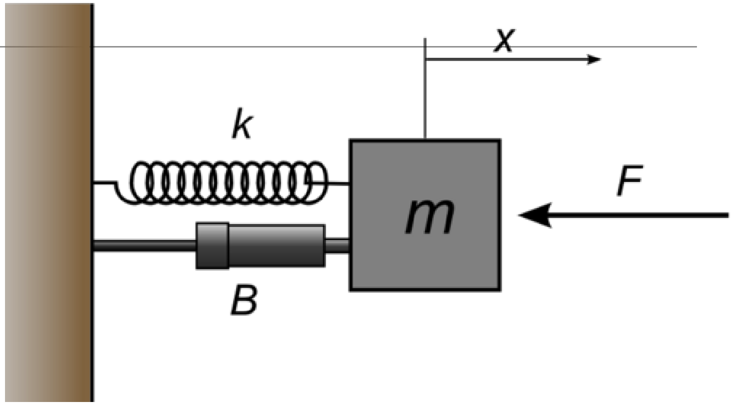
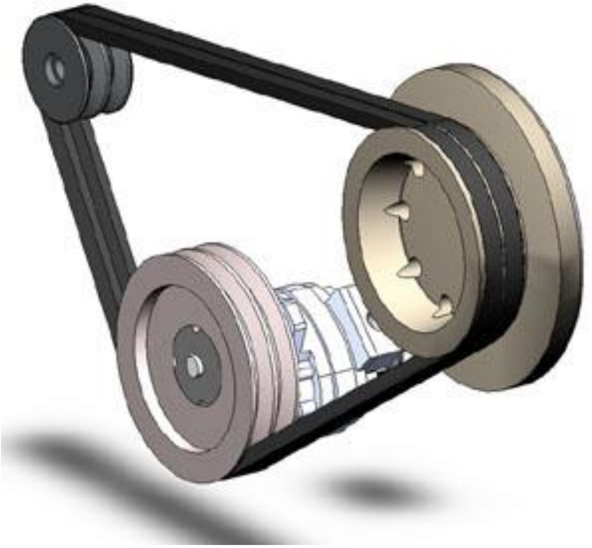
Basic Types of Mechanical Systems

Translational
- Linear Motion



Rotational
- Rotational Motion



Translational Mechanical System Vs Rotational System

- The analogous quantities of translational and rotational mechanical system are tabulated as below:

S.No.	Translational	Rotational
1	Force, F	Torque, T
2	Acceleration, a	Angular Acceleration
3	Velocity, v	Angular velocity
4	Displacement, x	Angular displacement
5	Mass, M	Moment of inertia
6	Damping coefficient B	Rotational damping coefficient
7	Stiffness	Torsion stiffness

Newton’s Second Law

Newton's law of motion states that the algebraic sum of external forces acting on a rigid body in a given direction is equal to the product of the mass of the body and its acceleration in the same direction. The law can be expressed as

$$\sum \boldsymbol{F} = \boldsymbol{Ma}$$

Translational Mechanical Systems

Part-I

Basic Elements of Translational Mechanical Systems

Translational Spring

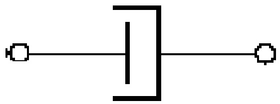
i)

Translational Mass

ii)

Translational Damper

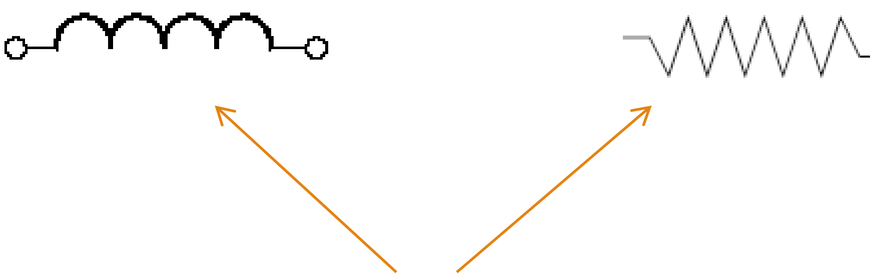
iii)



Translational Spring

A translational spring is a mechanical element that can be deformed by an external force such that the deformation is directly proportional to the force applied to it.

i) **Translational Spring**

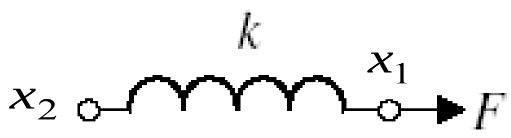


Circuit Symbols

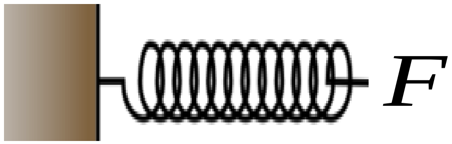
Translational Spring

Translational Spring

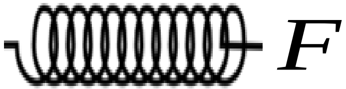
If *F* is the applied force



Then x_1 is the deformation if $x_2 = 0$



Or $(x_1 - x_2)$ is the deformation.



The equation of motion is given as

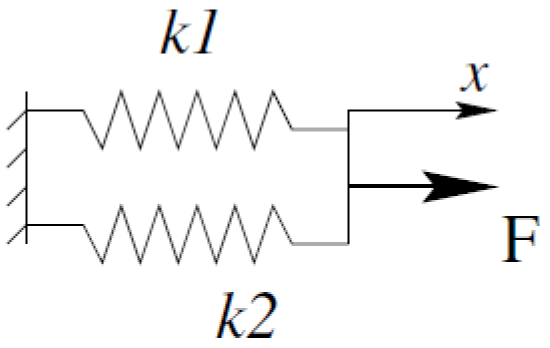
$$F = k(x_1 - x_2)$$

Where k is stiffness of spring expressed in N/m

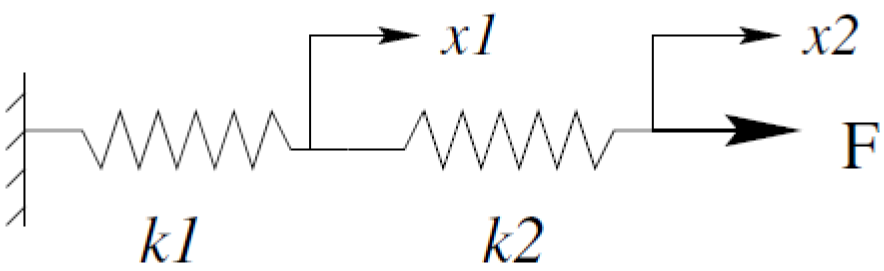
Translational Spring

Given two springs with spring constant k_1 and k_2, obtain the equivalent spring constant k_{eq} for the two springs connected in:

(1) Parallel



(2) Series



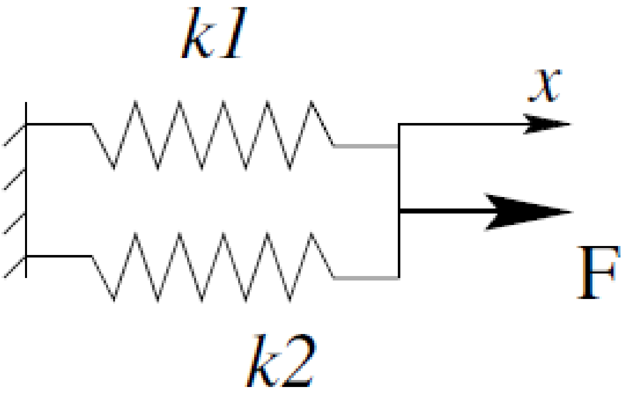
Translational Spring

- The two springs have same displacement therefore:

$$k_1 x + k_2 x = F$$

(1) Parallel

$$(k_1 + k_2)x = F$$



$$k_{eq} x = F$$

$$k_{eq} = k_1 + k_2$$

- If *n* springs are connected in parallel then:

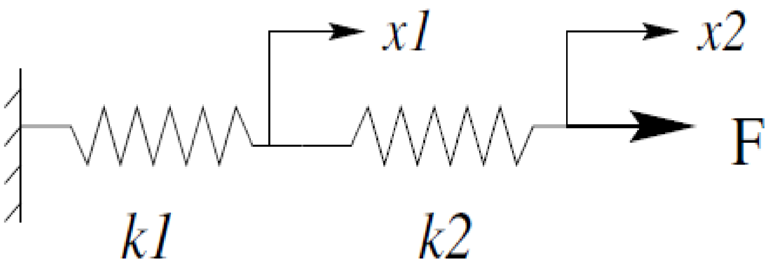
$$k_{eq} = k_1 + k_2 + \cdots + k_n$$

Translational Spring

- The forces on two springs are same, *F*, however displacements are different therefore:

(2) Series

$$k_1 x_1 = k_2 x_2 = F$$



$$x_1 = \frac{F}{k_1} \qquad x_2 = \frac{F}{k_2}$$

- Since the total displacement is $x = x_1 + x_2$, and we have $F = k_{eq} x$

$$x = x_1 + x_2 \Rightarrow \frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2}$$

Translational Spring

$$\frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2}$$

- Then we can obtain

$$k_{eq} = \frac{1}{\dfrac{1}{k_1} + \dfrac{1}{k_2}} = \frac{k_1 k_2}{k_1 + k_2}$$

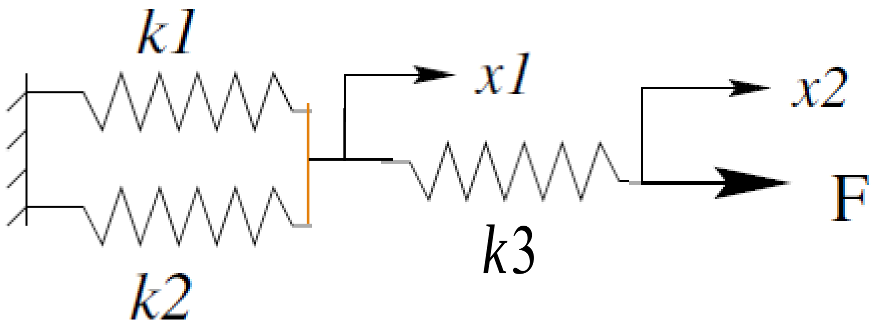
- If *n* springs are connected in series then:

$$k_{eq} = \frac{k_1 k_2 \cdots k_n}{k_1 + k_2 + \cdots + k_n}$$

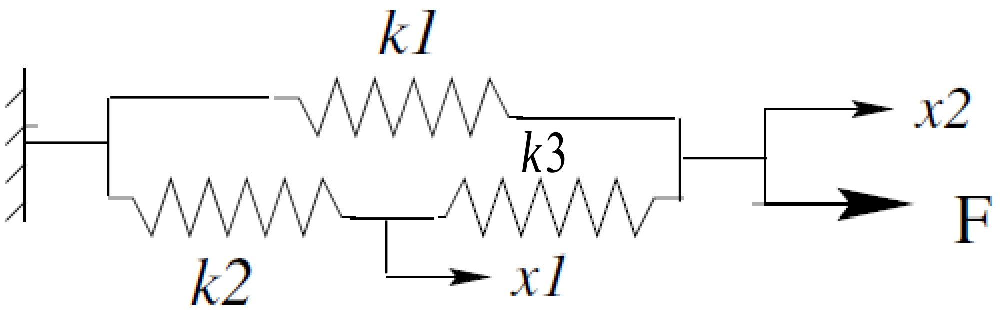
Translational Spring

Exercise: Obtain the equivalent stiffness for the following spring networks.

i)

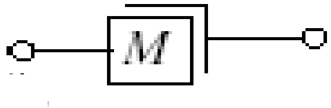


ii)

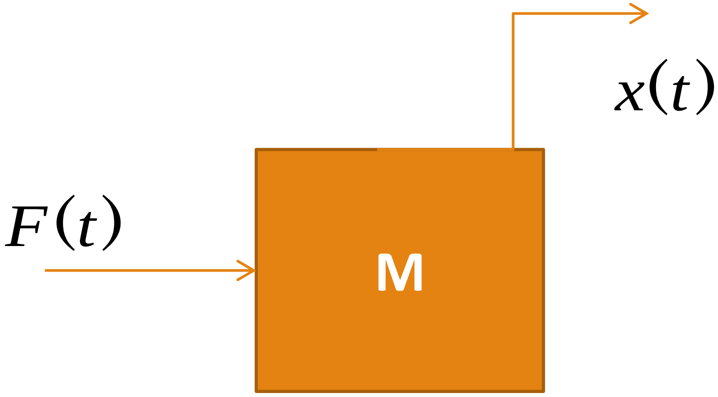


Translational Mass

ii) **Translational Mass**



- Translational Mass is an inertia element.
- A mechanical system without mass does not exist.
- If a force *F* is applied to a mass and it is displaced to *x* meters then the relation b/w force and displacements is given by Newton's law.



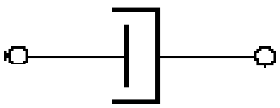
$$F = M\ddot{x}$$

Translational Damper

- When the viscosity or drag is not negligible in a system, we often model them with the damping force.
- All the materials exhibit the property of damping to some extent.
- If damping in the system is not enough then extra elements (e.g. Dashpot) are added to increase damping.

