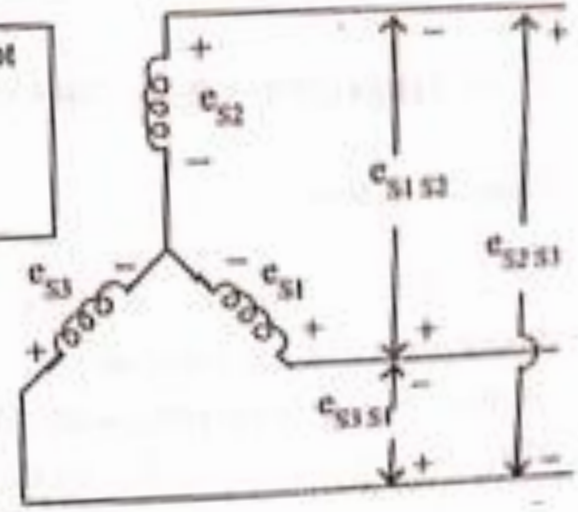
$$e_{S2} = K_t K_1 \cos\theta \, E_r \sin\omega t = K \, E_r \cos\theta \sin\omega t$$

$$e_{S3} = K_t K_1 \cos(\theta - 120^o) \, E_r \sin\omega t = K \, E_r \cos(\theta - 120^o) \sin\omega t$$

$$e_{S1} = K_t K_1 \cos(\theta - 240^o) \, E_r \sin\omega t = K \, E_r \cos(\theta - 240^o) \sin\omega t$$

Where, $K = K_t K_1$

$$
\begin{aligned}
e_{S1S2} = e_{S1} - e_{S2} &= K\,E_r \operatorname{Cos}(\theta - 240^\circ)\sin\omega t - K\,E_r \cos\theta \sin\omega t \\
&= K\,E_r \left[\cos\theta\cos 240^\circ + \sin\theta\sin 240^\circ - \cos\theta\right]\sin\omega t \\
&= K\,E_r \left[\cos\theta\,(-0.5) + \sin\theta\left(-\frac{\sqrt{3}}{2}\right) - \cos\theta\right]\sin\omega t \\
&= \sqrt{3}\,K\,E_r \left[\sin\theta\left(-\frac{1}{2}\right) + \cos\theta\left(-\frac{\sqrt{3}}{2}\right)\right]\sin\omega t \\
&= \sqrt{3}\,K\,E_r \left[\sin\theta\cos 240^\circ + \cos\theta\sin 240\right]\sin\omega t \\
&= \sqrt{3}\,KE_r \sin(\theta + 240^\circ)\sin\omega t
\end{aligned}
$$



$$
\begin{aligned}
e_{S2S3} = e_{S2} - e_{S3} &= K\,E_r \operatorname{Cos}\theta \sin\omega t - K\,E_r \cos(\theta - 120^\circ)\sin\omega t \\
&= K\,E_r \left[\cos\theta - \cos\theta\cos 120^\circ - \sin\theta\sin 120^\circ\right]\sin\omega t \\
&= K\,E_r \left[\cos\theta - \cos\theta(-0.5) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)\right]\sin\omega t \\
&= \sqrt{3}\,K\,E_r \left[\sin\theta\left(-\frac{1}{2}\right) + \cos\theta\left(\frac{\sqrt{3}}{2}\right)\right]\sin\omega t \\
&= \sqrt{3}\,K\,E_r \left[\sin\theta\cos 120^\circ + \cos\theta\sin 120\right]\sin\omega t \\
&= \sqrt{3}\,KE_r \sin(\theta + 120^\circ)\sin\omega t
\end{aligned}
$$

$$
\begin{aligned}
e_{S3S1} &= e_{S3} - e_{S1} = K\,E_r \cos(\theta - 120^\circ)\ \sin\omega t - K\,E_r \cos(\theta - 240^\circ)\sin\omega t \\
&= K\,E_r \left[\cos\theta\ \cos 120^\circ + \sin\theta \sin 120^\circ - \cos\theta \cos 240^\circ - \sin\theta \sin 240^\circ\right]\sin\omega t \\
&= K\,E_r \left[\cos\theta(-0.5) + \sin\theta\left(\frac{\sqrt{3}}{2}\right) - \cos\theta(-0.5) - \sin\theta\left(-\frac{\sqrt{3}}{2}\right)\right]\sin\omega t \\
&= \sqrt{3}\,K\,E_r\ \sin\theta \sin\omega t
\end{aligned}
$$