Translational Damper

iii)



Common Uses of Dashpots

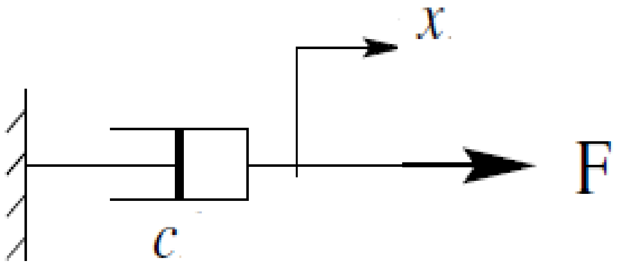
Door Stoppers

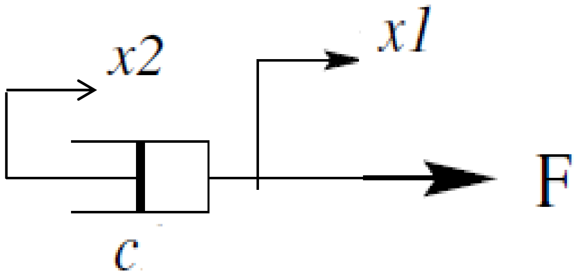
Vehicle Suspension

Bridge Suspension

Flyover Suspension

Translational Damper





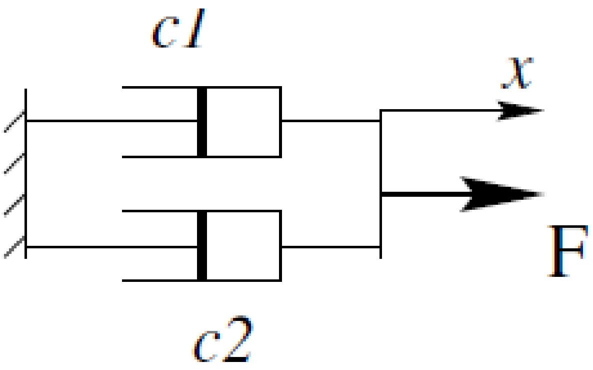
$$F = C\dot{x}$$

$$F = C(\dot{x}_1 - \dot{x}_2)$$

- Where *C* is damping coefficient (*N/ms*$^{-1}$).

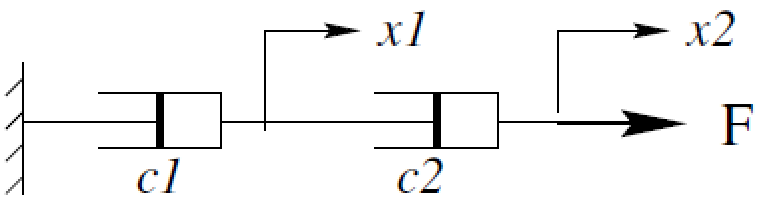
Translational Damper

Translational Dampers in Parallel



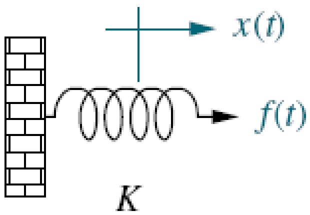
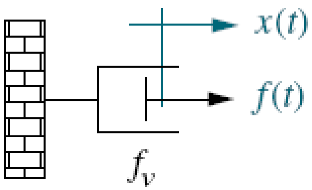
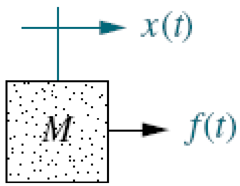
$$C_{eq} = C_1 + C_2$$

Translational Dampers in Series



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Force-velocity, force-displacement, and impedance relationships for springs, viscous dampers, and mass

Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

where, K, f v, and M are called spring constant, coefficient of viscous friction, and mass, respectively.

Analogies Between Electrical and Mechanical Components

- Mechanical systems, like electrical networks, have three passive, linear components.
- Two of them, the **spring** and the **mass**, are energy-storage elements;
- One of them, the **viscous damper**, dissipates energy.
- The two energy-storage elements are analogous to the two electrical energy-storage elements, the inductor and capacitor.
- The energy dissipater is analogous to electrical resistance.
- The motion of translation is defined as a motion that takes place along a straight or curved path. The variables that are used to describe translational motion are **acceleration**, **velocity**, and **displacement**.

Procedure for writing Mathematical Model for Mechanical Systems

Step 1: Write all Elements and Forces in the given in the Diagram.

Step 2: Write all displacements, corresponding velocity and accelerations.

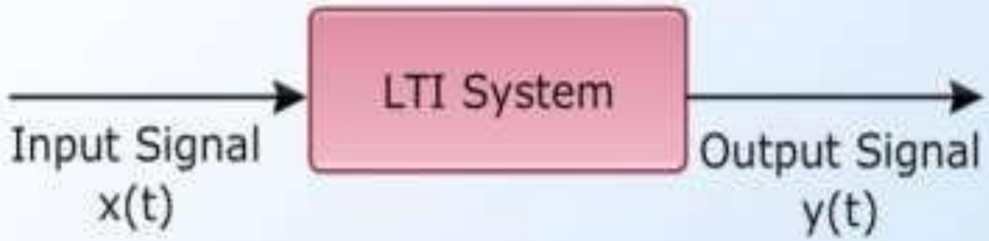
Step 3: Draw the Free Body Diagram by considering Mass as Node and Marking Acting and Opposing Forces on it.

Step 4: Write the differential equation for each Node based on Newtons Laws.

Step 5: Rearrange the equations in suitable form and calculate the Transfer Function.

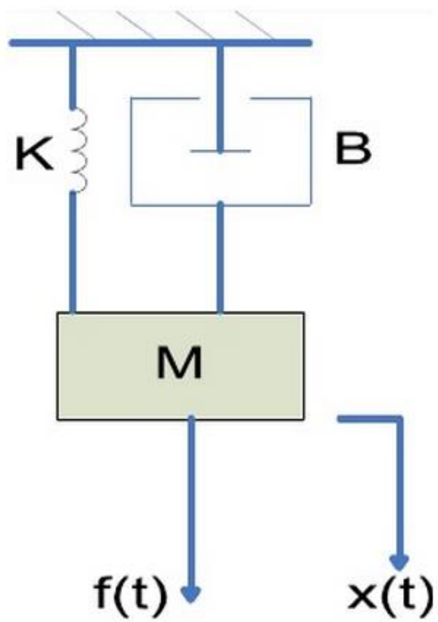
Introduction

- The I/O relationship in a "linear time invariant" system is defined by the transfer function
- Features of the transfer functions
- Steps involved in obtaining the transfer function
  - Write the differential equation of the system
  - Replace the terms "d/dt" by 's' and ∫dt by 1/s
  - Eliminate all the variables except the output and the input variables



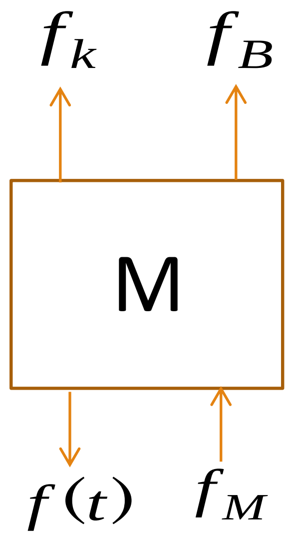
Linear Time Invariant System

Example-1(a): Find the transfer function of the mechanical translational system given in the Figure.



Figure

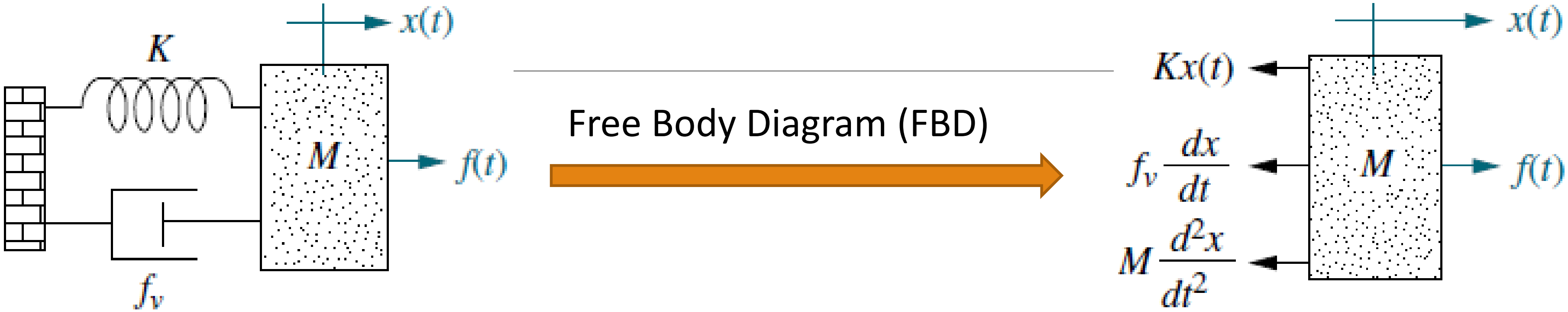
Free Body Diagram



$$f(t) = f_k + f_M + f_B$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

Example: Find the transfer function, *X(s)/F(s)*, of the system.



- First step is to draw the free-body diagram.
- Place on the mass all forces felt by the mass.
- We assume the mass is traveling toward the right. Thus, only the applied force points to the right; all other forces impede the motion and act to oppose it. Hence, the spring, viscous damper, and the force due to acceleration point to the left.

- Second step is to write the differential equation of motion using Newton's law to sum to zero all of the forces shown on the mass.

$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{dt} + Kx(t) = f(t)$$

Example: Continue.

- Third step is to take the Laplace transform, assuming zero initial conditions,

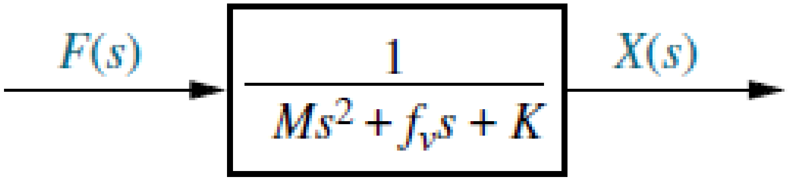
$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s)$$

or

$$(Ms^2 + f_v s + K)X(s) = F(s)$$

- Finally, solving for the transfer function yields

$$\boxed{G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}}$$



Block Diagram

Impedance Approach to Obtain the Transfer Function of Mechanical System

- Taking the Laplace transform of the force-displacement terms of mechanical components , we get

For the spring, $\boxed{F(s) = KX(s)}$

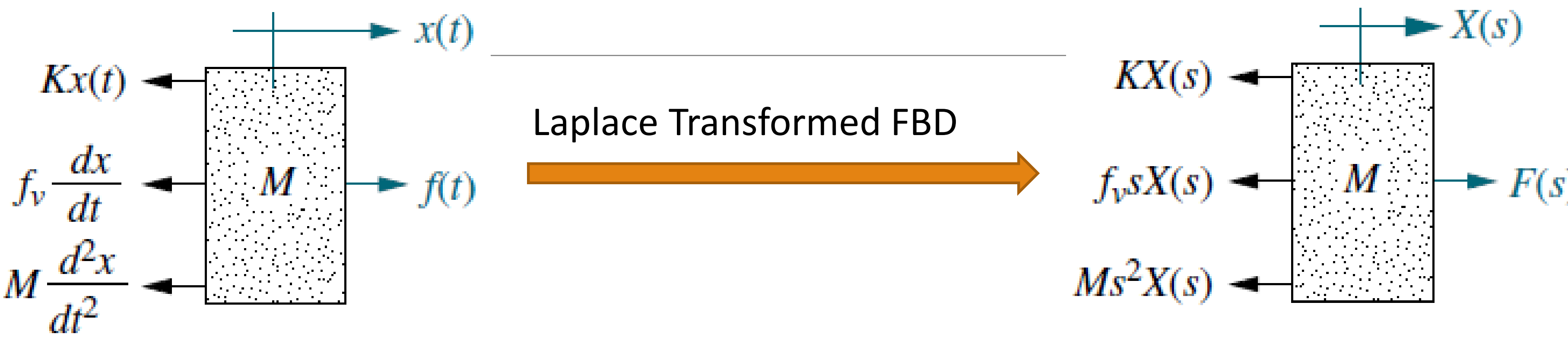
For the viscous damper, $\boxed{F(s) = f_v sX(s)}$

and for the mass, $\boxed{F(s) = Ms^2X(s)}$

- We can define impedance for mechanical components as

$$\boxed{Z_M(s) = \frac{F(s)}{X(s)}}$$

Example: Solve example-1 using the Impedance Approach.



- Summing the forces in the Laplace Transformed FBD, we get

$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s)$$

$$(Ms^2 + f_v s + K)X(s) = F(s)$$

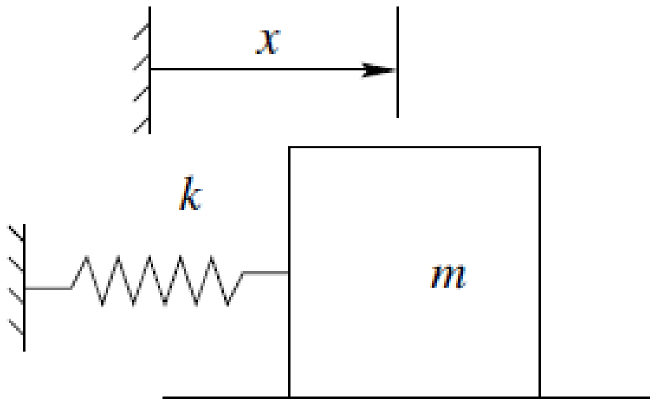
or

$$\boxed{G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}}$$

- Which is in the form of

$$[\text{Sum of impedances}]X(s) = [\text{Sum of applied forces}]$$

Example: Consider a simple horizontal spring-mass system on a frictionless surface, as shown in figure below.



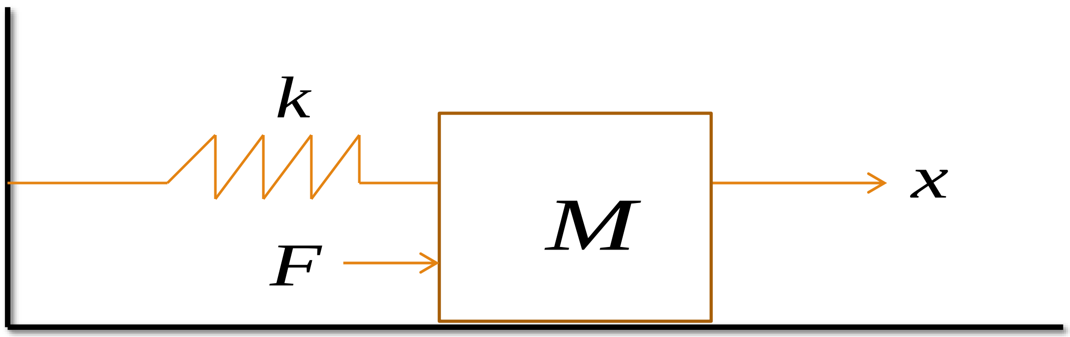
The differential equation of the above system is

$$m\ddot{x} = -kx$$

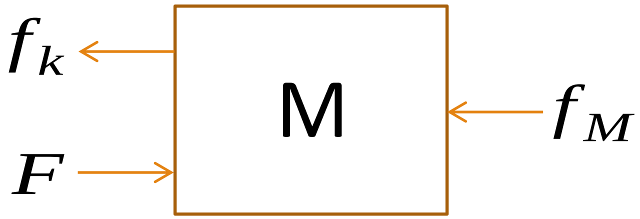
or

$$m\ddot{x} + kx = 0$$

Example: Find the transfer function, *X(s)/F(s),* of the system. Consider the system friction is negligible.

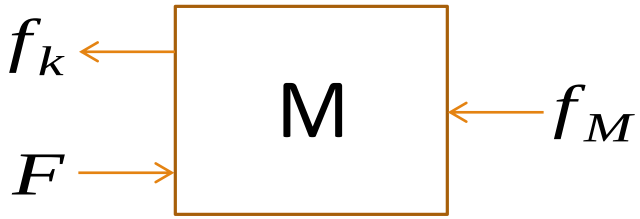


- Free Body Diagram



- Where f_k and f_M are force applied by the spring and inertial force respectively.

Example: continue



$$F = f_k + f_M$$

- Then the differential equation of the system is:

$$F = M\ddot{x} + kx$$

- Taking the Laplace Transform of both sides and ignoring initial conditions we get

$$F(s) = Ms^2X(s) + kX(s)$$

Example: continue.

$$F(s) = Ms^2 X(s) + kX(s)$$

- The transfer function of the system is

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + k}}$$

- if

$$M = 1000\, kg$$

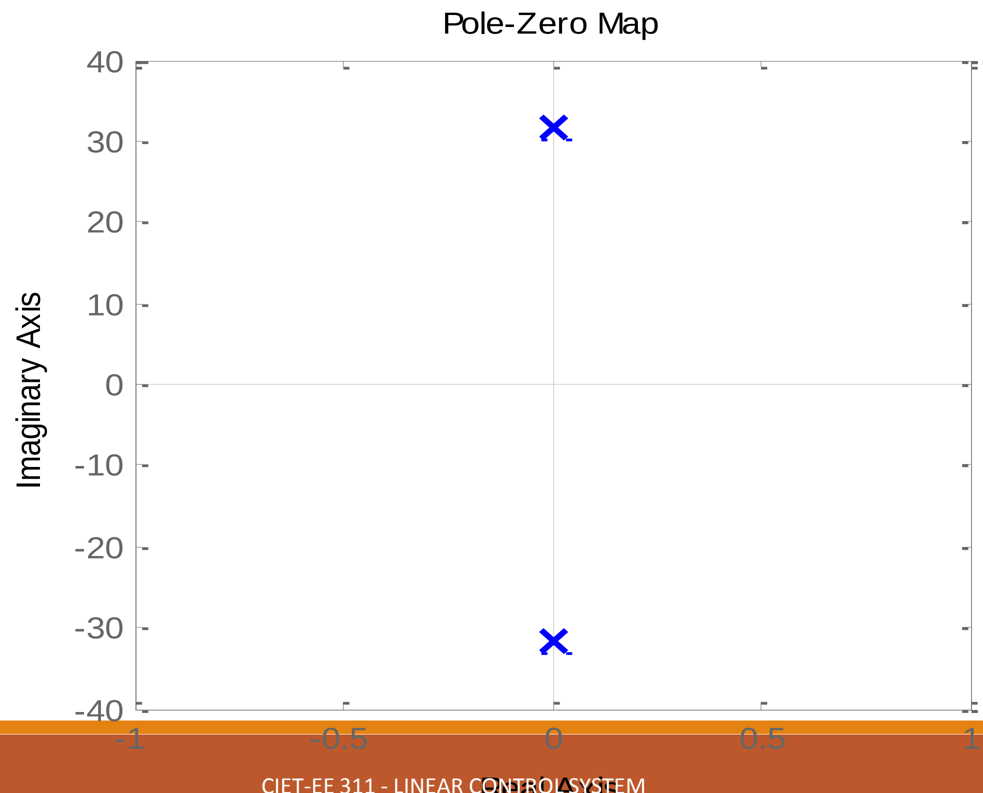
$$k = 2000\, Nm^{-1}$$

$$\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + 2}$$

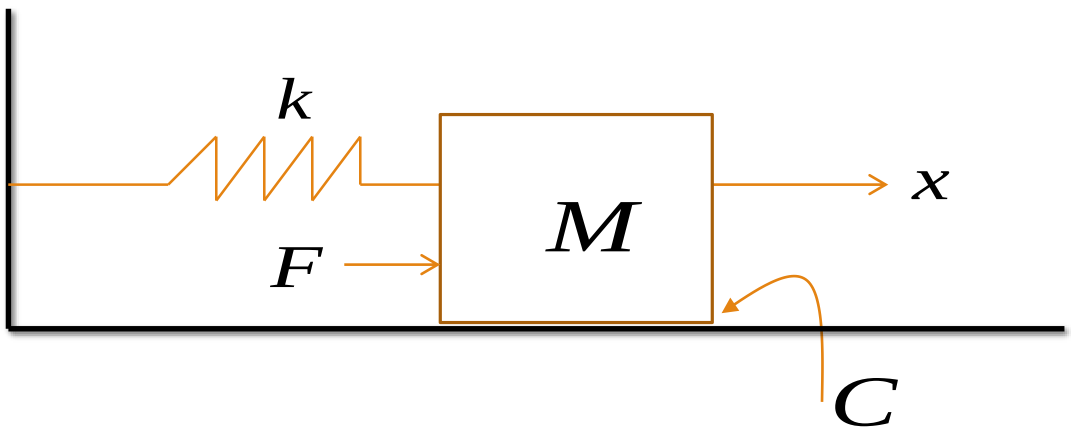
Example: continue.

$$\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + 2}$$

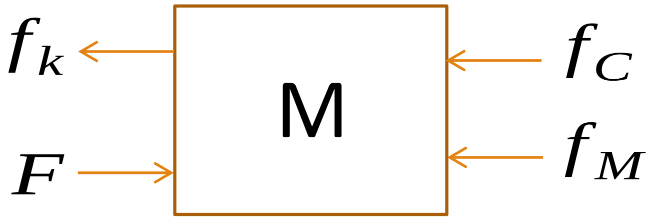
- The pole-zero map of the system is



Example: Find the transfer function, *X(s)/F(s),* of the following system, where the system friction is negligible.



- Free Body Diagram



$$F = f_k + f_M + f_C$$

Example: continue.

Differential equation of the system is:

$$F = M\ddot{x} + C\dot{x} + kx$$

Taking the Laplace Transform of both sides and ignoring Initial conditions we get

$$F(s) = Ms^2 X(s) + CsX(s) + kX(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Cs + k}}$$

Example: continue.

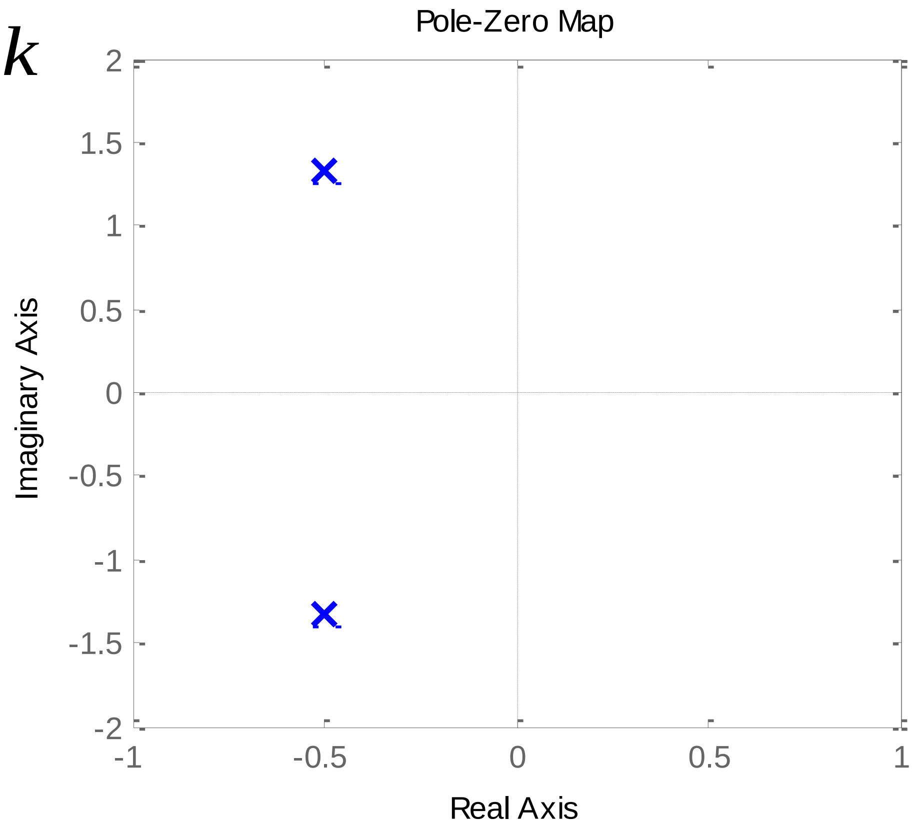
- if $$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Cs + k}$$

$$M = 1000\,kg$$

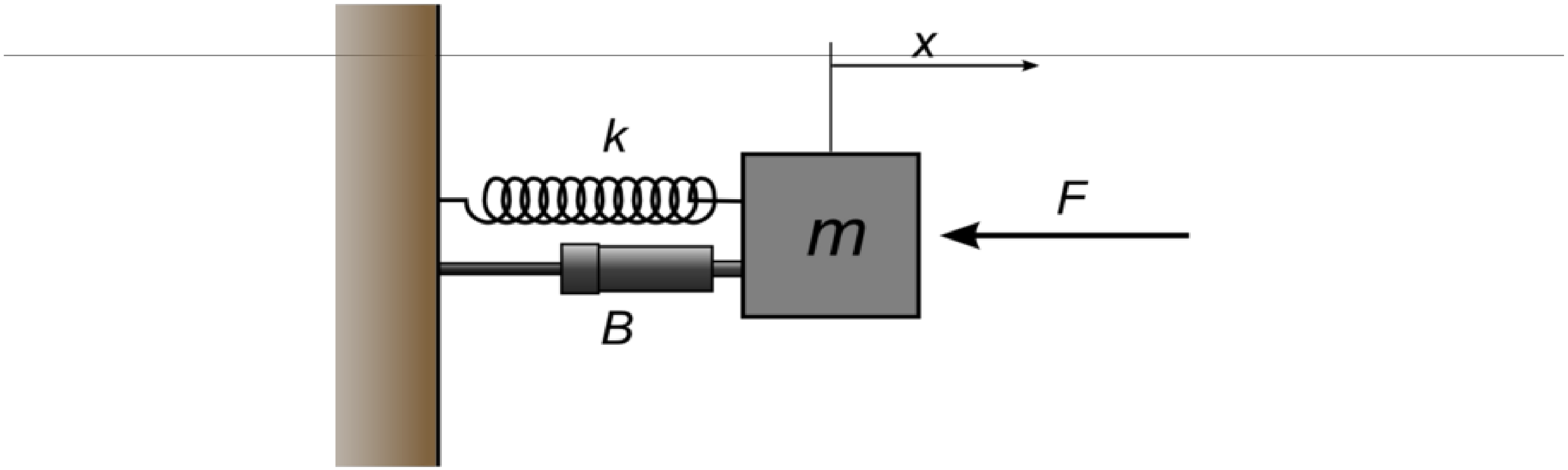
$$k = 2000\,Nm^{-1}$$

$$C = 1000\,N/ms^{-1}$$

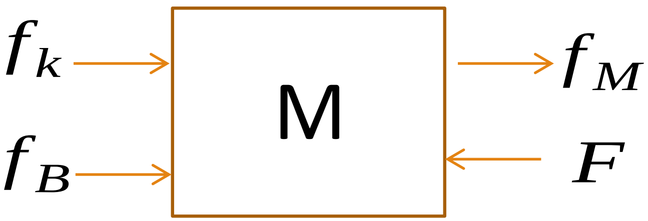
$$\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + s + 1000}$$



Example: Find the transfer function, *X(s)/F(s)*, of the following system.



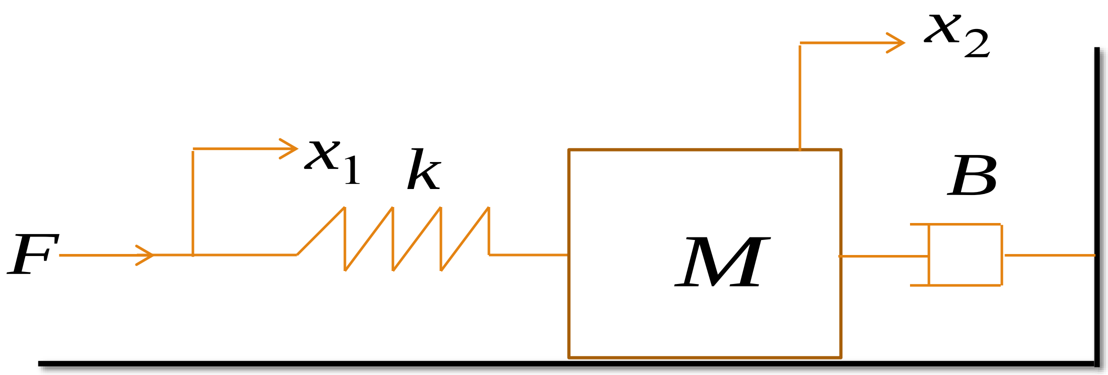
- Free Body Diagram



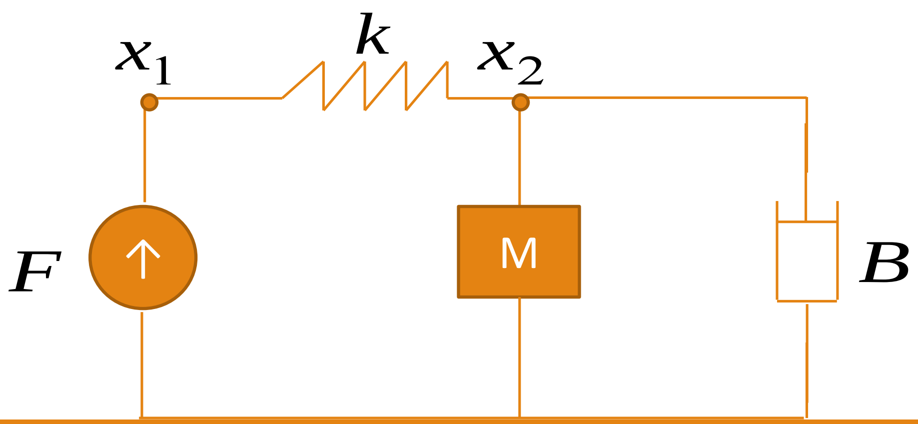
$$F = f_k + f_M + f_B$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}}$$

Example: Write the differential equations of the following system.