The position of rotor where the coil to coil voltage eS3S1 = 0, is defined as the electrical Zero of the transmitter.

It is used as reference for specifying the angular position of the rotor.

Synchro Receiver

- The receiver or Synchro motor, is electrically similar to the Synchro Generator
- The Synchro receiver uses the voltage generated by each of the Synchro generator windings to position the receiver rotor

Synchro Receiver

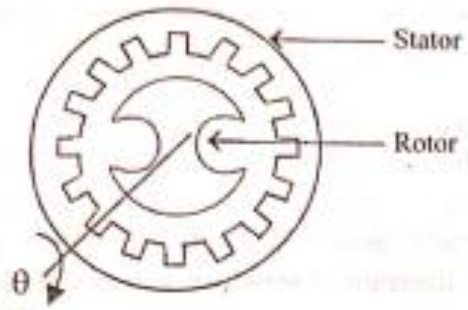


Fig a : Constructional features

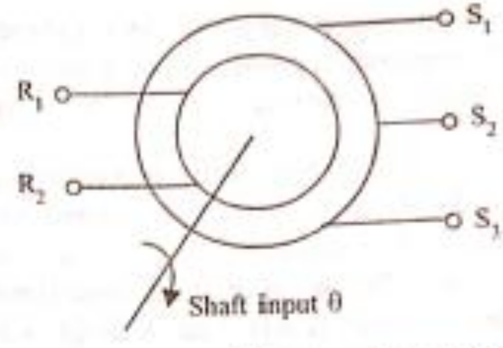


Fig b : Schematic symbol of a synchro control transformer

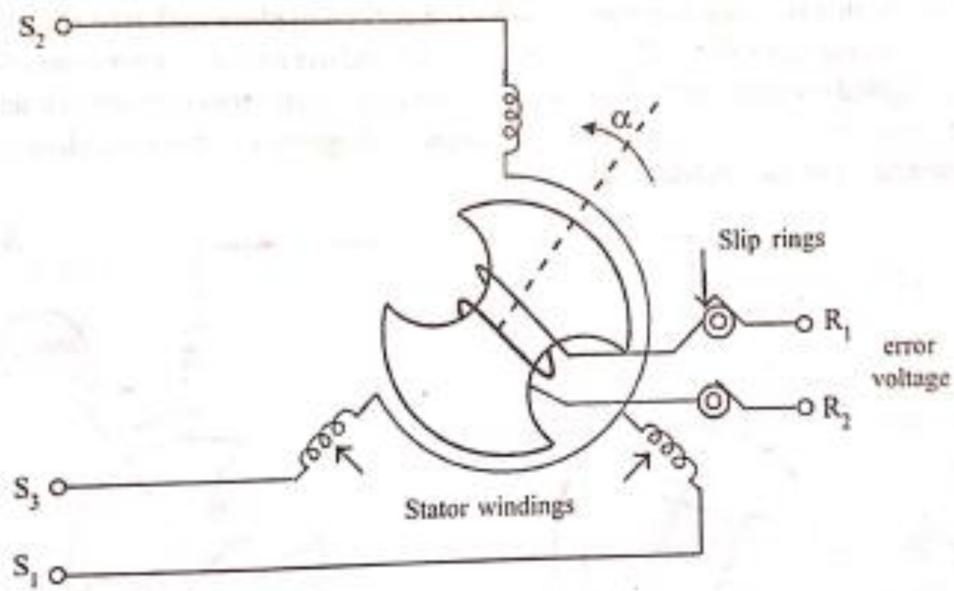
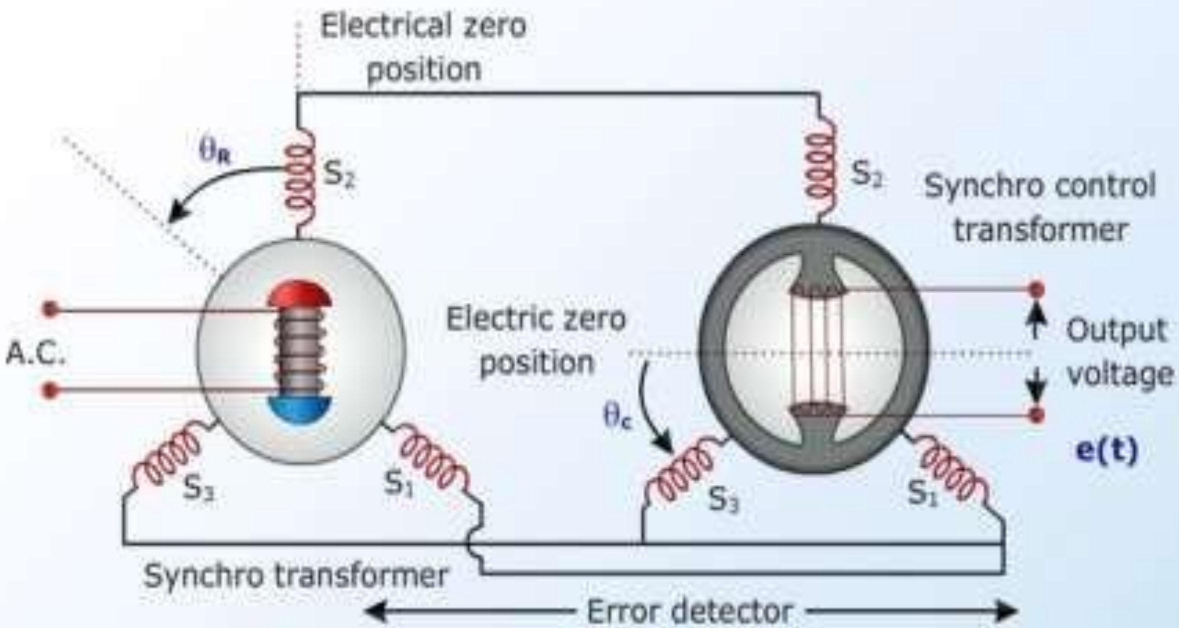