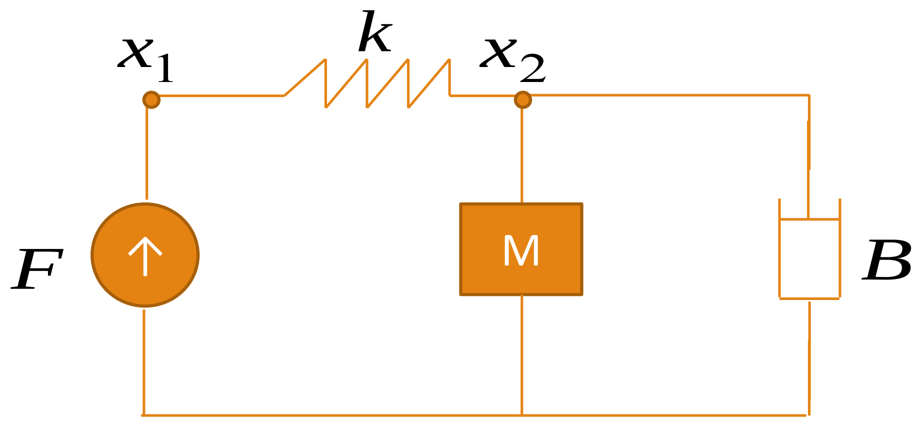


- Mechanical Network



Example: continue.

- Mechanical Network



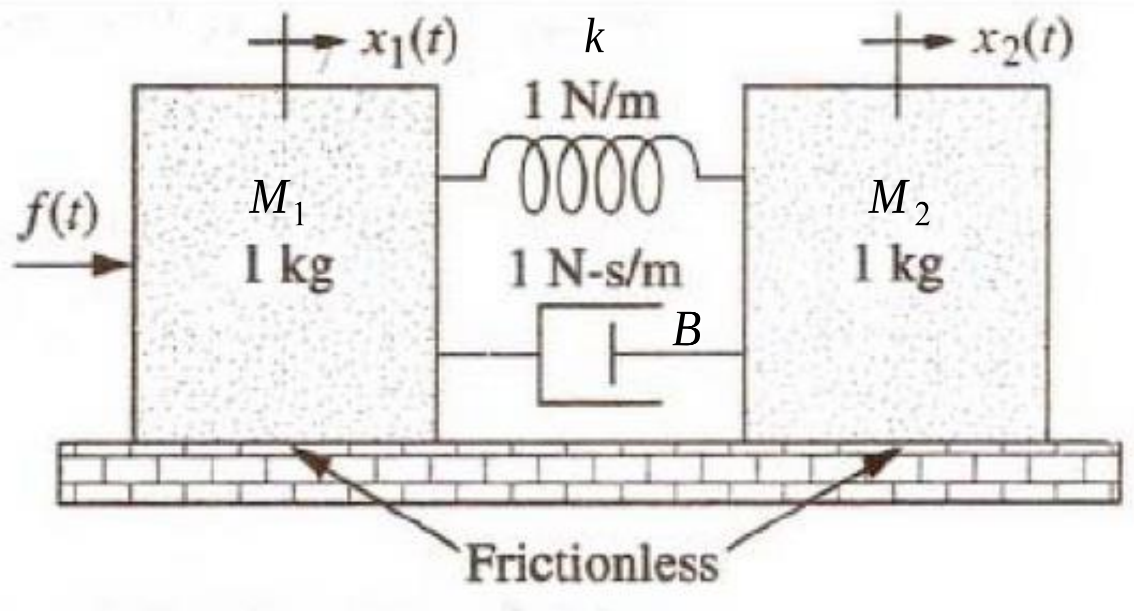
At node x_1

$$F = k(x_1 - x_2)$$

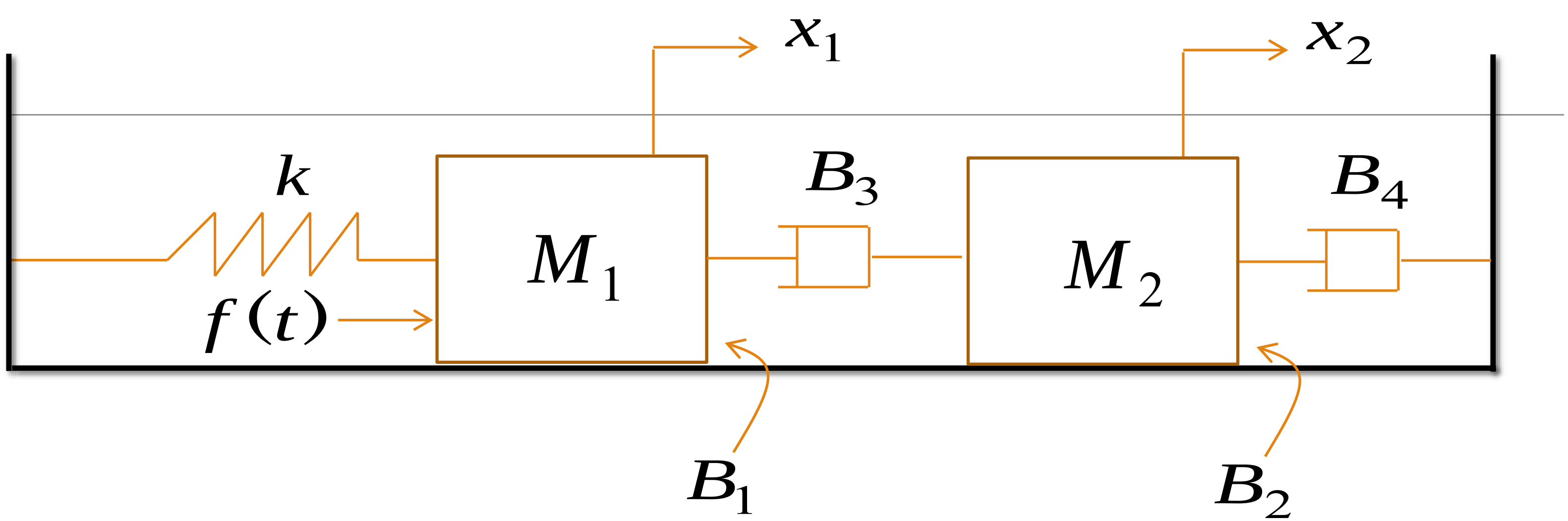
At node x_2

$$0 = k(x_2 - x_1) + M\ddot{x}_2 + B\dot{x}_2$$

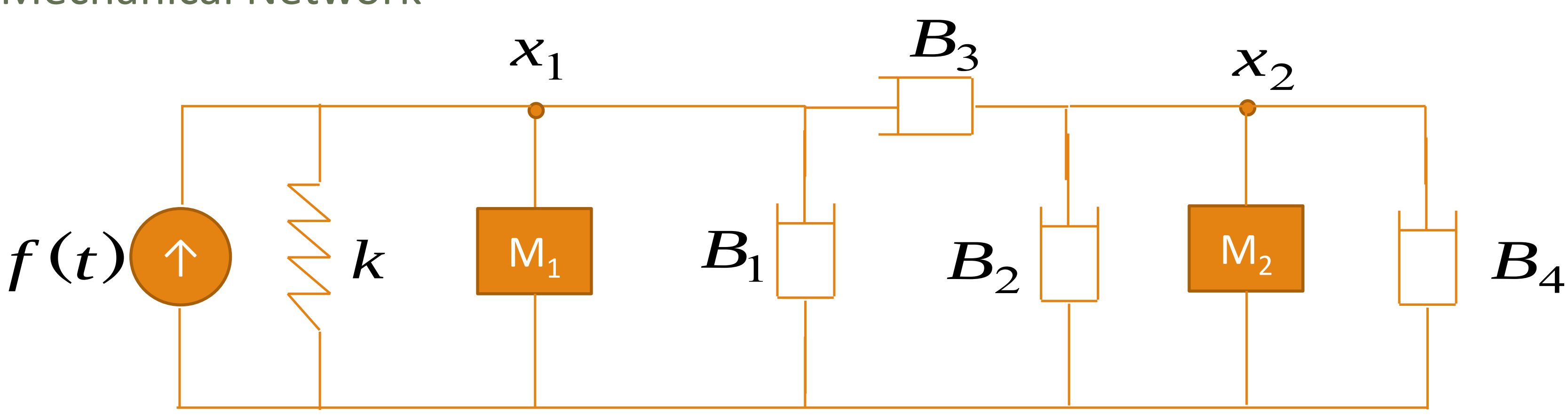
Example: Find the transfer function X2(s)/F(s) of the following system.



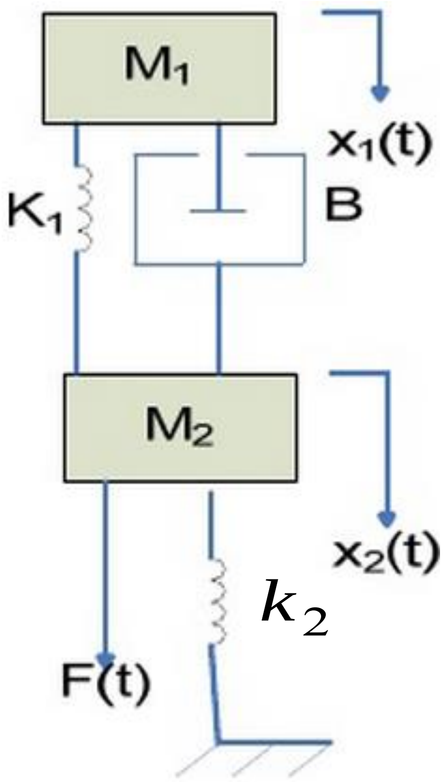
Example: Write the differential equations of the following system.



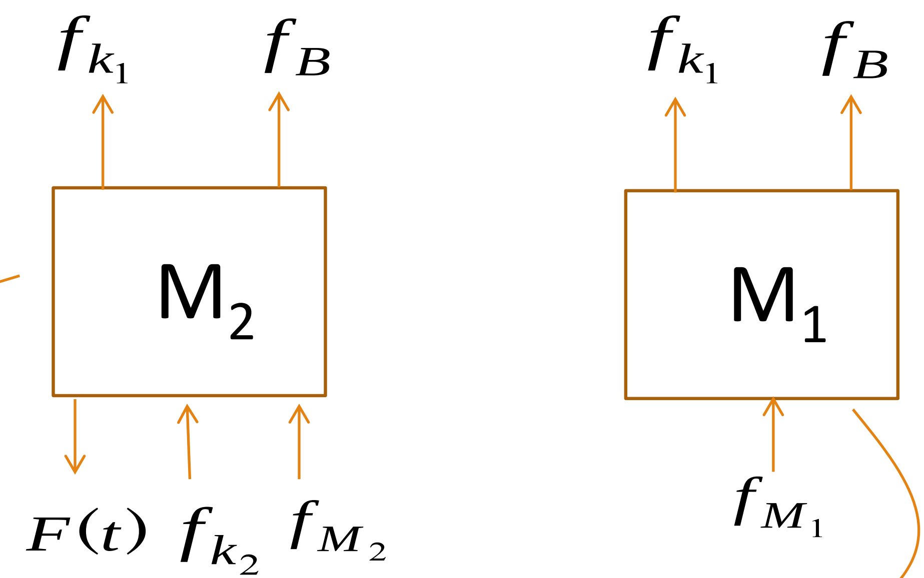
- Mechanical Network



Example: Find the transfer function X2(s)/F(s) of the following system.



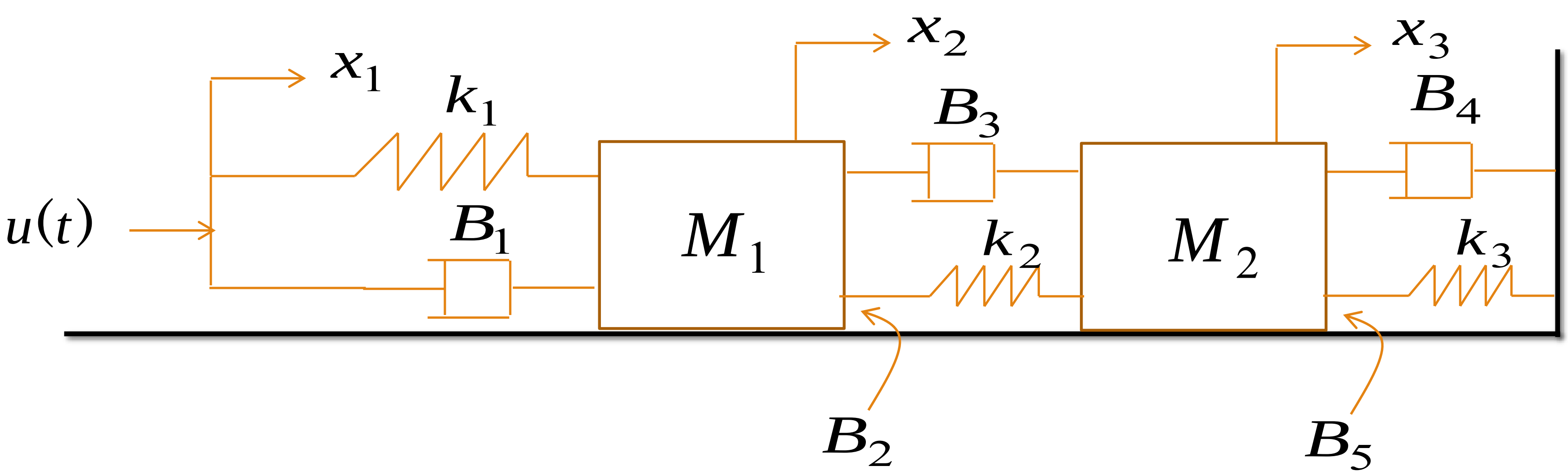
Free Body Diagram



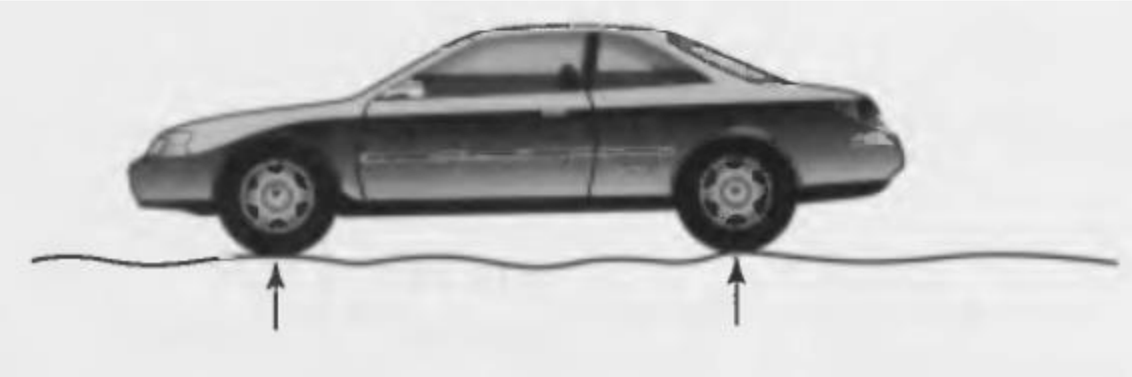
$$F(t) = f_{k_1} + f_{k_2} + f_{M_2} + f_B \longrightarrow (1)$$

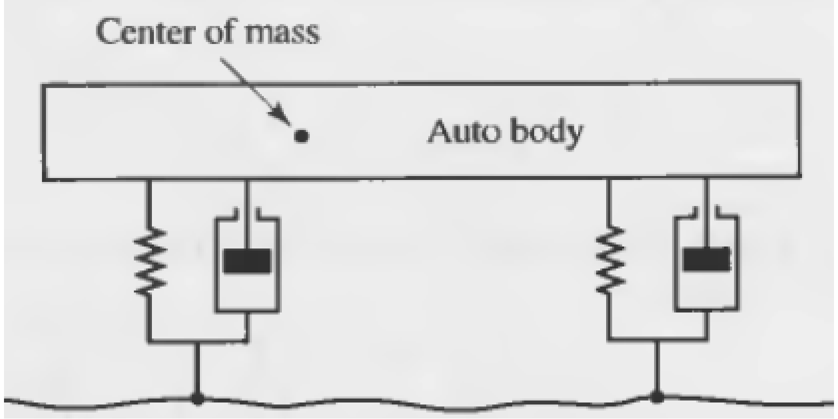
$$0 = f_{k_1} + f_{M_1} + f_B \longrightarrow (2)$$

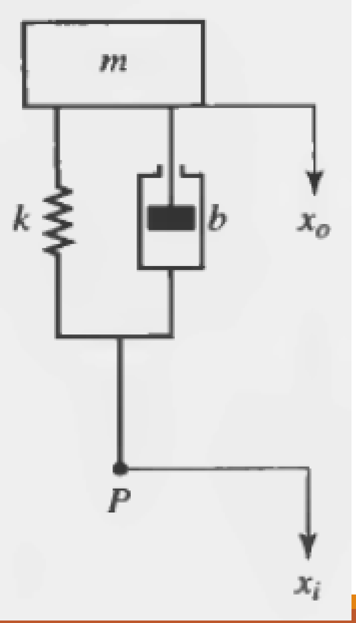
Example: Draw a mechanical network and write the differential equations of the following system.



Example: continue.



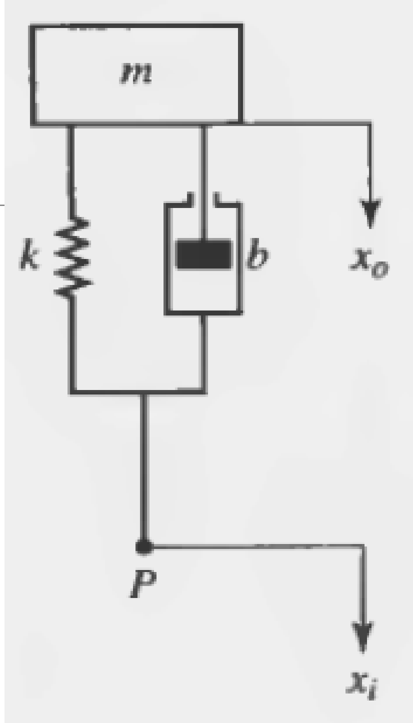




Example: continue.

$$m\ddot{x}_o + b(\dot{x}_o - \dot{x}_i) + k(x_o - x_i) = 0 \qquad \text{(eq .1)}$$

$$m\ddot{x}_o + b\dot{x}_o + kx_o = b\dot{x}_i + kx_i \qquad \text{eq. 2}$$



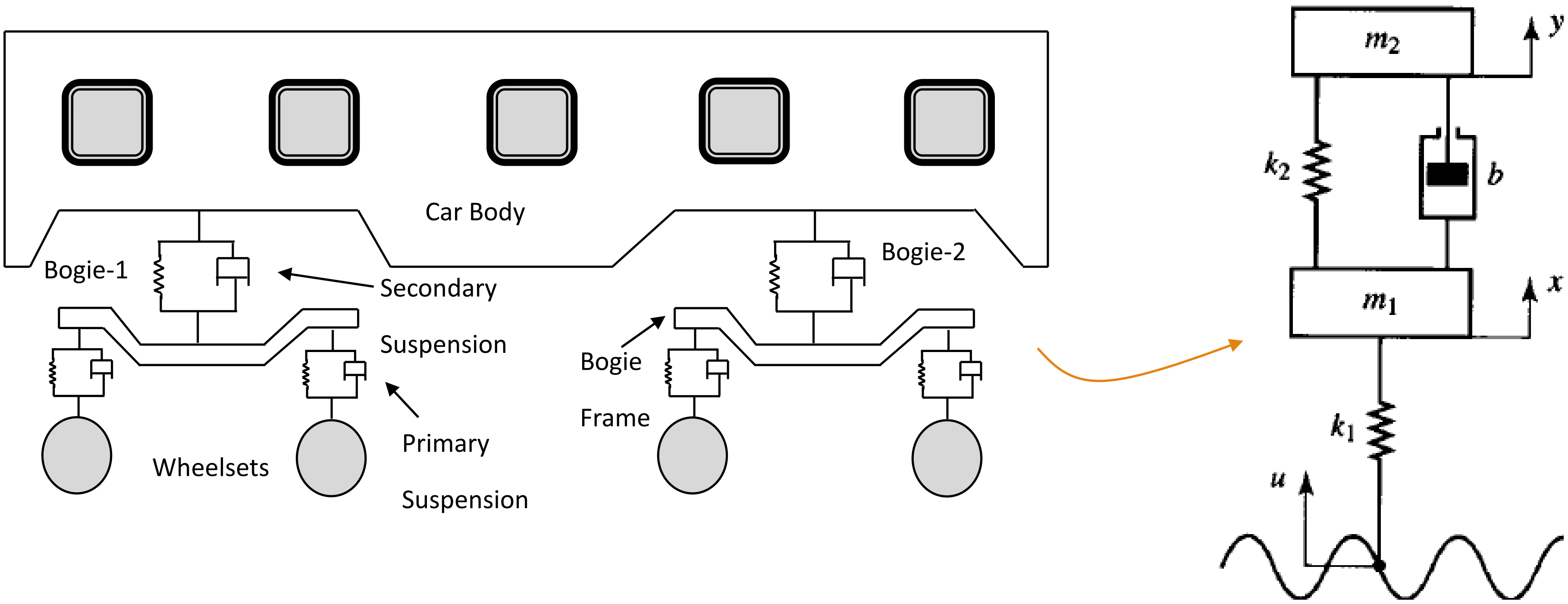
Taking Laplace Transform of the equation (2)

$$ms^2 X_o(s) + bsX_o(s) + kX_o(s) = bsX_i(s) + kX_i(s)$$

The transfer function of the system is

$$\boxed{\frac{X_o(s)}{X_i(s)} = \frac{bs + k}{ms^2 + bs + k}}$$

Example: Find the transfer function $Y(s)/U(s)$ of the train suspension system.

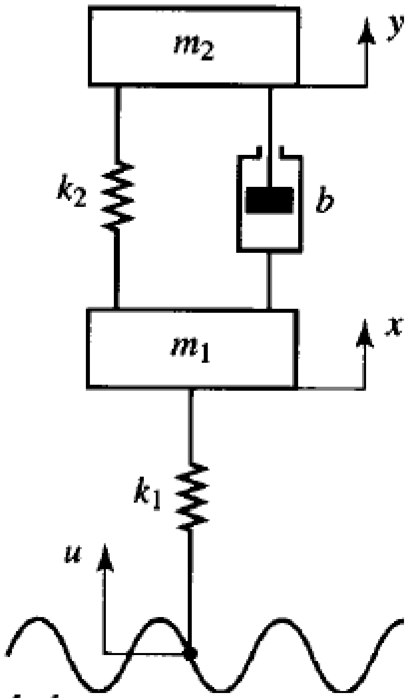


Example continue:

$$m_1 \ddot{x} + b\dot{x} + (k_1 + k_2)x = b\dot{y} + k_2 y + k_1 u$$

$$m_2 \ddot{y} + b\dot{y} + k_2 y = b\dot{x} + k_2 x$$

Taking Laplace transforms of these two equations, assuming zero initial conditions, we obtain



$$[m_1 s^2 + bs + (k_1 + k_2)]X(s) = (bs + k_2)Y(s) + k_1 U(s)$$

$$[m_2 s^2 + bs + k_2]Y(s) = (bs + k_2)X(s)$$

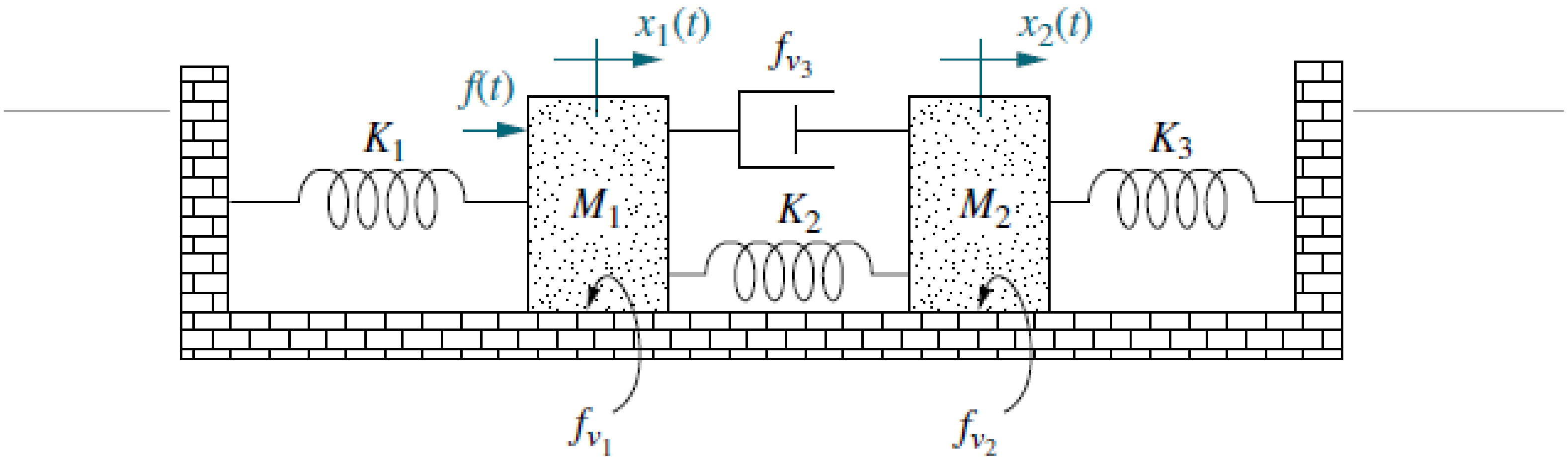
Eliminating $X(s)$ from the last two equations, we have

$$(m_1 s^2 + bs + k_1 + k_2)\frac{m_2 s^2 + bs + k_2}{bs + k_2} Y(s) = (bs + k_2)Y(s) + k_1 U$$

which yields

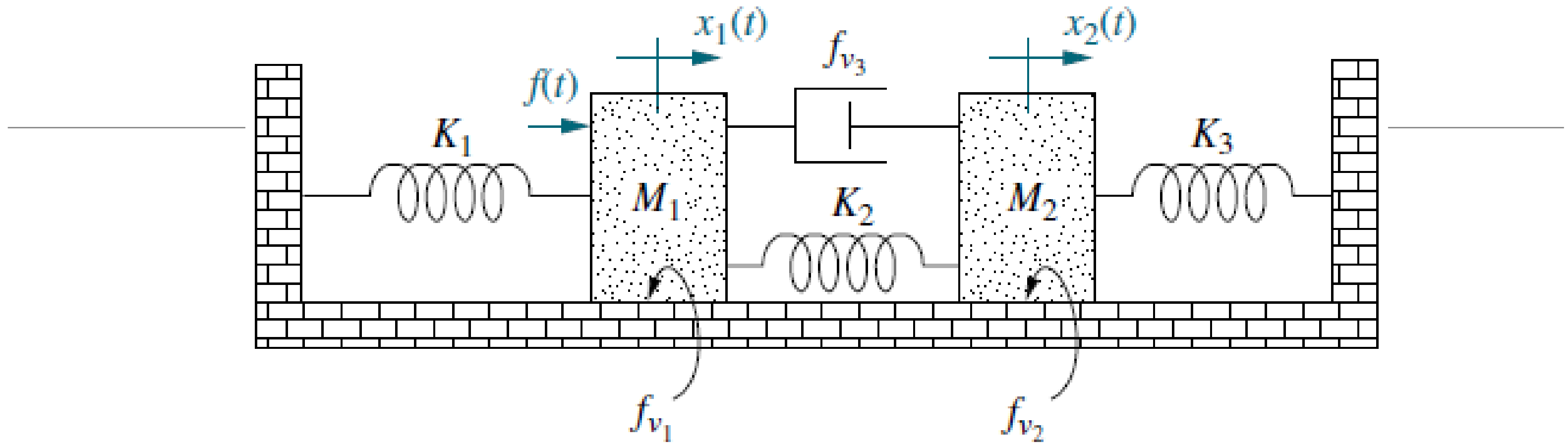
$$\frac{Y(s)}{U(s)} = \frac{k_1(bs + k_2)}{m_1 m_2 s^4 + (m_1 + m_2)bs^3 + [k_1 m_2 + (m_1 + m_2)k_2]s^2 + k_1 bs + k_1 k_2}$$

Example: Find the transfer function, $X_2(s)/F(s)$, of the system.