Fig c : Electrical circuit

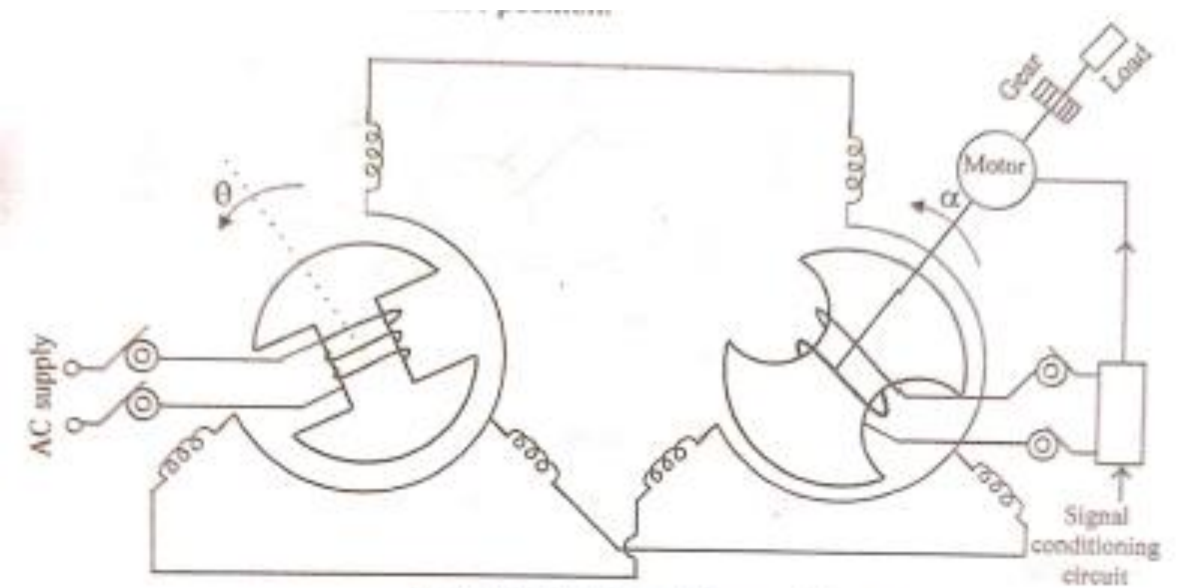
Synchro control transformer

Synchros as Error Detector

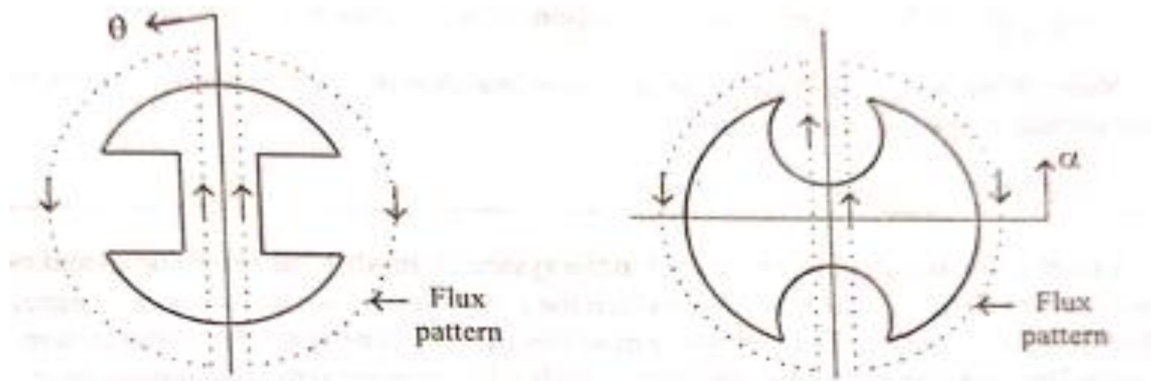
- The synchro "transmitter-receiver pair" acts as an error detector because the voltage signal of the rotor terminal of the synchro receiver is proportional to the difference in transmitter and receiver rotor shaft Position



Synchros as Error Detector



Servo system using synchro error detector



Rotor positions and flux patterns

$\therefore$ Voltage across slip rings of control transformer (modulated error voltage) $\Big\}\ e_m = K^{'} E_r \cos(90 - \theta + \alpha) \sin\omega t$

$$= K' E_r \cos(90 - (\theta - \alpha)) \sin\omega t$$

$$= K^{'} E_r \sin(\theta - \alpha) \sin\omega t$$

Where $K^{'}$ is a proportional constant

Let $\phi(t) = \theta - \alpha$

For small values of $\phi(t)$, $\sin(\theta - \alpha) = \sin\phi(t) \approx \phi(t)$

$$\therefore e_m = K^{'} E_r \phi(t) \sin\omega t$$

The demodulated error voltage, $e = K_s \phi(t)$

Where K_s = sensitivity of the synchro error detector in volts/deg.

On taking laplace transform of equation, We get,

$$E(s) = K_s\, \phi(s) \qquad \therefore \frac{E(s)}{\phi(s)} = K_s$$