- The system has two degrees of freedom, since each mass can be moved in the horizontal direction while the other is held still.
- Thus, two simultaneous equations of motion will be required to describe the system.
- The two equations come from free-body diagrams of each mass.
- Superposition is used to draw the free body diagrams.
- **For example**, the forces on M1 are due to (1) its own motion and (2) the motion of M2 transmitted toM1 through the system.
- We will consider these two sources separately.

Example: Continue.



Case-I: Forces on M1

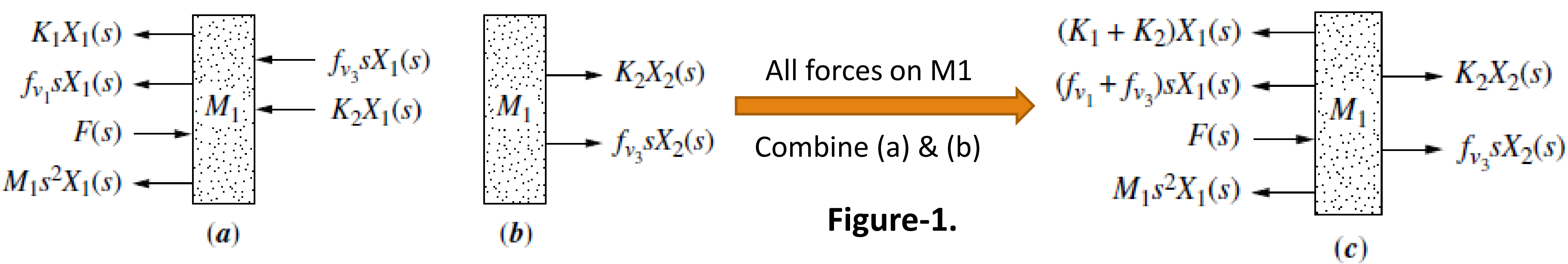


Figure-1.

Figure-1:

a. Forces on M1 due only to motion of M1;

b. Forces on M1 due only to motion of M2;

c. All forces on M1.

Example: Continue.

Case-I: Forces on M1

- If we hold M2 still and move M1 to the right, we see the forces shown in Figure-1(a).
- If we holdM1 still and moveM2 to the right, we see the forces shown in Figure 1(b).
- The total force on M1 is the superposition, or sum of the forces, as shown in Figure-1(c).

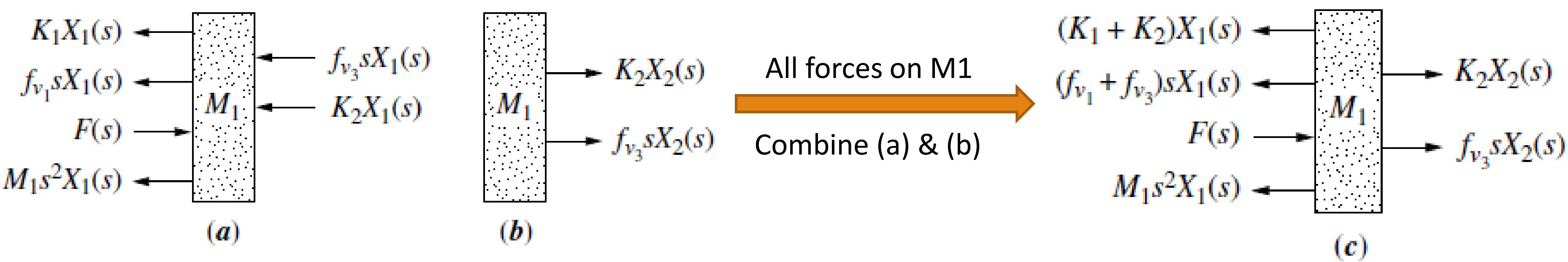
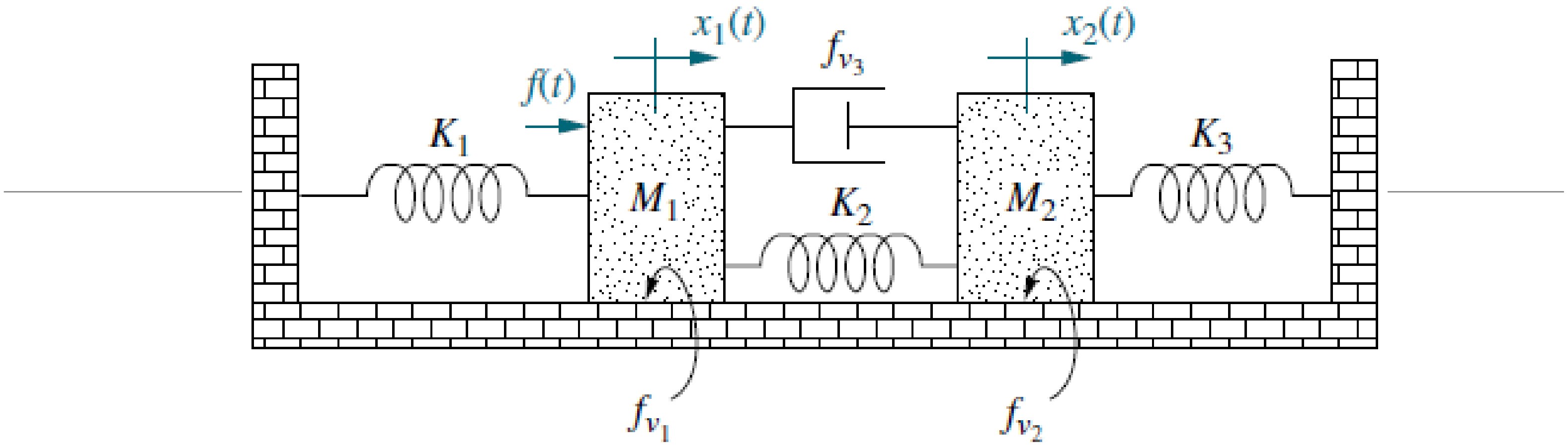


Figure-1:
a. Forces on M1 due only to motion of M1;
b. Forces on M1 due only to motion of M2;
c. All forces on M1.

- The Laplace transform of the equations of motion can be written from Figure-1 (c) as;

$$\left[M_1s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)\right]X_1(s) - (f_{v_3}s + K_2)X_2(s) = F(s) \quad (1)$$

Example-: Continue.



Case-II: Forces on M2

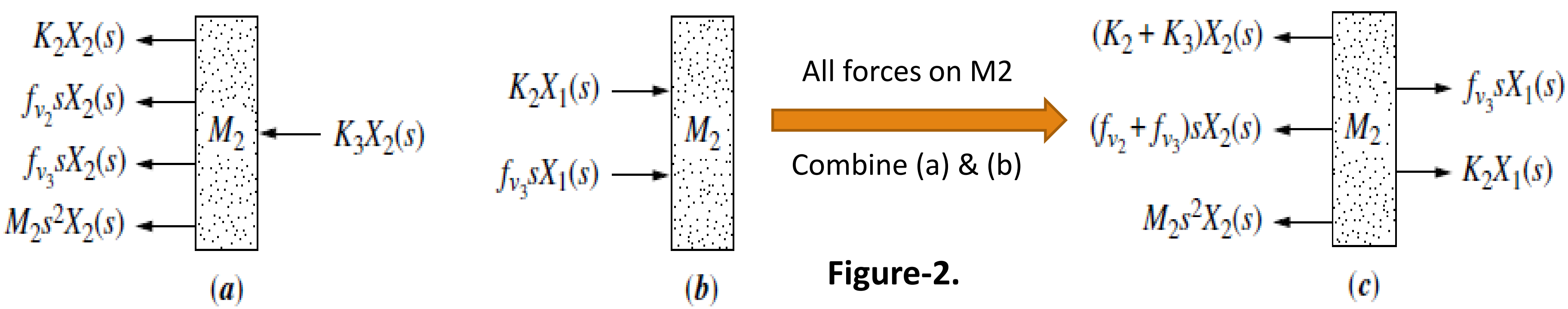


Figure-2.

Figure-2:
a. Forces on M2 due only to motion of M2;
b. Forces on M2 due only to motion of M1;
c. All forces on M2.

Example: Continue.

Case-II: Forces on M2

- If we hold M1 still and move M2 to the right, we see the forces shown in Figure-2(a).
- If we move M1 to the right and hold M2 still, we see the forces shown in Figure-2(b).
- For each case we evaluate the forces on M2.
- The total force on M2 is the superposition, or sum of the forces, as shown in Figure-2(c).

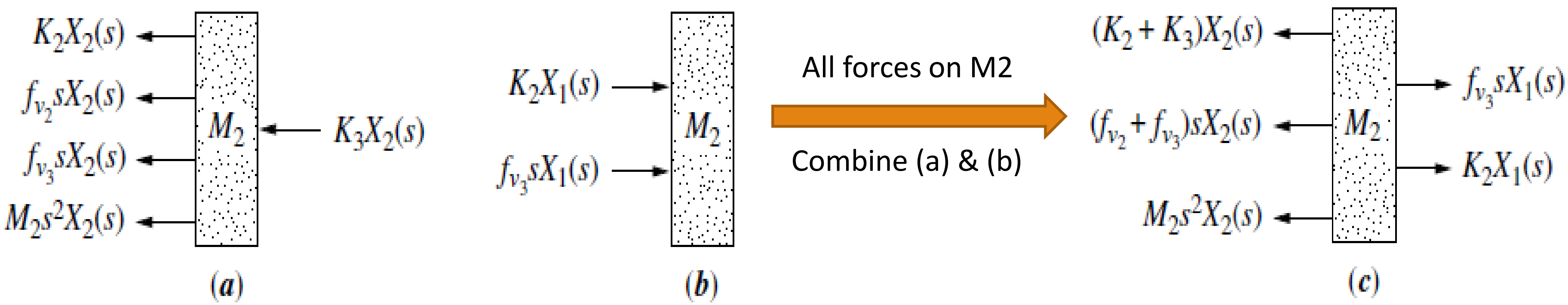


Figure-2:
a. Forces on M2 due only to motion of M2;
b. Forces on M2 due only to motion of M1;
c. All forces on M2.

- The Laplace transform of the equations of motion can be written from Figure-2 (c) as;

$$-(f_{v_3}s+K_2)X_1(s)+\left[M_2s^2+(f_{v_2}+f_{v_3})s+(K_2\;+\;K_3)\right]X_2(s)=0 \qquad (2)$$

Example-: Continue.

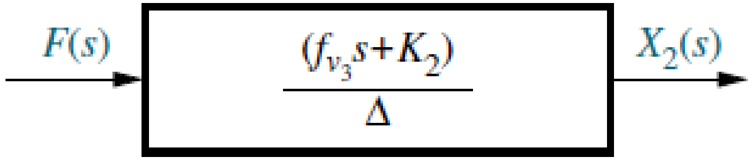
$$[M_1s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)]X_1(s) - (f_{v_3}s + K_2)X_2(s) = F(s) \longrightarrow (1)$$

$$-(f_{v_3}s + K_2)X_1(s) + [M_2s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)]X_2(s) = 0 \longrightarrow (2)$$

- From equation (1) and (2), the transfer function, *X2(s)/F(s),* is

$$\boxed{\frac{X_2(s)}{F(s)} = G(s) = \frac{(f_{v_3}s + K_2)}{\Delta}}$$

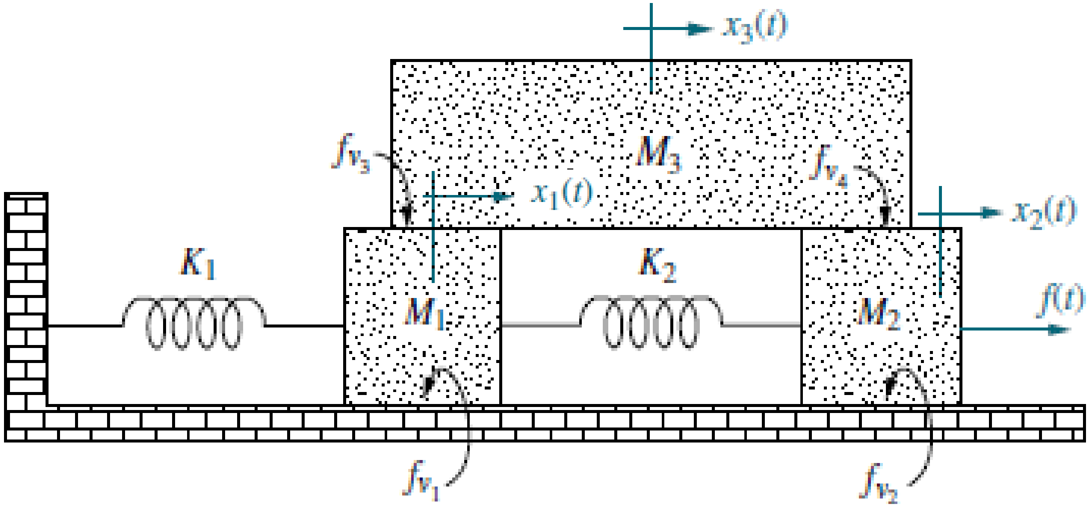
Block Diagram



- Where,

$$\Delta = \begin{vmatrix} [M_1s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)] & -(f_{v_3}s + K_2) \\ -(f_{v_3}s + K_2) & [M_2s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)] \end{vmatrix}$$

Example: Write, but do not solve, the equations of motion for the mechanical network shown below.



- The system has three degrees of freedom, since each of the three masses can be moved independently while the others are held still.
- M1 has two springs, two viscous dampers, and mass associated with its motion.
- There is one spring between M1 and M2 and one viscous damper between M1 and M3.

For M_1, $$[M_1s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)]X_1(s) - K_2X_2(s) - f_{v_3}sX_3(s) = 0$$

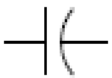
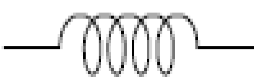
for M_2, $$-K_2X_1(s) + [M_2s^2 + (f_{v_2} + f_{v_4})s + K_2]X_2(s) - f_{v_4}sX_3(s) = F(s)$$

for M_3, $$-f_{v_3}sX_1(s) - f_{v_4}sX_2(s) + [M_3s^2 + (f_{v_3} + f_{v_4})s]X_3(s) = 0$$

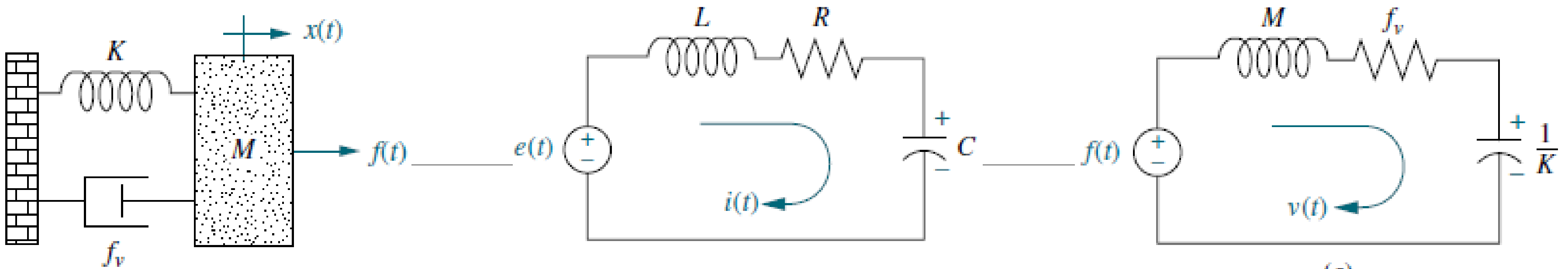
Electric Circuit Analogs

- An electric circuit that is analogous to a system from another discipline is called an **electric circuit analog**.
- The mechanical systems with which we worked can be represented by equivalent electric circuits.
- Analogs can be obtained by comparing the equations of motion of a mechanical system, with either electrical mesh or nodal equations.
- When compared with mesh equations, the resulting electrical circuit is called a **series analog**.
- When compared with nodal equations, the resulting electrical circuit is called a **parallel analog**.

Voltage, Current, Charge Relationship for Capacitor, Resistor, and Inductor.

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Series Analog / Force – Voltage Analogy



(a) (b) (c)

Equation of motion of the above translational mechanical system is;

$$(Ms^2 + f_v s + K)X(s) = F(s)$$

→ (1)

Kirchhoff's mesh equation for the above simple series RLC network is;

$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = E(s)$$

→ (2)

For a direct analogy b/w Eq (1) & (2), **convert displacement to velocity** by divide and multiply the left-hand side of Eq (1) by s, yielding;

$$\left(Ms + f_v + \frac{K}{s}\right)V(s) = F(s)$$

→ (3)

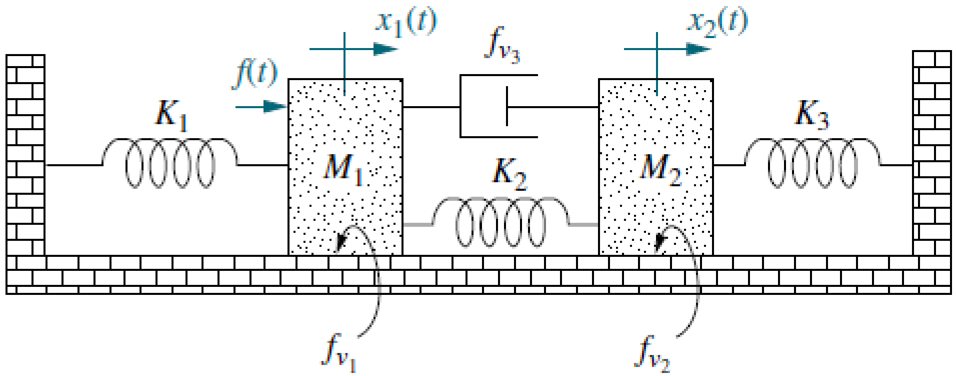
Comparing Eqs. (2) & (3), we recognize the **sum of impedances** & draw the circuit shown in Figure (c). The conversions are summarized in Figure (d).

mass = M	⟶	inductor	= M henries
viscous damper = f_v	⟶	resistor	= f_v ohms
spring = K	⟶	capacitor	= $\frac{1}{K}$ farads
applied force = $f(t)$	⟶	voltage source	= $f(t)$
velocity = $v(t)$	⟶	mesh current	= $v(t)$

(d)

Converting a Mechanical System to a FV Analog

Example-17: Draw a series analog for the mechanical system.



- The equations of motion in the Laplace transform domain are;

$$\left[M_1 s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)\right]X_1(s) - (f_{v_3}s + K_2)X_2(s) = F(s) \longrightarrow (1)$$

$$-(f_{v_3}s + K_2)X_1(s) + \left[M_2 s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)\right]X_2(s) = 0 \longrightarrow (2)$$

- Eqs (1) & (2) are analogous t0 electrical mesh equations after conversion to velocity. Thus,

$$\left[M_1 s + (f_{v_1} + f_{v_3}) + \frac{(K_1 + K_2)}{s}\right]V_1(s) - \left(f_{v_3} + \frac{K_2}{s}\right)V_2(s) = F(s) \longrightarrow (3)$$

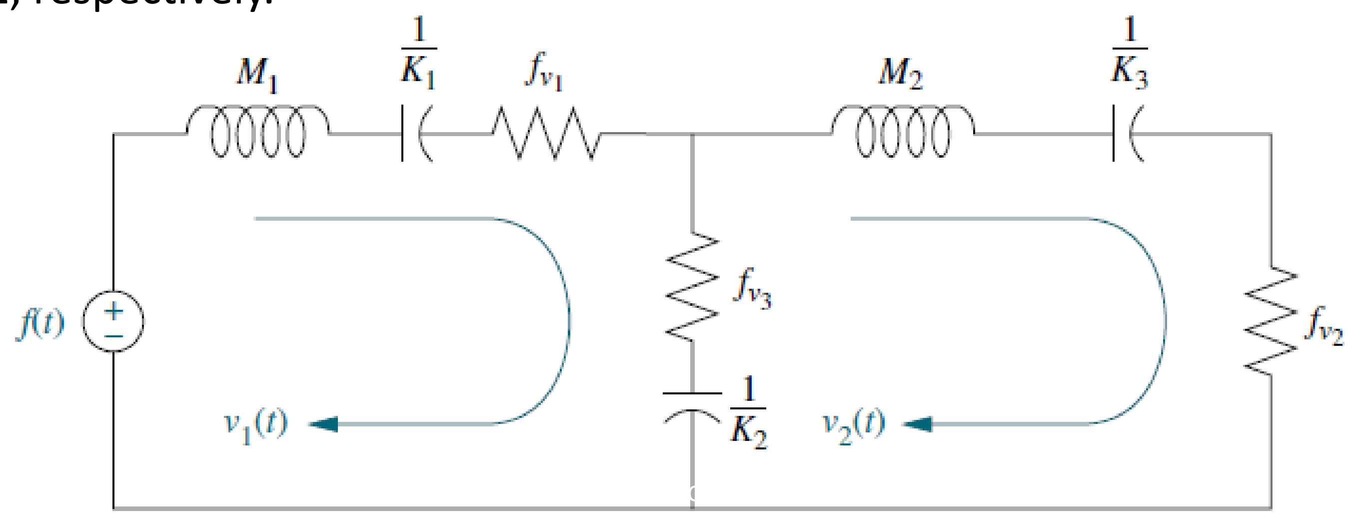
$$-\left(f_{v_3} + \frac{K_2}{s}\right)V_1(s) + \left[M_2 s + (f_{v_2} + f_{v_3}) + \frac{(K_2 + K_3)}{s}\right]V_2(s) = 0 \longrightarrow (4)$$

Example: Continue.

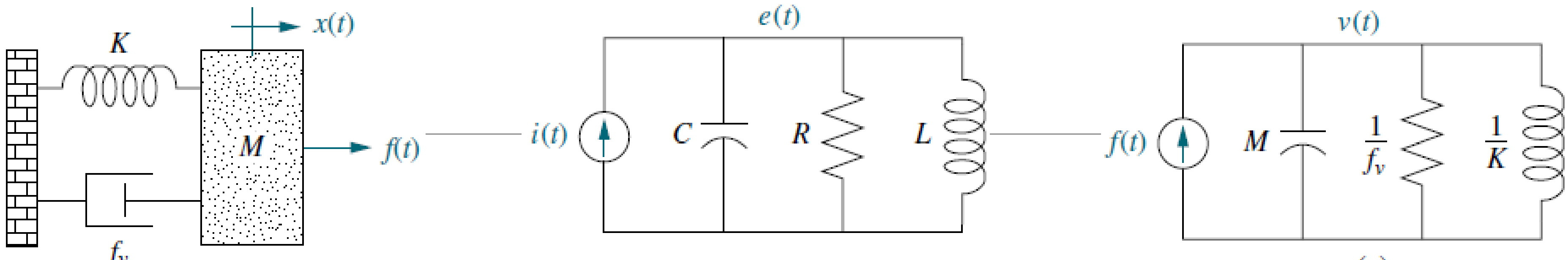
$$\left[M_1 s + (f_{v_1} + f_{v_3}) + \frac{(K_1 + K_2)}{s}\right] V_1(s) - \left(f_{v_3} + \frac{K_2}{s}\right) V_2(s) = F(s) \longrightarrow (3)$$

$$-\left(f_{v_3} + \frac{K_2}{s}\right) V_1(s) + \left[M_2 s + (f_{v_2} + f_{v_3}) + \frac{(K_2 + K_3)}{s}\right] V_2(s) = 0 \longrightarrow (4)$$

- Coefficients represent sums of electrical impedance.
- Mechanical impedances associated withM1 form the first mesh,
- whereas impedances between the two masses are common to the two loops.
- Impedances associated with M2 form the second mesh.
- The result is shown in Figure below, where v1(t) and v2(t) are the velocities of M1 and M2, respectively.



Parallel Analog/ Force Current Analogy



(a) (b) (c)

- Equation of motion of the above translational mechanical system is;

$$\left(Ms + f_v + \frac{K}{s}\right)V(s) = F(s) \quad \rightarrow (1)$$

- Kirchhoff's nodal equation for the simple parallel RLC network shown above is;

$$\boxed{\left(Cs + \frac{1}{R} + \frac{1}{Ls}\right)E(s) = I(s)} \quad \rightarrow (2)$$

- Comparing Eqs. (1) & (2), we identify the **sum of admittances** & draw the circuit shown in Figure (c).

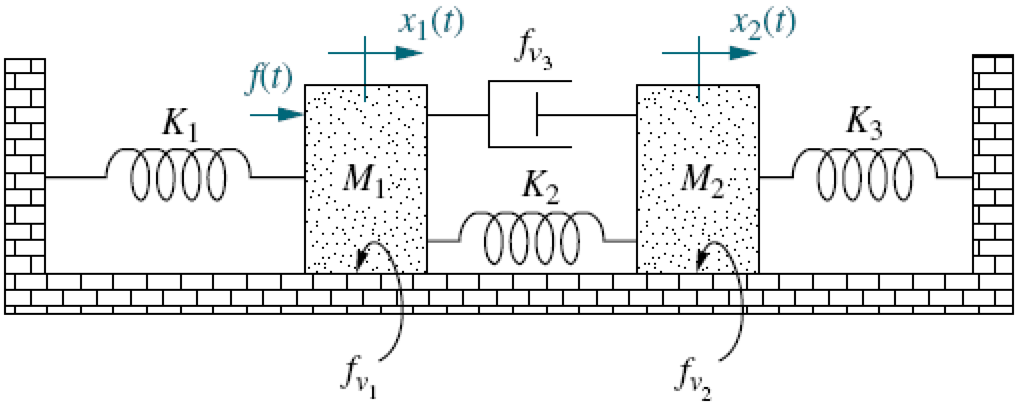
- The conversions are summarized in Figure 2.43(d).

mass $= M$	$\longrightarrow$	capacitor	$= M$ farads
viscous damper $= f_v$	$\longrightarrow$	resistor	$= \frac{1}{f_v}$ ohms
spring $= K$	$\longrightarrow$	inductor	$= \frac{1}{K}$ henries
applied force $= f(t)$	$\longrightarrow$	current source	$= f(t)$
velocity $= v(t)$	$\longrightarrow$	node voltage	$= v(t)$

(d)

Converting a Mechanical System to a F-I Analog

Example-18: Draw a parallel analog for the mechanical system.



- Equations of motion after conversion to velocity are;

$$\left[M_1 s + (f_{v_1} + f_{v_3}) + \frac{(K_1 + K_2)}{s}\right] V_1(s) - \left(f_{v_3} + \frac{K_2}{s}\right) V_2(s) = F(s) \longrightarrow (1)$$

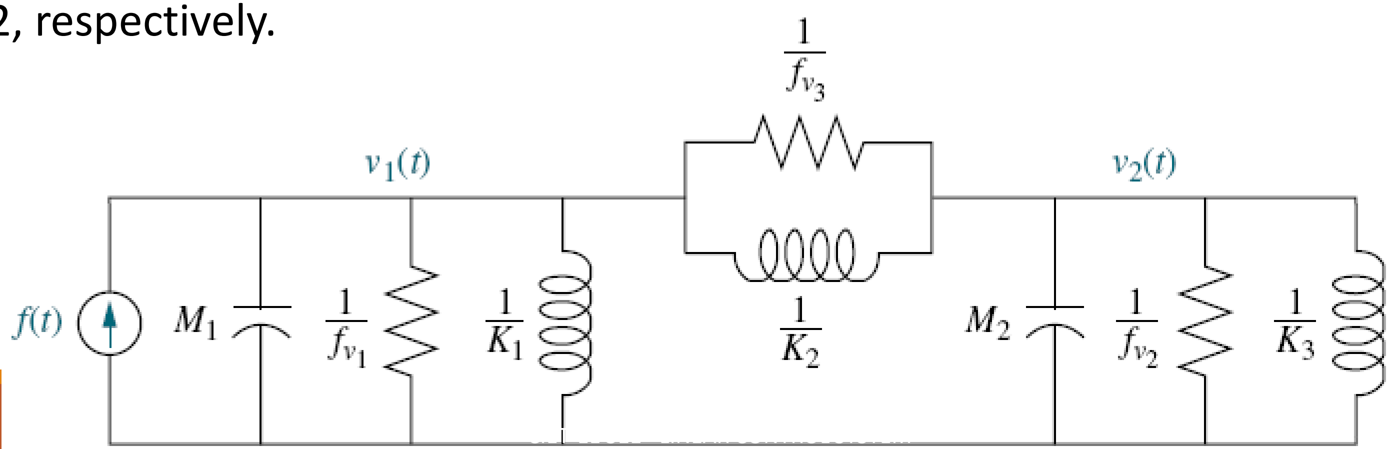
$$-\left(f_{v_3} + \frac{K_2}{s}\right) V_1(s) + \left[M_2 s + (f_{v_2} + f_{v_3}) + \frac{(K_2 + K_3)}{s}\right] V_2(s) = 0 \longrightarrow (2)$$

Example: Continue.

$$\left[M_1 s + (f_{v_1} + f_{v_3}) + \frac{(K_1 + K_2)}{s}\right] V_1(s) - \left(f_{v_3} + \frac{K_2}{s}\right) V_2(s) = F(s) \longrightarrow (1)$$

$$-\left(f_{v_3} + \frac{K_2}{s}\right) V_1(s) + \left[M_2 s + (f_{v_2} + f_{v_3}) + \frac{(K_2 + K_3)}{s}\right] V_2(s) = 0 \longrightarrow (2)$$

- The Equation (1) and (2) are also analogous to electrical node equations.
- Coefficients represent sums of electrical admittances.
- Admittances associated with M1 form the elements connected to the first node,
- whereas mechanical admittances b/w the two masses are common to the two nodes.
- Mechanical admittances associated with M2 form the elements connected to the second node.
- The result is shown in the Figure below, where v1(t) and v2(t) are the velocities of M1 and M2, respectively.

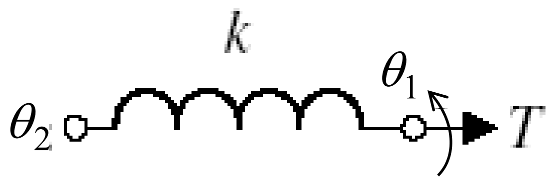


Rotational Mechanical Systems

Part-II

Basic Elements of Rotational Mechanical Systems

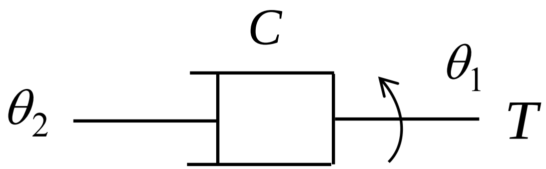
Rotational Spring



$$T = k(\theta_1 - \theta_2)$$

Basic Elements of Rotational Mechanical Systems

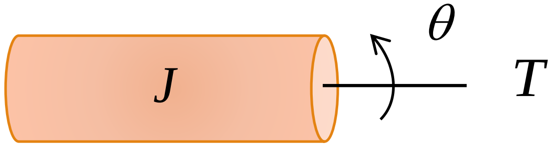
Rotational Damper



$$T = C(\dot{\theta}_1 - \dot{\theta}_2)$$

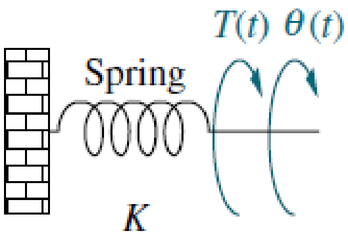
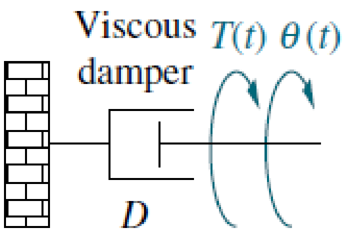
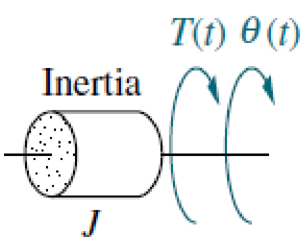
Basic Elements of Rotational Mechanical Systems

Moment of Inertia



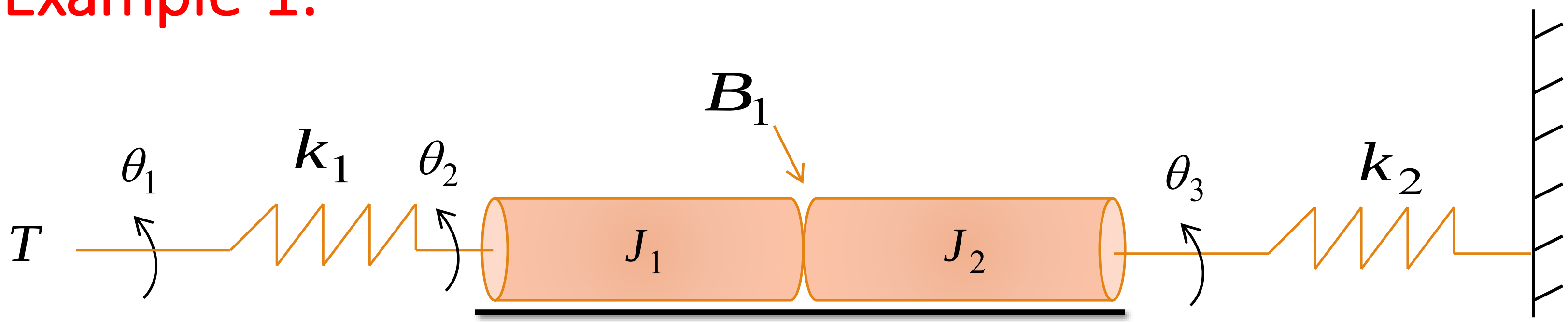
$$T = J\ddot{\theta}$$

Table: Torque-angular velocity, torque-angular displacement, and impedance rotational relationships for springs, viscous dampers, and inertia.

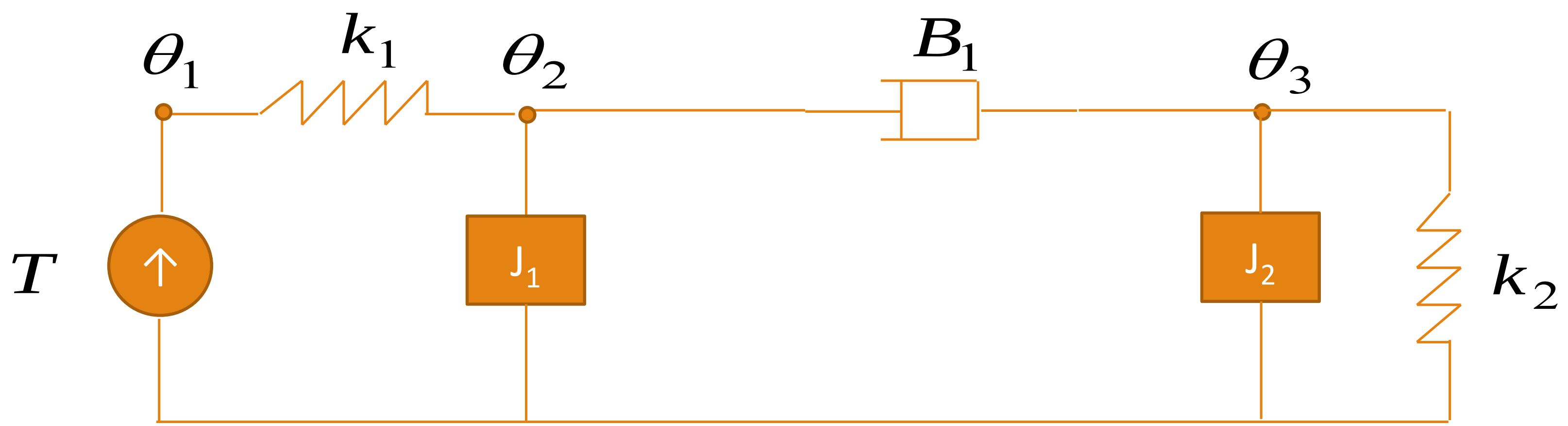
Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Note: The following set of symbols and units is used throughout this book: $T(t)$ – N-m (newton-meters), $\theta(t)$ – rad(radians), $\omega(t)$ – rad/s(radians/second), K – N-m/rad(newton- meters/radian), D – N-m-s/rad (newton- meters-seconds/radian). J – kg-m^2(kilograms-meters^2 – newton-meters-seconds^2/radian).

Example-1:

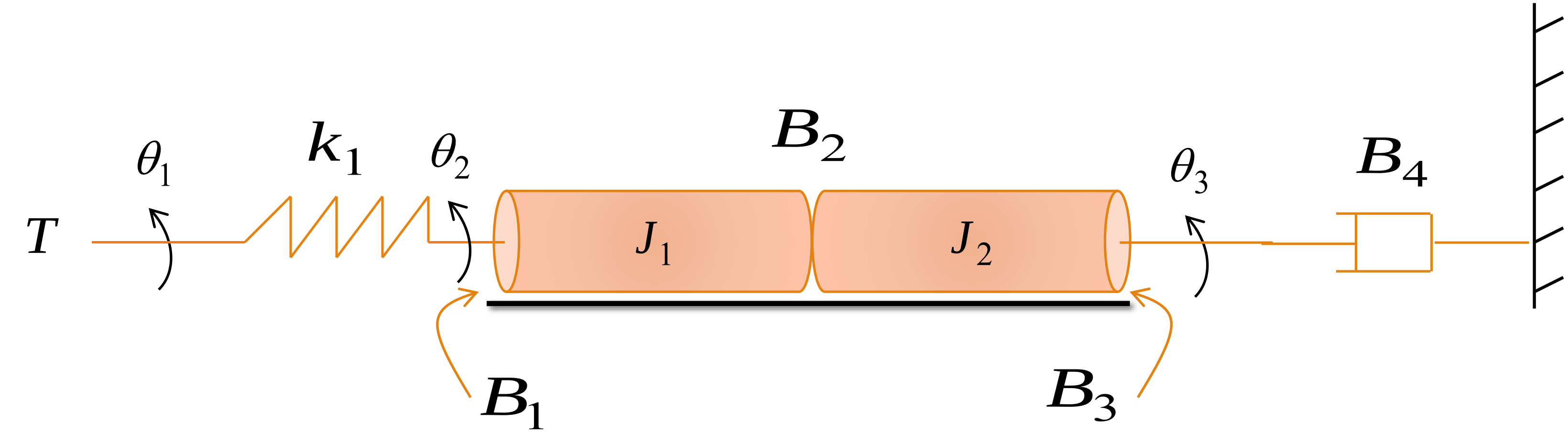


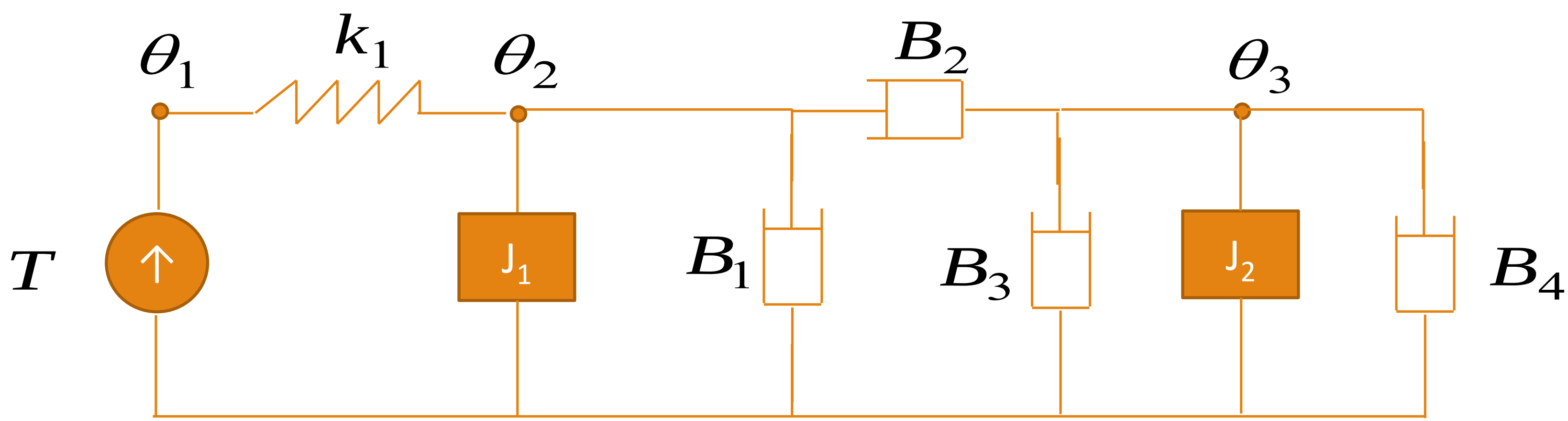
B_1
θ_1
k_1
θ_2
θ_3
k_2
T
J_1
J_2



θ_1
k_1
θ_2
B_1
θ_3
T
J_1
J_2
k_2

Example-2:


T
θ_1
k_1
θ_2
B_2
J_1
J_2
θ_3
B_4
B_1
B_3


θ_1
k_1
θ_2
B_2
θ_3
T
J_1
B_1
B_3
J_2
B_4